

Mass inequalities for baryons with heavy quarks

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(Received 13 December 2019; accepted 6 February 2020; published 18 February 2020)

Baryons with one or more heavy quarks have been shown, in the context of a nonrelativistic description, to exhibit mass inequalities under permutations of their quarks, when spin averages are taken. These inequalities sometimes are invalidated when spin-dependent forces are taken into account. A notable instance is the inequality $2E(Mmm) > E(MMm) + E(mmm)$, where $m = m_u = m_d$, satisfied for $M = m_b$ or $M = m_c$ but not for $M = m_s$, unless care is taken to remove effects of spin-spin interactions. Thus in the quark-level analog of nuclear fusion, the reactions $\Lambda_b\Lambda_b \rightarrow \Xi_{bb}N$ and $\Lambda_c\Lambda_c \rightarrow \Xi_{cc}^+n$ are exothermic, releasing respectively 138 and 12 MeV, while $\Lambda\Lambda \rightarrow \Xi N$ is endothermic, requiring an input of between 23 and 29 MeV. Here we explore such mass inequalities in the context of an approach, previously shown to predict masses successfully, in which contributions consist of additive constituent-quark masses, spin-spin interactions, and additional binding terms for pairs each member of which is at least as heavy as a strange quark.

DOI: 10.1103/PhysRevD.101.036015

I. INTRODUCTION

Quantum chromodynamics (QCD) predicts the existence of baryons containing not only one but two or three heavy quarks (c or b). The LHCb Collaboration at CERN has discovered the first doubly heavy baryon, a $\Xi_{cc}^{++} = ccu$ state in the decay modes $\Lambda_c K^- \pi^+ \pi^+$ [1] and $\Xi_c^+ \pi^+$ [2], at a mass $M(\Xi_{cc}^{++}) = 3621.24 \pm 0.65 \pm 0.31$ MeV, very close to that predicted in Ref. [3]. (Hints of a possible isospin partner Ξ_{cc}^+ decaying to $\Lambda_c K^- \pi^+$ have also been found [4,5].) In this approach, one adds up constituent-quark masses and spin-spin interactions [6–8] and corrects for the additional binding in any quark pair both of whose members are at least as heavy as a strange quark [3].

Under some circumstances hadrons satisfy mass inequalities associated with permutations of their quarks [9–16]. For example, under some conditions one expects $2E(Mmm) > E(MMm) + E(mmm)$, where E denotes the mass, $m = m_u = m_d$, and M is the mass of a heavy quark, to apply to spin-averaged states [cf. Eq. (3) in Ref. [15]]. This inequality may be invalidated when spin-dependent forces are taken into account; it holds for $M = m_b$ or $M = m_c$ but not for $M = m_s$. Thus, in the quark-level

analog of nuclear fusion [17], the reactions $\Lambda_b\Lambda_b \rightarrow \Xi_{bb}N$ and $\Lambda_c\Lambda_c \rightarrow \Xi_{cc}^+n$ are exothermic, releasing respectively 138 and 12 MeV, while $\Lambda\Lambda \rightarrow \Xi N$ is endothermic, requiring energy input of between 23 and 29 MeV, depending on which members of the N and Ξ doublets one uses.

Here we give some examples of inequalities involving baryon masses in our constituent-quark approach. We outline in Sec. II some of the relations and their origin. In Sec. III we treat light-quark systems in which the strange quark plays the role of a heavy quark. Baryons with one and two heavy quarks (c , b) are described in Secs. IV and V, respectively, while Sec. VI concludes.

II. INEQUALITIES AND THEIR ORIGIN

A number of mass inequalities involving ground-state mesons and baryons were noted by Nussinov [11], respectively,

$$m_{x\bar{y}} > \frac{1}{2}(m_{x\bar{x}} + m_{y\bar{y}}), \quad m_{xyy} > \frac{1}{2}(m_{xxy} + m_{yyy}). \quad (1)$$

We shall motivate these relations in a simple case where quark masses enter through their nonrelativistic kinetic energy, but they are much more general (see many of the references quoted above, in particular [15]). Consider systems governed by the Hamiltonians

$$H_{ij} = \frac{\mathbf{p}^2}{2\mu_{ij}} + V(\mathbf{x}), \quad \mu_{ij} \equiv \frac{m_1 m_2}{m_1 + m_2}. \quad (2)$$

Then, since

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$$\frac{1}{\mu_{12}} = \frac{1}{m_1} + \frac{1}{m_2}, \quad (3)$$

we have $H_{12} = \frac{1}{2}(H_{11} + H_{22})$. The ground-state energy in the 12 system is

$$\begin{aligned} E_{12} &= \text{Min}_{\psi} \langle \psi_{12} | H_{12} | \psi_{12} \rangle \\ &= \langle \psi_{12} | \frac{1}{2}(H_{11} + H_{22}) | \psi_{12} \rangle. \end{aligned} \quad (4)$$

Now

$$\langle \psi_{12} | H_{11} | \psi_{12} \rangle > E_1; \quad \langle \psi_{12} | H_{22} | \psi_{12} \rangle > E_2, \quad (5)$$

since ψ_{12} is not the ground state (assumed nondegenerate) of H_{11} or H_{22} . Hence we have the result $E_{12} > (E_1 + E_2)/2$, implying Eq. (1).

When spin-spin interactions are taken into account, the energy shift due to the interaction of quarks i and j may be written

$$\Delta E_{ij, \text{HFs}} = b \langle \sigma_i \cdot \sigma_j \rangle / (m_i m_j). \quad (6)$$

Inclusion of such terms in the Hamiltonian in general spoils the relation $H_{12} = (H_{11} + H_{22})/2$.

III. BARYONS WITH LIGHT QUARKS

If we adopt the semiempirical model of hadron masses [3,18] in which the constituent u , d , s quarks have constituent masses of several hundred MeV, the only corrections to the sum of quark masses are the sum of hyperfine terms (6) and terms $B(ss)$ expressing the stronger binding of one strange quark with another. Here the tilde stands for a mass without hyperfine interaction terms. The Gell-Mann–Okubo relation $\frac{1}{2}[M(N) + M(\Xi)] = \frac{1}{4}[3M(\Lambda) + M(\Sigma)]$, derived under the assumption that SU(3) breaking is linear in hypercharge, then becomes an inequality

$$\begin{aligned} \frac{1}{2}[\tilde{M}(N) + \tilde{M}(\Xi)] &= 2m_q + m_s - B(ss)/2 \\ &< \frac{1}{4}[3\tilde{M}(\Lambda) + \tilde{M}(\Sigma)] \\ &= 2m_q + m_s, \end{aligned} \quad (7)$$

and the second inequality in Eq. (1) is satisfied. (It turns out that the experimental values of octet baryon masses [19] also satisfy this relation, with $[M(N) + M(\Xi)]/2 = 1128.6$ MeV and $[3M(\Lambda) + M(\Sigma)]/4 = 1135.0$ MeV.)

When spin-spin interactions [Eq. (6)] are taken into account as in Ref. [3], one has

$$\begin{aligned} \frac{1}{2}[\tilde{M}(N) + \tilde{M}(\Xi)] - \frac{1}{4}[3\tilde{M}(\Lambda) + \tilde{M}(\Sigma)] \\ = -\frac{B(ss)}{2} + b \frac{(m_s - m_q)^2}{2m_s^2 m_q^2}, \end{aligned} \quad (8)$$

where m_q stands for the isospin-averaged mass of u and d quarks. A fit to masses of light-quark baryons [18] gives $b/m_q^2 = 50.0$ MeV, $m_q = 362.1$ MeV, $m_s = 543.9$ MeV, $B(ss) = 9.23$ MeV, so the right-hand side of Eq. (8) is -4.6 MeV + 2.8 MeV, with the spin-spin term working against the predicted inequality but not enough to counteract it.

IV. BARYONS WITH ONE HEAVY QUARK

The masses of ground state baryons cqq , cqs , and css containing a single charmed quark satisfy the inequality

$$[\tilde{M}(\Lambda_c) + \tilde{M}(\Omega_c)]/2 < \tilde{M}(\Xi_c), \quad (9)$$

thanks to the binding correction $B(ss) = 9.2$ MeV [18]¹:

$$\begin{aligned} [\tilde{M}(\Lambda_c) + \tilde{M}(\Omega_c)]/2 &= m_q + m_s + m_c - B(cs) - B(ss)/2 \\ &< \tilde{M}(\Xi_c) = m_q + m_s + m_c - B(cs). \end{aligned} \quad (10)$$

The experimental values [19] do not satisfy the inequality,

$$\begin{aligned} \frac{1}{2}[M(\Lambda_c) + M(\Omega_c)] &= 2490.8 \text{ MeV} > M(\Xi_c) \\ &= 2468.9 \text{ MeV}, \end{aligned} \quad (11)$$

indicating a significant contribution from spin-spin interactions. (Here we have used the isospin-averaged Ξ_c mass.)

The corresponding relation for states containing a b quark is

$$\begin{aligned} [\tilde{M}(\Lambda_b) + \tilde{M}(\Omega_b)]/2 &= m_q + m_s + m_b - B(bs) - B(ss)/2 \\ &< \tilde{M}(\Xi_b) = m_q + m_s + m_b - B(bs), \end{aligned} \quad (12)$$

with $B(bs) = 41.8$ MeV [3] common to both sides. Again, the term $B(ss)$ turns the equality into an inequality. The experimental values, again thanks to a significant spin-spin contribution, satisfy the opposite inequality:

$$\begin{aligned} \frac{1}{2}[M(\Lambda_b) + M(\Omega_b)] &= 5832.9 \text{ MeV} > M(\Xi_b) \\ &= 5791.3 \text{ MeV}. \end{aligned} \quad (13)$$

¹A term $B(cs) = 35.0$ MeV [3] is common to both sides.

V. BARYONS WITH TWO OR MORE HEAVY QUARKS

In this section we discuss inequalities between binding energies. In our convention the binding energies are positive, but they contribute to the hadron mass with a minus sign, so the direction of the inequalities between binding energies is flipped vs inequalities between hadron masses.

We have seen that the second of Eqs. (1) holds for baryons with zero or one heavy (c , b) quarks. Inequalities also hold in our semiempirical approach when two quarks are heavy. For example [3,18], $B(ss) = 9.2$ MeV, $B(cs) = 35.0$ MeV, $B(cc) = 129.0$ MeV, so

$$B(cs) < [B(ss) + B(cc)]/2. \quad (14)$$

Similarly $B(bs) = 41.8$ MeV, $B(bb) = 281.4$ MeV, so

$$B(bs) < [B(ss) + B(bb)]/2. \quad (15)$$

It was also found [3] that $B(bc) = 170.8$ MeV, so

$$\begin{aligned} B(bc) &= 170.8 \text{ MeV} < [B(cc) + B(bb)]/2 \\ &= 205.2 \text{ MeV}. \end{aligned} \quad (16)$$

A related inequality for the spin-weighted averages \bar{M} of heavy $c\bar{c}$, $b\bar{b}$, and $b\bar{c}$ ground-state mesons,

$$[\bar{M}(c\bar{c}) + \bar{M}(b\bar{b})]2 < \bar{M}(b\bar{c}), \quad (17)$$

is comfortably satisfied. The left-hand side is $[3068.7 + 9444.9]/2 = 6256.8$ MeV, while the right-hand side is likely [20] to be tens of MeV above the mass $M(B_c) = 6274.9 \pm 0.8$ MeV [19] of the spin-zero pseudoscalar $b\bar{c}$ ground state.

VI. CONCLUSIONS

A semiempirical method [3] successfully determines hadron masses, including the mass of the first doubly charmed baryon. This method sums constituent-quark masses, quark-quark hyperfine interactions, and terms $B(qq')$ expressing the binding of quarks both of which are at least as heavy as the strange quark. These binding terms are seen to satisfy inequalities $B(qq') < [B(qq) + B(q'q')]/2$, with the consequence that when hyperfine contributions are removed, baryons satisfy the inequality $m_{xyy} > \frac{1}{2}(m_{xxy} + m_{yyy})$ [9–16]. This constitutes a useful consistency check of the semiempirical method, and enables rough estimates, independent of potential models, of unseen hadron masses.

ACKNOWLEDGMENTS

The research of M. K. was supported by the ISF-NSFC joint research program (Grant No. 3423/19). We thank J.-M. Richard for a helpful discussion.

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