# Supersymmetric tensor model at large $N$ and small $\epsilon$

Fedor K. Popov
*Department of Physics, Princeton University, Princeton, New Jersey 08544, USA*

(Received 6 November 2019; published 29 January 2020)

We study the $O(N)^3$ supersymmetric quantum field theory of a scalar superfield $\Phi_{abc}$ with a tetrahedral interaction. In the large $N$ limit, the theory is dominated by the melonic diagrams. We solve the corresponding Dyson-Schwinger equations in continuous dimensions below 3. For sufficiently large $N$, we find an IR stable fixed point and computed the $3-\epsilon$ expansion up to the second order of perturbation theory, which is in agreement with the solution of DS equations. We also describe the $1+\epsilon$ expansion of the model and discuss the possibility of adding the Chern-Simons action to gauge the supersymmetric model.

DOI: 10.1103/PhysRevD.101.026020

## I. INTRODUCTION AND SUMMARY

In recent literature, there has been strong interest in theories whose dynamical fields are tensors of rank 3 or higher (for reviews, see [1–3]). Such theories possess a number of interesting features. For example, only the melonic diagrams dominate in the large $N$ limit, in contrast to the vector models, where only snail diagrams dominate [3], and the matrix models, where all the planar diagrams survive in the large $N$ limit. This fact makes the tensor models similar to the famous Sachdev-Ye-Kitaev (SYK) model [4–6]. The SYK model contains a disordered coupling constant, making it hard to use standard tools of quantum field theory. The SYK model is believed to describe quantum properties of the extremal charged black holes [7–9] and therefore may help to serve as a toy model for understanding the AdS/CFT correspondence [10–12]. It is already used for understanding the properties of the traversable wormholes [13–16]. While the tensor models [1] exhibit the same properties at the large $N$ limit, they do not have disorder therefore giving us hope that they can be understood at finite $N$ via standard techniques of quantum field theories. These techniques have already brought many interesting results [17–27].

We shall consider a supersymmetric analog of such theories, which has been recently considered as a generalization of SYK model [28–30] or as a quantum mechanical supersymmetric tensor model [31–34]. Here we will present a similar model in continuous dimension $d$. We consider a minimal $\mathcal{N}=1$ supersymmetric model, where we have some number of scalar superfields $\Phi_{abc}(x,\theta)$, and indices $a$, $b$, $c$ run from 1 to $N$. These fields are coupled via a "tetrahedral" superpotential,[1]

$$S=\int d^dxd^2\theta\left[\frac{1}{2}(D_\alpha\Phi_{abc})^2+g\Phi_{abc}\Phi_{ab'c'}\Phi_{a'bc'}\Phi_{a'b'c}\right]. \tag{1.1}$$

This theory, which is renormalizable in $d<3$, possesses $O(N)\times O(N)\times O(N)$ symmetry rather than $O(N^3)$ [the superpotential breaks such a symmetry, while the free theory, of course, possesses the $O(N^3)$ symmetry]. This model has been proposed in the paper [21] as a generalization of the scalar melonic theory. It was proved that the nonsupersymmetric analog of this theory has a so-called melonic dominance in the limit when $N\to\infty$, $g\to 0$ but $gN^{\frac{3}{2}}$ is kept fixed [36]. The proof of this peculiar fact relies on the combinatorial properties of the potential, and therefore is applicable in any dimensions and in various theories, provided that the combinatorial properties are left the same. In the case of the supersymmetric theories, the Feynman diagrams, written down in terms of the components, look quite complicated and, at first glance, do not possess a melonic limit as in the case of scalar model or the SYK model. However, one can develop a supersymmetric version of the usual Feynman diagrams technique and work explicitly with the superfields $\Phi_{abc}$ and see that the combinatorial and topological properties are the same as in the case of the scalar tensor models. Therefore, the proof of the dominance of melonic diagrams [20,21,36–38] is applicable in this case and the theory (1.1) also possesses a

[1] Here we will refer to the Appendix and the paper [35] for the notations and the other helpful formulas that will be used through the paper.

melonic dominance in the large $N$ limit. We generalize the theory (1.1) where the tetrahedral term is replaced by $q$-valent maximally single-trace operator to study models with different numbers of the internal propagators in each melon [37,39].

The properties of such theories in the IR limit can be investigated by solving the Dyson-Schwinger (DS) equations, which are drastically simplified if the theory is melonic. Namely, the dominance of the melonic diagrams in the large $N$ limit can be understood as a suppression of the corrections to the vertex operators in the system of DS equations. The solution of the DS equation in the IR yields a conformal propagator, suggesting that the theory in the IR flows to the fixed point, which is described by some conformal field theory. The existence of the stress-energy tensor with the correct dimension and the spectra of the operators confirm this hypothesis. Therefore, one can wonder whether it is possible to describe such a transition from the UV scale (where we have a bare conformal propagator determined by commutation relations) to the IR region by means of renormalization group (RG) flow and $\epsilon$ expansion. Several attempts have been made toward this idea. For example, the melonic scalar theory in four dimensions [40] has been considered at the second order of the perturbation theory. For this theory, a melonic fixed point of RG flow was found, even though the corresponding couplings are complex. The complex couplings indicate that the theory is unstable. For example, the dimensions of some operators have imaginary part. One of the reasons of instability could be that the potential is unbounded from below, leading to the decay of the vacuum state. The theory (1.1), being supersymmetric, lacks such a disadvantage.

It is quite interesting that if one drops the fermionic part of the action (1.1) and integrates out the auxiliary field, the theory still possesses the melonic dominance in the large $N$ limit. Such a "prismatic" theory was considered in the paper [41]. The solution of this theory was found in the large $N$ limit, and the RG properties were investigated at two loops. As opposed to the standard melonic theory [40], the fixed point is real and first order of $\epsilon$ expansion recovers the exact solution in the large $N$ limit.

In this paper, we solve the model (1.1) in the large $N$ limit, assuming that the supersymmetry is not broken and that in the IR region the theory is described by the conformal propagator. The solution is found for general dimension $d$ and general $q$-valent maximally single trace (MST) potential [37,39]. The dimension of the operators at given $d$ and spin $s$ can be found as a solution of the corresponding transcendental equation. It is shown that at any dimension $d$, there is always a stress-energy operator of dimension $d$ and a supercurrent operator of dimension $d-\frac{1}{2}$, which indicates that the theory is indeed described by a conformal field theory. While the model (1.1) exists only in the fractional dimensions between one and three dimensions, the counterpart SYK model with $q=3$ can work at the integer dimension $d=3$ and describe a good conformal field theory with the melonic dominance in the large $N$ limit. After that we derive a perturbation theory in $3-\epsilon$ dimensions of the theory (1.1) to find a fixed point that could describe the IR solution of the large $N$ limit of the model (1.1). We find that the $\epsilon$ expansion is consistent with the exact large $N$ solution up to the first order in $\epsilon$. The two-loop analysis also suggests that the found melonic fixed point is IR stable.

The structure of the paper is as follows: in Sec. II, we discuss the properties of the theory (1.1) in the large $N$ limit. The dimensions of the operators are found and the DS equation is solved in the superspace formalism. In Sec. III, we consider $q=3$ supersymmetric SYK model and study the stability of such a theory. In Sec. IV, we study the RG properties of the quartic super theories in three dimensions and compare to the exact solutions in the large $N$ limit. In Sec. V, we discuss the possibility of introducing higher order supersymmetry and speculate about the consequences of gauging the supersymmetric tensor models. The Appendix provides supplemental materials including the notations and useful formulas that are used throughout the paper.

## II. SOLUTION OF THE LARGE $N$ THEORY

In this section, we will try to find the solution of DS equations for the theory (1.1) in the large $N$ limit. As mentioned in the introduction, the theory possesses a melonic dominance in the large $N$ limit. This means that only specific diagrams survive in the large $N$ limit, namely the ones generated recursively by the DS equation (schematically depicted in Fig. 1). The resulting equation for scalar or fermion field theories was investigated analytically and numerically for many different theories [6,21,42]. For example, the DS equation can be solved in the IR limit and the solution possesses a conformal symmetry in that limit. In the case of the supersymmetric theories, one of the important differences is that one can demand the solution to respect supersymmetry. In order to do it manifestly, the DS equation should be formulated in terms of the superfields. Of course, one can do this calculation in terms of the components as in the paper [31] and check that these two approaches give the same answers. To make the discussion more general, we consider the case where there are $q-1$ internal propagators in the melon diagrams and suitable MST operator is considered [37]. The DS equation in the supersymmetric case reads as

$$G(p;\theta,\theta') = G_0(p;\theta,\theta') + \frac{1}{16}\lambda^2 \int d^2\theta_1 d^2\theta_2 G_0(p;\theta,\theta_1) \times \int \prod_{i=1}^{q-1} \frac{d^d k_i}{(2\pi)^d} G(k_i;\theta_1,\theta_2)(2\pi)^d \delta^d \times \left(p - \sum_{i=1}^{q-1} k_i\right) G(p;\theta_2,\theta'), \tag{2.1}$$

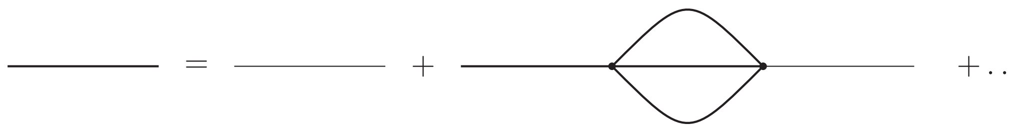

FIG. 1. A supersymmetric version of the Dyson-Schwinger equation for melonic theories in the large $N$ limit.

where $G_0(p;\theta,\theta')$ is a bare superpropagator (A10), $G(p;\theta,\theta')$ is an exact superpropagator, and $g=\lambda N^{\frac{3}{2}}$ is a 't Hooft coupling. Analogously to the scalar case, we consider a conformal propagator as an ansatz for the solution. But if we also demand to preserve supersymmetry and $O(N)\times O(N)\times O(N)$ symmetry, that yields only one form of the solution

$$\langle\Phi_{abc}(p,\theta)\Phi_{a'b'c'}(-p,\theta')\rangle=\delta_{aa'}\delta_{bb'}\delta_{cc'}G(p;\theta,\theta'),$$
$$G(p;\theta,\theta')=A\frac{D^2\delta(\theta-\theta')}{p^{2\Delta}}, \tag{2.2}$$

where $\Delta<\Delta_0=1$ for the solution to be valid in the IR limit [7] (namely, we can neglect by bare propagator in comparison to the exact one $G_0^{-1}\ll G^{-1},\quad p\to 0$). Substituting the ansatz in the DS equation (2.1), we get

$$A\frac{D^2\delta(\theta-\theta')}{p^{2\Delta}}=\frac{D^2\delta(\theta-\theta')}{p^2}+A^q\lambda^2\int d^2\theta_1 d^2\theta_2\frac{D^2\delta(\theta-\theta_1)}{p^2}$$
$$\times\prod_{i=1}^{q-1}\int\frac{d^dk_i}{(2\pi)^d}(2\pi)^d\delta^d\left(p-\sum_{i=1}^{q-1}k_i\right)$$
$$\times\frac{D^2\delta(\theta_1-\theta_2)}{k_i^{2\Delta}}\frac{D^2\delta(\theta_2-\theta')}{p^{2\Delta}}. \tag{2.3}$$

As soon as $\Delta<1$, we can neglect the lhs of the equation by the rhs in the limit $p\to 0$. After that, one can integrate out Grassman variables using identities for the superderivative to get

$$\lambda^2A^q\prod_{i=1}^{q-1}\int\frac{d^dk_i}{(2\pi)^d}\frac{1}{k_i^{2\Delta}}(2\pi)^d\delta^d\left(p-\sum_{i=1}^{q-1}k_i\right)\frac{1}{p^{2\Delta-2}}=-1. \tag{2.4}$$

This equation gives the dimension of the superfield to be $\Delta=\frac{d(q-2)+2}{2q}$ and

$$A^q=\frac{(4\pi)^{\frac{d(q-2)}{2}}}{\lambda^2}\frac{\Gamma^{q-1}(\frac{d}{2}-\frac{d-1}{q})\Gamma(d-1-\frac{d-1}{q})}{\Gamma^{q-1}(\frac{d-1}{q})\Gamma(\frac{d-1}{q}-\frac{d}{2}+1)}. \tag{2.5}$$

The solution suggests that we cannot work directly in $d_{\rm crit}(q)=\frac{2q-2}{q-2}$ dimensions because the bare propagator is not suppressed in the IR limit and change the solution. For example, for the case of tetrahedral potential $q=4$, $d_{\rm crit}=3$; therefore, the tensorial melonic theory is not conformal in three dimensions. Nevertheless, we can still study the theory slightly below three dimensions and compare it with the $\epsilon$ expansion.

If one chooses the case of $q=3$, the critical dimension is $d_{\rm crit}=4$ and such a melonic theory should describe a conformal field theory in three dimensions. In the next section, we will review this model in more details.

We calculated the propagator (2.2) in the momentum representation. One can carry out the calculation in the coordinate space. With the use of the relation,

$$\int\frac{d^dk}{(2\pi)^d}e^{ikx}D^2\delta(\theta-\theta')$$
$$=\int\frac{d^dk}{(2\pi)^d}e^{ikx}(1-ik^\mu\bar{\theta}'\gamma_\mu\theta+k^2\bar{\theta}'\theta'\bar{\theta}\theta)$$
$$=e^{\bar{\theta}'\gamma^\mu\theta\frac{\partial}{\partial x^\mu}}, \tag{2.6}$$

the propagator in the coordinate representation is

$$G(x,\theta,\theta')=\frac{B}{|x_\mu-\bar{\theta}'\gamma_\mu\theta|^{\frac{2(d-1)}{q}}},$$
$$B^q=\frac{1}{4\pi^d\lambda^2}\frac{\Gamma(\frac{d-1}{q})\Gamma(d-1-\frac{d-1}{q})}{\Gamma(\frac{d}{2}-\frac{d-1}{q})\Gamma(\frac{d-1}{q}-\frac{d}{2}+1)}. \tag{2.7}$$

Another way to see that the dimension of the superfield is $\frac{d-1}{q}$ is to rewrite the action in terms of the components and impose the conditions $\Delta_\psi=\Delta_\phi+\frac{1}{2}$, then the action contains a term

$$W(\Phi)=\Phi^q\Rightarrow W(\phi)=\phi^{q-2}\psi^2\Rightarrow[W]$$
$$=d\Rightarrow(q-2)\Delta_\phi+2\Delta_\psi=d,\quad\Delta_\phi=\frac{d-1}{q}. \tag{2.8}$$

The solution (2.2) suggests that in the IR limit, the theory is described by some conformal field theory (CFT). One of the interesting questions that one may ask is, what is the spectrum of the bipartite conformal operators in this theory? The supersymmetric theory (1.1) has different types of the bipartite operators, as the prismatic one [41]. We should consider these families separately. The most simple ones have the following structure [29]:

$$V_{\rm FF}=\Phi_{abc}(x,\theta)\Box^h\Phi_{abc}(x,\theta),$$
$$V_{\rm BB}=\Phi_{abc}(x,\theta)\Box^hD^2\Phi_{abc}(x,\theta). \tag{2.9}$$

These operators should be considered as a collection of operators with different spins and dimensions that transforms through each other when the supersymmetry transformations are applied. For shorthand, we will omit the indices, assuming that the operators are singlet under the action of $O(N)$'s groups. These operators could be rewritten in the terms of components (A3) as

$$\begin{aligned} V_{\mathrm{FF}}(x,\theta) &= \phi(x)\Box^h\phi(x) + \phi(x)\Box^h\psi^\alpha(x)\theta_\alpha \\ &\quad + \theta^2(\phi(x)\Box^h F(x) + \Box^h\phi(x)F(x) \\ &\quad + \bar\psi(x)\Box^h\psi(x)), \\ V_{\mathrm{BB}}(x,\theta) &= \bar\psi\Box^h\psi + (F\Box^h\psi_\alpha + \Box^h F\psi_\alpha \\ &\quad + (\gamma^\mu\psi)_\alpha\partial_\mu\Box^h\phi + (\gamma^\mu\Box^h\psi)_\alpha\partial_\mu\Box^h\phi)\theta^\alpha \\ &\quad + \theta^2(\partial_\mu\phi\Box^h\partial_\mu\phi + i\bar\psi\gamma^\mu\Box^h\partial_\mu\psi + F\Box^h F). \end{aligned} \tag{2.10}$$

A similar set of the operators was considered in the paper [29] in two dimensions and [31] in one dimension. Later, we shall compare the results of these papers with the continuous solution for arbitrary $d$. We can try to put more $D^2$ in (2.9) to get more families, but with the use of the identity $(D^2)^2 = \Box$, one can descend these operators to the BB or FF series. That is why we can consider only these two families to get the whole spectrum of bipartite operators with the lowest component having spin $s = 0$.

As usual, the corrections to the bilinear operators in the large $N$ limit are given by the ladder diagrams (but again, in comparison to [7,21], these diagrams should be considered to be in superspace). We assume the following ansatz in momentum space for the three-point correlation function for these families:

$$\begin{aligned} G_{\mathrm{FF}}(k,\theta,\theta') &= \langle V_{\mathrm{FF}}\Phi(-k,\theta)\Phi(k,\theta')\rangle \\ &= \frac{\delta(\theta-\theta')}{k^{\Delta_V+2\Delta}}, \\ G_{\mathrm{BB}}(k,\theta,\theta') &= \langle V_{\mathrm{BB}}\Phi(-k,\theta)\Phi(k,\theta')\rangle = \frac{D^2\delta(\theta-\theta')}{k^{\Delta_V+2\Delta}}, \end{aligned} \tag{2.11}$$

where we have set the operators $V_{\mathrm{BB}}$, $V_{\mathrm{FF}}$ to be at infinity and made a Fourier transformation with respect to the spatial coordinates, and $\Delta_V$ is the corresponding dimensions of the operator. The derivation of the equations for the dimensions $\Delta_V$ is just a straightforward generalization of the analogous calculation for the scalar model [21] or the SYK model [6]. Here we will show the derivation of such equation for the BB operators.

The addition of one step of the ladder can be considered as the action of the *kernel* operator (Fig. 2)

$$\hat K = K(p,k;\theta,\theta',\theta_1,\theta_2) = (q-1)\prod_{i=1}^{q-2}\int\frac{d^dq_i}{(2\pi)^d}\frac{D^2\delta(\theta_1-\theta_2)}{q_i^{2\Delta}}\frac{D^2\delta(\theta-\theta_1)}{p^{2\Delta}}\frac{D^2\delta(\theta_2-\theta')}{p^{2\Delta}}(2\pi)^d\delta^d\left(\sum q_i-(p-k)\right). \tag{2.12}$$

We act on the (2.9) by one step of the ladder,

$$(\hat K G_{\mathrm{BB}})(p,\theta,\theta') = \int d^2\theta_1 d^2\theta_2\int\frac{d^dk}{(2\pi)^d}K(p,k;\theta,\theta',\theta_1,\theta_2)G_{\mathrm{BB}}(k,\theta_1,\theta_2). \tag{2.13}$$

The Grassman variables can be integrated out with the use of identities from Sec. I. After that, we are left with a simple integral

$$\begin{aligned} (\hat K G_{\mathrm{BB}})(p,\theta,\theta') &= (q-1)A^q\lambda^2 D^2\delta(\theta-\theta')\int\frac{d^dk}{(2\pi)^d}\prod_{i=1}^{q-2}\frac{d^dq_i}{(2\pi)^d}\frac{1}{q_i^{2\Delta}}\frac{1}{k^{\Delta_V+2\Delta}p^{2\Delta-2}}(2\pi)^d\delta^d\left(\sum q_i-(p-k)\right) \\ &= g_B(\Delta_V)G_{\mathrm{BB}}(p,\theta,\theta'), \end{aligned} \tag{2.14}$$

where

$$g_B(\Delta_V) = -(q-1)\frac{\Gamma(\frac{2+d(q-2)}{2q})\Gamma(\frac{(q-1)(d-1)}{4})\Gamma(\frac{d}{4}-\frac{1}{q}-\frac{\Delta_V}{2})\Gamma(1-\frac{d}{2}-\frac{1}{q}+\frac{d}{q}+\frac{\Delta_V}{2})}{\Gamma(1-\frac{d}{2}+\frac{d-1}{q})\Gamma(\frac{d-1}{q})\Gamma(\frac{(q-1)(d-1)}{q}-\frac{\Delta_V}{2})\Gamma(\frac{d}{2}+\frac{1}{q}-\frac{d}{q}+\frac{\Delta_V}{2})}. \tag{2.15}$$

In order for the operator to be primary, the equation $g_B(\Delta_V) = 1$ must hold. An analogous equation can be written for the $V_{\mathrm{FF}}$ operator, but one can see that

$$g_F(\Delta_V) = g_B(\Delta_V - 1). \tag{2.16}$$

This suggests that we might build a bigger multiplet and enhance the supersymmetry to be $\mathcal{N} = 2$ (later we shall see that this does not actually happen, because there is no additional fermionic counterparts to finish supermultiplet).

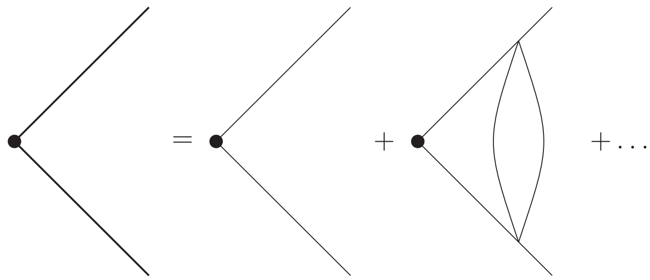


FIG. 2. The corrections to the bipartite conformal operator can be summed with the use of the Bethe-Salpeter equation. The diagrams should be considered to be in the superspace.

From now on, we shall consider the case only $q = 4$ to get $3 - \epsilon$ expansion unless the other is specified. Thus, we can get the $\epsilon$ expansion in the large $N$ limit of the $\Phi^2$ operator (Fig. 4)

$$\Delta_{\Phi^2} = 1 + \epsilon + 3\epsilon^2 - \frac{\pi^2 + 24}{4}\epsilon^3 + \mathcal{O}(\epsilon^4). \tag{2.17}$$

The plot of the $\Delta_{\Phi^2}$ as a function of the dimension is depicted in Fig. 3. Analogously, we get the dimension of $\Phi D^2 \Phi$ operator

$$\Delta_{\Phi D^2 \Phi} = 2 + \epsilon + 3\epsilon^2 - \frac{\pi^2 + 24}{4}\epsilon^3 + \mathcal{O}(\epsilon^4). \tag{2.18}$$

We can discuss dimensions of nonsinglet operators of the form $\Phi_{abc}\Phi_{a'bc}$. The equation for the dimension of this operator can be rewritten as

$$g_B(\Delta_{aa'}) = q - 1, \tag{2.19}$$

where a factor $q - 1$ appears from the combinatorics [43], and $\Delta_{aa'}$ is the dimension of the operator. The $\epsilon$ expansion near three dimensions for $q = 4$ has the following form:

$$\Delta_{aa'} = 1 - \frac{1}{2}\epsilon^2 + \frac{\pi^2}{24}\epsilon^3 + \mathcal{O}(\epsilon^4). \tag{2.20}$$

Later, we shall show that the solution coincides with the $\epsilon$ expansion in the second level of perturbation theory.

From this, the next step would be to study the spectrum of the higher-spin operators. A generalization for the higher-spin operators is

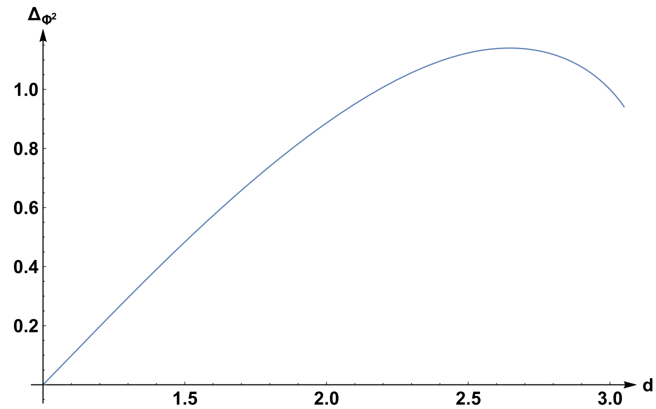


FIG. 3. The dimension of the operator $\Phi^2$ as a function of the dimension. As $d \to 1$, the dimension goes to zero.

$$\begin{aligned} V^s_{\mathrm{FF}} &= \Phi(x,\theta)\Box\partial_{\mu_1}\ldots\partial_{\mu_s}\Phi(x,\theta), \\ V^s_{\mathrm{BB}} &= \Phi(x,\theta)\Box\partial_{\mu_1}\ldots\partial_{\mu_s}D^2\Phi(x,\theta), \end{aligned} \tag{2.21}$$

with the corresponding modifications for the ansatz. For example, for higher-spin spectrum of the BB operators, the ansatz is

$$\begin{aligned} G^s_{\mu_1\ldots\mu_s,\mathrm{BB}}(k,\theta,\theta') &= \langle V^s_{\mu_1\mu_2\ldots\mu_s,\mathrm{BB}}\Phi(-k,\theta)\Phi(k,\theta')\rangle \\ &= \frac{D^2\delta(\theta-\theta')k_{\mu_1}\ldots k_{\mu_s}}{k^{\Delta_V + \frac{d+1}{2} + s}}. \end{aligned} \tag{2.22}$$

In this case, we consider an arbitrarily chosen null-vector $\xi^\mu$ and consider the convolution of the ansatz (2.22) with the vector $\xi$. After that, one can integrate out the Grassman variables and carry out the integration over the real pace with the use of a relation [40]

$$\begin{aligned} &\int d^d x \frac{(\xi\cdot x)^s}{x^{2\alpha}(x-y)^{2\beta}} \\ &= \pi^{\frac{d}{2}} \frac{\Gamma(\frac{d}{2}-\alpha+s)\Gamma(\frac{d}{2}-\beta)\Gamma(\alpha+\beta-\frac{d}{2})}{\Gamma(\alpha)\Gamma(\beta)\Gamma(d+s-\alpha-\beta)} \frac{(\xi\cdot y)^s}{y^{2\alpha+2\beta-d}}. \end{aligned} \tag{2.23}$$

Eventually, the equation for the dimension at given spin $s$ reads as

$$g_B(d,\Delta_V,s) = -(q-1)\frac{\Gamma(\frac{2+d(q-2)}{2q})\Gamma(\frac{(q-1)(d-1)}{4})\Gamma(\frac{d}{4}-\frac{1}{q}-\frac{\Delta_V-s}{2})\Gamma(1-\frac{d}{2}-\frac{1}{q}+\frac{d}{q}+\frac{\Delta_V+s}{2})}{\Gamma(1-\frac{d}{2}+\frac{d-1}{q})\Gamma(\frac{d-1}{q})\Gamma(\frac{(q-1)(d-1)}{q}-\frac{\Delta_V-s}{2})\Gamma(\frac{d}{2}+\frac{1}{q}-\frac{d}{q}+\frac{\Delta_V+s}{2})} = 1. \tag{2.24}$$

One would expect that there is a solution at any $d$ and $s = 2$ with $\Delta = d$, because of the existence of the stress-energy tensor. However, one cannot find this solution. The reason is quite simple. First of all, there is no stress-energy tensor in the field decomposition of the BB and FF operators. Second, the stress-energy tensor has a superpartner $\mathcal{S}^\alpha_\mu$

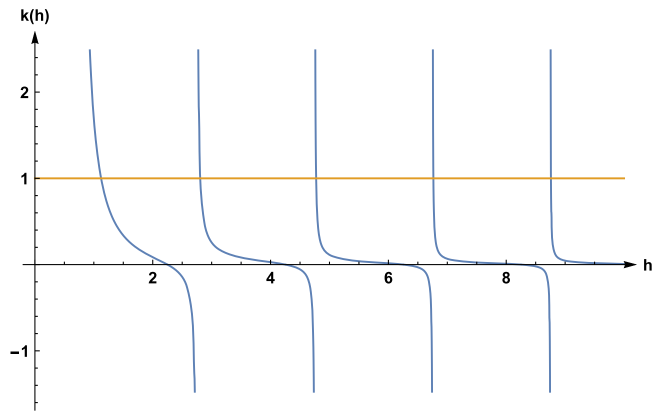


FIG. 4. The dimension of the operator $\Phi^2$ can be found graphically. The plot of $k(h)$ is drawn for the case of $d = 2.5$.

(corresponding to supertranslations) that has spin $\frac{3}{2}$, and therefore to find it we should consider a whole different family of operators, with lowest component being a Rarita-Schwinger field. Namely, let us consider a Fermi conformal primary operator

$$V_{\mathrm{BF},\mu_1\ldots\mu_{2n+1}}(x,\theta) = \partial_{\mu_i}^{2n+1}\Phi(x,\theta)D_\alpha\Phi(x,\theta), \qquad (2.25)$$

where the odd number of the space-time derivatives should be inserted to get a primary operator. Indeed, if we consider a zero number of the derivatives

$$V_{\mathrm{BF}} = \Phi_{abc}D_\alpha\Phi_{abc} = \frac{1}{2}D_\alpha(\Phi_{abc}^2), \qquad (2.26)$$

it is just a descendant of the FF operator. To get a supercurrent multiplet, we have to project the operators (2.25) on the specific component. The ansatz for the three-point function has the following form:

$$\langle V_{\mathrm{BF}}\Phi(k,\theta)\Phi(-k,\theta')\rangle = \frac{D_\alpha\delta(\theta-\theta')}{k^{\Delta_V+2\Delta}}. \qquad (2.27)$$

The derivation of the equation for the spectrum of the dimensions is straightforward,

$$g_{\mathrm{BF}}(d,\Delta_V,s) = -g_B\left(d,\Delta_V-\frac{1}{2},s-\frac{1}{2}\right) = 1, \qquad (2.28)$$

where the spin should be chosen to be of the form $s = 2n - \frac{1}{2}$. Now we can try to find the stress-energy momentum and its partner. And indeed, at any $d$, $q$, and $s = \frac{3}{2}$, there is an operator with dimension $\Delta = d - \frac{1}{2}$ that corresponds to the usual stress-energy supermultiplet.

At this point, one can wonder whether the current $J_{aa'}$, responsible for the $O(N)$'s transformations, is a primary operator. The supersymmetric multiplet containing the current should be also a Fermi supermultiplet with spin $s = 1/2$ [this operator is not a singlet operator and therefore (2.25) is not applicable]. The current should satisfy the equation [43]

$$g_{\mathrm{BF}}^{aa'}(d,\Delta_V,s) = \frac{1}{3}g_{\mathrm{BF}}(d,\Delta_V,s) = 1, \qquad (2.29)$$

at any $d$ and $q$ there is always a solution $\Delta_V = d - 3/2$. One can see that the dimension of square of this operator is given by the direct sum of the dimensions $\Delta_{J\tilde{J}} = 2\Delta_V = 2d-3$. This operator becomes relevant when $\Delta_{J\tilde{J}} = 2d - 3 \leq d - 1$, where minus 1 comes from accounting the dimension of the superspace. From this, one can see the operator becomes marginal in $d = 2$ and relevant as $d < 2$. This extra marginal operator in $d = 2$ may destabilize the CFT. The only exception is the case $N = 1$, where the theory does not have any continuous symmetry and has superpotential $\Phi^4$. In $d = 2$, this theory flows to the $m = 4$ superconformal minimal model, which has central charge $c = 1$.[2]

The relation (2.24) can be thought as a generalization of the equation for the kernel at two dimensions derived by Murugan *et al.* [29]. In this case, they introduced two dimensions, $h = \frac{\Delta+s}{2}$ and $\tilde{h} = \frac{\Delta-s}{2}$, and one can check that

$$k(h,\tilde{h}) = g_B(d=2,h+\tilde{h},h-\tilde{h}) = -(q-1)\frac{\Gamma^2(1-1/q)\Gamma(1/q-\tilde{h})\Gamma(1/q+h)}{\Gamma^2(1/q)\Gamma(1-1/q-\tilde{h})\Gamma(h+1-1/q)}, \qquad (2.30)$$

which coincides with Eq. (7.17) in [29].

The relation (2.28) also shows that if there is a scalar bilinear multiplet with dimension $h$, there is no BF operator with higher spin and the dimension $\Delta = \Delta + \frac{1}{2}$. This shows that we cannot complete the $\mathcal{N} = 2$ supermultiplet and the enhancement does not happen. It is interesting that there is an argument in $d = 1$ stating that it actually must happen. Basically, it comes from the fact that group of diffeomorphism of supertransformations in one dimension comprises the $\mathcal{N} = 2$ superalgebra [29].

Finally, we discuss the dimension of the quartic operators, because there is a fundamental relation between their dimensions and the eigenvalues of the matrix $\frac{\partial\beta_i}{\partial g_j}$. We can find the dimensions of some quartic operators in the large $N$ limit. For example, in the matrix models, the anomalous dimension of a double trace operator is just the sum of the anomalous dimensions of the corresponding single trace operators. By the same analysis, we get that the anomalous dimension of the double trace operator is

$$\Delta_{\Phi^4} = 2\Delta_{\Phi^2} = 2 + 2\epsilon + \mathcal{O}(\epsilon^2). \qquad (2.31)$$

Analogous analysis gives that

$$\Delta_{\mathrm{Pillow}} = 2\Delta_{aa'} = 2 + \mathcal{O}(\epsilon^2). \qquad (2.32)$$

[2] I would like to thank I.R. Klebanov for pointing out these facts.

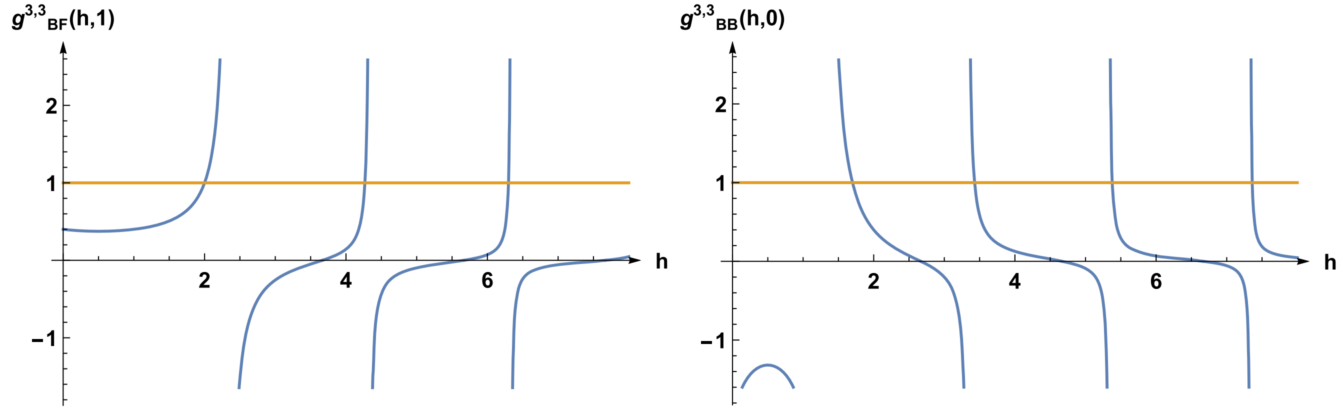


FIG. 5. Plots for $g^{3,3}_{\mathrm{BF}}(h,1)$ and $g^{3,3}_{\mathrm{BB}}(h,0)$ that can help to understand the structure of the spectrum of the theory (3.1).

Finally, the dimension of the tetrahedral operator can be determined as the dimension of the operator $\Phi_{abc}D^2\Phi_{abc}$ (namely, it follows from the equations of motion) and it gives us

$$\Delta_{\mathrm{Tetra}} = 2 + \epsilon + \mathcal{O}(\epsilon^2). \tag{2.33}$$

### A. The $\epsilon$ expansion near one dimension

One can try to study the behavior of the model (1.1) near one dimension. The case of $d = 1$ supersymmetric tensor models was considered recently (see [31]). It was found that the supersymmetry is broken in the IR region. The easiest way to see this is to assume a conformal ansatz and plug it in the DS equation (2.1). The solution suggests $\Delta = 0$ in one dimension, but constant or logarithmic function does not satisfy the DS equation. The conformal solution found in [31] shows that the dimensions of the superfield components are not related to each other by usual supersymmetric relations. It might be the case that for the system in one dimension the conformal solution does not describe the true vacuum state, while the true vacuum respects supersymmetry and the propagators exponentially decay at large distances. It might be shown by studying the stability of the conformal solution in a way described in [16] for two coupled SYK models.

Also, if one considers a limit $d \to 1$ in the equations derived in the previous sections, the propagator does not have a smooth limit in one dimension and the kernel is equal to the constant $\lim_{d\to 1} g_B(d,h,s) = -1$. The last fact confirms that in one dimension the conformal IR solution does not respect the supersymmetry. But, in the vicinity of dimension one, everything works fine. Thus, one can study the $1+\epsilon$ expansion. We shall consider the case of tensor models and set $q = 4$. For example, the dimension of the $\Phi^2$ operator is

$$\Delta_{\Phi^2} = \epsilon - \frac{\pi^2}{48}\epsilon^3 + \frac{3\zeta(3)}{16}\epsilon^4 + \mathcal{O}(\epsilon^5),$$
$$\Delta_{\Phi D^2\Phi} = 1 + \epsilon - \frac{\pi^2}{48}\epsilon^3 + \frac{3\zeta(3)}{16}\epsilon^4 + \mathcal{O}(\epsilon^5). \tag{2.34}$$

And the dimension of the colored operators $\Phi_{abc}\Phi_{a'bc}$ is

$$\Delta_{aa'} = \frac{3}{4}\epsilon - \frac{3\pi^2}{256}\epsilon^3 + \frac{9\zeta(3)}{128}\epsilon^4 + \mathcal{O}(\epsilon^5). \tag{2.35}$$

It would be interesting to derive these results by considering a one-dimensional supersymmetric melonic quantum mechanics and lift the solution to $1+\epsilon$ dimension. Or just derive these results starting with the conformal solution found in one dimension [31] and show that in higher dimensions the supersymmetry is immediately restored.

## III. SUPERSYMMETRIC SYK MODEL WITH $q = 3$ IN $d = 3$

In the previous section, we mostly work with the tensor models in noninteger dimensions. The main problem that did not allow us to work directly in three dimensions was that the critical dimension for such an interaction is $d_{\mathrm{cr}} = \frac{2q-2}{q-2} = 3$, meaning that directly at three dimensions the conformal IR solution does not work. Nevertheless, if one considers $q = 3$ case, the critical dimension becomes $d_{\mathrm{cr}} = 4$ and therefore should work perfectly in three dimensions. Unfortunately, we do not know any $q = 3$ tensor model and in order to somehow study this melonic model we shall consider a SYK-like model with disorder, which is a special case of the models [29].

Thus, we shall try to study the following model:

$$S = \int d^dx d^2\theta \left[\frac{1}{2}(D\Phi_i)^2 + C_{ijk}\Phi_i\Phi_j\Phi_k\right],$$
$$\langle C^2_{ijk}\rangle = \frac{J^2}{3N^2}, \quad i,j,k = 1,\ldots,N, \tag{3.1}$$

where we consider a quenched disorder for the coupling $C_{ijk}$. One might worry that such a theory violates the causality, because the field $C_{ijk}$ is assumed to have the same value across the space-time and therefore the excitation of such a field changes the value of it everywhere, thus violating causality. But the procedure of quenching requires first to fix the value of $C_{ijk}$ that makes the theory casual and after that average over this field. It means that we cannot excite the field $C_{ijk}$ and violate causality.

This model is similar to the tensor one considered in the previous section, because again only melonic diagrams survive in the large $N$ limit, but with two internal propagators in each melon. Therefore, the formulas derived in the previous section are applicable in this case and with the replacement of $\lambda \to J$ and setting $q = 3$, we can recover the large $N$ solution of this model. For example, the propagator in this case is

$$G(x,\theta,\theta') = \frac{B}{|x_\mu - \bar{\theta}'\gamma_\mu\theta|^{\frac{4}{3}}}, \qquad B^3 = \frac{1}{12\sqrt{3}\pi^3 J^2}, \tag{3.2}$$

and the dimension of the field $\Phi_i$ is $\Delta = \frac{2}{3}$. Again, the spectrum of the operators could be separated into three sectors, described in the previous section. The equation for the BB operators is determined by the equation

$$g^{3,3}_{\mathrm{BB}}(h,s) = -\frac{2^{\frac{4}{3}}\sqrt{\pi}\Gamma(\frac{2}{3}-\frac{h}{2}+\frac{s}{2})\Gamma(\frac{1}{6}+\frac{h}{2}+\frac{s}{2})}{3\Gamma(\frac{1}{6})\Gamma(\frac{4}{3}-\frac{h}{2}+\frac{s}{2})\Gamma(\frac{5}{6}+\frac{h}{2}+\frac{s}{2})}, \tag{3.3}$$

where $s$ is the spin and should be chosen even. One can try to find the spectra of low lying states (5) (Fig. 5).

| | |
|---|---|
| $[\Phi^2]_{\theta=0} s = 0$ | $h = 1.69944, 3.42951, 5.38013, 7.36259, 9.354, \ldots$ |
| $[D_\alpha(\Phi^2)]_{\theta=0} s = 1/2$ | $h = 2.19944, 3.92951, 5.88013, 7.86259, 9.854, \ldots$ |
| $[\Phi\partial_{\mu_1}\partial_{\mu_2}\Phi]_{\theta=0} s = 2$ | $h = 3.51911, 5.39016, 7.3654, 9.35514, 11.3496, \ldots$ |
| $[D_\alpha(\Phi\partial_{\mu_1}\partial_{\mu_2}\Phi)]_{\theta=0} s = 5/2$ | $h = 4.01911, 5.89016, 7.8654, 9.85514, 11.8496, \ldots$ |

It is easy to see that the spectrum has the following asymptotic behavior at large spins:

$$h \approx \frac{4}{3} + 2n + s + \mathcal{O}(1/n, 1/s), \quad n \to \infty, s \to \infty.$$

On a principal line $h = \frac{d}{2} + i\alpha$, the kernel is complex; it is connected to the fact that there is no well-defined metric in the space of two-point functions [29]. Therefore, there are no problems with the complex modes, that could possibly destroy the conformal solution in the IR [16]. Thus, $q = 3$ supersymmetric SYK model is stable at least in the BB channel. Also, one can check there are no additional solutions to the equation $g^{3,3}_{\mathrm{BB}}(h,s) = 1$ in the complex plane except the ones on the real line. The spectrum of the FF operators coincides with the spectrum of the BB operators but shifted with $h \to h + 1$; therefore, we do not have to worry about the instabilities of the theory in this sector.

Analogous calculations could be conducted for the BF series

$$g^{3,3}_{\mathrm{BF}}(h,s) = -g^{3,3}_{\mathrm{BB}}\left(h - \frac{1}{2}, s - \frac{1}{2}\right), \tag{3.4}$$

where the spin $s$ should be in the form $s = 2n - \frac{1}{2}$. One can notice that there is a solution $g^{3,3}_{\mathrm{BF}}(5/2, 3/2) = 1$ corresponding to the existence of the supercurrent and energy momentum tensor (the energy momentum is not seen directly because it belongs to the supermultiplet of the supercurrent, but if one studies the theory in terms of the components, he or she will of course find the energy momentum tensor). There is a list of some low lying operators in the FF sector (5).

| | |
|---|---|
| $[\partial_\mu\Phi D_\alpha\Phi]_{\theta=0}$ | $s = \frac{3}{2}: h = 2.5, 4.76759, 6.79738, 8.80934, 10.8157, \ldots$ |
| $[D_\beta(\partial_\mu\Phi D_\alpha\Phi)]_{\theta=0}$ | $s = 2: h = 3, 5.26759, 7.29738, 9.30934, 11.3157, \ldots$ |
| $[\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\Phi D_\alpha\Phi]_{\theta=0}$ | $s = \frac{7}{2}: h = 4.15398, 6.28752, 8.30627, 10.3143, 12.3189, \ldots$ |
| $[\partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3} D_\beta(\Phi D_\alpha\Phi)]_{\theta=0}$ | $s = 4: h = 4.65398, 6.78752, 8.80627, 10.8143, 12.8189, \ldots$ |

The spectrum has the following form of asymptotic behavior:

$$h \approx \frac{5}{6} + 2n + s + \mathcal{O}(1/n, 1/s), \quad n \to \infty.$$

The kernel is again complex on the principal line, but if one chooses $s = \frac{1}{2}$, there would be an additional solution of the equation $g^{3,3}_{\mathrm{BF}} = 1$ at $h = 1 + 0.496i$, but as soon as it is not on the principal line and $s$ is not permissible, we do not have to worry about this complex mode and expect that it could break the conformal solution. Thus, this $q = 3$ supersymmetric SYK model could provide us with a conformal field theory that is melonic and stable at integer dimensions. It would be interesting to study the $4 - \epsilon$ expansion for this model, where it will be close to its critical dimension.

## IV. $3-\epsilon$ EXPANSION

In this section, we continue the investigation of the supersymmetric tensor model (1.1) from the point of view of the $\epsilon$ expansion. The calculation is similar to the ones performed in the papers [40,41,44]. We include all possible $O(N)^3$ symmetric marginal interactions that respect the supersymmetry. Thus, the superpotential has the following form:

$$W(\Phi)=g_1\Phi_{abc}\Phi_{ab'c'}\Phi_{a'bc'}\Phi_{a'b'c}+\frac{g_2}{3}(\Phi_{abc}\Phi_{a'bc}\Phi_{ab'c'}\Phi_{a'b'c'}+\Phi_{abc}\Phi_{ab'c}\Phi_{a'bc'}\Phi_{a'b'c}+\Phi_{abc}\Phi_{abc'}\Phi_{a'b'c}\Phi_{a'b'c'})+g_3(\Phi_{abc}^2)^2, \tag{4.1}$$

where we imposed a symmetry under the exchange of the colors. In comparison to the "prismatic" theory [41], which has eight coupling constants, the supersymmetric theory has only three; this is a significant simplification.

Let us first consider the general renormalizable $d=3$ theory of $\mathcal{N}=1$ superfields $\Phi^i$, $i=1,\ldots n$,

$$S[\Phi_i]=\int d^3xd^2\theta\left[\frac{1}{2}(D\Phi_i)^2+\frac{Y_{ijkl}}{4!}\Phi_i\Phi_j\Phi_k\Phi_l\right], \tag{4.2}$$

where $Y_{ijkl}$ is a real symmetric tensor. Adapting the results from [45,46], we find that the two-loop corrections to the gamma and beta functions are

$$\begin{aligned}\gamma_{ab}^{(2)}&=\frac{1}{3(8\pi)^2}Y_{ajkl}Y_{bjkl},\\ \beta_{abcd}^{(2)}&=\frac{1}{3(8\pi)^2}Y_{ijkl}(Y_{jkla}Y_{bcdi}+Y_{jklb}Y_{cdai}+Y_{jklc}Y_{acdi}+Y_{jkld}Y_{abci})\\&\quad+\frac{2}{(8\pi)^2}(Y_{anom}Y_{bfom}Y_{nfcd}+Y_{anom}Y_{cfom}Y_{nfbd}+Y_{anom}Y_{dfom}Y_{nfbc}+Y_{bnom}Y_{cfom}Y_{nfad}\\&\quad+Y_{bnom}Y_{dfom}Y_{nfac}+Y_{cnom}Y_{dfom}Y_{nfab}).\end{aligned} \tag{4.3}$$

These two-loop results are closely related to those in a nonsupersymmetric theory with Yukawa coupling $\frac{1}{4}Y_{ijkl}\psi^i\psi^j\phi^k\phi^l$ (see [46]), except the supersymmetry requires $Y_{ijkl}$ to be fully symmetric.

Substituting $Y_{ijkl}$ corresponding to the superpotential (4.1), we find from (4.3)

$$\begin{aligned}\gamma^{\Phi}_{abc,a'b'c'}&=\delta_{aa'}\delta_{bb'}\delta_{cc'}\gamma^{\Phi}\\ \gamma^{\Phi}&=\frac{1}{6\pi^2}[12g_2g_1(1+N+N^2)+6g_3^2(2+N^3)+3g_1^2(2+3N+N^3)\\&\quad+g_2^2(5+9N+3N^2+N^3)+36g_3g_1N+12g_3g_2(1+N+N^2)]\end{aligned} \tag{4.4}$$

and

$$\begin{aligned}\beta_1&=-\epsilon g_1+\frac{2}{9\pi^2}(6g_1(12g_3^2(N^3+11)+g_2^2(N^3+6N^2+30N+29)+12g_3g_2(2N^2+5N+5))\\&\quad+9g_1^3(N^3+12N+8)+18g_1^2(g_2(4N^2+7N+16)+24g_3N)+2g_2^2(g_2(2N^2+13N+24)+72g_3)),\\ \beta_2&=-\epsilon g_2+\frac{2}{9\pi^2}(g_2(72g_3^2(N^3+11)+g_2^2(7N^3+36N^2+162N+194)+36g_3g_2((5N^2+9N+16))+54g_1^3(N^2+N+4)\\&\quad+18g_1^2(g_2(N^3+3N^2+27N+26)+18g_3(N+2))++18g_2g_1(g_2(7N^2+21N+32)+48g_3(N+1))),\\ \beta_3&=-\epsilon g_3+\frac{2}{9\pi^2}(108g_3^3(N^3+4)+252g_2g_3^2(N^2+N+1)+7g_2^3(N^2+3N+5)\\&\quad+18g_1^2(2g_3(N^3+3N+2)+g_2(N^2+N+4))+27g_1^3N+12g_2^2g_3(N^3+3N^2+15N+14)\\&\quad+36g_1(2g_2^2(N+1)+2g_3g_2(2N^2+2N+5)+21g_3^2N)).\end{aligned} \tag{4.5}$$

If one sets $g_1=g_2=0$, the symmetry gets enhanced to $O(N^3)$ and corresponds to the $O(n)$ vector model, which was considered in [45].[3] For the supersymmetric $O(n)$ model with superpotential $g(\Phi^i\Phi^i)^2$,

[3] Please note that they considered $SU(n)$ case that corresponds to $N^3=2n$ and their definition of $\gamma^{\Phi}$ includes a factor of 2.

$$\beta_g = -\epsilon g + \frac{24(n+4)}{\pi^2} g^3 + O(g^5), \tag{4.6}$$

in agreement with [45].

If we choose $N = 1$, the couplings $g_1$, $g_2$, $g_3$ becomes degenerate because they describe the same operator. Therefore, the beta functions should be added to get the right expression. And indeed, if we choose $N = 1$ and sum up the couplings, we get

$$\beta_1 + \beta_2 + \beta_3 = \mu \frac{d(g_1 + g_2 + g_3)}{d\mu} = -\epsilon(g_1 + g_2 + g_3) + \frac{120}{\pi^2}(g_1 + g_2 + g_3)^3, \tag{4.7}$$

which is the correct beta function for the theory with superpotential $(g_1 + g_2 + g_3)\Phi^4$ for a single chiral superfield $\Phi$. This special case of our theory is conformal in the entire range $2 \leq d < 3$. Indeed, in $d = 2$, the $\mathcal{N} = 1$ supersymmetric theory with superpotential $\Phi^m$ for one superfield $\Phi$ flows to the superconformal minimal model with central charge

$$c = \frac{3}{2}\left(1 - \frac{8}{m(m+2)}\right). \tag{4.8}$$

Therefore, the $N = 1$ case of the supertensor model gives the $m = 4$, $c = 1$ superminimal model in $d = 2$. For $N > 2$, the $O(N)^3$ supertensor model is expected to be conformal in $2 < d < 3$, but not in $d = 2$.

Let us consider the large $N$ limit where we scale the coupling constants in the following way:

$$g_1 = \frac{\pi}{2}\frac{\sqrt{2\epsilon}\lambda_1}{N^{\frac{3}{2}}}, \quad g_2 = \frac{\pi}{2}\frac{\sqrt{2\epsilon}\lambda_2}{N^{\frac{5}{2}}}, \quad g_3 = \frac{\pi}{2}\frac{\sqrt{2\epsilon}\lambda_3}{N^{\frac{7}{2}}}. \tag{4.9}$$

The scaling is taken to be the same as in the paper [40]. Applying this scaling to the formula (4.5), we get

$$\gamma_\Phi = \epsilon\frac{\lambda_1^2}{4}, \quad \beta_1 = -\lambda_1 + \lambda_1^3,$$
$$\beta_2 = -\lambda_2 + 2\lambda_2\lambda_1^2 + 6\lambda_1^3, \quad \beta_3 = -\lambda_3 + 2(2\lambda_3 + \lambda_2)\lambda_1^2 + 3\lambda_1^3. \tag{4.10}$$

From this, one can find the fixed point in the large $N$ limit. Namely,

$$\lambda_1^\infty = \pm 1, \qquad \lambda_2^\infty = \mp 6, \qquad \lambda_3^\infty = \pm 3,$$
$$\Delta_\Phi = \frac{d-2}{2} + \gamma_\Phi = \frac{1}{2} - \frac{\epsilon}{4}. \tag{4.11}$$

We may try to compute the $1/N$ corrections to these results to get

TABLE I. The approach of the finite $N$ fixed points in $3-\epsilon$ dimensions to the large $N$ limit. We note that the fixed point exists for all values of $N$.

| $N$ | $\frac{\lambda_1}{\lambda_1^\infty}$ | $\frac{\lambda_2}{\lambda_2^\infty}$ | $\frac{\lambda_3}{\lambda_3^\infty}$ |
|---|---|---|---|
| 100 000 | 1.000 | 1.000 | 1.000 |
| 10 000 | 1.000 | 1.001 | 1.002 |
| 1000 | 1.000 | 0.995 | 0.995 |
| 100 | 1.001 | 0.953 | 0.950 |
| 10 | 1.033 | 0.691 | 0.670 |
| 5 | 1.068 | 0.546 | 0.527 |
| 2 | 1.049 | 0.350 | 0.322 |
| 1 | 1.093 | 0.273 | 0.139 |

$$\lambda_1 = 1 + \mathcal{O}\left(\frac{1}{N^2}\right), \quad \lambda_2 = -6 + \frac{20}{N} + \mathcal{O}\left(\frac{1}{N^2}\right),$$
$$\lambda_3 = 3 - \frac{16}{N} + \mathcal{O}\left(\frac{1}{N^2}\right), \quad \gamma^\Phi = \frac{1}{2} - \frac{\epsilon}{4} + \mathcal{O}\left(\frac{1}{N^2}\right). \tag{4.12}$$

The anomalous dimension of the matter field operator $\Phi$ coincides with the exact dimension of the field by solving the DS equation found above. This might indicate that the higher-loop corrections to the RG equations (4.5) are suppressed in the large $N$ limit. It would be interesting to study these suppressions in $N$ for a general superpotential (4.1) from a combinatorial diagrammatic point of view and compare the results with the investigation of the finite $N$ solutions of Eq. (4.5).

If one considers the large $N$ fixed point (4.11) of the RG flow governed by Eq. (4.5) and tries to descend to finite $N$, one can find that the solution always exists (see Table I) and quite close to the found fixed point (4.11) (of course with the appropriate chosen scaling), in comparison to the "prismatic" model, where the melonic fixed point exists only at $N > 54$ [41].

We can study the dimension of various operators in the fixed point (4.11). One of these operators is $\Phi_{abc}^2$, which belongs to the BB spectrum. We can find that the anomalous dimension of this operator is

$$\Delta_{\Phi^2} = \Delta_{\Phi^2}^0 + 2\gamma_\Phi + \gamma_{\Phi^2} = 1 + \epsilon + \mathcal{O}(\epsilon^2), \tag{4.13}$$

where we have used the relation $\gamma_{\Phi^2} = 6\gamma_\Phi$, which is true only at the second level of perturbation theory. The answer coincides with the exact solution found earlier (2.17).

As one can see, the fixed point (4.11) is IR stable, which means that the dimensions of the operators are bigger than the dimension of the space-time. Indeed, the linearized equations of RG flow near the fixed point (4.11) have the following eigenvalues:

$$\left(\frac{\partial\beta_i}{\partial\lambda_j}\right)=\begin{pmatrix} -1+3\lambda_1^2 & 0 & 0 \\ 4\lambda_2\lambda_1+18\lambda_1^2 & -1+2\lambda_1^2 & 0 \\ 4(2\lambda_3+\lambda_2)\lambda_1+9\lambda_1^2 & 2\lambda_1^2 & -1+4\lambda_1^2 \end{pmatrix},$$
$$\Lambda=[2,1,3], \tag{4.14}$$

but as it is known the eigenvalues of this matrix give the dimensions of quartic operators

$$\Delta_i = d-\epsilon+\Lambda_i. \tag{4.15}$$

Thus, we get

$$\Delta_{\Phi^4} = 2-\epsilon+3\epsilon = 2+2\epsilon+\mathcal{O}(\epsilon^2),$$
$$\Delta_{\mathrm{pillow}} = 2-\epsilon+\epsilon = 2+\mathcal{O}(\epsilon^2),$$
$$\Delta_{\mathrm{tetra}} = 2-\epsilon+2\epsilon = 2+\epsilon+\mathcal{O}(\epsilon^2). \tag{4.16}$$

This is in the agreement with the large $N$ solution. As one can see, $\Lambda_i > 0$, indicating that the fixed point is IR stable. The agreement found between the exact large $N$ solution and perturbative $\epsilon$ expansion indicates that there is a nice flow from the UV scale to the IR one where the bare, free propagator flows to the one found by direct solving the DS equations (2.1). The study of the higher loop corrections might help to understand this relation better.

## V. $\mathcal{N}=2$ SUPERSYMMETRY AND GAUGING

One can try to consider $\mathcal{N}=2$ supersymmetry and study the properties of such a model. Here we are not going to present the solution of the corresponding DS equation, but we will just calculate the beta functions and find the fixed point of the resulting equations. The SYK model with $\mathcal{N}=2$ supersymmetry at two dimensions was considered in the paper [30].

The theory is built analogously to the $\mathcal{N}=1$ case. It can be obtained by dimensional reduction from $\mathcal{N}=1$ supersymmetry in four dimensions. In this case, we have a set of chiral superfields $\Psi_{abc}$ with the action

$$S=\int d^3x d^2\theta d^2\bar\theta \bar\Psi_{abc}\Psi_{abc}+\int d^3x d^2\theta W(\Psi_{abc})+\mathrm{H.c.},$$
$$\bar D_\alpha \Psi_{abc}=0, \tag{5.1}$$

where the superpotential is taken to be the same as in the case of $\mathcal{N}=1$ supersymmetry. The beta function for a general quartic superpotential was considered in the paper [47]. The beta function receives corrections only from the field renormalizations, meaning that it has the following form:

$$\beta_{1,2,3} = (-\epsilon+4\gamma^\Phi)g_{1,2,3}$$
$$\gamma^\Phi = \frac{1}{6\pi^2}(12g_2g_1(1+N+N^2)+6g_3^2(2+N^3)$$
$$+3g_1^2(2+3N+N^3)+g_2^2(5+9N+3N^2+N^3)$$
$$+36g_3g_1N+12g_3g_2(1+N+N^2)). \tag{5.2}$$

The fixed point is determined by demanding that the anomalous dimension of the field must be $\Delta_\Phi=\Delta_\Phi^0+\gamma^\Phi=\frac{d-1}{4}$, as we got for a general melonic theory in arbitrary dimensions. Apparently, for $\mathcal{N}=2$ models, this fact comes not from the melonic dominance, but from the consideration of the supersymmetric algebra that fixes the dimensions to be proportional to the $R$ charge of the corresponding operator. This condition defines a whole manifold in the space of marginal couplings. Applying the scaling (4.9), in the large $N$ limit, we get the equation

$$\gamma(\lambda_1,\lambda_2,\lambda_3)=\frac{\lambda_1^2}{4}=\frac{1}{4},\quad \lambda_1=1. \tag{5.3}$$

It is quite interesting that this equation does not fix $\lambda_2, \lambda_3$ in the large $N$ limit. One can study the stability of these fixed points at arbitrary $\lambda_{2,3}$. The RG flow near the fixed point could be linearized to get the stability matrix

$$\left(\frac{\partial\beta_i}{\partial g_j}\right)=\begin{pmatrix} 2 & 0 & 0 \\ 2\lambda_2 & 0 & 0 \\ 2\lambda_3 & 0 & 0 \end{pmatrix},\ \text{with eigenvalues}\ \Lambda=[2,0,0]. \tag{5.4}$$

The given solution is marginally stable, because of the existence of two marginal operators. These two zero directions correspond to the previously discussed existence of a whole manifold of IR fixed points.

From this consideration, it would be interesting to study the large $N$ limit of the considered $\mathcal{N}=2$ theory and corresponding DS equations. This model must have the same combinatorial properties as the $\mathcal{N}=1$ and scalar tensor model, but some cancellation happens that drastically simplifies the theory.

One can try to examine a gauged version of $\mathcal{N}=2$ theory. The gauging of the tensor models is one of the important aspects that makes them different from the SYK model. In the latter, due to the presence of the disorder in the system, the theory can possess only the global $O(N)$ symmetry and cannot be gauged, while in the tensor models there are no such obstructions and one can add gauge field and couple to the tensor models at any dimensions.

Gauging should be important for understanding the actual AdS/CFT correspondence. In one dimension, the gauging singles out from the spectrum all nonsinglet states from the Hilbert states. There have been many attempts to understand of the structure of the tensorial quantum mechanics of

Majorana fermions from numerical and analytical calculations [48–51]. These gave some interesting results, such as the structure of the spectrum of the matrix quantum mechanics and the importance of the discrete symmetries for explaining huge degeneracies of the spectra. Still, the general impact of gauging of the tensorial theory is not clear and demands a new approach. Here, we will give some comments of the combinatorial character and study how the gauging of $\mathcal{N} = 2$ theory, studied in the previous section, changes.

In three dimensions, one can gauge a theory by adding a Chern-Simons term instead of the usual Yang-Mills term

$$S = \int d^3x d^2\theta[-k(D_\alpha\Gamma^a_\beta)^2 + |(D_\alpha\delta^a_b + g\Gamma^a_{b\alpha})T^a\Phi_{abc}|^2 + W(\Phi_{abc})], \tag{5.5}$$

where $W(\Phi_{abc})$ is the same as in the (4.1), $T^a$ are the generators of the group $O(N)\times O(N)\times O(N)$, and $\Gamma^\alpha$ are vector superfields that have a gauge potential $A^a_{b\mu}$ as one of the components. If one rewrites the kinetic term for the gauge field in terms of usual components, he will get a usual Chern-Simons theory. Since the theory is gauge invariant, we can choose an axial gauge to simplify the action[4] $A^a_{b3} = 0$, which eliminates the nonlinear term from the theory and the Fadeev-Popov ghosts decouple from the theory. Therefore, the $A^a_{b1}$, $A^a_{b2}$ can be integrated out to get an effective potential. For example, such a term appears in the action

$$W_{\mathrm{eff}} \sim \frac{1}{k}\int \frac{d^3q}{(2\pi)^3}\frac{(\Phi_{abc}D_\alpha\Phi_{ab'c'})(q)(\Phi_{a'bc}D_\alpha\Phi_{a'b'c'})(-q)}{q_\perp} + \text{perm.}, \tag{5.6}$$

which can be considered as a nonlocal pillow operator with the wrong scaling, because the level of Chern-Simons (CS) action usually scales as $k = \lambda N$. Therefore, some diagrams would have large $N$ factor and diverge in the large $N$ limit. To fix it, we should consider the unusual scaling for the CS level $k = \lambda N^2$.

One can check that only specific Feynman propagators containing the nonlocal vertex (5.6) contribute in the large $N$ limit [44]. Namely, only snail diagrams contribute in the large $N$ limit and usually are equal to zero by dimensional regularization for massless fields. Therefore, one can suggest that the gauge field in the large $N$ limit does not get any large corrections and does not change the dynamics of the theory. This argument being purely combinatorial should be applied for any theory coupled to the CS action.

We can confirm this argument by direct calculation of the dimensions of the fields in the $\epsilon$ expansion for the $\mathcal{N} = 2$ supertensor model at two-loops and see whether the dimensions of the fields gets modified. The beta functions for a general $\mathcal{N} = 2$ theory coupled to a CS action was considered in the paper [47] and have the following form at finite $N$:

$$\beta_{1,2,3} = (-\epsilon + 4\gamma^\Phi_k)g_{1,2,3}, \quad \gamma^\Phi_k = \gamma^\Phi - \frac{3N(N-1)}{64\pi^2k^2}, \tag{5.7}$$

where $\gamma^\Phi$ is the same as in Eq. (5.2). As $k \sim N^2$, $N \to \infty$ the corrections to the gamma-functions vanish in the large $N$ limit. Thus, the gauging in three dimensions indeed does not bring any new corrections to the theory. It would be interesting to study such a behavior in different dimensions. For example, if in one dimension the gauging does not change structure of the solutions, one may conclude that the main physical degrees of freedom are singlets and there is a gap between the nonsinglet and singlet sectors. Also, it would be interesting to confirm this observation by a direct computation for the prismatic theories and for Yang-Mills theories.

## VI. CONCLUSION

We have studied supersymmetric extensions of the tensor field theories with $O(N)^3$ symmetry, such as the one with the tetrahedral superpotential [21]. Using the Dyson-Schwinger equations, we solved for operator scaling dimensions in the large $N$ limits as a function of dimension of space-time $d$. It is useful to compare our results with those in the prismatic model [41] where there are no fermions, while the scalar interactions are the same as in the supersymmetric model. Both models are renormalizable in $d < 3$, but in the large $N$ prismatic model, there is a transition in behavior at $d_{\mathrm{crit}} \approx 2.8$. For $d < d_{\mathrm{crit}}$, the dimension of one of the operators becomes complex so that the prismatic model becomes a complex CFT [52,53]. There is no such transition in the supersymmetric models, and they seem to be conventional CFTs in the range $1 < d < 3$. However, in $d = 2$, the theory may get destabilized by the presence of current-current marginal operators. Since in $d = 1$, the supersymmetry is not consisted with a conformal solution; it therefore appears that there are no superconformal tensor models in any integer dimensions.

Continuing the search for melonic CFTs in integer dimensions, we considered the SYK-like supersymmetric theory with random cubic superpotentials. This theory is renormalizable for $d < 4$ and appears to be stable in $d = 3$. This provides us with an example of the higher-dimensional field theory which possesses a melonic large $N$ limit. It would be interesting to explore the AdS/CFT correspondence for this $d = 3$ superconformal model.

## ACKNOWLEDGMENTS

I would like to thank Igor R. Klebanov for suggesting this problem and for the guidance throughout the project. This research was supported in part by the US NSF under Grant No. PHY-1620059. I am very grateful to S. Giombi and G. Tarnopolsky for collaboration at the early stages of

[4]I would like to thank S. Prakash for the suggested argument.

this project. The author also thanks A. M. Polyakov, S. Prakash, E. Akhmedov, and A. Milekhin for useful discussions. Also, I would like to thank M. Grinberg and P. Pallegar for the careful reading of the first drafts. I thank the organizers of the conference "Quantum Gravity 2019" in Paris for hospitality and stimulating atmosphere during some of the work of this project. I dedicate this work to the memory of my physics teacher, Polyanskii Sergey Evgenievich. Sergey Evgenievich guided me throughout School No. 146 in Perm, Moscow Institute of Physics and Technology and shared his profound wisdom, that helped me to come to the point where I am.

## APPENDIX: SUPERSYMMETRY IN THREE DIMENSIONS

In this section, we will introduce the notations and useful identities for the $\mathcal{N}=1$ supersymmetric theories in three dimensions. We will mostly follow the lectures [35]. The Lorentz group in three dimensions is $SL(2,\mathbb{R})$; that is a group of all unimodular real matrices of dimension two. The gamma matrices can be chosen to be real

$$\gamma^0=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix},\qquad \gamma^1=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},$$
$$\gamma^2=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\qquad \{\gamma^\mu,\gamma^\nu\}=2\eta^{\mu\nu}. \tag{A1}$$

There is no $\gamma^5$ matrix, so we cannot split the spinor representation into small Weyl ones. Because of this, the smallest spinor representation is two-dimensional and real. It is endowed with a scalar product defined as

$$\bar{\xi}\eta=\xi^\alpha\eta_\alpha=i\xi^\alpha\gamma^0_{\alpha\beta}\eta^\beta,\qquad \theta^2=\frac{1}{2}\bar{\theta}\theta. \tag{A2}$$

Because of these facts, the $\mathcal{N}=1$ superspace, in addition to the usual space-time coordinates, will include two real Grassman variables $\theta^\pm$. The fields on the superspace can be decomposed in terms of fields in the usual Minkowski space. For instance, a scalar superfield (that is our major interest) has the following decomposition:

$$\Phi(x,\theta^\alpha)=\phi(x)+\bar{\theta}\psi(x)+\theta^2F(x). \tag{A3}$$

As usual, the algebra supersymmetry in superspace can be realized via the derivatives that act on the superfields (A3) and mix different components

$$Q_\alpha=\partial_\alpha+i\gamma^\mu_{\alpha\beta}\theta^\beta\partial_\mu,\qquad \{Q_\alpha,Q_\beta\}=2i\gamma^\mu_{\alpha\beta}\partial_\mu, \tag{A4}$$

where $\partial_\mu$ stands for differentiation with respect to the usual space-time variables and $\partial_\alpha$ for the anticommuting ones. One can define a superderivative that anticommutes with supersymmetry generators and therefore preserves the supersymmetry

$$D_\alpha=\partial_\alpha-i\gamma^\mu_{\alpha\beta}\theta^\beta\partial_\mu,\qquad \{D_\alpha,Q_\beta\}=0. \tag{A5}$$

Out of these ingredients, namely (A3) and (A5), we can build an explicit version of a supersymmetric Lagrangian. For example, we can consider the following Lagrangian:

$$S=\int d^3xd^2\theta\left[-\frac{1}{2}(D_\alpha\Phi)^2+W(\Phi)\right], \tag{A6}$$

where the integral over Grassman variables is defined in the usual way with the normalization $\int d^2\theta\bar{\theta}\theta=1$. Writing out the explicit form of (A6), we get

$$S=\int d^3x\left[\frac{1}{2}(\partial_\mu\phi)^2+i\psi^\alpha\gamma^\mu_{\alpha\beta}\partial_\mu\psi^\beta+F^2 + W'(\phi)F+W''(\phi)\psi^2\right]. \tag{A7}$$

The field $F$ does not have a kinetic term and therefore is not dynamical and can be integrated out (that we will not do). For a further investigation, we have to develop the technique of super Feynman graphs. We start with considering the partition function of the theory (A6),

$$Z[J]=\int[d\Phi]\exp\left[\int d^3xd^2\theta\left(\frac{1}{2}(D_\alpha\Phi)^2+W(\Phi)+J\Phi\right)\right]$$
$$=\exp\left(W\left(\frac{\delta}{\delta J}\right)\right)$$
$$\times\int[d\Phi]\exp\left[\int d^3xd^2\theta\left(\frac{1}{2}\Phi D^2\Phi+J\Phi\right)\right]. \tag{A8}$$

The last integral is Gaussian and therefore can be evaluated and is equal to

$$Z[J]=\exp\left(W\left(\frac{\delta}{\delta J}\right)\right)\exp\left(-\int d^3xd^2\theta\left[\frac{1}{2}J\frac{1}{D^2}J\right]\right). \tag{A9}$$

From this, one can recover the usual Feynman diagrammatic technique, where the vertex is taken from the superpotential $W(\Phi)$ rather than the integrated version, and the propagator is defined as

$$\langle\Phi(x_1,\theta_1)\Phi(x_2,\theta_2)\rangle=\frac{1}{D^2}\delta^2(\theta_1-\theta_2)=\frac{D^2}{\Box}\delta^2(\theta_1-\theta_2), \tag{A10}$$

which can be calculated by double differentiation of the partition function (A8), and the operator $\Box$ is the usual Laplacian.

[1] R. Gurau, Invitation to random tensors, SIGMA **12**, 094 (2016).
[2] N. Delporte and V. Rivasseau, The tensor track V: Holographic tensors, arXiv:1804.11101.
[3] I. R. Klebanov, F. Popov, and G. Tarnopolsky, TASI lectures on large $N$ tensor models, Proc. Sci., TASI2017 (**2018**) 004 [arXiv:1808.09434].
[4] S. Sachdev and J. Ye, Gapless Spin Fluid Ground State in a Random, Quantum Heisenberg Magnet, Phys. Rev. Lett. **70**, 3339 (1993).
[5] A. Kitaev, A simple model of quantum holography, Proceedings of the KITP, April 7, 2015 and May 27, 2015, http://online.kitp.ucsb.edu/online/entangled15/kitaev/, http://online.kitp.ucsb.edu/online/entangled15/kitaev2/.
[6] J. Maldacena and D. Stanford, Comments on the Sachdev-Ye-Kitaev model, Phys. Rev. D **94**, 106002 (2016).
[7] J. Maldacena, D. Stanford, and Z. Yang, Conformal symmetry and its breaking in two dimensional nearly anti-de-Sitter space, Prog. Theor. Exp. Phys. **2016**, 12C104 (2016).
[8] J. Engelsöy, T. G. Mertens, and H. Verlinde, An investigation of $AdS_2$ backreaction and holography, J. High Energy Phys. 07 (2016) 139.
[9] K. Jensen, Chaos in $AdS_2$ Holography, Phys. Rev. Lett. **117**, 111601 (2016).
[10] J. M. Maldacena, The Large N limit of superconformal field theories and supergravity, Int. J. Theor. Phys. **38**, 1113 (1999); , Adv. Theor. Math. Phys. **2**, 231 (1998).
[11] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Gauge theory correlators from noncritical string theory, Phys. Lett. B **428**, 105 (1998).
[12] E. Witten, Anti-de Sitter space and holography, Adv. Theor. Math. Phys. **2**, 253 (1998).
[13] P. Gao, D. L. Jafferis, and A. Wall, Traversable wormholes via a double trace deformation, J. High Energy Phys. 12 (2017) 151.
[14] J. Maldacena, D. Stanford, and Z. Yang, Diving into traversable wormholes, Fortsch. Phys. **65**, 1700034 (2017).
[15] J. Maldacena and X.-L. Qi, Eternal traversable wormhole, arXiv:1804.00491.
[16] J. Kim, I. R. Klebanov, G. Tarnopolsky, and W. Zhao, Symmetry Breaking in Coupled SYK or Tensor Models, Phys. Rev. X **9**, 021043 (2019).
[17] S. Choudhury, A. Dey, I. Halder, L. Janagal, S. Minwalla, and R. Poojary, Notes on melonic $SO(N)^{q-1}$ tensor models, J. High Energy Phys. 06 (2018) 094..
[18] A. Mironov and A. Morozov, Correlators in tensor models from character calculus, Phys. Lett. B **774**, 210 (2017).
[19] R. Gurau and V. Rivasseau, The $1/N$ expansion of colored tensor models in arbitrary dimension, Europhys. Lett. **95**, 50004 (2011).
[20] E. Witten, An SYK-like model without disorder, J. Phys. A **52**, 474002 (2019).
[21] I. R. Klebanov and G. Tarnopolsky, Uncolored random tensors, melon diagrams, and the Sachdev-Ye-Kitaev models, Phys. Rev. D **95**, 046004 (2017).
[22] S. Prakash and R. Sinha, A complex fermionic tensor model in $d$ dimensions, J. High Energy Phys. 02 (2018) 086.
[23] T. Azeyanagi, F. Ferrari, P. Gregori, L. Leduc, and G. Valette, More on the new large $D$ limit of matrix models, Ann. Phys. (Berlin) **393**, 308 (2018).
[24] D. Benedetti, S. Carrozza, R. Gurau, and M. Kolanowski, The $1/N$ expansion of the symmetric traceless and the antisymmetric tensor models in rank three, Commun. Math. Phys. **371**, 55 (2019).
[25] D. Benedetti and R. Gurau, 2PI effective action for the SYK model and tensor field theories, J. High Energy Phys. 05 (2018) 156.
[26] H. Itoyama, A. Mironov, and A. Morozov, Ward identities and combinatorics of rainbow tensor models, J. High Energy Phys. 06 (2017) 115.
[27] H. Itoyama, A. Mironov, and A. Morozov, Cut and join operator ring in tensor models, Nucl. Phys. **B932**, 52 (2018).
[28] W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, Supersymmetric Sachdev-Ye-Kitaev models, Phys. Rev. D **95**, 026009 (2017); **95**, 069904(A) (2017).
[29] J. Murugan, D. Stanford, and E. Witten, More on supersymmetric and 2d analogs of the SYK model, J. High Energy Phys. 08 (2017) 146.
[30] K. Bulycheva, $\mathcal{N} = 2$ SYK model in the superspace formalism, J. High Energy Phys. 04 (2018) 036.
[31] C.-M. Chang, S. Colin-Ellerin, and M. Rangamani, On melonic supertensor models, J. High Energy Phys. 10 (2018) 157.
[32] C. Peng, M. Spradlin, and A. Volovich, A supersymmetric SYK-like tensor model, J. High Energy Phys. 05 (2017) 062.
[33] C. Peng, M. Spradlin, and A. Volovich, Correlators in the $\mathcal{N} = 2$ supersymmetric SYK model, J. High Energy Phys. 10 (2017) 202.
[34] C.-M. Chang, S. Colin-Ellerin, and M. Rangamani, Supersymmetric Landau-Ginzburg tensor models, J. High Energy Phys. 11 (2019) 007.
[35] S. J. Gates, Jr., M. T. Grisaru, M. Rocek, and W. Siegel, Superspace, or one thousand and one lessons in supersymmetry, Front. Phys. **58**, 1 (1983).
[36] S. Carrozza and A. Tanasa, $O(N)$ random tensor models, Lett. Math. Phys. **106**, 1531 (2016).
[37] I. R. Klebanov, P. N. Pallegar, and F. K. Popov, Majorana fermion quantum mechanics for higher rank tensors, Phys. Rev. D **100**, 086003 (2019).
[38] S. Carrozza, Large $N$ limit of irreducible tensor models: $O(N)$ rank-3 tensors with mixed permutation symmetry, J. High Energy Phys. 06 (2018) 039.
[39] F. Ferrari, V. Rivasseau, and G. Valette, A new large $N$ expansion for general matrix-tensor models, Commun. Math. Phys. **370**, 403 (2019).
[40] S. Giombi, I. R. Klebanov, and G. Tarnopolsky, Bosonic tensor models at large $N$ and small $\epsilon$, Phys. Rev. D **96**, 106014 (2017),
[41] S. Giombi, I. R. Klebanov, F. Popov, S. Prakash, and G. Tarnopolsky, Prismatic large $N$ models for bosonic tensors, Phys. Rev. D **98**, 105005 (2018).
[42] A. Z. Patashinskii and V. L. Pokrovskii, Second order phase transitions in a bose fluid, JETP **19**, 677 (1964).

[43] K. Bulycheva, I. R. Klebanov, A. Milekhin, and G. Tarnopolsky, Spectra of operators in large $N$ tensor models, Phys. Rev. D **97**, 026016 (2018).

[44] D. Benedetti, R. Gurau, and S. Harribey, Line of fixed points in a bosonic tensor model, J. High Energy Phys. 06 (2019) 053.

[45] L. Avdeev, G. Grigoryev, and D. Kazakov, Renormalizations in Abelian Chern-Simons field theories with matter, Nucl. Phys. **B382**, 561 (1992).

[46] I. Jack, D. R. T. Jones, and C. Poole, Gradient flows in three dimensions, J. High Energy Phys. 09 (2015) 061.

[47] J. Gracey, I. Jack, C. Poole, and Y. Schröder, A-function for $N = 2$ supersymmetric gauge theories in three dimensions, Phys. Rev. D **95**, 025005 (2017).

[48] I. R. Klebanov, A. Milekhin, F. Popov, and G. Tarnopolsky, Spectra of eigenstates in fermionic tensor quantum mechanics, Phys. Rev. D **97**, 106023 (2018).

[49] K. Pakrouski, I. R. Klebanov, F. Popov, and G. Tarnopolsky, Spectrum of Majorana Quantum Mechanics with $O(4)^3$ Symmetry, Phys. Rev. Lett. **122**, 011601 (2019).

[50] C. Krishnan and K. V. P. Kumar, Towards a finite-$N$ hologram, J. High Energy Phys. 10 (2017) 099.

[51] C. Krishnan and K. V. P. Kumar, Exact Solution of a Strongly Coupled Gauge Theory in $0 + 1$ Dimensions, Phys. Rev. Lett. **120**, 201603 (2018).

[52] V. Gorbenko, S. Rychkov, and B. Zan, Walking, weak first-order transitions, and complex CFTs II. Two-dimensional Potts model at $Q > 4$, SciPost Phys. **5**, 050 (2018).

[53] V. Gorbenko, S. Rychkov, and B. Zan, Walking, weak first-order transitions, and complex CFTs, J. High Energy Phys. 10 (2018) 108.