S, P and D-wave resonance contributions to $B_{(s)} \to \eta_c(1S,2S)K\pi$ decays in the perturbative QCD approach

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In this work, we analyze the three-body $B_{(s)} \to \eta_c(1S,2S)K\pi$ decays within the framework of the perturbative QCD approach (PQCD) under the quasi-two-body approximation, where the kaon-pion invariant mass spectra are dominated by the $K_0^*(1430)^0$, $K_0^*(1950)^0$, $K^*(892)^0$, $K^*(1410)^0$, $K^*(1680)^0$, and $K_2^*(1430)^0$ resonances. The time-like form factors are adopted to parametrize the corresponding S, P, D-wave kaon-pion distribution amplitudes for the concerned decay modes, which describe the final-state interactions between the kaon and pion in the resonant region. The $K\pi$ S-wave component at low $K\pi$ mass is described by the LASS line shape, while the time-like form factors of other resonances are modeled by the relativistic Breit-Wigner function. We find the following main points: (a) the PQCD predictions of the branching ratios for most considered $B \to \eta_c(1S)(K^{*0} \to)K^+\pi^-$ decays agree well with the currently available data within errors; (b) for $\mathcal{B}(B^0 \to \eta_c(K_0^*(1430) \to)K^+\pi^-)$ and $\mathcal{B}(B^0 \to \eta_c K^+\pi^-(\mathrm{NR}))$ (where NR means nonresonant), our predictions of the branching ratios are a bit smaller than the measured ones; and (c) the PQCD results for the D-wave contributions considered in this work can be tested once precise data from the future LHCb and Belle-II experiments become available.

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I. INTRODUCTION

The $B_{(s)}$ meson decays into charmonia and a kaon-pion pair are of great interest since only a few color-suppressed modes in hadronic B decays have been measured so far. Some standard model (SM) parameters can be extracted from the $b \to c\bar{c}s$ transitions, while the studies of these decay channels can also provide an ideal place to find a signal for physics beyond the SM. The meson η_c and J/ψ have the same quark content but with different spin angular momentum. As expected, the B meson decays involving the η_c will garner considerable experimental attention with the development of experiments. In recent years, significant improvements in understanding the heavy quarkonium production mechanism have been achieved [1]. The $B^0 \to \eta_c(K^*(892)^0 \to)K\pi$ decay has been observed by the Belle [2] and *BABAR* collaborations [3,4]. Very recently, the $B^0 \to \eta_c K^+\pi^-$ decay was measured for the first time by the LHCb Collaboration [5], with the η_c meson reconstructed using the $p\bar{p}$ decay mode. This decay is expected to proceed through $K^{*0} \to K\pi$ intermediate states as well as the nonresonant (NR) S-wave component, where the K^{*0} refers to various partial-wave resonances, such as $K_0^*(1430)^0$, $K_0^*(1950)^0$, $K^*(892)^0$, $K^*(1410)^0$, $K^*(1680)^0$, and $K_2^*(1430)^0$. The P-wave $K^*(892)^0$ is the largest component, ~50%, while the $K^*(1410)^0$, $K^*(1680)^0$, and D-wave $K_2^*(1430)^0$ states amount to only a few percent.

The theoretical study and experimental measurement of the three-body B meson decays is still in an early stage. On the theoretical side, compared with those two-body decay modes, these three-body B decays are less tractable due to the entangled resonant and nonresonant contributions, as well as the possible final-state interactions [6–8]. An important breakthrough in the theory of three-body B meson decays was the confirmation of the validity of factorization. We restrict ourselves to the specific kinematical configurations in which the three mesons are

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quasialigned in the rest frame of the B meson. This condition is particularly natural in the low-effective-$K\pi$-mass region of the Dalitz plot, where most of the $K\pi$ resonant structures are seen. When the two particles among the three final-state mesons move collinearly and generate a small invariant mass recoiling against the third one, the three-body interactions are expected to be suppressed. Then, it seems reasonable to assume the validity of the factorization for these quasi-two-body B decays. In the quasi-two-body mechanism, the two-body scattering and all possible interactions between the two involved particles are included, but the interactions between the third particle and the pair of mesons are ignored. In recent years, several different theoretical frameworks based on the factorization theorems and symmetry principles have been proposed to deal with the three-body B meson decays. The QCD-improved factorization approach [9–12] has been widely used in the study of the three-body charmless hadronic B meson decays [13–23]. The U-spin and flavor $SU(3)$ symmetries were also adopted in Refs. [24–29].

It has been known that the collinear factorization of the charmed and charmless two-body B meson decays suffer from end-point singularities. The perturbative QCD (PQCD) factorization approach relying on the k_T factorization theorem [30,31] was proposed in Refs. [32–34], which has been shown to be infrared finite, gauge invariant, and consistent with the factorization assumption in the heavy-quark limit [35–38]. The operator-level definition of the transverse-momentum-dependent hadronic wave functions is highly nontrivial in order to avoid the potential light-cone divergence and the rapidity singularity [39,40]. The Sudakov factors from the k_T resummation have been included to suppress the long-distance contributions from the large-b region, with b being a variable conjugate to k_T. Therefore, the PQCD approach is a self-consistent framework and has good predictive power. Based on the PQCD approach, the quasi-two-body B meson decays have been studied in Refs. [41–52] by introducing the two-meson distribution amplitudes (DAs) [53–59], which catch the dynamics associated with the pair of mesons.

In this paper, we continue to study the quasi-two-body decays $B \to \eta_c K^{*0} \to \eta_c K\pi$ involving the S, P, and D-wave kaon-pion pairs as shown in Fig. 1, within the framework of the PQCD factorization approach. Some studies of $B \to \eta_c K^*$ decays have used the two-body framework [60–62]. From Refs. [41,43,45], we know that the width of the resonant state and the interactions between the final-state meson pair will show their effects on the branching ratios, especially on the direct CP violations of the quasi-two-body decays. Thus, it seems more appropriate to treat the K^{*0} as an intermediate resonance. As addressed before, this process is dominated by a series of resonances in S, P, and D waves. The S-wave kaon-pion DAs for the resonance $K_0^*(1430)^0$ have been studied in Ref. [63], and we will further investigate the dependence of the branching ratios in different scalar scenarios, as proposed in Refs. [64–67]. Besides, we have roughly determined the possible range of the first odd Gegenbauer moment B_1 for the $K_0^*(1950)^0$ resonance by fitting to the existing data, which must be tested in the future. We intend to adopt the same fitted parameters as those of the longitudinal kaon-pion DAs in Ref. [52], where the $SU(3)$ flavor-symmetry-breaking effect has been considered and plays an important role in the longitudinal polarization fractions. The D-wave resonance $K_2^*(1430)^0$ is investigated for the first time in our work. Due to the limited studies on the tensor resonant states, we treat the D-wave DAs of the $K_2^*(1430)^0$ in the same way as those of $f_2(1270)$ [45].

This paper is organized as follows. In Sec. II we give a brief introduction of the theoretical framework. The numerical values, some discussions, and our conclusions will be given in last two sections. The explicit PQCD factorization formulas for all of the decay amplitudes are collected in the Appendix.

II. FRAMEWORK

In the framework of the PQCD factorization approach, the nonperturbative dynamics associated with the pair of mesons can be absorbed into two-meson DAs; then, the relevant decay amplitude $\mathcal{A}$ for the quasi-two-body decays $B \to \eta_c K^{*0} \to \eta_c K\pi$ can be written in the following form:

$$\mathcal{A} = \Phi_B \otimes H \otimes \Phi_{K\pi} \otimes \Phi_{\eta_c}, \qquad (1)$$

where Φ_B and Φ_{η_c} are the B meson and charmonium DAs, respectively. The kaon-pion DA $\Phi_{K\pi}$ absorbs the nonperturbative dynamics in the $K\pi$ hadronization process.

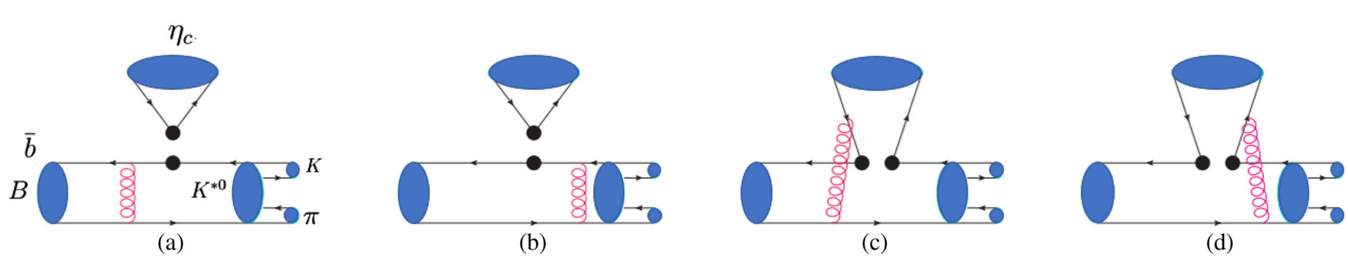

FIG. 1. Typical diagrams for the quasi-two-body decays $B \to \eta_c(1S, 2S)(K^{*0} \to)K\pi$, where the symbol (Bullet) denotes the weak vertex. K^{*0} represents various partial-wave intermediate states.

The hard kernel H contains only one hard gluon and describes the dynamics of the strong and electroweak interactions in the three-body hadronic decays, as in the formalism for the corresponding two-body decays.

In the light-cone coordinates, we make the kaon-pion pair and the final-state η_c move along the directions $n = (1, 0, 0_{\rm T})$ and $v = (0, 1, 0_{\rm T})$, respectively. The B meson momentum p_B, the total momentum of the kaon-pion pair, $p = p_1 + p_2$, the final-state η_c momentum p_3, and the quark momentum k_i in each meson are

$$\begin{aligned} p_B &= \frac{m_B}{\sqrt{2}}(1, 1, 0_{\rm T}), & k_B &= \left(0, x_B\frac{m_B}{\sqrt{2}}, \mathbf{k}_{B\rm T}\right), \\ p &= \frac{m_B}{\sqrt{2}}(1 - r^2, \eta, 0_{\rm T}), & k &= \left(z(1 - r^2)\frac{m_B}{\sqrt{2}}, 0, \mathbf{k}_{\rm T}\right), \\ p_3 &= \frac{m_B}{\sqrt{2}}(r^2, 1 - \eta, 0_{\rm T}), & k_3 &= \left(r^2 x_3\frac{m_B}{\sqrt{2}}, (1 - \eta)x_3\frac{m_B}{\sqrt{2}}, \mathbf{k}_{3\rm T}\right), \end{aligned} \quad (2)$$

where m_B is the mass of the B meson, $\eta = \frac{\omega^2}{m_B^2(1-r^2)}$ with $r = m_{\eta_c}/m_B$, m_{η_c} is the mass of charmonia, and the invariant mass squared $\omega^2 = p^2$. The momentum fractions x_B, z, and x_3 run from zero to unity, respectively.

We also define the momenta p_1 and p_2 in the kaon-pion pair as

$$\begin{aligned} p_1 &= (\zeta p^+, (1 - \zeta)\eta p^+, \mathbf{p}_{1\rm T}), \\ p_2 &= ((1 - \zeta)p^+, \zeta\eta p^+, \mathbf{p}_{2\rm T}), \end{aligned} \quad (3)$$

with $\zeta = p_1^+/P^+$ characterizing the distribution of the longitudinal momentum of the kaon and $p_{1\rm T}^2 = p_{2\rm T}^2 = \zeta(1-\zeta)\omega^2$.

The $B_{(s)}$ meson wave function and the charmonium DAs are the same as those widely adopted in the PQCD approach [42,44,51,68]. Very recently, a new method was proposed to calculate the B meson light-cone DA from lattice QCD, which can be used as an updated input for the B meson DA in the future [69]. Below, we briefly describe the S, P, and D-wave kaon-pion DAs and the corresponding time-like form factors. The S-wave kaon-pion DAs are introduced analogously to the two-pion ones [70],

$$\begin{aligned} \Phi_S = \frac{1}{\sqrt{2N_c}}[&\not{p}\phi_S^0(z, \zeta, \omega^2) + \omega\phi_S^s(z, \zeta, \omega^2) \\ &+ \omega(\not{n}\not{v} - 1)\phi_S^t(z, \zeta, \omega^2)]. \end{aligned} \quad (4)$$

In what follows the subscripts S, P, and D are always associated with the corresponding partial waves.

We will use the asymptotic forms for the twist-3 DAs, but no knowledge on the twist-2 DAs is available at present. We shall adopt similar formulas as those for a scalar meson [66,71], bearing in mind large uncertainties that may be introduced by this approximation. The detailed expressions of DAs of various twists are as follows:

$$\begin{aligned} \phi_S^0(z, \zeta, \omega^2) = &\frac{6}{2\sqrt{2N_c}}F_S(\omega^2)z(1 - z) \\ &\times \left[\frac{1}{\mu_S} + B_1 C_1^{3/2}(t) + B_3 C_3^{3/2}(t)\right], \end{aligned} \quad (5)$$

$$\phi_S^s(z, \zeta, \omega^2) = \frac{1}{2\sqrt{2N_c}}F_S(\omega^2), \quad (6)$$

$$\phi_S^t(z, \zeta, \omega^2) = \frac{1}{2\sqrt{2N_c}}F_S(\omega^2)(1 - 2z), \quad (7)$$

where the Gegenbauer polynomials $C_1^{3/2}(t) = 3t$, $C_3^{3/2}t = \frac{5}{2}t(7t^2 - 3)$ with $t = 1–2z$ and $\mu_S = \omega/(m_2 - m_1)$. The Gegenbauer moments $B_{1,3}$ and the related running current quark masses can be found in Refs. [66,67,72]. It should be stressed that there is less information about the scalar resonance $K_0^*(1950)^0$, and we only test the sensitivity of the branching ratios on the first odd Gegenbauer moment B_1 in our work.

If there are overlapping resonances or there is significant interference with a nonresonant component in the same partial wave, the relativistic Breit-Wigner (RBW) function leads to unitarity violation within the isobar model [73]. This is the case for the $K\pi$ S-wave at low $K\pi$ mass, where the $K_0^*(1430)^0$ resonance interferes strongly with a slowly varying NR S-wave component. In this work, the time-like scalar form factor $F_S(\omega^2)$ for the S-wave $K\pi$ system is parametrized by using a modified LASS line shape [74] for the S-wave resonance $K_0^*(1430)^0$, which has been widely used in experimental analyses [5],

$$F_S(\omega^2) = \frac{\omega}{|\vec{p}_1|[\cot(\delta_B) - i]} + e^{2i\delta_B}\frac{m_0^2\Gamma_0/|\vec{p}_0|}{m_0^2 - \omega^2 - im_0^2\frac{\Gamma_0}{\omega}\frac{|\vec{p}_1|}{|\vec{p}_0|}}, \quad (8)$$

with

$$\cot(\delta_B) = \frac{1}{a|\vec{p}_1|} + \frac{r|\vec{p}_1|}{2}, \tag{9}$$

where the first term in Eq. (8) is an empirical term from the elastic kaon-pion scattering and the second term is the resonant contribution with a phase factor to retain unitarity. Here m_0 and Γ_0 are the pole mass and width of the $K_0^*(1430)$ state. $|\vec{p}_1|$ is the momentum vector of the resonance decay product measured in the resonance rest frame, and $|\vec{p}_0|$ is the value of $|\vec{p}_1|$ when $\omega = m_{K^*}$. The parameters $a = (3.1 \pm 1.0)\ \mathrm{GeV}^{-1}$ and $r = (7.0 \pm 2.4)\ \mathrm{GeV}^{-1}$ are the scattering length and effective range [5], respectively, which are universal in application to the description of different processes involving a kaon-pion pair. The slowly varying part [the first term in the Eq. (8)] is not well modeled at high masses and it is set to zero for $m(K\pi)$ values above 1.7 GeV [5]. For the $K_0^*(1950)^0$, we use the relativistic BW line shape to parametrize the time-like form factors $F_S(\omega^2)$, which is adopted in the experimental data analyses [5].

The P-wave kaon-pion DAs related to both longitudinal and transverse polarizations have been studied in Ref. [52]. In the quasi-two-body $B \to \eta_c K^*(892)^0 \to \eta_c K\pi$ decay, the vector meson $K^*(892)$ should be completely polarized in the longitudinal direction due to the angular momentum conservation requirement. The explicit expressions of the P-wave kaon-pion DAs associated with longitudinal polarization are listed as follows:

$$\Phi_P = \frac{1}{\sqrt{2N_c}}\left[\not{p}\phi_P^0(z,\zeta,\omega^2) + \omega\phi_P^s(z,\zeta,\omega^2) + \frac{\not{p}_1\not{p}_2 - \not{p}_2\not{p}_1}{\omega(2\zeta-1)}\phi_P^t(z,\zeta,\omega^2)\right]. \tag{10}$$

The DAs of various twists in Eq. (10) can be expanded in terms of the Gegenbauer polynomials:

$$\phi_P^0(z,\zeta,\omega^2) = \frac{3F_P^{\parallel}(\omega^2)}{\sqrt{2N_c}} z(1-z)\left[1 + a_{1K^*}^{\parallel} 3t + a_{2K^*}^{\parallel}\frac{3}{2}(5t^2-1)\right](2\zeta - 1 - \alpha), \tag{11}$$

$$\phi_P^s(z,\zeta,\omega^2) = \frac{3F_P^{\perp}(\omega^2)}{2\sqrt{2N_c}}\{t[1 + a_{1s}^{\perp}t] - a_{1s}^{\perp}2z(1-z)\}P_1(2\zeta-1), \tag{12}$$

$$\phi_P^t(z,\zeta,\omega^2) = \frac{3F_P^{\perp}(\omega^2)}{2\sqrt{2N_c}} t[t + a_{1t}^{\perp}(3t^2-1)]P_1(2\zeta-1), \tag{13}$$

where the Legendre polynomial $P_1(2\zeta-1) = 2\zeta-1$ and the factor $\alpha = (m_K^2 - m_\pi^2)/\omega^2$ is treated as the $SU(3)$ asymmetry factor.

For the Gegenbauer moments $a_{1K^*}^{\parallel}$, $a_{2K^*}^{\parallel}$, $a_{1s}^{\perp}$, and $a_{1t}^{\perp}$ we adopt the same values as those determined in Ref. [52]:

$$a_{1K^*}^{\parallel} = 0.2, \qquad a_{2K^*}^{\parallel} = 0.5, \qquad a_{1s}^{\perp} = -0.2, \qquad a_{1t}^{\perp} = 0.2. \tag{14}$$

The relativistic BW line shape is adopted for the P-wave resonances $K^*(892)$, $K^*(1410)$, and $K^*(1680)$ to parametrize the time-like form factors $F_P^{\parallel}(\omega^2)$. The explicit expression is [5]

$$F_P^{\parallel}(\omega^2) = \frac{c_1 m_{K^*(892)}^2}{m_{K^*(892)}^2 - \omega^2 - i m_{K^*(892)}\Gamma_1(\omega^2)} + \frac{c_2 m_{K^*(1410)}^2}{m_{K^*(1410)}^2 - \omega^2 - i m_{K^*(1410)}\Gamma_2(\omega^2)} + \frac{c_3 m_{K^*(1680)}^2}{m_{K^*(1680)}^2 - \omega^2 - i m_{K^*(1680)}\Gamma_3(\omega^2)}, \tag{15}$$

where the three terms describe the contributions from $K^*(892)$, $K^*(1410)$, and $K^*(1680)$, respectively. The weight coefficients $c_1 = 0.72$, $c_2 = 0.135$, and $c_3 = 0.145$ are the same as those determined previously [52].

The mass-dependent width $\Gamma_i(\omega)$ is defined as

$$\Gamma_i(\omega^2) = \Gamma_i\left(\frac{m_i}{\omega}\right)\left(\frac{|\vec{p}_1|}{|\vec{p}_0|}\right)^{(2L_R+1)}. \tag{16}$$

The m_i and Γ_i are the pole mass and width of the corresponding resonances, where $i = 1, 2, 3$ represents the resonances $K^*(892)$, $K^*(1410)$, and $K^*(1680)$, respectively. L_R is the orbital angular momentum in the $K^+\pi^-$ system and $L_R = 0, 1, 2, \ldots$ corresponds to the $S, P, D, \ldots$ partial-wave resonances. Following Ref. [41], we also assume that

$$F_P^{\perp}(\omega^2)/F_P^{\parallel}(\omega^2) \approx (f_{K^*}^T/f_{K^*}), \tag{17}$$

with $f_{K^*} = 0.217 \pm 0.005$ GeV and $f_{K^*}^T = 0.185 \pm 0.010$ GeV [75]. Due to the limited studies on the decay constants of $K^*(1410)$ and $K^*(1680)$, we use the two decay constants of $K^*(892)$ to determine the ratio $F_P^{\perp}(\omega^2)/F_P^{\parallel}(\omega^2)$.

The D-wave kaon-pion DAs are introduced analogously to the two-pion ones [45],

$$\Phi_D = \frac{1}{\sqrt{2N_c}}\left[\not p\phi_D^0(z,\zeta,\omega^2) + \omega\phi_D^s(z,\zeta,\omega^2) + \frac{\not p_1\not p_2 - \not p_2\not p_1}{\omega(2\zeta-1)}\phi_D^t(z,\zeta,\omega^2)\right]. \tag{18}$$

The D-wave $K\pi$ system has similar asymptotic DAs as those for a tensor meson [76–78], but with the time-like form factor replacing the tensor decay constants:

$$\phi_D^0(z,\zeta,\omega^2) = \frac{6F_D^{\parallel}(\omega^2)}{2\sqrt{2N_c}} z(1-z)[3a_1^0(2z-1)]P_2(2\zeta-1), \tag{19}$$

$$\phi_D^s(z,\zeta,\omega^2) = -\frac{9F_D^{\perp}(\omega^2)}{4\sqrt{2N_c}}[a_1^0(1-6z+6z^2)]P_2(2\zeta-1), \tag{20}$$

$$\phi_D^t(z,\zeta,\omega^2) = \frac{9F_D^{\perp}(\omega^2)}{4\sqrt{2N_c}}[a_1^0(1-6z+6z^2)(2z-1)]P_2(2\zeta-1), \tag{21}$$

where the Legendre polynomial $P_2(2\zeta-1) = 1–6\zeta(1-\zeta)$ and the Gegenbauer moment $a_1^0 = 0.4 \pm 0.1$ [45]. The time-like form factor $F_D^{\parallel}(\omega^2)$ for the D-wave $K\pi$ resonance is also described by the relativistic BW function as given in Eq. (15). Besides, the approximate relation $F_D^{\perp}(\omega^2)/F_D^{\parallel}(\omega^2) \approx f_{K_2^*(1430)}^T/f_{K_2^*(1430)}$ is used in the following calculation with $f_{K_2^*(1430)} = 0.118 \pm 0.005$ GeV and $f_{K_2^*(1430)}^T = 0.077 \pm 0.014$ GeV [76].

III. NUMERICAL RESULTS AND DISCUSSIONS

In our numerical calculations, besides the quantities specified before, the following input parameters (where the masses and decay constants are in units of GeV) are used [79]:

$$\begin{aligned}
&m_{B^0} = 5.28, \quad m_{B_s^0} = 5.367, \quad m_b = 4.8, \quad m_c = 1.275,\\
&m_{\pi^\pm} = 0.140, \quad m_{K^\pm} = 0.494, \quad m_{\eta_c(1S)} = 2.9834, \quad m_{\eta_c(2S)} = 3.6392,\\
&f_{B^0} = 0.19, \quad f_{B_s} = 0.23, \quad f_{\eta_c} = 0.42, \quad f_{\eta_c(2S)} = 0.243,\\
&\tau_{B^0} = 1.519\ \text{ps}, \quad \tau_{B_s^0} = 1.512\ \text{ps}.
\end{aligned} \tag{22}$$

The pole masses and widths of various partial-wave resonances are summarized in Table I, while we adopt the values of the Wolfenstein parameters given in Ref. [79]: $A = 0.836 \pm 0.015$, $\lambda = 0.22453 \pm 0.00044$, $\bar\rho = 0.122^{+0.018}_{-0.017}$, $\bar\eta = 0.355^{+0.012}_{-0.011}$.

For the decays $B \to \eta_c(K^{*0} \to)K\pi$, the differential decay rate is expressed as

$$\frac{d\mathcal{B}}{d\omega} = \frac{\tau_B\omega|\vec{p}_1||\vec{p}_3|}{32\pi^3 m_B^3}|\mathcal{A}|^2, \tag{23}$$

where the kaon and charmonium three-momenta in the $K\pi$ center-of-mass frame are given by

$$|\vec{p}_1| = \frac{1}{2\omega}\sqrt{\lambda(\omega^2, m_K^2, m_\pi^2)}, \qquad |\vec{p}_3| = \frac{1}{2\omega}\sqrt{\lambda(m_B^2, m_{\eta_c}^2, \omega^2)}, \tag{24}$$

with the kaon (pion) mass m_K (m_π) and the Källén function $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$.

TABLE I. The pole masses and widths of the various partial-wave resonances [5].

Resonance	Mass [MeV]	Width [MeV]	J^P	Model
$K^*(892)^0$	895.55 ± 0.20	47.3 ± 0.5	1^-	RBW
$K^*(1410)^0$	1414 ± 15	232 ± 21	1^-	RBW
$K_0^*(1430)^0$	1425 ± 50	270 ± 80	0^+	LASS
$K_2^*(1430)^0$	1432.4 ± 1.3	109 ± 5	2^+	RBW
$K^*(1680)^0$	1717 ± 27	322 ± 110	1^-	RBW
$K_0^*(1950)^0$	1945 ± 22	201 ± 90	0^+	RBW

By using Eqs. (23)–(24), the decay amplitudes from the Appendix, and all of the input quantities, the resultant branching ratios $\mathcal{B}$ and the available experimental results for the considered $B^0_{(s)} \to \eta_c K\pi$ decays involving the S-wave resonances are summarized in Table II, while those for P- and D-wave resonances are shown in Tables III and IV. Since the charged B meson decays differ from the neutral ones only in the lifetimes and the isospin factor in our formalism, we can derive the branching ratios for the B^+ decay modes by multiplying those for the B^0 decay modes by the ratio τ_{B^+}/τ_{B^0}.

In our calculations for the various partial-wave resonances, the first uncertainty is induced by the Gegenbauer moments in the S, P, and D-wave kaon-pion DAs, as aforementioned. The second error comes from the variations of the shape parameter $\omega_{B_{(s)}}$ of the $B_{(s)}$ meson DA. We adopt the value $\omega_B = 0.40 \pm 0.04$ or $\omega_{B_s} = 0.50 \pm 0.05$ GeV and vary its value within a 10% range, which is supported by intensive PQCD studies [33,34]. The last

TABLE II. PQCD results for the branching ratios of the S-wave resonances in the $B^0_{(s)} \to \eta_c(1S,2S)K^\pm\pi^\mp$ decays in Scenario I and Scenario II together with experimental data [5]. The theoretical errors are attributed to the variation of the Gegenbauer moments B_1 and B_3, the shape parameters $\omega_{B_{(s)}}$ in the wave function of the $B_{(s)}$ meson, and the hard scale t, respectively.

Modes	Quasi-two-body $\mathcal{B}$ (in 10^{-5}) Scenario I	Scenario II	Exp. [5]
$B^0 \to \eta_c K^+\pi^-(\mathrm{NR})$	$0.85^{+0.43+0.32+0.10}_{-0.41-0.26-0.15}$	$1.85^{+0.94+0.50+0.56}_{-0.59-0.24-0.31}$	$5.90^{+1.23}_{-1.29}$
$B^0 \to \eta_c(K_0^*(1430)^0 \to)K^+\pi^-$	$1.69^{+0.71+0.22+0.42}_{-0.69-0.17-0.24}$	$4.75^{+2.10+1.17+1.05}_{-1.95-1.10-0.68}$	$14.50^{+3.36}_{-3.14}$
$B^0_s \to \eta_c K^-\pi^+(\mathrm{NR})$	$0.03^{+0.01+0.01+0.00}_{-0.01-0.01-0.00}$	$0.09^{+0.04+0.03+0.02}_{-0.04-0.02-0.02}$	...
$B^0_s \to \eta_c(\bar{K}_0^*(1430)^0 \to)K^-\pi^+$	$0.08^{+0.03+0.02+0.01}_{-0.02-0.01-0.01}$	$0.21^{+0.11+0.07+0.05}_{-0.08-0.05-0.03}$	...
$B^0 \to \eta_c(2S)K^+\pi^-(\mathrm{NR})$	$0.17^{+0.07+0.03+0.04}_{-0.06-0.02-0.05}$	$0.41^{+0.22+0.12+0.11}_{-0.11-0.09-0.06}$	...
$B^0 \to \eta_c(2S)(K_0^*(1430)^0 \to)K^+\pi^-$	$0.56^{+0.28+0.17+0.12}_{-0.25-0.24-0.15}$	$0.79^{+0.41+0.24+0.17}_{-0.19-0.13-0.12}$	...
$B^0_s \to \eta_c(2S)K^-\pi^+(\mathrm{NR})$	$0.007^{+0.004+0.003+0.001}_{-0.002-0.001-0.002}$	$0.02^{+0.01+0.01+0.00}_{-0.01-0.01-0.00}$	...
$B^0_s \to \eta_c(2S)(\bar{K}_0^*(1430)^0 \to)K^-\pi^+$	$0.02^{+0.01+0.01+0.00}_{-0.01-0.00-0.00}$	$0.04^{+0.02+0.01+0.01}_{-0.02-0.01-0.01}$	...

TABLE III. PQCD results for the branching ratios of the P-wave resonances in the $B^0_{(s)} \to \eta_c(1S,2S)K^\pm\pi^\mp$ decays together with experimental data [5,79]. The theoretical errors are attributed to the variation of the Gegenbauer moments ($a^{||}_{1K^*}$, $a^{||}_{2K^*}$ and $a^\perp_{1s}$, $a^\perp_{1t}$), the shape parameters $\omega_{B_{(s)}}$ in the wave function of the $B_{(s)}$ meson, and the hard scale t, respectively.

Modes	Quasi-two-body $\mathcal{B}$ (in 10^{-5})	Exp. (in 10^{-5})
$B^0 \to \eta_c(K^*(892)^0 \to)K^+\pi^-$	$46.49^{+18.07+12.63+12.13}_{-14.80-9.67-8.63}$	35 ± 5 [79][a]
$B^0 \to \eta_c(K^*(1410)^0 \to)K^+\pi^-$	$1.35^{+0.50+0.24+0.43}_{-0.40-0.23-0.26}$	1.20 ± 0.90 [5]
$B^0 \to \eta_c(K^*(1680)^0 \to)K^+\pi^-$	$1.44^{+0.63+0.34+0.55}_{-0.49-0.26-0.30}$	$1.26^{+1.44}_{-1.51}$ [5]
$B^0_s \to \eta_c(\bar{K}^*(892)^0 \to)K^-\pi^+$	$2.13^{+0.89+0.66+0.54}_{-0.73-0.46-0.39}$	...
$B^0_s \to \eta_c(\bar{K}^*(1410)^0 \to)K^-\pi^+$	$0.07^{+0.02+0.02+0.02}_{-0.02-0.02-0.01}$	...
$B^0_s \to \eta_c(\bar{K}^*(1680)^0 \to)K^-\pi^+$	$0.08^{+0.02+0.02+0.03}_{-0.02-0.02-0.02}$	...
$B^0 \to \eta_c(2S)(K^*(892)^0 \to)K^+\pi^-$	$13.33^{+4.72+3.70+3.74}_{-3.51-2.96-2.08}$	< 26 [79][a]
$B^0 \to \eta_c(2S)(K^*(1410)^0 \to)K^+\pi^-$	$0.24^{+0.11+0.06+0.10}_{-0.06-0.04-0.05}$	...
$B^0 \to \eta_c(2S)(K^*(1680)^0 \to)K^+\pi^-$	$0.11^{+0.04+0.02+0.04}_{-0.03-0.02-0.03}$	...
$B^0_s \to \eta_c(2S)(\bar{K}^*(892)^0 \to)K^-\pi^+$	$0.60^{+0.20+0.21+0.16}_{-0.19-0.16-0.11}$	...
$B^0_s \to \eta_c(2S)(\bar{K}^*(1410)^0 \to)K^-\pi^+$	$0.01^{+0.01+0.01+0.01}_{-0.00-0.00-0.00}$	...
$B^0_s \to \eta_c(2S)(\bar{K}^*(1680)^0 \to)K^-\pi^+$	$0.008^{+0.002+0.002+0.002}_{-0.002-0.002-0.002}$	...

[a]The experimental results are obtained by multiplying the relevant measured two-body branching ratios according to Eq. (27).

TABLE IV. PQCD results for the branching ratios of the D-wave resonances in the $B^0_{(s)} \to \eta_c(1S,2S)K^{\pm}\pi^{\mp}$ decays together with experimental data [5]. The theoretical errors are attributed to the variation of the Gegenbauer moment a_1^0, the shape parameters $\omega_{B_{(s)}}$ in the wave function of the $B_{(s)}$ meson, and the hard scale t, respectively.

Modes	Quasi-two-body $\mathcal{B}$ (in 10^{-5})	Exp. (in 10^{-5})
$B^0 \to \eta_c(K_2^*(1430)^0 \to)K^+\pi^-$	$3.98^{+1.24+0.59+0.11}_{-1.74-0.55-0.04}$	$2.35^{+1.08}_{-1.29}$ [5]
$B^0_s \to \eta_c(\bar{K}_2^*(1430)^0 \to)K^-\pi^+$	$0.23^{+0.13+0.04+0.01}_{-0.10-0.05-0.01}$	…
$B^0 \to \eta_c(2S)(K_2^*(1430)^0 \to)K^+\pi^-$	$0.55^{+0.31+0.12+0.02}_{-0.24-0.09-0.01}$	…
$B^0_s \to \eta_c(2S)(\bar{K}_2^*(1430)^0 \to)K^-\pi^+$	$0.04^{+0.02+0.01+0.01}_{-0.02-0.01-0.01}$	…

one is caused by the variation of the hard scale t from $0.75t$ to $1.25t$ (without changing $1/b_i$), which characterizes the next-to-leading-order (NLO) effects in the PQCD approach. In Tables II, III, and IV, it is shown that the main uncertainties in our approach come from the Gegenbauer moments, which can reach a total magnitude of about 60%. The scale-dependent uncertainty is less than 25% due to the inclusion of the NLO vertex corrections. The other possible errors from the uncertainties of m_c and the Cabibbo-Kobayashi-Maskawa matrix elements are actually very small and can be safely neglected.

Combined with the Clebsch-Gordan Coefficients, we can write the relation

$$\left|K\pi, I=\frac{1}{2}\right\rangle = \sqrt{\frac{1}{3}}|K^0\pi^0\rangle - \sqrt{\frac{2}{3}}|K^+\pi^-\rangle. \quad (25)$$

Isospin conservation is assumed for the strong decays of an $I=1/2$ resonance K^{*0} to $K\pi$ when we compute the branching fraction of the quasi-two-body process $B\to\eta_c K^{*0}\to\eta_c K^+\pi^-$, namely,

$$\frac{\Gamma(K^{*0}\to K^+\pi^-)}{\Gamma(K^{*0}\to K\pi)}=2/3, \quad \frac{\Gamma(K^{*0}\to K^0\pi^0)}{\Gamma(K^{*0}\to K\pi)}=1/3. \quad (26)$$

Therefore, the corresponding two-body branching fraction $\mathcal{B}(B\to\eta_c K^{*0})$ can be extracted directly from the quasi-two-body decay modes in Table III under the narrow-width approximation relation:

$$\mathcal{B}(B\to\eta_c K^{*0}\to\eta_c K^+\pi^-)=\mathcal{B}(B\to\eta_c K^{*0})\cdot\mathcal{B}(K^{*0}\to K\pi)\cdot\frac{2}{3}. \quad (27)$$

There already exist some results for $B^0_{(s)}\to\eta_c K^*(892)^0$ in the two-body framework [60–62]. One can see that the branching ratios of the quasi-two-body decay modes match well with the two-body analyses presented in Ref. [61] using the PQCD approach. These results suggest that the PQCD factorization approach is suitable for describing the quasi-two-body B meson decays through analyzing various resonances by reconstructing $K\pi$ final states and reproducing the invariant mass spectra of Dalitz plots.

For the S-wave resonance $K_0^*(1430)^0$, it should be mentioned that two scenarios have been proposed to describe the scalar mesons above 1 GeV using the QCD sum rules method [66,67]. In Scenario I, the $K_0^*(1430)^0$ is treated as the first excited state, while $a_0(980)$ and $f_0(980)$ are regarded as the lowest-lying states. In Scenario II, we assume that $K_0^*(1430)^0$ is the lowest-lying resonance and the corresponding first excited states lie between (2.0–2.3) GeV. Scenario II corresponds to the case that light scalar mesons are four-quark bound states. The Gegenbauer moments $B_1=0.58\pm0.07$ and $B_3=-1.20\pm0.08$ are adopted in Scenario I, while $B_1=-0.57\pm0.13$ and $B_3=-0.42\pm0.22$ are adopted in Scenario II [67]. In this work, we consider two scenarios for the S-wave components and list the relevant results in Table II. One can see that the predicted $\mathcal{B}$ in Scenario I are always smaller than those in Scenario II. This phenomenon is mainly caused by the different signs of the Gegenbauer moment B_1 in different scenarios, which indicates that there is a large cancellation in Scenario I.

From Table II, one can see that our predictions for the branching ratios for the $K_0^*(1430)^0$ resonance and NR components in Scenario II are $\mathcal{B}_{K_0^*(1430)^0}=(4.75^{+2.62}_{-2.34})\times10^{-5}$ and $\mathcal{B}_{\rm NR}=(1.85^{+1.20}_{-0.71})\times10^{-5}$, respectively, which are a bit smaller than the experimental measurements within errors. Anyway, as is well known, in contrast to the vector and tensor mesons the identification of scalar mesons is a long-standing puzzle. It is difficult to deal with scalar resonances since some of them have wide decay widths, which cause a strong overlap between resonances and background. Furthermore, the underlying structure of scalar mesons is not theoretically well established (for a review, see Ref. [79]). We hope that the situation can be improved using nonperturbative QCD tools including lattice QCD simulations. Nonetheless, we define the PQCD prediction of the corresponding ratio for a more direct comparison with the available experimental data,

$$R_1^{\rm PQCD}=\frac{\mathcal{B}(B^0\to\eta_c(K_0^*(1430)^0\to)K^+\pi^-)}{\mathcal{B}(B^0\to\eta_c K^+\pi^-({\rm NR}))}=2.56^{+1.98}_{-1.71}, \quad (28)$$

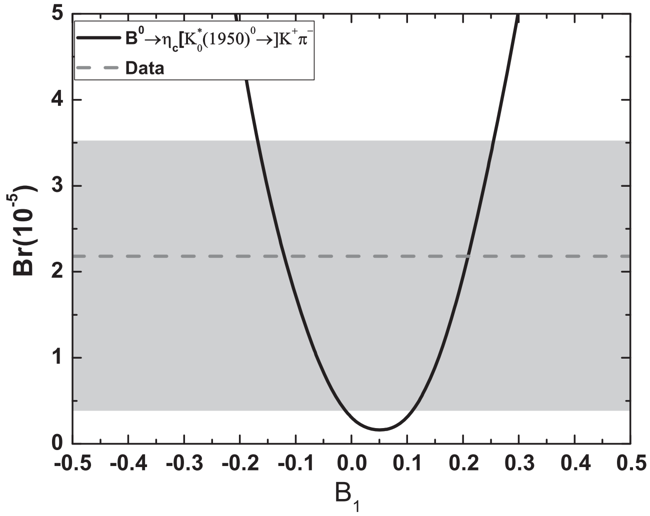

FIG. 2. The branching fraction of the $B^0 \to \eta_c[K^*(1950)^0 \to]K^+\pi^-$ decay as a function of the Gegenbauer moment B_1. Shaded bands show the experimental uncertainties.

where the branching fraction of the $B^0 \to \eta_c(K_0^*(1430)^0 \to) K^+\pi^-$ decay is measured relative to that of the corresponding NR contributions by the LHCb Collaboration [5]:

$$R_1^{\rm LHCb} = \frac{\mathcal{B}(B^0 \to \eta_c(K_0^*(1430)^0 \to)K^+\pi^-)}{\mathcal{B}(B^0 \to \eta_c K^+\pi^-(\rm NR))} = 2.45^{+0.81}_{-0.68}. \tag{29}$$

Our prediction is quite consistent with the LHCb measurement. Combined analyses from the LHCb and Belle-II measurements for these decays in the near future could help us to further study the scalar resonances.

For the phenomenological study of the scalar meson $K_0^*(1950)^0$, we still lack the distribution amplitudes of the $K_0^*(1950)^0$ state at present. Fortunately, we are allowed to single out the $K_0^*(1950)^0$ component according to the kaon-pion DAs. In Fig. 2, we plot the variation of the branching fraction with the first odd Gegenbauer moment B_1 for the $B^0 \to \eta_c(K_0^*(1950)^0 \to)K^+\pi^-$ decay mode, as well as the experimental data $\mathcal{B}_{\rm exp} = (2.18^{+1.32}_{-1.79}) \times 10^{-5}$ [5]. One can see that the theoretical prediction of the branching ratio shows a strong dependence on the variation of B_1. Combined with the available data, we can roughly determine that the possible range of the first odd Gegenbauer moment is from −0.15 to 0.05 or 0.10 to 0.25, which should be examined both theoretically and experimentally in the future.

In Fig. 3, we show the ω dependence of the differential decay rates $d\mathcal{B}(B^0 \to \eta_c K^+\pi^-)/d\omega$ (the solid curve) and $d\mathcal{B}(B^0 \to \eta_c(2S)K^+\pi^-)/d\omega$ (short-dotted curve) after the inclusion of the P-wave resonance $K^*(892)^0$, which exhibit a peak at the $K^*(892)^0$ meson mass. For the considered decay modes $B^0_{(s)} \to \eta_c(1S,2S)K\pi$, the dynamical limit on the value of the invariant mass ω is $(m_K + m_\pi) \le \omega \le (m_{B_{(s)}} - m_{\eta_c(1S,2S)})$. Although $m_{K^*(1680)^0} > (m_B - m_{\eta_c(2S)})$, the resonance $K^*(1680)^0$ can still contribute to the $B^0_{(s)} \to \eta_c(2S)K\pi$ decay due to its large width ($\Gamma_{K^*(1680)^0} = 322$ MeV). It is shown that the main portion of the differential branching ratio lies in the region around the resonance in Fig. 3, as expected. For $B^0 \to \eta_c(1S)(K^*(892)^0 \to)K^+\pi^-$ decay, the central values of the branching ratio $\mathcal{B}$ are 23.36×10^{-5} and 34.46×10^{-5} when the integration over ω is limited to the range $\omega = [m_{K^*} - 0.5\Gamma_{K^*}, m_{K^*} + 0.5\Gamma_{K^*}]$ or $\omega = [m_{K^*} - \Gamma_{K^*}, m_{K^*} + \Gamma_{K^*}]$, respectively, which amounts to 50% and 74% of the total branching ratio $\mathcal{B} = 46.49 \times 10^{-5}$ as listed in Table III.

From Table III, one can see that our PQCD prediction for the branching ratio of the $B^0 \to \eta_c(1S)K^*(892)^0 \to \eta_c K^+\pi^-$ decay is $\mathcal{B} = (46.49^{+25.16}_{-19.67}) \times 10^{-5}$, the central value of which is a little larger than that in PDG2018: $(35 \pm 5) \times 10^{-5}$ [79]. The measurements from the Belle [2], *BABAR* [3,4], and LHCb [5] collaborations, as well as the average value from HFLAV [80], are the following:

$$\mathcal{B}(B^0 \to \eta_c(K^*(892)^0 \to)K^+\pi^-) = \begin{cases} (108^{+42}_{-46}) \times 10^{-5} & \text{Belle[2]}, \\ (53^{+28}_{-19}) \times 10^{-5} & BABAR\text{[3]}, \\ (38 \pm 7) \times 10^{-5} & BABAR\text{[4]}, \\ (29.5^{+3.8}_{-4.7}) \times 10^{-5} & \text{LHCb[5]}, \\ (41.3 \pm 6.6) \times 10^{-5} & \text{HFLAV[80]}. \end{cases} \tag{30}$$

The data point $(41.3 \pm 6.6) \times 10^{-5}$ from HFLAV [80] is obtained by multiplying the relevant measured two-body branching ratio according to Eq. (27). One can see that the central values of the measured branching ratio for the $K^*(892)^0$ resonance from different experiments vary in the wide range $(29\text{–}108) \times 10^{-5}$, while the HFLAV world average of the measured values from the Belle and *BABAR* collaborations [2–4] leads to $(41.3 \pm 6.6) \times 10^{-5}$, which is in good agreement with our prediction.

For the $B^0 \to \eta_c(2S)(K^*(892)^0 \to)K^+\pi^-$ decay, the *BABAR* Collaboration has measured an upper limit on the branching ratio $\mathcal{B}(B^0 \to \eta_c(2S)(K^*(892)^0 \to)K^+\pi^-) < 26 \times 10^{-5}$ at the 90% confidence level [4]. The PQCD prediction of $\mathcal{B}(B^0 \to \eta_c(2S)(K^*(892)^0 \to)K^+\pi^-) = (13.33^{+7.07}_{-5.04}) \times 10^{-5}$ agrees with the limit. Meanwhile, one can see in Fig. 3 that the branching fractions of $B^0 \to \eta_c(2S)(K^{*0} \to)K^+\pi^-$ decays are always smaller than those for $B^0 \to \eta_c(1S)(K^{*0} \to)K^+\pi^-$ decays, which is mainly induced by the difference between the DAs of $\eta_c(2S)$ and $\eta_c(1S)$ mesons: the tighter phase space and the smaller decay constant of the $\eta_c(2S)$ meson result in the suppression.

From the numerical results given in Table III, we obtain the relative ratio R_2 between the branching ratio of B meson decays involving $\eta_c(2S)$ and $\eta_c(1S)$ and the resonance $K^*(892)^0$,

$$R_2(K^*(892)^0) = \frac{\mathcal{B}(B^0 \to \eta_c(2S)(K^*(892)^0 \to)K^+\pi^-)}{\mathcal{B}(B^0 \to \eta_c(1S)(K^*(892)^0 \to)K^+\pi^-)} = 0.29^{+0.21}_{-0.15}, \quad (31)$$

which can be tested by the forthcoming LHCb and Belle-II experiments.

The branching ratios of the considered D-wave resonance are presented in Table IV. We emphasized that our predictions of these decay channels are only rough estimates. Although there is not enough data at present, the calculated value $\mathcal{B}(B^0 \to \eta_c(1S)(K_2^*(1430)^0 \to)K^+\pi^-) = (3.98^{+1.24}_{-1.74}) \times 10^{-5}$ is compatible with the measurement $(2.35^{+1.08}_{-1.29}) \times 10^{-5}$ [5] within the large errors. Future experimental measurements with high precision can provide a better understanding of the properties of the tensor resonances.

For all of the $B_s^0 \to \eta_c(1S, 2S)K\pi$ decays, such decay modes can be theoretically related to the corresponding B^0 decays since they have identical topologies and similar kinematic properties in the limit of $SU(3)$ flavor symmetry. At the quark level, the B^0 and B_s^0 decays correspond to the $b \to c\bar{c}s$ and $b \to c\bar{c}d$ transitions, respectively. The relative ratios of the branching fractions for B_s^0 and B^0 decay modes are dominated by the Cabibbo suppression factor of $|V_{cd}|^2/|V_{cs}|^2 \sim \lambda^2$ under the naive factorization approximation. It is reasonable to see that the branching fractions of the B_s^0 decays are smaller than those of the corresponding B^0 decays. Though the B_s^0 channels have relatively small branching ratios, some of them can be potentially measurable at future experiments.

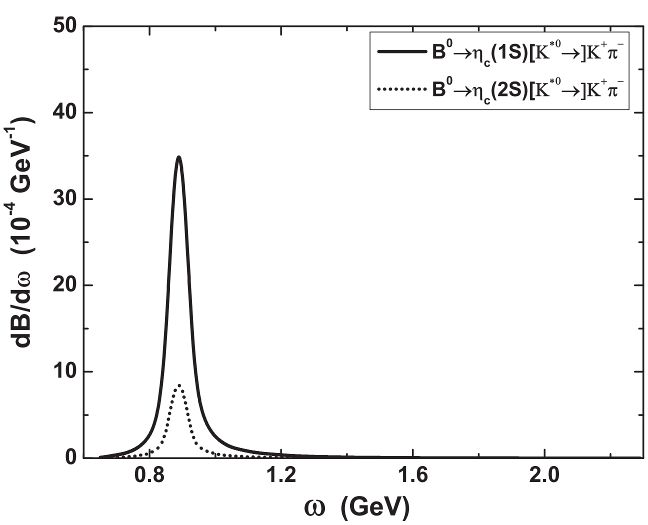

FIG. 3. Differential branching ratio for the $B^0 \to \eta_c(1S, 2S)$ $[K^*(892)^0 \to]K^+\pi^-$ decays.

IV. CONCLUSION

In this work, by introducing the kaon-pion DAs, we studied the quasi-two-body decays $B^0_{(s)} \to \eta_c(1S, 2S)(K^{*0} \to)K\pi$ in the PQCD approach, in which the kaon-pion invariant mass spectra are dominated by the $K_0^*(1430)^0$, $K_0^*(1950)^0$, $K^*(892)^0$, $K^*(1410)^0$, $K^*(1680)^0$, and $K_2^*(1430)^0$ resonances. These six resonances fall into three partial waves according to their spins, namely, S, P, and D-wave states. The contributions from each partial wave can be parametrized into the corresponding time-like form factors involved in the kaon-pion DAs. The $K\pi$ S-wave component at low mass is described by the LASS line shape, while the time-like form factors of other resonances are modeled by the relativistic BW function.

It has been shown that our predictions of the branching ratios for most of the considered $B^0 \to \eta_c(1S)(K^{*0} \to)K^+\pi^-$ decays are in good agreement with the existing data within the errors. For the $B^0 \to \eta_c(1S)(K_0^*(1430)^0 \to)K^+\pi^-$ decay, although there exists a clear difference between the central value of the PQCD calculation for $\mathcal{B}_{K_0^*(1430)^0}$, $\mathcal{B}_{\rm NR}$, and the measured ones, they are still consistent within three standard deviations due to the large experimental errors, which should be examined by forthcoming experiments. The new ratio $R_2(K^*(892)^0)$ among the branching ratios of the considered decay modes has been defined and will be confronted with future measurements.

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APPENDIX: DECAY AMPLITUDES

The total decay amplitudes for the considered decay modes $B^0_{(s)} \to \eta_c K\pi$ in this work are given as follows:

$$\begin{aligned}\mathcal{A}(B^0_{(s)} \to \eta_c K\pi) = \frac{G_F}{\sqrt{2}}\Big\{ & V^*_{cb}V_{cs(cd)}\Big[\Big(C_1+\frac{1}{3}C_2\Big)\mathcal{F}^{LL}+C_2\mathcal{M}^{LL}\Big] \\ & - V^*_{tb}V_{ts(td)}\Big[\Big(C_3+\frac{1}{3}C_4+C_9+\frac{1}{3}C_{10}\Big)\mathcal{F}^{LL}+\Big(C_5+\frac{1}{3}C_6+C_7+\frac{1}{3}C_8\Big)\mathcal{F}^{LR} \\ & +(C_4+C_{10})\mathcal{M}^{LL}+(C_6+C_8)\mathcal{M}^{SP}\Big]\Big\}, \end{aligned} \tag{A1}$$

where $G_F = 1.16639 \times 10^{-5}$ GeV^{-2} is the Fermi coupling constant and the V_{ij}'s are the Cabibbo-Kobayashi-Maskawa matrix elements. The superscripts LL, LR, and SP refer to the contributions from $(V-A)\otimes(V-A)$, $(V-A)\otimes(V+A)$, and $(S-P)\otimes(S+P)$ operators, respectively. The explicit amplitudes $F(M)$ from the factorizable (nonfactorizable) diagrams in Fig. 1 can be obtained straightforwardly by replacing the twist-2 or twist-3 DAs of the $\pi\pi$ and KK systems with the corresponding twists of the $K\pi$ ones in Eqs. (5)–(7), (11)–(13), and (19)–(21), since the kaon-pion distribution amplitudes considered in this work [Eqs. (4), (10), and (18)] have the same Lorentz structure as the two-pion (kaon) ones in Refs. [42,51].

The Wilson coefficients C_i are evaluated at the corresponding scale t. At the m_W scale, the Wilson coefficients at the NLO level can be written in the following form (as in Ref. [81]):

$$\begin{aligned} & C_1(m_W)=\frac{11}{2}\frac{\alpha_s(m_W)}{4\pi}, \qquad C_2(m_W)=1-\frac{11}{6}\frac{\alpha_s(m_W)}{4\pi}-\frac{35}{8}\frac{\alpha_{em}}{4\pi}, \\ & C_3(m_W)=-\frac{\alpha_s(m_W)}{24\pi}\Big[E_0(x)-\frac{2}{3}\Big]+\frac{\alpha_{em}}{6\pi}\frac{1}{\sin^2\theta_W}[2B_0(x)+C_0(x)], \qquad C_4(m_W)=\frac{\alpha_s(m_W)}{8\pi}\Big[E_0(x)-\frac{2}{3}\Big], \\ & C_5(m_W)=-\frac{\alpha_s(m_W)}{24\pi}\Big[E_0(x)-\frac{2}{3}\Big], \qquad C_6(m_W)=\frac{\alpha_s(m_W)}{8\pi}\Big[E_0(x)-\frac{2}{3}\Big], \\ & C_7(m_W)=\frac{\alpha_{em}}{6\pi}\Big[4C_0(x)+D_0(x)-\frac{4}{9}\Big], \\ & C_9(m_W)=\frac{\alpha_{em}}{6\pi}\Big[4C_0(x)+D_0(x)-\frac{4}{9}+\frac{1}{\sin^2\theta_W}(10B_0(x)-4C_0(x))\Big], \qquad C_i(m_W)=0, \quad i=8,10, \end{aligned} \tag{A2}$$

where the relevant Inami-Lim functions $B_0(x), C_0(x), D_0(x)$, and $E_0(x)$ [82] are

$$\begin{aligned} & B_0(x)=\frac{1}{4}\Big(\frac{x}{1-x}-\frac{x\ln x}{(x-1)^2}\Big), \qquad C_0(x)=\frac{6x-x^2}{8(1-x)}-\frac{(3x^2+2x)\ln x}{8(x-1)^2}, \\ & D_0(x)=\frac{-25x^2+19x^3}{36(1-x)^3}-\frac{(8-32x+54x^2-30x^3+3x^4)\ln x}{18(x-1)^4}, \\ & E_0(x)=\frac{18x-11x^2-x^3}{12(1-x)^3}-\frac{(4-16x+9x^2)\ln x}{6(x-1)^4}, \end{aligned} \tag{A3}$$

with $x = m_t^2/m_W^2$. In the region $m_b < t < m_W$, we evaluate the Wilson coefficients at the t scale using the following renormalization group equation:

$$\mathbf{C}(t) = U(t, m_W)\mathbf{C}(m_W), \tag{A4}$$

where $\mathbf{C}(m_W) = (C_1(m_W), ..., C_{10}(m_W))^T$ and $U(t, m_W)$ is the renormalization group running matrix at the NLO level (for details, see Ref. [81]). The Wilson coefficients $C_i(t)$ evaluated at the scale $t = m_b = 4.8$ GeV (as given, for example, in Ref. [83]) are as follows:

$$
\begin{aligned}
&C_1 = -0.17474, \quad C_2 = 1.07737, \quad C_3 = 0.01249, \quad C_4 = -0.03304,\\
&C_5 = 0.00942, \quad C_6 = -0.03929, \quad C_7 = -0.00003, \quad C_8 = 0.00023,\\
&C_9 = -0.00999, \quad C_{10} = 0.00201.
\end{aligned} \tag{A5}
$$

If the scale $t < m_b$, we can evaluate the Wilson coefficients at the t scale using the evolution equation $\mathbf{C}(t) = U(t, m_b)\mathbf{C}(m_b)$, where $\mathbf{C}(m_b) = (C_1(m_b), ..., C_{10}(m_b))^T$ are given in Eq. (A5).

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