

Warm brane inflation with an exponential potential: A consistent realization away from the swampland

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It has very recently been realized that coupling branes to higher dimensional quantum gravity theories and considering the consistency of what lives on the branes, one is able to understand whether such theories can belong either to the swampland or to the landscape. In this regard, in the present work, we study a warm inflation model embedded in the Randall-Sundrum brane-world scenario. It is explicitly shown that this model belongs to the landscape by supporting a strong dissipative regime with an inflaton steep exponential potential. The presence of extra dimension effects from the braneworld allow achieving this strong dissipative regime, which is shown to be both theoretically and observationally consistent. In fact, such strong dissipation effects, which decrease towards the end of inflation, together with the extra dimension effect, allow the present realization to simultaneously satisfy all previous restrictions imposed on such a model and to evade the recently proposed swampland conjectures. The present implementation of this model, in terms of an exponential potential for the scalar field, makes it also a possible candidate for describing the late-time Universe in the context of a dissipative quintessential inflation model, and we discuss this possibility in the Conclusions.

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I. INTRODUCTION

Flattening, isotropizing, and homogenizing the Universe and reproducing the adiabatic, nearly Gaussian, and quasiscale invariant spectrum of primordial density fluctuations in accordance with the observational cosmological data [1] require implementing a mechanism complementing the standard big bang cosmology to portray the very early Universe. There are today several scenarios attempting to achieve all of these features, namely, ekpyrotic/cyclic theories [2], matter-bounce scenario [3], pre-big bang cosmology [4], string gas cosmology [5], pseudoconformal cosmology [6], to name but a few. Yet, inflation [7], an exponentially accelerated expansion driven by a scalar field ϕ rolling down a sufficiently flat region of a potential $V(\phi)$, is still the simplest and most successful one among all other alternatives. Despite its tremendous success, inflation suffers from some long-lasting conceptual problems, namely, the fine-tuning problem [8], the initial condition problem [9], the trans-Planckian problem [10], the measure problem [11], singularity problems [12], etc, which makes inflation to be known as a paradigm rather than a well-established theory after three decades.

Cosmologists believe that some, if not all, of the above problems might be solved by consistently embedding an

inflation model, taken as an effective field theory of gravity, into a M/string theory, taken as a candidate for a quantum theory of gravity. If this construction is possible, then one could in principle build an ultraviolet (UV) complete model for inflation. Although great efforts have been made to accomplish such a goal, there is still no conclusive result for such possibility so far (see, e.g., discussions in Ref. [13]). The most recent efforts led into the so-called swampland conjectures [14], which requests steep potentials V , such that $M_{\text{Pl}}|V'|/V \gtrsim \mathcal{O}(1)$, with a sub-Planckian field excursion, $\Delta\phi \lesssim M_{\text{Pl}}$ (for a general review on the swampland conjectures, see, e.g., Ref. [15] and also the references therein for the many implications in cosmology). While the latter condition can be satisfied with some inflationary models [16], the former condition given on the potential rules out most slow-roll single field inflationary models, requesting the violation of the slow-roll regime.

Among the many recent discussions concerning these conjectures, the authors in Ref. [17] have very recently explained the necessity of the swampland constraints, utilizing the completeness of the spectrum of charged branes in a quantum theory of gravity and the consistency of what lives on the branes. In fact, they have shown that coupling branes, as a consistent ingredient of higher

dimensional theories, to quantum theories of gravity in higher dimensions, are useful to separate theories which are in the landscape from those which belong to the swampland. Braneworld type of models have been extensively used in cosmology. In particular, the braneworld models from Randall and Sundrum [18], which we will focus in the present work, also known as Randall-Sundrum (RS) type I and II models. In such theories, all the standard particles and their interactions are confined to the brane and just gravity can propagate along a fifth dimension. The extra dimension modifies the Friedmann equation on the brane, leading in particular to a quadratic term proportional to the energy density, which have some significant cosmological implications [19]. The scenario of inflation in RS II braneworld was studied in Ref. [20], in which the authors have shown that brane corrections at the high energy regime allow steep potentials to be embedded in the braneworld scenario. More recently, the authors in Ref. [21] have also discussed about the swampland in this model.

In the present work, we will focus on an inflation model with an exponential potential in the context of the RS II braneworld scenario. Let us recall that exponential potentials for scalar fields can naturally emerge in M/string theory from the compactification of the higher dimensions into a four-dimensional effective field theory, that encodes the size of the extra dimensions [22]. It has been shown that an exponential potential form for the scalar inflaton field (and with canonical kinetic term) leads to a power-law type of solutions, with the scale factor satisfying $a(t) = t^q$ and $q > 1$. This type of potential can also describe different cosmological epochs, including, for instance, the behavior of the late-time Universe [23]. Moreover, it leads to an everlasting power law inflation at the very early Universe, with constant slow-roll parameters [24,25]. Despite all of this, several attempts have been made towards overcoming such difficulties, like, for instance, implementing these exponential potential models in the braneworld scenario [26–28]. Although inflation with an exponential potential in the braneworld scenario can have a graceful exit, it is still in tension with the observational data. Even in a braneworld scenario, an exponential potential model for inflation produces a large tensor-to-scalar ratio, so it is considered excluded by the observational data [28]. It also requires an efficient reheating mechanism [26]. Besides, it suffers from a long kination period after inflation, leading to a large amount of gravitational particle production and relic gravity waves, disrupting and even undermining the big bang nucleosynthesis (BBN) [27].

Although dissipation is an indispensable part of any physical system interacting with its environment, conventional inflationary scenarios, namely cold inflation, typically neglect such effects during the inflationary phase. Thus, a (p)reheating phase at the end of inflation is required so to connect with the big bang radiation dominated regime. In the warm inflation (WI) picture [29], the

possible dissipative effects that results from the interactions of the inflaton field with other field degrees of freedom can sustain a subdominant radiation bath throughout the inflationary dynamics. If the dissipative effects can become strong enough, the Universe can smoothly enter into the radiation dominated era, with no separate reheating phase being in general required [30]. There is by now an extensive literature on different aspects of WI, the models giving origin to it and on its applications [31–48]. For instance, the inflaton itself can be a source and responsible for cosmic magnetic field generation [31]. In combination with the intrinsic dissipative effects lead to a successful baryogenesis scenario [32]. There can also be a remnant inflaton field behaving like dark matter [33] and dark energy [34–36]. Furthermore, such dissipation effects change the dynamics of inflation due to the supplementary friction term, making it possible to construct models with steep potentials [37,38]. Moreover, the dissipation and radiation effects modify the primordial spectrum of curvature perturbations [39–41], resulting in a lower energy scale of inflation and making it also possible to reconcile steep potentials with the observational data [42,45]. Such appealing features of WI allow it to simultaneously satisfy the so-called swampland conjectures, provided WI can occur with a sufficient strong dissipation [46,47]. Furthermore, strong dissipation effects are also able to suppress the energy scale of brane inflation, making it consistent with the observations even in the high energy regime [48]. Although it is enormously challenging achieving strong dissipative regimes in WI, a recently proposed new model was built using first principles in quantum field theory and shown explicitly to overcome such difficulties [47].

It is the purpose of the present work to investigate WI in a model with an exponential potential in the RS II braneworld scenario. We are going to explicitly see how it is possible to make this WI model realization fully functional and consistent with both observations and also evading the theoretical restrictions faced by this type of model in its previous constructions done in the context of cold inflation. As an additional welcome feature, we also show that it can support strong enough dissipation to evade the previous, but also the more recent, swampland conjectures.

The paper is organized as follows. In Sec. II, we briefly review the WI dynamics constructed in the RS II braneworld scenario. In Sec. III, we discuss several limitations imposed on the exponential potential considered in this work. We also give the motivations for considering such a potential and discuss how WI is going to remove the discussed previous limitations imposed on the model. In Sec. IV, we give the explicit implementation of the model in WI and demonstrate its viability through explicit numerical analysis. Finally, in Sec. V, we give our conclusions and also discuss the viability of the model to unify the cosmology of the very early Universe with the present epoch through a possible quintessential inflation scenario.

II. WARM INFLATION DYNAMICS IN THE BRANEWORLD SCENARIO

In this section, we consider the RS II braneworld scenario in which our four-dimensional world has its space dimensions like a 3 brane embedded in a five-dimensional bulk. In such a braneworld scenario, adopting a Friedmann-Lemaître-Robertson-Walker (FLRW) metric, the cosmological evolution can be described by a generalized Friedmann equation as follows [20,49]:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_{\text{Pl}}^2}\rho + \frac{1}{36M_5^6}\rho^2 + \frac{\Lambda_4}{3} + \frac{\mu}{a^4}, \quad (2.1)$$

where $a(t)$ is the usual scale factor, ρ is the total energy density in the three-dimensional space, $M_{\text{Pl}} \equiv 1/\sqrt{8\pi G} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass defined in terms of the usual Newton gravitational constant G , Λ_4 is an effective cosmological constant, μ is an integration constant which behaves as dark radiation term, and M_5 is the five-dimensional Planck mass, defined as

$$M_5^6 = \frac{\lambda M_{\text{Pl}}^2}{6}, \quad (2.2)$$

where λ is the intrinsic tension on the brane. The cosmological constant term in Eq. (2.1) can be tuned to zero by an appropriate choice of the brane tension and bulk cosmological constant. Also, the dark radiation term quickly redshifts during inflation so we can also neglect it. Thus, Eq. (2.1) can then be expressed simply as

$$H^2 = \frac{1}{3M_{\text{Pl}}^2}\rho \left(1 + \frac{\rho}{2\lambda}\right). \quad (2.3)$$

In the WI scenario, the total energy density is given by

$$\rho = \frac{\dot{\phi}^2}{2} + V(\phi) + \rho_R, \quad (2.4)$$

which accounts for the radiation fluid, with energy density ρ_R , and the scalar field (the inflaton) ϕ , with a potential $V(\phi)$. The background dynamics for the inflaton field ϕ and the radiation energy density ρ_R , which are coupled to each other, are, respectively, given by

$$\ddot{\phi} + 3(1 + Q)H\dot{\phi} + V_{,\phi} = 0, \quad (2.5)$$

$$\dot{\rho}_R + 4H\rho_R = 3HQ\dot{\phi}^2, \quad (2.6)$$

where dots denote temporal derivatives and Q is the dissipation ratio in WI, defined as

$$Q = \frac{\Upsilon(T, \phi)}{3H}, \quad (2.7)$$

where $\Upsilon(T, \phi)$ is the dissipation coefficient in WI, which can be a function of the temperature and/or the background inflaton field, depending on the specifics of the microscopic physics behind the construction of WI. For a radiation bath of relativistic particles, the radiation energy density is given by $\rho_R = \pi^2 g_* T^4/30$, where g_* is the effective number of light degrees of freedom (g_* is fixed according to the dissipation regime and interactions form used in WI).

A typical dissipation coefficient functional form in terms of the temperature and the inflaton field amplitude found in many WI models can be expressed as [37,38,50,51],

$$\Upsilon(T, \phi) = CT^c \phi^p M^{1-c-p}, \quad (2.8)$$

where C is a dimensionless constant (that carries the details of the microscopic model used to derive the dissipation coefficient, e.g., the different coupling constants of the model), M is a mass scale in the model and depends of its construction, while c and p are numerical powers, which can be either positive or negative numbers [the dimensionality of the dissipation coefficient in Eq. (2.8) is of course preserved, i.e., $[\Upsilon] = [\text{energy}]$]. Some typical dependencies found in the literature are the cases with $c = -1$, $p = 2$ (see Ref. [52]), $c = 1$, $p = 0$ (see Ref. [45]), $c = -1$, $p = 0$ (in the asymptotic high temperature limit for the dissipation coefficient found in the model of Ref. [47]) and $c = 3$, $p = -2$ (see Refs. [37,50]). The case with $c = 3$, $p = -2$ was in fact the first microscopic construction for WI from a particle physics model building perspective and giving consistent observables [38]. In the present work, we will not be interested in the specific origin of these dissipation coefficients from a microscopic quantum field theory derivation (for such details, we refer the interested reader to the above cited references), but on the phenomenological consequences of them.

In terms of Eqs. (2.3), (2.5), and (2.6) the slow-roll parameters in the braneworld scenario are given by

$$\epsilon_{\text{brane}} = \epsilon_V \frac{1 + V/\lambda}{[1 + V/(2\lambda)]^2}, \quad (2.9)$$

$$\eta_{\text{brane}} = \eta_V \frac{1}{1 + V/(2\lambda)}, \quad (2.10)$$

where ϵ_V and η_V are the usual slow-roll inflaton potential parameters, defined as

$$\epsilon_V = \frac{M_{\text{Pl}}^2}{2} \left(\frac{V_{,\phi}}{V}\right)^2, \quad (2.11)$$

$$\eta_V = M_{\text{Pl}}^2 \frac{V_{,\phi\phi}}{V}. \quad (2.12)$$

In terms of Eqs. (2.9) and (2.10), the slow-roll conditions in WI are defined by the requirement that $\epsilon_{\text{brane}} \ll 1 + Q$ and

$\eta_{\text{brane}} \ll 1 + Q$. The accelerated inflationary dynamics terminates when $\epsilon_{\text{brane}} = 1 + Q$.

By assuming the slow-roll approximation, with energy density $\rho \sim V$, the Eqs. (2.3), (2.5), and (2.6) reduce to

$$H^2 \simeq \frac{1}{3M_{\text{Pl}}^2} V \left(1 + \frac{V}{2\lambda} \right), \quad (2.13)$$

$$3(1 + Q)H\dot{\phi} \simeq -V_{,\phi}, \quad (2.14)$$

$$4\rho_R \simeq 3Q\dot{\phi}^2. \quad (2.15)$$

Given the dissipation coefficient with the functional form given by Eq. (2.8) and expressing the evolution in terms of the number of e -folds, $dN = Hdt$, we can deduce how the dissipation ratio Q and the ratio of temperature of the radiation bath by the Hubble rate, T/H , evolve during WI. After some straightforward algebra, we find that their evolution are determined by the equations,

$$\frac{d \ln Q}{dN} = \frac{2[(2 + c)\epsilon_{\text{brane}} - c\eta_{\text{brane}} - 2p\kappa_{\text{brane}}]}{4 - c + (4 + c)Q}, \quad (2.16)$$

$$\frac{d \ln(T/H)}{dN} = \frac{[7 + c(Q - 1) + 5Q]\epsilon_{\text{brane}} - 2(1 + Q)\eta_{\text{brane}} + (Q - 1)p\kappa_{\text{brane}}}{(1 + Q)[4 - c + (4 + c)Q]}, \quad (2.17)$$

where κ_{brane} is defined as

$$\kappa_{\text{brane}} = M_{\text{Pl}}^2 \frac{V_{,\phi}}{\phi V [1 + V/(2\lambda)]}. \quad (2.18)$$

Moreover, the ratio of radiation to inflaton energy density in the slow-roll regime is roughly given by

$$\frac{\rho_R}{\rho_\phi} \approx \frac{1}{2} \frac{\epsilon_{\text{brane}}}{1 + Q} \frac{Q}{1 + Q}. \quad (2.19)$$

Although the ratio of the radiation to inflaton energy density is very small at the beginning of WI, it can be large at the end of inflation (when $\epsilon_{\text{brane}} = 1 + Q$), even for an initially small dissipation ratio Q . Consequently, the Universe can smoothly enter into the radiation dominated epoch at the end of WI, with no need for a separate reheating phase *a priori*.

III. THE MODEL

As explained in the Introduction, in this paper, we will be working with the exponential potential form for the inflaton field,

$$V(\phi) = V_0 \exp(-\alpha\phi/M_{\text{Pl}}), \quad (3.1)$$

where V_0 is the normalization for the potential and which can be fixed by the amplitude of primordial curvature perturbations as usual. The study of inflation with this potential is diverse, specially motivated by the fact that a potential like Eq. (3.1) can also serve as a quintessential inflation model, describing both the inflationary early Universe period, as also the late Universe, working as an evolving dark energy model at the present epoch [26]. The exponential potential for the inflaton has also already been studied before in the presence of dissipative effects

from particle production [25], yet with no connection to the WI scenario, as it is the focus in the present study.

Despite the many uses of the potential Eq. (3.1), it is, however, hard to find consistent ways of ending inflation in such models in the traditional ways (i.e., ending the acceleration regime). For instance, with the standard slow-roll parameters given by Eqs. (2.11) and (2.12), the accelerated expansion requires $\alpha < \sqrt{2}$. However, this leads to a power-law type of inflation, which has long been found to be inconsistent with the observations [53,54], unless there are either modifications to the form of the potential, to the dynamics, or to both. In the braneworld scenario, it is possible to have inflation even when $\alpha > \sqrt{2}$, provided that the dynamics is dominated by the high energy brane regime, $V > 2\lambda$. In this case, the modification of the dynamics because of the brane corrections are strong, and we also have a stronger Hubble friction. As the inflaton field evolves and $V < 2\lambda$, the effects of the brane corrections lessen, the usual general relativity (GR) dynamics is restored, and inflation ends. But the inflaton evolution in a steep potential like Eq. (3.1) will at some point be dominated by the kinetic energy of the field, which gives start to a kination regime. During this regime, the energy density falls off like stiff matter, $\rho \propto 1/a^6$. The abrupt change in the dynamics can produce a large amount of gravitational particle production that might disrupt BBN later on [26]. Furthermore, a large amount of relic gravity waves is also predicted to be generated [27]. The energy density on these produced gravitational waves can be boosted by the kination period and can also disrupt the BBN later on. As an additional problem, the model leads yet to predictions on the tensor-to-scalar ratio and on the spectral tilt of the primordial scalar spectrum that is also excluded by the observations [28]. All these issues together have rendered this inflaton potential model unsuitable from an observational point of view.

In the following, we discuss the choice for the dissipation coefficient used in the present study. We will also discuss the theoretical and observational constraints on the parameters of the braneworld scenario and on the exponential potential. Then, we explain under which conditions the WI scenario realization of the model may allow it to overcome all of the above mentioned difficulties, and we then give an explicit example in Sec. IV.

A. Warm inflation implementation

Considering the model given by Eq. (3.1) in the context of WI, we find, in particular, that the evolution equation for the dissipation ratio Eq. (2.16) becomes

$$\frac{d \ln Q}{dN} = \frac{4\lambda\alpha\{2p(2\lambda + V) + [(2 - c)\lambda + 2V]\alpha\phi/M_{\text{Pl}}\}}{[4 - c + (4 + c)Q](2\lambda + V)^2\phi/M_{\text{Pl}}}. \quad (3.2)$$

We are interested in the strong dissipative regime of WI, i.e., $Q \gg 1$, which ensures that the swampland conditions are satisfied [46], with the inflaton excursion typically sub-Planckian, $\Delta\phi < M_{\text{Pl}}$. Furthermore, we have the constraint to be discussed below and given by Eq. (3.6), on the inflaton potential constant α , requiring rather large values. At the same time, we do not want the dissipation ratio to grow throughout the evolution; otherwise, even by arranging inflation to end due to the braneworld effects, it might reinitiate again or even never end in the first place. Hence, we would ideally want a *decreasing* Q with the number of e -folds, i.e., the right-hand side of Eq. (3.2) should become negative, soon after the braneworld effects become subdominant. Thus, we need to analyze the region of parameters for which the right-hand side of Eq. (3.2) can be either positive or negative. In particular, from previous studies on the stability of the WI dynamics [55], we have that the power c should satisfy $-4 < c < 4$, which results in the denominator of Eq. (3.2) to be always positive. Thus, for $2 < c < 4$ and $-3 < p < 0$, we find that the numerator in the right-hand side of Eq. (3.2) can be positive in the high energy regime, $V \gg 2\lambda$, but then it becomes negative as one enters in the low energy regime, $V < 2\lambda$. This is, in particular, exactly the behavior expected with a dissipation coefficient with a functional form with a cubic power in the temperature, as found, e.g., in Refs. [37,50]. In this case, we find that Q will grow initially during the inflationary evolution, provided that $V > 2\lambda$, decreasing later on in the evolution when $V < 2\lambda$. Thus, by having initially that $\epsilon_{\text{brane}}/(1 + Q) \ll 1$, this will ensure that the accelerated expansion will happen both because of the brane effect and because of a growing Q , further facilitating the accelerated expansion, even if α is large. On the other hand, as the brane effects lessen and we recover the usual general relativity evolution, the dissipation ratio will start to decrease, eventually ending the accelerated inflationary

regime due to the large value of α . Note that this effect of the dissipation on the evolution does not happen when $c = 1$, as in the case of the dissipation coefficient found in Ref. [45], or in the case of the dissipation coefficient recently found and correspondingly, WI dynamics studied in Ref. [47], which favors a dissipation coefficient with $c = -1$. In both of these two cases, Q will always increase, in general, making it more difficult to end inflation. In fact, such decreasing Q allowed by a $\Upsilon \propto T^3/\phi^2$ dissipation coefficient not only plays a pivotal role in bringing the inflation to an end but also it will be shown to allow the model studied here to be consistent with the observations, even for large values of the dissipation ratio Q . Thus, in the following, we will restrict our study to the cubic in the temperature dissipation coefficient form, where¹

$$\Upsilon(T, \phi) = C \frac{T^3}{\phi^2}. \quad (3.3)$$

In the next section, we will then show our explicit and detailed results for the WI model with the dissipation coefficient Eq. (3.3) in the context of the braneworld scenario. But before entering in the explicit results derived from this model, let us discuss some important constraints that the model might be subjected.

B. Constraints

In the braneworld scenario, the quadratic term of the energy density in the Hubble parameter Eq. (2.3) becomes dominant in the high energy regime, provided the brane tension λ has a moderate value. However, BBN implies that it has to be subdominant, since it decays as a^{-8} and becomes rapidly negligible thereafter. Therefore, the BBN bounds put a lower limit on the brane tension such that it has to satisfy $\lambda \gtrsim (1 \text{ MeV})^4$, which in combination with Eq. (2.2) gives [20]

$$M_5 \gtrsim \left(\frac{1 \text{ MeV}}{M_{\text{Pl}}}\right)^{2/3} \sim 10 \text{ TeV}. \quad (3.4)$$

However, considering the fifth dimension to be infinite and requiring relative corrections to the Newtonian law of gravity to be also small [56], one obtains a more stringent constraint as given by $M_5 \gtrsim 10^5 \text{ TeV}$, or, equivalently, $\lambda \gtrsim 100 \text{ GeV}$.

¹The attentive reader may question what happens with this dissipation coefficient with the exponential potential we use here and where ϕ might cross zero. This is not a problem since, as shown in Refs. [37,38,50,51], this dissipation coefficient is derived when the inflaton field is coupled to heavy intermediate fields χ , where the mass $m_\chi \propto \phi$, but we could as well have a nonvanishing bare mass for χ , which regulates any possible infrared divergence and avoids Υ to diverge when $\phi \rightarrow 0$.

As discussed in Refs. [57,58], the abundance of gravitinos is related to the reheating temperature through the Boltzmann equation. Hence, by constraining the gravitino abundance, one can obtain an upper bound on the reheating temperature as $T_R \lesssim 10^6\text{--}10^8$ GeV. However, the authors in Ref. [58] have shown that the relation between gravitino abundance and the reheating temperature breaks in the high energy limit, $V/(2\lambda) > 1$, and allows inflation to occur even with a higher reheating temperature.

The late-time behavior for steep exponential potentials produces a scaling solution, where the scalar field exhibits the same redshift dependence as the dominant fluid in the Universe. Thus, the energy density fraction for the scalar field, assuming spatial flatness, has a scaling solution given by [23,26]

$$\Omega_\phi = \frac{3(w+1)}{\alpha^2}, \quad (3.5)$$

where w is the equation of state of the dominant fluid. If the inflaton potential Eq. (3.1) remains unchanged till late in the Universe, it will act like a quintessence field. Early dark energy can influence the cosmic microwave background (CMB) peaks among to other changes in the CMB power spectrum. Hence, its fraction can be strongly constrained when including small-scale measurements and CMB lensing, for instance. The strongest constraint on the fraction of dark energy at last scattering time has been produced by the Planck data and leading to an upper bound [59], $\Omega_\phi \lesssim 0.0036$ at 95% confidence level (for Planck TT, TE, EE – lowP + BSH). Applying this to Eq. (3.5), we are lead to the lower bound on the coefficient α for the inflaton potential,

$$\alpha \gtrsim 33.3. \quad (3.6)$$

Furthermore, the recently proposed swampland conjectures restrict both the dynamics and potential form of the inflationary models [14,15]. In fact, the de Sitter swampland conjecture requests steep potentials, $M_{\text{Pl}}V'/V \geq \mathcal{O}(1)$, with no extrema, as a result of which not only inflationary models but also the usual reheating mechanism, due to an oscillatory phase around the minimum at the end of inflation, are ruled out [60]. Moreover, the distance swampland conjecture requests sub-Planckian field excursions during inflation, $\Delta\phi \lesssim M_{\text{Pl}}$, whereby all large field inflationary models are excluded when the conjecture is taken at face value. As discussed in Ref. [46], to have a large slow-roll parameter ϵ_V , one needs to violate the relation between $\epsilon_H \equiv -\dot{H}/H^2$ and ϵ_V in the conventional single field slow-roll inflation. Besides, one also needs to find another mechanism to heat up the Universe at the end of inflation. The WI scenario has the appealing feature to give us both of these in a single frame due to the presence of the intrinsic dissipation effects, provided that WI occurs in the strong dissipative

regime [46], $Q \gg 1$. Moreover, the WI scenario also gives us the possibility of suppressing the tensor-to-scalar ratio by several orders of magnitude for a large dissipation ratio, due to effects of the dissipation on the primordial perturbations and the change in the scalar curvature spectrum (to be discussed below). As a result, the inflaton field can remain sub-Planckian during inflation, evading the Lyth bound even for steep potentials. Therefore, dissipation effects make WI able to satisfy the swampland conjectures, while also allowing for the accelerated expansion, even when $\alpha > 1$ in the exponential potential and braneworld scenario as considered here.

In our numerical study to be presented in the next section, we will keep in mind all of the above constraints and conditions and aim at satisfying them all.

IV. NUMERICAL RESULTS

As previously mentioned, both the background dynamics and the perturbations get modified due to the presence of dissipation and a radiation bath during WI [39,40]. As a matter of fact, the primordial power spectrum for WI at horizon crossing can be expressed in the form (see, e.g., Ref. [41] and references therein),

$$\Delta_{\mathcal{R}}(k/k_*) = \left(\frac{H_*^2}{2\pi\dot{\phi}_*} \right)^2 \mathcal{F}(k/k_*), \quad (4.1)$$

where the subindex “*” stands for those quantities evaluated at the Hubble radius crossing, $k_* = a_*H_*$. The function $\mathcal{F}(k/k_*)$ in Eq. (4.1) is given by

$$\mathcal{F}(k/k_*) \equiv \left(1 + 2n_* + \frac{2\sqrt{3}\pi Q_* T_*}{\sqrt{3+4\pi Q_*} H_*} \right) G(Q_*), \quad (4.2)$$

where n_* denotes the inflaton statistical distribution due to the presence of the radiation bath and $G(Q_*)$ accounts for the effect of the coupling of the inflaton fluctuations to radiation [40]. $G(Q_*)$, in general, can only be determined by numerically solving the full set of perturbation equations in WI and fitting it to an appropriate function. Following an analogous derivation as considered, e.g., in the papers in Ref. [40], we find that an appropriate functional form for $G(Q_*)$ that is valid for the present exponential model in the braneworld construction and with dissipation coefficient (3.3), is well described by

$$G(Q_*) = \frac{1 + 0.413Q_*^{0.85}}{(1 + 0.18Q_*^{0.859})^{26.5}} + \frac{0.00692 \exp(6.44Q_*^{0.292})}{1 + 0.00082 \exp(0.1763Q_*^{0.66})}. \quad (4.3)$$

The above equation for $G(Q_*)$ is found to hold for rather very large values for Q_* , up to around $Q_* \simeq 2000$. Besides, since the behavior of the spectrum with Q_* is smooth and

well behaved, we can always do this procedure (numerical fitting) with a sufficient precision such that any arbitrariness in the numerical fitting does not change the observable quantities, e.g., the spectral index n_s and the tensor-to-scalar ratio r , both defined below.

In our numerical results, we fix the scalar spectral amplitude value at the pivot scale k_* as $\ln(10^{10}\Delta_{\mathcal{R}}) \simeq 3.047$, according to the Planck Collaboration [1] (in the TT, TE, EE – lowE + lensing + BAO 68% limits data set).

While the primordial scalar curvature perturbation in WI gets modified according to Eq. (4.1), the tensor perturbations spectrum is unchanged in WI because of the weakness of the gravitational interactions.² However, the spectrum of tensor perturbations is modified in the braneworld scenario due to the presence of the extra dimension, where the graviton resides, with respect to the standard four-dimensional (GR) Universe. The tensor perturbations power spectrum in the braneworld scenario has been determined to be given by [62]

$$\Delta_T = \frac{2H^2}{\pi^2 M_{\text{Pl}}^2} F^2(x), \quad (4.4)$$

where

$$F(x) = \left[\sqrt{1+x^2} - x^2 \ln \left(\frac{1}{x} + \sqrt{1 + \frac{1}{x^2}} \right) \right]^{-1/2}, \quad (4.5)$$

with $x = \sqrt{6M_{\text{Pl}}^2 H^2 / \lambda}$.

From Eqs. (4.1) and (4.4), the tensor-to-scalar ratio r is defined as usual,

$$r = \frac{\Delta_T}{\Delta_{\mathcal{R}}}, \quad (4.6)$$

while the spectral tilt n_s is defined as

$$n_s - 1 = \lim_{k \rightarrow k_*} \frac{d \ln \Delta_{\mathcal{R}}(k/k_*)}{d \ln(k/k_*)}. \quad (4.7)$$

We recall that the recent data from the Planck Collaboration [63] has placed the upper bound $r < 0.056$ (95% C.L., Planck TT, TE, EE + lowE + lensing + BK15, at the pivot scale $k_p = 0.002/\text{Mpc}$), while for the spectral tilt the result is $n_s = 0.9658 \pm 0.0040$ (95% C.L., Planck TT, TE, EE + lowE + lensing + BK15 + BAO + running).

In the results shown below, the number of e -folds for inflation, in particular, the number of e -folds before Hubble radius crossing, N_* , is always computed self-consistently. This is done by noticing that length scales crossing the

²See, however, Ref. [61] for possible changes in the tensor spectrum due to WI. However, even for the very large values of Q_* considered here by us, those corrections found in Ref. [61] are completely negligible and can be safely neglected.

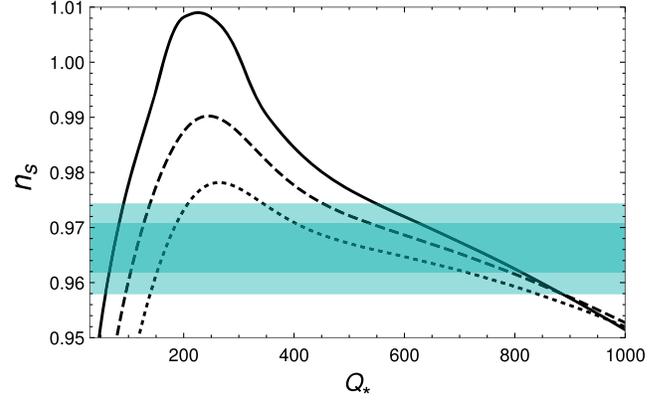


FIG. 1. The spectral index n_s as a function of the dissipation ratio Q_* for the cases of $\alpha = 30$ (solid line), $\alpha = 40$ (dashed line), and $\alpha = 50$ (dotted line), for a fixed value of (normalized) brane tension, $\lambda/V_0 = 5 \times 10^{-5}$. The shaded areas are for the 68% and 95% C.L. results from Planck 2018 (TT + TE + EE + lowE + lensing + BK15 + BAO data).

Hubble radius during inflation and reentering today will satisfy $k_* = a_i H_i = a_0 H_0$, such that [64]

$$\frac{k_*}{a_0 H_0} = e^{-N_*} \frac{T_0 H_i}{T_{\text{end}} H_0}, \quad (4.8)$$

where 0-index quantities mean that they are evaluated today, while i -index quantities are those evaluated at N_* e -folds before the end of inflation. The CMB temperature today, T_0 , is set to the value $T_0 = 2.725 \text{ K} = 2.349 \times 10^{-13} \text{ GeV}$. In our convention, we use $a_0 = 1$ and for the Hubble parameter today, we assume the Planck result, $H_0 = 67.66 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [from the Planck Collaboration [1], TT, TE, EE – lowE + lensing + BAO 68% limits, $H_0 = (67.66 \pm 0.42) \text{ km s}^{-1} \text{ Mpc}^{-1}$]. T_{end} is the temperature at the beginning of the radiation dominated regime, which in the WI, turns out to be simply the temperature at the end of inflation, since WI ends by the time the radiation energy density takes over the inflaton one. For the exponential potential Eq. (3.1) in the cold inflation case and in the braneworld scenario, there is a prediction for N_* given by [27] $N_* \simeq 70$. In the WI case studied here, we always find a smaller value for N_* due to the effect of dissipation.

In Fig. 1, we show the behavior of the spectral index n_s as a function of the dissipation ratio Q_* at a Hubble radius crossing, when keeping the ratio of the brane tension by the normalization of the inflaton potential, λ/V_0 , fixed but for three different values for the exponent α in the inflaton potential. For a strong dissipation ratio, $Q_* \gg 1$, the tensor-to-scalar ratio is always quite very small, $r < 10^{-14}$. We recall that such very small values for r are typical for WI in the strong dissipative regime. This is so because the scalar curvature power spectrum Eq. (4.1) is completely dominated by the dissipation, thus strongly suppressing the

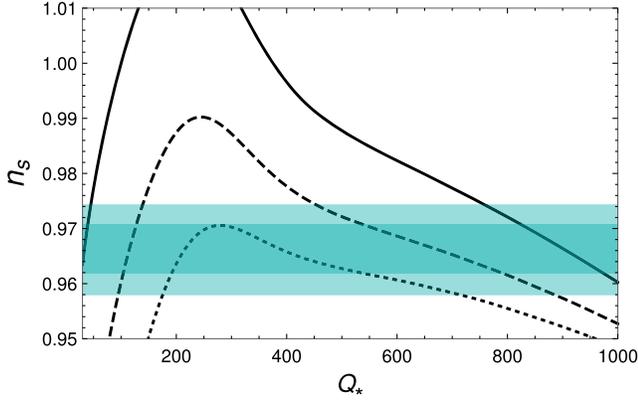


FIG. 2. The spectral index n_s as a function of the dissipation ratio Q_* for the cases of $\lambda/V_0 = 10^{-5}$ (solid line), $\lambda/V_0 = 5 \times 10^{-5}$ (dashed line) and $\lambda/V_0 = 10^{-4}$ (dotted line), for the fixed value of $\alpha = 40$. The shaded areas are for the 68% and 95% C.L. results from Planck 2018 (TT + TE + EE + lowE + lensing + BK15 + BAO data).

tensor-to-scalar ratio. Whereby, when $Q_* \gg 1$, the inflaton statistical distribution term n_* in Eq. (4.1) also has a negligible effect, and we can set it as vanishing. From Fig. 1, we see that the larger is the α , the steeper is the inflaton potential; therefore, we can find a larger range of dissipation values for which n_s agrees with the observational data. We also see that there are always two ranges of Q_* values satisfying the observations.

In Fig. 2, we show the behavior of the spectral index n_s as a function of the dissipation ratio Q_* at the Hubble radius crossing, where now we keep the constant α in the exponential inflaton potential fixed and give the results for three different values for the ratio of the brane tension by the normalization of the inflaton potential. We have a similar behavior as seen in Fig. 1 when considering λ/V_0 fixed. However, the larger the λ/V_0 , the smaller is the range of dissipation ratios satisfying the observations. In particular, for $\alpha = 40$, we have obtained that when $\lambda/V_0 \gtrsim 10^{-3}$ there are no longer values for n_s found to be compatible with the Planck data, with n_s being too red tilted.

In Fig. 3(a), we show the evolution of the ratio $V(\phi)/(2\lambda)$ with the number of e -folds, while in Fig. 3(b), we show the evolution of the dissipation ratio with the number of e -folds. We have chosen the particular case of $\alpha = 40$ and $\lambda/V_0 = 5 \times 10^{-5}$, but there is little change in the results when considering other values of parameters close to these. This is found to be true for all the background quantities in general. From these results, we can confirm the behavior for the dissipation ratio with the cubic dissipation coefficient anticipated in the previous section. In the high energy regime, $V > 2\lambda$, i.e., when the brane corrections prevail, Q grows with the number of e -folds, while when the brane corrections lessen, $V < 2\lambda$, Q fast decreases.

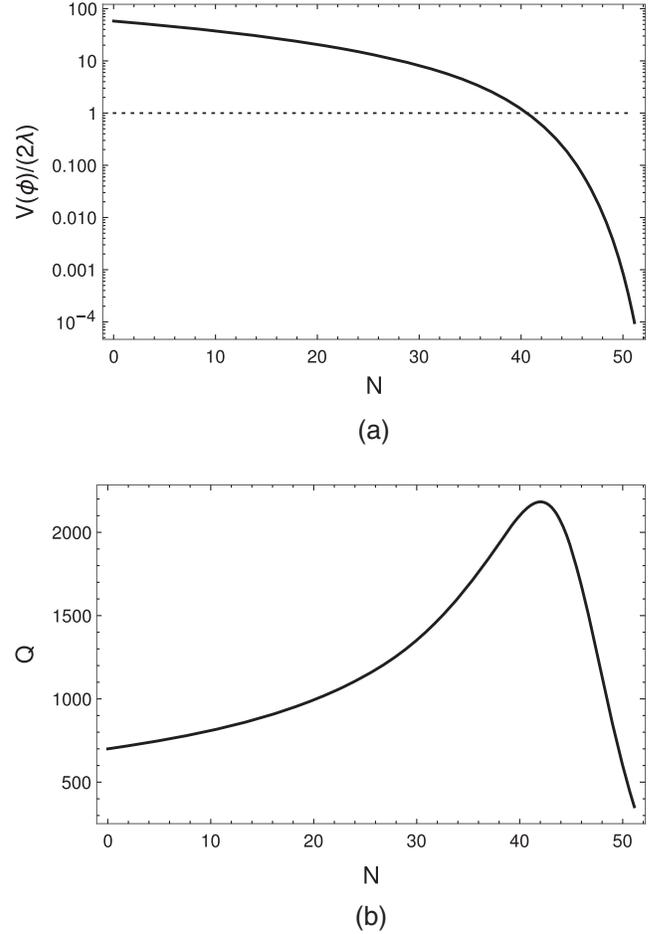


FIG. 3. The evolution of the ratio $V(\phi)/(2\lambda)$ [panel (a)] and the dissipation ratio Q [panel (b)], with the number of e -folds, for the choice of parameters $\alpha = 40$ and $\lambda/V_0 = 5 \times 10^{-5}$.

Considering the same parameters as those used in the previous figure, in Fig. 4 we show the behavior of the temperature at the end of inflation as a function of the dissipation ratio. The thin horizontal dotted line at $T = 10^7$ GeV has been drawn as reference only. From the results shown in Fig. 4, we see that the gravitino bound can comfortably be satisfied for a sufficient large dissipation, and the result is also consistent with the Planck data, as seen from Figs. 1 and 2.

We note that by taking as an explicit example for illustration purposes, the choice of parameters $\alpha = 40$, $\lambda/V_0 = 5 \times 10^{-5}$, and $Q_* = 700$, we find that $n_s \simeq 0.965$; that is well within the central value obtained from the Planck legacy data. Furthermore, as expected for such a large dissipation ratio, we have an extremely small tensor-to-scalar ratio, $r \simeq 1.3 \times 10^{-27}$. We also find for these parameter values that the inflaton potential normalization is $V_0^{1/4} \simeq 2.219 \times 10^9$ GeV and $N_* \simeq 51.1$. For these parameters, we also find that $V_*^{1/4} \simeq 6.12 \times 10^8$ GeV and that at the end of the inflationary regime, $V_{\text{end}} \simeq 2.20 \times 10^7$ GeV.

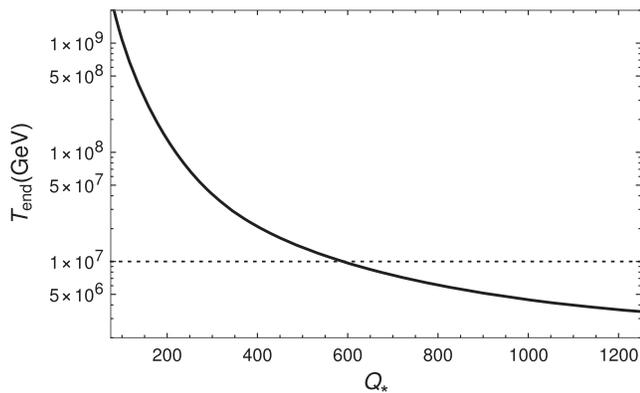


FIG. 4. The temperature at the end of inflation as a function of the dissipation ratio Q_* for $\alpha = 40$ and $\lambda/V_0 = 5 \times 10^{-5}$.

Furthermore, we find that $T_{\text{end}} \simeq 7 \times 10^6$ GeV at the end of inflation. The same parameters produce a sub-Planckian inflaton field excursion, found to be $\Delta\phi \simeq 0.3M_{\text{Pl}}$. In addition, the inflaton potential slow-roll parameter, ϵ_V , can be larger than one, as a consequence of the large value of the dissipation coefficient, while still allowing accelerated expansion by having $\epsilon_H \equiv -\dot{H}/H^2 \ll 1$. These results alone already ensure that the distance and the de Sitter (along also with the refined) swampland conjectures [14,46] are satisfied. In addition, given the lower energy scale for inflation allowed for the present model in the large dissipation regime, we also find the recently proposed trans-Planckian censorship conjecture (TCC) [65] is also satisfied here.³ Hence, to find that the results found here can satisfy all previously mentioned constraints is quite a pleasant feature.⁴

There is also another important consistent check that needs to be verified here. Since we are working with very steep potentials, and consequently, also with very efficient and large dissipation, where $Q_* \gg 1$, one might worry how such a large dissipation might affect the spectrum beyond the linear order. Since in WI the inflaton and radiation perturbations get very strongly coupled at large dissipation ratios, we expect that large nonlinearities will emerge and adversely affect the spectrum of perturbations. As far as

³This recent TCC condition was also recently analyzed in the context of WI in Ref. [66]. However, we find that this conjecture (along with the older ones) requires in fact a much larger dissipation than the $Q \sim 20$ value estimated in Ref. [66]. This is also backed up by the model studied in Ref. [47]. This is so because the conjecture TCC requires a very small scale for inflation in general, and in WI, this can only be achieved if very large Q_* values are considered, at least for the presently known and treated models in the literature.

⁴Note also that as recently claimed in Ref. [67], a larger scale for inflation than the one found in [65] might in fact be required from the TCC. Though this would lessen the condition on Q here, we still need a sufficiently large Q so as to have a temperature low enough at the end of inflation to satisfy the other bounds on the model.

non-Gaussianities in WI are concerned, the Planck team has produced constraints for the so-called warm shape of the bispectrum in WI [68], with the non-Gaussianity coefficient denoted by $f_{\text{NL}}^{\text{warm}}$. In the analysis done in Ref. [68] and valid for the strong dissipative regime of WI (but for a temperature independent and constant dissipation ratio Q), the expression found for $f_{\text{NL}}^{\text{warm}}$, valid in the strong dissipative regime, was⁵ $f_{\text{NL}}^{\text{warmS}} = -15 \ln(1 + Q/14) - 5/2$. The Planck 2018 analysis [69] based on this expression has given the results, $f_{\text{NL}}^{\text{warmS}} = -48 \pm 27$ (from SMICA + T + E, 68% C.L.) and $f_{\text{NL}}^{\text{warmS}} = -39 \pm 44$ (from SMICA + T, 68% C.L.), which then can be translated into the upper bounds for the dissipation ratio, $\log_{10} Q \leq 3.5$ and $\log_{10} Q \leq 3.6$, respectively, at 95% C.L. The largest values for Q_* that we have and that are shown in Figs. 1 and 2 and to still to have a consistent value for n_s , $Q_* \lesssim 10^3$, all fall safely inside these upper bounds from the non-Gaussianity analysis. In particular, using the simple formula for the non-Gaussianity parameter derived in Ref. [68] that was considered by the Planck team, we find $|f_{\text{NL}}^{\text{warm}}| \simeq 61.5$, when using the parameters of the explicit numerical example given above, with $Q_* = 700$. Later on, in Ref. [70], the authors generalized the computation of $f_{\text{NL}}^{\text{warm}}$ for a temperature dependent Q (and it was also pointed a sign error in the previous work). In Ref. [71], the authors made use of the δN formalism to study the non-Gaussianity. However, the δN formalism better probes the non-Gaussianity of the local shape, while the warm shape of WI, valid in the strong dissipative regime, is very weakly correlated with the local shape [72]. In Ref. [72], a complete analysis of non-Gaussianity in WI was performed, including the full effect of the coupling of the inflaton and radiation perturbations, which is essential to gauge the effect of the large dissipation on the spectrum. Following the numerical procedure explained in Ref. [72] and using the parameters values we have for the cubic form of the dissipation coefficient used in this work and the numerical example given above, for $Q_* = 700$, we find the result $|f_{\text{NL}}^{\text{warm}}| \simeq 5.5$, which is small enough to satisfy the Planck bounds, but still large enough to possibly be probed in the future through more precise observations from both fourth generation CMB observatories and on also future large scale structure surveys, which are expected to bring down the present upper bounds on non-Gaussianities.

V. CONCLUSIONS

In this work, we have shown that dissipation effects, in the WI context, assist to achieve a consistent inflationary model with an exponential potential in the RS II brane-world. It also allows us to simultaneously satisfy all the theoretical and observational restrictions given previously

⁵Note that in the early Ref. [68] and also in the Planck papers on the non-Gaussianities, the notation used for the dissipation coefficient Q was r_d .

on these type of exponential inflation potential in the braneworld construction. In fact, this is the first model of warm inflation with the specific form of a cubic temperature dependent dissipation coefficient that is found to be consistent with the observations in the strong dissipative regime of WI.

Achieving the strong dissipative regime in the present model is a consequence of both the braneworld high energy change of the GR evolution, combined with the exponential form for the primordial inflaton potential. As the result of achieving such a consistent dynamics in the strong dissipative regime, where $Q \gg 1$, it turns out that one can easily satisfy all the swampland conditions that have been recently proposed. Moreover, the model has a graceful exit from inflation for a large value of the constant parameter α in the inflaton exponential potential, with the dissipation ratio Q decreasing and the brane correction disappearing at the end of inflation. Furthermore, although the brane correction can make the tensor-to-scalar ratio r larger in the high energy level, due to the change in the tensor spectrum, the dissipation effects suppress the energy scale of inflation more significantly in the strong dissipative regime and the model turns out to be fully consistent with observational data, with appropriate values for the spectral tilt and a highly suppressed tensor-to-scalar ratio, which is one of the main results of the present work. Additionally, the temperature of the Universe at the end of inflation is inside the range that the model does not suffer from potential gravitino overproduction, and we can also avoid a large amount of relic gravity waves. We also note that these results follow for a broad range of the model parameter values, like for the constant α in the inflaton potential and for the brane tension λ , yet always satisfying the constrain bounds for these parameters. As also seen from the results shown in Figs. 1 and 2, there is a broad range of values for the dissipation coefficient that can be found to be consistent with the Planck data. In this sense, there is no special imposition or need for specific fine-tunings on these parameters. Likewise, as we study a regime of strong dissipation in WI, the inflationary slow-roll trajectory can be approached much faster, which reinforces the attractor like behavior of the corresponding

slow-roll solution (see, e.g., Refs. [73,74] for earlier studies of the effect of dissipation on the inflationary slow-roll trajectories).

We recall that exponential type of potentials have been considered as a possible candidate for describing the late-time acceleration [23,75] observed in the recent Universe. However, by having a tracking behavior, it fails to properly act in the present time as dark energy, with an equation of state that must be $\omega_\phi \simeq -1$. Besides, even in the context of the braneworld scenario, problems with this type of potential has been exposed [28], like a too large tensor-to-scalar ratio, a too red-tilted spectral index, and a possible excess of gravitational waves production that can destroy the BBN. From the results we have presented in this work, we see that all these problems can be overcome. Furthermore, typical interactions in the dark sector recently proposed in Ref. [36], and fully motivated from WI, could also be used in the present context. Since the Universe can smoothly enter into the radiation dominated epoch due to the presence of the WI dissipative effects, the reheating regime can be evaded, and the remaining inflaton field can be thawed and can behave like quintessence at late time, like in the models studied in Ref. [36]. Hence, the exponential potential can still be practical for unifying both the recent and the very early Universe histories, in the so-called dissipative quintessential inflation scenario. Therefore, our model can be a first step towards reconciling inflation and dark energy, with the bonus of being consistent with a high-energy UV completion within a theory of quantum gravity. We will study this possibility in the near future as a separate work.

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