Strong first order electroweak phase transition in gauge-Higgs unification at finite temperature

Yuki Adachi [1] and Nobuhito Maru [2]

[1] *Department of Sciences, Matsue College of Technology, Matsue 690-8518, Japan*
[2] *Department of Mathematics and Physics, Osaka City University, Osaka 558-8585, Japan and Nambu Yoichiro Institute of Theoretical and Experimental Physics (NITEP), Osaka City University, Osaka 558-8585, Japan*

(Received 15 November 2019; accepted 3 February 2020; published 19 February 2020)

We analyze the electroweak phase transition at finite temperature in a model of gauge-Higgs unification where the fermion mass hierarchy including top quark mass, a viable electroweak symmetry breaking, and an observed Higgs mass are successfully reproduced. To study the phase transition, we employ the ζ function regularization method which is a well-known technique because the Kaluza-Klein mass spectrum is not determined exactly. Applying to our model of gauge-Higgs unification, the strong first order phase transition is realized in our model.

DOI: 10.1103/PhysRevD.101.036013

I. INTRODUCTION

Gauge-Higgs unification (GHU) [1,2] is one of the attractive scenarios that solves the hierarchy problem without invoking supersymmetry, where the Standard Model (SM) Higgs boson mass and its potential are calculable thanks to the higher dimensional gauge symmetry [2]. These characteristic properties have been studied and verified in models with various types of compactification at one-loop level [3] and at the two-loop level [4,5]. The calculability of other physical observables have been also investigated [6–8]. The flavor physics which is a very nontrivial in GHU has been studied in [9].

In five dimensional (5D) GHU, since Higgs potential at the tree level is forbidden by the gauge symmetry in higher dimensions, but it is radiatively generated, it is nontrivial to obtain a realistic electroweak symmetry breaking and the observed Higgs mass. In GHU, Higgs quartic coupling is provided by the gauge coupling squared and is 1-loop suppressed. Therefore, Higgs mass squared is likely to be light. In order to obtain an observed Higgs mass and a realistic electroweak symmetry breaking, a very small Higgs vacuum expectation value (VEV) is required in GHU. It is well known for getting small Higgs VEV that Higgs potential has to be generated by various contributions from higher rank representations of the gauge group [10,11].

As for the SM fermion masses, embedding the SM fermions except for top quark into some massive bulk fermions, Yukawa couplings can be obtained from the overlaps of zero mode functions of the gauge coupling. The fermion masses are easily reproduced by mild tuning of the bulk masses. Top quark should be embedded into massless bulk fermion to avoid a suppression, and into a fermion with higher rank representation [10,11].

In our previous paper [12], we have proposed a new model with a greatly simplified fermion content, where the fermion mass hierarchy including top quark mass, a successful electroweak symmetry breaking, and 125 GeV Higgs mass are reproduced. The point of the model is that we have employed another mechanism of generating Yukawa coupling for the third generation quarks. The third generation quarks are introduced as the brane-localized fermions not bulk fermions and have couplings with bulk fermions through the mass term on the brane. Integrating out the bulk fermions lead to Yukawa coupling and we have succeeded in reproducing top quark mass.

As a familiar application of the electroweak model to the finite temperature theory, there exists an electroweak baryogenesis for the generation of baryon asymmetry. It is well known that an application of the SM to the electroweak baryogenesis does not work because the 125 GeV Higgs mass is not compatible with the strong first order phase transition. Therefore, the application to the electroweak baryogenesis is well motivates to consider the physics beyond the SM.

In a paper by one of the authors [13], an application of GHU to the electroweak phase transition at finite temperature has been considered. Although the models discussed in the paper were not realistic ones, we have investigated whether strong first order electroweak phase transition takes place. The results were positive, namely, the strong first order phase transition in GHU is relatively easy to happen and a compatibility with 125 GeV Higgs mass was suggested. The reason is that the higher dimensional gauge boson contribution to the cubic term in Higgs potential at finite temperature is large. The application of a realistic GHU to the electroweak phase transition at finite temperature was found in [14]. Their result was that the phase transition becomes of first order, but not strong enough for 125 GeV Higgs mass.

In this paper, we investigate the phase transition at finite temperature in our realistic GHU model mentioned above [12]. In our model, it is very nontrivial to calculate the 1-loop effective potential at finite temperature because of the brane localized fermions and their coupling to the bulk fermions. In such a case, the Kaluza-Klein (KK) mass spectrum cannot be exactly solved in general. Therefore, Poisson resummation technique familiar with the calculation of the 1-loop effective potential in higher dimensions cannot be utilized. Instead, we employ the ζ function regularization method which was derived in [15] and [16] and extend it to calculate the 1-loop effective potential at finite temperature. The advantage of this method is that the functions determining the KK mass spectrum have only to be known, but the KK mass spectrum themselves are not necessary. Along this line, we calculate a 1-loop effective potential at finite temperature. Applying this potential to our model [12], we analyze the electroweak phase transition at finite temperature and examine whether the strong first order is realized.

This paper is organized as follows. In Sec. II, we briefly describe our model. In Sec. III, we calculate the 1-loop effective potential at finite temperature by exploiting ζ function regularization method. The electroweak phase transition at finite temperature is analyzed in Sec. IV. A summary is given in Sec. V.

II. THE MODEL

We begin with a brief review of the model proposed in our previous paper [12]. The $SU(3) \otimes U(1)_X$ gauge theory in five-dimensional flat space-time is considered. The fifth spatial extra dimension is compactified on an orbifold S^1/Z_2 with the radius R of S^1. The $U(1)_X$ gauge symmetry is introduced in order to realize the correct weak mixing angle θ_W.

The top (t) and the bottom (b) quarks in our setup are brane-localized fermions at the $y = \pi R$ brane. The SM fermions other than the top and bottom quarks are embedded in the bulk fermions Ψ_l for leptons and Ψ_q for quarks, which are assigned to the fundamental representation **3** of $SU(3)$. They obtain a mass through the five-dimensional gauge interaction, which is Yukawa interaction in the context of the gauge-Higgs unification scenario. Since the t and b quarks cannot interact directly with the Higgs boson (A_y) being the

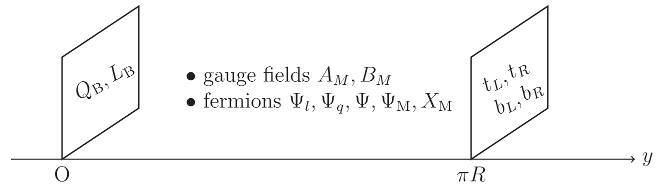

FIG. 1. Setup of the model.

fifth component of the gauge field in five dimensions, two extra bulk fermions Ψ (referred to as messenger fermions) are introduced, where one (the other) messenger fermion is embedded in the **3** (**15**) representation of $SU(3)$ coupling to the bottom (top) quark on the $y = \pi R$ brane. We also introduce a pair of fermions (referred to as mirror fermions) Ψ_M and X_M in **15** representation of $SU(3)$ to realize the realistic electroweak symmetry breaking. Such fermions may be a possible candidate of the dark matter as pointed out in [17]. The outline of the model is shown in Fig. 1.

As was pointed out in our previous paper, this simple matter content can explain the fermion mass hierarchy including top quark mass and the suitable electroweak symmetry breaking. The reason for introducing fermions of higher dimensional representation such as **15** is to reproduce the Higgs mass. In GHU, Higgs mass is likely to be light because Higgs potential is generated by quantum corrections. In order to obtain 125 GeV Higgs mass, the small VEV is required and realized by utilizing their nature of high frequency of Higgs potential.

To discuss the phase transition at finite temperature, we need the mass spectrum of the fields in our model because the nonzero KK modes contribute to the Higgs potential at 1-loop. In general, the KK mass spectrum becomes very complicated in the presence of the brane-localized terms. It is therefore very hard to find the exact mass eigenvalues. However, as will be explained in the next section, if we employ the ζ function regularization method, the mass eigenvalues are not required explicitly, but only the conditions for the mass spectrum to be satisfied are necessary. Then, it allows us to compute the Higgs potential in detail. These conditions are determined from the boundary conditions such as the Z_2 symmetry and/or the periodic/antiperiodic boundary conditions.

III. 1-LOOP EFFECTIVE POTENTIAL AT FINITE TEMPERATURE IN ζ FUNCTION REGULARIZATION METHOD

In this section, we analyze the 1-loop effective potential at finite temperature in our model by using the ζ function regularization instead of the Poisson resummation method. As pointed out in the paper [15] and [16], the ζ function regularization is powerful even in the case where the mass spectrum cannot be exactly found. First of all, we provide the formula to calculate the 1-loop effective potential at finite temperature along with their strategy. A particle with the mass m_n contributes to the 1-loop effective potential as

$$V=(-1)^F\frac{N_{\mathrm{DOF}}}{\beta}\sum_{l=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\int\frac{\mathrm{d}^{D-1}p}{(2\pi)^{D-1}}\frac{1}{2}\ln(\vec{p}^2+\omega^2+m_n^2) \tag{3.1}$$

where $\beta=T^{-1}$ is an inverse of temperature T. ω stands for the Matsubara frequency, which is given by $\omega_B=\frac{2\pi}{\beta}l$ for the bosonic field and $\omega_F=\frac{\pi}{\beta}(2l-1)$ for the fermionic field. The N_{DOF} stands for the degrees of freedom of the particles running in the loop. F means a fermion number, $F=0(1)$ for bosons (fermions). Note that the momentum integration covers $D-1$ dimensional momentum space because the time direction is compactified on a circle in an imaginary time formalism of the finite temperature theory. The Poisson resummation formula can be applied as usual for the simple form of KK mass spectrum such as n/R, but it is very difficult to do when the exact mass eigenvalues are not found. This is why we adopt their method instead of the Poisson resummation formula. The above 1-loop effective potential can be rewritten as follows.

$$V=-(-1)^F\frac{N_{\mathrm{DOF}}}{2\beta}\sum_{l=-\infty}^{\infty}\frac{1}{(2\sqrt{\pi})^{D-1}}\frac{(\pi R)^{-D+1}}{\Gamma(\frac{D+1}{2})}\times\int_0^\infty\mathrm{d}u(u^2+2\pi R|\omega|u)^{\frac{D-1}{2}}\times\frac{\mathrm{d}}{\mathrm{d}u}[\ln N(iu+i\pi R|\omega|)+\ln N(-iu-i\pi R|\omega|)]. \tag{3.2}$$

In this method, the mass eigenvalues m_n are not need to calculate the potential, namely, the functions $N(z)$ of determining the mass spectrum are only required. Thus, the above formula allows us to calculate the contributions from the complicated KK mass spectrum.

To check the validity of the general potential derived above (3.2), we show an example of the calculation of the contributions from the W boson by the ζ function renormalization method. We note that the W boson mass is given as a familiar form $m_n=\frac{n+\alpha}{R}$, which satisfies the following relation

$$\sin(\pi R m_n)-\sin\pi\alpha=0. \tag{3.3}$$

In this case, the function $N(z)$ that the KK mass spectrum satisfies is found as

$$N(z)=\sin z-\sin\pi\alpha. \tag{3.4}$$

Substituting the $D=4$ and $N_{\mathrm{DOF}}=3$, we obtain

$$V_{\mathrm{eff}}=-\frac{3}{2\beta}\sum_{l=-\infty}^{\infty}\frac{1}{(2\sqrt{\pi})^3}\frac{(\pi R)^{-3}}{\Gamma(\frac{5}{2})}\int_0^\infty\mathrm{d}u(u^2+2\pi R|\omega|u)^{\frac{3}{2}}\times\frac{\mathrm{d}}{\mathrm{d}u}[\ln\{\sin(iu+i\pi R|\omega|)-\sin\pi\alpha\}+\ln\{\sin(-iu-i\pi R|\omega|)-\sin\pi\alpha\}] \tag{3.5}$$

$$=-\frac{1}{4\pi^5\beta R^3}\sum_{l=-\infty}^{\infty}\int_0^\infty\mathrm{d}u(u^2+2\pi R|\omega|u)^{\frac{3}{2}}\frac{\mathrm{d}}{\mathrm{d}u}\ln\frac{\cosh(2u+2\pi R|\omega|)+\cos 2\pi\alpha}{2} \tag{3.6}$$

where $\omega=\frac{2\pi l}{\beta}$. Since this potential has a divergence independent of α, we subtract $V(\alpha=0)$ corresponding to the vacuum energy as the regularization. The effective potential at finite temperature is finally obtained as

$$V_{\mathrm{eff}}=-\frac{1}{4\pi^5\beta R^3}\sum_{l=-\infty}^{\infty}\int_0^\infty\mathrm{d}u\left(u^2+4\pi^2R\frac{|l|}{\beta}u\right)^{3/2}\frac{\mathrm{d}}{\mathrm{d}u}\ln\frac{\cosh(2u+4\pi^2R\frac{|l|}{\beta})-\cos(2\pi\alpha)}{\mathrm{e}^{2u+4\pi^2R\frac{|l|}{\beta}}}. \tag{3.7}$$

Next, we calculate the contributions from the W boson to 1-loop effective potential at finite temperature in terms of the Poisson resummation formula and compare it with the above result calculated by the ζ function regularization method. The result is given by

$$V_{\mathrm{eff}}^{\mathrm{Poisson}}=-\frac{9}{2\pi}R\sum_{n=1}^{\infty}\frac{1}{(2\pi Rn)^5}\cos(2\pi n\alpha)-\frac{9}{\pi}R\sum_{n=1}^{\infty}\sum_{l=1}^{\infty}\frac{1}{\{(2\pi Rn)^2+(\beta l)^2\}^{5/2}}\cos(2\pi n\alpha). \tag{3.8}$$

The first and second term corresponds to the zero temperature and the finite temperature part of the effective potential, respectively.

The comparison between the results calculated in two different methods is depicted in the Fig. 2. In the left plot, the summation in both effective potentials by ζ function

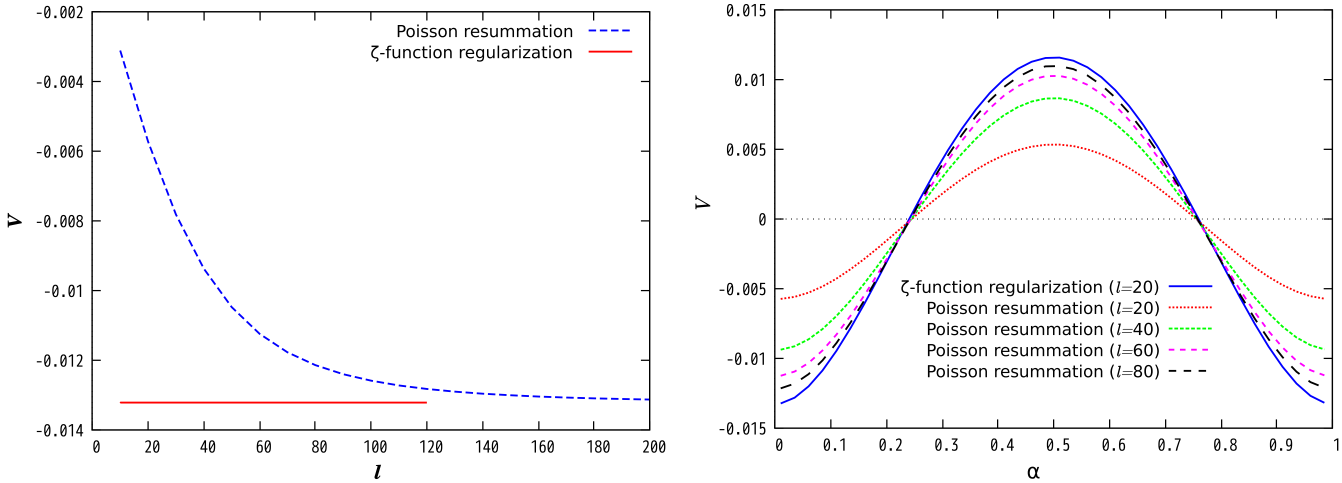

FIG. 2. The comparison of convergence nature between two expressions Eqs. (3.7) and (3.8) The left figure shows the values of effective potential for $R^{-1} = 1$ TeV, $\beta = 0.1/\mathrm{TeV}$, $\alpha = 0.01$. The horizontal axis describes an upper limit of summation (n and l). The right one shows the effective potential in the various upper limit.

regularization method and Poisson resummation is cut off up to the finite number of modes. The plot expresses a dependence of the both potential on the maximum values. A good agreement between two potentials is seen for the maximum values more than $n = l = 150$. In the right plot, the dependencies of the outline of the both potentials on the various maximum values of the summation are shown. The integral in the potential calculated by ζ function regularization method converges more rapidly. From these comparisons, we confirm a validity of the 1-loop effective potential derived by ζ function regularization method.

IV. ANALYSIS OF THE ELECTROWEAK PHASE TRANSITION AT FINITE TEMPERATURE

Before studying the phase transition at finite temperature in our model, we discuss general properties of the phase transition at finite temperature. The discussion in the case of GHU at finite temperature is found in [13,14]. In order to realize the first order phase transition, the cubic term in the 1-loop Higgs potential plays an essential role. The cubic terms arise only from the massless bosonic field contributions. In the case of higher dimensional gauge theories, the phase transition is likely to be first order, since the massless gauge boson contribution is present model independently. The discussions by use of approximate forms of the potential for the case of simple KK mass spectrum were given in [13,14]. Also, this property must be valid for the general potential derived in this paper. It is nontrivial and model dependent whether the phase transition is strong enough for the electroweak baryogenesis and whether it is compatible with 125 GeV Higgs mass.

Now, we are ready to discuss the electroweak phase transition at finite temperature in our GHU model. In order to compute Higgs potential of our model by using the result Eq. (3.2), We need seven kinds of functions, $N_Z, N_W, N_{\mathrm{BOT}}, N_{\mathrm{TOP}}, N_{\mathrm{LSM}}, N_{\mathrm{exotic}}$, and N_{M}, which are functions to determine the mass spectrum for the Z and W bosons, the bottom quark, top quark, the SM fermions except for the top and bottom quarks, the exotic fermions and the mirror fermions. The explicit forms for the W and Z boson $N_{W,Z}$ were given in [12] and the other functions can be similarly obtained but are very complicated. Subtracting $V(\alpha = 0)$ as was done in the above example, the four dimensional effective potential is obtained as

$$
\begin{aligned}
V_{\mathrm{R}} = &-\frac{1}{64\pi^5\beta R^3}\sum_{l=-\infty}^{\infty}\int_0^\infty \mathrm{d}u\Big[3(u^2+2\pi R|\omega_B|u)^{\frac{3}{2}}\frac{\mathrm{d}}{\mathrm{d}u}\{\ln N_Z(iu+i\pi R|\omega_B|)+\ln N_Z(-iu-i\pi R|\omega_B|)\\
&+\ln N_W(iu+i\pi R|\omega_B|)+\ln N_W(-iu-i\pi R|\omega_B|)\} - 3\cdot 4(u^2+2\pi R|\omega_F|u)^{\frac{3}{2}}\frac{\mathrm{d}}{\mathrm{d}u}\{\ln N_{\mathrm{BOT}}(iu+i\pi R|\omega_F|)\\
&+\ln N_{\mathrm{BOT}}(-iu-i\pi R|\omega_F|)+\ln N_{\mathrm{TOP}}(iu+i\pi R|\omega_F|)+\ln N_{\mathrm{TOP}}(-iu-i\pi R|\omega_F|)\\
&+\ln N_{\mathrm{LSM}}(iu+i\pi R|\omega_F|)+\ln N_{\mathrm{LSM}}(-iu-i\pi R|\omega_F|)+\ln N_{\mathrm{exotic}}(iu+i\pi R|\omega_F|)+\ln N_{\mathrm{exotic}}(-iu-i\pi R|\omega_F|)\\
&+\ln N_{\mathrm{M}}(iu+i\pi R|\omega_F|)+\ln N_{\mathrm{M}}(-iu-i\pi R|\omega_F|)\}\Big] - (\alpha\to 0). \qquad (4.1)
\end{aligned}
$$

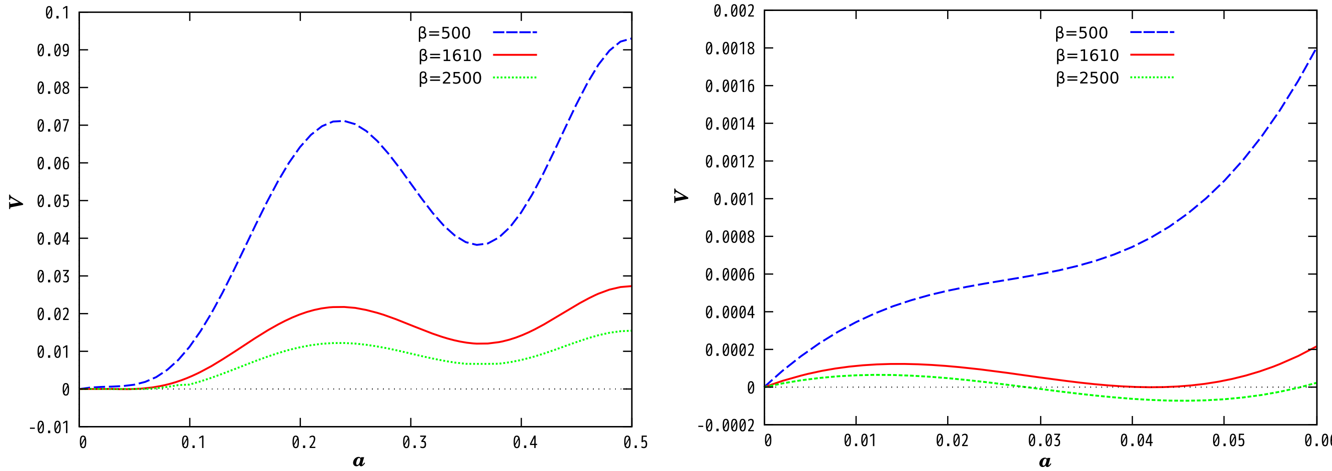

FIG. 3. The effective potential at some particular temperatures. Electroweak symmetry is restored at $\beta = 1610/\mathrm{TeV}$.

Although their explicit expressions of the potential is omitted here since they are very lengthy and complicated, they are written in our previous paper [12]. We choose the compactification scale and the bulk mass for the third generation quarks as $R^{-1} = 1.43$ TeV, $M = 1.95$ TeV, which succeeds in explaining the quark mass parameters including top quark mass [12]. The resultant figures are depicted in Fig. 3. The left plot shows the 1-loop Higgs effective potential at some temperatures. The temperature changes lower accordingly from the blue curve to the green one. The right plot zooms up around the minimum of the same potentials in the left plot. The potential minimum is at origin in the case of blue potential and the electroweak symmetry is unbroken. As the temperature is lowered, the red potential minimum at origin and some finite VEV are degenerate at the critical temperature. Lowering the temperature further, the green potential minimum is located at some VEV, which means that the electroweak symmetry is broken. The critical temperature T_C and the corresponding VEV at the critical temperature $\alpha(T_C)$ can be read off as $\beta_C = T_C^{-1} = 1610/\mathrm{TeV}$, $\alpha(T_C) = 0.0422$. By using the relation $v = \frac{\alpha}{Rg_4}$, the ratio between the VEV at the critical temperature and the critical temperature, which gives a signal of the first order phase transition, is found

$$\frac{v(T_C)}{T_C} = \beta_C \frac{\alpha(T_C)}{Rg_4} = 1610 \times \frac{0.0422}{1.43 g_4} = 47.5 \frac{1}{g_4}, \quad (4.2)$$

where the 4D $SU(2)$ gauge coupling is smaller than the unity $g_4 < 1$. This result ensures the strong first order phase transition in our model, but the critical temperature is too low and our result is inconsistent with the observations.

Such a relatively lower critical temperature compared with the compactification scale is the general feature in our model. As we mentioned in the Sec. II, the small VEV compared with the compactification scale is achieved by introducing the higher rank representations such as the **15** representations. It indicates that the magnitude of the minimum of the effective potential is much smaller than the model such as [14], so that the electroweak symmetry is restored at low temperature in our model.

V. SUMMARY

In this paper, we have studied the electroweak phase transition at finite temperature in a model proposed by the authors, 5D $SU(3) \otimes U(1)_X$ GHU with a realistic fermion mass hierarchy including top quark mass, a successful electroweak symmetry breaking and an observed Higgs boson mass [12]. The purpose of the analysis is to investigate whether the phase transition is strong first order or not. As an application of the electroweak phase transition, the electroweak baryogenesis is very familiar. In order to work the mechanism, the famous Sakharov's conditions must be satisfied. One of the conditions is that the phase transition should be strong first order.

In order to study the phase transition in our model, the calculation of the 1-loop effective potential is not trivial since our model has the brane localized fermions and their couplings to bulk fermions and the KK mass spectrum of such fermions cannot be exactly found in general. Therefore, we cannot use Poisson resummation formula in calculation of the 1-loop effective potential. Instead, we adopted the ζ function regularization method which is well-known technique and extend it to analyze the 1-loop effective potential at finite temperature in our model. The advantage of this method is that even if the KK mass spectrum cannot be found, the 1-loop effective potential can be calculated if we have the functions determining the KK mass spectrum. Applying the above method to our GHU model, we found the too small critical temperature $T_C \sim \mathcal{O}(1)$ GeV compared with the compactification scale. It indicates that the strong first order phase transition occurs in our model, but it does not agree with the observation. Such a relatively lower critical temperature compared with the compactification scale is the general feature in our model. As we mentioned in the Sec. II, the small VEV compared with the compactification scale is achieved by introducing fermions in the higher rank representations of the gauge group. It implies that the magnitude of the effective potential minimum is much smaller than the model such as [14]. This is a reason why the electroweak symmetry is restored at low temperature in our model. To avoid this difficulty, we must extend our model. For example, the GUT extension [18–20] may change such a situation since the gauge bosons which correspond to the broken symmetry contribute to the cubic terms in the effective potential at finite temperature.

If the strong first order phase transition is realized at electroweak scale in the model of GUT extension, *CP* violation is the next condition to be satisfied. *CP* violation is one of the most nontrivial issues in GHU since Yukawa coupling is originated from the gauge coupling. Namely, Yukawa couplings are real as they stands. Furthermore, it is also nontrivial to obtain an enough *CP* phase required for the baryon asymmetry, which must be an additional *CP* phase other than Kobayashi-Maskawa *CP* phase. Our previous work on *CP* violation in the context of GHU [21] might help to overcome this issue.

ACKNOWLEDGMENTS

The work of N. M. is supported in part by JSPS KAKENHI Grant No. JP17K05420.

[1] N. S. Manton, Nucl. Phys. **B158**, 141 (1979); D. B. Fairlie, Phys. Lett. **82B**, 97 (1979), J. Phys. G **5**, L55 (1979); Y. Hosotani, Phys. Lett. **126B**, 309 (1983), **129B**, 193 (1983), Ann. Phys. (N.Y.) **190**, 233 (1989).

[2] H. Hatanaka, T. Inami, and C. S. Lim, Mod. Phys. Lett. A **13**, 2601 (1998).

[3] I. Antoniadis, K. Benakli, and M. Quiros, New J. Phys. **3**, 20 (2001); G. von Gersdorff, N. Irges, and M. Quiros, Nucl. Phys. **B635**, 127 (2002); R. Contino, Y. Nomura, and A. Pomarol, Nucl. Phys. **B671**, 148 (2003); C. S. Lim, N. Maru, and K. Hasegawa, J. Phys. Soc. Jpn. **77**, 074101 (2008); C. S. Lim, N. Maru, and T. Miura, Prog. Theor. Exp. Phys. **2015**, 43B02 (2015) K. Hasegawa, C. S. Lim, and N. Maru, Phys. Lett. B **604**, 133 (2004).

[4] N. Maru and T. Yamashita, Nucl. Phys. **B754**, 127 (2006); Y. Hosotani, N. Maru, K. Takenaga, and T. Yamashita, Prog. Theor. Phys. **118**, 1053 (2007).

[5] J. Hisano, Y. Shoji, and A. Yamada, arXiv:1908.09158.

[6] C. S. Lim and N. Maru, Phys. Rev. D **75**, 115011 (2007).

[7] N. Maru, Mod. Phys. Lett. A **23**, 2737 (2008); N. Maru and N. Okada, Phys. Rev. D **77**, 055010 (2008); **87**, 095019 (2013); **88**, 037701 (2013); arXiv:1310.3348.

[8] Y. Adachi, C. S. Lim, and N. Maru, Phys. Rev. D **76**, 075009 (2007); **79**, 075018 (2009); **80**, 055025 (2009).

[9] Y. Adachi, N. Kurahashi, C. S. Lim, and N. Maru, J. High Energy Phys. 11 (2010) 150; 01 (2012) 047; Y. Adachi, N. Kurahashi, N. Maru, and K. Tanabe, Phys. Rev. D **85**, 096001 (2012); Y. Adachi, N. Kurahashi, and N. Maru, arXiv:1404.4281.

[10] C. A. Scrucca, M. Serone, and L. Silvestrini, Nucl. Phys. **B669**, 128 (2003).

[11] G. Cacciapaglia, C. Csaki, and S. C. Park, J. High Energy Phys. 03 (2006) 099.

[12] Y. Adachi and N. Maru, Phys. Rev. D **98**, 015022 (2018).

[13] N. Maru and K. Takenaga, Phys. Rev. D **72**, 046003 (2005); **74**, 015017 (2006).

[14] G. Panico and M. Serone, J. High Energy Phys. 05 (2005) 024.

[15] A. Falkowski, Phys. Rev. D **75**, 025017 (2007).

[16] H. Hatanaka and Y. Hosotani, Phys. Lett. B **713**, 481 (2012).

[17] N. Maru, T. Miyaji, N. Okada, and S. Okada, J. High Energy Phys. 07 (2017) 048; N. Maru, N. Okada, and S. Okada, Phys. Rev. D **96**, 115023 (2017).

[18] C. S. Lim and N. Maru, Phys. Lett. B **653**, 320 (2007); N. Maru and Y. Yatagai, Prog. Theor. Exp. Phys. **2019**, 083B03 (2019); arXiv:1911.03465.

[19] K. Kojima, K. Takenaga, and T. Yamashita, Phys. Rev. D **84**, 051701 (2011); **95**, 015021 (2017); J. High Energy Phys. 06 (2017) 018.

[20] Y. Hosotani and N. Yamatsu, Prog. Theor. Exp. Phys. **2015**, 111B01 (2015); **2017**, 091B01 (2017); **2018**, 023B05 (2018); A. Furui, Y. Hosotani, and N. Yamatsu, Prog. Theor. Exp. Phys. **2016**, 093B01 (2016).

[21] Y. Adachi, N. Kurahashi, N. Maru, and K. Tanabe, arXiv:1201.2290.