White dwarfs as a probe of dark energy

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We investigate the radial density distribution of the dynamical dark energy inside the white dwarfs (WDs) and its possible impact on their intrinsic structure. The minimally coupled dark energy with the barotropic equation of state, which has three free parameters (density, equation of state, and effective sound speed), is used. We analyze how such dark energy affects the mass-radius relation for the WDs because of its contribution to the joint gravitational potential of the system. For this we use the Chandrasekhar model of the WDs, where model parameters are the parameter of the chemical composition and the relativistic parameter. To evaluate the dark energy distribution inside a WD we solve the conservation equation in the spherical static metric. The obtained distribution is used to find the parameters of dark energy for which the deviation from the Chandrasekhar model mass-radius relation become non-negligible. We conclude also that the absence of observational evidence for the existence of WDs with untypical intrinsic structure (mass-radius relation) gives us lower limits for the value of an effective sound speed of dark energy $c_s^2 \gtrsim 10^{-4}$ (in units of speed of light).

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I. INTRODUCTION

The nature of dark energy, a substance which causes the observable accelerated expansion of Universe, has become a highly studied subject in cosmology in the last two decades. A significant part of models that explain it are (usually, scalar) field models of the dark energy [1–8]. Unlike the cosmological constant, scalar field dark energy is assumed to be dynamical and perturbable, changing its density across time and space, and having, as a result, restrained impact on the evolution of the large scale structures [9]. Such dark energy can be modeled as the perfect or imperfect fluid, which is effectively described by hydrodynamical parameters: the density ρ_{de}, the equation of state parameter w_{de}, and the effective speed of sound c_s. The most conservative models assume that only the density of the dark energy varies with cosmological time, while models with more degrees of freedom assume that all three parameters are dynamical. Depending on its properties, dark energy is referred to as the quintessence ($w_{de} > -1$), the phantom ($w_{de} < -1$), or the quintom, in which w_{de} changes its sign during the evolution. Current observable data does not give strong preference to any of these types of dark energy [8–11].

It was first noticed by Babichev and co-authors [12] that scalar field dark energy can influence the compact objects through accretion. They analyzed how the infall of the phantom dark energy "screens" a black hole's (BH) gravitational field, eventually leading even to its disappearing.

The idea, that hidden components of the Universe can influence the compact objects, like BHs or white dwarfs (WDs), through gravity has developed further. In the last decade, a number of works appeared, in which alternative theories of gravity, also capable of explaining the accelerated expansion of the Universe, were tested on deviation from the general relativity at small scales through impact on the observable features of compact objects. For example, the Vainshtein mechanism [13,14], which restores the equivalence of Einstein's general relativity and some alternative gravity theories at small (Solar System) scales, can be broken inside matter for some cases, such as beyond Horndeski models [15–17]. With the purpose of probing this scenario, different kinds of compact astrophysical objects were chosen. For instance, in work [18] the red and brown dwarfs were used as probes for the modified gravity theories through impact on the mass-radius relation, the Chandrasekhar mass limit, and the mass-radius relation for the WDs were used in works [19,20] in order to obtain independent constraints on the Vainshtein breaking parameter. A similar work [17] was devoted to the study of relativistic objects, such as WDs and neutron stars, in which it was shown the importance of post-Newtonian corrections in the equilibrium equation for WDs while calculating macroscopic characteristics in the frame of the theory of modified gravity. In work [21] the authors attempted to explain the existence of sub- and super-Chandrasekhar

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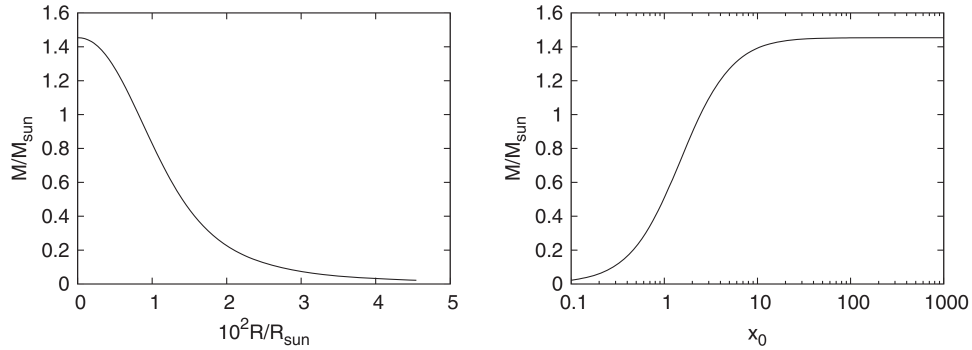


FIG. 1. Characteristics of white dwarfs in Chandrasekhar model: the mass-radius relation (left panel), the Chandrasekhar limit (right).

WDs as possible progenitors of peculiar supernova of Ia type with the help of modified gravity theory. BHs and relativistic stars in scalar-tensor theories of gravity are studied in [22,23]. In both works, authors find constraints from observing compact objects on theories. WDs are used for similar purpose in papers [24,25]: the corrections to equations that describe them are evaluated and constraints on the possible modifications of gravity are given.

The goal of this paper is to investigate the radial density distribution of the dynamical dark energy inside the WDs and estimate its possible impact on their intrinsic structure. It will give the possibility to estimate the lower limit for the value of the effective sound speed c_s of the dark component. We do it by considering the static solution of dark energy distribution in a spherically symmetrical metric. We take into account in the equation of the Chandrasekhar model, that the dark energy changes the joint gravitational potential of the system "white dwarf + dark energy," hence, changing the mass-radius relation for WDs, depending on its parameters, in particular, on the effective speed of sound.

Though the local behavior of dark energy clustering is an object of study in a lot of works lately (see [26,27,28], for example), and there are even examples of "mixed star" (dark energy + baryon matter) solutions [29,30], our work, as of our knowledge, is the first attempt to constrain dark energy parameters through observable WD properties.

The paper is organized as follows. In Sec. II we briefly recall the Chandrasekhar model of WDs and its main results. Section III contains the equation of state for the dark energy and the calculation of its radial distribution inside WDs. In Sec. IV we discuss the possibility of setting the constraint on the effective speed of sound of the dark energy using the WDs and in Sec. V we present our conclusions.

II. CHANDRASEKHAR MODEL OF WHITE DWARFS

A typical WD is a spherical object with mass of one half that of the Sun and a radius of the order of Earth's. In such extreme dense objects hydrostatic equilibrium is maintained by the pressure of relativistic degenerated electron gas [31–33]. The high density of matter causes the equation of state of the electron gas to be almost independent on temperature, which is why the mechanical and thermal structure of WDs can be treated separately. Such a zero-temperature approximation is especially applicable to massive WDs where finite temperature effects are negligible [34]. In approximation of complete degeneration, the equation of state of noninteracting relativistic electron gas can be written in the parametric form as follows:

$$P_e(r) = \frac{\pi m_e^4 c^5}{3h^3} f(x(r)),$$
$$f(x) = 8\int_0^x \frac{y^4 dy}{\sqrt{1+y^2}} = x(2x^2-3)\sqrt{1+x^2} + 3\ln\left(x+\sqrt{1+x^2}\right);$$
$$\rho(r) = \mu_e m_u \frac{8\pi(m_e c)^3}{3h^3} x^3(r). \quad (1)$$

Here m_e is the electron rest mass, m_u stands for atomic mass unit, and the dimensionless chemical composition parameter μ_e determines the number of nucleons per free electron for an averaged nucleus in a star (we assumed here $\mu_e = 2$). The dimensionless Fermi momentum of electrons $x = p_F/m_e c$, called the relativistic parameter, plays the role of the parameter in the above mentioned equation of state.

Assuming the hydrostatic equilibrium of a nonrotating gaseous sphere, Chandrasekhar obtained the model of the WDs with two parameters [33]: the relativistic parameter in a stellar center x_0 and the chemical composition parameter μ_e (which is close to 2 for all elements except hydrogen). Within this model two important outcomes became famous: the peculiar mass-radius relation—the radius of WDs decreases with increasing mass, which is contrary to normal stars (left panel of Fig. 1); and the existence of the maximum mass of the WD $\sim 1.5\ M_\odot$, known as the Chandrasekhar mass limit. This is the formal limit for WDs with the central density approaching infinity (right panel of Fig. 1). The latter one played the crucial role in the discovery of the accelerating expansion of the Universe through observations of the distant supernovae of Ia type. These kinds of superluminous events are believed to be explosions of WDs exceeding the Chandrasekhar limit due to the accretion of matter from another component of the binary system.

III. DARK ENERGY IN WHITE DWARFS

A. Dark energy model

In this work we analyze the scalar field model of dark energy with the barotropic equation of state

$$p_{de} = w(\rho_{de})\rho_{de}c^2, \tag{2}$$

where p_{de} and ρ_{de} are the pressure and density of dark energy, respectively. We consider a model for which the relation between the equation of state parameter w and the effective speed of sound c_s^2 (in the units of speed of light c) is as follows:

$$w = c_s^2 - (c_s^2 - w_\infty)\frac{\rho_\infty}{\rho_{de}}. \tag{3}$$

Here ρ_∞ is the background density of dark energy (at $r \to \infty$), for which we adopted the value $10^{-26}\ \mathrm{kg/m^3}$ [35]. Also, we considered two types of dark energy, quintessence with $w_\infty = -0.8$ and phantom with $w_\infty = -1.2$.

We chose hydrodynamical (phenomenological) representation of the scalar field dark energy as usual, for convenience. Scalar field dark energy can be represented as a perfect or imperfect fluid with the barotropic equation of state ([36,37]). Indeed, given the Lagrangian of the field $\mathcal{L}(X, U)$ with kinetic term X and potential U, the connection with phenomenological quantities is as follows:

$$\rho_{de} = 2X\mathcal{L}_{,X} - \mathcal{L}, \qquad p_{de} = \mathcal{L},$$
$$w_{de} = \frac{p_{de}}{c^2\rho_{de}} = \frac{\mathcal{L}}{2X\mathcal{L}_{,X}}, \qquad c_s^2 = \frac{\delta p_{de}}{c^2\delta\rho_{de}} = \frac{\mathcal{L}_{,X}}{2X\mathcal{L}_{,XX} - \mathcal{L}_{,X}}.$$

One can obtain the linear equation of state for the stationary Minkowski or Schwarzschild world and the scalar field dark energy with conditions $c_s^2 = \mathrm{const} > 0$ and $w_{de} < 0$ [35]. The properties of such dark energy in the vicinities of compact objects were also studied in [12,35,37]. In the case of static space-time the equation of state parameter (3) corresponds to a static scalar field with a constant potential U and a density-dependent kinetic term X [38].

B. Dark energy distribution inside a white dwarf

In order to analyze the behavior of dark energy inside a compact astrophysical object, we consider the simplest model of WDs without rotation and the neglected effects of the magnetic field, finite temperature, and Coulomb interactions on the mechanical structure. Consequently, we expect a spherically symmetric distribution of dark energy inside a star. Also, in this work we do not aim to describe the dynamical evolution of dark energy in the gravitational field of a compact object, but instead focus on the static configuration of the system, which consists of two components: the matter of WDs and dark energy.

The space-time metric for the spherically symmetric case can be written in the form

$$ds^2 = e^{\nu(r)}c^2d\tau^2 - e^{\lambda(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{4}$$

In our case, the components of the metric do not depend on time and can be obtained from Einstein equations with boundary condition $\lambda(r=0) = 0$

$$e^{-\lambda(r)} = 1 - \frac{8\pi G}{c^2 r}\int_0^r [\rho_m(r') + \rho_{de}(r')]r'^2 dr',$$
$$\nu(r) + \lambda(r) = -\frac{8\pi G}{c^2}\int_r^R \left[\rho_m(r') + \rho_{de}(r') + \frac{p_m(r') + p_{de}(r')}{c^2}\right] e^{\lambda(r')} r' dr'. \tag{5}$$

Here ρ_m, p_m are the local density and pressure of stellar matter and ρ_{de}, p_{de} denote the corresponding characteristics of dark energy.

In the case when the equilibrium of the gravitational force and pressure gradient is fulfilled, the following equations hold for both components of the considered system:

$$\frac{dp_m}{dr} + \frac{1}{2}(\rho_m c^2 + p_m)\frac{d\nu}{dr} = 0,$$
$$\frac{dp_{de}}{dr} + \frac{1}{2}(\rho_{de} c^2 + p_{de})\frac{d\nu}{dr} = 0. \tag{6}$$

If the density of dark energy is essentially lower than the matter density and metric function ν is defined by distribution of matter mainly, then the last equation gives the radial distribution of dark energy inside a star [35]

$$\rho_{de}(r) = \rho_\infty\left(\frac{c_s^2 - w_\infty}{1 + c_s^2} + \frac{1 + w_\infty}{1 + c_s^2}[e^{\nu(r)}]^{-\frac{1+c_s^2}{2c_s^2}}\right). \tag{7}$$

In order to solve the system of equations, (5) and (6), in the general case, we have to know the value for $\rho_{de}(0)$ or the value of $\nu(0)$ in the stellar center. For this we applied the iterative procedure: in the zero approximation we assumed no influence of dark energy on WDs and, having the results of the Chandrasekhar model [given in Eq. (1)], we calculated the second equation in (5) at the point $r = 0$. Following this, we use found potential $\nu(0)$ to evaluate the density of the dark energy at the center with formula (7). At the next step, this value was used to solve the system, (5) and (6), where both baryon matter and dark energy are

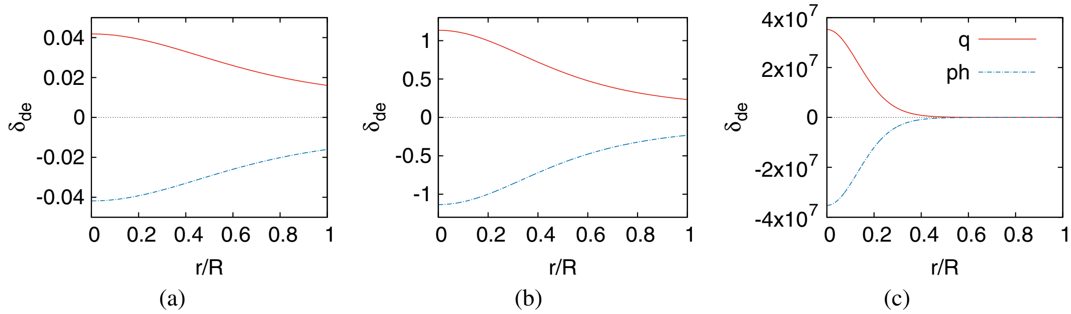


FIG. 2. The relative deviation $\delta_{de}(r) = (\rho_{de}(r) - \rho_\infty)/\rho_\infty$ as a function of the radial coordinate r inside a WD with radius R for fixed value $x_0 = 1$ and different values of c_s: (a) $c_s^2 = 10^{-3}$; (b) $c_s^2 = 10^{-4}$; (c) $c_s^2 = 10^{-5}$. Solid lines correspond to quintessential dark energy with $w_\infty = -0.8$, dash-dotted correspond to phantom dark energy with $w_\infty = -1.2$.

taken into account when calculating the potential, and recalculate a new value of $\rho_{de}(0)$. Such a procedure was repeated until the convergence was reached or the iteration limit exceeded. The algorithm stops when relative change of the potential at consecutive iterations is less than 10^{-5}. Usually it takes less then 10 iterations to reach the convergence.

The system ceases to converge when the effective sound speed c_s^2 approaches to zero, meaning that dark energy with very small c_s^2 does not allow static solutions. The specific value of c_s^2 when convergence is lost depends on the relativistic parameter in the center of the star x_0 and this value of c_s^2 increases with the growth of x_0. Technically divergence is manifested through very rapid growth of density and thus, the mass of dark energy in the system up to infinity (or negative infinity in the case of phantom dark energy).

Figure 2 illustrates the calculated relative deviation of the density of dark energy from the background for different values of the effective speed of sound c_s and fixed central density of the matter (or relativistic parameter x_0). It can be seen that with decreasing value of c_s the amount of dark energy inside WDs increases for quintessential dark energy (decreases for phantom one) by orders and becomes concentrated towards the stellar center.

One can see similar behavior of dark energy when we fixed the value of c_s but varied the central density (or x_0) of a star (see Fig. 3). Relative change of ρ_{de} is very sensitive to the relativistic parameter in the stellar center x_0 and is even more abrupt with the growing central density of the stellar matter.

The radial dependence of the WD mass, as well as the mass of dark energy inside a star, are shown in Fig. 4. It shows that the presence of quintessential dark energy reduces the mass of WDs in comparison to the result of the Chandrasekhar model, whereas the phantom dark energy causes its increase. Also, the stellar radius changes pettily, it grows in the case of quintessential dark energy and shrinks in the case of the phantom one.

The obtained negative values of density and mass for phantom dark energy needs some comments. The possibility of negative density for this model was already mentioned in [35] (see also [39] for cosmological consequences of phantom models). It was shown in [40,41], that the presence of such dark energy can cause UV quantum instability of vacuum through producing a pair of phantom particles and one nonphantom, having conserved energy. The brief discussion of the problem of the existence of the physical essence with a negative density or mass of particles from the general relativity point of view can be found in the recent paper [42]. Though we consider here classical behavior of the dark energy and the field behind it, this means that one should be careful when trying to obtain real observable constraints from the solutions for phantom dark energy, remembering there are also quantum limits of such models. In our case, constraints for quintessence and

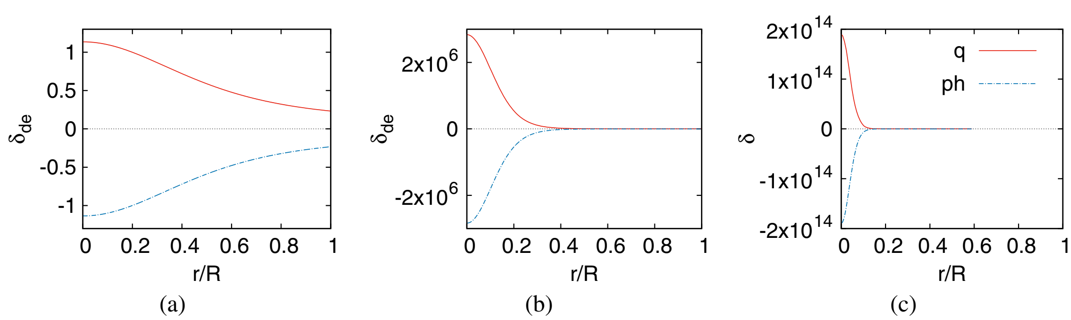


FIG. 3. The same as in Fig. 2 but for fixed value $c_s^2 = 10^{-4}$ and different values of x_0: (a) $x_0 = 1$; (b) $x_0 = 5$; (c) $x_0 = 10$.

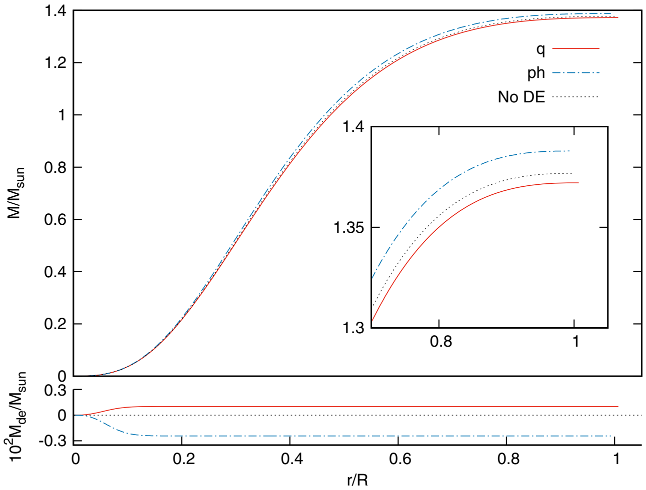


FIG. 4. Radial dependence of the white dwarf mass (top) and the mass of dark energy inside a star (bottom). Also, the zoomed-in part of the dependence near the stellar surface is shown on the top panel. Solid lines correspond to quintessential dark energy with $w_\infty = -0.8$, dash-dotted correspond to the phantom one with $w_\infty = -1.2$. The short-dashed line corresponds to the result of the Chandrasekhar model without taking into account dark energy. Here we assumed $x_0 = 10.2$, $c_s^2 = 4 \times 10^{-5}$.

phantom models are almost the same, as divergence occurs almost for the same values of x_0 and c_s^2.

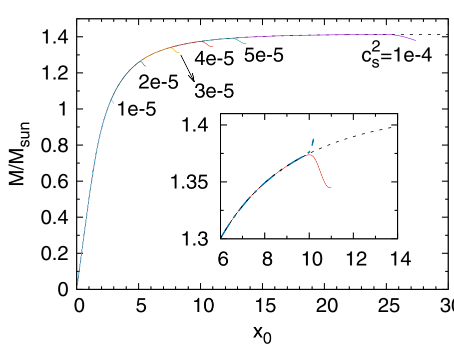
IV. CONSTRAINT ON c_s BY THE WD'S MASS-RADIUS RELATION

A. Influence on the mass-radius relation

We calculated the WD masses and radii for various values of a relativistic parameter in the stellar center x_0 and effective speed of sound c_s. Because we are interested in masses of WDs that can be obtained from observations, our results shown in Fig. 5 are represented in the form of the object's mass as a function of x_0.

As we can see from the figure, the dark energy inside WDs does not reveal itself unless some critical value of x_0 (dependent on c_s) is reached. In the vicinity of this value, the dark energy accumulated inside a star causes abrupt deviations from the results of the Chandrasekhar model (depicted with dotted line). The lower is the value of the effective speed of sound, the smaller is the critical value of x_0 at which the dark energy begins to make a significant contribution into the hydrostatic equilibrium of the WD by lowering its mass in the case of quintessential dark energy and increasing it in the case of the phantom one. The reason of such behavior is the following. The higher x_0 corresponds to the higher density ρ_m of matter in the center of WDs and hence, the deeper potential well of the system. The deeper potential well causes the growth of ρ_{de}. In the case of quintessential dark energy, given equation of state of dark energy makes its behavior similar to that of the matter, which means that for satisfying the hydrostatic equilibrium the lower mass of matter is necessary. Indeed, when $\rho_{de} \gg \bar{\rho}_{de}$ then $w \to c_s^2$, $p_{de} \to c_s^2 \rho_{de} c^2$ and dark energy contributes into the metric functions, as it follows from Eqs. (3) and (5). This breaks the hydrostatic equilibrium and causes the gravitational collapse of the system. In the case of phantom dark energy, ρ_{de} changes its sign and becomes negative. As a consequence, for satisfying the hydrostatic equilibrium, the larger mass of matter is necessary. In both cases the process is rapid, as sort of positive feedback loops are created, changing the properties of dark energy inside and in the nearest vicinities of the WD.

In the left panel of Fig. 5 the dependences $M(x_0)$ are presented for WDs in the models without dark energy (short-dashed line) and with quintessential dark energy with different values of c_s^2 (main part) and with phantom and quintessential dark energy in the insert which is a zoom-in of the central part of the figure. One can see that deviations for both quintessential and phantom dark energy take place approximately at the same values of x_0.



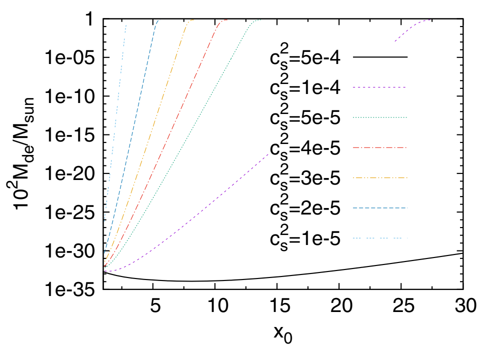


FIG. 5. Left panel: The dependence of the WD mass on x_0 for a set of values of effective speed of sound of quintessential dark energy c_s^2 (10^{-5}, 2×10^{-5}, 3×10^{-5}, 4×10^{-5}, 5×10^{-5}, 10^{-4}). Zoomed-in part of the dependence near $x_0 = 10$ for both types of dark energy with $c_s^2 = 4 \times 10^{-5}$ is shown in the central part of the figure. The downward deviation corresponds to quintessential dark energy, and upward corresponds to the phantom one. The short-dashed line corresponds to the result of the Chandrasekhar model. Right panel: The dependence of the mass of the quintessential type dark energy on parameter x_0 for the same set of values for c_s^2.

TABLE I. The critical values of the relativistic parameter in the stellar center x_0 for different values of the effective speed of sound for the dark energy of both considered types.

c_s^2	x_0^q	x_0^{ph}
10^{-4}	27.4	25.9
5×10^{-5}	13.7	12.7
4×10^{-5}	11.0	10.2
3×10^{-5}	8.3	7.6
2×10^{-5}	5.6	5.2
10^{-5}	3.0	2.8

The corresponding masses of dark energy of the quintessential type (in log-scale) as a function of the relativistic parameter in the stellar center x_0 are shown in the right panel of Fig. 5. The amount of dark energy inside a dwarf steeply increases with x_0 and strongly depends on the effective speed of sound c_s. Our solutions yield infinite values of densities when $c_s = 0$. This is a consequence of considering the equation of state of the dark energy given in Eqs. (2) and (3) and, correspondingly, solution (7), where c_s is in the denominator of the exponent. One can conclude that dark energy with $c_s = 0$ is excluded from possible models.

B. Constraints

As we saw in Fig. 5, there are some critical values of x_0 depending on c_s at which the dark energy changes the $M - x_0$ relation for the WD. These values are given in Table I for both considered types of dark energy.

In the papers [43,44], the authors employed the Chandrasekhar model of WDs to solve the inverse problem for a large sample (~3000) of spectroscopically confirmed WDs of type DA from SDSS DR4 [45]. Using known values of masses and radii, the relativistic parameter in the stellar center in the frame of the Chandrasekhar model $x_0 \lesssim 2.5$ (left panel of Fig. 6) for the vast majority of WDs from the sample was found. However, there exist the outliers for which x_0 can be sufficiently high—up to $x_0^{\max} \approx 8.5$ (indicated by arrows in figure). This value can be treated as the maximal value of x_0 for real WDs.

Also, WDs in binary systems can be used to estimate the maximal value of x_0. As can be seen in the right panel of Fig. 6, both tails of the distribution by mass (or by x_0) are more populated than for field (single) dwarfs. The reason is a mass transfer between the components of binary systems. As was mentioned above, it is believed that progenitors of Ia type supernovae events are WDs in close binaries with masses near the Chandrasekhar limit. Formally, in the frame of the Chandrasekhar model, they occur at $x_0 \to \infty$. However, it was shown first in [47] that mass accretion onto WDs can cause the instability before reaching the Chandrasekhar limit due to the effects of the general theory of relativity and/or neutronization. The critical values of the central density in the case of carbon WDs were found to be of the same order for both effects (2.65×10^{13} and 3.90×10^{13} kg/m^3, respectively) [48]. The recent values for the general theory of relativity effects are very similar (see, e.g., [49]). The corresponding values of the relativistic parameter $x_0^{\max}$ are 23.8 and 27.1, meaning x_0 cannot exceed it as the supernovae explosion occurs.

Thus, supposing that there exist field WDs with such high masses that correspond up to $x_0^{\max} \approx 10$ and/or that Ia type supernovae events are explosions of WDs in the binaries with relativistic parameter in their centers $x_0^{\max} \approx 25$, we can conclude that the dark energy inside this objects did not reveal itself. Having limits on x_0, one can constrain c_s^2. For each value of the former one there is a corresponding value of c_s^2 for which the solution for the hydrostatical equilibrium equation ceases to exist, i.e., dark energy with lower values of c_s^2 would destroy the WD. Within this assumption we have found the lower limit for the value of effective speed of sound when the deviation from the Chandrasekhar model becomes non-negligible: for field (single) WDs $c_s^2 \gtrsim 4\times10^{-5}$ and for WDs in binary systems $c_s^2 \gtrsim 10^{-4}$. It is interesting to point out that these constraints are close to the ones obtained in work [50], where we have used the current precision of the measuring

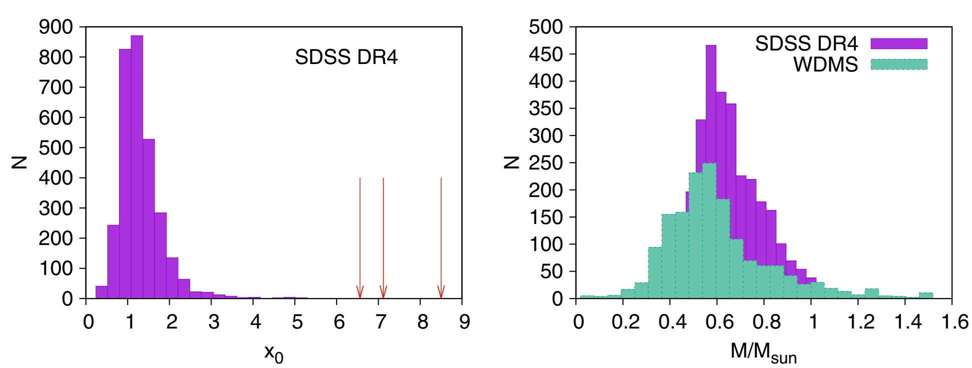


FIG. 6. Left panel: Distribution of white dwarfs of spectral class DA from SDSS DR4 [45] by x_0. Right panel: Comparison of distributions by a mass of white dwarfs from SDSS DR4 and white dwarfs in binary systems with main sequence stars (labeled as WDMS) [46].

gravitating mass in the Solar System to constrain the value of the speed of sound of the dark energy.

V. CONCLUSIONS

In this work we have considered a dark energy with the barotropic equation of state in the static gravitational field of WDs. We have obtained the distribution of dark energy in a WD using the Chandrasekhar model and calculated its impact on an object's characteristics in a self-consistent way. An investigation of the "mass-radius" relation for WDs with dark energy inside has shown that deviation of the WD mass from the one in the model without dark energy appears to be tiny, unless some critical value of the relativistic parameter x_0 is reached, though deviation of the density of dark energy in the center of the star from the background dark energy density can be noticeable. The deviation of the mass of WDs in comparison to the Chandrasekhar model is negative for the quintessential type of dark energy and positive for the phantom one. The critical value of x_0 decreases with the decreasing value of the dark energy effective speed of sound c_s.

Using this, we have compared the critical values of the relativistic parameter when the concentration of dark energy is too high to maintain the equilibrium of WDs with a maximum value of x_0 obtained in the Chandrasekhar model for the observed (single) WDs $x_0^{\max} \lesssim 10$ and with the value at which another effect affects the stellar structure such as neutronization or the effects of general theory of relativity for massive WDs in binary systems, $x_0^{\max} \lesssim 25$. Supposing that the dark energy has no or has negligible influence on the WD structure, which allows WDs with such relativistic parameters to exist, we can conclude that the minimal value of the squared effective speed of sound is $c_s^2 \approx 4 \times 10^{-5}$ in the case of field WDs and $c_s^2 \approx 10^{-4}$ for dwarfs near the Chandrasekhar limit in the binary systems.

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