Quantum spacetime instability and breakdown of semiclassical gravity

Hiroki Matsui[1,*] and Naoki Watamura[2,†]

[1]*Department of Physics, Tohoku University, Sendai, Miyagi 980-8578, Japan*
[2]*Department of Mathematics, Shanghai University, Shanghai 200444, China*

(Received 24 October 2019; published 31 January 2020)

The semiclassical gravity describes gravitational backreactions of the classical spacetime interacting with quantum matter fields, but the quantum effects on the background are formally defined as higher-derivative curvatures. These induce catastrophic instabilities and classic solutions become unstable under small perturbations or their evolutions. In this paper, we discuss validity of the semiclassical gravity from the perspective of the spacetime instabilities and consider cosmological dynamics of the Universe in this theory. We clearly show that the homogenous and isotropic flat Universe is unstable and the solutions either grow exponentially or oscillate even in Planckian time $t_{\mathrm{I}} = (\alpha_1 G_N)^{1/2} \approx \alpha_1 10^{-43}$ sec. The subsequent curvature evolution leads to Planck-scale spacetime curvature in a short time and causes a catastrophe of the Universe unless one takes extremely large values of the gravitational couplings. Furthermore, we confirm the above suggestion by comparing the semiclassical solutions and ΛCDM with the Planck data and it is found that the semiclassical solutions are not consistent with the cosmological observations. Thus, the standard semi-classical gravity using quantum energy momentum tensor $\langle T_{\mu\nu} \rangle$ is not appropriate to describe our Universe.

DOI: 10.1103/PhysRevD.101.025014

I. INTRODUCTION

There are many essential difficulties to construct a consistent theory of quantum gravity. Almost certainly, quantum gravity where metric is also quantized together with matter fields would change even fundamental concept of the spacetime and requires a completely different theory from classical general relativity. However, in the regime where the curvature is small, one usually regards the gravity as a classic field and matters move on the background. Therefore, the semiclassical approximation is usually expected to be sufficient. Based on this assumption, thermal Hawking radiation around black holes [1] or amplification of primordial quantum fluctuations during inflation is correctly performed by quantum field theory in curved spacetime [2].

However, not only quantum fluctuations of the matter fields, but also quantum backreaction on the spacetime must be considered in full semiclassical approximation. The standard semiclassical gravity replaces energy momentum tensor in Einstein equations by the expectation values of some quantum state,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} \equiv 8\pi G_N \langle \Psi | T_{\mu\nu} | \Psi \rangle. \tag{1}$$

The semiclassical gravity naturally includes quantum effects of the matters on spacetime such as vacuum polarization or quantum particle creation. This theory is regarded as a first approximation to quantum gravity [3] and yields some insight into quantum nature of gravity. For instance, it describes evaporation of the black holes [1,4–7] and provides new classes of the cosmological solutions [8–15]. However, the quantum energy momentum tensor has nontrivial structures which depend on the curvature tensor or its derivatives and introduce higher-derivative corrections. In the semiclassical gravity, these higher-derivative curvatures always appear and are necessary to take account of the interaction of the classical gravitational field with quantum matter fields. Even in quantum gravity the higher-derivative curvatures are necessary for the renormalizability of the theory [16]. However, these induce instabilities of classical spacetime drawn from general relativity and produce unphysical massive ghosts which lead to the nonunitary graviton S matrix [17,18]. Although we usually assume that the semiclassical gravity would be applicable below the Planck regime, it has several undesired properties and the validity is still unknown.

In fact, Refs. [19–26] have shown that the Minkowski spacetime in the semiclassical gravity is unstable under small perturbations and it is not the ground state [21]. The perturbations either grow exponentially or oscillate even in the Planck time, and the subsequent curvature evolution leads to the Planck-scale spacetime curvature [19,20]. By using large N expansion of quantum gravity [21] or effective action approach [23], it was shown that the curvature instabilities occur at the frequencies far below the Planck regime. The quantum de Sitter instability for scalar fields or graviton has been also discussed by Refs. [27–55]. These facts strongly indicate that the semiclassical gravity is not a good theory of the gravity, and the instabilities cannot be easily rescued by full quantum theory of gravity [21,23]. However, it has not been clearly shown whether

*hiroki.matsui.c6@tohoku.ac.jp
†watamura@shu.edu.cn

the quantum instabilities are incompatible with the cosmological observations.

In this paper, we will thoroughly investigate the spacetime instabilities induced by quantum backreaction and reconsider cosmological dynamics of the Universe in the semiclassical equations. Due to the spacetime instabilities, we obtain exactly nontrivial cosmological constraints on the semiclassical gravity. We clearly show that Minkowski spacetime or homogenous and isotropic flat spacetime are unstable, and the corresponding solutions either grow exponentially or oscillate even in the Planckian time $t_{\rm I} = (\alpha_1 G_N)^{1/2} \approx \alpha_1 10^{-43}$ sec where α_1 is defined by the quantum energy momentum tensor or higher-derivative gravitational action. Furthermore, we confirm the above proposition by comparing the cosmological solutions of the ΛCDM and the semiclassical Einstein equations with the recent Planck data [56] and then, we show that the semiclassical gravity is inconsistent with the cosmological observations unless one takes extremal values of the gravitational couplings. Thus, the standard semiclassical gravity using quantum energy momentum tensor is not appropriate to describe our Universe.

The present paper is organized as follows. In Sec. II, we review the semiclassical gravity and introduce our formulation for this theory. In particular, we explain how the higher-derivative corrections appear in the semiclassical gravity. Furthermore, we discuss several problems of the semiclassical gravity such as violations of gravitational thermodynamical laws and (averaged) null energy condition. In Sec. III, we investigate the spacetime instabilities induced by quantum backreaction. First, we consider some results of Refs [19,24,25] and investigate the instability of Minkowski spacetime under small perturbations. Next, we investigate quantum instabilities of the homogenous and isotropic FLRW Universe and obtain cosmological constraints on the semiclassical gravity. Finally, in Sec. IV, we discuss the validity of this theory and draw the conclusion of our work.

II. SEMICLASSICAL GRAVITY

In this section, we review how quantum matter fields interact with the spacetime and introduce our formulation for the semiclassical gravity. At the quantum level, the classical action of gravity is replaced by the effective action $\Gamma_{\rm eff}[g_{\mu\nu}]$, that is a functional of quantum matter fields ϕ in the classical background metric,[1]

$$e^{i\Gamma_{\rm eff}[g_{\mu\nu}]} = e^{iS_{\rm G}[g_{\mu\nu}]} \int \mathcal{D}\phi e^{iS_{\rm M}[\phi, g_{\mu\nu}]}, \tag{2}$$

where $S_{\rm G}[g_{\mu\nu}]$ is the gravitational action and $S_{\rm M}[g_{\mu\nu}]$ is the classical action of matter. This procedure also corresponds to the large N approximation of quantum gravity [21]. When quantum corrections of graviton are smaller than the corrections of large number of matter fields, one can neglect the graviton loops. In particular, if one considers the early Universe, there were certainly a large number of matter fields. Thus, the large N approximation is applicable and semiclassical gravity is valid. For the current Universe, the corresponding energy scale is much smaller than the Planck scale and the semiclassical approximation should be valid.

The gravitational action is constructed by the Einstein-Hilbert term and the cosmological constant

$$S_{\rm EH}[g_{\mu\nu}] \equiv -\frac{1}{16\pi G_N} \int d^4x \sqrt{-g}(R + 2\Lambda) \tag{3}$$

and the fourth-derivative curvature terms

$$S_{\rm HD}[g_{\mu\nu}] \equiv \int d^4x \sqrt{-g}(c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R_{\mu\nu\kappa\lambda}R^{\mu\nu\kappa\lambda} + c_4 \Box R). \tag{4}$$

The fourth-derivative terms are indispensable for renormalization to eliminate one-loop divergences in curved spacetime. Without the higher-derivative terms, the semiclassical gravity becomes nonrenormalizable and is not consistent as fundamental (not effective) quantum field theory. If one regards the semiclassical gravity as the fundamental, higher-derivative curvatures always exist at the classical level. Even if these terms are not included into the classical action, they will emerge from quantum energy momentum tensors. The effective action of Eq. (2) derives the semiclassical Einstein's equations [2],

$$\frac{1}{8\pi G_N}\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu}\right) + a_1 H^{(1)}_{\mu\nu} + a_2 H^{(2)}_{\mu\nu} + a_3 H_{\mu\nu} = \langle T_{\mu\nu}\rangle, \tag{5}$$

where $\langle T_{\mu\nu}\rangle$ is the vacuum expectation value of the quantum energy momentum tensor,

$$\langle T_{\mu\nu}\rangle = -\frac{2}{\sqrt{-g}}\frac{\delta\Gamma_{\rm eff}[g_{\mu\nu}]}{\delta g^{\mu\nu}}, \tag{6}$$

and the geometric tensors $H^{(1,2)}_{\mu\nu}$ are defined as

$$\begin{aligned}
H^{(1)}_{\mu\nu} &\equiv \frac{1}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}\int d^4x\sqrt{-g}R^2 \\
&= 2\nabla_\nu\nabla_\mu R - 2g_{\mu\nu}\Box R - \frac{1}{2}g_{\mu\nu}R^2 + 2RR_{\mu\nu}, \\
H^{(2)}_{\mu\nu} &\equiv \frac{1}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}\int d^4x\sqrt{-g}R_{\mu\nu}R^{\mu\nu} \\
&= 2\nabla_\alpha\nabla_\nu R^\alpha_\mu - \Box R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\Box R - \frac{1}{2}g_{\mu\nu}R_{\alpha\beta}R^{\alpha\beta} + 2R^\rho_\mu R_{\rho\nu}, \\
H_{\mu\nu} &\equiv \frac{1}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}\int d^4x\sqrt{-g}R_{\mu\nu\kappa\lambda}R^{\mu\nu\kappa\lambda} = -H^{(1)}_{\mu\nu} + 4H^{(2)}_{\mu\nu}.
\end{aligned} \tag{7}$$

[1]In theory involving gravitational fields, anomalies may exist and break general covariance or local Lorentz invariance. These are called gravitational anomalies and generally exist when the spacetime dimension is $D=4k+2$, $k = 0, 1, 2\ldots$ [57]. For four-dimensional semiclassical gravity, they do not appear in general.

For the flat Friedmann-Lemaitre-Robertson-Walker (FLRW) Universe, the geometrical tensors $H^{(1)}_{\mu\nu}$ and $H^{(2)}_{\mu\nu}$ have a relation $H^{(1)}_{\mu\nu} = 3H^{(2)}_{\mu\nu}$, and we get

$$a_1 H^{(1)}_{\mu\nu} + a_2 H^{(2)}_{\mu\nu} + a_3 H_{\mu\nu} = \left(a_1 + \frac{1}{3}a_2 + \frac{1}{3}a_3\right) H^{(1)}_{\mu\nu} = \alpha_1 H^{(1)}_{\mu\nu}, \quad (8)$$

where we note $\alpha_1 = a_1 + \frac{1}{3}a_2 + \frac{1}{3}a_3$. Furthermore, the quantum energy momentum tensor $\langle T_{\mu\nu}\rangle$ introduces more additional geometric tensors (for the detailed discussion, see Ref. [2]). For instance, the renormalized vacuum energy momentum tensor for a massless conformally coupled scalar field is given by the conformal anomaly

$$\langle T_{\mu\nu}\rangle_{\text{conformal}} = \frac{1}{2880\pi^2}\left(-\frac{1}{6}H^{(1)}_{\mu\nu} + H^{(3)}_{\mu\nu}\right), \quad (9)$$

where

$$H^{(3)}_{\mu\nu} \equiv \frac{1}{12}R^2 g_{\mu\nu} - R^{\rho\sigma}R_{\rho\mu\sigma\nu} = R^{\rho}_{\mu}R_{\rho\nu} - \frac{2}{3}RR_{\mu\nu} - \frac{1}{2}R_{\rho\sigma}R^{\rho\sigma}g_{\mu\nu} + \frac{1}{4}R^2 g_{\mu\nu}.$$

The semiclassical gravity introduces more additional geometric tensor which depends on quantum states [2]. In fact, the renormalized energy momentum tensor for a massless minimally coupled scalar field in Bunch-Davies vacuum state is given by [58]

$$\langle T_{\mu\nu}\rangle_{\text{ren}} = \frac{(-\frac{1}{6}H^{(1)}_{\mu\nu} + H^{(3)}_{\mu\nu})}{2880\pi^2} - \frac{H^{(1)}_{\mu\nu}\log(\frac{R}{\mu^2})}{1152\pi^2} + \frac{(-32\nabla_\nu\nabla_\mu R + 56\Box R g_{\mu\nu} - 8RR_{\mu\nu} + 11R^2 g_{\mu\nu})}{13824\pi^2}, \quad (10)$$

which have higher-derivative corrections.

In the next subsection, we review the renormalization of the quantum energy momentum tensor and see how the higher-derivative curvatures appear in semiclassical gravity.

A. Quantum backreaction

First, we consider adiabatic (Wentzel-Kramers-Brillouin) approximation for the conformally massless fields and derive the renormalized quantum energy momentum tensors in this method. It is found that the derivation of adiabatic (WKB) approximation reduces the ambiguity of the UV divergences in renormalization, and it is more significant than any other regularization in curved spacetime.

In this paper, we consider a spatially flat FLRW spacetime,

$$ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j, \quad (11)$$

where $a(t)$ is the scale factor and t is the cosmic time. We introduce conformal time η defined by $d\eta = dt/a$.

Let us consider the matter action for the conformally coupled scalar field ϕ with mass m,

$$S_{\text{M}} = \int d^4x\sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\left(m^2 + \frac{R}{6}\right)\phi^2\right), \quad (12)$$

which leads to the Klein-Gordon equation given as

$$\Box\phi - \left(m^2 + \frac{R}{6}\right)\phi = 0. \quad (13)$$

The operator $\phi(\eta, x)$ can be decomposed as

$$\phi(\eta, x) = \int \frac{d^3k}{(2\pi)^{3/2}}\left(a_k \frac{e^{ik\cdot x}\varphi_k(\eta)}{a(\eta)} + a_k^\dagger \frac{e^{-ik\cdot x}\varphi_k^*(\eta)}{a(\eta)}\right), \quad (14)$$

where a_k, $a_k^\dagger$ are the annihilation and creation operators, respectively. In curved spacetime, quantum states are determined by the choice of the mode functions. The mode function $\varphi_k(\eta)$ should satisfy the Wronskian condition

$$\varphi_k'^*(\eta)\varphi_k(\eta) - \varphi_k'(\eta)\varphi_k^*(\eta) = i, \quad (15)$$

which ensures the canonical commutation relations. We adopt adiabatic (WKB) approximation to the mode function $\phi(\eta, x)$, which is written by [59]

$$\varphi_k(\eta) = \frac{1}{a(\eta)\sqrt{2W_k(\eta)}}(\alpha_k \cdot e^{-i\int W_k(\eta)d\eta} + \beta_k \cdot e^{i\int W_k(\eta)d\eta}), \quad (16)$$

where the background changes slowly and must satisfy the adiabatic (WKB) conditions ($\omega_k^2 > 0$ and $|\omega'_k/\omega_k^2| \ll 1$ where $\omega_k^2(\eta) = k^2 + a^2(\eta)m^2$). The coefficients α_k and β_k satisfy the following conditions:

$$|\alpha_k|^2 - |\beta_k|^2 = 1. \quad (17)$$

The adiabatic function $W_k(\eta)$ is given by

$$
\begin{aligned}
W_k(\eta) \simeq \omega_k &- \frac{m^2 C}{8\omega_k^3}(D' + D^2) + \frac{5m^4 C^2 D^2}{32\omega^5} + \frac{m^2 C}{32\omega_k^5}(D''' + 4D'D + 3D'^2 + 6D'D^2 + D^4) \\
&- \frac{m^4 C^2}{128\omega_k^7}(28D''D + 19D'^2 + 122D'^2 + 47D^4) + \frac{221 m^6 C^3}{256\omega_k^9}(D'D^2 + D^4) - \frac{1105 m^8 C^4 D^4}{2048\omega_k^{11}} + \cdots, \qquad (18)
\end{aligned}
$$

where $C(\eta) = a^2(\eta)$ and $D = C'/C$. The mode function $\varphi_k(\eta)$ with $\alpha_k = 1$ and $\beta_k = 0$ is a reasonable choice for a sufficiently slow and smooth background [59],

$$
\varphi_k(\eta) = \frac{1}{\sqrt{2W_k(\eta)C(\eta)}} \cdot e^{-i\int W_k(\eta)d\eta}, \qquad (19)
$$

which defines the adiabatic vacuum state $|\Psi_A\rangle$ which is annihilated by all the operators a_k. Here, we assume that the adiabatic vacuum state $|\Psi_A\rangle$ is adequate initial vacuum in flat FLRW spacetime.

Let us consider the quantum energy momentum tensor. The classical energy momentum tensor is given by [60]

$$
T_{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} = \frac{2}{3}\partial_\mu\phi\partial_\nu\phi - \frac{1}{6}g_{\mu\nu}g^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi - \frac{1}{3}\phi\nabla_\mu\nabla_\nu\phi + \frac{1}{3}g_{\mu\nu}\phi\Box\phi - \frac{1}{6}G_{\mu\nu}\phi^2 + \frac{1}{2}m^2 g_{\mu\nu}\phi^2 \qquad (20)
$$

and the corresponding trace

$$
T^\mu{}_\mu = m^2\phi^2, \qquad (21)
$$

which is exactly zero when $m \to 0$. Thus, the conformally massless scalar field has the vanishing trace of the stress tensor classically. It is found that the conformal invariance is broken in quantum field theory. The vacuum expectation values of the energy momentum tensor $\langle T_{\mu\nu}\rangle$ for $\varphi_k(\eta)$ are given by

$$
\begin{aligned}
\langle T_{00}\rangle &= \frac{1}{4\pi^2 C(\eta)}\int dk k^2[|\varphi_k'(\eta)|^2 + \omega_k^2|\varphi_k(\eta)|^2], \\
\langle T^\mu{}_\mu\rangle &= \frac{1}{2\pi^2 C^2(\eta)}\int dk k^2[Cm^2|\varphi_k(\eta)|^2], \qquad (22)
\end{aligned}
$$

where $\langle T_{00}\rangle$ is the time component of the energy momentum tensor and $\langle T^\mu{}_\mu\rangle$ is the trace. These quantum energy momentum tensors have UV divergences and one must proceed the renormalization.

Next, we rewrite the vacuum expectation values $\langle T_{\mu\nu}\rangle$ of the energy momentum tensor by the adiabatic approximation [60] and renormalize the quantum energy momentum tensors as follows:

$$
\begin{aligned}
\langle T_{00}\rangle &= \frac{1}{8\pi^2 C(\eta)}\int dk k^2\left[2\omega_k + \frac{C^2 m^4 D^2}{16\omega_k^5} - \frac{C^2 m^4}{64\omega_k^7}(2D''D - D'^2 + 4D'D^2 + D^4) + \frac{7C^3 m^6}{64\omega_k^9}(D'D^2 + D^4) - \frac{105 C^4 m^8 D^4}{1024\omega_k^{11}}\right], \\
\langle T^\mu{}_\mu\rangle &= \frac{1}{4\pi^2 C^2(\eta)}\int dk k^2\left[\frac{Cm^2}{\omega_k} + \frac{C^2 m^4}{8\omega_k^5}(D' + D^2) - \frac{5C^3 m^6 D^2}{32\omega_k^7} - \frac{C^2 m^4}{32\omega_k^7}(D''' + 4D''D + 3D'^2 + 6D'D^2 + D^4)\right. \\
&\quad \left. + \frac{C^3 m^6}{128\omega_k^9}(28D''D + 21D'^2 + 126D'D^2 + 49D^4) - \frac{231 C^4 m^8}{256\omega_k^{11}}(D'D^2 + D^4) + \frac{1155 C^5 m^{10} D^4}{2048\omega_k^{13}}\right], \qquad (23)
\end{aligned}
$$

where the lowest-order term of the quantum energy momentum tensor $\langle T_{\mu\nu}\rangle$ actually diverges,

$$
\langle T_{00}\rangle_{\text{diverge}} = \frac{1}{4\pi^2 C(\eta)}\int dk k^2\omega_k \to \infty. \qquad (24)
$$

By adopting the dimensional regularization, the quantum energy momentum tensor $\langle T_{\mu\nu}\rangle$ can be regularized as

$$\langle T_{00}\rangle_{\rm reg} = -\frac{m^4 C}{64\pi^2}\left[\frac{1}{\epsilon}+\frac{3}{2}-\gamma+\ln 4\pi+\ln\frac{\mu^2}{m^2}\right]+\frac{m^2D^2}{384\pi^2} -\frac{1}{2880\pi^2 C}\left(\frac{3}{2}D''D-\frac{3}{4}D'^2-\frac{3}{8}D^4\right),$$
$$\langle T^\mu{}_\mu\rangle_{\rm reg} = -\frac{m^4}{32\pi^2 C}\left[\frac{1}{\epsilon}+1-\gamma+\ln 4\pi+\ln\frac{\mu^2}{m^2}\right] +\frac{m^2D^2}{192\pi^2 C}(2D'+D^2)-\frac{1}{960\pi^2C^2}(D'''-D'D^2), \tag{25}$$

where μ is the renormalization parameter and γ is the Euler-Mascheroni constant. The $1/\epsilon$ terms represent the UV divergences, and they must be absorbed by the counter-terms of the gravitational action.

Hence, the renormalized energy momentum tensor for flat spacetime is

$$\langle T_{00}\rangle_{\rm ren} = \frac{m^4 C}{64\pi^2}\left(\ln\frac{m^2}{\mu^2}-\frac{3}{2}\right)+\frac{m^2D^2}{384\pi^2} -\frac{1}{2880\pi^2 C}\left(\frac{3}{2}D''D-\frac{3}{4}D'^2-\frac{3}{8}D^4\right),$$
$$\langle T^\mu{}_\mu\rangle_{\rm ren} = \frac{m^4}{32\pi^2 C}\left(\ln\frac{m^2}{\mu^2}-1\right)+\frac{m^2D^2}{192\pi^2 C}(2D'+D^2) -\frac{1}{960\pi^2C^2}(D'''-D'D^2), \tag{26}$$

where the first terms are the running cosmological constant corrections which originate from the lowest adiabatic term. On the other hand, the latter parts express vacuum polarization or quantum particle creation in curved spacetime. The anomaly term of Eq. (26) is consistent with using dimensional regularization [61,62] and it is equal to $a_2(x)/16\pi^2$ [3], where $a_2(x)$ is a coefficient of the DeWitt-Schwinger formalism. The conformal anomaly is given by the massless limit of Eq. (26),

$$\langle T^\mu{}_\mu\rangle_{\rm anomaly} = \lim_{m\to 0}\langle T^\mu{}_\mu\rangle_{\rm ren} = -\frac{1}{960\pi^2C^2}(D'''-D'D^2) = -\frac{1}{2880\pi^2}\left[\left(R_{\mu\nu}R^{\mu\nu}-\frac{1}{3}R^2\right)+\Box R\right] = \frac{1}{360(4\pi)^2}E-\frac{1}{180(4\pi)^2}\Box R. \tag{27}$$

By using the adiabatic approximation, we obtain the following expression for a massless fermion [63,64]:

$$\langle T^\mu{}_\mu\rangle^{\rm fermion}_{\rm anomaly} = -\frac{1}{2880\pi^2}\left[11\left(R_{\mu\nu}R^{\mu\nu}-\frac{1}{3}R^2\right)+6\Box R\right] = \frac{11}{360(4\pi)^2}E-\frac{6}{180(4\pi)^2}\Box R. \tag{28}$$

The conformal anomaly for the gauge field in adiabatic expansion is given by [65]

$$\langle T^\mu{}_\mu\rangle^{\rm gauge\ boson}_{\rm anomaly} = -\frac{1}{2880\pi^2}\left[62\left(R_{\mu\nu}R^{\mu\nu}-\frac{1}{3}R^2\right) -(18+15\log\xi)\Box R\right] = \frac{62}{360(4\pi)^2}E+\frac{(18+15\log\xi)}{180(4\pi)^2}\Box R, \tag{29}$$

where ξ is a gauge fixing parameter defined by the covariant gauge fixing term [65]

$$\mathcal{L}_{\rm gf} = -\frac{\sqrt{-g}}{2\xi}(\nabla^\mu A_\mu)^2. \tag{30}$$

The gauge dependence of Eq. (29) also exists in the DeWitt-Schwinger expansion formalism [66–68]. The adiabatic approximation reproduces the gauge dependence of the $\Box R$ term which has also the regularization-scheme dependence. However, the gauge fixing parameter can be removed by the gravitational coupling constants in the Einstein equation and we can drop the gauge fixing parameter ξ. It is found out that the adiabatic expressions for the conformal anomaly precisely match the expression derived by effective action using the dimensional regularization [3].

The renormalized energy momentum tensor $\langle T_{\mu\nu}\rangle_{\rm ren}$ is generally given as follows:

$$\langle T_{\mu\nu}\rangle_{\rm ren} = \alpha_1 H^{(1)}_{\mu\nu}+\alpha_3 H^{(3)}_{\mu\nu}+\alpha_4 H^{(4)}_{\mu\nu}, \tag{31}$$

where the geometric tensor $H^{(4)}_{\mu\nu}$ depends on quantum states [2], and the above equations are defined as fourth-order derivative equations. The dimensionless parameters $\alpha_{1,3}$ for massless fields are given by [2]

$$\alpha_1 = \frac{-1}{2880\pi^2}\left(\frac{N_S}{6}+N_F-3N_G\right), \tag{32a}$$

$$\alpha_3 = \frac{1}{2880\pi^2}\left(N_S+\frac{11}{2}N_F+62N_G\right), \tag{32b}$$

where we consider N_S scalars (spin-0), N_F Dirac fermions (spin-1/2), and N_G abelian gauge fields (spin-1). For instance, the minimal supersymmetric Standard Model takes the following values: $N_S=104$, $N_F=32$, and $N_G=12$. On the other hand, the Standard Model takes the following values: $N_S=4$, $N_F=24$, and $N_G=12$ where the right-handed neutrinos are assumed. Finally, the current Universe has only photon, $N_S=0$, $N_F=0$, and $N_G=1$. It is found that the large number of the scalar fields or fermions lead to the negative α_1 and induce the runaway solutions.

B. Covariant conservation laws

Briefly, we comment covariant conservation laws for the quantum energy momentum tensor [8]. The energy momentum tensor in both classical and quantum cases must satisfy the covariant conservation laws,

$$\nabla_\mu \langle T^{\mu\nu} \rangle_{\rm ren} = 0, \tag{33}$$

which means energy and momentum conservations. We rewrite the trace of Eq. (31) for massless conformally invariant fields in terms of the scale factor $a(t)$,

$$\langle T^\mu{}_\mu \rangle_{\rm ren} = -36\alpha a^{-3}[a^2 a^{(4)} + 3a\dot{a}a^{(3)} + a\ddot{a}^2 - 5\dot{a}^2\ddot{a}] + 12\beta a^{-3}\ddot{a}\dot{a}^2, \tag{34}$$

where the dots and bracketed superscripts denote differentiation with respect to t. Using the above expression, Eq. (33) derives the renormalized vacuum energy density,

$$\rho_{\rm ren} = -36\alpha a^{-4}\left[a^2\dot{a}a^{(3)} + a\dot{a}^2\ddot{a} - \frac{1}{2}a^2\ddot{a}^2 - \frac{3}{2}\dot{a}^4\right] + 3\beta a^{-4}\dot{a}^4 + Ca^{-4}, \tag{35}$$

where C is a constant from integration and the last term corresponds to the thermal radiation $\rho \propto a^{-4}$. Hence, the renormalized expression of the energy momentum tensor of Eq. (31) satisfies the covariant conservation laws. From here, we drop the constant C for simplicity.

C. Gravitational thermodynamics and averaged null energy condition

In this section, let us briefly discuss several problems of the semiclassical gravity such as violations of the gravitational thermodynamical law which is characterized by thermodynamical entropy S and the averaged null energy condition. Famously, the black hole thermodynamics assumes that black holes have the entropy, quantified by the area of the event horizon,

$$S_{\rm BH} = \frac{A}{4G_N}, \tag{36}$$

where the horizon area A is quantified by the surface gravity κ and the mass $M_{\rm BH}$ of stationary black holes,

$$dM_{\rm BH} = \frac{\kappa dA}{8\pi G_N} + (\text{rotation and charge terms}). \tag{37}$$

Classically, the black holes acquire mass from other massive objects and $S_{\rm BH}$ always increases. This fact matches the thermodynamical interpretation of the entropy. However, the black holes may lose its mass due to the Hawking radiation with the temperature,

$$T_{\rm H} = \frac{\kappa}{2\pi}, \tag{38}$$

and thus $S_{\rm BH}$ decreases. On the other hand, thermal character of the event horizon in de Sitter space formally defines de Sitter entropy,

$$S_{\rm dS} = \frac{\pi H^{-2}}{G_N}, \tag{39}$$

where the horizon area is given by $A = 4\pi H^{-2}$ and the time evolution is written as follows:

$$\frac{dS_{\rm dS}}{dt} = -\frac{2\pi H^{-3}\dot{H}}{G_N}. \tag{40}$$

By using the de Sitter entropy $S_{\rm dS}$, one can get interesting consequences such as a no-go theorem for slow-roll eternal inflation. Let us consider slow-roll inflation driven by an inflaton field ϕ. For the slow-roll inflation, we have

$$\dot{H} = -4\pi G_N \dot{\phi}^2. \tag{41}$$

Hence, the de Sitter entropy $S_{\rm dS}$ is rewritten as

$$\frac{dS_{\rm dS}}{dN} = -\frac{2\pi\dot{H}}{G_N H^4} = \frac{8\pi^2\dot{\phi}^2}{H^4} \sim \left(\frac{\delta\rho}{\rho}\right)^{-2} \gtrsim 1, \tag{42}$$

where N is the number of e-foldings defined by $dN = Hdt$, ρ is the energy density, and $\delta\rho$ is the energy density perturbation satisfying $|\delta\rho/\rho| \lesssim 1$. The total number of e-folding $N_{\rm tot}$ is bounded as follows [69]:

$$N_{\rm tot} \lesssim \Delta S = S_{\rm end} - S_{\rm ini}, \tag{43}$$

where $S_{\rm end}$ and $S_{\rm ini}$ are the de Sitter entropy at the end and the beginning of the inflation, respectively. For the large field inflation, the entropy at the beginning is much smaller than that at the end, and then one get $\Delta S \sim S_{\rm end}$ but $\Delta S \ll S_{\rm end}$ for the small field inflation. In any case, the total e-folding number $N_{\rm tot}$ is strictly restricted.

Let us discuss a more general case. For flat FLRW Universe, the Friedmann equations yield a simple equation

$$\dot{H} = -4\pi G_N(\rho + P). \tag{44}$$

Hence, the de Sitter entropy can be written as follows:

$$\frac{dS_{\rm dS}}{dt} = 8\pi^2 H^{-3}(\rho + P) \geq 0, \tag{45}$$

which always increases and matches gravitational thermodynamical laws when the null energy condition (NEC) is satisfied,

$$T_{\mu\nu}k^\mu k^\nu \geq 0 \Rightarrow \rho + P \geq 0, \tag{46}$$

where k^μ is the null (lightlike) vector. It is known that the NEC and the gravitational thermodynamical laws are closely related with each other [69]. In general relativity, the NEC is a necessary condition to eliminate any pathological spacetime or unphysical consequences such as wormhole, geometric instability, and superluminal propagation. It is well known that the classical matters always satisfy the NEC, and in this sense the classical general relativity does not violate any gravitational principles. However, these classical conditions can be easily violated in quantum field theory (QFT) and the NEC is broken even for quantum fields in Minkowski spacetime [70] (e.g., squeezed vacuum states [71]). More generally, the averaged null energy condition (ANEC) [72], which is satisfied in Minkowski spacetime [73] and prohibits a traversable wormhole [74], has been proposed,

$$\int_\gamma T_{\mu\nu}k^\mu k^\nu dl \geq 0 \Rightarrow \int_{-\infty}^{\infty} \frac{1}{a}(\rho + P)dt \geq 0, \tag{47}$$

where the integral is taken over a null geodesic γ, k^μ is the parameterized tangent vector to the geodesic, and l is the affine parameter [75]. However, it has been known that curved spacetime, i.e., semiclassical gravity violates the NEC or ANEC [76–79] and the de Sitter entropy decreases [80]. This seems very plausible because the energy density undergoes quantum vacuum fluctuations and the variances of the energy density would be allowed to be both negative and positive in QFT. Although the so-called quantum inequalities [81] have been proposed, in principle, there is no lower limit for the negative vacuum energy and one can take any nonphysical spacetime. In a nutshell, such quantum effects on gravity require careful discussion and theory should not break these basic gravitational principles.

Finally, let us briefly see the violations of the NEC, ANEC, and gravitational thermodynamical laws in semiclassical gravity and simply consider conformal anomaly $\langle T^\mu{}_\mu\rangle_{\rm anomaly}$ in Eq. (67). For the semiclassical gravity, the NEC, ANEC, and de Sitter entropy can be violated with various conditions as follows:

$$\rho + P = 12\alpha_1(6\dot{H}^2 + 3H\ddot{H} + \dddot{H}) - 4\alpha_3 H^2\dot{H} \ngeq 0, \tag{48}$$

$$\int_{-\infty}^{\infty} \frac{1}{a}(\rho + P)dt = \int_{-\infty}^{\infty}\left\{\frac{12\alpha_1}{a}(6\dot{H}^2 + 3H\ddot{H} + \dddot{H}) - \frac{4\alpha_3}{a}H^2\dot{H}\right\}dt \ngeq 0, \tag{49}$$

$$\frac{dS_{\rm dS}}{dt} = 96\pi^2\alpha_1(6H^{-3}\dot{H}^2 + 3H^{-2}\ddot{H} + H^{-3}\dddot{H}) - 32\pi^2\alpha_3 H^{-1}\dot{H} \ngeq 0, \tag{50}$$

which suggests that semiclassical gravity is incompatible with basic gravitational principles [69].

III. QUANTUM SPACETIME INSTABILITY

We now turn to a more quantitative discussion of the validity of the semiclassical gravity. The quantum energy momentum tensor holding higher-derivative terms modifies the Einstein's equations and destabilizes the classical solutions. In this section, we investigate the quantum spacetime instabilities in semiclassical gravity and consider the cosmological dynamics of the Universe.

First, we will revisit some results of Refs. [19,24,25] and discuss the instability of the Minkowski spacetime under small perturbations. Next, we discuss the instabilities of the homogenous and isotropic FLRW Universe, and consider the cosmological dynamics. Our results suggest that the corresponding solutions of the semiclassical gravity cannot be incompatible with the cosmological observations.

For simplicity, we consider the semiclassical Einstein's equations for the massless conformally invariant fields and the classical radiation or nonrelativistic matters,

$$\frac{1}{8\pi G_N}\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu}\right) = \langle T_{\mu\nu}\rangle_{\rm ren} + T^{\rm c}_{\mu\nu} = \alpha_1 H^{(1)}_{\mu\nu} + \alpha_3 H^{(3)}_{\mu\nu} + T^{\rm c}_{\mu\nu}, \tag{51}$$

where $T^{\rm c}_{\mu\nu}$ is the energy momentum tensor for ordinary radiation or nonrelativistic matters.

A. Instability of Minkowski spacetime under conformally flat perturbations

Let us consider the Minkowski spacetime with no matter fields and investigate whether the spacetime is stable under the perturbations. Although in classical general relativity, the Minkowski spacetime should be completely stable, semiclassical gravity does not ensure this important fact. References [19–26] have shown that the Minkowski spacetime is unstable under small perturbations, and the perturbations either grow exponentially or oscillate even in Planck time. This leads to disaster. Before studying the FRLW instabilities and considering the influence on the current Universe, let us reconsider some results of the Minkowski instability of Refs. [19,24,25].

In the Minkowski spacetime, the geometric tensors in semiclassical Einstein's equations satisfy

$$G_{\mu\nu} = H^{(1)}_{\mu\nu} = H^{(3)}_{\mu\nu} = 0. \tag{52}$$

Now, we consider only conformally flat perturbations and write the metric as follows:

$$g_{\mu\nu} = \Omega^2\eta_{\mu\nu}, \tag{53}$$

where $\eta_{\mu\nu}$ is the Minkowski metric ($+---$ convention) and Ω is the conformal parameter.

The corresponding Ricci tensor $R_{\mu\nu}$ is given by

$$R_{\mu\nu} = 4\Omega^{-2}(\partial_\mu\Omega)(\partial_\nu\Omega) - 2\Omega^{-1}\partial_\mu\partial_\nu\Omega - \Omega^{-1}\eta_{\mu\nu}(\partial^\alpha\partial_\alpha\Omega) - \Omega^{-2}\eta_{\mu\nu}(\partial^\alpha\Omega\partial_\alpha\Omega), \tag{54}$$

where $\Omega = 1$ reproduces the Minkowski spacetime solution. Now, we rewrite Eq. (51) using this expression with the conformally flat perturbations $\Omega = 1 + \gamma$ and one can obtain [19]

$$-\partial_\mu\partial_\nu\gamma + (\Box\gamma)\eta_{\mu\nu} + 48\pi\alpha_1 G_N[-\partial_\mu\partial_\nu(\Box\gamma) + \Box(\Box\gamma)\eta_{\mu\nu}] = 0, \tag{55}$$

where $\Box \equiv \eta^{\alpha\beta}\partial_\alpha\partial_\beta = \partial^\alpha\partial_\alpha$. This perturbation equation for γ can be rewritten by a simple equation

$$(\partial_\mu\partial_\nu - \eta_{\mu\nu}\Box)f = 0, \tag{56}$$

where we define $f \equiv (\gamma + 48\pi\alpha_1 G_N\Box\gamma)$ and the general solution is clearly

$$f = k^\alpha x_\alpha + \text{const.}, \tag{57}$$

where k^α expresses a constant vector field and x^α is a position vector field in the Minkowski spacetime. Hence, we can get the following equation:

$$(1 + 48\pi\alpha_1 G_N\Box)\gamma = k^\alpha x_\alpha + \text{const.} \tag{58}$$

The most general solution to the above equation is

$$\gamma = k^\alpha x_\alpha + \chi + \text{const.}, \tag{59}$$

where inhomogeneous solution $\gamma = k^\alpha x_\alpha + \text{const.}$ is pure gauge in the Minkowski spacetime and the metric perturbation χ satisfies the Klein-Gordon equation

$$\Box\chi + \frac{1}{48\pi\alpha_1 G_N}\chi = 0, \tag{60}$$

which admits the spatially homogeneous solutions

$$\chi = C_1 \sin(\omega t) + C_2 \cos(\omega t), \qquad C_3 e^{t/\tau}, \tag{61}$$

where $\omega = (48\pi\alpha_1 G_N)^{1/2}$ and $C_{1,2,3}$ are constants. The first solution is for $\alpha_1 > 0$ and consistent with the ordinary Klein-Gordon equation. However, if one takes $48\pi\alpha_1 \sim \mathcal{O}(1)$, the perturbations oscillate in the Planck time $t_\mathrm{P} = (G_N)^{1/2} = 10^{-43}$ sec and they emit the Planck energy photons, $E \sim 10^{19}$ GeV [19], which is unreasonable for the observed Universe. The last possible solution is given for $\alpha_1 < 0$. We denote $\tau = (48\pi\alpha_1 G_N)^{1/2}$. This corresponds to the Klein-Gordon equation with a negative mass and suggests that the perturbations exponentially grow even in the Planck time.

Once the Minkowski spacetime is perturbed, the perturbations lead to a catastrophe. For $\alpha_1 < 0$, the instability timescale can be summarized as

$$t_\mathrm{I} = (48\pi\alpha_1 G_N)^{1/2} = (48\pi\alpha_1)^{1/2} \cdot 10^{-43}\ \text{sec} = (48\pi\alpha_1/10^{118})^{1/2}\ \text{Gyr}. \tag{62}$$

The instability time t_I must be as large as the age of the observed Universe, $t_\mathrm{Age} = 13.787 \pm 0.020$ Gyr [56] (Planck 2018, TT, TE, EE + lowE + lensing + BAO 68% limits). Otherwise, the perturbations or scale factor exponentially grows and our Universe is seriously destabilized. Hence, we obtain the stable condition against quantum backreaction

$$\alpha_1 \gtrsim 10^{118}, \tag{63}$$

which requires a large value of the gravitational curvature coupling or a large number of the particle species $\mathcal{N} \sim 10^{118}$ for the high energy theory. Hence, the $\alpha_1 < 0$ case is trouble for the homogenous and isotropic flat Universe. We will confirm these results in different methods in the next subsection.[2]

B. Numerical analysis for Minkowski spacetime instability under perturbations

Let us consider the FLRW spacetime with matter fields and study the spacetime insatiabilities from the quantum backreaction.[3] For the Minkowski spacetime, we have no classical matter and no cosmological constant.

By using Eq. (51), we obtain the following semiclassical equation [82]:

$$\frac{dR}{dt} = \frac{1}{12H}R^2 - HR - \frac{H}{16\pi G_N\alpha_1} + \frac{\alpha_3}{\alpha_1}H^3. \tag{64}$$

If we assume that the third term of Eq. (64) can dominate due to the smallness of $16\pi G_N\alpha_1$, we can approximately rewrite

$$\frac{d^2H}{dt^2} \approx -\frac{1}{16\pi G_N\alpha_1}H, \tag{65}$$

where we consider $H(0) \approx 0$, $R(0) \approx 0$. This admits the exponential or oscillating solutions for the Planck time t_P

[2] Taking the following conformal parameter,

$$g_{\mu\nu} = \Omega^2\eta_{\mu\nu} = a^2(\eta)\eta_{\mu\nu},$$

one can get a similar consequence for the scale factor $a(\eta)$.

[3] The similar analysis was given recently by one of the authors [80].

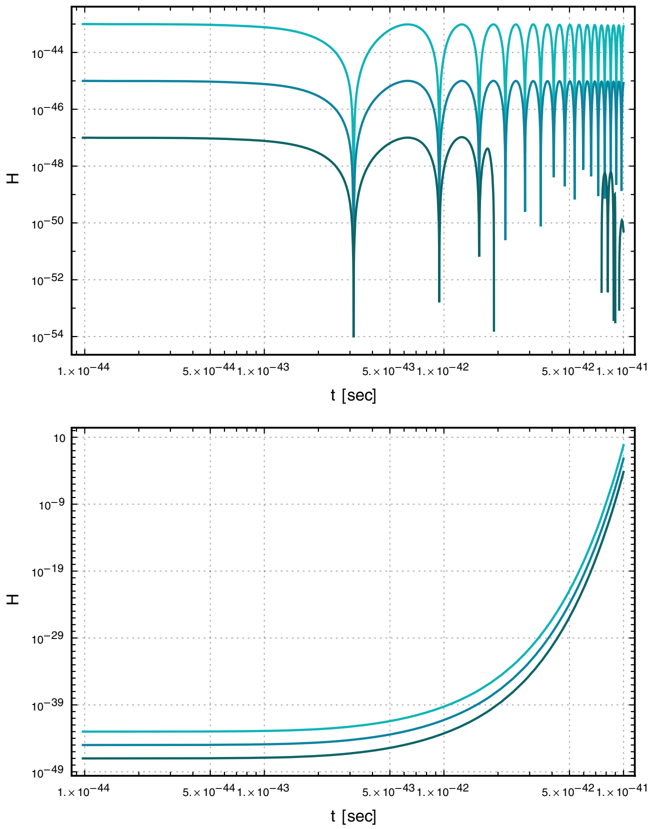

FIG. 1. We consider the instability of the Minkowski spacetime under perturbations and show that the dynamics of the Hubble perturbation $H(t)$ around the Planck time $t_{\mathrm{P}} = 10^{-43}$ sec. We assume the initial conditions and the couplings of Eq. (66). The top figure assumes $\alpha_1 = 10^{-86}$, whereas the bottom figure takes $\alpha_1 = -10^{-86}$. The top-to-bottom lines correspond to $H_0 = 10^{-43,-45,-48}$, respectively.

for $16\pi\alpha_1 \sim \mathcal{O}(1)$. Thus, these are consistent with the previous discussion. Next, we numerically confirm the above analytical estimations. Let us assume the Minkowski spacetime perturbed by the small Hubble variations. Using Eq. (64) and $R = 6(\dot{H} + 2H^2)$, we obtain the differential equation. We investigate the system of equations starting at $t = 0$ with various conditions and perturbations. It is found that the numerical solutions of the system show that the Minkowski spacetime is unstable and they are consistent with the above analytical estimation.

In Fig. 1, we demonstrate numerical results for the Hubble perturbation $H(t)$ determined by Eq. (72) with the following conditions and couplings:

$$\begin{aligned} &\text{Fig. 1:}\ H_0 = 10^{-43,-45,-48}, \qquad \dot{H}_0 = 0, \\ &48\pi\alpha_1 G_N = \pm 10^{-86}, \qquad 8\pi G_N \alpha_3 = 0, 10^{-86}, \end{aligned} \tag{66}$$

where we set these parameters with respect to the Planck time $t_{\mathrm{P}} = 10^{-43}$ and it is found that magnitudes of the perturbation $\Upsilon(t)$ and values of α_3 are irreverent for the dynamics. We found that the Hubble oscillations with the Planck frequency occur for $\alpha_1 > 0$, whereas for $\alpha_1 < 0$, the Hubble perturbations exponentially grow even in the Planck time t_{P}. The small values of α_1 lead to faster destabilization, and the stability of the Minkowski spacetime requires a large value of $|\alpha_1|$.

C. Numerical analysis for FLRW spacetime instability

Let us consider the de Sitter spacetime under the Hubble perturbation and discuss the cosmological evolution. Now using Eq. (51) for the FLRW metric, we obtain the semiclassical Friedmann equations

$$\begin{aligned} H^2 = {}& \frac{\Lambda}{3} - 48\pi G_N \alpha_1 (6H^2\dot{H} + 2H\ddot{H} - \dot{H}^2) \\ & + 8\pi G_N \alpha_3 H^4 + \frac{8\pi G_N}{3}\rho_{\mathrm{m}}, \end{aligned} \tag{67}$$

where ρ_{m} is the energy density of the classical matter and satisfies the covariant conservation law

$$\dot{\rho}_{\mathrm{m}} = -3H(\rho_{\mathrm{m}} + P_{\mathrm{m}}) = -3H(1+\omega)\rho_{\mathrm{m}}, \tag{68}$$

where $w = P/\rho$ is an equation-of-state parameter. For the nonrelativistic matter, radiation, and cosmological constant, one takes $w = 0, 1/3, -1$, respectively.

We rewrite the above semiclassical equations in terms of the dimensionless parameters

$$\begin{aligned} h^2 &= -x(6h^2h' + 2hh'' - h'^2) + yh^4 + z, \\ z' &= -3h(1+\omega)z, \end{aligned} \tag{69}$$

where these parameters are given by

$$\begin{aligned} &\tau = H_0 t, \qquad h = H/H_0, \\ &x = 48\pi G_N \alpha_1 H_0^2, \qquad y = 8\pi G_N \alpha_3 H_0^2, \\ &z = \Lambda/3H_0^2 + 8\pi G_N \rho_{\mathrm{m}}/3H_0^2, \end{aligned} \tag{70}$$

where H_0 is the initial Hubble parameter. We note the following relations of these parameters:

$$\begin{aligned} &H_0 \sim 10^{14}\ \mathrm{GeV}, \qquad M_{\mathrm{P}} \sim 10^{18}\ \mathrm{GeV}, \qquad \alpha_{1,3} \sim 10^{-2} \\ &\Rightarrow x, y \sim 10^{-10}, \\ &H_0 \sim 10^{-42}\ \mathrm{GeV}, \qquad M_{\mathrm{P}} \sim 10^{18}\ \mathrm{GeV}, \qquad \alpha_{1,3} \sim 10^{-2} \\ &\Rightarrow x, y \sim 10^{-122}, \end{aligned} \tag{71}$$

where the former and latter Hubble parameters correspond to an example consistent with typical inflation and current Universe, respectively [56]. The dynamics of the dimensionless Hubble parameter h with $w = -1$ for the spacetime is determined by

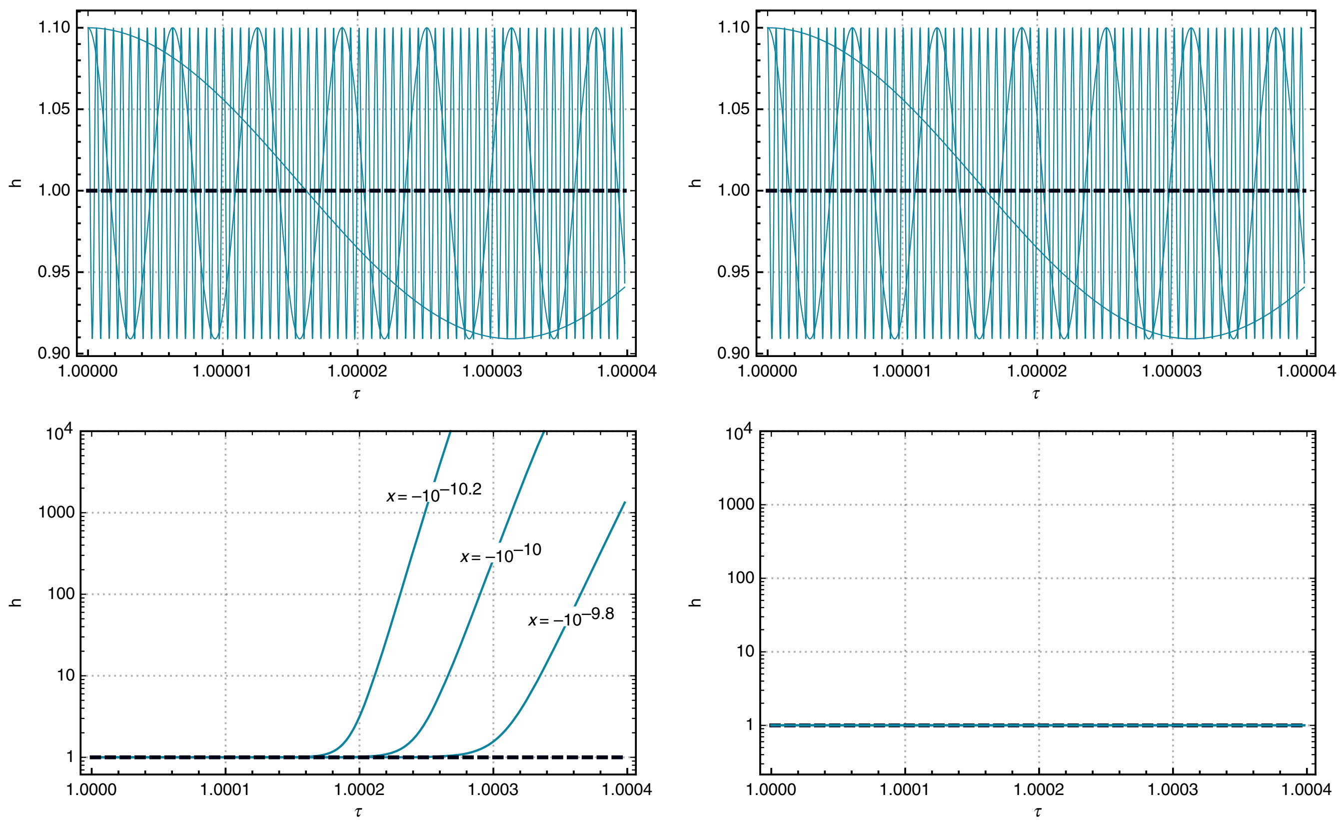

FIG. 2. We show the numerical solution of Eq. (72) with the initial conditions and the higher-derivative couplings of Eq. (74). These figures show that the dynamics of the dimensionless Hubble parameter $h(\tau)$ in a few normalization time τ. The dashed line shows the de Sitter solution $h(\tau) = 1$ from the general relativity. The right panel is $y = 0$, whereas the tight panel is $y = 0$. The top panels show that the solutions oscillate where the top-to-bottom lines correspond to $x = 10^{-10.0,-12.0,-14.0}$. The bottom panels on the other hand show the solutions exponentially grow where the top-to-bottom lines correspond to $x = -10^{-9.8,-10.0,-10.2}$. We found out that the timescale τ_I is $\tau_I \approx |x|^{1/2} \approx 10^{-5}$.

$$h^2 = -x(6h^2h' + 2hh'' - h'^2) + yh^4 + z, \qquad (72)$$

where z is constant and prime expresses the derivative with respect to the dimensionless time τ. The suitable de Sitter initial conditions for Eq. (72) are

$$\tau_0 = 1, \qquad h_0 = 1, \qquad h'_0 = 0, \qquad z_0 = 1. \qquad (73)$$

In order to investigate the de Sitter spacetime instabilities, we consider the numerical solutions of the de Sitter system starting at $\tau_0 = 1$ with various initial conditions and perturbations. In Fig. 2, we present the numerical results for the dimensionless parameter $h(\tau)$ from Eq. (72) with the following conditions:

$$\begin{aligned} \text{Fig. 2}:\ & h_0 = 1 + 10^{-1.0}, \qquad h'_0 = 0, \\ & x = 10^{-10.0,-12.0,-14.0}, \qquad y = 0, 10^{-10.0}, \\ & h_0 = 1, \qquad h'_0 = 0, \\ & x = -10^{-9.8,-10.0,-10.2}, \qquad y = 0, 10^{-10.0}, \end{aligned} \qquad (74)$$

where we compare these results with the de Sitter solution $h(\tau) = 1$ from the general relativity. The top panels show that the Hubble perturbations in the de Sitter spacetime oscillate, and the oscillation timescale becomes shorter for the small values of $|x|$. On the other hand, the bottom panels show that the de Sitter spacetime is destabilized in $\tau \approx |x|^{1/2}$ and the smallness of $|x|$ amplifies the instabilities. The dynamics for $y > 0$ and $y < 0$ shows the similar results, and it is found that y is irreverent for the dynamics.

Next, we consider matter-dominated stage of the Universe with $w = 0$. The natural initial conditions for the system are given by

$$\tau_0 = 2/3, \qquad h_0 = 1, \qquad h'_0 = -3/2, \qquad z_0 = 1, \qquad (75)$$

where we take $z = 8\pi G_N \rho_m / 3H_0^2$ and $\Lambda = 0$. In Fig. 3, we investigate the system of equations starting at $\tau_0 = 2/3$ with the following conditions:

$$\begin{aligned} \text{Fig. 3}:\ & h_0 = 1 + 0.3, \qquad h'_0 = -3/2, \\ & x = 10^{-1.0,-3.0,-5.0} \qquad y = 10^{-1.0}, \\ & h_0 = 1, \qquad h'_0 = -3/2, \\ & x = -10^{-1.0,-1.5,-2.0}, \qquad y = 10^{-1.0}, \end{aligned} \qquad (76)$$

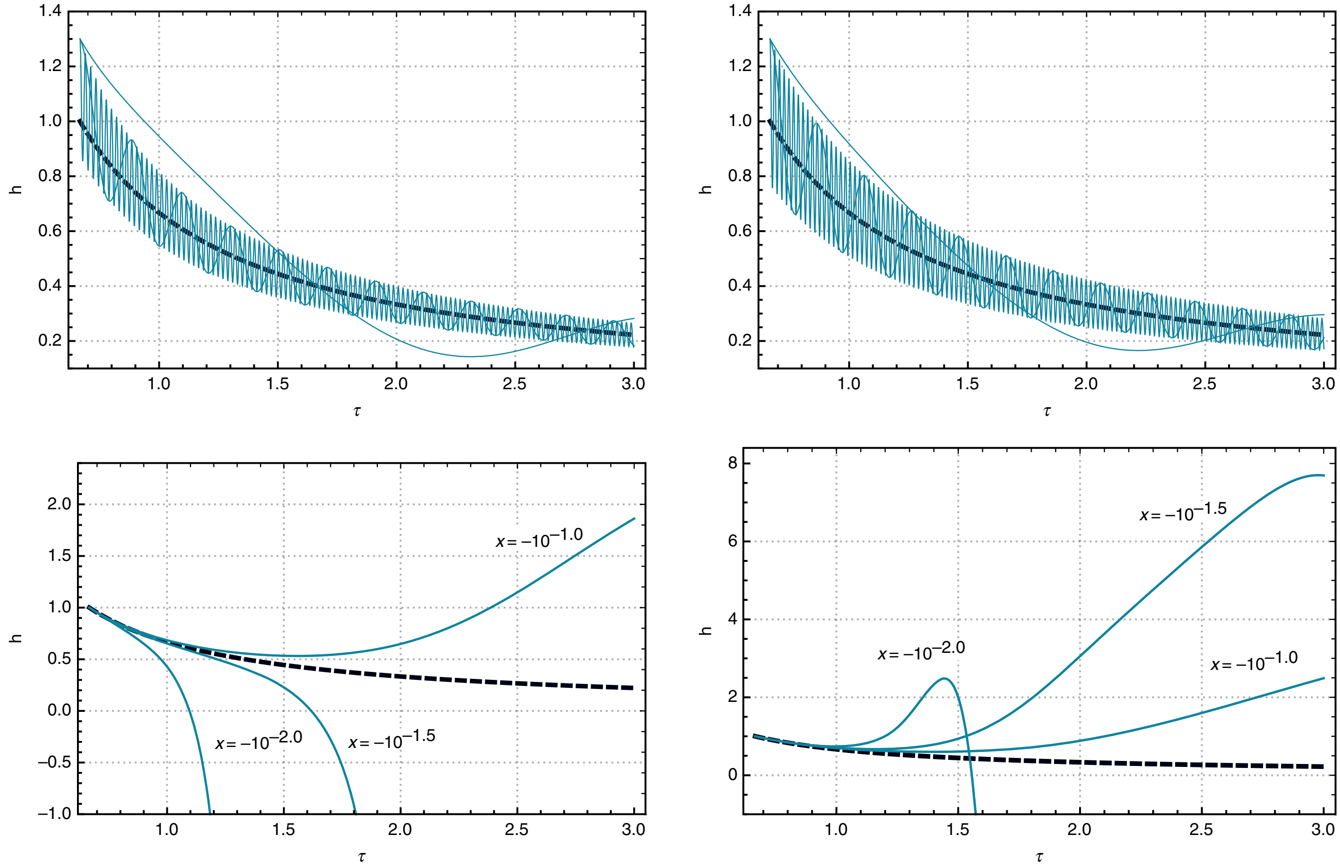

FIG. 3. We compare the numerical solution of Eq. (72) with the conditions of Eq. (76) and the standard solution $h(\tau) = 1/2\tau$ from the general relativity is presented by the dashed line. The right panel is $y = 0$, whereas the tight panel is $y = 0$. The top panels show that the solutions oscillate for $x = 10^{-1.0,-3.0,-5.0}$. The bottom panels where the top-to-bottom lines correspond to $x = -10^{-1.0,-1.5,-2.0}$ show the instabilities for the matter-dominated Universe and the solutions are inconsistent with the standard general relativity.

where we compare them with the standard cosmic solution $h(\tau) = 2/3\tau$. The top panels show that the matter-dominated Universe is unstable for the small perturbations but the solutions converge the general relativity. The bottom panels show that the matter-dominated Universe is unstable for $\tau \approx |x|^{1/2}$ and inconsistent with the usual general relativity. In this case, the instability timescale τ_I should be larger than of order unity,

$$\tau_I \approx |x|^{1/2} \gtrsim \mathcal{O}(1), \tag{77}$$

and thus using the current value of the Hubble parameter $H_0 \sim 10^{-42}$ GeV, we obtain the following constraint:

$$|\alpha_1| \gtrsim 10^{118}, \tag{78}$$

which is consistent with Eq. (63). Now, we confirmed the results of the previous subsection and found out that the FLRW spacetime is unstable under the perturbations for $\alpha_1 > 0$ or the evolution for $\alpha_1 < 0$. Since the theoretically expected value of α_1 is given by Eq. (32), the above constraint is unacceptably large.

D. Instability of the current Universe

Finally, we consider cosmological evolution of the Universe with the quantum backreaction and derive a strict cosmological constraint for the semiclassical gravity. Before considering the detail, let us review the standard cosmological history of the Universe based on ΛCDM with inflation. First, the Universe proceeds inflation [10,83–86] (around 10^{-35}–10^{-32} sec). Second, the inflation ends and thermal radiation dominates up to the recombination with the cosmic microwave background radiation (around 379,000 years). After the radiation-dominated stage, the Universe is dominated by nonrelativistic matters, and then galaxies and clusters are gradually formed. From about 9.8 Gyr, the expansion of the Universe begins to accelerate via unknown dark energy. The age of the current Universe is about 13.8 Gyr and the present value of the Hubble parameter is $H_0 \approx 67.7$ km/s.MPc.

We consider the semiclassical Friedmann equations of Eq. (67) for the conditions of ΛCDM,

$$H^2(t_0) = H_0^2[\Omega_{\Lambda,0} + \Omega_{\mathrm{m},0} + \Omega_{\mathrm{r},0}],$$
$$\Omega_{\Lambda,0} + \Omega_{\mathrm{m},0} + \Omega_{\mathrm{r},0} \approx 1, \tag{79}$$

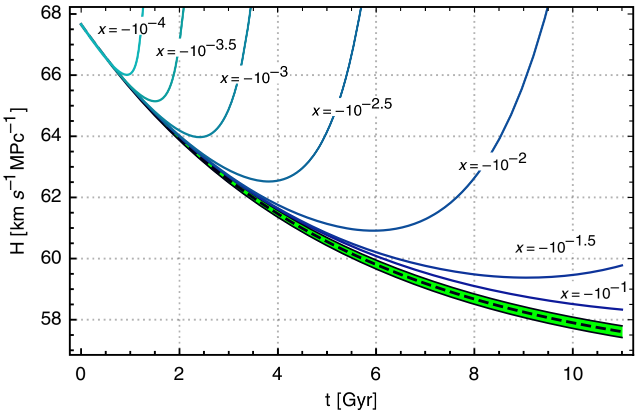

FIG. 4. In this figure, we compare standard result of the ΛCDM and the numerical solutions of Eq. (81) with the conditions of Eqs. (80) and (83). The dashed line corresponds to the central value of the Planck data [56], and the green line expresses the allowed region of the Planck data (Planck 2018, TT, TE, EE + lowE + lensing + BAO 68% limits). The semiclassical gravity or conformal anomaly expects $|x| \approx 10^{-122}$ and it is not consistent with the observations.

where $\Omega_{\Lambda,0}$ is the density parameter for the cosmological constant and $\Omega_{\mathrm{m},0}$, $\Omega_{\mathrm{r},0}$ are the current density values of nonrelativistic matters, including dark matter and radiation. From the recent Planck data [56] (Planck 2018, TT, TE, EE + lowE + lensing + BAO 68% limits), we have

$$\begin{aligned} H_0 &\approx 67.66 \pm 0.42\ [\mathrm{km\,s^{-1}\,MPc^{-1}}] \\ &\approx 7.25 \times 10^{-2}\ [\mathrm{Gyr^{-1}}], \\ \Omega_{\Lambda,0} &= 0.6889 \pm 0.0056, \\ \Omega_{\mathrm{m},0} &= 0.3111 \pm 0.0056, \end{aligned} \tag{80}$$

where $(G_N)^{1/2} = 10^{-59}$ Gyr. For simplicity, we rewrite the semiclassical Friedmann equations as follows:

$$\begin{aligned} h^2 &= -x(6h^2h' + 2hh'' - h'^2) + yh^4 + \lambda_0 + z, \\ z' &= -3hz, \end{aligned} \tag{81}$$

where the dimensionless parameters are given by

$$\begin{aligned} &\tau = H_0 t, \qquad h = H/H_0, \\ &x = 48\pi G_N \alpha_1 H_0^2, \qquad y = 8\pi G_N \alpha_3 H_0^2, \\ &\lambda_0 = \Lambda/3H_0^2, \qquad z = 8\pi G_N \rho_\mathrm{m}/3H_0^2. \end{aligned} \tag{82}$$

In Fig. 4, we assume the following initial conditions and various couplings based on the ΛCDM:

$$\begin{aligned} \text{Fig. 4:}\ & x = -10^{-1.0,-2.0,-3.0,-4.0}, \qquad y = 0, \\ & h_0 = 1, \qquad h_0' = -3/2\Omega_{\mathrm{m},0}, \\ & \lambda_0 = \Omega_{\Lambda,0} \qquad z_0 = \Omega_{\mathrm{m},0}. \end{aligned} \tag{83}$$

Figure 4 shows the spacetime instabilities for the small $|x|$ in a short time, and the corresponding spacetime solutions are inconsistent with the future or current evolution of the Universe unless one takes $|x|^{1/2} \approx \mathcal{O}(1)$. However, the semiclassical gravity expects the extremal small value $|x|^{1/2} \approx 10^{-61}$ for the current Universe and that is not consistent with the observations.

The spacetime instability of the semiclassical gravity is certainly a serious problem and the solutions are not consistent with the cosmological observation. It has been argued that the semiclassical solutions must be given by the truncating perturbative expansions [87,88] and the quantum higher-derivative corrections can be regarded as small perturbations from the classical solution. However, this procedure is *ad hoc* approach for the semiclassical equations much below the Planck scale and ineffective near the Planck regime. That is a problem when one considers large N expansion where the semiclassical gravity could be adequate to describe Planckian phenomenon due to the suppression of the graviton loops [21]. Also, the Euclidean formulation of quantum gravity imposes the boundary condition and the curvature instability or runaway solutions might be removed [89]. However, it is not clear how to handle the quantum energy momentum tensor $\langle T_{\mu\nu} \rangle$ in these procedures and the problem of the quantum instability is still left open.

IV. CONCLUSION

Semiclassical gravity describe the interactions between classical gravity and quantum matters, and the quantum backreaction is formally defined as the higher-derivative curvatures. These induce instabilities of the classic solutions and Refs. [19–26] presented that the Minkowski spacetime is unstable under small perturbations. The spacetime instability was seen as a serious problem in semiclassical gravity. However, it has not been discussed whether the semiclassical instabilities are inconsistent in our Universe.

In this paper, we have shown that the homogenous and isotropic FLRW Universe interacting with quantum matter fields is unstable under small perturbations or the evolutions. We have analytically and numerically demonstrated that the homogenous and isotropic cosmological solutions either grow exponentially or oscillate even in the Planckian time $t_\mathrm{I} = \alpha_1 10^{-43}$ sec. For $\alpha_1 > 0$, the curvature perturbations oscillate rapidly and would emit the Planck energy photons [19] which is unacceptable for the observed Universe. On the other hand, for $\alpha_1 < 0$, the evolution of the curvature perturbation leads to the Planckian curvature or singularity. These instabilities induce a catastrophe unless one takes extremal values of the gravitational couplings or fundamental particle species $|\alpha_1| \gtrsim 10^{118}$. We have also confirmed these results based on the cosmological evolution by comparing ΛCDM and the semiclassical Einstein solutions using the Planck data, and it is found that these solutions of the semiclassical gravity including conformal anomaly are not consistent with cosmological observations.

ACKNOWLEDGMENTS

N. W. would like to thank Shingo Kukita for the valuable discussions.

[1] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975).
[2] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space*, Cambridge Monographs on Mathematical Physics (Cambridge University Press, Cambridge, United Kingdom, 1984).
[3] I. L. Buchbinder, S. D. Odintsov, and I. L. Shapiro, *Effective Action in Quantum Gravity* (Taylor and Francis, New York, 1992).
[4] W. G. Unruh, Phys. Rev. D **14**, 870 (1976).
[5] S. M. Christensen and S. A. Fulling, Phys. Rev. D **15**, 2088 (1977).
[6] P. Candelas, Phys. Rev. D **21**, 2185 (1980).
[7] P.-M. Ho, H. Kawai, Y. Matsuo, and Y. Yokokura, J. High Energy Phys. 11 (2018) 056.
[8] P. C. W. Davies, Phys. Lett. **68B**, 402 (1977).
[9] M. V. Fischetti, J. B. Hartle, and B. L. Hu, Phys. Rev. D **20**, 1757 (1979).
[10] A. A. Starobinsky, Phys. Lett. **91B**, 99 (1980).
[11] P. Anderson, Phys. Rev. D **28**, 271 (1983).
[12] P. R. Anderson, Phys. Rev. D **29**, 615 (1984).
[13] T. Azuma and S. Wada, Prog. Theor. Phys. **75**, 845 (1986).
[14] S. Nojiri and S. D. Odintsov, Phys. Lett. B **562**, 147 (2003).
[15] S. Nojiri and S. D. Odintsov, Phys. Lett. B **595**, 1 (2004).
[16] R. Utiyama and B. S. DeWitt, J. Math. Phys. (N.Y.) **3**, 608 (1962).
[17] K. S. Stelle, Phys. Rev. D **16**, 953 (1977).
[18] F. d. O. Salles and I. L. Shapiro, Phys. Rev. D **89**, 084054 (2014); **90**, 129903(E) (2014).
[19] G. T. Horowitz and R. M. Wald, Phys. Rev. D **17**, 414 (1978).
[20] G. T. Horowitz, Phys. Rev. D **21**, 1445 (1980).
[21] J. B. Hartle and G. T. Horowitz, Phys. Rev. D **24**, 257 (1981).
[22] S. Randjbar-Daemi, J. Phys. A **14**, L229 (1981).
[23] R. D. Jordan, Phys. Rev. D **36**, 3593 (1987).
[24] W.-M. Suen, Phys. Rev. D **40**, 315 (1989).
[25] W. M. Suen, Phys. Rev. Lett. **62**, 2217 (1989).
[26] P. R. Anderson, C. Molina-Paris, and E. Mottola, Phys. Rev. D **67**, 024026 (2003).
[27] L. H. Ford, Phys. Rev. D **31**, 710 (1985).
[28] E. Mottola, Phys. Rev. D **31**, 754 (1985).
[29] E. Mottola, Phys. Rev. D **33**, 1616 (1986).
[30] I. Antoniadis, J. Iliopoulos, and T. N. Tomaras, Phys. Rev. Lett. **56**, 1319 (1986).
[31] I. Antoniadis and E. Mottola, J. Math. Phys. (N.Y.) **32**, 1037 (1991).
[32] A. Higuchi, Classical Quantum Gravity **4**, 721 (1987).
[33] D. Polarski, Phys. Rev. D **41**, 442 (1990).
[34] N. C. Tsamis and R. P. Woodard, Phys. Lett. B **301**, 351 (1993).
[35] N. C. Tsamis and R. P. Woodard, Ann. Phys. (N.Y.) **238**, 1 (1995).
[36] N. C. Tsamis and R. P. Woodard, Commun. Math. Phys. **162**, 217 (1994).
[37] N. C. Tsamis and R. P. Woodard, Nucl. Phys. **B474**, 235 (1996).
[38] N. C. Tsamis and R. P. Woodard, Phys. Rev. D **54**, 2621 (1996).
[39] V. F. Mukhanov, L. R. W. Abramo, and R. H. Brandenberger, Phys. Rev. Lett. **78**, 1624 (1997).
[40] L. R. W. Abramo, R. H. Brandenberger, and V. F. Mukhanov, Phys. Rev. D **56**, 3248 (1997).
[41] N. Goheer, M. Kleban, and L. Susskind, J. High Energy Phys. 07 (2003) 056.
[42] R. H. Brandenberger, in *18th IAP Colloquium on the Nature of Dark Energy: Observational and Theoretical Results on the Accelerating Universe Paris, France, 2002* (CERN, Paris, France, 2002), http://cds.cern.ch/record/522549.
[43] S. Kachru, R. Kallosh, A. D. Linde, and S. P. Trivedi, Phys. Rev. D **68**, 046005 (2003).
[44] F. Finelli, G. Marozzi, G. P. Vacca, and G. Venturi, Phys. Rev. D **71**, 023522 (2005).
[45] T. Janssen and T. Prokopec, Classical Quantum Gravity **25**, 055007 (2008).
[46] T. Janssen and T. Prokopec, Ann. Phys. (N.Y.) **325**, 948 (2010).
[47] T. Janssen, S.-P. Miao, and T. Prokopec, arXiv:0807.0439.
[48] A. M. Polyakov, Nucl. Phys. **B834**, 316 (2010).
[49] A. Shukla, S. P. Trivedi, and V. Vishal, J. High Energy Phys. 12 (2016) 102.
[50] P. R. Anderson and E. Mottola, Phys. Rev. D **89**, 104038 (2014).
[51] P. R. Anderson and E. Mottola, Phys. Rev. D **89**, 104039 (2014).
[52] R. Myrzakulov, S. Odintsov, and L. Sebastiani, Phys. Rev. D **91**, 083529 (2015).
[53] G. Cusin, F. de O. Salles, and I. L. Shapiro, Phys. Rev. D **93**, 044039 (2016).
[54] P. Peter, F. D. O. Salles, and I. L. Shapiro, Phys. Rev. D **97**, 064044 (2018).
[55] I. Kuntz and R. da Rocha, Eur. Phys. J. C **79**, 447 (2019).
[56] N. Aghanim *et al.* (Planck Collaboration), arXiv:1807.06209.
[57] L. Alvarez-Gaume and E. Witten, Nucl. Phys. **B234**, 269 (1984).
[58] T. S. Bunch and P. C. W. Davies, J. Phys. A **11**, 1315 (1978).
[59] L. Parker and S. A. Fulling, Phys. Rev. D **9**, 341 (1974).
[60] T. S. Bunch, J. Phys. A **13**, 1297 (1980).
[61] S. Deser, M. J. Duff, and C. J. Isham, Nucl. Phys. **B111**, 45 (1976).

[62] M. J. Duff, Nucl. Phys. **B125**, 334 (1977).
[63] A. Landete, J. Navarro-Salas, and F. Torrenti, Phys. Rev. D **89**, 044030 (2014).
[64] A. del Rio, J. Navarro-Salas, and F. Torrenti, Phys. Rev. D **90**, 084017 (2014).
[65] C.-S. Chu and Y. Koyama, Phys. Rev. D **95**, 065025 (2017).
[66] R. Endo, Prog. Theor. Phys. **71**, 1366 (1984).
[67] D. J. Toms, Phys. Rev. D **90**, 044072 (2014).
[68] A. R. Vieira, J. C. C. Felipe, G. Gazzola, and M. Sampaio, Eur. Phys. J. C **75**, 338 (2015).
[69] N. Arkani-Hamed, S. Dubovsky, A. Nicolis, E. Trincherini, and G. Villadoro, J. High Energy Phys. 05 (2007) 055.
[70] H. Epstein, V. Glaser, and A. Jaffe, Nuovo Cimento **36**, 1016 (1965).
[71] C.-I. Kuo, Nuovo Cimento B **112**, 629 (1997).
[72] A. Borde, Classical Quantum Gravity **4**, 343 (1987).
[73] G. Klinkhammer, Phys. Rev. D **43**, 2542 (1991).
[74] J. L. Friedman, K. Schleich, and D. M. Witt, Phys. Rev. Lett. **71**, 1486 (1993); **75**, 1872(E) (1995).
[75] L.-F. Li and J.-Y. Zhu, Phys. Rev. D **79**, 044011 (2009).
[76] M. Visser, Phys. Lett. B **349**, 443 (1995).
[77] M. Visser, Phys. Rev. D **54**, 5116 (1996).
[78] D. Urban and K. D. Olum, Phys. Rev. D **81**, 024039 (2010).
[79] D. Urban and K. D. Olum, Phys. Rev. D **81**, 124004 (2010).
[80] H. Matsui, arXiv:1901.08785.
[81] L. H. Ford, Proc. R. Soc. A **364**, 227 (1978).
[82] J. Z. Simon, Phys. Rev. D **43**, 3308 (1991).
[83] A. H. Guth, Phys. Rev. D **23**, 347 (1981); *Quantum Cosmology*, edited by L.-Z. Fang and R. Ruffini, Advanced Series in Astrophysics and Cosmology Vol. 3 (World Scientific, Singapore, 1987), p. 139.
[84] K. Sato, Mon. Not. R. Astron. Soc. **195**, 467 (1981).
[85] A. D. Linde, Phys. Lett. **108B**, 389 (1982); *Quantum Cosmology*, edited by L.-Z. Fang and R. Ruffini, Advanced Series in Astrophysics and Cosmology Vol. 3 (World Scientific, Singapore, 1987), p. 149.
[86] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982); *Quantum Cosmology*, edited by L.-Z. Fang and R. Ruffini, Advanced Series in Astrophysics and Cosmology Vol. 3 (World Scientific, Singapore, 1987), p. 158.
[87] J. Z. Simon, Phys. Rev. D **45**, 1953 (1992).
[88] L. Parker and J. Z. Simon, Phys. Rev. D **47**, 1339 (1993).
[89] S. W. Hawking and T. Hertog, Phys. Rev. D **65**, 103515 (2002).