

White dwarf cooling via gravity portals

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We investigate the impact of the gravity portal on the properties of white dwarfs, such as the equation of state and the cooling time. We find that the interaction between dark matter spin-zero bosons and electrons in the mean-field approximation softens the equation of state, and the objects cool down more slowly compared to the usual case.

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I. INTRODUCTION

Current cosmological and astrophysical observational data indicates that ordinary, luminous matter comprises only a tiny percentage of the total energy budget of the Universe [1]. Einstein's general relativity (GR) [2] can be made compatible with modern data only if another form of matter—which has neither strong nor electromagnetic interactions, and manifests itself via gravity—is postulated to exist. This new form of matter, now called dark matter (DM), is nonrelativistic in nature, roughly 5 times more abundant than baryonic matter [1], and it should be searched for in extensions of the Standard Model (SM) of particle physics. Its nature and origin still remains a mystery, and it comprises one of the biggest challenges of modern theoretical cosmology and particle physics. For reviews see, e.g., Refs. [3–7] and references therein, and for a list of good DM candidates see Ref. [8].

Since the spin of the particle that plays the role of DM in the Universe is still unknown, the simplest way to extend the SM to include a good DM candidate is to introduce a scalar field, i.e., a spin-zero boson. The treatment of scalar fields is simpler as they do not carry spinor or Lorentz indices, and they arise in many different setups in modern particle physics, such as (i) the Higgs sector needed to break electroweak symmetry and give masses to particles [9,10], (ii) pseudo-Goldstone bosons (pNGBs) associated with the explicit breaking of additional global symmetries [11], (iii) moduli from superstring theory compactifications [12–16], and (iv) supermultiplets in supersymmetric theories [17] and supergravity [18] which contain several scalar fields.

There is one more reason why one may consider the possibility of DM consisting of a scalar field. The Λ CDM model, based on cold DM and a positive cosmological

constant, has become the concordance cosmological model. Dark matter in the standard parametrization is assumed to be made of weakly interacting massive particles, a conjecture which works very well on large (cosmological) scales (\geq Mpc), but unfortunately on smaller (galactic) scales a few problems arise, such as the missing satellites problem, the core-cusp problem, and the too-big-to-fail problem [19]. These problems may be tackled in the context of self-interacting dark matter [20,21], as any cuspy feature will be smoothed out by the dark matter collisions. In addition, if dark matter consists of ultralight scalar particles with a mass $m \leq$ eV and a small repulsive quartic self-interaction, a Bose-Einstein condensate may be formed with a long-range correlation. This scenario has been proposed as a possible solution to the aforementioned problems on galactic scales [22–24]. For a review see, e.g., Ref. [25].

Since the presence of DM is inferred only via gravitational interactions, it seems more than natural to couple the Lagrangian of particle physics to GR. Then, one possibility that should not be ignored is a nonminimal coupling of the scalar field to gravity. Higgs inflation is a notable example of this type of scenario [26,27]. After all, as is well known, even if this term is absent at tree level it will be generated via quantum loop corrections [28]. Even if the scalar field that plays the role of DM does not have any direct interactions with the SM fields in the Jordan frame, its nonminimal coupling to gravity will induce nonvanishing interaction vertices in the Einstein frame after a conformal transformation is performed; see the discussion in the next section. The predictions and consequences of the gravity portal for the lifetime of the DM particle for different values of its mass and nonminimal coupling have been investigated in Refs. [29,30].

In the present work we propose to investigate for the first time the impact of the gravity portal on properties of white dwarfs (WDs), such as the equation of state (EoS) and cooling time. Being compact enough, WDs serve as ideal stellar laboratories for new gravitational effects.

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Their advantage over other compact objects, e.g., neutron stars, is that their equation of state is relatively well understood. The Fermi pressure of the degenerate electron gas prevents the collapse of the star due to its own gravity, and thus hydrostatic equilibrium is achieved. For these reasons the choice of WDs as cosmic laboratories for gravity is quite popular in the literature; see, e.g., Refs. [31,32].

The plan of our work is as follows. In the two subsections of the next section we briefly review the gravity portal scenario, and summarize the standard EoS of an ideal Fermi gas and the time dependence of the luminosity of WDs. In Sec. III we discuss the impact of the gravity portal on the EoS and the cooling time of WDs, and finally we conclude our work in the last section.

II. FORMALISM

A. The gravity portal scenario

First let us briefly review the gravity portal following Refs. [29,30]. We extend the Lagrangian of the SM to include dark matter by adding a scalar field ϕ , $\mathcal{L}_T = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{DM}}$, where \mathcal{L}_{SM} is the usual Lagrangian of the SM [33,34], while the scalar field is described by the Lagrangian

$$\mathcal{L}_{\text{DM}} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi), \quad (1)$$

where the scalar potential includes a mass term and possible self-interactions. For instance, for repulsive dark matter it may have the form [35,36]

$$V(\phi) = \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{24} \lambda \phi^4 + \dots, \quad (2)$$

where m_ϕ is the mass of the DM particle, and $\lambda = (m_\phi/F)^2 > 0$, with F being a high mass scale, is the self-interaction coupling constant; for pNGBs it has the form [37]

$$V(\phi) = \Lambda^4 [1 \pm \cos(\phi/F)], \quad (3)$$

which upon expansion around the minimum leads to an attractive force [38].

In Refs. [39,40] it was shown that a spin-zero particle can form a Bose-Einstein condensate, solving the core/cusp problem on galactic scales. However, this requires a repulsive force, and this is why in the following we shall focus on potentials of the first kind, i.e., Eq. (2). Moreover, if it is assumed that the scattering length is of the order of 1 fm, the mass of the DM particle is computed to be 10 meV.

Then, we couple the particle physics Lagrangian to gravity. In the physical (Jordan) frame the model is described by the action

$$S_J = \int d^4x \sqrt{-g} \left[-\frac{1}{2\kappa^2} R + \mathcal{L}_T - \xi R f(\phi) \right], \quad (4)$$

where R is the Ricci scalar, g is the determinant of the metric tensor $g_{\mu\nu}$, $\kappa^2 = 8\pi G$, and we allow for a non-minimal coupling to gravity. We remind the reader that this type of coupling is not optional; rather, it is inevitable, since it will be generated via quantum loop corrections even if it is absent in the classical action. The functional form of the factor $f(\phi)$ depends on the specific model considered.

Performing a conformal transformation

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad (5)$$

where $\Omega^2 = 1 + 2\xi\kappa^2 f(\phi)$, the action in the Einstein frame takes the form [29,30]

$$S_E = \int d^4x \sqrt{-\tilde{g}} \left[-\frac{1}{2\kappa} \tilde{R} + \mathcal{L}_{\phi,\text{SM}} + \dots \right], \quad (6)$$

where the dots denote terms that are not of interest here, while the interaction Lagrangian between the SM particles and the dark matter particle ϕ is found to be [29,30]

$$\mathcal{L}_{\phi,\text{SM}} = -2\xi\kappa^2 \left. \frac{\partial f}{\partial \phi} \right|_{\phi=0} \phi \left[\frac{3}{2} T_f + \dots \right], \quad (7)$$

where for the time being we are interested only in the coupling between the spin-zero boson and the SM fermions.

We see that even if in the Jordan frame there are no direct couplings between ϕ and the SM particles, in the Einstein frame there are interaction terms induced due to the nonminimal coupling ξ . For heavy DM particles, $m_\phi \geq 1$ GeV, in the gravity portal studied in Refs. [29,30] two specific models were considered, namely, the scalar singlet DM [41,42], where

$$\Omega^2(\phi) = 1 + 2\kappa^2 \xi M \phi, \quad (8)$$

and the inert doublet model [43,44], where

$$\Omega^2(h, \eta) = 1 + 2\kappa^2 \xi (v + h)\eta, \quad (9)$$

where h is the SM Higgs boson, v is its vacuum expectation value, and η is the CP -even Higgs boson coming from the second doublet. Therefore, the DM particle may decay into SM particle pairs (if kinematically allowed), $\phi \rightarrow X_1 X_2$, and the precise expressions for the partial decay widths depend on the conformal factor Ω^2 [or on the function $f(\phi)$, if you wish]. The lifetime τ of the DM particle is given by

$$\tau^{-1} = \Gamma_T = \sum_i \Gamma_i, \quad (10)$$

where the index i runs over all possible decay channels, and Γ_T is the total decay width of the scalar field ϕ . Since the DM particle must be quasistable (cosmologically stable), the lifetime of ϕ should be larger than the age of the Universe, $t_0 \simeq 4 \times 10^{17}$ sec. In reality, however, the lifetime of the DM is further constrained by telescopes that have been designed to detect its decay products, such as neutrinos (IceCube [45]) or γ rays (*Fermi*-LAT [46]). In Ref. [30], for the models studied there and for masses $m_\phi \geq 1$ GeV, the authors imposed the conservative limit $\tau \geq 10^{24}$ sec. In the framework considered here, and if the scalar field ϕ is very light ($m_\phi \sim \text{meV}$ or lower), as suggested by the cusp/core problem, the only kinematically open channel is the one to photons. However, the scalar field is coupled to massive vector bosons only (see the Feynman rules in the Appendix of Ref. [30]), and therefore in the scenario adopted here ϕ cannot decay into a pair of photons at tree level. The decay process will necessarily take place at the one-loop level via charged particles circulating in the loop (this is the case of, for instance, the QCD axion [47–50]), and therefore the lifetime of the scalar field is expected to exceed the age of the Universe.

At this point a remark is in order. In the simplified scalar field framework we consider here, ϕ does not need to be identified with the DM particle. As a matter of fact, it would be more interesting if we suggested a possible way to constrain the more general case of any new scalar field with certain couplings to electrons. Therefore, to keep the discussion as general as possible, in the rest of our work the scalar field does not necessarily serve as the DM particle. We shall only make a couple of minimal assumptions, namely, (i) that ϕ is a new scalar field beyond the SM of particle physics with a self-interaction potential of the Higgs-like form (2), and (ii) that it is real and very light, $m_\phi \sim \text{meV}$ or lower.

B. EoS and cooling time of WDs: Standard treatment

1. EoS of an ideal Fermi gas

White dwarf stars are old compact objects that mark the final evolutionary stage of the vast majority of stars [51,52]. Indeed, more than 95% (perhaps up to 98%) of all stars will die as white dwarfs [53]. They were discovered in 1914 when Russell noticed that the star now known as 40 Eridani B was located well below the main sequence on the Hertzsprung-Russell diagram. About 80% of WDs have a hydrogen atmosphere (DA type), while 20% have a helium atmosphere (DB type) [54]. Low-mass white dwarfs are expected to harbour He cores, while average-mass white dwarfs most likely contain carbon/oxygen cores [51].

In the zeroth-order approximation, ignoring the Coulomb interactions of electrons, the essential features of the EoS of WDs are captured by the Chandrasekhar model [55]. In the standard case without a spin-zero boson, electrons with mass

m inside a WD form an ideal Fermi gas, the energy density and pressure of which are given by the well-known expressions [55–57]

$$\epsilon_{\text{st}} = \frac{2}{(2\pi)^3} \int_0^{k_F} d^3\vec{k} \sqrt{k^2 + m^2}, \quad (11)$$

$$p_{\text{st}} = \frac{1}{3} \frac{2}{(2\pi)^3} \int_0^{k_F} d^3\vec{k} \frac{k^2}{\sqrt{k^2 + m^2}}, \quad (12)$$

where the Fermi wave number k_F is related to the fermion number density n as follows:

$$n = \frac{k_F^3}{3\pi^2}. \quad (13)$$

The integrals above can be computed exactly, and therefore one can obtain analytical expressions for the pressure and energy density of an ideal Fermi gas as follows:

$$\epsilon_{\text{st}} = \frac{m^4}{8\pi^2} \left((x_F + 2x_F^3) \sqrt{1 + x_F^2} - \sinh^{-1}(x_F) \right), \quad (14)$$

$$p_{\text{st}} = \frac{m^4}{24\pi^2} \left((-3x_F + 2x_F^3) \sqrt{1 + x_F^2} + 3 \sinh^{-1}(x_F) \right), \quad (15)$$

where we have defined $x_F = k_F/m$. In addition, we define the scalar baryon density as

$$n_s = \frac{\partial \epsilon_{\text{st}}(m)}{\partial m} = \frac{2}{(2\pi)^3} \int_0^{k_F} d^3\vec{k} \frac{m}{\sqrt{k^2 + m^2}} \quad (16)$$

for later use, and it is given by

$$n_s = \frac{m^3}{2\pi^2} \left[x_F \sqrt{1 + x_F^2} - \ln \left(x_F + \sqrt{1 + x_F^2} \right) \right]. \quad (17)$$

In the nonrelativistic limit, $x_F \ll 1$, the expression for the pressure takes the approximate form [58]

$$p_{\text{st}} \sim \frac{m^4 x_F^5}{15\pi^2}, \quad (18)$$

while the density is given by [58]

$$n = \frac{x_F^3 m^3}{3\pi^2} = \frac{\rho}{m_u \mu_e}, \quad (19)$$

where $m_u = 1 \text{ amu} = 1.66 \times 10^{-24} \text{ g}$ is the unified atomic mass unit [59], and $\mu_e = A/Z$, with A being the atomic number of the element of the core, is the molecular weight per electron. There is no explicit dependence on Z , and $\mu_e = 2$ irrespective of the core composition [32]. Therefore, one obtains an EoS of the form

$$p_{\text{st}} = \frac{(3\pi^2)^{5/3}}{15\pi^2 m (\mu_e m_u)^{5/3}} \rho^{5/3} = K_{\text{st}} \rho^{5/3}, \quad (20)$$

where the constant $K_{\text{st}} = 3.1 \times 10^{12}$ in cgs units.

2. Time dependence of WD luminosity

Since there are no thermonuclear reactions for WDs, these objects cool down via radiation, and their emitted energy is due to stored thermal energy. As the pressure drops to zero close to the surface, there must be a non-degenerate atmosphere, which provides a very thin insulating layer (of the order of 10^{-3} times the radius of the star) that regulates the rate of heat loss from the object [60]. The cooling time of WDs in the standard case can be found, e.g., in Refs. [60,61]. Combining the equations that describe hydrostatic equilibrium and the EoS with the well-known laws [60]

$$p = \frac{\mathcal{R}\rho T}{\mu}, \quad (21)$$

$$L = -MC_\nu \frac{dT}{dt}, \quad (22)$$

where t is time, L is the luminosity of the WD, T is the constant temperature of its isothermal core, M is the mass of the star, μ is the mean molecular weight, $\mathcal{R} = k_B/m_u = 8.315 \times 10^7 \text{ erg K}^{-1} g^{-1}$ is the ideal gas constant) with k_B being the Boltzmann constant) [59,60], and $C_\nu = (3\mathcal{R})/(2\mu)$, one finally obtains the time dependence of the WD luminosity,

$$\frac{L}{L_0} = \left[1 + \frac{t}{\tau_{\text{st}}} \right]^{-7/5}, \quad (23)$$

where L_0 is the initial luminosity, while the characteristic cooling time τ_{st} is computed to be

$$\tau_{\text{st}} = \frac{3\mathcal{R}}{5\mu} \left[\frac{51\kappa_0\mu\mathcal{R}^4}{64\pi G 4\sigma\mu_e^5 K_{\text{st}}^3} \right]^{2/7} \left(\frac{L_0}{M} \right)^{-5/7}, \quad (24)$$

where M is the mass of the WD star, G is Newton's constant, $\kappa_0 = 2 \times 10^{20} \text{ m}^5 \text{ kg}^{-2} \text{ K}^{7/2}$ is the opacity [60], $\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ is the Stefan-Boltzmann constant [59], and $\mu = 1.35$ for realistic compositions [60]. In Eq. (23) the term t/τ_{st} is dimensionless. Accordingly, the first and second terms of Eq. (24) have units $\text{m}^{10/7} \text{ s}^{-8/7}$ and $\text{m}^{-10/7} \text{ s}^{15/7}$ if the luminosity and mass of the star are expressed in kg and W (in SI units), respectively.

III. IMPACT OF THE GRAVITY PORTAL ON WDs

Here we shall study the impact of the DM particle on the EoS of WDs and its cooling time. The treatment is similar to the relativistic mean field theory of neutron stars

[62–64], where nucleons interact by exchanging mesons, the value of which are taken to be constants. Here we study WDs instead of neutron stars, nucleons are replaced by electrons, and finally mesons are replaced by the DM particle ϕ .

In the gravity portal, and in the Einstein frame, there is a Yukawa coupling between the DM boson ϕ and the electrons, which are Dirac fermions ψ . The system is described by the Lagrangian density

$$\mathcal{L}_{e\text{-DM}} = \bar{\psi}(i\gamma_\mu\partial^\mu - m + g\phi)\psi + \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m_\phi^2\phi^2), \quad (25)$$

where a possible self-interaction coupling constant λ has been ignored, since it will have a negligible effect on the numerical results. The Yukawa coupling constant g depends on the two free parameters of the model, namely, the nonminimal coupling ξ and the mass of the scalar field m_ϕ . However, it is more convenient to trade ξ for g and take in the following (m_ϕ, g) to be the two free parameters of the model.

In the mean-field approximation [62–64] it is assumed that ϕ is a constant, ϕ_0 , and therefore the system looks like an ideal Fermi gas where electrons acquire an effective mass

$$m_* = m - g\phi_0. \quad (26)$$

Since the kinetic term of the DM particle vanishes, the total pressure and energy density of the system is given by

$$p = p_{\text{st}}(m_*) - \frac{m_\phi^2\phi_0^2}{2}, \quad (27)$$

$$\epsilon = \epsilon_{\text{st}}(m_*) + \frac{m_\phi^2\phi_0^2}{2}, \quad (28)$$

where $p_{\text{st}}(m_*)$ and $\epsilon_{\text{st}}(m_*)$ are the standard expressions for the pressure and energy density, respectively, of an ideal Fermi gas evaluated at the effective mass m_* . Finally, the constant value of the DM boson is given by

$$\phi_0 = \frac{gn_s(m_*)}{m_\phi^2}, \quad (29)$$

where the scalar density n_s is evaluated at the electron effective mass m_* . The expression for ϕ_0 can be obtained from the thermodynamic argument that a closed, isolated system will minimize its energy with respect to the field or the effective mass.

The effective mass of the electrons is determined by solving the equation

$$m_* = m - \frac{g^2 n_s(m_*)}{m_\phi^2}, \quad (30)$$

and the new EoS is obtained.

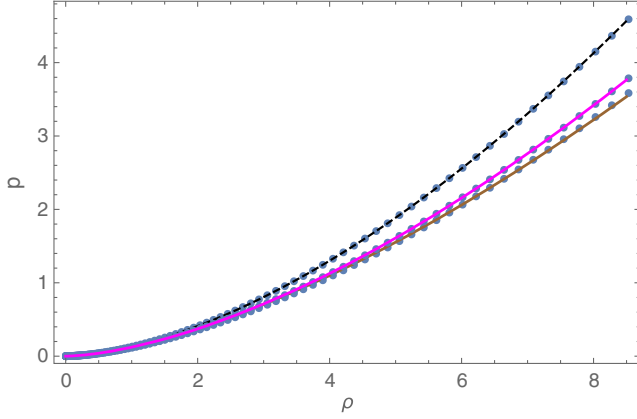


FIG. 1. Modification of the EoS due to the electron-DM interaction in the gravity portal for $g = 4.4 \times 10^{-9}$ and $m_\phi = 4.4 \times 10^{-13}$ GeV (brown curve), and $g = 4.1 \times 10^{-9}$ and $m_\phi = 4.6 \times 10^{-13}$ GeV (magenta curve). The standard EoS (black curve) is also shown for comparison.

In Fig. 1 we show the modification of the EoS in the gravity portal. The points are generated from the numerical solution, while the continuous curves are the fitting curves that correspond to polytropic EoSs $p = K\rho^{(1+1/n)}$ with appropriate K, n . In particular, the black curve corresponds to the standard polytropic EoS with index $n = 1.5$,

$$p_{\text{st}} = K_{\text{st}}\rho^{5/3}, \quad (31)$$

the brown curve (obtained assuming $g = 4.4 \times 10^{-9}$ and $m_\phi = 4.4 \times 10^{-13}$ GeV) corresponds to a new EoS with index $n = 1.82$,

$$p_1 = K_1\rho^{1.55}, \quad (32)$$

with $K_1 = 5.75 \times 10^{12}$ in cgs units, and the magenta curve (obtained assuming $g = 4.1 \times 10^{-9}$ and $m_\phi = 4.6 \times 10^{-13}$ GeV) corresponds to a modified EoS with index $n = 1.67$,

$$p_2 = K_2\rho^{1.6}, \quad (33)$$

with $K_2 = 4.19 \times 10^{12}$ in cgs units.

Going through the same steps for the generic polytropic EoS $p = K\rho^{(1+1/n)}$, one obtains the following expression for the luminosity:

$$\frac{L}{L_0} = \left[1 + \frac{t}{\tau_a}\right]^{1/a}, \quad (34)$$

where a is found to be

$$a = -\frac{(17/2) - 3 - 2n}{2((17/4) - n - 1)}, \quad (35)$$

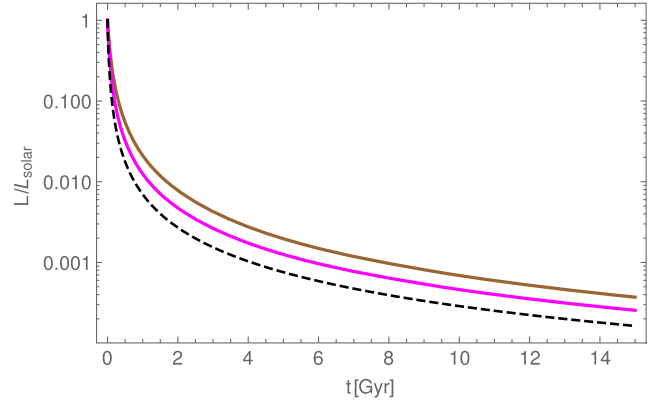


FIG. 2. Cooling of white dwarfs in the gravity portal (solid curves) in comparison with their cooling in the standard case (dashed curve) for $M = 0.5 M_\odot$ and $L_0 = L_\odot$.

while the new characteristic cooling time τ_a is computed to be

$$\tau_a = \left(\frac{1+a}{-a}\right) \frac{3\mathcal{R}}{2\mu} \left[\kappa_a \left(\frac{\mathcal{R}}{\mu_e}\right)^{2(n+1)}\right]^{1+a} \left(\frac{L_0}{M}\right)^a, \quad (36)$$

where $\kappa_a = (51\kappa_0\mu)/(64\pi G4\sigma\mathcal{R}K^{2n})$. It is easy to verify that when $n = 3/2 = 1.5$, we recover the expressions of the previous subsection, which are valid in the usual case for $\xi = 0 = g$. In Fig. 2 we show the time dependence of the WD luminosity for the standard case and for the modified EoS for $M = 0.5 M_\odot$ and $L_0 = L_\odot$.

Figure 3 shows the path of WD stars that are going through a cooling process in a simplified Hertzsprung-Russell diagram (luminosity decreasing with time). The two dashed black curves correspond to the standard cooling process [as computed using Eq. (23)] of two WDs with masses $0.3 M_\odot$ and $0.6 M_\odot$. Both stars have an initial luminosity $L_0 = L_\odot$ as computed using Eq. (23).

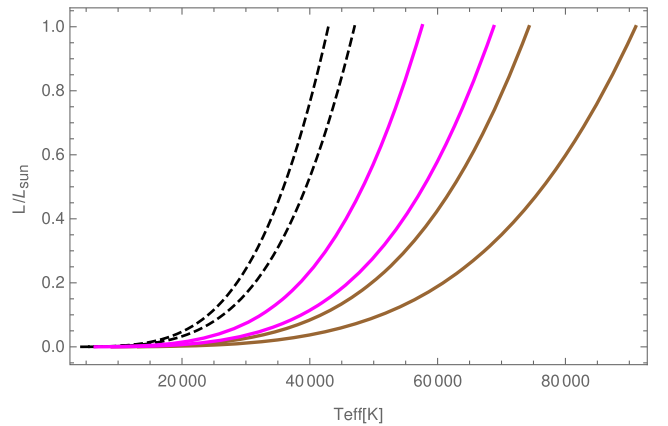


FIG. 3. Luminosity of WDs (in units of solar luminosity) versus effective temperature (in Kelvin) (i) for the standard case (in black) and (ii) for the two models in the gravity portal scenario (colors are as in Figs. 1 and 2).

The colored curves correspond to the same stars, but for the models discussed in Figs. 1 and 2.

It would be interesting to use the predictions of the model and the results obtained here to put constraints on the free parameters of the model using current observational data related to white dwarf stars. We hope to be able to address this issue, and perform a thorough analysis along these lines in a future work.

IV. CONCLUSIONS

In summary, in this work we have studied the impact of the gravity portal scenario on the cooling time of white dwarfs. Our results show that the electron–scalar DM

interaction leads to a softer equation of state, which in turn implies a slower cooling time in comparison with the standard case.

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