Behavior of observables for neutral meson decaying to two vectors in the presence of T, CP, and CPT violation in mixing only

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When a neutral meson (P^0 or $\bar{P}^0$) decays to two vector particles, a large number of observables can be constructed from the differential decay rate based on the polarization of the final state. But, theoretically, all of them are not independent to each other and hence, some relations among observables emerge. These relations have been well studied in the scenario with no T and CPT violations in neutral meson mixing and no direct CP violation as well. In this paper, we have studied the relations among observables in the presence of T, CP, and CPT violating effects in mixing only. We find that except for four of them, all the other old relations get violated and new relations appear if T and CPT violations in mixing are present. The invalidity of these relation will signify the presence of direct violation of T, CP, and CPT (i.e., a violation in the decay itself).

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I. INTRODUCTION

CPT invariance is believed to be a sacred principle of any locally Lorentz invariant quantum field theory. In any axiomatic quantum field theory, this discrete symmetry emerges to be exact up to any order. It has a direct connection with the preservation of Lorentz symmetry [1,2]. Because of its great theoretical importance, it is necessary to test the validity of this principle experimentally. CPT invariance predicts the masses or lifetimes of any particle and its anti-particle to be the same, which has been tested for lots of particles through direct experiments [3]. But one can argue that these quantities are usually dominated by strong or electromagnetic interactions and hence there exits a possibility for tiny CPT violating effects, mediated by weak interactions, to be undetectable in those direct experiments. In this regard, the mixing of the neutral pseudoscalar meson (K^0, D^0, B_d^0, B_s^0) with its own anti-particle is a promising area [4] to search for CPT violating effects as this phenomenon is a second order electroweak process. However, since the most general mixing matrix includes T and CP violating parameters as well, we have to study the effects of CP, T, and CPT violation together.

Searches for CP, T, and CPT violation using leptonic and semi-leptonic channels as well as the modes where neutral pseudoscalar meson decays to two other pseudoscalars or one vector and one pseudoscalar have been performed extensively [5–21]. However, the effects of CPT violation on the modes where neutral pseudoscalar meson decays to two vectors (P^0 or $\bar{P}^0 \to V_1V_2$) are not very well studied. Though Refs. [22–24] discuss these modes involving two vectors, they only consider the standard model (SM) scenario (i.e., only CP violation in mixing) and its extension to a model with CPT conserving generic new physics effects. However, Ref. [25] has taken CPT violation into account for describing the mode $B_s^0 \to J/\psi\phi$ and Ref. [26] has discussed triple products and angular observables for $B \to V_1V_2$ decays in light of CPT violation. In this paper, we have revisited the prospect of searching CPT violation in mixing through $P^0 \to V_1V_2$ decays using a helicity-based analysis for the time-dependent differential decay rate. We would also like to emphasize that we have taken a model-independent approach in the sense that we do not specify any definite model that might lead to CPT violation.

The usual technique to deal with the oscillations of neutral pseudoscalar mesons is to consider a final state f to which both P^0 and $\bar{P}^0$ can decay. If f consists of two vectors, a large number of observables can be constructed from the time-dependent differential decay rate depending on the polarization or orbital angular momentum of the

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final state. But, all of these observables will not be independent to each other and hence there emerge various relations among them. In Refs. [23,24], these relations have been discussed in the context of the SM scenario only for the modes B_d^0 or $\bar{B}_d^0$ decaying to two vectors. In this paper, we study these relations in the presence of T, CP, and CPT violations in mixing only. We have confined our analysis to the case where CPT violation is small compared to the SM amplitude, which is justified based on the data from several experiments [7,8,12,18,21]. Since independent theoretical parameters for this case are more in number than the SM scenario, it is expected to obtain a fewer number of relations among observables. We find that except for four, all the other old relations in the SM get violated and new relations appear if T and CPT violations in mixing are present. These new relations will hold true even if the T, CP, and CPT violations become zero; however, they will not form the complete set of relations in that case as they are fewer in number. These new relations will break down only if T, CP, and CPT violating effects are present in decay too (i.e., direct violation).

The paper is organized as follows. In the next section, we briefly describe the theoretical formalism for CPT violation in $P^0 - \bar{P}^0$ mixing and express the time dependent differential decay rate of P^0 and $\bar{P}^0$ in terms of the mixing parameters. In Sec. III, we construct helicity-dependent observables from the differential decay rates and express them in terms of T, CP, and CPT violating parameters assuming T and CPT violations in mixing to be very small. We also solve for all the unknown theoretical parameters as functions of the observables. In. Sec. IV, we establish the independent relations among these observables in the SM case and the scenario with the presence of T and CPT violations in mixing separately. We also discuss how these relations can help us in distinguishing three different scenarios: (a) the SM case; (b) T, CP, and CPT violation in mixing; and (c) direct violation of T, CP, and CPT. Finally, we summarize and conclude in Sec. V.

II. THEORETICAL FORMALISM

We begin by reviewing the most general formalism for $P^0 - \bar{P}^0$ mixing, in which CPT and T violations are incorporated. This formalism has already been discussed in Ref. [19]; however, for the sake of completeness we present it in this section. In the $(P^0 - \bar{P}^0)$ basis, the generic mixing Hamiltonian can be expressed in terms of two 2×2 Hermitian matrices $\mathbf{M}$ and $\mathbf{\Gamma}$, respectively, the mass and decay matrices, as $\mathbf{M} - (i/2)\mathbf{\Gamma}$. It should be noticed that the mixing matrix $\mathbf{M} - (i/2)\mathbf{\Gamma}$ is non-Hermitian and it is justified as the probability of finding P^0 and $\bar{P}^0$ decreases with time due to the presence of the non-null decay matrix $\mathbf{\Gamma}$. Now, since any 2×2 matrix can be expanded in terms of three Pauli matrices $\boldsymbol{\sigma_j}$ and identity matrix $\mathbf{I}$ with complex coefficients, we can write

$$\mathbf{M} - \frac{i}{2}\mathbf{\Gamma} = E\sin\theta\cos\phi\boldsymbol{\sigma_1} + E\sin\theta\sin\phi\boldsymbol{\sigma_2} + E\cos\theta\boldsymbol{\sigma_3} - iD\mathbf{I} \tag{1}$$

where E, θ, ϕ, and D are complex entities in general. Comparing both sides of this equation, we obtain

$$\begin{aligned} D &= \frac{i}{2}(M_{11} + M_{22}) + \frac{1}{4}(\Gamma_{11} + \Gamma_{22}), \\ E\cos\theta &= \frac{1}{2}(M_{11} - M_{22}) - \frac{i}{4}(\Gamma_{11} - \Gamma_{22}), \\ E\sin\theta\cos\phi &= \mathrm{Re}M_{12} - \frac{i}{2}\mathrm{Re}\Gamma_{12}, \\ E\sin\theta\sin\phi &= -\mathrm{Im}M_{12} + \frac{i}{2}\mathrm{Im}\Gamma_{12}, \end{aligned} \tag{2}$$

where M_{ij} and Γ_{ij} are (i,j)-th elements of $\mathbf{M}$ and $\mathbf{\Gamma}$ matrices, respectively.

The eigenvectors of the mixing Hamiltonian $\mathbf{M} - (i/2)\mathbf{\Gamma}$ are the mass eigenstates ($|P_L\rangle$ and $|P_H\rangle$) and they can be expressed as linear combinations of the flavor eigenstates ($|P^0\rangle$ and $|\bar{P}^0\rangle$) as follows:

$$\begin{aligned} |P_L\rangle &= p_1|P^0\rangle + q_1|\bar{P}^0\rangle, \\ |P_H\rangle &= p_2|P^0\rangle - q_2|\bar{P}^0\rangle, \end{aligned} \tag{3}$$

where $p_1 = N_1\cos\frac{\theta}{2}, q_1 = N_1 e^{i\phi}\sin\frac{\theta}{2}, p_2 = N_2\sin\frac{\theta}{2}$, and $q_2 = N_2 e^{i\phi}\cos\frac{\theta}{2}$ with N_1, N_2 being two normalization factors and the L, H tags indicate light and heavy physical states, respectively. Since, the physical states, as given by Eq. (3), depend only on the parameters θ and ϕ, they are called the mixing parameters for the $P^0 - \bar{P}^0$ system. It should be noticed that the physical states are not orthogonal in general since the mixing matrix is non-Hermitian.

The time evolution of flavor states ($|B^0\rangle \equiv |B^0(t=0)\rangle$ and $|\bar{B}^0\rangle \equiv |\bar{B}^0(t=0)\rangle$) are given by

$$\begin{aligned} |P^0(t)\rangle &= h_+|P^0\rangle + h_-\cos\theta|P^0\rangle + h_- e^{i\phi}\sin\theta|\bar{P}^0\rangle, \\ |\bar{P}^0(t)\rangle &= h_+|\bar{P}^0\rangle - h_-\cos\theta|\bar{P}^0\rangle + h_- e^{-i\phi}\sin\theta|P^0\rangle, \end{aligned} \tag{4}$$

$$\begin{aligned} \text{where } h_+ &= e^{-i(M-i\frac{\Gamma}{2})t}\cos\left[\left(\Delta M - i\frac{\Delta\Gamma}{2}\right)\frac{t}{2}\right], \\ h_- &= e^{-i(M-i\frac{\Gamma}{2})t}\, i\sin\left[\left(\Delta M - i\frac{\Delta\Gamma}{2}\right)\frac{t}{2}\right]. \end{aligned} \tag{5}$$

Here $M = (M_H + M_L)/2$, $\Delta M = M_H - M_L$, $\Gamma = (\Gamma_H + \Gamma_L)/2$, and $\Delta\Gamma = \Gamma_H - \Gamma_L$ with $M_{L,H}$ and $\Gamma_{L,H}$ to be masses and decay widths of the light and heavy mass eigenstates, respectively.

Let us now consider a final state f to which both P^0 and $\bar{P}^0$ can decay. Using Eq. (4), the time-dependent decay amplitudes for the neutral mesons are given by

$$\mathcal{A}mp(P^0(t) \to f) = h_+ \mathcal{A}_f + h_- \cos\theta \mathcal{A}_f + h_- e^{i\phi} \sin\theta \bar{\mathcal{A}}_f,$$
$$\mathcal{A}mp(\bar{P^0}(t) \to f) = h_+ \bar{\mathcal{A}}_f - h_- \cos\theta \bar{\mathcal{A}}_f + h_- e^{-i\phi} \sin\theta \mathcal{A}_f, \tag{6}$$

where $\mathcal{A}_f = \langle f|\mathcal{H}_{\Delta F=1}|P^0\rangle$ and $\bar{\mathcal{A}}_f = \langle f|\mathcal{H}_{\Delta F=1}|\bar{P^0}\rangle$. Hence, the decay rates $\Gamma_f(P^0(t) \to f)$ and $\bar{\Gamma}_f(\bar{P^0}(t) \to f)$ can be expressed as

$$\begin{aligned}\frac{d\Gamma}{dt}(P^0(t) \to f) = \frac{1}{2} e^{-\Gamma t}[&\sinh(\Delta\Gamma t/2)\{2\mathrm{Re}(\cos\theta|\mathcal{A}_f|^2 + e^{i\phi} \sin\theta \mathcal{A}_f^* \bar{\mathcal{A}}_f)\} \\ &+ \cosh(\Delta\Gamma t/2)\{|\mathcal{A}_f|^2 + |\cos\theta|^2|\mathcal{A}_f|^2 + |e^{i\phi}\sin\theta|^2|\bar{\mathcal{A}}_f|^2 + 2\mathrm{Re}(e^{i\phi}\cos\theta^* \sin\theta \mathcal{A}_f^*\bar{\mathcal{A}}_f)\} \\ &+ \cos(\Delta M t)\{|\mathcal{A}_f|^2 - |\cos\theta|^2|\mathcal{A}_f|^2 - |e^{i\phi}\sin\theta|^2|\bar{\mathcal{A}}_f|^2 - 2\mathrm{Re}(e^{i\phi}\cos\theta^*\sin\theta\mathcal{A}_f^*\bar{\mathcal{A}}_f)\} \\ &- \sin(\Delta M t)\{2\mathrm{Im}(\cos\theta|\mathcal{A}_f|^2 + e^{i\phi}\sin\theta\mathcal{A}_f^*\bar{\mathcal{A}}_f)\}],\end{aligned} \tag{7}$$

$$\begin{aligned}\frac{d\Gamma}{dt}(\bar{P}^0(t) \to f) = \frac{1}{2} e^{-\Gamma t}[&\sinh(\Delta\Gamma t/2)\{2\mathrm{Re}(-\cos\theta^*|\bar{\mathcal{A}}_f|^2 + e^{i\phi^*}\sin\theta^*\mathcal{A}_f^*\bar{\mathcal{A}}_f)\} \\ &+ \cosh(\Delta\Gamma t/2)\{|\bar{\mathcal{A}}_f|^2 + |\cos\theta|^2|\bar{\mathcal{A}}_f|^2 + |e^{-i\phi}\sin\theta|^2|\mathcal{A}_f|^2 - 2\mathrm{Re}(e^{i\phi^*}\cos\theta\sin\theta^*\mathcal{A}_f^*\bar{\mathcal{A}}_f)\} \\ &+ \cos(\Delta M t)\{|\bar{\mathcal{A}}_f|^2 - |\cos\theta|^2|\bar{\mathcal{A}}_f|^2 - |e^{-i\phi}\sin\theta|^2|\mathcal{A}_f|^2 + 2\mathrm{Re}(e^{i\phi^*}\cos\theta\sin\theta^*\mathcal{A}_f^*\bar{\mathcal{A}}_f)\} \\ &+ \sin(\Delta M t)\{2\mathrm{Im}(-\cos\theta^*|\bar{\mathcal{A}}_f|^2 + e^{i\phi^*}\sin\theta^*\mathcal{A}_f^*\bar{\mathcal{A}}_f)\}].\end{aligned} \tag{8}$$

III. OBSERVABLES

A. Decay rates

Any final state consisting of two vectors can have three different values for the orbital angular momentum quantum number $\{0, 1, 2\}$ corresponding to the polarization states $\{0, \perp, \|\}$, respectively. As we are not considering CPT violation in decay, we can express the decay amplitudes for modes and conjugate modes in terms of transversity amplitudes as [22–24,26]

$$\mathcal{A}_f(P^0 \to V_1V_2) = \mathcal{A}_0 g_0 + \mathcal{A}_\| g_\| + i\mathcal{A}_\perp g_\perp = \sum_\lambda \mathcal{A}_\lambda g_\lambda \zeta_\lambda,$$
$$\bar{\mathcal{A}}_f(\bar{P^0} \to V_1V_2) = \bar{\mathcal{A}}_0 g_0 + \bar{\mathcal{A}}_\| g_\| - i\bar{\mathcal{A}}_\perp g_\perp = \sum_\lambda \bar{\mathcal{A}}_\lambda g_\lambda \zeta_\lambda^*, \tag{9}$$

where the helicity index λ takes the value $\{0, \|, \perp\}$ and ζ_λ takes the value $\{1, 1, i\}$ for these three helicities, respectively. The factors g_λ are the coefficients of helicity amplitudes ($\mathcal{A}_\lambda$ or $\bar{\mathcal{A}}_\lambda$) in the linear polarization basis and only depend on kinematic angles [27]. In the absence of a direct violation for CP, T, and CPT, these helicity amplitudes can be expressed as

$$\mathcal{A}_\lambda = \bar{\mathcal{A}}_\lambda = a_\lambda e^{i\delta_\lambda}, \tag{10}$$

where a_λ and δ_λ are two real quantities indicating the magnitudes and phases for different helicity amplitudes.

Now, using Eqs. (7)–(10), the time-dependent decay rates for $P^0 \to V_1V_2$ and $\bar{P}^0 \to V_1V_2$ modes can be written as [22–26]

$$\frac{d\Gamma}{dt}(P^0(t) \to V_1V_2) = e^{-\Gamma t}\sum_{\lambda\le\sigma}\left[\Lambda_{\lambda\sigma}\cosh\left(\frac{\Delta\Gamma t}{2}\right) + \eta_{\lambda\sigma}\sinh\left(\frac{\Delta\Gamma t}{2}\right) + \Sigma_{\lambda\sigma}\cos(\Delta M t) - \rho_{\lambda\sigma}\sin(\Delta M t)\right]g_\lambda g_\sigma, \tag{11}$$

$$\frac{d\Gamma}{dt}(\bar{P}^0(t) \to V_1V_2) = e^{-\Gamma t}\sum_{\lambda\le\sigma}\left[\bar{\Lambda}_{\lambda\sigma}\cosh\left(\frac{\Delta\Gamma t}{2}\right) + \bar{\eta}_{\lambda\sigma}\sinh\left(\frac{\Delta\Gamma t}{2}\right) + \bar{\Sigma}_{\lambda\sigma}\cos(\Delta M t) + \bar{\rho}_{\lambda\sigma}\sin(\Delta M t)\right]g_\lambda g_\sigma, \tag{12}$$

where both λ and σ take the value $\{0, \|, \perp\}$.

From Eq. (11) we see that for each helicity combination, there are four observables $(\Lambda_{\lambda\sigma}, \eta_{\lambda\sigma}, \Sigma_{\lambda\sigma}, \rho_{\lambda\sigma})$ and six such helicity combinations are possible. Hence, we get a total 24 observables for the $P^0 \to V_1V_2$ mode. Similarly, there will be 24 different observables $(\bar{\Lambda}_{\lambda\sigma}, \bar{\eta}_{\lambda\sigma}, \bar{\Sigma}_{\lambda\sigma}, \bar{\rho}_{\lambda\sigma})$ for the $\bar{P}^0 \to V_1V_2$ mode too. These observables can be measured by performing a time-dependent angular analysis of $P^0(t) \to V_1V_2$ and $\bar{P}^0(t) \to V_1V_2$ [22–24]. The procedure described in Ref. [26] can be helpful in this regard. On the other hand, probing polarizations of the final state particles may also aid in the measurement of these observables. One important point to notice here is that Refs. [22–24] did not consider $\sinh(\frac{\Delta\Gamma t}{2})$ terms in the decays of B_d^0 and $\bar{B}_d^0$ since $\Delta\Gamma$ is consistent with zero [3] for these modes. In that case, $\eta_{\lambda\sigma}$ and $\bar{\eta}_{\lambda\sigma}$ remain undetermined and one should work with the remaining $(18+18)=36$ observables for a mode and its conjugate mode. However, since we are considering a general scenario here, we keep all the terms and proceed.

B. Parametric expansion

In Ref. [28], T. D. Lee discusses the *CPT* and *T* properties of $\mathbf{M}$ and $\mathbf{\Gamma}$ matrices. First, if the *CPT* invariance holds, then, independently of the *T* symmetry,

$$M_{11}=M_{22}, \qquad \Gamma_{11}=\Gamma_{22}\Rightarrow\theta=\frac{\pi}{2} \quad [\text{Using Eq.}(2)]. \tag{13}$$

In addition, if the *T* invariance holds, then, independently of the *CPT* symmetry,

$$\frac{\Gamma_{12}^*}{\Gamma_{12}}=\frac{M_{12}^*}{M_{12}}\Rightarrow \mathrm{Im}\,\phi=0 \quad [\text{Using Eq.}(2)]. \tag{14}$$

Hence, incorporating the *T*, *CP*, and *CPT* violations in $P^0-\bar{P}^0$ mixing, we parametrize θ and ϕ as [19]

$$\theta=\frac{\pi}{2}+\epsilon_1+i\epsilon_2 \quad \text{and} \quad \phi=-2\beta+i\epsilon_3 \tag{15}$$

where β is the *CP* violating weak phase, ϵ_1 and ϵ_2 are *CPT* violating parameters, and ϵ_3 is a *T* violating parameter. The notations of Belle, *BABAR*, and LHCb Collaborations [7,8,12,18] are a bit different from ours; however, the two notations are related to each other by the following transformation [19]:

$$\cos\theta\leftrightarrow -z, \qquad \sin\theta\leftrightarrow\sqrt{1-z^2}, \qquad e^{i\phi}\leftrightarrow\frac{q}{p}, \tag{16}$$

$$\text{or, equivalently}:\ \epsilon_1=\mathrm{Re}(z), \quad \epsilon_2=\mathrm{Im}(z), \quad \epsilon_3=1-\left|\frac{q}{p}\right|. \tag{17}$$

Now, comparing Eq. (7) to Eq. (11), one can easily infer that all of the observables will be functions of the complex quantities θ and ϕ. As *T* and *CPT* violations are expected to be very small [7,8,12,18,21], we can expand all the observables in terms of ϵ_j $(j\in\{1,2,3\})$. So, using Eqs. (10) and (15), we expand all of the 24 helicity-dependent observables for $P^0\to V_1V_2$ in terms of ϵ_j $(j\in\{1,2,3\})$ keeping up to the linear terms as follows:

$$\begin{aligned}
\Lambda_{ii}&=a_i^2(1-\epsilon_3-\epsilon_1\cos2\beta+\epsilon_2\sin2\beta),\\
\Lambda_{\perp\perp}&=a_\perp^2(1-\epsilon_3+\epsilon_1\cos2\beta-\epsilon_2\sin2\beta),\\
\Lambda_{0\parallel}&=2a_0a_\parallel\cos(\Delta_0-\Delta_\parallel)\\
&\quad\times(1-\epsilon_3-\epsilon_1\cos2\beta+\epsilon_2\sin2\beta),\\
\Lambda_{\perp i}&=2a_\perp a_i((\epsilon_2\cos2\beta+\epsilon_1\sin2\beta)\cos\Delta_i\\
&\quad+\epsilon_3\sin\Delta_i),
\end{aligned} \tag{18}$$

$$\begin{aligned}
\eta_{ii}&=-a_i^2(\epsilon_1-\cos2\beta+\epsilon_3\cos2\beta),\\
\eta_{\perp\perp}&=-a_\perp^2(\epsilon_1+\cos2\beta-\epsilon_3\cos2\beta),\\
\eta_{0\parallel}&=-2a_0a_\parallel\cos(\Delta_0-\Delta_\parallel)(\epsilon_1-\cos2\beta+\epsilon_3\cos2\beta),\\
\eta_{\perp i}&=-2a_\perp a_i((1-\epsilon_3)\sin2\beta\cos\Delta_i+\epsilon_1\sin\Delta_i),
\end{aligned} \tag{19}$$

$$\begin{aligned}
\Sigma_{ii}&=a_i^2(\epsilon_3+\epsilon_1\cos2\beta-\epsilon_2\sin2\beta),\\
\Sigma_{\perp\perp}&=a_\perp^2(\epsilon_3-\epsilon_1\cos2\beta+\epsilon_2\sin2\beta),\\
\Sigma_{0\parallel}&=2a_0a_\parallel\cos(\Delta_0-\Delta_\parallel)\\
&\quad\times(\epsilon_3+\epsilon_1\cos2\beta-\epsilon_2\sin2\beta),\\
\Sigma_{\perp i}&=-2a_\perp a_i((\epsilon_2\cos2\beta+\epsilon_1\sin2\beta)\cos\Delta_i\\
&\quad-(1-\epsilon_3)\sin\Delta_i),
\end{aligned} \tag{20}$$

$$\begin{aligned}
\rho_{ii}&=-a_i^2(\epsilon_2+\sin2\beta-\epsilon_3\sin2\beta),\\
\rho_{\perp\perp}&=-a_\perp^2(\epsilon_2-\sin2\beta+\epsilon_3\sin2\beta),\\
\rho_{0\parallel}&=-2a_0a_\parallel\cos(\Delta_0-\Delta_\parallel)(\epsilon_2+\sin2\beta-\epsilon_3\sin2\beta),\\
\rho_{\perp i}&=-2a_\perp a_i((1-\epsilon_3)\cos2\beta\cos\Delta_i+\epsilon_2\sin\Delta_i),
\end{aligned} \tag{21}$$

where $i\in\{0,\parallel\}$ and $\Delta_i=\delta_i-\delta_\perp$. Similarly, it is also possible to expand the observables of the conjugate mode $\bar{P}^0\to V_1V_2$ in terms of ϵ_j (given in the Appendix).

C. Solutions

As can be seen from the expansion of the observables given by Eqs. (18)–(21), there are a total of nine unknown parameters (i.e., three of a_λ, three of ϵ_j, two of Δ_i, and β). In the SM case, there are six unknown parameters (three of a_λ, two of Δ_i, and β) as stated in Refs. [23,24]; however, for our scenario, we have three extra parameters emerging due to *T* and *CPT* violation in mixing, namely, $\epsilon_{1,2,3}$, thus resulting in nine theoretical parameters. It should be noted that Refs. [23,24] originally deal with the SM scenario plus *CP* violation in decay, not *T* and *CPT* violations in mixing; hence, in addition to six unknown SM parameters, they have three more amplitudes (b_λ), three more strong phases (δ_λ^b), and one extra weak phase related to the *CP* violating part of the decay amplitudes ($\mathcal{A}_\lambda$ or $\bar{\mathcal{A}}_\lambda$). Now, we go back to our scenario and solve the nine theoretical parameters in terms of the observables as follows:

$$a_\lambda = \sqrt{\Lambda_{\lambda\lambda} + \Sigma_{\lambda\lambda}}, \tag{22}$$

$$\epsilon_1 = -\frac{1}{2}\left(\frac{\eta_{ii}}{\Lambda_{ii} + \Sigma_{ii}} + \frac{\eta_{\perp\perp}}{\Lambda_{\perp\perp} + \Sigma_{\perp\perp}}\right), \tag{23}$$

$$\epsilon_2 = -\frac{1}{2}\left(\frac{\rho_{ii}}{\Lambda_{ii} + \Sigma_{ii}} + \frac{\rho_{\perp\perp}}{\Lambda_{\perp\perp} + \Sigma_{\perp\perp}}\right), \tag{24}$$

$$\epsilon_3 = \frac{1}{2}\left(\frac{\Sigma_{ii}}{\Lambda_{ii} + \Sigma_{ii}} + \frac{\Sigma_{\perp\perp}}{\Lambda_{\perp\perp} + \Sigma_{\perp\perp}}\right), \tag{25}$$

$$\sin 2\beta = -\frac{1}{2}\left(\frac{\rho_{ii}}{\Lambda_{ii}} - \frac{\rho_{\perp\perp}}{\Lambda_{\perp\perp}}\right), \tag{26}$$

$$\cos 2\beta = \frac{1}{2}\left(\frac{\eta_{ii}}{\Lambda_{ii}} - \frac{\eta_{\perp\perp}}{\Lambda_{\perp\perp}}\right), \tag{27}$$

$$\cos(\Delta_0 - \Delta_\parallel) = \frac{1}{2}\left[\frac{\Lambda_{0\parallel} + \Sigma_{0\parallel}}{\sqrt{\Lambda_{00} + \Sigma_{00}}\sqrt{\Lambda_{\parallel\parallel} + \Sigma_{\parallel\parallel}}}\right], \tag{28}$$

$$\sin \Delta_i = \frac{1}{2}\left[\frac{\Lambda_{\perp i} + \Sigma_{\perp i}}{\sqrt{\Lambda_{ii} + \Sigma_{ii}}\sqrt{\Lambda_{\perp\perp} + \Sigma_{\perp\perp}}}\right], \tag{29}$$

$$\cos \Delta_i = X_i \Lambda_{ii}\Lambda_{\perp\perp}\left[\frac{\sqrt{\Lambda_{ii} + \Sigma_{ii}}\sqrt{\Lambda_{\perp\perp} + \Sigma_{\perp\perp}}}{\Lambda_{\perp\perp}\Sigma_{ii} + \Lambda_{ii}\Sigma_{\perp\perp} + 2\Lambda_{\perp\perp}\Lambda_{ii}}\right], \tag{30}$$

$$\text{where}\ ,\ X_i = \left[\frac{(\Lambda_{\perp i} - \Sigma_{\perp i})(\Lambda_{\perp\perp}\Sigma_{ii} + \Lambda_{ii}\Sigma_{\perp\perp}) + 2(\Lambda_{ii}\Lambda_{\perp\perp}\Lambda_{\perp i} - \Sigma_{ii}\Sigma_{\perp\perp}\Sigma_{\perp i})}{(\eta_{\perp\perp}\rho_{ii} - \eta_{ii}\rho_{\perp\perp})(\Lambda_{ii} + \Sigma_{ii})(\Lambda_{\perp\perp} + \Sigma_{\perp\perp})}\right], \tag{31}$$

with $\lambda \in \{0, \parallel, \perp\}$ and $i \in \{0, \parallel\}$. In principle, we should present only nine equations as the solutions for nine unknown parameters. But, we have listed more than nine relations from Eq. (22) to Eq. (30) because the observables involve several angular parameters. Actually, to specify any angular variable without any ambiguity, one must quantify both sin and cos of that angle. However, as can be seen in Sec. IV B, the extra equations will result in some relations among observables by applying various trigonometric identities.

IV. OBSERVABLE RELATIONS

A. SM relations

In the SM scenario, all three ϵ_j become zero and there remain only six unknown parameters (3 of a_λ, 2 of Δ_i, and β) in the theory. But the number of observables for the $P^0 \to V_1 V_2$ mode is 24. Hence, 18 independent relations among observables must emerge and they are the following:

$$\Sigma_{\lambda\lambda} = 0, \qquad \Sigma_{0\parallel} = 0, \qquad \Lambda_{\perp i} = 0, \tag{32}$$

$$\frac{\rho_{ii}}{\Lambda_{ii}} = \frac{\rho_{0\parallel}}{\Lambda_{0\parallel}} = -\frac{\rho_{\perp\perp}}{\Lambda_{\perp\perp}}, \tag{33}$$

$$\frac{\rho_{\perp i}^2}{4\Lambda_{\perp\perp}\Lambda_{ii} - \Sigma_{\perp i}^2} = \frac{\Lambda_{0\parallel}^2 - \rho_{0\parallel}^2}{\Lambda_{0\parallel}^2}, \tag{34}$$

$$\Lambda_{0\parallel} = \frac{1}{2\Lambda_{\perp\perp}}\left[\Sigma_{\perp 0}\Sigma_{\perp\parallel} + \rho_{\perp 0}\rho_{\perp\parallel}\left(\frac{\Lambda_{0\parallel}^2}{\Lambda_{0\parallel}^2 - \rho_{0\parallel}^2}\right)\right]. \tag{35}$$

$$\frac{\eta_{ii}}{\Lambda_{ii}} = \frac{\eta_{0\parallel}}{\Lambda_{0\parallel}} = -\frac{\eta_{\perp\perp}}{\Lambda_{\perp\perp}}, \tag{36}$$

$$\frac{\eta_{\perp i}}{\rho_{\perp i}} + \frac{\eta_{0\parallel}}{\rho_{0\parallel}} = 0, \tag{37}$$

$$\eta_{0\parallel}^2 + \rho_{0\parallel}^2 = \Lambda_{0\parallel}^2, \tag{38}$$

with $\lambda \in \{0, \parallel, \perp\}$ and $i \in \{0, \parallel\}$. Here, Eq. (32) contains six relations (for three different λ and two different i), Eqs. (33) and (36) contain three relations each (for two different i) whereas there are two relations (for two different i) inside of Eqs. (34) and (37).

However, for vanishing $\Delta\Gamma$, only 18 observables will be accessible to us (as discussed in the Sec. III A) and hence, in that case, we should obtain 12 independent relations among observables. Those 12 relations are given by Eqs. (32)–(35), as discussed in Refs. [23,24].

One important point to state is that one can use the solutions, given by Eqs. (22)–(29), in the SM scenario also. But, X_i, given by Eq. (31), takes the form $\frac{0}{0}$ in this case and it causes problems in finding $\cos\Delta_i$ from Eq. (30). Still, one can express $\cos\Delta_i$ ($i \in \{0, \|\}$) in this scenario as follows:

$$\cos\Delta_i = -\left(\frac{\Lambda_{0\|}\rho_{\perp i}}{2\eta_{0\|}\sqrt{\Lambda_{ii}\Lambda_{\perp\perp}}}\right), \tag{39}$$

which can easily be verified by substituting vanishing ϵ_j into Eqs. (18)–(21). Hence, using Eqs. (30), (32), and (39), one can write X_i ($i \in \{0, \|\}$) in the limit $\epsilon_j \to 0$ ($j \in \{1,2,3\}$) as

$$X_i = -\left(\frac{\Lambda_{0\|}\rho_{\perp i}}{\eta_{0\|}\Lambda_{ii}\Lambda_{\perp\perp}}\right). \tag{40}$$

Nevertheless, we shall see in the next section that most of these 18 relations from Eqs. (32)–(38) will get violated if T and CPT violations in mixing are also present. On the other hand, if there exists direct violation of T, CP, or CPT instead of T and CPT violating effects in mixing, then most of these relations also get violated. Hence, it is impossible to infer from this set of relations whether CPT violation (if it exists at all) is present in mixing or in decay.

B. *T* and *CPT* violation

In addition to the CP violating weak phase, if there exists T and CPT violation in mixing, we have nine unknown theoretical parameters (three of ϵ_j, three of a_λ, two of Δ_i, and β). But the number of observables is still 24. So, there should appear $(24-9) = 15$ relations among observables. In order to find them we substitute the solutions of unknown parameters, given by Eqs. (22)–(30), back to the expansion of observables, given by Eqs. (18)–(21). Thus we get 11 independent relations, which are given below:

$$\frac{\Lambda_{0\|}}{\Lambda_{ii}} = \frac{\Sigma_{0\|}}{\Sigma_{ii}} = \frac{\rho_{0\|}}{\rho_{ii}} = \frac{\eta_{0\|}}{\eta_{ii}}, \tag{41}$$

$$\frac{\rho_{0\|}^2 + \eta_{0\|}^2}{\Lambda_{0\|}^2} = \frac{\rho_{\perp\perp}^2 + \eta_{\perp\perp}^2}{\Lambda_{\perp\perp}^2}, \tag{42}$$

$$\eta_{\perp i} = \frac{1}{2}\left[\frac{\Sigma_{\perp i}}{\Lambda_{ii}\Lambda_{\perp\perp}}\{\eta_{\perp\perp}(\Lambda_{ii}+\Sigma_{ii}) + \eta_{ii}(\Lambda_{\perp\perp}+\Sigma_{\perp\perp})\} + X_i\{\Lambda_{\perp\perp}\rho_{ii} - \Lambda_{ii}\rho_{\perp\perp}\}\right], \tag{43}$$

$$\rho_{\perp i} = \frac{1}{2}\left[\frac{\Sigma_{\perp i}}{\Lambda_{ii}\Lambda_{\perp\perp}}\{\rho_{\perp\perp}(\Lambda_{ii}+\Sigma_{ii}) + \rho_{ii}(\Lambda_{\perp\perp}+\Sigma_{\perp\perp})\} - X_i\{\Lambda_{\perp\perp}\eta_{ii} - \Lambda_{ii}\eta_{\perp\perp}\}\right], \tag{44}$$

with $i \in \{0, \|\}$. It should be noticed that there are six independent relations in Eq. (41), two relations in Eq. (43), and two relations in Eq. (44).

There are four more such independent relations among observables which emerge due to the following trigonometric identities:

$$\sin^2\alpha + \cos^2\alpha = 1 \quad (\text{where } \alpha = \Delta_0, \Delta_\| \text{ or } 2\beta), \tag{45}$$

$$\cos(\Delta_0 - \Delta_\|) = \cos\Delta_0\cos\Delta_\| + \sin\Delta_0\sin\Delta_\|. \tag{46}$$

Substituting expressions for different angular variables from Eqs. (26)–(30) into the above trigonometric identities, given by Eqs. (45) and (46), we get the remaining four relations as

$$\left[\frac{(\Lambda_{\perp i}+\Sigma_{\perp i})^2}{(\Lambda_{ii}+\Sigma_{ii})(\Lambda_{\perp\perp}+\Sigma_{\perp\perp})}\right] + 4X_i^2\Lambda_{ii}^2\Lambda_{\perp\perp}^2\left[\frac{(\Lambda_{ii}+\Sigma_{ii})(\Lambda_{\perp\perp}+\Sigma_{\perp\perp})}{(\Lambda_{\perp\perp}\Sigma_{ii}+\Lambda_{ii}\Sigma_{\perp\perp}+2\Lambda_{\perp\perp}\Lambda_{ii})^2}\right] = 4, \tag{47}$$

$$\left(\frac{\rho_{00}}{\Lambda_{00}} - \frac{\rho_{\perp\perp}}{\Lambda_{\perp\perp}}\right)^2 + \left(\frac{\eta_{00}}{\Lambda_{00}} - \frac{\eta_{\perp\perp}}{\Lambda_{\perp\perp}}\right)^2 = 4, \tag{48}$$

$$(\Lambda_{0\|}+\Sigma_{0\|}) - \frac{1}{2}\left[\frac{(\Lambda_{\perp 0}+\Sigma_{\perp 0})(\Lambda_{\perp\|}+\Sigma_{\perp\|})}{(\Lambda_{\perp\perp}+\Sigma_{\perp\perp})}\right] = \left[\frac{2X_0X_\|\Lambda_{00}\Lambda_{\|\|}\Lambda_{\perp\perp}^2(\Lambda_{00}+\Sigma_{00})(\Lambda_{\|\|}+\Sigma_{\|\|})(\Lambda_{\perp\perp}+\Sigma_{\perp\perp})}{(\Lambda_{\perp\perp}\Sigma_{00}+\Lambda_{00}\Sigma_{\perp\perp}+2\Lambda_{\perp\perp}\Lambda_{00})(\Lambda_{\perp\perp}\Sigma_{\|\|}+\Lambda_{\|\|}\Sigma_{\perp\perp}+2\Lambda_{\perp\perp}\Lambda_{\|\|})}\right], \tag{49}$$

with $i \in \{0, \|\}$. Equation (47) contains two relations (for two different i). However, it should be noticed that though $\sin 2\beta$ and $\cos 2\beta$ can be expressed in two ways using the helicities 0 and $\|$ separately [as shown in Eqs. (26) and (27)], we obtain only one relation among observables from the trigonometric identity: $\sin^2 2\beta + \cos^2 2\beta = 1$. It happens because Eq. (41) ensures the following: $(\rho_{00}/\Lambda_{00}) = (\rho_{\|\|}/\Lambda_{\|\|})$ and $(\eta_{00}/\Lambda_{00}) = (\eta_{\|\|}/\Lambda_{\|\|})$.

However, one should keep in mind that the relations in Eqs. (41)–(46) will not hold true for all orders in ϵ_j as we are computing the observables perturbatively up to the first order in ϵ_j. The corrections to these relations are quadratic or of higher order in ϵ_j and hence can be neglected for sufficiently small values of ϵ_j. Now, if one wants to check the validity of the 18 relations of last section [given by Eqs. (32)–(38)] in this scenario, he/she would find ϵ_j order correction terms in 14 of them. The four relations, which remain intact in both the scenarios are $(\rho_{ii}/\Lambda_{ii}) = (\rho_{0\parallel}/\Lambda_{0\parallel})$ and $(\eta_{ii}/\Lambda_{ii}) = (\eta_{0\parallel}/\Lambda_{0\parallel})$, which can easily be observed from Eqs. (33), (36), and (41).

It should be noted that the 15 relations of this section [Eqs. (41)–(44) and Eqs. (47)–(49)] hold true even if all ϵ_j become zero. It can be verified straightforwardly by setting $\epsilon_j = 0$ in a parametric expansion of observables [Eqs. (18)–(21)] and then substituting those expressions for observables into these 15 relations. But it does not mean that we have 15 more independent relations in the SM case. One can easily check that the 18 relations in last section automatically satisfy the 15 relations of this section. In other words, the 18 relations of the previous section are embedded in a complicated form inside the 15 relations of the present section. However, as discussed in last section, one has to be careful in dealing with X_i while verifying since it takes the $\frac{0}{0}$ form in SM scenario.

Now, if direct violations of T, CP, and CPT are present in the decay mode, most of these 15 relations will not hold true and that can be used as a smoking gun signal of confirming those effects. In that case, the 18 relations of the SM scenario will be disobeyed too. On the other hand, if these 15 relations are satisfied, then one becomes sure that there is no direct violations of T, CP, and CPT, but it cannot be confirmed whether T and CPT violations in mixing are present or not since those 15 relations are satisfied on both the occasions. In this circumstance, the validity of the 18 relations in the last section should be examined. If those 18 relations hold true, it would signify the absence of T and CPT violation in mixing and if they get violated, the presence of them will be confirmed.

There is another way to confirm the existence of T, CP, and CPT violation in decay. In this analysis, we have used the observables of the $P^0 \to V_1 V_2$ mode only for solving all of the nine unknown parameters, as shown in Eqs. (22)–(30). Similarly, it is also possible to solve them by using the observables of the $\bar{P}^0 \to V_1 V_2$ mode, as given in the Appendix. These two sets of solutions should match numerically in the absence of new physics effects in decay. Hence, significant deviations in the numerical values of the nine unknown parameters from these two sets of solutions will definitely indicate sizeable contributions of T, CP, and CPT violations in decay.

V. CONCLUSION

In conclusion, we have studied the behavior of observables for neutral meson decaying to two vectors in the presence of T, CP, and CPT violation in mixing. Polarizations of the final state with two vectors provide us with a large number of observables in these modes. We choose the final state in such a way that both P^0 and $\bar{P}^0$ can decay to it. We establish the complete set of 15 relations among observables which must be obeyed if there do not exist any direct violations of T, CP, and CPT and these relations can be used as the smoking gun signal to confirm their presence or absence. In addition to that we also listed the full set of 18 relations among observables which should be satisfied if there is no violation of T and CPT in the mixing of $P^0 - \bar{P}^0$ and these relations can be used to probe their existence unambiguously.

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APPENDIX: OBSERVABLES FOR $\bar{P}^0 \to V_1 V_2$ AND THE SOLUTIONS

The expansion of observables for the $\bar{P}^0 \to V_1 V_2$ mode in terms of ϵ_j ($j \in \{1,2,3\}$) is given by

$$\begin{aligned}
\bar{\Lambda}_{ii} &= a_i^2(1+\epsilon_3+\epsilon_1\cos 2\beta+\epsilon_2\sin 2\beta),\\
\bar{\Lambda}_{\perp\perp} &= a_\perp^2(1+\epsilon_3-\epsilon_1\cos 2\beta-\epsilon_2\sin 2\beta),\\
\bar{\Lambda}_{0\parallel} &= 2a_0a_\parallel\cos(\Delta_0-\Delta_\parallel)(1+\epsilon_3+\epsilon_1\cos 2\beta\\
&\quad +\epsilon_2\sin 2\beta),\\
\bar{\Lambda}_{\perp i} &= 2a_\perp a_i((\epsilon_2\cos 2\beta-\epsilon_1\sin 2\beta)\cos\Delta_i\\
&\quad +\epsilon_3\sin\Delta_i),
\end{aligned}\tag{A1}$$

$$\begin{aligned}
\bar{\eta}_{ii} &= a_i^2(\epsilon_1+\cos 2\beta+\epsilon_3\cos 2\beta),\\
\bar{\eta}_{\perp\perp} &= a_\perp^2(\epsilon_1-\cos 2\beta-\epsilon_3\cos 2\beta),\\
\bar{\eta}_{0\parallel} &= 2a_0a_\parallel\cos(\Delta_0-\Delta_\parallel)(\epsilon_1+\cos 2\beta+\epsilon_3\cos 2\beta),\\
\bar{\eta}_{\perp i} &= -2a_\perp a_i((1+\epsilon_3)\sin 2\beta\cos\Delta_i+\epsilon_1\sin\Delta_i),
\end{aligned}\tag{A2}$$

$$\begin{aligned}
\bar{\Sigma}_{ii} &= -a_i^2(\epsilon_3+\epsilon_1\cos 2\beta+\epsilon_2\sin 2\beta),\\
\bar{\Sigma}_{\perp\perp} &= -a_\perp^2(\epsilon_3-\epsilon_1\cos 2\beta-\epsilon_2\sin 2\beta),\\
\bar{\Sigma}_{0\parallel} &= -2a_0a_\parallel\cos(\Delta_0-\Delta_\parallel)(\epsilon_3+\epsilon_1\cos 2\beta\\
&\quad +\epsilon_2\sin 2\beta),\\
\bar{\Sigma}_{\perp i} &= -2a_\perp a_i((\epsilon_2\cos 2\beta-\epsilon_1\sin 2\beta)\cos\Delta_i\\
&\quad +(1+\epsilon_3)\sin\Delta_i),
\end{aligned}\tag{A3}$$

$$\begin{aligned}
\bar{\rho}_{ii} &= -a_i^2(\epsilon_2 + \sin 2\beta + \epsilon_3 \sin 2\beta),\\
\bar{\rho}_{\perp\perp} &= -a_\perp^2(\epsilon_2 - \sin 2\beta - \epsilon_3 \sin 2\beta),\\
\bar{\rho}_{0\parallel} &= -2a_0 a_\parallel \cos(\Delta_0 - \Delta_\parallel)(\epsilon_2 + \sin 2\beta + \epsilon_3 \sin 2\beta),\\
\bar{\rho}_{\perp i} &= -2a_\perp a_i((1+\epsilon_3)\cos 2\beta \cos \Delta_i - \epsilon_2 \sin \Delta_i),
\end{aligned} \tag{A4}$$

with $\lambda \in \{0, \parallel, \perp\}$ and $i \in \{0, \parallel\}$.

The solutions for nine unknown parameters (i.e., three of a_λ, three of ϵ_j, two of Δ_i, and β) in terms of the observables of the $\bar{P}^0 \to V_1 V_2$ mode are given by

$$a_\lambda = \sqrt{\bar{\Lambda}_{\lambda\lambda} + \bar{\Sigma}_{\lambda\lambda}}, \tag{A5}$$

$$\epsilon_1 = \frac{1}{2}\left(\frac{\bar{\eta}_{ii}}{\bar{\Lambda}_{ii} + \bar{\Sigma}_{ii}} + \frac{\bar{\eta}_{\perp\perp}}{\bar{\Lambda}_{\perp\perp} + \bar{\Sigma}_{\perp\perp}}\right), \tag{A6}$$

$$\epsilon_2 = -\frac{1}{2}\left(\frac{\bar{\rho}_{ii}}{\bar{\Lambda}_{ii} + \bar{\Sigma}_{ii}} + \frac{\bar{\rho}_{\perp\perp}}{\bar{\Lambda}_{\perp\perp} + \bar{\Sigma}_{\perp\perp}}\right), \tag{A7}$$

$$\epsilon_3 = -\frac{1}{2}\left(\frac{\bar{\Sigma}_{ii}}{\bar{\Lambda}_{ii} + \bar{\Sigma}_{ii}} + \frac{\bar{\Sigma}_{\perp\perp}}{\bar{\Lambda}_{\perp\perp} + \bar{\Sigma}_{\perp\perp}}\right), \tag{A8}$$

$$\sin 2\beta = -\frac{1}{2}\left(\frac{\bar{\rho}_{ii}}{\bar{\Lambda}_{ii}} - \frac{\bar{\rho}_{\perp\perp}}{\bar{\Lambda}_{\perp\perp}}\right), \tag{A9}$$

$$\cos 2\beta = \frac{1}{2}\left(\frac{\bar{\eta}_{ii}}{\bar{\Lambda}_{ii}} - \frac{\bar{\eta}_{\perp\perp}}{\bar{\Lambda}_{\perp\perp}}\right), \tag{A10}$$

$$\cos(\Delta_0 - \Delta_\parallel) = \frac{1}{2}\left[\frac{\bar{\Lambda}_{0\parallel} + \bar{\Sigma}_{0\parallel}}{\sqrt{\bar{\Lambda}_{00} + \bar{\Sigma}_{00}}\sqrt{\bar{\Lambda}_{\parallel\parallel} + \bar{\Sigma}_{\parallel\parallel}}}\right], \tag{A11}$$

$$\sin \Delta_i = -\frac{1}{2}\left[\frac{\bar{\Lambda}_{\perp i} + \bar{\Sigma}_{\perp i}}{\sqrt{\bar{\Lambda}_{ii} + \bar{\Sigma}_{ii}}\sqrt{\bar{\Lambda}_{\perp\perp} + \bar{\Sigma}_{\perp\perp}}}\right], \tag{A12}$$

$$\cos \Delta_i = \bar{X}_i \bar{\Lambda}_{ii} \bar{\Lambda}_{\perp\perp}\left[\frac{\sqrt{\bar{\Lambda}_{ii} + \bar{\Sigma}_{ii}}\sqrt{\bar{\Lambda}_{\perp\perp} + \bar{\Sigma}_{\perp\perp}}}{\bar{\Lambda}_{\perp\perp}\bar{\Sigma}_{ii} + \bar{\Lambda}_{ii}\bar{\Sigma}_{\perp\perp} + 2\bar{\Lambda}_{\perp\perp}\bar{\Lambda}_{ii}}\right], \tag{A13}$$

$$\text{where , } \bar{X}_i = \left[\frac{(\bar{\Lambda}_{\perp i} - \bar{\Sigma}_{\perp i})(\bar{\Lambda}_{\perp\perp}\bar{\Sigma}_{ii} + \bar{\Lambda}_{ii}\bar{\Sigma}_{\perp\perp}) + 2(\bar{\Lambda}_{ii}\bar{\Lambda}_{\perp\perp}\bar{\Lambda}_{\perp i} - \bar{\Sigma}_{ii}\bar{\Sigma}_{\perp\perp}\bar{\Sigma}_{\perp i})}{(\bar{\eta}_{\perp\perp}\bar{\rho}_{ii} - \bar{\eta}_{ii}\bar{\rho}_{\perp\perp})(\bar{\Lambda}_{ii} + \bar{\Sigma}_{ii})(\bar{\Lambda}_{\perp\perp} + \bar{\Sigma}_{\perp\perp})}\right], \tag{A14}$$

with $\lambda \in \{0, \parallel, \perp\}$ and $i \in \{0, \parallel\}$.

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