Sequential hyperon decays in the reaction $e^+e^- \to \Sigma^0\bar{\Sigma}^0$

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(Received 30 October 2019; accepted 21 January 2020; published 18 February 2020)

We report on a study of the sequential hyperon decay $\Sigma^0 \to \Lambda\gamma$; $\Lambda \to p\pi^-$ and its corresponding antihyperon decay. We derive a multidimensional and model-independent formalism for the case when the hyperons are produced in the reaction $e^+e^- \to \Sigma^0\bar{\Sigma}^0$. Cross-section distributions are calculated using the folding technique. We also study sequential decays of single-tagged hyperons.

DOI: 10.1103/PhysRevD.101.033002

I. INTRODUCTION

The BESIII experiment [1] has created new opportunities for research into hyperon physics, based on e^+e^- annihilation into hyperon-antihyperon pairs. Such possibilities are interesting, and for several reasons:

(i) They offer the currently only feasible way for investigating the electromagnetic structure of hyperons [2].
(ii) By measuring in the vicinity of vector-charmonium states, one gains information on the strong baryon-antibaryon decay processes of charmonia.
(iii) They offer a model-independent method for measuring weak-decay-asymmetry parameters, which can probe CP symmetry [3].

The basic reaction, $e^+e^- \to Y\bar{Y}$, is graphed in Fig. 1. In the continuum region, i.e., in energy regions that do not overlap with energies of vector charmonia like J/ψ, ψ' and $\psi(2S)$, the production process is dominated by one-photon exchange, $e^+e^- \to \gamma^* \to Y\bar{Y}$. The reaction amplitude is then governed by the electromagnetic form factors G_E and G_M. In the vicinity of vector resonances, the electromagnetic form factors are replaced by hadronic form factors G_E^ψ and G_M^ψ. However, the shapes of the differential-cross-section distributions are the same in the two cases: all physics of the production mechanism is contained within the form factors, or equivalently, the strength of form factors, $D_\psi(s)$; the ratio of form-factor magnitudes, $\eta_\psi(s)$; and the relative phase of form factors, $\Delta\Phi_\psi(s)$.

Analyses of joint-decay distributions of hyperons, such as $\Lambda(\to p\pi^-)\bar{\Lambda}(\to \bar{p}\pi^+)$, enables us to determine the weak-interaction-decay parameters, $\alpha\beta\gamma$. For a complete determination we need to know the bayon-final-state polarizations.

The theoretical description of the annihilation reaction of Fig. 1 is described in Ref. [4], and the corresponding annihilation reaction mediated by J/ψ in Ref. [5]. Accurate experimental results for the form-factor parameters η_ψ and $\Delta\Phi_\psi$ and the weak-interaction parameters $\alpha_\Lambda(\alpha_{\bar{\Lambda}})$ for the latter annihilation process are all reported in Ref. [3]. A precise knowledge of the asymmetry parameters $\alpha_\Lambda(\alpha_{\bar{\Lambda}})$ is needed for studies of spin polarization in Ω^-, Ξ^-, and Λ_c^+ decays, and for tests of the Standard Model.

The graph of Fig. 1 can be generalized in the sense that it can include hyperons that decay sequentially. It can also include cases where the produced hyperon is of a different kind than the produced antihyperon, i.e., $e^+e^- \to Y_1\bar{Y}_2$.

In this paper we shall consider annihilation into $\Sigma^0\bar{\Sigma}^0$ pairs, in a way similar to that of Ref. [6]. The Σ^0 decays electromagnetically, $\Sigma^0 \to \Lambda\gamma$, and subsequently the Lambda hyperon decays weakly, $\Lambda \to p\pi^-$. The interest of such a study is many-fold:

(i) The form factors provide information about the production process. So far, literature has focused on electromagnetic form factors whose interpretation is

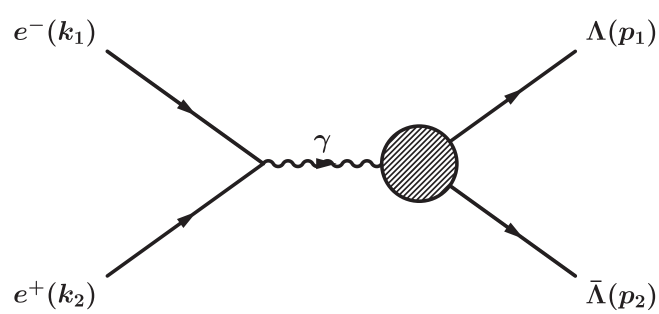

FIG. 1. Graph describing the electromagnetic annihilation reaction $e^+e^- \to \bar{\Lambda}\Lambda$. The same reaction can also proceed hadronicly via vector charmonium states such as J/ψ, ψ', or $\psi(2S)$, replacing the photon.

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straightforward [2,7]. However, recent experimental advances call for an interpretation also of the hadronic form factors. In particular, it would be interesting to compare the decay of J/ψ into various hyperon-antihyperon pairs with the corresponding decays of other vector charmonia.

(ii) The BESIII Collaboration plans to perform a first measurement of the branching fraction of the Σ^0 Dalitz decay $\Sigma^0 \to \Lambda\gamma^*$, $\gamma^* \to e^+e^-$ using the large data sample available for the $e^+e^- \to J/\psi \to \bar{\Sigma}^0\Sigma^0$ process. Then, the most important background will come from $e^+e^- \to J/\psi \to \bar{\Sigma}^0\Sigma^0$; ($\Sigma^0 \to \Lambda\gamma$; $\Lambda \to p\pi^- +$ c.c), where one of the photons undergoes external conversion into an e^+e^- pair. This is because the branching ratio of the $\Sigma^0 \to \Lambda\gamma$, according to QED, is 3 orders of magnitude larger than that of the Dalitz decay. In order to properly account for the background, precise knowledge of the joint angular distribution is required.

(iii) It can provide an independent measurement of the Lambda asymmetry parameters α_Λ and $\alpha_{\bar{\Lambda}}$.

(iv) It can provide a first test of strong CP symmetry in the $\Sigma^0 \to \Lambda\gamma$ decay [8].

Our calculation is performed in steps. First, we review some important facts; the spin structure of the $e^+e^- \to \Sigma^0\bar{\Sigma}^0$ annihilation reaction [4]; the classical $\alpha\beta\gamma$ description of hyperon decays [9]; the description of the electromagnetic $\Sigma^0 \to \Lambda\gamma$ decay, both for real and virtual photons [6,10]. The virtual photons decay into Dalitz lepton pairs. An important element of our calculation is the factorization of the squared amplitudes into a spin-independent fractional decay rate and a spin-density distribution.

Following these reviews we demonstrate how the folding method of Ref. [11] is adapted to sequential decays. Both simple and double decay chains are treated. Finally, we join production and decay steps to give the cross-section distributions.

The information we are hoping to gain resides in the angular distributions, and we are therefore not overly concerned with absolute normalizations, although they may be obtained without too much effort.

II. BARYON FORM FACTORS

The diagram in Fig. 1 describes the annihilation reaction $e^-(k_1)e^+(k_2) \to Y(p_1)\bar{Y}(p_2)$ and involves two vertex functions: one of them leptonic, the other one baryonic. The strength of the lepton-vertex function is determined by the electric charge e_e, but two form factors $G_M(s)$ and $G_E(s)$ are needed for describing the baryonic vertex function. Here, $s = (p_1 + p_2)^2$ with p_1 and p_2 as defined in Fig. 1.

The strength of the baryon form factors is measured by the function $D(s)$,

$$D(s) = s|G_M|^2 + 4M^2|G_E|^2, \tag{2.1}$$

with the M-variable representing the hyperon mass. The ratio of form factors is measured by $\eta(s)$,

$$\eta(s) = \frac{s|G_M|^2 - 4M^2|G_E|^2}{s|G_M|^2 + 4M^2|G_E|^2}, \tag{2.2}$$

with $\eta(s)$ satisfying $-1 \le \eta(s) \le 1$. The relative phase of form factors is measured by $\Delta\Phi(s)$,

$$\frac{G_E}{G_M} = e^{i\Delta\Phi(s)}\left|\frac{G_E}{G_M}\right|. \tag{2.3}$$

In Ref. [5] annihilation in the region of the J/ψ and $\psi(2S)$ masses is considered. The photon propagator of Fig. 1 is then replaced by the appropriate vector-meson propagator.

III. CROSS SECTION FOR $e^-e^+ \to Y(s_1)\bar{Y}(s_2)$

Our first task is to review the calculation of the cross-section distribution for e^+e^- annihilation into baryon-antibaryon pairs, with baryon-four-vector polarizations s_1 and s_2 [4,5]. From the squared matrix element of this process, $|\mathcal{M}|^2$, we remove a factor e_e^4/s^2, which is the square of the propagator, and get

$$\mathrm{d}\sigma = \frac{1}{2s}\frac{e_e^4}{s^2}|\mathcal{M}_{\mathrm{red}}(s_1, s_2)|^2 \mathrm{dLips}(k_1 + k_2; p_1, p_2), \tag{3.1}$$

with $s = (p_1 + p_2)^2$, and dLips denotes the phase-space element of Ref. [12], as described in Appendix A. For a baryon of momentum $\mathbf{p}$ the four-vector spin s is related to the three-vector spin $\mathbf{n}$, the spin in the rest system, by

$$s(\mathbf{p}, \mathbf{n}) = \frac{n_\parallel}{M}(|\mathbf{p}|, E\hat{\mathbf{p}}) + (0, \mathbf{n}_\perp). \tag{3.2}$$

Longitudinal and transverse directions of vectors are relative to the $\hat{\mathbf{p}}$ direction.

In the global c.m. system kinematics simplifies. There, three-momenta $\mathbf{p}$ and $\mathbf{k}$ are defined such that

$$\mathbf{p}_1 = -\mathbf{p}_2 = \mathbf{p}, \tag{3.3}$$

$$\mathbf{k}_1 = -\mathbf{k}_2 = \mathbf{k}, \tag{3.4}$$

and with scattering angle θ defined by

$$\cos\theta = \hat{\mathbf{p}} \cdot \hat{\mathbf{k}}. \tag{3.5}$$

Furthermore, according to Appendix B, in the global c.m. system the phase-space element reads

$$\mathrm{dLips}(k_1 + k_2; p_1, p_2) = \frac{p}{32\pi^2 k}\mathrm{d}\Omega, \tag{3.6}$$

with $p = |\mathbf{p}|$ and $k = |\mathbf{k}|$.

The matrix element in Eq. (3.1) can be written as a sum of terms that depends on the baryon and antibaryon spin directions in their respective rest systems, $\mathbf{n}_1$ and $\mathbf{n}_2$,

$$|\mathcal{M}_{\rm red}(e^+e^- \to Y(s_1)\bar{Y}(s_2))|^2 = sD(s)S(\mathbf{n}_1, \mathbf{n}_2), \tag{3.7}$$

with the strength function $D(s)$ defined in Eq. (2.1). We call a function such as $S(\mathbf{n}_1, \mathbf{n}_2)$ a spin density. In the present case, the spin density is a sum of seven mutually orthogonal contributions [4],

$$\begin{aligned} S(\mathbf{n}_1, \mathbf{n}_2) = {} & \mathcal{R} + \mathcal{S}\mathbf{N}\cdot\mathbf{n}_1 + \mathcal{S}\mathbf{N}\cdot\mathbf{n}_2 + \mathcal{T}_1\mathbf{n}_1\cdot\hat{\mathbf{p}}\mathbf{n}_2\cdot\hat{\mathbf{p}} \\ & + \mathcal{T}_2\mathbf{n}_{1\perp}\cdot\mathbf{n}_{2\perp} + \mathcal{T}_3\mathbf{n}_{1\perp}\cdot\hat{\mathbf{k}}\mathbf{n}_{2\perp}\cdot\hat{\mathbf{k}} \\ & + \mathcal{T}_4(\mathbf{n}_1\cdot\hat{\mathbf{p}}\mathbf{n}_{2\perp}\cdot\hat{\mathbf{k}} + \mathbf{n}_2\cdot\hat{\mathbf{p}}\mathbf{n}_{1\perp}\cdot\hat{\mathbf{k}}), \end{aligned} \tag{3.8}$$

where $\mathbf{N}$ is the normal to the scattering plane,

$$\mathbf{N} = \frac{1}{\sin\theta}\hat{\mathbf{p}}\times\hat{\mathbf{k}}. \tag{3.9}$$

The six structure functions $\mathcal{R}$, $\mathcal{S}$, and $\mathcal{T}$ of Eq. (3.8) depend on the scattering angle θ, the ratio function $\eta(s)$, and the phase function $\Delta\Phi(s)$. Their detailed expressions are given in Appendix C.

The cross-section distribution for polarized final-state hyperons becomes

$$\frac{d\sigma}{d\Omega} = \frac{p}{k}\frac{\alpha_e^2 D(s)}{4s^2} S(\mathbf{n}_1, \mathbf{n}_2), \tag{3.10}$$

where α_e is the fine-structure constant. Summing over baryon and antibaryon final-state polarizations gives as a result

$$\frac{d\sigma}{d\Omega}(e^+e^- \to \gamma^\star \to Y\bar{Y}) = \frac{p}{k}\frac{\alpha_e^2 D(s)}{s^2}\mathcal{R}. \tag{3.11}$$

Summing only over the antibaryon polarizations gives

$$\frac{d\sigma}{d\Omega} = \frac{p}{k}\frac{\alpha_e^2 D(s)}{2s^2}(\mathcal{R} + \mathcal{S}\mathbf{N}\cdot\mathbf{n}_1). \tag{3.12}$$

This result tells us that the baryon is polarized and that its polarization is directed along the normal to the scattering plane, $\hat{\mathbf{p}}\times\hat{\mathbf{k}}$, and that the value of the polarization is

$$P_Y(\theta) = \frac{\mathcal{S}}{\mathcal{R}} = \frac{\sqrt{1-\eta^2}\cos\theta\sin\theta}{1+\eta\cos^2\theta}\sin(\Delta\Phi). \tag{3.13}$$

From Eq. (3.8) we conclude that there is a corresponding result for the antibaryon, but it should then be remembered that $\mathbf{p}$ is the momentum of the baryon, but $-\mathbf{p}$ that of the antibaryon.

Baryon and antibaryon polarizations in e^+e^- annihilation were first discussed by Dubničkova *et al.* [13], but with results slightly different from ours, and later by Czyż *et al.* [14]. For details see Ref. [4].

IV. WEAK BARYON DECAYS

Weak decays of spin one-half baryons, such as $\Lambda \to p\pi^-$, involve two amplitudes, one S-wave and one P-wave amplitude, and the decay distribution is commonly parametrized by three parameters, denoted $\alpha\beta\gamma$, and which fulfill a relation

$$\alpha^2 + \beta^2 + \gamma^2 = 1. \tag{4.1}$$

Details of this description can be found in Refs. [15] or [4,9].

Since we shall encounter several weak baryon decays of the same structure as the $\Lambda \to p\pi^-$ decay, we shall use a generic notation, $c \to d\pi$, for those decays.

The matrix element describing the decay of a polarized c baryon into a polarized d baryon is

$$\mathcal{M}(c \to d\pi) = \bar{u}(p_d, s_d)(A + B\gamma_5)u(p_c, s_c), \tag{4.2}$$

with p and s with appropriate indices denoting momenta and spin four-vectors of the baryons. The square of this matrix element we factorize, writing

$$\begin{aligned} |\mathcal{M}(c \to d\pi)|^2 &= \mathrm{Tr}\Big[\frac{1}{2}(1+\gamma_5\not{s}_d)(\not{p}_d + m_d)(A + B\gamma_5) \\ &\quad \times (\not{p}_c + m_c)\frac{1}{2}(1+\gamma_5\not{s}_c)(A^\star - B^\star\gamma_5)\Big] \\ &= R(c \to d\pi)G(\mathbf{n}_c, \mathbf{n}_d), \end{aligned} \tag{4.3}$$

where $\mathbf{n}_c$ and $\mathbf{n}_d$ are the spin vectors of baryons c and d in their rest frames, Eq. (3.2). The R-factor is a spin independent factor, defined by

$$\begin{aligned} R(c \to d\pi) &= 2m_c\Gamma(c \to d\pi)/\Phi(c \to d\pi), \\ &= |A|^2((m_c + m_d)^2 - m_\pi^2) \\ &\quad + |B|^2((m_c - m_d)^2 - m_\pi^2), \end{aligned} \tag{4.4}$$

where $\Phi(c \to d\pi) = \Phi(m_c; m_d, m_\pi)$ is the phase-space volume of Appendix B. We refer to $R(c \to d\pi)$ as the fractional decay rate, since it is a decay rate per unit phase space. Further inspection of Eq. (4.3) tells us that $\Gamma(c \to d\pi)$ is defined as an average over the spins of both initial- and final-state baryon.

The spin-density-distribution function, $G(\mathbf{n}_c, \mathbf{n}_d)$ of Eq. (4.3), is a Lorentz scalar, which we choose to evaluate in the rest system of the mother baryon, c,

$$G(c,d) = 1 + \alpha_c\mathbf{n}_c\cdot\mathbf{l}_d + \alpha_c\mathbf{n}_d\cdot\mathbf{l}_d + \mathbf{n}_c\cdot\mathbf{L}_c(\mathbf{n}_d, \mathbf{l}_d), \tag{4.5}$$

with

$$\mathbf{L}_c(\mathbf{n}_d, \mathbf{l}_d) = \gamma_c \mathbf{n}_d + [(1-\gamma_c)\mathbf{n}_d \cdot \mathbf{l}_d]\mathbf{l}_d + \beta_c \mathbf{n}_d \times \mathbf{l}_d. \tag{4.6}$$

The vector $\mathbf{l}_d$ is a unit vector in the direction of motion of the daughter baryon, d, in the rest system of mother baryon c. The indices on the $\alpha\beta\gamma$ parameters remind us they characterize baryon c. A spin density is normalized if the spin-independent term is unity.

We observe an important symmetry,

$$\mathbf{n}_c \cdot \mathbf{L}_c(\mathbf{n}_d, \mathbf{l}_d) = \mathbf{n}_d \cdot \mathbf{L}_c(\mathbf{n}_c, -\mathbf{l}_d). \tag{4.7}$$

Since the spin of baryon d is usually not measured, the interesting spin-density is obtained by taking the average over the spin directions $\mathbf{n}_d$,

$$\begin{aligned} W_c(\mathbf{n}_c; \mathbf{l}_d) &= \langle G_c(c,d) \rangle_{\mathbf{n}_d} \\ &= U_c + \mathbf{n}_c \cdot \mathbf{V}_c, \end{aligned} \tag{4.8}$$

with

$$U_c = 1, \qquad \mathbf{V}_c = \alpha_c \mathbf{l}_d. \tag{4.9}$$

For an initial state polarization $\mathbf{P}_c$ we put $\mathbf{n}_c = \mathbf{P}_c$, and get an angular distribution known from the weak hyperon decay $\Lambda \to p\pi^-$ [4,9].

The matrix element describing the decay of a polarized $\bar{c}$ (anti)baryon into a polarized $\bar{d}$ (anti)baryon is similar to that of Eq. (4.2),

$$\mathcal{M}(\bar{c} \to \bar{d}\pi) = \bar{v}(p_{\bar{c}}, s_{\bar{c}})(A' + B'\gamma_5)v(p_{\bar{d}}, s_{\bar{d}}). \tag{4.10}$$

The relation between the parameters A, B and A', B' is clarified in Refs. [16,17].

The square of the antibaryon matrix element of Eq. (4.10) is factorized exactly as the baryon-matrix element of Eq. (4.3),

$$|\mathcal{M}(\bar{c} \to \bar{d}\pi)|^2 = R(\bar{c} \to \bar{d}\pi) G(\mathbf{n}_{\bar{c}}, \mathbf{n}_{\bar{d}}), \tag{4.11}$$

where $\mathbf{n}_{\bar{c}}$ and $\mathbf{n}_{\bar{d}}$ are the spin vectors of baryons $\bar{c}$ and $\bar{d}$ in their rest systems.

The functions $R(\bar{c} \to \bar{d}\pi)$ and $G(\mathbf{n}_{\bar{c}}, \mathbf{n}_{\bar{d}})$ are tied to hyperons $\bar{c}$ and $\bar{d}$ in exactly the same way as those tied to hyperons c and d, Eqs. (4.4) and (4.5), or to be specific,

$$G(\bar{c}, \bar{d}) = 1 + \alpha_{\bar{c}} \mathbf{n}_{\bar{c}} \cdot \mathbf{l}_{\bar{d}} + \alpha_{\bar{c}} \mathbf{n}_{\bar{d}} \cdot \mathbf{l}_{\bar{d}} + \mathbf{n}_{\bar{c}} \cdot \mathbf{L}_{\bar{c}}(\mathbf{n}_{\bar{d}}, \mathbf{l}_{\bar{d}}). \tag{4.12}$$

For CP conserving interactions the asymmetry parameters of the hyperon pair c, d are related to those of antihyperon pair $\bar{c}$, $\bar{d}$ by [16,17]

$$\alpha_c = -\alpha_{\bar{c}}, \qquad \beta_c = -\beta_{\bar{c}}, \qquad \gamma_c = \gamma_{\bar{c}}. \tag{4.13}$$

V. ELECTROMAGNETIC HYPERON DECAYS: REAL PHOTONS

Electromagnetic transitions such as $\Sigma^0 \to \Lambda\gamma$ and $\Xi^0 \to \Sigma^0\gamma$ are readily investigated in e^+e^- annihilation. The electromagnetic $\Sigma^0 \to \Lambda$ transition is caused by the four-vector current [12]

$$\begin{aligned} J_\mu(c \to d) = \frac{1}{m_c + m_d}\bigg[& F_1(k^2)\left\{\frac{k^2}{m_d - m_c}\gamma_\mu + k_\mu\right\} \\ & + F_2(k^2) i\sigma_{\mu\nu} k^\nu \bigg], \end{aligned} \tag{5.1}$$

with $k = p_c - p_d$. This transition current is gauge invariant, inasmuch as $k \cdot J = 0$. In fact, the $F_1(k^2)$ and $F_2(k^2)$ contributions are each, by themselves, gauge invariant. For real photons $k^2 = 0$ and the F_1 contribution vanishes, since F_1 itself vanishes, $F_1(0) = 0$. Thus, for this case it is sufficient to consider the F_2 term. We denote by μ_{cd},

$$\mu_{cd} = eF_2(0)/(m_c + m_d), \tag{5.2}$$

the strength of the M1 magnetic transition. As a consequence, the expression for the matrix element for any electromagnetic $\Sigma^0 \to \Lambda\gamma$ like decay, becomes

$$\begin{aligned} \mathcal{M}_\gamma(c \to d\gamma) &= \mu_{cd} \bar{u}_d(p_d, s_d)(\sigma^{\mu\nu} e^\star_\mu(-ik_\nu)) u_c(p_c, s_c) \\ &= \mu_{cd} \bar{u}_d(p_d, s_d)(\not{e}^\star \not{k}) u_c(p_c, s_c), \end{aligned} \tag{5.3}$$

where s_c and s_d are the spin four-vectors of the two baryons.

It is convenient to write the square of this matrix element on the form

$$\begin{aligned} |\mathcal{M}_\gamma(c \to d\gamma)|^2 &= \mu_{cd}^2 \mathrm{Tr}\bigg[\frac{1}{2}(1 + \gamma_5 \not{s}_d)(\not{p}_d + m_d)\not{e}^\star \not{k} \\ &\quad \times (\not{p}_c + m_c)\frac{1}{2}(1 + \gamma_5 \not{s}_c)\not{e}\not{k}\bigg] \\ &= H^{\mu\nu}_\gamma(k) e_\mu(k) e^\star_\nu(k), \end{aligned} \tag{5.4}$$

with $H^{\mu\nu}_\gamma(k)$ referred to as the hadron tensor. We have also made use of the simplifying identity

$$e^\mu i\sigma_{\mu\nu} k^\nu = -\not{e}\not{k}, \tag{5.5}$$

valid for real photons.

Summation over the two photon-spin directions entails replacing $e_\mu(k) e^\star_\nu(k)$ by $-g_{\mu\nu}$. This leads to

$$\sum_{e_\gamma} |\mathcal{M}_\gamma(c \to d\gamma)|^2 = R(c \to d\gamma) G_\gamma(\mathbf{n}_c, \mathbf{n}_d), \tag{5.6}$$

and again $\mathbf{n}_c$ and $\mathbf{n}_d$ are the spin vectors of baryons c and d in their rest systems. Photon polarizations are summed over. There are also electromagnetic transitions between charged baryons, but in this section we limit ourselves to electromagnetic transitions between neutral baryons.

The factorization of Eq. (5.6) is chosen so that the fractional decay rate $R(c \to d\gamma)$ is the unpolarized part of Eq. (5.6) and its $G_\gamma(\mathbf{n}_c, \mathbf{n}_d)$ factor the normalized spin-density-distribution function. Here, unpolarized means averaged over the spin directions of both initial and final baryons.

The fractional decay rate, $R(c \to d\gamma)$ of Eq. (5.7), has the same structure as the corresponding one for weak baryon decays, Eq. (4.4),

$$\begin{aligned} R(c \to d\gamma) &= 2m_c\Gamma(c \to d\gamma)/\Phi(c \to d\gamma), \\ &= \mu_{cd}^2(m_c^2 - m_d^2)^2, \end{aligned} \tag{5.7}$$

where $\Phi(c \to d\gamma) = \Phi(m_c; m_d, m_\gamma)$ is the phase-space volume.

The electromagnetic decay width is

$$\Gamma(c \to d\gamma) = \frac{1}{2\pi}\mu_{cd}^2\omega^3, \tag{5.8}$$

where ω is the photon energy. Remember, that this width is obtained after averaging over both initial and final baryon spin states.

The spin-density-distribution function of Eq. (5.6) involves an implicit summation over photon polarizations. For such a case

$$G_\gamma(\mathbf{n}_c, \mathbf{n}_d) = 1 - \mathbf{n}_c \cdot \mathbf{l}_\gamma \mathbf{l}_\gamma \cdot \mathbf{n}_d, \tag{5.9}$$

where $\mathbf{l}_\gamma$ is a unit vector in the direction of motion of the photon, and $\mathbf{l}_d = -\mathbf{l}_\gamma$ a unit vector in the direction of motion of baryon d, both in the rest system of baryon c.

We notice that when both hadron spins are parallel or antiparallel to the photon momentum, then the decay probability vanishes, a property of angular-momentum conservation. We also notice that expression (5.9) cannot be written in the $\alpha\beta\gamma$ representation of Eqs. (4.5) and (4.6).

When the spin of the final-state baryon d is not measured, the relevant spin density is obtained by forming the average over the spin directions $\mathbf{n}_d$,

$$\begin{aligned} W_\gamma(\mathbf{n}_c; \mathbf{l}_d) &= \langle G_\gamma(c, d)\rangle_{\mathbf{n}_d} \\ &= U_c + \mathbf{n}_c \cdot \mathbf{V}_c, \end{aligned} \tag{5.10}$$

with

$$U_c = 1, \qquad \mathbf{V}_c = 0. \tag{5.11}$$

Thus, the decay-distribution function is independent of the initial-state baryon spin vector $\mathbf{n}_c$.

The antiparticle matrix element corresponding to the particle matrix element of Eq. (5.3) is simply

$$\mathcal{M}_\gamma(\bar{c} \to \bar{d}\gamma) = \mu_{cd}\bar{v}_{\bar{c}}(p_{\bar{c}}, s_{\bar{c}})(\not{\epsilon}^\star \not{k})v_{\bar{d}}(p_{\bar{d}}, s_{\bar{d}}). \tag{5.12}$$

We assume the parameter μ is the same for particle transitions $c \to d$ as for antiparticle transitions $\bar{c} \to \bar{d}$.

The normalized spin density corresponding to the antiparticle-matrix element of Eq. (5.12) is the same as that corresponding to the particle matrix element of Eq. (5.3), as given in Eq. (5.9), provided we replace the particle spin vectors $\mathbf{n}_c$ and $\mathbf{n}_d$ by the antiparticle spin vectors $\mathbf{n}_{\bar{c}}$ and $\mathbf{n}_{\bar{d}}$.

The possibility to search for P-violating admixtures in the electromagnetic decay $\Sigma^0 \to \Lambda\gamma$ was advocated by Nair *et al.* [8]. Such contributions are created by making the substitution

$$\not{\epsilon}^* \not{k} \to (1 - b\gamma_5)\not{\epsilon}^* \not{k}, \tag{5.13}$$

in the decay amplitude of Eq. (5.3). This substitution is gauge invariant and changes the normalized spin density (5.9) into

$$G_\gamma(\mathbf{n}_c, \mathbf{n}_d) = 1 - \mathbf{n}_c \cdot \mathbf{l}_\gamma \mathbf{l}_\gamma \cdot \mathbf{n}_d + \rho_c[\mathbf{n}_c \cdot \mathbf{l}_\gamma - \mathbf{n}_d \cdot \mathbf{l}_\gamma], \tag{5.14}$$

with asymmetry parameter

$$\rho_c = \frac{2\Re(b)}{1 + |b|^2}. \tag{5.15}$$

Similarly, the decay width of Eq. (5.8) is changed into

$$\Gamma(c \to d\gamma) = \frac{1}{2\pi}(1 + |b|^2)\mu_{cd}^2\omega^3. \tag{5.16}$$

Parity violating admixtures in the antiparticle decay $\bar{\Sigma}^0 \to \bar{\Lambda}\gamma$ can also be simulated by the substitution of Eq. (5.13). Replacing the parameter b by $\bar{b}$, the spin density for the antiparticle decay becomes

$$G_\gamma(\mathbf{n}_{\bar{c}}, \mathbf{n}_{\bar{d}}) = 1 - \mathbf{n}_{\bar{c}} \cdot \mathbf{l}_\gamma \mathbf{l}_\gamma \cdot \mathbf{n}_{\bar{d}} - \rho_{\bar{c}}[\mathbf{n}_{\bar{c}} \cdot \mathbf{l}_\gamma - \mathbf{n}_{\bar{d}} \cdot \mathbf{l}_\gamma], \tag{5.17}$$

where

$$\rho_{\bar{c}} = \frac{2\Re(\bar{b})}{1 + |\bar{b}|^2}. \tag{5.18}$$

The P-violating interference term now enters with the opposite sign. If CP is conserved then $\bar{b} = -b$. For a full discussion of P and CP conservation in this context we refer to Ref. [8].

VI. ELECTROMAGNETIC HYPERON DECAYS: VIRTUAL PHOTONS

The leptonic decay $\Sigma^0 \to \Lambda e^+ e^-$ is a small fraction of the electromagnetic decay $\Sigma^0 \to \Lambda\gamma$ [18,19]. The lepton pair of the leptonic decay is interpreted as the decay product of a virtual, massive photon. This pair is often referred to as a Dalitz lepton pair.

The form factors $F_1(k^2)$ and $F_2(k^2)$ have been calculated in chiral perturbation theory [20,21]. The form factor $F_1(k^2)$ remains small for virtual photons and it is therefore reasonable to neglect its contribution.

The steps to follow in order to find the cross-section distribution for virtual photons are well known. The square of the reduced matrix element is written as

$$|\mathcal{M}_e(c \to de^+e^-)|^2 = \frac{1}{m_\gamma^4} H_e^{\mu\nu} L_{\mu\nu}, \tag{6.1}$$

where $H_e^{\mu\nu}$ is the hadron tensor and $L_{\mu\nu}$ the lepton tensor.

The hadron tensor can be extracted from Eq. (5.4),

$$H_e^{\mu\nu}(c \to de^+e^-) = \mu_{cd}^2 \mathrm{Tr}\Big[\frac{1}{2}(1+\gamma_5 \not{s}_d)(\not{p}_d + m_d)\sigma^{\mu\tau}k_\tau \times (\not{p}_c + m_c)\frac{1}{2}(1+\gamma_5\not{s}_c)\sigma^{\nu\lambda}k_\lambda\Big]. \tag{6.2}$$

We need the square of $\mathcal{M}_e$ for fixed baryon spins but summed over lepton spins. The summation over lepton spins leads to a lepton tensor,

$$\begin{aligned} L_{\mu\nu}(k_1,k_2) &= e^2 \sum_{l\,spin} \bar{v}(k_2)\gamma_\mu u(k_1)\bar{u}(k_1)\gamma_\nu v(k_2) \\ &= 4e^2\Big[k_\mu k_\nu - k_{1\mu}k_{1\nu} - k_{2\mu}k_{2\nu} - \frac{1}{2}g_{\mu\nu}k^2\Big], \end{aligned} \tag{6.3}$$

where k_1 and k_2 are the lepton momenta, and $k = k_1 + k_2$ the four momentum of the virtual photon.

Next, we integrate over the lepton momenta. For this purpose we rewrite the phase-space element as

$$\begin{aligned} &\mathrm{dLips}(p_c; p_d, k_1, k_2) \\ &= \frac{1}{2\pi} \mathrm{d}m_\gamma^2 \mathrm{dLips}(p_c; p_d, k)\mathrm{dLips}(k; k_1, k_2), \end{aligned} \tag{6.4}$$

with $k^2 = m_\gamma^2$ and $\mathrm{dLips}(k; k_1, k_2)$ the phase-space element for the lepton pair, as in Appendix B.

The integration over the lepton phase space affects only the lepton tensor. Thus, we note that

$$\langle k_{1\mu}k_{1\nu}\rangle = \Big[\frac{1}{3}\Big(1-\frac{m_e^2}{k^2}\Big)k_\mu k_\nu - \frac{1}{12}k^2\Big(1-\frac{4m_e^2}{k^2}\Big)g_{\mu\nu}\Big]\langle 1\rangle, \tag{6.5}$$

and similarly for $\langle k_{2\mu}k_{2\nu}\rangle$, with brackets denoting integration over lepton phase space, $\mathrm{dLips}(k; k_1, k_2)$, and $\langle 1\rangle$ denoting the phase-space volume itself. The term proportional to $k_\mu k_\nu$ in Eq. (6.5) vanishes due to gauge invariance. As a consequence, we get as average of the lepton tensor,

$$\langle L_{\mu\nu}\rangle = L(k^2)(-g_{\mu\nu}), \tag{6.6}$$

$$L(k^2) = \alpha_e k^2 \sqrt{1-\frac{4m_e^2}{k^2}}\Big[1-\frac{1}{3}\Big(1-\frac{4m_e^2}{k^2}\Big)\Big]. \tag{6.7}$$

The lepton tensor $L_{\mu\nu}$ of Eq. (6.6) comes with a factor $(-g_{\mu\nu})$. Contracting it with the hadron tensor $H_{\mu\nu}(c \to dg)$, with g representing the virtual photon, is equivalent to summing over photon polarizations. We write

$$\begin{aligned} |\mathcal{M}_e(c \to dg)|^2 &= -H_\mu^\mu(c \to dg), \\ &= R(c \to dg)G(\mathbf{n}_c, \mathbf{n}_d). \end{aligned} \tag{6.8}$$

The factorization is chosen so that $R(c \to dg)$ is spin independent, and so that the spin-independent term of $G(\mathbf{n}_c, \mathbf{n}_d)$ is unity.

The functions R and G are easily calculated. Neglecting terms unimportant for the $\Sigma^0 \to \Lambda\gamma$ transition, we get for the fractional decay rate of Eq. (6.8),

$$\begin{aligned} R(c \to dg) &= 2m_c\Gamma(c \to dg)/\Phi(c \to dg), \\ &= \mu_{cd}^2[(m_c - m_d)^2 - m_\gamma^2](m_c + m_d)^2, \end{aligned} \tag{6.9}$$

where $\Phi(c \to dg) = \Phi(m_c; m_d, m_\gamma)$ is the phase-space volume. For $m_\gamma = 0$ we recover $R(c \to d\gamma)$ for real photons, Eq. (5.7).

Again neglecting terms unimportant for the $\Sigma^0 \to \Lambda\gamma$ transition, the properly normalized spin density reads

$$G(\mathbf{n}_c, \mathbf{n}_d) = 1 - \mathbf{n}_c \cdot \mathbf{l}_\gamma \mathbf{l}_\gamma \cdot \mathbf{n}_d. \tag{6.10}$$

Thus, it is in this approximation also equal to the normalized spin density for real photons, Eq. (5.9). The exact expressions for R and G are given in Appendix D.

Next, we combine the matrix elements for the transitions $c \to dg$ and $g \to e^+e^-$, g representing a virtual photon of mass m_γ.

Since the lepton tensor of Eq. (6.7) lacks spin dependence, so that $G(g \to e^+e^-) = 1$, we have the spin-density relation

$$G(c \to de^+e^-) = G(c \to dg)G(g \to e^+e^-), \tag{6.11}$$

and a corresponding R-factor relation

$$R(c \to de^+e^-) = R(c \to dg)R(g \to e^+e^-). \tag{6.12}$$

The function $R(g\to e^+e^-)$ collects the remains, the lepton tensor of Eq. (6.7) multiplied by the propagator $1/k^4$ of Eq. (6.1),

$$R(g\to e^+e^-)=\frac{\alpha_e}{k^2}\sqrt{1-\frac{4m_e^2}{k^2}}\left[1-\frac{1}{3}\left(1-\frac{4m_e^2}{k^2}\right)\right]. \tag{6.13}$$

This expression comes with the phase-space element

$$\mathrm{dLips}=\frac{1}{2\pi}\mathrm{d}m_\gamma^2\mathrm{dLips}(p_c;p_d,k), \tag{6.14}$$

where k is the four-momentum of the virtual photon and $k^2=m_\gamma^2$. Remember that $m_\gamma^2\geq 4m_e^2$ so there is no singularity in $R(g\to e^+e^-)$.

Deviations from the Dalitz-distribution function of Eq. (6.13) signals the importance of electromagnetic form factors in the virtual photon exchange.

VII. FOLDING

Our general aim is to calculate the cross-section distributions for e^+e^- annihilation into $\Sigma^0\bar{\Sigma}^0$ pairs that subsequently decay, as $\Sigma^0\to\Lambda\to p$ or $\bar{\Sigma}^0\to\bar{\Lambda}\to\bar{p}$, and as illustrated in Fig. 2. The first step in this endeavour is to perform the folding of a product of spin densities, a technique especially adapted to spin one-half baryons.

A folding procedure implies forming an average over intermediate-spin directions $\mathbf{n}$ according to the prescription

$$\langle 1\rangle_{\mathbf{n}}=1,\qquad \langle\mathbf{n}\rangle_{\mathbf{n}}=0,\qquad \langle\mathbf{n}\cdot\mathbf{k}\mathbf{n}\cdot\mathbf{l}\rangle_{\mathbf{n}}=\mathbf{k}\cdot\mathbf{l}. \tag{7.1}$$

For more details see Ref. [11].

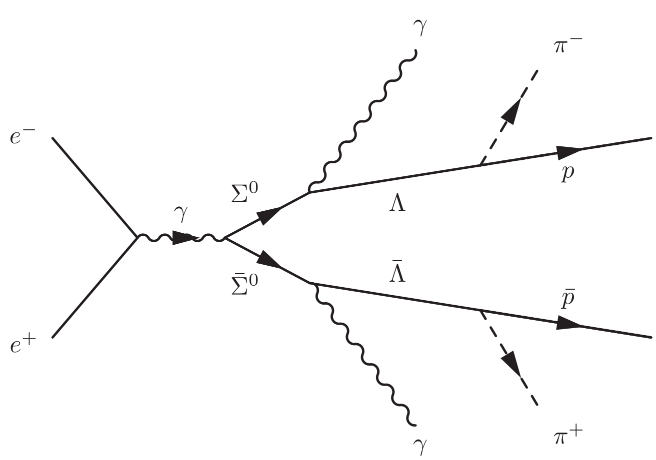

FIG. 2. Graph describing the reaction $e^+e^-\to\bar{\Sigma}^0\Sigma^0$, and the subsequent decays, $\Sigma^0\to\Lambda\gamma$; $\Lambda\to p\pi^-$ and $\bar{\Sigma}^0\to\bar{\Lambda}\gamma$; $\bar{\Lambda}\to\bar{p}\pi^+$. The reaction graphed can, in addition to photons, be mediated by vector charmonia, such as J/ψ, ψ' and $\psi(2S)$. Solid lines refer to baryons, dashed to mesons, and wavy to photons.

In the present case there are five spin densities; the annihilation spin density $S(\mathbf{n}_\Sigma,\mathbf{n}_{\bar{\Sigma}})$ of Eq. (3.8); the spin densities of the electromagnetic and weak decays, Eqs. (5.9) and (4.5),

$$G(\Sigma^0\to\Lambda\gamma)=1-\mathbf{n}_\Sigma\cdot\mathbf{l}_\gamma\mathbf{l}_\gamma\cdot\mathbf{n}_\Lambda, \tag{7.2}$$

$$G(\Lambda\to p\pi^-)=1+\alpha_\Lambda\mathbf{n}_\Lambda\cdot\mathbf{l}_p+\alpha_\Lambda\mathbf{n}_p\cdot\mathbf{l}_p + \mathbf{n}_\Lambda\cdot\mathbf{L}_\Lambda(\mathbf{n}_p,\mathbf{l}_p), \tag{7.3}$$

with $\mathbf{L}_\Lambda(\mathbf{n}_p,\mathbf{l}_p)$ defined in Eq. (4.6); and the antihyperon versions of the last two spin densities. Remember that the symbol $\mathbf{l}$ represents a unit vector.

The spin density for the $\Sigma^0\to p$ transition is obtained by folding a product of spin densities. Averaging over the Lambda and final-state proton spins, according to the folding prescription Eq. (7.1), gives us

$$\begin{aligned}G(\Sigma^0\to p)&=\langle G(\Sigma^0\to\Lambda\gamma)G(\Lambda\to p\pi^-)\rangle_{\mathbf{n}_\Lambda,\mathbf{n}_p}\\&=1-\alpha_\Lambda\mathbf{n}_\Sigma\cdot\mathbf{l}_\gamma\mathbf{l}_\gamma\cdot\mathbf{l}_p.\end{aligned} \tag{7.4}$$

We notice that this spin density does not depend on the asymmetry parameters β_Λ and γ_Λ, a consequence of the average over the final-state-proton-spin directions.

To the baryon decay chain $\Sigma^0\to\Lambda\to p$ there is a corresponding antibaryon decay chain $\bar{\Sigma}^0\to\bar{\Lambda}\to\bar{p}$, and a corresponding transition-spin density.

To go from the baryon to the antibaryon case, we simply replace the baryon variables by their antibaryon counterparts, $\mathbf{n}_\Sigma\to\mathbf{n}_{\bar{\Sigma}}$, $\alpha_\Lambda\to\alpha_{\bar{\Lambda}}$, etc.

The inclusion of parity violation in the $\Sigma^0\to\Lambda\gamma$ decay is straightforward. We simply replace $G(\Sigma^0\to\Lambda\gamma)$ of Eq. (7.2) by

$$G(\Sigma^0\to\Lambda\gamma)=1-\mathbf{n}_\Sigma\cdot\mathbf{l}_\gamma\mathbf{l}_\gamma\cdot\mathbf{n}_\Lambda+\rho_\Sigma[\mathbf{n}_\Sigma\cdot\mathbf{l}_\gamma-\mathbf{n}_\Lambda\cdot\mathbf{l}_\gamma] \tag{7.5}$$

of Eq. (5.14), and get

$$\begin{aligned}G(\Sigma^0\to p)&=\langle G(\Sigma^0\to\Lambda\gamma)G(\Lambda\to p\pi^-)\rangle_{\mathbf{n}_\Lambda,\mathbf{n}_p}\\&=(1-\rho_\Sigma\alpha_\Lambda\mathbf{l}_\gamma\cdot\mathbf{l}_p)-\mathbf{n}_\Sigma\cdot\mathbf{l}_\gamma(\alpha_\Lambda\mathbf{l}_\gamma\cdot\mathbf{l}_p-\rho_\Sigma).\end{aligned} \tag{7.6}$$

From this expression the angular distribution in the decay of a Σ^0 of polarization $\mathbf{P}_\Sigma$ is obtained by the substitution $\mathbf{n}_\Sigma\to\mathbf{P}_\Sigma$. The angular distribution in the decay of an unpolarized Σ^0 hyperon becomes $(1-\rho_\Sigma\alpha_\Lambda\mathbf{l}_\gamma\cdot\mathbf{l}_p)$. Hence, as a consequence of parity violation the cross-section distribution acquires a small angular dependent term.

VIII. SINGLE CHAIN DECAYS

Single-chain decays of Σ^0 hyperons can be studied in the e^+e^- annihilation into $\Sigma^0\bar{\Sigma}^0$ pairs, provided the $\bar{\Sigma}^0$ is somehow identified, e.g., as a missing hyperon [6]. The spin-density state of the Σ^0 will then be obtained from Eq. (3.8) as

$$\langle S(\mathbf{n}_\Sigma, \mathbf{n}_{\bar{\Sigma}})\rangle_{\mathbf{n}_{\bar{\Sigma}}} = \mathcal{R} + \mathcal{S}\mathbf{N}\cdot\mathbf{n}_\Sigma. \tag{8.1}$$

A Σ^0 hyperon in a state of polarization $\mathbf{P}_\Sigma$, subject to the condition $|\mathbf{P}_\Sigma| \leq 1$, is characterized by a normalized spin-density function,

$$S_\Sigma(\mathbf{n}_\Sigma) = 1 + \mathbf{P}_\Sigma\cdot\mathbf{n}_\Sigma. \tag{8.2}$$

Therefore, by Eq. (8.1), it follows that

$$\mathbf{P}_\Sigma = \mathcal{S}\mathbf{N}/\mathcal{R}. \tag{8.3}$$

If a Σ^0 hyperon of polarization $\mathbf{P}_\Sigma$ undergoes an electromagnetic decay, $\Sigma^0 \to \Lambda\gamma$, we can determine the spin-density distribution of the Λ hyperon by folding the initial state Σ^0 spin density of Eq. (8.2) with the Σ^0 decay distribution of Eq. (5.9), to get

$$\begin{aligned} W_\Lambda(\mathbf{n}_\Lambda; \mathbf{l}_\Lambda) &= \langle S_\Sigma(\mathbf{n}_\Sigma) G_\gamma(\mathbf{n}_\Sigma, \mathbf{n}_\Lambda)\rangle_{\mathbf{n}_\Sigma} \\ &= 1 - \mathbf{P}_\Sigma\cdot\mathbf{l}_\Lambda\mathbf{l}_\Lambda\cdot\mathbf{n}_\Lambda, \end{aligned} \tag{8.4}$$

with $\mathbf{l}_\Lambda = -\mathbf{l}_\gamma$ and a Λ polarization

$$\mathbf{P}_\Lambda = -\mathbf{P}_\Sigma\cdot\mathbf{l}_\Lambda\mathbf{l}_\Lambda. \tag{8.5}$$

Consequently, the Λ polarization is directed along the Λ momentum $\mathbf{l}_\Lambda$, a fact which is independent of the initial Σ^0 hyperon spin.

Let us now consider also the weak decay of the Λ-hyperon, $\Lambda \to p\pi^-$, which is described by the spin density $G_\Lambda(\mathbf{n}_\Lambda, \mathbf{n}_p)$ of Eq. (4.5). Since the spin of the final-state proton is usually not measured, we form the average over the proton-spin directions. Then, the spin-density-distribution function of Eq. (8.4) is expanded to

$$\begin{aligned} W_p(\mathbf{l}_\Lambda, \mathbf{l}_p) &= \langle S_\Sigma(\mathbf{n}_\Sigma) G_\gamma(\mathbf{n}_\Sigma, \mathbf{n}_\Lambda) G_p(\mathbf{n}_\Lambda, \mathbf{n}_p)\rangle_{\mathbf{n}_\Sigma, \mathbf{n}_\Lambda, \mathbf{n}_p} \\ &= 1 - \alpha_\Lambda\mathbf{P}_\Sigma\cdot\mathbf{l}_\Lambda\mathbf{l}_\Lambda\cdot\mathbf{l}_p, \\ &= 1 + \alpha_\Lambda\mathbf{P}_\Lambda\cdot\mathbf{l}_p. \end{aligned} \tag{8.6}$$

The decay chain $\Sigma^0 \to \Lambda\gamma \to p\pi^-$ makes part of our annihilation process and it is therefore of interest to investigate what additional information may be obtained by measuring the spin of the final-state proton. Thus, instead of the spin density of Eq. (7.6) we investigate the spin density

$$G(\Sigma^0 \to p) = \langle G(\Sigma^0 \to \Lambda\gamma) G(\Lambda \to p\pi^-)\rangle_{\mathbf{n}_\Lambda}. \tag{8.7}$$

Invoking the vector-function identity of Eq. (4.6) we get

$$\begin{aligned} G(\Sigma^0 \to p) = 1 + \alpha_\Lambda\mathbf{n}_p\cdot\mathbf{l}_p - \mathbf{n}_\Sigma\cdot\mathbf{l}_\gamma[\alpha_\Lambda\mathbf{l}_\gamma\cdot\mathbf{l}_p \\ + \mathbf{n}_p\cdot\mathbf{L}_\Lambda(\mathbf{l}_\gamma, -\mathbf{l}_p)]. \end{aligned} \tag{8.8}$$

Finally, the spin-density-distribution function for the final state proton is obtained as

$$\begin{aligned} S(\mathbf{n}_p) &= \langle S(\mathbf{n}_\Sigma) S(\mathbf{n}_\Sigma, \mathbf{n}_p)\rangle_{\mathbf{n}_\Lambda, \mathbf{n}_\Sigma} \\ &= U_p + \mathbf{V}_p\cdot\mathbf{n}_p, \end{aligned} \tag{8.9}$$

$$U_p = 1 - \alpha_\Lambda\mathbf{P}_\Sigma\cdot\mathbf{l}_\gamma\mathbf{l}_\gamma\cdot\mathbf{l}_p, \tag{8.10}$$

$$\mathbf{V}_p = \alpha_\Lambda\mathbf{l}_p - \mathbf{P}_\Sigma\cdot\mathbf{l}_\gamma\mathbf{L}_\Lambda(\mathbf{l}_\gamma, -\mathbf{l}_p). \tag{8.11}$$

This result describes a proton polarization which is $\mathbf{V}_p/U_p$. It is explicitly dependent on α_Λ, but there is a hidden dependence on β_Λ and γ_Λ in the vector function $\mathbf{L}_\Lambda$.

IX. PRODUCTION AND DECAY OF $\Sigma^0\bar{\Sigma}^0$ PAIRS

Now, we come to the main task of our investigation: production and decay of $\Sigma^0\bar{\Sigma}^0$ pairs. The starting point is the reaction $e^+e^- \to \Sigma^0\bar{\Sigma}^0$, the spin-density distribution of which was calculated in Sec. III. We name it $S(\mathbf{n}_\Sigma, \mathbf{n}_{\bar{\Sigma}})$. The explicit expression is given by Eq. (3.8), with $\mathbf{n}_1$, $\mathbf{n}_2$ replaced by $\mathbf{n}_\Sigma, \mathbf{n}_{\bar{\Sigma}}$.

The spin-density distribution $W_\Sigma(\mathbf{n}_\Sigma, \mathbf{n}_p)$ for the decay chain $\Sigma^0 \to \Lambda\gamma$; $\Lambda \to p\pi^-$ is given in Eq. (8.8). We write

$$W_\Sigma(\mathbf{n}_\Sigma, \mathbf{n}_p) = U_\Sigma + \mathbf{n}_\Sigma\cdot\mathbf{V}_\Sigma, \tag{9.1}$$

$$U_\Sigma = 1 + \alpha_\Lambda\mathbf{n}_p\cdot\mathbf{l}_p \tag{9.2}$$

$$\mathbf{V}_\Sigma = -\mathbf{l}_\gamma[\alpha_\Lambda\mathbf{l}_\gamma\cdot\mathbf{l}_p + \mathbf{n}_p\cdot\mathbf{L}_\Lambda(\mathbf{l}_\gamma, -\mathbf{l}_p)], \tag{9.3}$$

and ditto for $W_{\bar{\Sigma}}(\mathbf{n}_{\bar{\Sigma}}, \mathbf{n}_{\bar{p}})$. We are only interested in decay chains of Σ^0 and $\bar{\Sigma}^0$ which are each other's antichains.

The final-state-angular distributions are obtained by folding the spin distributions for production and decay, according to prescription (7.1). Invoking Eq. (3.8) for the production step and Eqs. (9.1) and its antidistribution for the decay steps, we get the angular distribution

$$\begin{aligned} W_{\Sigma\bar{\Sigma}}(\mathbf{l}_a) &= \langle S(\mathbf{n}_\Sigma, \mathbf{n}_{\bar{\Sigma}}) W_\Sigma(\mathbf{n}_\Sigma, \mathbf{n}_p) W_{\bar{\Sigma}}(\mathbf{n}_{\bar{\Sigma}}, \mathbf{n}_{\bar{p}}) \rangle_{\mathbf{n}_\Sigma, \mathbf{n}_{\bar{\Sigma}}} \\ &= \mathcal{R} U_\Sigma U_{\bar{\Sigma}} + \mathcal{S} U_{\bar{\Sigma}} \mathbf{N}\cdot\mathbf{V}_\Sigma + \mathcal{S} U_\Sigma \mathbf{N}\cdot\mathbf{V}_{\bar{\Sigma}} \\ &\quad + \mathcal{T}_1 \mathbf{V}_\Sigma\cdot\hat{\mathbf{p}}\mathbf{V}_{\bar{\Sigma}}\cdot\hat{\mathbf{p}} + \mathcal{T}_2 \mathbf{V}_{\Sigma\perp}\cdot\mathbf{V}_{\bar{\Sigma}\perp} \\ &\quad + \mathcal{T}_3 \mathbf{V}_{\Sigma\perp}\cdot\hat{\mathbf{k}}\mathbf{V}_{\bar{\Sigma}\perp}\cdot\hat{\mathbf{k}} \\ &\quad + \mathcal{T}_4(\mathbf{V}_\Sigma\cdot\hat{\mathbf{p}}\mathbf{V}_{\bar{\Sigma}\perp}\cdot\hat{\mathbf{k}} + \mathbf{V}_{\bar{\Sigma}}\cdot\hat{\mathbf{p}}\mathbf{V}_{\Sigma\perp}\cdot\hat{\mathbf{k}}), \end{aligned} \tag{9.4}$$

where $\mathbf{l}_a$ represents the ensemble of $\mathbf{l}$ values in the decays.

The angular distributions of Eq. (9.4) still depend on the spin vectors $\mathbf{n}_p$ and $\mathbf{n}_{\bar{p}}$ which are difficult to measure. If we are willing to consider proton- and antiproton-spin averages, then variables U and $\mathbf{V}$ simplify,

$$\begin{aligned} U_\Sigma &= 1, \qquad \mathbf{V}_\Sigma = -\alpha_\Lambda \mathbf{l}_\Lambda\cdot\mathbf{l}_p\mathbf{l}_\Lambda, \\ U_{\bar{\Sigma}} &= 1, \qquad \mathbf{V}_{\bar{\Sigma}} = -\alpha_{\bar{\Lambda}} \mathbf{l}_{\bar{\Lambda}}\cdot\mathbf{l}_{\bar{p}}\mathbf{l}_{\bar{\Lambda}}. \end{aligned} \tag{9.5}$$

Since $U_\Sigma = U_{\bar{\Sigma}} = 1$ the effect of the folding is to make, in the spin-density function $S(\mathbf{n}_\Sigma, \mathbf{n}_{\bar{\Sigma}})$ of Eq. (3.8), the replacements $\mathbf{n}_\Sigma \to \mathbf{V}_\Sigma$ and $\mathbf{n}_{\bar{\Sigma}} \to \mathbf{V}_{\bar{\Sigma}}$. We notice that the U and $\mathbf{V}$ variables are independent of the weak-asymmetry parameters β_Λ and γ_Λ. Their dependence is hidden in the vector function $\mathbf{L}_\Lambda(\mathbf{l}_\gamma, -\mathbf{l}_p)$ of Eq. (9.3), and which is absent in Eq. (9.4).

Inserting the expressions of Eq. (9.5) into the spin-density function of Eq. (9.4) we get

$$\begin{aligned} W_{\Sigma\bar{\Sigma}}(\mathbf{l}_a) &= \mathcal{R} - \alpha_\Lambda \mathcal{S}\mathbf{N}\cdot\mathbf{l}_\Lambda\mathbf{l}_\Lambda\cdot\mathbf{l}_p - \alpha_{\bar{\Lambda}}\mathcal{S}\mathbf{N}\cdot\mathbf{l}_{\bar{\Lambda}}\mathbf{l}_{\bar{\Lambda}}\cdot\mathbf{l}_{\bar{p}} \\ &\quad + \alpha_\Lambda\alpha_{\bar{\Lambda}}\mathbf{l}_\Lambda\cdot\mathbf{l}_p\mathbf{l}_{\bar{\Lambda}}\cdot\mathbf{l}_{\bar{p}}[\mathcal{T}_1\mathbf{l}_\Lambda\cdot\hat{\mathbf{p}}\mathbf{l}_{\bar{\Lambda}}\cdot\hat{\mathbf{p}} \\ &\quad + \mathcal{T}_2\mathbf{l}_{\Lambda\perp}\cdot\mathbf{l}_{\bar{\Lambda}\perp} + \mathcal{T}_3\mathbf{l}_{\Lambda\perp}\cdot\hat{\mathbf{k}}\mathbf{l}_{\bar{\Lambda}\perp}\cdot\hat{\mathbf{k}} \\ &\quad + \mathcal{T}_4(\mathbf{l}_\Lambda\cdot\hat{\mathbf{p}}\mathbf{l}_{\bar{\Lambda}\perp}\cdot\hat{\mathbf{k}} + \mathbf{l}_{\bar{\Lambda}}\cdot\hat{\mathbf{p}}\mathbf{l}_{\Lambda\perp}\cdot\hat{\mathbf{k}})]. \end{aligned} \tag{9.6}$$

Thus, this is the angular distribution obtained when folding the product of spin densities for production and decay.

X. DIFFERENTIAL DISTRIBUTIONS

Explicit expressions for the structure functions $\mathcal{R}$, $\mathcal{S}$, and $\mathcal{T}$ are given in Appendix C. With their help we can rewrite the differential distribution function of Eq. (9.6) as

$$\begin{aligned} \mathcal{W}(\boldsymbol{\xi}) &= [\mathcal{F}_0 + \eta\mathcal{F}_1] \\ &\quad - \sqrt{1-\eta^2}\sin(\Delta\Phi)\sin\theta\cos\theta \\ &\quad \times [\alpha_\Lambda\mathcal{F}_2\mathcal{F}_5 + \alpha_{\bar{\Lambda}}\mathcal{F}_3\mathcal{F}_6] \\ &\quad + \alpha_\Lambda\alpha_{\bar{\Lambda}}\mathcal{F}_2\mathcal{F}_3[(\eta+\cos^2\theta)\mathcal{F}_4 - \eta\sin^2\theta\mathcal{F}_7 \\ &\quad + (1+\eta)\sin^2\theta\mathcal{F}_8 \\ &\quad + \sqrt{1-\eta^2}\cos(\Delta\Phi)\sin\theta\cos\theta\mathcal{F}_9], \end{aligned} \tag{10.1}$$

where the argument $\boldsymbol{\xi}$ of the angular functions is a nine-dimensional vector $\boldsymbol{\xi} = (\theta, \Omega_\Lambda, \Omega_p, \Omega_{\bar{\Lambda}}, \Omega_{\bar{p}})$.

The ten angular functions $\mathcal{F}_k(\boldsymbol{\xi})$ are defined as

$$\begin{aligned} \mathcal{F}_0(\boldsymbol{\xi}) &= 1, \\ \mathcal{F}_1(\boldsymbol{\xi}) &= \cos^2\theta, \\ \mathcal{F}_2(\boldsymbol{\xi}) &= \mathbf{l}_\Lambda\cdot\mathbf{l}_p, \\ \mathcal{F}_3(\boldsymbol{\xi}) &= \mathbf{l}_{\bar{\Lambda}}\cdot\mathbf{l}_{\bar{p}}, \\ \mathcal{F}_4(\boldsymbol{\xi}) &= \mathbf{l}_\Lambda\cdot\hat{\mathbf{p}}\mathbf{l}_{\bar{\Lambda}}\cdot\hat{\mathbf{p}}, \\ \mathcal{F}_5(\boldsymbol{\xi}) &= \mathbf{N}\cdot\mathbf{l}_\Lambda, \\ \mathcal{F}_6(\boldsymbol{\xi}) &= \mathbf{N}\cdot\mathbf{l}_{\bar{\Lambda}}, \\ \mathcal{F}_7(\boldsymbol{\xi}) &= \mathbf{l}_{\Lambda\perp}\cdot\mathbf{l}_{\bar{\Lambda}\perp}, \\ \mathcal{F}_8(\boldsymbol{\xi}) &= \mathbf{l}_{\Lambda\perp}\cdot\hat{\mathbf{k}}\mathbf{l}_{\bar{\Lambda}\perp}\cdot\hat{\mathbf{k}}/\sin^2\theta, \\ \mathcal{F}_9(\boldsymbol{\xi}) &= (\mathbf{l}_\Lambda\cdot\hat{\mathbf{p}}\mathbf{l}_{\bar{\Lambda}\perp}\cdot\hat{\mathbf{k}} + \mathbf{l}_{\bar{\Lambda}}\cdot\hat{\mathbf{p}}\mathbf{l}_{\Lambda\perp}\cdot\hat{\mathbf{k}})/\sin\theta. \end{aligned} \tag{10.2}$$

The cross-section distribution (9.6), and also the ten angular functions above, depend on a number of unit vectors; $\hat{\mathbf{p}}$ and $-\hat{\mathbf{p}}$ are unit vectors along the directions of motion of the Σ^0 and the $\bar{\Sigma}^0$ in the c.m. system; $\hat{\mathbf{k}}$ and $-\hat{\mathbf{k}}$ are unit vectors along the directions of motion of the incident electron and positron in the c.m. system; $\mathbf{l}_\Lambda$ and $\mathbf{l}_{\bar{\Lambda}}$ are unit vectors along the directions of motion of the Λ and $\bar{\Lambda}$ in the rest systems of the Σ^0 and the $\bar{\Sigma}^0$; $\mathbf{l}_p$ and $\mathbf{l}_{\bar{p}}$ are unit vectors along the directions of motion of the p and the $\bar{p}$ in the rest systems of the Λ and the $\bar{\Lambda}$. Longitudinal and transverse components of vectors are defined with respect to the $\hat{\mathbf{p}}$ direction.

The differential distribution function $\mathcal{W}(\boldsymbol{\xi})$ of Eq. (10.1) involves two parameters related to the $e^+e^- \to \Sigma^0\bar{\Sigma}^0$ reaction that can be determined by data: the ratio of form factors η, and the relative phase of form factors $\Delta\Phi$. In addition, the distribution function $\mathcal{W}(\boldsymbol{\xi})$ depends on the weak-asymmetry parameters α_Λ and $\alpha_{\bar{\Lambda}}$ of the two Lambda-hyperon decays. The dependence on the weak-asymmetry parameters β and γ drops out, since final-state-proton and antiproton spins are not measured.

An important conclusion to be drawn from the differential distribution of Eq. (10.1) is that when the phase $\Delta\Phi$ is small, the parameters α_Λ and $\alpha_{\bar{\Lambda}}$ are strongly correlated and therefore difficult to separate. In order to contribute to the experimental precision of α_Λ and $\alpha_{\bar{\Lambda}}$ a nonzero value of $\Delta\Phi$ is required.

The sequential differential decay distribution of a single-tagged Σ^0 produced in e^+e^- annihilation can be obtained form Eq. (10.1) by suitably integrating over the angular variables $\Omega_{\bar{\Lambda}}$ and $\Omega_{\bar{p}}$. As a result we get the differential distribution for Σ^0 production and decay,

$$\begin{aligned} d\sigma &\propto [\mathcal{R} - \alpha_\Lambda\mathcal{S}\mathbf{N}\cdot\mathbf{l}_\Lambda\mathbf{l}_\Lambda\cdot\mathbf{l}_p]d\Omega d\Omega_\Lambda d\Omega_p \\ &= [1 + \eta\cos^2\theta - \alpha_\Lambda\sqrt{1-\eta^2}\sin(\Delta\Phi)\sin\theta\cos\theta \\ &\quad \times \cos\theta_{\Lambda p}\sin\theta_\Lambda\sin\phi_\Lambda]d\Omega d\Omega_\Lambda d\Omega_p. \end{aligned} \tag{10.3}$$

Here, θ is the Σ^0 production angle, $\theta_{\Lambda p}$ the relative angle between the vectors $\mathbf{l}_\Lambda$ and $\mathbf{l}_p$, and θ_Λ and ϕ_Λ the directional angles of $\mathbf{l}_\Lambda$ in the global coordinate system of Appendix E. From the angular distribution of Eq. (10.3) we can determine the product $\alpha_\Lambda \sin(\Delta\Phi)$, and from the corresponding $\bar{\Sigma}^0$ distribution the product $\alpha_{\bar{\Lambda}} \sin(\Delta\Phi)$.

In this application the final-state-proton spin can be included in a formula of finite length. From Eq. (9.4) we get

$$\begin{aligned} \mathrm{d}\sigma \propto [&\mathcal{R} - \alpha_\Lambda \mathcal{S}\mathbf{N}\cdot\mathbf{l}_\Lambda \mathbf{l}_\Lambda\cdot\mathbf{l}_p + \alpha_\Lambda \mathcal{R}\mathbf{n}_p\cdot\mathbf{l}_p \\ &+ \mathcal{S}\mathbf{N}\cdot\mathbf{l}_\Lambda \mathbf{n}_p\cdot\mathbf{L}_\Lambda(-\mathbf{l}_\Lambda, -\mathbf{l}_p)]\mathrm{d}\Omega\mathrm{d}\Omega_\Lambda\mathrm{d}\Omega_p, \end{aligned} \quad (10.4)$$

with $\mathbf{n}_p$ the final-state-proton-spin vector, and the function $\mathbf{L}_\Lambda(-\mathbf{l}_\Lambda, -\mathbf{l}_p)$ defined in Eq. (4.6), and dependent on the weak interaction parameters β_Λ and γ_Λ.

Important information can be retrieved from Eq. (10.3). Denoting its right-hand side $\mathcal{W}_\mathbf{\Sigma}$, and forming the average over the final-state-phase space, we get

$$\langle \mathcal{W}_\mathbf{\Sigma} \rangle = \langle 1 + \eta\cos^2\theta \rangle = 1 + \frac{1}{3}\eta. \quad (10.5)$$

The correlation between the scattering angle θ and the angle θ_{Np}, with $\cos\theta_{Np} = \mathbf{N}\cdot\mathbf{l}_p$, can also be determined, and

$$\langle \cos\theta \cos\theta_{Np} \mathcal{W}_\mathbf{\Sigma} \rangle = -\frac{\pi}{144}\alpha_\Lambda \sqrt{1-\eta^2}\sin(\Delta\Phi). \quad (10.6)$$

Thus, knowledge of the weak interaction parameter α_Λ, and the ratio of form factors η, allows us to determine the relative phase Φ between form factors, by considering the ratio of expressions (10.6) and (10.5). Since the absolute value of cross sections are usually unknown it is essential to consider cross-section ratios for information.

XI. CROSS-SECTION DISTRIBUTIONS

We shall now consider the phase-space imbedding of the differential-distribution function of Eq. (10.1). We start with the cross-section-distribution function for creation of a pair of baryons, $e^+e^- \to \Sigma^0\bar{\Sigma}^0$. Combining Eqs. (3.1), (3.6), and (3.7), we get

$$\mathrm{d}\sigma(e^+e^- \to \gamma^\star \to \Sigma^0\bar{\Sigma}^0) = \frac{p}{k}\frac{\alpha_e^2 D(s)}{4s^2} S(\mathbf{n}_{\Sigma^0}, \mathbf{n}_{\bar{\Sigma}^0})\mathrm{d}\Omega, \quad (11.1)$$

where Ω are the baryon scattering angles in the c.m. system.

Next we consider the propagator factors associated with the sequential decays of the baryons Σ^0 and $\bar{\Sigma}^0$ produced in the e^+e^- annihilation process. These sequential decays are illustrated in Fig. 2. There are three factors associated with the square of each propagator. Let us consider the decay $c \to dg$, where g can represent a pion or a photon. Other decay modes are also possible to incorporate. Then, we have

$$\mathcal{P}_c = \left[\frac{\pi}{m_c\Gamma_c}\delta(s_c - m_c^2)\right]\left[\frac{\mathrm{d}s_c}{2\pi}\mathrm{dLips}(p_c; p_d, p_g)\right][R_c G_c]. \quad (11.2)$$

Here, the first factor comes from squaring the propagator in the Feynman diagram; the second factor from dividing the phase-space element into a product of two-body phase-space elements; and the third factor is the reduced matrix element squared for the decay $c \to dg$, and the product of the normalized spin density G_c and the fractional decay rate R_c.

The fractional decay rate R_c is defined in Eq. (5.7) as

$$R(c \to dg) = 2m_c\Gamma(c \to dg)/\Phi(c \to dg), \quad (11.3)$$

where Φ is the two-body phase-space volume, and $\Gamma(c \to dg)$ the channel width for the decay $c \to dg$. It was defined to be spin averaged for both initial and final baryon states. However, in a sequential decay both final spin-state contributions must be included. This is achieved by multiplying $R(c \to d\gamma)$ by a factor of 2. This factor can be incorporated in the channel width $\Gamma(c \to dg)$, reinterpreting it to include the sum over final baryon spin states. Finally, we observe that

$$\mathrm{dLips}(p_c; p_d, p_g) = \Phi_c(c \to dg)\frac{\mathrm{d}\Omega_c}{4\pi}, \quad (11.4)$$

giving as a consequence a $\mathcal{P}$ factor

$$\mathcal{P}_c = G_c \frac{\Gamma(c \to dg)}{\Gamma(c \to \mathrm{all})}\frac{\mathrm{d}\Omega_c}{4\pi}, \quad (11.5)$$

with Ω_c the angular variable in the rest system of baryon c. In our application index c represents one of the four mother hyperons Σ^0, Λ and $\bar{\Sigma}^0$, $\bar{\Lambda}$. Similarly, index d represents one of the four daughter hyperons Λ, p and $\bar{\Lambda}$, $\bar{p}$.

The differential-distribution function $\mathcal{W}(\boldsymbol{\xi})$ of Eq. (10.1) is obtained by *folding* a product of five spin densities

$$\mathcal{W}(\boldsymbol{\xi}) = \left\langle S(\mathbf{n}_{\Sigma^0}, \mathbf{n}_{\bar{\Sigma}^0})\prod_c G_c(\mathbf{n}_c, \mathbf{n}_d)\right\rangle_\mathbf{n}. \quad (11.6)$$

Folding involves averages over spin directions, but as remarked, cross-section distributions require summing over the spin directions. Thus, an average over the spin density $S(\mathbf{n}_{\Sigma^0}, \mathbf{n}_{\bar{\Sigma}^0})$ is accompanied by an extra factor of 4, and it is not normalized to unity either but to $\mathcal{R}$.

The folding formula Eq. (11.6) combined with Eqs. (11.1) and (3.11) gives the *master equation*

$$\mathrm{d}\sigma = \mathrm{d}\sigma(e^+e^- \to \gamma^\star \to \Sigma^0\bar{\Sigma}^0) \times \left[\frac{\mathcal{W}(\boldsymbol{\xi})}{\mathcal{R}}\right]\prod_c \left[\frac{\Gamma(c \to dg)}{\Gamma(c \to \mathrm{all})}\frac{\mathrm{d}\Omega_c}{4\pi}\right]. \tag{11.7}$$

This readily understood structure agrees with that reported in Ref. [11].

Since spin densities are normalized, except for the annihilation density $S(\mathbf{n}_{\Sigma^0}, \mathbf{n}_{\bar{\Sigma}^0})$, the overall normalization condition reads

$$\int \mathcal{W}(\boldsymbol{\xi})\prod_c \frac{\mathrm{d}\Omega_c}{4\pi} = \mathcal{R}. \tag{11.8}$$

This normalization is checked explicitly in single-chain-sequential decay in Ref. [6].

When the $\Sigma^0 \to \Lambda\gamma$ reaction is involved, and the photon is real, then the channel width in Eq. (11.7) is for all practical purposes equal to the total width, $\Gamma(\Sigma^0 \to \Lambda\gamma) = \Gamma(\Sigma^0 \to \mathrm{all})$.

We also point out that for virtual photons the cross-section distribution of Eq. (11.7) receives an additional lepton factor,

$$\frac{1}{2\pi}\mathrm{d}m_\gamma^2 R(g \to e^+e^-), \tag{11.9}$$

where m_γ is the virtual photon mass, $m_\gamma^2 = k^2$, and R the Dalitz function

$$R(g \to e^+e^-) = \frac{\alpha_e}{k^2}\sqrt{1 - \frac{4m_e^2}{k^2}}\left[1 - \frac{1}{3}\left(1 - \frac{4m_e^2}{k^2}\right)\right]. \tag{11.10}$$

The e^+e^- annihilation reactions described above are all concerned with annihilation through ordinary photons, as illustrated in Fig. 1. However, the same reactions can be initiated by other vector mesons as well. Of special interest is the J/ψ case, which is treated in Ref. [5], and which is accessible to the BESIII experiment. By making the replacement

$$\frac{\alpha_e^2}{s^2} \to \frac{\alpha_\psi \alpha_g}{(s - m_\psi^2)^2 + m_\psi^2\Gamma(m_\psi)} \tag{11.11}$$

in the photon-induced reaction, Eq. (11.1), we get the cross-section-distribution formula for annihilation through the J/ψ meson. The meaning of the parameters α_ψ and α_g is explained in Ref. [5]. This replacement is equivalent to replacing in the master formula of Eq. (11.7) the corresponding photon-induced $e^+e^- \to \Sigma^0\bar{\Sigma}^0$ annihilation cross section by the J/ψ induced cross section,

$$\mathrm{d}\sigma(e^+e^- \to \gamma^\star \to \Sigma^0\bar{\Sigma}^0) \to \mathrm{d}\sigma(e^+e^- \to J/\psi \to \Sigma^0\bar{\Sigma}^0). \tag{11.12}$$

XII. SUMMARY

In two previous publications we analyzed the hyperon decay $\Lambda \to p\pi^-$ and its corresponding antihyperon decay, $\bar{\Lambda} \to \bar{p}\pi^+$, for hyperons produced in the reaction $e^+e^- \to \Lambda\bar{\Lambda}$. Annihilation via one-photon states $e^+e^- \to \gamma^* \to \Lambda\bar{\Lambda}$ was analyzed in Ref. [4], and annihilation via vector-charmonium states $e^+e^- \to J/\psi, \psi', \psi(2S) \to \Lambda\bar{\Lambda}$ in Ref. [5].

In the present investigation we analyze the sequential hyperon decay $\Sigma^0 \to \Lambda\gamma$; $\Lambda \to p\pi^-$, and its corresponding sequential antihyperon decay, again when simultaneously taking place in the reaction $e^+e^- \to \Sigma^0\bar{\Sigma}^0$. The structure of the cross-section distribution for annihilation, whether via one-photon states or vector-charmonium states, is the same.

The aim of the present investigation was, among other things, to discuss how to relate measured observables to observables used in theoretical analyses. In studies of reactions like $e^+e^- \to \Sigma^0\bar{\Sigma}^0(\Lambda\bar{\Lambda})$ it is customary to use different coordinate systems for observables referring to the $\Sigma^0(\Lambda)$ and $\bar{\Sigma}^0(\bar{\Lambda})$ hyperons, whereas we prefer the use of a single-common-global-coordinate system.

The cross-section distribution for production and subsequent decay of a $\Sigma^0\bar{\Sigma}^0$ pair is described by an easy to understand *master formula*,

$$\mathrm{d}\sigma = \mathrm{d}\sigma(e^+e^- \to \Sigma^0\bar{\Sigma}^0)\left[\frac{\mathcal{W}(\boldsymbol{\xi})}{\mathcal{R}}\right]\mathrm{d}\Phi(\Sigma^0, \Lambda, p; \bar{\Sigma}^0, \bar{\Lambda}, \bar{p}), \tag{12.1}$$

already encountered in Ref. [4]. The master formula is a product of three factors, describing the *annihilation* of lepton pairs into hyperon pairs, the *folded product* of spin densities representing hyperon production and decay, and the *phase space* of sequential hyperon decays. Each event is specified by a nine-dimensional vector $\boldsymbol{\xi} = (\theta, \Omega_\Lambda, \Omega_p, \Omega_{\bar{\Lambda}}, \Omega_{\bar{p}})$, with θ the scattering angle in the $e^+e^- \to \Sigma^0\bar{\Sigma}^0$ subprocess.

According to Eq. (3.11), the cross-section distribution for the reaction $e^+e^- \to \Sigma^0\bar{\Sigma}^0$ can in the one-photon approximation be written as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_{\Sigma^0}}(e^+e^- \to \gamma^\star \to \Sigma^0\bar{\Sigma}^0) = \frac{p}{k}\frac{\alpha_e^2 D(s)}{s^2}\mathcal{R}, \tag{12.2}$$

where α_e is the fine-structure constant, and $\mathcal{R}$ a function defined by the equation

$$\mathcal{R} = 1 + \eta(s)\cos^2\theta. \tag{12.3}$$

Two complex form factors, $G_M(s)$ and $G_E(s)$, are needed for a unique characterization of the $\Sigma^0\bar{\Sigma}^0\gamma$-electromagnetic-vertex function, but it is more convenient to work with the three real combinations thereof, $D(s)$, $\eta(s)$, and $\Delta\Phi(s)$ defined in Sec. II.

The reaction $e^+e^- \to \Sigma^0\bar{\Sigma}^0$ can also be initiated by a vector-charmonium state, such as the J/ψ. Since the photon and the J/ψ are both vector mesons the structures of the corresponding cross-section distributions will be similar. In fact we obtain the J/ψ cross-section distribution by making the replacement

$$\frac{\alpha_e^2}{s^2} \to \frac{\alpha_\psi \alpha_g}{(s - m_\psi^2)^2 + m_\psi^2 \Gamma(m_\psi)} \tag{12.4}$$

in the photon-induced cross-section distribution. The constant α_ψ is determined by the electromagnetic-decay width $\Gamma(J/\psi \to e^+e^-)$, and the constant α_g similarly by the hadronic-decay width $\Gamma(J/\psi \to \Sigma^0\bar{\Sigma}^0)$. The redefined real form-factor functions are now denoted $D^\psi(s)$, $\eta^\psi(s)$, and $\Delta\Phi^\psi(s)$.

The differential-spin-distribution function $\mathcal{W}(\boldsymbol{\xi})$ of Eq. (12.1) is obtained by *folding* a product of five spin densities,

$$\mathcal{W}(\boldsymbol{\xi}) = \langle S(\mathbf{n}_{\Sigma^0}, \mathbf{n}_{\bar{\Sigma}^0}) G(\mathbf{n}_{\Sigma^0}, \mathbf{n}_\Lambda) G(\mathbf{n}_\Lambda, \mathbf{n}_p) \times G(\mathbf{n}_{\bar{\Sigma}^0}, \mathbf{n}_{\bar{\Lambda}}) G(\mathbf{n}_{\bar{\Lambda}}, \mathbf{n}_{\bar{p}}) \rangle_{\mathbf{n}}, \tag{12.5}$$

in accordance with the prescription of Eq. (7.1). The folding operation $\langle \ldots \rangle_{\mathbf{n}}$ applies to each of the six hadron spin vectors, $\mathbf{n}_{\Sigma^0}, \ldots, \mathbf{n}_{\bar{p}}$.

The function $S(\mathbf{n}_{\Sigma^0}, \mathbf{l}_{\Sigma^0}; \mathbf{n}_{\bar{\Sigma}^0}, \mathbf{l}_{\bar{\Sigma}^0})$ represents the spin-density distribution for the hyperon pair produced in the $e^+e^- \to \Sigma^0\bar{\Sigma}^0$ reaction, and $\mathbf{l}_{\Sigma^0}$ and $\mathbf{l}_{\bar{\Sigma}^0}$ are unit vectors in their directions of motion in the global c.m. system. The four remaining spin-density distributions $G(\mathbf{n}_\Lambda, \mathbf{n}_p)$ etc., represent spin-density distributions for the hyperon decays $\Sigma^0 \to \Lambda\gamma$, $\Lambda \to p\pi^-$, or their antihyperon counterparts. The simplest spin-density-decay distribution of the four is that for the decay $\Sigma^0 \to \Lambda\gamma$,

$$G(\mathbf{n}_{\Sigma^0}; \mathbf{n}_\Lambda, \mathbf{l}_\Lambda) = 1 - \mathbf{n}_{\Sigma^0} \cdot \mathbf{l}_\Lambda \mathbf{l}_\Lambda \cdot \mathbf{n}_\Lambda. \tag{12.6}$$

The decay-distribution functions $G(\mathbf{n}_{Y_1}, \mathbf{n}_{Y_2})$ are normalized to unity, i.e., the spin independent terms are unity, but the density-distribution function $S(\mathbf{n}_{\Sigma^0}, \mathbf{n}_{\bar{\Sigma}^0})$ is normalized to $\mathcal{R}$.

The phase-space factor, $\mathrm{d}\Phi(\Sigma^0, \Lambda, p; \bar{\Sigma}^0, \bar{\Lambda}, \bar{p})$ of the master equation, describes the normalized phase-space element for the sequential decays of the two baryons Σ^0 and $\bar{\Sigma}^0$,

$$\begin{aligned} &\mathrm{d}\Phi(\Sigma^0, \Lambda, p; \bar{\Sigma}^0, \bar{\Lambda}, \bar{p}) \\ &= \frac{\Gamma(\Sigma^0 \to \Lambda\gamma)}{\Gamma(\Sigma^0 \to \text{all})} \frac{\mathrm{d}\Omega_\Lambda}{4\pi} \cdot \frac{\Gamma(\Lambda \to p\pi^-)}{\Gamma(\Lambda \to \text{all})} \frac{\mathrm{d}\Omega_p}{4\pi} \\ &\quad \cdot \frac{\Gamma(\bar{\Sigma}^0 \to \bar{\Lambda}\gamma)}{\Gamma(\bar{\Sigma}^0 \to \text{all})} \frac{\mathrm{d}\Omega_{\bar{\Lambda}}}{4\pi} \cdot \frac{\Gamma(\bar{\Lambda} \to \bar{p}\pi^-)}{\Gamma(\bar{\Lambda} \to \text{all})} \frac{\mathrm{d}\Omega_{\bar{p}}}{4\pi}. \end{aligned} \tag{12.7}$$

The widths are defined in the usual way. For $\Gamma(\Sigma^0 \to \Lambda\gamma)$ this means forming an average over the Σ^0 spin direction, and summing over the Lambda and gamma spin directions. The angles Ω_Λ define the direction of motion of the Λ hyperon in the Σ^0 rest system. And so on.

In addition to the reactions mentioned above we have also calculated the cross-section distributions obtained when one of the final-state photons is materialized as a Dalitz e^+e^- pair. We have also investigated how parity violating contributions affect the $\Sigma^0 \to \Lambda\gamma$ amplitude.

ACKNOWLEDGMENTS

K. S is supported by the Knut and Alice Wallenberg Foundation (Sweden) under Contract No. 2016.0157. We have greatly benefitted from discussions with Patrik Adlarson and Stefan Leupold.

APPENDIX A: GRAPH CALCULATION

In this appendix we shall work out the phase-space density for the two-step case. Our notation follows Pilkuhn [12]. The cross-section distribution can be written as

$$\mathrm{d}\sigma = \frac{1}{2\sqrt{\lambda(s, m_e^2, m_e^2)}} \overline{|\mathcal{M}|^2} \mathrm{dLips}(k_1 + k_2; \{\mathbf{l}_i\}, \{\mathbf{l}_{i'}\}), \tag{A1}$$

where $\{\mathbf{l}_i\}$ are the final-state momenta in the hyperon decay chain and $\{\mathbf{l}_{i'}\}$ the final-state momenta in the antihyperon decay chain. The average over the squared matrix element indicates summation over final-state spins and average over initial-state lepton. The definitions of the particle momenta are explained in Fig. 1.

Since $\Gamma \ll M$ for the intermediate propagators, their squares may be approximated as

$$\frac{1}{(s - M^2)^2 + M^2\Gamma^2(\sqrt{s})} = \frac{\pi}{M\Gamma(M)} \delta(s - M^2). \tag{A2}$$

This makes it convenient to pull out a factor $\mathcal{K}$ from the squared matrix element,

$$\mathcal{K} = \prod_i \frac{1}{(s_i - M_i^2)^2 + M_i^2 \Gamma_i^2(M_i)}, \tag{A3}$$

and plug it into the phase-space density. In Eq. (A3) the product runs over the four intermediate-state hyperons.

After some manipulations we can write the modified phase-space density as

$$\mathcal{K}\mathrm{dLips}(k_1 + k_2; \{\mathbf{l}_i\}, \{\mathbf{l}_{i'}\}) = \left[\frac{p}{(4\pi)^2\sqrt{s}} \mathrm{d}\Omega\right]_{\mathrm{CM}} \times \prod_i \left[\frac{q_i}{8\pi M_i^2 \Gamma_i(M_i)} \mathrm{d}\Omega_i\right]_{\mathrm{Y}}, \tag{A4}$$

where index CM refers to the two-body reaction $e^+e^- \to Y\bar{Y}$, and index Y to each of the four intermediate-state hyperon decays, in their respective hyperon rest systems.

APPENDIX B: PHASE-SPACE VOLUME

The Lorentz invariant two-body phase-space element is by definition

$$\mathrm{dLips}(k;k_1,k_2) = \frac{\mathrm{d}^3k_1}{(2\pi)^3 2\omega_1}\frac{\mathrm{d}^3k_2}{(2\pi)^3 2\omega_2}(2\pi)^4\delta(k-k_1-k_2). \tag{B1}$$

Integration exploiting the delta functions leads to

$$\int \mathrm{dLips}(k;k_1,k_2) = \frac{k_c}{4\pi\sqrt{s}}\frac{\mathrm{d}\Omega_c}{4\pi} \tag{B2}$$

where $\sqrt{s}=M$, k_c the momentum, and Ω_c the angular variable, both in the c.m. system. In terms of the mass variables

$$k_c^2 = \frac{1}{4M^2}[(M^2+m_1^2-m_2^2)^2 - 4M^2m_1^2]. \tag{B3}$$

The phase-space volume Φ is obtained from Eq. (B2) by integration over $\mathrm{d}\Omega_c$,

$$\Phi(M;m_1,m_2) = \frac{k_c}{4\pi\sqrt{s}}. \tag{B4}$$

For equal masses $m_1=m_2=m$ the value of the phase-space volume becomes

$$\Phi(M;m,m) \equiv \langle 1\rangle = \frac{1}{8\pi}\sqrt{1-\frac{4m^2}{M^2}}. \tag{B5}$$

APPENDIX C: STRUCTURE FUNCTIONS

The six structure functions $\mathcal{R}$, $\mathcal{S}$, and $\mathcal{T}$ of Eq. (3.8) depend on the scattering angle θ, in the c.m. system, the ratio function $\eta(s)$, and the phase function $\Delta\Phi(s)$. To be specific [4,5],

$$\mathcal{R} = 1+\eta\cos^2\theta, \tag{C1}$$

$$\mathcal{S} = \sqrt{1-\eta^2}\sin\theta\cos\theta\sin(\Delta\Phi), \tag{C2}$$

$$\mathcal{T}_1 = \eta+\cos^2\theta, \tag{C3}$$

$$\mathcal{T}_2 = -\eta\sin^2\theta, \tag{C4}$$

$$\mathcal{T}_3 = 1+\eta, \tag{C5}$$

$$\mathcal{T}_4 = \sqrt{1-\eta^2}\cos\theta\cos(\Delta\Phi). \tag{C6}$$

The parameters η and $\Delta\Phi$ are defined in Eqs. (2.2) and (2.3).

APPENDIX D: DECAY INTO VIRTUAL GAMMA

The squared matrix element $|\mathcal{M}(c\to dg)|^2$ for the decay of a baryon c into a baryon d and a virtual gamma g of mass m_γ is given in Eq. (6.8). It can be factorized into factors $R(c\to dg)$ and $G_g(\mathbf{n}_c,\mathbf{n}_d)$. The exact expression for the fractional width is

$$R(c\to dg) = \mu_{cd}^2[(m_c-m_d)^2-m_\gamma^2]\left[(m_c+m_d)^2+\frac{1}{2}m_\gamma^2\right], \tag{D1}$$

with $2m_e \le m_\gamma \le (m_c-m_d)$. In the limit $m_\gamma=0$ we recover $R(c\to d\gamma)$ for real photons, Eq. (5.7). The exact expression for the normalized spin density is

$$G_g(\mathbf{n}_c,\mathbf{n}_d) = 1 + B\mathbf{n}_c\cdot\mathbf{l}_\gamma\mathbf{l}_\gamma\cdot\mathbf{n}_d + C\mathbf{n}_c\cdot\mathbf{n}_d, \tag{D2}$$

$$\begin{aligned} A &= (m_c+m_d)^2+\frac{1}{2}m_\gamma^2,\\ B &= -(m_c+m_d)^2/A,\\ C &= \frac{1}{2}m_\gamma^2/A. \end{aligned} \tag{D3}$$

Here, we can without qualm put $B=-1$ and $C=0$. In this limit we recover the normalized spin density for real photons, Eq. (5.9).

APPENDIX E: ANGULAR FUNCTIONS

The cross-section distribution (9.6) is a function of two hyperon unit vectors: $\mathbf{l}_\Lambda$, the direction of motion of the Lambda hyperon in the rest system of the Sigma hyperon, and $\mathbf{l}_p$ the direction of motion of the proton in the rest system of the Lambda hyperon. Plus the corresponding vectors for the antihyperon chain. In order to handle these vectors we introduce a common global coordinate system, which we define as follows.

The scattering plane of the reaction $e^+e^-\to\Sigma^0\bar{\Sigma}^0$ is spanned by the unit vectors $\hat{\mathbf{p}}=\mathbf{l}_\Sigma$ and $\hat{\mathbf{k}}=\mathbf{l}_e$, as measured in the c.m. system. The scattering plane makes up the xz-plane, with the y-axis along the normal to the scattering plane. We choose a right-handed coordinate system with basis vectors

$$\begin{aligned} \mathbf{e}_z &= \hat{\mathbf{p}},\\ \mathbf{e}_y &= \frac{1}{\sin\theta}(\hat{\mathbf{p}}\times\hat{\mathbf{k}}),\\ \mathbf{e}_x &= \frac{1}{\sin\theta}(\hat{\mathbf{p}}\times\hat{\mathbf{k}})\times\hat{\mathbf{p}}. \end{aligned} \tag{E1}$$

Expressed in terms of them the initial-state lepton momentum becomes

$$\hat{\mathbf{k}} = \sin\theta \mathbf{e}_x + \cos\theta \mathbf{e}_z. \tag{E2}$$

This coordinate system is used for defining the directional angles of the Lambda and the proton. The directional angles of the Lambda hyperon in the Sigma hyperon rest system are

$$\mathbf{l}_\Lambda = (\cos\phi_\Lambda \sin\theta_\Lambda, \sin\phi_\Lambda \sin\theta_\Lambda, \cos\theta_\Lambda), \tag{E3}$$

whereas the directional angles of the proton in the Lambda hyperon rest system are

$$\mathbf{l}_p = (\cos\phi_p \sin\theta_p, \sin\phi_p \sin\theta_p, \cos\theta_p). \tag{E4}$$

And so for the antihyperons.

An event of the reaction $e^+e^- \to \Sigma^0\bar{\Sigma}^0$, $\Sigma^0 \to \Lambda \to p$; $\bar{\Sigma}^0 \to \bar{\Lambda} \to \bar{p}$ is specified by a nine-dimensional vector $\boldsymbol{\xi} = (\theta, \Omega_\Lambda, \Omega_p, \Omega_{\bar{\Lambda}}, \Omega_{\bar{p}})$. The differential-cross-section distribution is proportional to a function $\mathcal{W}(\boldsymbol{\xi})$, which according to Eq. (10.1) can be decomposed as

$$\begin{aligned}
\mathcal{W}(\boldsymbol{\xi}) = {} & [\mathcal{F}_0(\boldsymbol{\xi}) + \eta\mathcal{F}_1(\boldsymbol{\xi})] \\
& - \sqrt{1-\eta^2}\sin(\Delta\Phi)\sin\theta\cos\theta[\alpha_\Lambda \mathcal{F}_2(\boldsymbol{\xi})\mathcal{F}_5(\boldsymbol{\xi}) \\
& + \alpha_{\bar{\Lambda}}\mathcal{F}_3(\boldsymbol{\xi})\mathcal{F}_6(\boldsymbol{\xi})] \\
& + \alpha_\Lambda\alpha_{\bar{\Lambda}}\mathcal{F}_2(\boldsymbol{\xi})\mathcal{F}_3(\boldsymbol{\xi})[(\eta + \cos^2\theta)\mathcal{F}_4(\boldsymbol{\xi}) \\
& - \eta\sin^2\theta\mathcal{F}_7(\boldsymbol{\xi}) + (1+\eta)\sin^2\theta\mathcal{F}_8(\boldsymbol{\xi}) \\
& + \sqrt{1-\eta^2}\cos(\Delta\Phi)\sin\theta\cos\theta\mathcal{F}_9(\boldsymbol{\xi})].
\end{aligned} \tag{E5}$$

The set of ten angular functions, $\mathcal{F}_0(\boldsymbol{\xi}) - \mathcal{F}_9(\boldsymbol{\xi})$, are defined in Eq. (10.2). The scalar products needed for their determination are as follows:

$$\begin{aligned}
\mathbf{N}\cdot\mathbf{l}_\Lambda &= \sin\theta_\Lambda \sin\phi_\Lambda, \\
\mathbf{l}_\Lambda\cdot\mathbf{l}_p &= \sin\theta_\Lambda\sin\theta_p\cos(\phi_\Lambda - \phi_p) + \cos\theta_\Lambda\cos\theta_p \\
\mathbf{l}_\Lambda\cdot\hat{\mathbf{p}} &= \cos\theta_\Lambda, \\
\mathbf{l}_{\Lambda\perp}\cdot\hat{\mathbf{k}} &= \sin\theta\sin\theta_\Lambda\cos\phi_\Lambda, \\
\mathbf{l}_{\Lambda\perp}\cdot\hat{\mathbf{p}} &= 0, \\
\mathbf{l}_{\Lambda\perp}\cdot\mathbf{l}_{\bar{\Lambda}\perp} &= \sin\theta_\Lambda\sin\theta_{\bar{\Lambda}}\cos(\phi_\Lambda - \phi_{\bar{\Lambda}}).
\end{aligned} \tag{E6}$$

We understand that the remaining scalar products are obtained from those above by the substitution $(\Lambda; p) \to (\bar{\Lambda}; \bar{p})$. With the scalar products of Eq. (E6) in hand one quickly determines the ten angular functions $\mathcal{F}_k(\boldsymbol{\xi})$ of Eq. (10.2),

$$\begin{aligned}
\mathcal{F}_0(\boldsymbol{\xi}) &= 1, \\
\mathcal{F}_1(\boldsymbol{\xi}) &= \cos^2\theta, \\
\mathcal{F}_2(\boldsymbol{\xi}) &= \sin\theta_\Lambda\sin\theta_p\cos(\phi_\Lambda - \phi_p) + \cos\theta_\Lambda\cos\theta_p, \\
\mathcal{F}_3(\boldsymbol{\xi}) &= \sin\theta_{\bar{\Lambda}}\sin\theta_{\bar{p}}\cos(\phi_{\bar{\Lambda}} - \phi_{\bar{p}}) + \cos\theta_{\bar{\Lambda}}\cos\theta_{\bar{p}}, \\
\mathcal{F}_4(\boldsymbol{\xi}) &= \cos\theta_\Lambda\cos\theta_{\bar{\Lambda}}, \\
\mathcal{F}_5(\boldsymbol{\xi}) &= \sin\theta_\Lambda\sin\phi_\Lambda, \\
\mathcal{F}_6(\boldsymbol{\xi}) &= \sin\theta_{\bar{\Lambda}}\sin\phi_{\bar{\Lambda}}, \\
\mathcal{F}_7(\boldsymbol{\xi}) &= \sin\theta_\Lambda\sin\theta_{\bar{\Lambda}}\cos(\phi_\Lambda - \phi_{\bar{\Lambda}}), \\
\mathcal{F}_8(\boldsymbol{\xi}) &= \sin\theta_\Lambda\cos\phi_\Lambda\sin\theta_{\bar{\Lambda}}\cos\phi_{\bar{\Lambda}}, \\
\mathcal{F}_9(\boldsymbol{\xi}) &= \cos\theta_\Lambda\sin\theta_{\bar{\Lambda}}\cos\phi_{\bar{\Lambda}} + \sin\theta_\Lambda\cos\phi_\Lambda\cos\theta_{\bar{\Lambda}}.
\end{aligned} \tag{E7}$$

The differential distribution of Eq. (E5) involves two parameters related to the $e^+e^- \to \Sigma^0\bar{\Sigma}^0$ reaction that can be determined by data: the ratio of form factors η, and the relative phase of form factors $\Delta\Phi$. In addition, the distribution function $\mathcal{W}(\boldsymbol{\xi})$ depends on the weak-decay parameters α_Λ and $\alpha_{\bar{\Lambda}}$ of the two Λ hyperon decays. The dependence on the weak decay parameters β and γ drops out, when final-state proton and antiproton spins are not measured.

APPENDIX F: FINDING ANGULAR VARIABLES

The angular functions and differential distributions of the previous appendix are expressed in terms of unit vectors such as $\mathbf{l}_p$ and $\mathbf{l}_\Lambda$, which are not directly measurable but which must be calculated. We suggest the following approach.

For each event we embed the particle momenta in its c.m. system and with coordinate axes as defined in Eq. (E1). For the Σ^0 hyperon the components of the momentum are, by definition,

$$\hat{\mathbf{p}}_{\Sigma^0} = (0, 0, 1). \tag{F1}$$

Then, let us consider the final-state proton with momentum $\mathbf{p}_p$ in the c.m. system. In the rest system of the Lambda hyperon the momentum of the same proton is denoted $\mathbf{L}_p$, and given by the expression

$$\mathbf{L}_p = \mathbf{p}_p + B_{\Lambda p}\mathbf{p}_\Lambda, \tag{F2}$$

$$B_{\Lambda p} = \frac{1}{m_\Lambda}\left[\frac{1}{E_\Lambda + m_\Lambda}\mathbf{p}_\Lambda\cdot\mathbf{p}_p - E_\Lambda\right]. \tag{F3}$$

Now, the length of the vector $\mathbf{L}_p$ is well known, being the momentum in the decay $\Lambda \to \pi N$, and therefore

$$|\mathbf{L}_p| = \frac{1}{2m_\Lambda}[(m_\Lambda^2 + m_\pi^2 - m_N^2)^2 - 4m_\Lambda^2 m_\pi^2]^{1/2}. \tag{F4}$$

Hence, the unit vector $\mathbf{l}_p$ appearing in our form-factor equations becomes

$$\mathbf{l}_p = \mathbf{L}_p/|\mathbf{L}_p|, \tag{F5}$$

$$= (\cos\phi_p \sin\theta_p, \sin\phi_p \sin\theta_p, \cos\theta_p). \tag{F6}$$

Corresponding equations for Λ in the decay $\Sigma^0 \to \Lambda\gamma$ are easily written down. In the rest system of the Σ^0 baryon the final-state Λ hyperon has momentum

$$\mathbf{L}_\Lambda = \mathbf{p}_\Lambda + B_{\Sigma\Lambda}\mathbf{p}_{\Sigma^0}, \tag{F7}$$

$$B_{\Sigma\Lambda} = \frac{1}{2m_{\Sigma^0}}\left[\frac{1}{E_{\Sigma^0} + m_{\Sigma^0}}\mathbf{p}_{\Sigma^0}\cdot\mathbf{p}_\Lambda - E_\Lambda\right], \tag{F8}$$

and the length of this vector is, as inferred from the Eq. (F4),

$$|\mathbf{L}_\Lambda| = \frac{1}{2m^0_\Sigma}(m^2_{\Sigma^0} - m^2_\Lambda). \tag{F9}$$

By the same reasoning the unit vector $\mathbf{l}_\Lambda$ appearing in our form-factor equations is

$$\mathbf{l}_\Lambda = \mathbf{L}_\Lambda/|\mathbf{L}_\Lambda|, \tag{F10}$$

$$= (\cos\phi_\Lambda \sin\theta_\Lambda, \sin\phi_\Lambda \sin\theta_\Lambda, \cos\theta_\Lambda). \tag{F11}$$

And so on for the antihyperons.

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