Baryon-number-violating nucleon and dinucleon decays in a model with large extra dimensions

Sudhakantha Girmohanta and Robert Shrock

C.N. Yang Institute for Theoretical Physics and Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794, USA

(Received 11 November 2019; published 27 January 2020)

It is known that limits on baryon-number-violating nucleon decays do not, in general, imply corresponding suppression of $n-\bar{n}$ transitions. Indeed, it has been shown, using a model with fermions propagating in higher dimensions, that even with nucleon decays suppressed far below observable levels, $n-\bar{n}$ oscillations can occur at a rate comparable to existing experimental limits, motivating new searches for such oscillations. In the context of this model we investigate a related question, namely the implications of limits on $\Delta L=-1$ proton and bound neutron decays mediated by four-fermion operators for rates of nucleon decays mediated by k-fermion operators with $k=6$ and $k=8$. These include a variety of nucleon and dinucleon decays to dilepton and trilepton final states with $\Delta L=-3,-2,1$, and 2. We carry out a low-energy effective field theory analysis of relevant operators for these decays and show that, in this extra-dimensional model, the rates for these decays are strongly suppressed and hence are in accord with experimental limits.

DOI: 10.1103/PhysRevD.101.015017

I. INTRODUCTION

Although the Standard Model (SM), as extended to include nonzero neutrino masses and lepton mixing, agrees with current data, there are many aspects of particle physics that it does not explain. Although this theory conserves baryon number B [1], many ultraviolet extensions of it predict baryon number violation (BNV). In general, one expects there to be some violation of baryon number in nature, because this is one of the necessary conditions for generating the observed baryon asymmetry in the universe [2]. A number of dedicated experiments have been carried out since the early 1980s to search for baryon-number-violating decays of protons and of neutrons bound in nuclei. (Henceforth, we shall refer to these as nucleon decays, with it being understood that the term excludes baryon-number-conserving weak decays of neutrons.) These experiments have obtained null results and have set resultant stringent upper limits for the rates of such nucleon decays [3].

It was pointed out early on that neutron-antineutron ($n-\bar{n}$) oscillations and the associated $|\Delta B|=2$ violation of baryon number could account for baryogenesis [4], and there has long been interest in this type of baryon number violation (some early works include [5–9]). The same physics beyond the Standard Model (BSM) that gives rise to $n-\bar{n}$ oscillations also leads to matter instability via the decays of nn and np dinucleon initial states to nonbaryonic final states, typically involving several pions. The reason for this is that a nonzero transition amplitude $\langle\bar{n}|\mathcal{L}_{\rm eff}|n\rangle$ means that a physical state $|n\rangle_{\rm phys}$ contains a small but nonzero $|\bar{n}\rangle$ component. In turn, this leads to the annihilation of the $|\bar{n}\rangle$ component with a neighboring neutron or proton in a nucleus, and thus produces $\Delta B=-2$ decays of dinucleons. There have been searches for $n-\bar{n}$ oscillations using neutron beams from reactors [10] and for matter instability and various dinucleon decay modes using large underground detectors [11–24].

The operators in the low-energy effective Lagrangian for nucleon decay are four-fermion operators with Maxwellian dimension 6 in mass units and hence coefficients of the form $1/({\rm mass})^2$. In contrast, the operators in $\mathcal{L}_{\rm eff}^{(n\bar{n})}$ are six-quark operators with dimension 9 and hence with coefficients of the form $1/({\rm mass})^5$. Consequently, if one were to assume that there is a single high mass scale $M_{\rm BNV}$ describing the physics responsible for baryon number violation, nucleon decay would be much more important than $n-\bar{n}$ oscillations and the corresponding dinucleon decays as a manifestation of baryon number violation. However, the actual situation might be quite different [6]. Reference [25] showed an example, using an extra-dimensional model [26,27], in which nucleon decays could be suppressed well below an observable level, while $n-\bar{n}$ oscillations could occur at a level comparable to existing experimental limits. In this case, it is the ($|\Delta B| = 2$) $n - \bar{n}$ oscillations and the corresponding ($\Delta B = -2$) nn and np dinucleon decays that are the main observable effects of baryon number violation, rather than ($\Delta B = -1$) decays of individual nucleons. This provides motivation for new experimental searches for $n - \bar{n}$ oscillations. Additional examples with baryon number violation but no proton decay were later discussed in [28]. Reviews of $n - \bar{n}$ oscillations include [29–31].

This finding in Ref. [25] naturally motivates one to ask a more general question: in this type of extra-dimensional model, are there baryon-number-violating processes mediated by k-fermion operators with higher values of k, in particular, $k = 6$ and $k = 8$, that could also be relatively unsuppressed, as was the case with the $k = 6$ operators responsible for $n - \bar{n}$ oscillations?

In this paper we address and answer this question. Using the same extra-dimensional model as in [25], we study a variety of nucleon and dinucleon decays that violate both B and total lepton number L and are mediated by k-fermion operators with $k = 6$ and $k = 8$, respectively. These include the $\Delta L = -3$ nucleon decays

$$p \to \ell^+ \bar{\nu} \bar{\nu}' \tag{1.1}$$

and

$$n \to \bar{\nu} \bar{\nu}' \bar{\nu}'' \tag{1.2}$$

and the $\Delta L = 1$ nucleon decays

$$p \to \ell^+ \nu \nu' \tag{1.3}$$

and

$$n \to \bar{\nu} \nu' \nu'', \tag{1.4}$$

both of which are mediated by six-fermion operators, and the following $\Delta L = -2$ dinucleon decays mediated by eight-fermion operators:

$$pp \to \ell^+ \ell'^+, \tag{1.5}$$

where ℓ^+ and ℓ'^+ can be e^+, μ^+, or τ^+, as allowed by phase space, i.e.,

$$pp \to (e^+e^+, \mu^+\mu^+, e^+\mu^+, e^+\tau^+, \text{ or } \mu^+\tau^+), \tag{1.6}$$

$$np \to \ell^+ \bar{\nu}, \tag{1.7}$$

and

$$nn \to \bar{\nu} \bar{\nu}'. \tag{1.8}$$

In addition, we consider the $\Delta L = 2$ dineutron decays

$$nn \to \nu \nu', \tag{1.9}$$

which are also mediated by eight-fermion operators. Here and below we use the symbol ν to denote either an electroweak-doublet (EW-doublet) neutrino or an EW-singlet neutrino. From experimental limits on nucleon decays, we first determine constraints on relevant parameters of the extra-dimensional model, namely distances separating centers of fermion wave functions in the extra dimensions. Then, for each of the various types of decays, we analyze relevant multifermion operators and apply these constraints to estimate the typical predictions of the model for the decay rates. Answering the question posed above, we show that these nucleon decays (1.1)–(1.4) and dinucleon decays (1.6)–(1.9) are safely smaller than the rates for the leading baryon-number-violating nucleon decays mediated by four-fermion operators and thus are in accord with experimental limits.

There are several motivations for the class of extra-dimensional theories that we consider. The possibility that our four-dimensional spacetime could be embedded in a higher-dimensional spacetime dates back at least to attempts to unify electromagnetism and gravity by Kaluza and Klein [32], and this embedding is implied by string theory, since the low-energy limit of a (super)string theory leads to a ten-dimensional pointlike field theory. Since all experimental data are consistent with spacetime being four-dimensional, the extra dimensions must be compactified on scale(s) that is (are) much shorter than those that have been probed experimentally. In this context, the Standard Model can be viewed as a low-energy effective field theory that describes physics at length scales much larger than the compactification scale(s). One of the most striking and perplexing features of the quarks and charged leptons is the great range of approximately 10^5 spanned by their masses, extending from 173 GeV for the top quark to 0.511 MeV for the electron. The Standard Model gives no insight into the reason for this large range of masses, and instead just accommodates it via a correspondingly large range of magnitudes of Yukawa couplings. This fermion mass hierarchy is even larger when one takes into account the tiny but nonzero masses of neutrinos. An intriguing suggestion was that this large range of SM fermion masses might be explained naturally if the SM is embedded in a spacetime of higher dimension $d = 4 + n$, with n extra additional spatial dimensions, and SM fermions have wave functions that are localized at different positions in the additional n-dimensional space [26,27]. Here we will use a model of this type in which the wave functions of the SM fermions are strongly localized, with Gaussian profiles of width $1/\mu$, at various points in this extra-dimensional space [25–27], [33–41]. As in Refs. [25–27], we do not make any specific assumption concerning possible ultraviolet completions of the model.

In addition to giving insight into various baryon- and lepton-number-violating processes in the context of a BSM model, our analysis is an interesting application of effective field theory in a more complicated case than usual, in which there are multiple mass scales relevant for the B and L violation, namely μ, a general scale $M_{\rm BNV}$ characterizing baryon number violation, and the inverse distances between the centers of the wave functions of various fermions in the extra dimensions. For each decay with a given $\ell = e$ or μ, there are at least $\binom{6}{2} = 15$ of these inverse distances, corresponding to the five SM quark and lepton fields Q_L, u_R, d_R, $L_{\ell,L}$, and ℓ_R, and one or more electroweak-singlet neutrinos, $\nu_{s,R}$. There is a correspondingly large variety of multifermion operators with different structures, which we analyze.

The present work complements our recent studies in [42], where we derived improved upper bounds on the rates for several nucleon-to-trilepton decay modes with $\Delta L = -1$, and in [43], where we similarly presented improved upper bounds on the rates for several dinucleon-to-dilepton decay channels with $\Delta L = 0$. These works [42,43] were model-independent phenomenological analyses, whereas our present paper is a study within the context of a specific type of extra-dimensional model.

This paper is organized as follows. In Sec. II we discuss the extra-dimensional model and low-energy effective field theory approach that serve as the theoretical framework for our calculations. In Sec. III we extract constraints on the fermion wave functions in the model from limits on nucleon decay modes. Section IV is devoted to a review of $n-\bar{n}$ oscillations in the model, as mediated by six-fermion operators. A discussion is given in Sec. V of $\Delta L = 0$ dinucleon decays to dileptons. In Secs. VI and VII we analyze six-fermion operators that contribute to $\Delta L = -3$ and $\Delta L = 1$ nucleon decays to trilepton final states, respectively. In Sec. VIII we present a general operator analysis of eight-fermion operators that contribute to $\Delta L = -2$ dinucleon decays to dileptons. Applications of this general analysis to the decays $pp \to \ell^+\ell'^+$, $np \to \ell^+\bar{\nu}$, and $nn \to \bar{\nu}\bar{\nu}'$ are given in Secs. IX–XI. Section XI also contains a discussion of the $\Delta L = 2$ dineutron decays $nn \to \nu\nu'$. Our conclusions are contained in Sec. XII. In the Appendixes A, B, and D we give relevant integral formulas, color $\mathrm{SU}(3)_c$ and weak $\mathrm{SU}(2)_L$ tensors, and present further information on relevant operators.

II. THEORETICAL FRAMEWORK

In this section we describe the theoretical framework for our study. Usual spacetime coordinates are denoted as x_ν, $\nu = 0$, 1, 2, 3, and the n extra coordinates as y_λ; for definiteness, the latter are assumed to be compact. The fermion fields are taken to have a factorized form:

$$\Psi(x,y) = \psi(x)\chi(y). \tag{2.1}$$

In the extra dimensions the SM fields are restricted to the interval $0 \le y_\lambda \le L$ for all λ. We define an energy corresponding to the inverse of the compactification scale as

$$\Lambda_L \equiv \frac{1}{L}. \tag{2.2}$$

We will give most results for general n, but note that only for even n are chiral projection operators defined, since they require there to be a γ_5 Dirac matrix that anticommutes with the other Dirac gamma matrices, and this is only possible for even n. The $d = (4+n)$-dimensional fields thus have Kaluza-Klein mode decompositions. We use a low-energy effective field theory approach that entails an ultraviolet cutoff, which we denote as M_*. The localization of the wave function of a fermion f in the extra dimensions has the form [26,27]

$$\chi_f(y) = Ae^{-\mu^2\|y-y_f\|^2}, \tag{2.3}$$

where A is a normalization factor and $y_f \in \mathbb{R}^n$ denotes the position vector of this fermion in the extra dimensions, with components $y_f = ((y_f)_1, \ldots, (y_f)_n)$ and with the standard Euclidean norm of a vector in $\mathbb{R}^n$, namely

$$\|y_f\| \equiv \left(\sum_{\lambda=1}^{n} y_{f,\lambda}^2\right)^{1/2}. \tag{2.4}$$

For $n = 1$ or $n = 2$, this fermion localization can result from appropriate coupling to a scalar with a kink or vortex solution, respectively [33]. One can also include corrections due to Coulombic gauge interactions between fermions [34] (see also [35,36]). The normalization factor A is determined by the condition that, after integration over the n higher dimensions, the four-dimensional fermion kinetic term has its canonical normalization. This yields the result

$$A = \left(\frac{2}{\pi}\right)^{n/4}\mu^{n/2}. \tag{2.5}$$

We define a distance inverse to the localization measure μ as

$$L_\mu \equiv \frac{1}{\mu}. \tag{2.6}$$

As noted, this type of model has the potential to yield an explanation for the hierarchy in the fermion mass matrices via the localization of fermion wave functions with half-width

$$L_\mu \ll L \tag{2.7}$$

at various points in the higher-dimensional space. The ratio of the compactification scale L divided by the scale characterizing the localization of the fermion wave functions in the extra dimensions is

$$\xi \equiv \frac{L}{L_\mu} = \frac{\mu}{\Lambda_L} = \mu L. \tag{2.8}$$

The choice

$$\xi \sim 30 \tag{2.9}$$

is made for sufficient separation of the various fermion wave functions while still fitting well within the size L of the compactified extra dimensions. The UV cutoff M_* satisfies $M_* > \mu$ for the validity the low-energy field theory analysis. The choice

$$\Lambda_L \gtrsim 100 \text{ TeV}, \tag{2.10}$$

i.e., $L \lesssim 2.0 \times 10^{-19}$ cm, is consistent with bounds on extra dimensions from precision electroweak constraints and collider searches [3] and produces adequate suppression of flavor-changing neutral-current (FCNC) processes [39,41]. With the ratio $\xi = 30$, this yields

$$\mu \sim 3 \times 10^3 \text{ TeV}, \tag{2.11}$$

i.e., $L_\mu \equiv \mu^{-1} = 0.67 \times 10^{-20}$ cm.

Starting from an effective Lagrangian in the $d = (4+n)$-dimensional spacetime, one obtains the resultant low-energy effective Lagrangian in four dimensions by integrating over the extra n dimensions. The integration over each of the n coordinates of a vector y runs from 0 to L, but, because of the restriction of the fermion wave functions to the form (2.3), with $L_\mu \ll L$, it follows that, to a very good approximation, the domain of integration can be extended to the interval $(-\infty, \infty)$: $\int_0^L d^n y \to \int_{-\infty}^{\infty} d^n y$. It is convenient to define the dimensionless variable

$$\eta = \mu y, \tag{2.12}$$

with components given by $\eta = (\eta_1, \ldots, \eta_n)$.

We first discuss the fermion mass terms. For the first generation of quarks and charged leptons, the Yukawa terms in the higher-dimension theory are

$$\mathcal{L}_{Yuk} = [h^{(d)} \bar{Q}_L d_R \phi + h^{(u)} \bar{Q}_L u_R \tilde{\phi} + h^{(e)} \bar{L}_{e,L} e_R \phi] + \text{H.c.}, \tag{2.13}$$

where $Q_L = \binom{u}{d}_L$, and $\phi = \binom{\phi^+}{\phi^0}$ is the SM Higgs field, with $\tilde{\phi} = i\sigma_2 \phi^\dagger = \binom{\phi^{0*}}{-\phi^-}$. With the inclusion of the second and third generations of SM fermions, the Yukawa couplings $h^{(f)}$ with $f = u$, d, e become 3×3 matrices. The diagonalization of the resultant quark mass matrices in the charge 2/3 and charge $-1/3$ sectors yields the quark masses and Cabibbo-Kobayashi-Maskawa quark mixing matrix. For our present purposes, it will often be adequate to neglect small off-diagonal elements in the Yukawa matrices. The vacuum expectation value of the Higgs field is written, in the standard normalization, as

$$\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \tag{2.14}$$

where $v = 246$ GeV. Given the factorization (2.1) and the Gaussian profiles of the fermion wave functions (2.3), the integration over the extra n dimensions of a given fermion bilinear operator product $h^{(f)}(v/\sqrt{2})[\bar{f}_L f_R]$ resulting from a Yukawa interaction involves the integral

$$A^2 h^{(f)} \frac{v}{\sqrt{2}} \int d^n y e^{-\|\eta - \eta_{f_L}\|^2 - \|\eta - \eta_{f_R}\|^2} = h^{(f)} \frac{v}{\sqrt{2}} \exp\left[-\frac{1}{2}\|\eta_{f_L} - \eta_{f_R}\|^2\right]. \tag{2.15}$$

Hence, for the fermions $f = u$, d and also $f = \ell = e, \mu, \tau$ (neglecting off-diagonal elements in the Yukawa matrices), we have

$$m_f = h^{(f)} \frac{v}{\sqrt{2}} \exp\left[-\frac{1}{2}\|\eta_{f_L} - \eta_{f_R}\|^2\right] \tag{2.16}$$

or, equivalently, the following constraint on the separation distance $\|\eta_{f_L} - \eta_{f_R}\|$:

$$\|\eta_{f_L} - \eta_{f_R}\| = \left[2 \ln\left(\frac{h^{(f)} v}{\sqrt{2} m_f}\right)\right]^{1/2}. \tag{2.17}$$

Note that this relation does not depend directly on the number of large extra dimensions, n. The relation (2.17) holds for the quarks and charged leptons. For neutrinos, the situation is more complicated because the neutrino mass eigenvalues and the lepton mixing matrix result, in general, from the diagonalization of the combined Dirac and Majorana mass terms involving electroweak-singlet neutrinos $\nu_{s,R}$, $s = 1, \ldots, n_s$. These Majorana neutrino mass terms violate L (as $|\Delta L| = 2$ operators) and lead to potentially observable L-violating processes. However, L violation can occur even with very small neutrino masses, as in R-parity-violating supersymmetric theories (e.g., [44]).

Since the relation (2.16) applies in the effective Lagrangian above the electroweak-symmetry-breaking scale, the values of m_f are the running masses evaluated at this high scale. In accord with the idea motivating this class of BSM theories, that the generational hierarchy in the SM fermion masses is not due primarily to a hierarchy in the dimensionless Yukawa couplings in the higher-dimensional space but instead to the different positions of the wave function centers in the extra dimensions, we will take $h^{(f)} \sim O(1)$ in the higher-dimensional space for the various SM fermions f. For technical simplicity, we actually set $h^{(f)} = 1$ for all f. It is straightforward to redo our analysis if one chooses to assign some of the generational mass hierarchy to these Yukawa couplings in the $(4+n)$-dimensional space. A calculation of the running quark masses at a scale $\Lambda_t = m_t$

gives [45] $m_u(\Lambda_t) = 2.2$ MeV and $m_d(\Lambda_t) = 4.5$ MeV. Combining these values with the known value $v = 246$ GeV from $G_F/\sqrt{2} = 1/(2v^2)$, we calculate the dimensionless separation distances

$$\|\eta_{Q_L} - \eta_{u_R}\| = 4.75 \tag{2.18}$$

and

$$\|\eta_{Q_L} - \eta_{d_R}\| = 4.60, \tag{2.19}$$

so that the ratio is $\|\eta_{Q_L} - \eta_{u_R}\|/\|\eta_{Q_L} - \eta_{d_R}\| = 1.03$ [46].

As noted, a major result from this type of model was the fact that with roughly equal dimensionless Yukawa couplings $h^{(f)} \sim O(1)$ for different generations of quark and charged leptons, the large hierarchy in the values of these SM fermion masses can be explained by moderate differences in the separation distances in the extra dimensions, $\|\eta_{f_L} - \eta_{f_R}\|$. This extra-dimensional model is minimal in the sense that we do not include additional fields aside from neutrinos that carry lepton number, such as Majorons.

A given baryon-number-violating decay involves a set of operators defined in four-dimensional spacetime, which, for our applications, are k-fold products of fermion fields. We denote these operators as $\mathcal{O}_{r,(k)}$ and write the effective Lagrangian in usual four-dimensional spacetime that is responsible for the BNV physics as

$$\mathcal{L}_{\rm eff}(x) = \sum_r c_{r,(k)}\mathcal{O}_{r,(k)}(x) + {\rm H.c.} \tag{2.20}$$

Each of the fermion fields in $\mathcal{O}_{r,(k)}$ has the factorized form (2.1). We denote the corresponding effective Lagrangian in the $d = (4+n)$-dimensional space as

$$\mathcal{L}_{{\rm eff},4+n}(x,y) = \sum_r \kappa_{r,(k)} O_{r,(k)}(x,y) + {\rm H.c.} \tag{2.21}$$

The factorization property (2.1) implies that the $O_{r,(k)}(x,y)$ also can be factored as

$$O_{r,(k)}(x) = U_{r,(k)}(x)V_{r,(k)}(y) \tag{2.22}$$

[with $\mathrm{SU}(3)_c$, $\mathrm{SU}(2)_L$, and Dirac structure implicit and with no sum on r]. We denote the integral over the extra dimensions of $V_{r,(k)}(y)$ as

$$I_{r,(k)} \equiv \int d^n y V_{r,(k)}(y). \tag{2.23}$$

This integral involves an integrand consisting of a k-fold product of Gaussian wave functions and is given by Eq. (A2) in Appendix A. Hence, for each r (with no sum on r)

$$c_{r,(k)} = \kappa_{r,(k)} I_{r,(k)}. \tag{2.24}$$

The coefficient $\kappa_{r,(k)}$ may depend on the generational indices of lepton fields that occur in $\mathcal{O}_{r,(k)}$; this is left implicit in the notation. In general, as a k-fold product of fermion fields in $d = 4+n$ spacetime dimensions, $O_{r,(k)}(x,y)$ has Maxwellian (free-field) operator dimension

$$\dim(O_{r,(k)}(x,y)) = \frac{k(d-1)}{2} = \frac{k(3+n)}{2} \tag{2.25}$$

in mass units. The condition that the action in the d-dimensional space must be dimensionless is $-d + \dim(\kappa_{r,(k)}) + \dim(O_{r,(k)}) = 0$, so

$$\begin{aligned}\dim(\kappa_{r,(k)}) &= d - k\left(\frac{d-1}{2}\right)\\ &= 4 + n - k\left(\frac{3+n}{2}\right).\end{aligned} \tag{2.26}$$

It is useful to write the coefficients $\kappa_{r,(k)}$ in a form that shows this dimensionality explicitly:

$$\kappa_{r,(k)} = \frac{\bar{\kappa}_{r,(k)}}{(M_{\rm BNV})^{(k(3+n)/2)-4-n}}, \tag{2.27}$$

where $\bar{\kappa}_{r,(k)}$ is dimensionless and $M_{\rm BNV}$ is an effective mass scale characterizing the baryon-number-violating physics. Then, making use of Eq. (A2), $I_{r,(k)}$ can be written as a prefactor b_k multiplying an exponential, namely

$$I_{r,(k)} = b_k e^{-S_{r,(k)}}, \tag{2.28}$$

where

$$\begin{aligned}b_k &= A^k \mu^{-n}\left(\frac{\pi}{k}\right)^{n/2}\\ &= [2^{k/4}\pi^{-(k-2)/4}k^{-1/2}\mu^{(k-2)/2}]^n.\end{aligned} \tag{2.29}$$

In Eq. (2.29), the factor A^k arises from the k-fold product of fermion fields, the factor μ^{-n} from the Jacobian $d^n y = \mu^{-n} d^n\eta$, and the factor $(\pi/k)^{n/2}$ from the integration [see Eq. (A2) in Appendix A]. By construction, $b_2 = 1$, independent of the number of large extra dimensions, n. Combining these results, we can write

$$\begin{aligned}c_{r,(k)} &= \kappa_{r,(k)} I_{r,(k)}\\ &= \frac{\bar{\kappa}_{r,(k)}}{(M_{\rm BNV})^{(3k-8)/2}}\left(\frac{\mu}{M_{\rm BNV}}\right)^{(k-2)n/2}\left(\frac{2^{k/4}}{\pi^{(k-2)/4}k^{1/2}}\right)^n\\ &\quad\times e^{-S_{r,(k)}}.\end{aligned} \tag{2.30}$$

For each of the various types of decays discussed below, the number k of fermions in the k-fermion operator products will be obvious, so henceforth, we suppress the subscript (k) in the notation for $I_{r,(k)}$ and $c_{r,(k)}$.

Before carrying out detailed analyses of various baryon-number-violating decays, it is useful to make some rough estimates of the expected ratios of resultant rates. The hadronic matrix elements that are relevant for decays mediated by operators with different numbers of fermions have different dimensions, but in comparing decay rates, this difference is compensated by the requisite powers of the quantum chromodynamics (QCD) mass scale $\Lambda_{\rm QCD}$. Thus, for the ratio of two BNV decays mediated by operators comprised of k_1 and k_2 fermions, respectively, we have the rough estimate

$$\frac{\Gamma_{(k_2)}}{\Gamma_{(k_1)}} \sim \left(\frac{\Lambda_{\rm QCD}}{M_{\rm BNV}}\right)^{3(k_2-k_1)} \left(\frac{\mu}{M_{\rm BNV}}\right)^{(k_2-k_1)n} \left(\frac{2^{(k_2-k_1)/2}k_1}{\pi^{(k_2-k_1)/2}k_2}\right)^n \times e^{-2(\langle S_{(k_2)}\rangle - \langle S_{(k_1)}\rangle)}, \tag{2.31}$$

where $\langle S_{(k)}\rangle$ denotes a typical size of the exponential factor occurring in Eqs. (2.28) and (2.30) for this decay. In particular, relative to BNV nucleon decays such as $p \to e^+\pi^0$, etc., mediated by four-fermion operators, the rough estimate (2.31) gives the ratio

$$\frac{\Gamma_{(6)}}{\Gamma_{(4)}} \sim \left(\frac{\Lambda_{\rm QCD}}{M_{\rm BNV}}\right)^6 \left(\frac{\mu}{M_{\rm BNV}}\right)^{2n} \left(\frac{4}{3\pi}\right)^n e^{-2(\langle S_{(6)}\rangle - \langle S_{(4)}\rangle)} \tag{2.32}$$

for decays such as (1.1)–(1.4) mediated by six-fermion operators. Similarly, for dinucleon decays mediated by eight-fermion operators, such as (1.6)–(1.9), Eq. (2.31) predicts

$$\frac{\Gamma_{(8)}}{\Gamma_{(4)}} \sim \left(\frac{\Lambda_{\rm QCD}}{M_{\rm BNV}}\right)^{12} \left(\frac{\mu}{M_{\rm BNV}}\right)^{4n} \left(\frac{2}{\pi^2}\right)^n e^{-2(\langle S_{(8)}\rangle - \langle S_{(4)}\rangle)}. \tag{2.33}$$

With $\Lambda_{\rm QCD} \simeq 0.25$ GeV, $\mu = 3\times 10^3$ TeV as in Eq. (2.11), and an illustrative value $M_{\rm BNV} \sim 10^2$ TeV and $n = 2$ extra dimensions, Eqs. (2.32) and (2.33) yield

$$\ln\left(\frac{\Gamma_{(6)}}{\Gamma_{(4)}}\right) \sim -65.5 - 2(\langle S_{(6)}\rangle - \langle S_{(4)}\rangle) \tag{2.34}$$

and

$$\ln\left(\frac{\Gamma_{(8)}}{\Gamma_{(4)}}\right) \sim -131 - 2(\langle S_{(8)}\rangle - \langle S_{(4)}\rangle). \tag{2.35}$$

The study of the sums $S_{r,(k)}$ requires a detailed analysis of the various k-fermion operators that contribute to specific baryon-number-violating processes. We discuss these below.

III. CONSTRAINTS FROM LIMITS ON BARYON-NUMBER-VIOLATING NUCLEON DECAYS

We discuss here the constraints on Standard-Model fermion wave function positions in the extra-dimensional model that follow from the upper limits on the rates for baryon-number-violating nucleon decays. The analysis begins with the observation that the mass scale characterizing the physics responsible for these decays must be large compared with the electroweak symmetry-breaking scale v, and therefore the effective Lagrangian must be invariant under the full Standard-Model gauge group $G_{\rm SM}$. To label the various (four-fermion) operators that contribute, we will use the abbreviations pd and nd to refer to proton and (otherwise stably bound) neutron decay, respectively, and Nd to subsume both of these types of decay, with the nucleon $N = p$ or $N = n$. (The use of the same symbol, n, to refer to neutron and the number of extra dimensions should not cause any confusion; the context will always make clear which is meant.) Then we can write

$$\mathcal{L}_{\rm eff}^{(Nd)}(x) = \sum_r c_r^{(Nd)} \mathcal{O}_r^{(Nd)}(x) + {\rm H.c.}, \tag{3.1}$$

where $c_r^{(Nd)}$ are coefficients and $\mathcal{O}_r^{(Nd)}(x)$ are operators. Correspondingly, in the $d = (4+n)$-dimensional space, the effective Lagrangian is

$$\mathcal{L}_{{\rm eff},4+n}^{(Nd)}(x,y) = \sum_r \kappa_r^{(Nd)} O_r^{(Nd)}(x,y) + {\rm H.c.} \tag{3.2}$$

We recall our notation for fermion fields. The ${\rm SU}(2)_L$-singlet and -doublet quark fields are denoted u_R^α, d_R^α, and $Q_L^\alpha = {u^\alpha \choose d^\alpha}_L$, where α is a color index. The ${\rm SU}(2)_L$-singlet and ${\rm SU}(2)_L$-doublet lepton fields are denoted ℓ_R and $L_{\ell,L} = {\nu_\ell \choose \ell}_L$, where $\ell = e,\ \mu,\ \tau$. In addition, we include electroweak-singlet neutrinos, written as $\nu_{s,R}$, with $s = 1,\ldots,n_s$, as is necessary to form Dirac and Majorana mass terms for the neutrinos. The upper and lower components of the quark and lepton ${\rm SU}(2)_L$ doublets are indicated by Roman indices $i, j, \ldots$, so $Q_L^{i\alpha} = u_L^\alpha$ for $i = 1$, $Q_L^{i\alpha} = d_L^\alpha$ for $i = 2$, $L_{\ell,L}^i = \nu_\ell$ for $i = 1$, and $L_{\ell,L}^i = \ell_L$ for $i = 2$. For each of these fields $f = Q_L, u_R, d_R, L_{\ell,L}, \ell_R$, and $\nu_{s,R}$, the wave function in the $(4+n)$-dimensional space has the form (2.1) with normalization factor A given by Eq. (2.5) and Gaussian profile given by Eq. (2.3).

With the original SM fermions, before the addition of any electroweak-singlet $\nu_{s,R}$ fields, the four-fermion operators $\mathcal{O}_r^{(Nd)}$ in $\mathcal{L}_{\rm eff}^{(Nd)}$ that contribute to nucleon decays are [47–50]

$$\mathcal{O}_1^{(Nd)} = \epsilon_{\alpha\beta\gamma}[u_R^{\alpha T} C d_R^{\beta}][u_R^{\gamma T} C \ell_R], \tag{3.3}$$

$$\begin{aligned}\mathcal{O}_2^{(Nd)} &= \epsilon_{ij}\epsilon_{\alpha\beta\gamma}[Q_L^{i\alpha T} C Q_L^{j\beta}][u_R^{\gamma T} C \ell_R] \\ &= 2\epsilon_{\alpha\beta\gamma}[u_L^{\alpha T} C d_L^{\beta}][u_R^{\gamma T} C \ell_R],\end{aligned} \tag{3.4}$$

$$\begin{aligned}\mathcal{O}_3^{(Nd)} &= \epsilon_{ij}\epsilon_{\alpha\beta\gamma}[Q_L^{i\alpha T} C L_{\ell,L}^{j}][u_R^{\beta T} C d_R^{\gamma}] \\ &= \epsilon_{\alpha\beta\gamma}([u_L^{\alpha T} C \ell_L] - [d_L^{\alpha T} C \nu_{\ell,L}])[u_L^{\beta T} C d_R^{\gamma}],\end{aligned} \tag{3.5}$$

and

$$\begin{aligned}\mathcal{O}_4^{(Nd)} &= \epsilon_{ij}\epsilon_{km}\epsilon_{\alpha\beta\gamma}[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C L_{\ell,L}^{m}] \\ &= 2\epsilon_{\alpha\beta\gamma}[u_L^{\alpha T} C d_L^{\beta}]([u_L^{\gamma T} C \ell_L] - [d_L^{\gamma T} C \nu_{\ell,L}]),\end{aligned} \tag{3.6}$$

where C is the Dirac charge conjugation matrix satisfying $C\gamma_\mu C^{-1} = -(\gamma_\mu)^T$, $C = -C^T$; and $\epsilon_{\alpha\beta\gamma}$ and ϵ_{ij} are totally antisymmetric $SU(3)_c$ and $SU(2)_L$ tensors, respectively. Two other operators would be present in a multigenerational context but vanish identically in the relevant case here, where the quarks are all of the first generation, i.e., u and d:

$$\epsilon_{\alpha\beta\gamma}[u_{a_1,R}^{\alpha T} C u_{a_2,R}^{\beta}][d_{a_3,R}^{\gamma T} C \ell_R] \tag{3.7}$$

and

$$(\epsilon_{ik}\epsilon_{jm} + \epsilon_{im}\epsilon_{jk})\epsilon_{\alpha\beta\gamma}[Q_{a_1,L}^{i\alpha T} C Q_{a_2,L}^{j\beta}][Q_{a_3,L}^{k\gamma T} C L_{\ell,L}^{m}], \tag{3.8}$$

where a_1, a_2, and a_3 are generation indices.

Including electroweak-singlet neutrinos $\nu_{s,R}$ with $s = 1, \ldots, n_s$, one has two additional types of operators for nucleon decays, namely

$$\mathcal{O}_5^{(Nd)} = \epsilon_{\alpha\beta\gamma}[u_R^{\alpha T} C d_R^{\beta}][d_R^{\gamma T} C \nu_{s,R}] \tag{3.9}$$

and

$$\begin{aligned}\mathcal{O}_6^{(Nd)} &= \epsilon_{ij}\epsilon_{\alpha\beta\gamma}[Q_L^{i\alpha T} C Q_L^{j\beta}][d_R^{\gamma T} C \nu_{s,R}] \\ &= 2\epsilon_{\alpha\beta\gamma}[u_L^{\alpha T} C d_L^{\beta}][d_R^{\gamma T} C \nu_{s,R}].\end{aligned} \tag{3.10}$$

For completeness, we also list a four-fermion operator that would be present in a multigenerational context but vanishes identically in the case considered here with first-generation quarks, namely

$$\epsilon_{\alpha\beta\gamma}[d_{a_1,R}^{\alpha T} C d_{a_2,R}^{\beta}][u_{a_3,R}^{\gamma T} C \nu_{s,R}]. \tag{3.11}$$

To each of the operators $\mathcal{O}_r^{(Nd)}$ there corresponds an operator $O_r^{(Nd)}$ in $\mathcal{L}_{\mathrm{eff},4+n}^{(Nd)}$. These are four-fermion operators, and, as the $k = 4$ special case of Eq. (2.27), we have

$$\kappa_r^{(Nd)} = \frac{\bar{\kappa}_r^{(Nd)}}{(M_{\mathrm{BNV}})^{2+n}}. \tag{3.12}$$

As noted before, in general, the coefficient $\kappa_r^{(Nd)}$ may depend on the generational indices of fermion fields that occur in $\mathcal{O}_r^{(Nd)}$; this is left implicit in the notation. The special case of Eq. (2.22) for nucleon decay is

$$O_r^{(Nd)}(x,y) = U_r^{(Nd)}(x) V_r^{(Nd)}(y). \tag{3.13}$$

We have

$$V_1^{(Nd)}(y) = A^4 \exp[-\{2\|\eta - \eta_{u_R}\|^2 + \|\eta - \eta_{d_R}\|^2 + \|\eta - \eta_{\ell_R}\|^2\}], \tag{3.14}$$

$$V_2^{(Nd)}(y) = A^4 \exp[-\{2\|\eta - \eta_{Q_L}\|^2 + \|\eta - \eta_{u_R}\|^2 + \|\eta - \eta_{\ell_R}\|^2\}], \tag{3.15}$$

$$V_3^{(Nd)}(y) = A^4 \exp[-\{\|\eta - \eta_{Q_L}\|^2 + \|\eta - \eta_{L_{\ell,L}}\|^2 + \|\eta - \eta_{u_R}\|^2 + \|\eta - \eta_{d_R}\|^2\}], \tag{3.16}$$

$$V_4^{(Nd)}(y) = A^4 \exp[-\{3\|\eta - \eta_{Q_L}\|^2 + \|\eta - \eta_{L_{\ell,L}}\|^2\}], \tag{3.17}$$

$$V_5^{(Nd)}(y) = A^4 \exp[-\{\|\eta - \eta_{u_R}\|^2 + 2\|\eta - \eta_{d_R}\|^2 + \|\eta - \eta_{\nu_{s,R}}\|^2\}], \tag{3.18}$$

and

$$V_6^{(Nd)}(y) = A^4 \exp[-\{2\|\eta - \eta_{Q_L}\|^2 + \|\eta - \eta_{d_R}\|^2 + \|\eta - \eta_{\nu_{s,R}}\|^2\}]. \tag{3.19}$$

To perform the integrals over y, we use the general integration formula given as Eq. (A2) in Appendix A. Carrying out the integration over the y components and using Eq. (2.5) for the relevant case $k = 4$, we obtain the following results for the nonvanishing operators:

$$I_1^{(Nd)} = b_4 \exp\left[-\frac{1}{4}\{2\|\eta_{u_R} - \eta_{d_R}\|^2 + 2\|\eta_{u_R} - \eta_{\ell_R}\|^2 + \|\eta_{d_R} - \eta_{\ell_R}\|^2\}\right], \tag{3.20}$$

$$I_2^{(Nd)} = b_4 \exp\left[-\frac{1}{4}\{2\|\eta_{Q_L} - \eta_{u_R}\|^2 + 2\|\eta_{Q_L} - \eta_{\ell_R}\|^2 + \|\eta_{u_R} - \eta_{\ell_R}\|^2\}\right], \tag{3.21}$$

$$I_3^{(Nd)} = b_4 \exp\left[-\frac{1}{4}\{\|\eta_{Q_L} - \eta_{L_{\ell,L}}\|^2 + \|\eta_{Q_L} - \eta_{u_R}\|^2 + \|\eta_{Q_L} - \eta_{d_R}\|^2 + \|\eta_{L_{\ell,L}} - \eta_{u_R}\|^2 + \|\eta_{L_{\ell,L}} - \eta_{d_R}\|^2 + \|\eta_{u_R} - \eta_{d_R}\|^2\}\right], \tag{3.22}$$

$$I_4^{(Nd)} = b_4 \exp\left[-\frac{3}{4}\|\eta_{Q_L} - \eta_{L_{\ell,L}}\|^2\right], \tag{3.23}$$

$$I_5^{(Nd)} = b_4 \exp\left[-\frac{1}{4}\{2\|\eta_{u_R} - \eta_{d_R}\|^2 + \|\eta_{u_R} - \eta_{\nu_{s,R}}\|^2 + 2\|\eta_{d_R} - \eta_{\nu_{s,R}}\|^2\}\right], \tag{3.24}$$

and

$$I_6^{(Nd)} = b_4 \exp\left[-\frac{1}{4}\{2\|\eta_{Q_L} - \eta_{d_R}\|^2 + 2\|\eta_{Q_L} - \eta_{\nu_{s,R}}\|^2 + \|\eta_{d_R} - \eta_{\nu_{s,R}}\|^2\}\right], \tag{3.25}$$

where $b_4 = (\pi^{-1/2}\mu)^n$, from the $k = 4$ special case of Eq. (2.29). It is convenient to write the integral $I_r^{(Nd)}$ in the form

$$I_r^{(Nd)} \equiv b_4 e^{-S_r^{(Nd)}}, \tag{3.26}$$

where $S_r^{(Nd)}$ denotes the sum of squares of fermion wave function separation distances (rescaled via multiplication by μ to be dimensionless) in the argument of the exponent in $I_r^{(Nd)}$. Thus, for example, in the case of $O_4^{(Nd)}$, the sum in the exponent is $S_4^{(Nd)} = (3/4)\|\eta_{Q_L} - \eta_{L_{\ell,L}}\|^2$, and similarly for the other $S_r^{(Nd)}$. Then, as the $k = 4$ special case of (2.30),

$$c_r^{(Nd)} = \frac{\bar{\kappa}_r^{(Nd)}}{(M_{\rm BNV})^2}\left(\frac{\mu}{\pi^{1/2}M_{\rm BNV}}\right)^n e^{-S_r^{(Nd)}}. \tag{3.27}$$

The amplitude for the decay of a nucleon $N = p$ or n to a final state $f.s.$ is given by $\langle f.s.|\mathcal{O}_{\rm eff}^{(Nd)}|N\rangle$. The hadronic matrix elements for various operators have been calculated by lattice gauge simulations [51,52]. We then use the experimental lower bound for the partial lifetime $(\tau/B)_{N\to f.s.} = \Gamma^{-1}_{N\to f.s.}$ for a given nucleon decay mode $N \to f.s.$ with branching ratio B to infer upper bounds on the magnitudes of the $c_r^{(Nd)}$ coefficients. Since in our low-energy effective field theory approach we do not assume any cancellation between different terms $c_r^{(Nd)}\mathcal{O}_r^{(Nd)}$ occurring in $\mathcal{L}_{\rm eff}^{(Nd)}$, we conservatively impose the bounds from a given decay individually on each term that contributes to it. For given values of μ, $M_{\rm BNV}$, and the dimensionless coefficients $\bar{\kappa}_r^{(Nd)}$, these constraints are upper bounds on the integrals $I_r^{(Nd)}$

$$I_r^{(nd)} < I_{\rm max}^{(Nd)}, \tag{3.28}$$

and hence lower bounds on the sums of squares of distances in $S_r^{(Nd)}$ for each operator $\mathcal{O}_r^{(Nd)}$,

$$S_r^{(Nd)} > S_{\rm min}^{(Nd)}, \tag{3.29}$$

where

$$S_{\rm min}^{(Nd)} = \ln\left(\frac{b_4}{I_{\rm max}^{(Nd)}}\right). \tag{3.30}$$

When comparing lower bounds from two different nucleon decay modes, denoted $Nd1$ and $Nd2$, to which the same operators contribute, a general relation is

$$S_{\rm min,1}^{(Nd)} - S_{\rm min,2}^{(Nd)} = \frac{1}{2}\ln\left[\frac{(\tau/B)_{Nd1,\rm min}}{(\tau/B)_{Nd2,\rm min}}\right]. \tag{3.31}$$

Some of the squared fermion separation distances $\|\eta_{f_i} - \eta_{f_j}\|^2$ occurring in the individual $S_r^{(Nd)}$ sums are already fixed by Standard-Model physics such as quark and lepton masses and mixing, and values of, or limits on, FCNC processes. These include the (rescaled) distances $\|\eta_{Q_L} - \eta_{q_R}\|$ with $q_R = u_R, d_R$, and for leptons, the distances $\|\eta_{L_{\ell,L}} - \eta_{\ell'_R}\|$ with $L_{\ell,L} = L_{e,L}, L_{\mu,L}, L_{\tau,L}$ and $\ell'_R = e_R, \mu_R, \tau_R$, respectively.

For example, for $\ell = e$, the inequality $S_2^{(Nd)} > S_{\rm min}^{(Nd)}$ is a quadratic inequality in the space $\mathbb{R}^{3n}$ spanned by the three n-dimensional vectors η_{Q_L}, η_{u_R}, and η_{ℓ_R}, with one distance $\|\eta_{Q_L} - \eta_{u_R}\|$ fixed by the u-quark mass. The (rescaled) separation distances between SM fermion wave function centers that enter into the $S_r^{(Nd)}$ of this type and are not already fixed by SM physics are

$$\begin{aligned} &\|\eta_{u_R} - \eta_{d_R}\|, \qquad \|\eta_{u_R} - \eta_{\ell_R}\|, \qquad \|\eta_{d_R} - \eta_{\ell_R}\|, \\ &\|\eta_{Q_L} - \eta_{\ell_R}\|, \qquad \|\eta_{\ell,L} - \eta_{u_R}\|, \qquad \|\eta_{\ell,L} - \eta_{d_R}\|, \\ &\|\eta_{Q_L} - \eta_{L_{\ell,L}}\| \quad \text{for } \ell = e, \mu. \end{aligned} \tag{3.32}$$

Hence, the full set of lower bounds on fermion separation distances from all of the inequalities $S_r^{(Nd)} > S_{\rm min}^{(Nd)}$ contributing to nucleon decays constitutes a set of coupled quadratic inequalities in the space spanned by the relevant fermion position vectors. For example, the most stringent lower bound on a partial lifetime, $(\tau/B)_{p\to e^+\pi^0}$, yields coupled quadratic inequalities in the $\mathbb{R}^{5n}$ space spanned by the vectors η_{Q_L}, η_{u_R}, η_{d_R}, $\eta_{L_{e,L}}$, and η_{e_R}, and similarly with nucleon decays involving $\ell = \mu$. With the inclusion of EW-singlet neutrino fields $\nu_{s,R}$, the set of separation distances that affect the rates for nucleon decay also includes

$$\|\eta_{q_R} - \eta_{\nu_{s,R}}\|, \qquad \|\eta_{Q_L} - \eta_{\nu_{s,R}}\| \quad \text{for } q = u, d. \tag{3.33}$$

The lower bounds on the partial lifetimes for some of the simplest proton decays are [53]

$$(\tau/B)_{p\to e^+\pi^0} > 1.6 \times 10^{34} \text{ yr} \tag{3.34}$$

and

$$(\tau/B)_{p\to \mu^+\pi^0} > 0.77 \times 10^{34} \text{ yr}. \tag{3.35}$$

These and the other bounds quoted here are at the 90% confidence level. Other bounds of comparable sensitivity include, e.g., $(\tau/B)_{p\to e^+\eta} > 1.0 \times 10^{34}$ yr and $(\tau/B)_{p\to \mu^+\eta} > 0.47 \times 10^{34}$ yr [54]. Comparable lower bounds apply for baryon-number-violating neutron decays, such as $(\tau/B)_{n\to e^+\pi^-} > 0.53 \times 10^{34}$ yr [53], $(\tau/B)_{n\to \mu^+\pi^-} > 0.35 \times 10^{34}$ yr [54], and $(\tau/B)_{n\to \bar\nu\pi^0} > 1.1 \times 10^{33}$ yr [55] (see also [56]). These bounds can easily be satisfied by separating the positions of the wave function centers of the quarks and first two generations of leptons [26].

The calculation of the rate for a nucleon decay to a given final state, $\Gamma_{N\to f.s.}$, depends on the ultraviolet physics responsible for the operators $\mathcal{O}_r^{(d)}$ and their coefficients $\kappa_r^{(Nd)}$ in the effective Lagrangian. In particular, it involves the integration of the square of the matrix element $\langle f.s.|\mathcal{L}_{\rm eff}|N\rangle$ with respect to the n-body phase space. Since this ultraviolet physics is not determined in the context of our low-energy effective Lagrangian approach, it is not possible to actually perform this integral precisely, but this will not be necessary for our estimates. Because the most stringent lower bounds on partial lifetimes of nucleon decays are for two-body final states, these two-body modes will determine the distance constraints, and hence we will only need the two-body phase-space factor R_2 (see Appendix C). The rate for the decay $N \to f.s.$ is

$$\begin{aligned} \Gamma_{N\to f.s.} &= \frac{1}{2m_N}\int dR_2 |A_{N\to f.s.}|^2 \\ &= \frac{1}{2m_N}\frac{1}{(M_{\rm BNV})^4}\left(\frac{\mu}{\pi^{1/2}M_{\rm BNV}}\right)^{2n}\left|\sum_r \bar\kappa_r^{(Nd)} e^{-S_r^{(Nd)}}\langle f.s.|\mathcal{O}_r^{(Nd)}|N\rangle\right|^2 R_2, \end{aligned} \tag{3.36}$$

where an average over initial spin and sum over final spins is understood. As noted above, the dimensionless coefficients $\bar\kappa_r^{(Nd)}$ depend on the UV completion of the extra-dimensional theory and the associated BSM physics responsible for the baryon number violation and are not determined within the framework of our low-energy effective field theory. We take $\bar\kappa_r^{(Nd)} \simeq O(1)$ and note that it is straightforward to recalculate bounds on separation distances in the context of a specific UV completion with different values of the dimensionless coefficients $\bar\kappa_r^{(Nd)}$. Given these sources of uncertainty, we limit ourselves to correspondingly rough estimates of lower bounds on fermion separation distances. From the most stringent bound on a two-body proton decay to $\ell^+ +$ meson, namely $(\tau/B)_{p\to e^+\pi^0}$ in (3.34) and $(\tau/B)_{p\to \mu^+\pi^0}$ in (3.35), using estimates of the hadronic matrix elements from lattice calculations [51,52] (and setting $\bar\kappa_r^{(Nd)} = 1$ as above), we derive the approximate lower bound, applicable for both of these types of decays:

$$S_r > (S_r^{(Nd)})_{\rm min}, \tag{3.37}$$

where

$$\begin{aligned} (S_r^{(Nd)})_{\rm min} &= 48 - \frac{n}{2}\ln\pi - 2\ln\left(\frac{M_{\rm BNV}}{100 \text{ TeV}}\right) \\ &\quad - n\ln\left(\frac{M_{\rm BNV}}{\mu}\right). \end{aligned} \tag{3.38}$$

The most direct bound on fermion separation distances arises from the contribution of the operator $O_4^{(Nd)}$, since the integral $I_4^{(Nd)}$ involves a single fermion separation distance, $\|\eta_{Q_L} - \eta_{L_{\ell,L}}\|$ for a given lepton generation $\ell = e$ or $\ell = \mu$. In this case, from the inequality (3.37) with (3.38), we obtain the lower bound, for both e^+ and μ^+ decay modes,

$$\|\eta_{Q_L} - \eta_{L_{\ell,L}}\|^2 > 62 - \frac{8}{3}\ln\left(\frac{M_{\rm BNV}}{100\ {\rm TeV}}\right) - \frac{8}{3}\ln\left(\frac{M_{\rm BNV}}{\mu}\right). \tag{3.39}$$

In a model having $n = 2$ extra dimensions [and value $\mu = 3\times 10^3$ TeV, as given in (2.11)], with the illustrative value $M_{\rm BNV} = 100$ TeV, this is the inequality $\|\eta_{Q_L} - \eta_{L_{\ell,L}}\| > 8.4$, while for $M_{\rm BNV} = \mu$, this is the inequality $\|\eta_{Q_L} - \eta_{L_{\ell,L}}\| > 7.3$. Since $S_{\rm min}^{(Nd)}$ depends only logarithmically on the mass scale $M_{\rm BNV}$, it follows that the lower bounds on the fermion separation distances also depend only logarithmically on $M_{\rm BNV}$, i.e., only rather weakly on this scale. A very conservative solution to the coupled quadratic inequalities would require that each of the relevant distances $\|\eta_{f_i} - \eta_{f_j}\|$ in Eq. (3.32) for both $\ell = e$ and $\ell = \mu$ would be larger than the square root of the right-hand side of Eq. (3.38):

$$\{\|\eta_{u_R} - \eta_{\ell_R}\|, \|\eta_{d_R} - \eta_{\ell_R}\|, \|\eta_{Q_L} - \eta_{\ell_R}\|, \|\eta_{Q_L} - \eta_{L_{\ell,L}}\|, \|\eta_{L_{\ell,L}} - \eta_{u_R}\|, \|\eta_{L_{\ell,L}} - \eta_{d_R}\|\} > [(S_r^{(Nd)})_{\rm min}]^{1/2}. \tag{3.40}$$

That is, this set of inequalities is sufficient, but not necessary, to satisfy experimental constraints on the model from lower limits on partial lifetimes for nucleon decays.

With inclusion of electroweak-singlet $\nu_{s,R}$ fields with small enough masses so that they could occur in nucleon decays involving (anti)neutrinos, an analogous conservative choice would be to impose the same lower bounds as in Eq. (3.40):

$$\{\|\eta_{u_R} - \eta_{\nu_{s,R}}\|, \|\eta_{d_R} - \eta_{\nu_{s,R}}\|, \|\eta_{Q_L} - \eta_{\nu_{s,R}}\|\} > [S_{\rm min}^{(Nd)}]^{1/2} \tag{3.41}$$

for all s such that the $\nu_{s,R}$ can occur in nucleon decays. We will assume that these inequalities on fermion separation distances hold in the following. It is straightforward to use Eq. (3.29) to calculate lower bounds on fermion wave function separation distances with values of $M_{\rm BNV}$ different from the illustrative value used above.

The limits on two-body nucleon decays involving (anti)neutrino emission are somewhat less stringent than the limits on nucleon decays yielding charged leptons. For example, $(\tau/B)_{p\to\bar{\nu}\pi^+} > 3.9\times 10^{32}$ yr and $(\tau/B)_{n\to\bar{\nu}\pi^0} > 1.1\times 10^{33}$ yr [55]. Hence, they do not add extra information to the constraints that we have derived on fermion separation distances involving the $L_{\ell,L}$ and ℓ_R fermions with $\ell = e$ or $\ell = \mu$. However, since a nucleon is kinematically forbidden from decaying to a real final state containing a τ lepton, these experimental limits are useful for deriving constraints on separation distances involving the $L_{\tau,L}$ and τ_R fermions. The relevant operators that would contribute to such decays would be the $\mathcal{O}_r^{(Nd)}$ listed above that contain $L_{\tau,L}$ or τ_R. The BSM physics responsible for baryon number violation determines the magnitude of the corresponding coefficients $\kappa_r^{(Nd)}$. Since the quark fields in these four-fermion operators are all of the first generation, a usual expectation would be that the resultant coefficients for operators in which the lepton field is of the third generation would be smaller than if the lepton field is of the first or second generation. However, to be as conservative as possible, we consider the possibility of substantial coefficients for such four-fermion operators with a third-generation lepton field, namely ν_τ (see also [57]). Using the above-mentioned experimental lower bounds on (τ/B) for the $p\to\bar{\nu}\pi^+$ and $n\to\bar{\nu}\pi^0$ decays in conjunction with Eqs. (3.31) and (3.38), we obtain the bound $(S_r^{(Nd)})_{{\rm min},\tau^+} \simeq (S_r^{(Nd)})_{\rm min} - 2$, where $(S_r^{(Nd)})_{\rm min}$ refers to decay modes such as $p\to e^+\pi^0$ and $p\to\mu^+\pi^0$ and was given in Eq. (3.38). This can be satisfied conservatively with the inequality

$$\{\|\eta_{q_R} - \eta_{\tau_R}\|, \|\eta_{Q_L} - \eta_{\tau_R}\|, \|\eta_{q_R} - \eta_{L_{\tau,L}}\|, \|\eta_{Q_L} - \eta_{L_{\tau,L}}\|\} > [(S_r^{(Nd)})_{{\rm min},\tau^+}]^{1/2} \quad \text{for } q = u, d. \tag{3.42}$$

IV. $n-\bar{n}$ OSCILLATIONS AND DINUCLEON DECAYS TO HADRONIC FINAL STATES

In this section we review the striking finding in Ref. [25], that in this extra-dimensional model, even with nucleon decays suppressed well below experimental limits, $n-\bar{n}$ oscillations can occur near to their experimental limits. Thus, let us consider a general theory in which BSM physics leads to $n-\bar{n}$ transitions and let us denote the relevant low-energy effective Lagrangian in 4D as $\mathcal{L}_{\rm eff}^{(n\bar{n})}$ and the transition matrix element $|\delta m| = |\langle\bar{n}|\mathcal{L}_{\rm eff}^{(n\bar{n})}|n\rangle|$. In (field-free) vacuum, an initial state which is $|n\rangle$ at time $t = 0$ has a nonzero probability to be an $|\bar{n}\rangle$ state at a later time $t > 0$. This probability is given by $P(n(t) = \bar{n}) = |\langle\bar{n}|n(t)\rangle|^2 = [\sin^2(t/\tau_{n\bar{n}})]e^{-t/\tau_n}$, where τ_n is the mean life of the neutron. The current direct limit on $\tau_{n\bar{n}}$ is from an experiment with a neutron beam from a nuclear reactor at the Institut Laue-Langevin (ILL) in Grenoble: $\tau_{n\bar{n}} \geq 0.86\times 10^8$ sec, i.e., $|\delta m| = 1/\tau_{n\bar{n}} < 0.77\times 10^{-29}$ MeV [10].

As noted above, a nonzero $n-\bar{n}$ transition amplitude $\langle\bar{n}|\mathcal{L}_{\rm eff}|n\rangle$ has the consequence that the resultant physical eigenstate for the neutron state in matter has a small component of $\bar{n}$, i.e., $|n\rangle_{\rm phys} = \cos\theta_{n\bar{n}}|n\rangle + \sin\theta_{n\bar{n}}|\bar{n}\rangle$.

The nonzero $|\bar{n}\rangle$ component in $|n\rangle_{\rm phys}$ leads to annihilation with an adjacent neutron or proton, and hence to the decays to zero-baryon, multimeson final states, consisting dominantly of several pions: $nn \to$ pions and $np \to$ pions. A number of experiments have searched for the resultant matter instability due to these dinucleon decays and have set lower limits on the matter instability (m.i.) lifetime $\tau_{\rm m.i.}$ [12–16]. This is related to $\tau_{n\bar{n}}$ by the formula $\tau_{\rm m.i} = R\tau_{n\bar{n}}^2$, where $R \sim O(10^2)$ MeV, or, equivalently, $R \simeq 10^{23}$ sec^{-1}, depending on the nucleus. The best current limit on matter instability is from the SuperKamiokande (SK) water Cherenkov experiment [16],

$$\tau_{\rm m.i} > 1.9 \times 10^{32} \ {\rm yr}. \tag{4.1}$$

Using the value $R \simeq 0.52 \times 10^{23}$ sec^{-1} for the ^{16}O nuclei in water (see, e.g., [31] and references therein), the SK experiment gives the lower limit

$$\tau_{n\bar{n}} > 2.7 \times 10^8 \ \ {\rm sec}, \tag{4.2}$$

or, equivalently,

$$|\delta m| < 2.4 \times 10^{-30} \ {\rm MeV}. \tag{4.3}$$

This lower bound on $\tau_{n\bar{n}}$ in (4.2) from the SK experiment [16] is comparable to, and stronger by approximately a factor of 3 than, the direct lower bound on $\tau_{n\bar{n}}$ from the ILL experiment [10]. The SK experiment has also searched for specific dinucleon decays and has obtained the limits [17]

$$\Gamma^{-1}_{np\to\pi^+\pi^0} > 1.70 \times 10^{32} \ {\rm yr} \tag{4.4}$$

and

$$\Gamma^{-1}_{nn\to\pi^0\pi^0} > 4.04 \times 10^{32} \ {\rm yr}. \tag{4.5}$$

An improvement in the search for $n-\bar{n}$ oscillations is anticipated if a new $n-\bar{n}$ search with requisite sensitivity could be carried out at the European Spallation Source [31].

The effective Lagrangian (in four-dimensional spacetime) that mediates $n-\bar{n}$ oscillations is a sum of six-quark operators:

$$\mathcal{L}^{(n\bar{n})}_{\rm eff}(x) = \sum_{r=1}^{4} c_r^{(n\bar{n})}\mathcal{O}_r^{(n\bar{n})}(x) + {\rm H.c.} \tag{4.6}$$

As with Eqs. (3.1) and (3.2), there is a corresponding Lagrangian in the $(4+n)$-dimensional space:

$$\mathcal{L}^{(n\bar{n})}_{{\rm eff},4+n}(x,y) = \sum_{r}\kappa_r^{(n\bar{n})}O_r^{(n\bar{n})}(x,y) + {\rm H.c.} \tag{4.7}$$

Since the mass scale characterizing the $|\Delta B| = 2$ baryon number violation is large compared with the electroweak symmetry-breaking scale, these six-quark operators must be invariant under the Standard-Model gauge symmetry. As indicated in Eq. (4.6), there are four $\mathcal{O}_r^{(n\bar{n})}$ of this type, namely

$$\mathcal{O}_1^{(n\bar{n})} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}[u_R^{\alpha T}Cu_R^{\beta}][d_R^{\gamma T}Cd_R^{\delta}][d_R^{\rho T}Cd_R^{\sigma}], \tag{4.8}$$

$$\mathcal{O}_2^{(n\bar{n})} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}[u_R^{\alpha T}Cd_R^{\beta}][u_R^{\gamma T}Cd_R^{\delta}][d_R^{\rho T}Cd_R^{\sigma}], \tag{4.9}$$

$$\begin{aligned}\mathcal{O}_3^{(n\bar{n})} &= \epsilon_{ij}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T}CQ_L^{j\beta}][u_R^{\gamma T}Cd_R^{\delta}][d_R^{\rho T}Cd_R^{\sigma}] \\ &= 2(T_a)_{\alpha\beta\gamma\delta\rho\sigma}[u_L^{\alpha T}Cd_L^{\beta}][u_R^{\gamma T}Cd_R^{\delta}][d_R^{\rho T}Cd_R^{\sigma}],\end{aligned} \tag{4.10}$$

and

$$\begin{aligned}\mathcal{O}_4^{(n\bar{n})} &= \epsilon_{ij}\epsilon_{km}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T}CQ_L^{j\beta}][Q_L^{k\gamma T}CQ_L^{m\delta}][d_R^{\rho T}Cd_R^{\sigma}] \\ &= 4(T_a)_{\alpha\beta\gamma\delta\rho\sigma}[u_L^{\alpha T}Cd_L^{\beta}][u_L^{\gamma T}Cd_L^{\delta}][d_R^{\rho T}Cd_R^{\sigma}],\end{aligned} \tag{4.11}$$

where, as before, Greek indices $\alpha, \beta, \ldots$ are SU$(3)_c$ color indices; $i, j\ldots$ are weak SU$(2)_L$ indices; and the SU$(3)_c$ color tensors are

$$\begin{aligned}(T_s)_{\alpha\beta\gamma\delta\rho\sigma} &= \epsilon_{\rho\alpha\gamma}\epsilon_{\sigma\beta\delta} + \epsilon_{\sigma\alpha\gamma}\epsilon_{\rho\beta\delta} \\ &\quad + \epsilon_{\rho\beta\gamma}\epsilon_{\sigma\alpha\delta} + \epsilon_{\sigma\beta\gamma}\epsilon_{\rho\alpha\delta}\end{aligned} \tag{4.12}$$

and

$$(T_a)_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\beta}\epsilon_{\sigma\gamma\delta} + \epsilon_{\sigma\alpha\beta}\epsilon_{\rho\gamma\delta}. \tag{4.13}$$

(See Appendix B for the symmetry properties of these tensors.)

To each of these operators there is a corresponding $V_r^{(n\bar{n})}$ function, as defined by Eq. (2.22). For example,

$$\begin{aligned}V_1^{(n\bar{n})} &= V_2^{(n\bar{n})} \\ &= A^6\exp[-\{2\|\eta-\eta_{u_R}\|^2 + 4\|\eta-\eta_{d_R}\|^2\}],\end{aligned} \tag{4.14}$$

and so forth for the other two operators. The resultant integrals (2.23) over the extra n dimensions comprise three classes. The integration of the $V_r^{(n\bar{n})}$ functions for the operators $\mathcal{O}_r^{(n\bar{n})}$ with $r = 1, 2$ are the same, defining class $C_1^{(n\bar{n})}$:

$$I_{C_1}^{(n\bar{n})} = b_6\exp\left[-\frac{4}{3}\|\eta_{u_R}-\eta_{d_R}\|^2\right], \tag{4.15}$$

where $b_6 = (2\cdot 3^{-1/2}\pi^{-1}\mu^2)^n$ from the $k=6$ special case of Eq. (2.29) and $I_{C_k}^{(n\bar{n})} \equiv I_{C_k^{(n\bar{n})}}$. The operator $\mathcal{O}_3^{(n\bar{n})}$ yields a second class:

$$I_{C_2}^{(n\bar{n})} = b_6 \exp\left[-\frac{1}{6}\{2\|\eta_{Q_L} - \eta_{u_R}\|^2 + 6\|\eta_{Q_L} - \eta_{d_R}\|^2 + 3\|\eta_{u_R} - \eta_{d_R}\|^2\}\right]. \quad (4.16)$$

Finally, the operator $\mathcal{O}_4^{(n\bar{n})}$ yields the third class:

$$I_{C_3}^{(n\bar{n})} = b_6 \exp\left[-\frac{4}{3}\|\eta_{Q_L} - \eta_{d_R}\|^2\right]. \quad (4.17)$$

From the $k = 6$ special cases of Eqs. (2.23)–(2.30), it follows that

$$c_r^{(n\bar{n})} = \frac{\bar{\kappa}_r^{(n\bar{n})}}{(M_{\rm BNV})^5}\left(\frac{2\mu^2}{3^{1/2}\pi M_{\rm BNV}^2}\right)^n e^{-S_r^{(n\bar{n})}}, \quad (4.18)$$

where

$$S_r^{(n\bar{n})} = \frac{4}{3}\|\eta_{u_R} - \eta_{d_R}\|^2 \quad \text{for } r = 1, 2, \quad (4.19)$$

$$S_3^{(n\bar{n})} = \frac{1}{6}\{2\|\eta_{Q_L} - \eta_{u_R}\|^2 + 6\|\eta_{Q_L} - \eta_{d_R}\|^2 + 3\|\eta_{u_R} - \eta_{d_R}\|^2\}, \quad (4.20)$$

and

$$S_4^{(n\bar{n})} = \frac{4}{3}\|\eta_{Q_L} - \eta_{d_R}\|^2. \quad (4.21)$$

Then

$$|\delta m| = \frac{1}{(M_{\rm BNV})^5}\left(\frac{\mu}{M_{\rm BNV}}\right)^{2n}\left(\frac{2}{3^{1/2}\pi}\right)^n \times\left|\sum_r \bar{\kappa}_r^{(n\bar{n})} e^{-S_r^{(n\bar{n})}}\langle n|\mathcal{O}_r^{(n\bar{n})}|n\rangle\right|. \quad (4.22)$$

Reference [25] used, as a specific framework, a model with $n = 2$ and, in addition to the values of $\|\eta_{Q_L} - \eta_{u_R}\|$ and $\|\eta_{Q_L} - \eta_{d_R}\|$ from (2.17), also the value $\|\eta_{u_R} - \eta_{d_R}\| = 7$ from [27]. It was shown in [25] that, with this input, the contributions of the $\mathcal{O}_r^{(n\bar{n})}$ with $r = 1$, 2, 3 are small compared with the contribution of $\mathcal{O}_4^{(n\bar{n})}$. Hence, $|\delta m| = |c_4^{(n\bar{n})}\langle\bar{n}|\mathcal{O}_4^{(n\bar{n})}|n\rangle|$; i.e., only the $r = 4$ term in Eq. (4.22) is non-negligible. The sum $S_4^{(n\bar{n})}$ is fixed, via Eq. (2.17), by the d quark mass, so, for the given μ and an input value of $M_{\rm BNV}$ (and with $\kappa_4^{(n\bar{n})} \simeq 1$), the coefficient $c_4^{(n\bar{n})}$ is also fixed. The matrix elements $\langle\bar{n}|\mathcal{O}_r^{(n\bar{n})}|n\rangle$ have dimensions of $(\text{mass})^6$, and since they are determined by hadronic physics, one expects on general grounds that they are $\sim\Lambda_{\rm QCD}^6$, where, as above, $\Lambda_{\rm QCD} \simeq 0.25$ GeV. This is borne out by quantitative studies [8,9,58]. Requiring that $|\delta m|$ must be less than the experimental upper bound (4.3) yields a lower bound on $M_{\rm BNV}$ (denoted M_X in [25]). With the illustrative value $n = 2$, this is

$$M_{\rm BNV} > (44\ \text{TeV})\left(\frac{\tau_{n\bar{n}}}{2.7\times 10^8\ \text{sec}}\right)^{1/9}\left(\frac{\mu}{3\times 10^3\ \text{TeV}}\right)^{4/9} \times\left(\frac{|\langle\bar{n}|\mathcal{O}_4^{(n\bar{n})}|n\rangle|}{\Lambda_{\rm QCD}^6}\right)^{1/9}. \quad (4.23)$$

Thus, as pointed out in [25], for values of $M_{\rm BNV}$ in the range relevant to our extra-dimensional model, although nucleon decays could easily be suppressed well below experimental limits, $n - \bar{n}$ oscillations could occur at a level comparable to current limits.

Since the value of the separation distance $\|\eta_{u_R} - \eta_{d_R}\|$ is not determined by quark masses or mixing (since these arise from bilinear operator products of Q_L with u_R and d_R), it is of interest to inquire what range of values this distance can have, subject to the condition that $|\delta m|$ be smaller than the experimental upper limit (4.3). With the input value of μ given in Eq. (2.11) and for a value of $M_{\rm BNV} = 50$ TeV, we find the bound $\|\eta_{u_R} - \eta_{d_R}\| \gtrsim 4.6$. As noted in Sec. III, because constraints on fermion separation distances enter in the sums $S_r^{(n\bar{n})}$, the lower bounds on these distances depend only rather weakly (logarithmically) on $M_{\rm BNV}$.

V. $\Delta L = 0$ DINUCLEON DECAYS TO DILEPTONS

The same baryon-number-violating physics that leads to $n - \bar{n}$ oscillations and hence also to the dinucleon decays $nn \to$ pions and $np \to$ pions also leads to dinucleon decays to dilepton final states. These decays are of several different types, characterized by different ΔL values: $\Delta L = 0$, $\Delta L = -2$, and $\Delta L = 2$. The $\Delta L = 0$ dinucleon decays are on a different footing from the $\Delta L = \pm 2$ decays, because a $\Delta L = 0$ dinucleon decay can occur via a combination of a $\Delta B = -1$ $n - \bar{n}$ transition followed by Standard-Model processes, namely the annihilation of the $\bar{n}$ (i) with a neighboring n to produce, respectively, a virtual photon or Z which then creates a final-state $\ell^+\ell^-$ or $\nu_\ell\bar{\nu}_\ell$, or (ii) with a neighboring p to produce a virtual W^+, which then creates the final-state $\ell^+\nu_\ell$.

In [43] we calculated rough lower bounds on the partial lifetimes for the above $\Delta L = 0$ dinucleon-to-dilepton decays by relating their rates to the rates for the decays $nn \to \pi^0\pi^0$, $nn \to \pi^+\pi^-$, and $np \to \pi^+\pi^0$ and using experimental lower bounds on the partial lifetimes of the latter dinucleon decays. Our study in [43] was a general phenomenological analysis and did not assume a particular BSM theory such as the extra-dimension model used in the present work. We obtained the estimated lower bounds

$$(\tau/B)_{nn\to\ell^+\ell^-} \gtrsim 5\times10^{34}\ \text{yr} \quad \text{for } \ell = e,\mu, \tag{5.1}$$

$$(\tau/B)_{nn\to\nu_\ell\bar\nu_\ell} \gtrsim 2\times10^{41}\ \text{yr} \quad \text{for } \nu_\ell = \nu_e,\nu_\mu,\nu_\tau, \tag{5.2}$$

$$(\tau/B)_{np\to\ell^+\nu_\ell} \gtrsim 10^{41}\ \text{yr} \quad \text{for } \ell = e,\mu, \tag{5.3}$$

and

$$(\tau/B)_{np\to\tau^+\nu_\tau} \gtrsim 10^{42}\ \text{yr}. \tag{5.4}$$

These bounds are considerably stronger than the corresponding experimental bounds from searches for these decays. Experiments use the notational convention of referring to their limits as limits on (τ/B) for $nn\to\pi^0\pi^0$, $n\to\pi^+\pi^-$, and $np\to\pi^+\pi^0$ although their limits actually apply to the nuclei in their detectors. We follow this convention here. These experimental bounds are as follows: $(\tau/B)_{nn\to e^+e^-} > 4.2\times10^{33}$ yr and $(\tau/B)_{nn\to\mu^+\mu^-} > 4.4\times10^{33}$ yr from SK [23] (per ^{16}O nucleus in the water); $(\tau/B)_{nn\to\text{inv}} > 1.4\times10^{30}$ yr from KamLAND [14,59] (per ^{12}C nucleus in the liquid scintillator), and $(\tau/B)_{np\to e^+x} > 2.6\times10^{32}$ yr, $(\tau/B)_{np\to\mu^+x} > 2.2\times10^{32}$ yr [21] (per ^{16}O nucleus), where x denotes a neutrino or antineutrino. Reference [19] used data from searches for dinucleon decays into multilepton final states involving e^+ and μ^+ plus (anti)neutrinos to obtain the bound $(\tau/B)_{np\to\tau^+\bar\nu_\tau} > 1\times10^{30}$ yr. A dedicated search by the SK experiment yielded the bound [21] $(\tau/B)_{np\to\tau^+x} > 2.9\times10^{31}$ yr, where, as above, x is a neutral, weakly interacting fermion, assumed to have a negligibly small mass. This subsumes the cases in which x is an electroweak-doublet neutrino or antineutrino of some undetermined flavor, or possibly an electroweak-singlet (sterile) neutrino.

VI. $\Delta L = -3$ NUCLEON DECAYS TO TRILEPTONS

In this section we consider the $\Delta L = -3$ nucleon decays to trileptons (1.1) and (1.2). We use the constraints on distances derived in Sec. III to obtain generic expectations for lower bounds on partial lifetimes for these decays in the extra-dimensional model. Operators that contribute to the decays (1.1) and (1.2) are six-fermion operators. In terms of fermion fields, the operators that we discuss comprise eight classes, which are listed in Table I. We denote these with a superscript $(pm3)$, $(nm3)$, or $(pm3,nm3)$, corresponding to the decays (1.1) and (1.2) to which the operator contributes, where $pm3$ stands for "proton decay to tripleptons, with ΔL equal to minus 3" and similarly for $nm3$. We list these operators below (with $\ell = e$ or μ), together with the class to which they belong:

TABLE I. Structures of classes $C_k^{(Nm3)}$ of operators contributing to $\Delta L = -3$ nucleon decays to trileptons. The first column lists the class number; the second column lists the number N_d of $SU(2)_L$ doublets in the operators in this class; and the third column lists the structure of operators in the class. As in the text, we use the abbreviations $pm3$ for $p\to\ell^+\bar\nu\bar\nu'$ and $nm3$ for $n\to\bar\nu\bar\nu'\bar\nu''$. The abbreviations used for the fermion fields are $Q=Q_L$, $L=L_L$, $u=u_R$, $d=d_R$, $\ell=\ell_R$, and $\nu=\nu_{s,R}$. The primes distinguishing different ν fields are suppressed in the notation.

Class $C_k^{(Nm3)}$	N_d	Structure
$C_1^{(pm3)}$	0	$u^2d\ell\nu^2$
$C_2^{(nm3)}$	0	$ud^2\nu^3$
$C_3^{(pm3)}$	2	$Q^2u\ell\nu^2$
$C_4^{(nm3)}$	2	$Q^2d\nu^3$
$C_5^{(pm3,nm3)}$	2	$QLud\nu^2$
$C_6^{(pm3,nm3)}$	2	$QLu^2\ell\nu$
$C_7^{(pm3,nm3)}$	4	$Q^3L\nu^2$
$C_8^{(pm3,nm3)}$	4	$Q^2L^2u\nu$

$$\mathcal{O}_1^{(pm3)} = \epsilon_{\alpha\beta\gamma}[u_R^{\alpha T}Cd_R^\beta][u_R^{\gamma T}C\ell_R][\nu_{s,R}^TC\nu_{s',R}] \in C_1^{(pm3)}, \tag{6.1}$$

$$\mathcal{O}_2^{(pm3)} = \epsilon_{\alpha\beta\gamma}[u_R^{\alpha T}Cd_R^\beta][u_R^{\gamma T}C\nu_{s,R}][\ell_R^TC\nu_{s',R}] \in C_1^{(pm3)}, \tag{6.2}$$

$$\mathcal{O}_3^{(nm3)} = \epsilon_{\alpha\beta\gamma}[u_R^{\alpha T}Cd_R^\beta][d_R^{\gamma T}C\nu_{s,R}][\nu_{s',R}^TC\nu_{s'',R}] \in C_2^{(nm3)}, \tag{6.3}$$

$$\begin{aligned}\mathcal{O}_4^{(pm3)} &= \epsilon_{ij}\epsilon_{\alpha\beta\gamma}[Q_L^{i\alpha T}CQ_L^{j\beta}][u_R^{\gamma T}C\ell_R][\nu_{s,R}^TC\nu_{s',R}]\\ &= 2\epsilon_{\alpha\beta\gamma}[u_L^{\alpha T}Cd_L^\beta][u_R^{\gamma T}C\ell_R][\nu_{s,R}^TC\nu_{s',R}] \in C_3^{(pm3)},\end{aligned} \tag{6.4}$$

$$\begin{aligned}\mathcal{O}_5^{(pm3)} &= \epsilon_{ij}\epsilon_{\alpha\beta\gamma}[Q_L^{i\alpha T}CQ_L^{j\beta}][u_R^{\gamma T}C\nu_{s,R}][\ell_R^TC\nu_{s',R}]\\ &= 2\epsilon_{\alpha\beta\gamma}[u_L^{\alpha T}Cd_L^\beta][u_R^{\gamma T}C\nu_{s,R}][\ell_R^TC\nu_{s',R}] \in C_3^{(pm3)},\end{aligned} \tag{6.5}$$

$$\begin{aligned}\mathcal{O}_6^{(nm3)} &= \epsilon_{ij}\epsilon_{\alpha\beta\gamma}[Q_L^{i\alpha T}CQ_L^{j\beta}][d_R^{\gamma T}C\nu_{s,R}][\nu_{s',R}^TC\nu_{s'',R}]\\ &= 2\epsilon_{\alpha\beta\gamma}[u_L^{\alpha T}Cd_L^\beta][d_R^{\gamma T}C\nu_{s,R}][\nu_{s',R}^TC\nu_{s'',R}] \in C_4^{(nm3)},\end{aligned} \tag{6.6}$$

$$\begin{aligned}\mathcal{O}_7^{(pm3,nm3)} &= \epsilon_{ij}\epsilon_{\alpha\beta\gamma}[Q_L^{i\alpha T}CL_{\ell,L}^j][u_R^{\beta T}Cd_R^\gamma][\nu_{s,R}^TC\nu_{s',R}]\\ &= \epsilon_{\alpha\beta\gamma}([u_L^{\alpha T}C\ell_L]-[d_L^{\alpha T}C\nu_{\ell,L}])[u_R^{\beta T}Cd_R^\gamma]\\ &\quad\times[\nu_{s,R}^TC\nu_{s',R}] \in C_5^{(pm3,nm3)},\end{aligned} \tag{6.7}$$

$$\mathcal{O}_8^{(pm3,nm3)} = \epsilon_{ij}\epsilon_{\alpha\beta\gamma}[Q_L^{i\alpha T}CL_{\ell,L}^{j}][u_R^{\beta T}C\nu_{s,R}][d^{\gamma T}C\nu_{s',R}]$$
$$= \epsilon_{\alpha\beta\gamma}([u_L^{\alpha T}C\ell_L] - [d_L^{\alpha T}C\nu_{\ell,L}])[u_R^{\beta T}C\nu_{s,R}] \times [d_R^{\gamma T}C\nu_{s',R}] \in C_5^{(pm3,nm3)}, \quad (6.8)$$

$$\mathcal{O}_9^{(pm3,nm3)} = \epsilon_{ij}\epsilon_{\alpha\beta\gamma}[Q_L^{i\alpha T}CL_{\ell',L}^{j}][u_R^{\beta T}C\ell_R][u_R^{\gamma T}C\nu_{s,R}]$$
$$= \epsilon_{\alpha\beta\gamma}([u_L^{\alpha T}C\ell'_L] - [d_L^{\alpha T}C\nu_{\ell',L}])[u_R^{\beta T}C\ell_R] \times [u_R^{\gamma T}C\nu_{s,R}] \in C_6^{(pm3,nm3)}, \quad (6.9)$$

$$\mathcal{O}_{10}^{(pm3,nm3)} = \epsilon_{ij}\epsilon_{km}\epsilon_{\alpha\beta\gamma}[Q_L^{i\alpha T}CQ_L^{j\beta}][Q_L^{k\gamma T}CL_{\ell,L}^{m}][\nu_{s,R}^TC\nu_{s',R}]$$
$$= 2\epsilon_{\alpha\beta\gamma}[u_L^{\alpha T}Cd_L^{\beta}]([u_L^{\gamma T}C\ell_L] - [d_L^{\gamma T}C\nu_{\ell,L}]) \times [\nu_{s,R}^TC\nu_{s',R}] \in C_7^{(pm3,nm3)}, \quad (6.10)$$

and

$$\mathcal{O}_{11}^{(pm3,nm3)} = \epsilon_{ij}\epsilon_{km}\epsilon_{\alpha\beta\gamma}[Q_L^{i\alpha T}CL_{\ell,L}^{j}][Q_L^{k\beta T}CL_{\ell',L}^{m}][u_R^{\gamma T}C\nu_{s,R}]$$
$$= \epsilon_{\alpha\beta\gamma}([u_L^{\alpha T}C\ell_L] - [d_L^{\alpha T}C\nu_{\ell,L}]) \times ([u_L^{\beta T}C\ell'_L] - [d_L^{\beta T}C\nu_{\ell',L}])[u_R^{\gamma T}C\nu_{s,R}] \in C_8^{(pm3,nm3)}. \quad (6.11)$$

The contributions of the operators are determined by the integrals over the n extra dimensions, which, in turn, only depend on the class to which a given operator belongs. A general remark relevant for these operators and also operators for other BNV processes is the following: in enumerating relevant operators contributing to some process, it is sometimes of interest to demonstrate that they are all linearly independent. However, for our present purposes, this is not necessary, since our actual analysis is based on the classes of operators and their resultant integrals, and these classes are manifestly independent of each other, since they are comprised of different fermion fields. This remark is also relevant for relations involving other operators with different Dirac structure.

Using our general formula (8.18), we calculate the integrals for these classes. With the notation $I_{C_1}^{(pm3)} \equiv I_{C_1^{(pm3)}}$, we have

$$I_{C_1}^{(pm3)} = b_6 \exp\Big[-\frac{1}{6}\{2\|\eta_{u_R}-\eta_{d_R}\|^2 + 2\|\eta_{u_R}-\eta_{\ell_R}\|^2 + 2\|\eta_{u_R}-\eta_{\nu_{s,R}}\|^2 + 2\|\eta_{u_R}-\eta_{\nu_{s',R}}\|^2 + \|\eta_{d_R}-\eta_{\ell_R}\|^2 + \|\eta_{d_R}-\eta_{\nu_{s,R}}\|^2 + \|\eta_{d_R}-\eta_{\nu_{s',R}}\|^2 + \|\eta_{\ell_R}-\eta_{\nu_{s,R}}\|^2 + \|\eta_{\ell_R}-\eta_{\nu_{s',R}}\|^2 + \|\eta_{\nu_{s,R}}-\eta_{\nu_{s',R}}\|^2\}\Big], \quad (6.12)$$

$$I_{C_2}^{(nm3)} = b_6 \exp\Big[-\frac{1}{6}\{2\|\eta_{u_R}-\eta_{d_R}\|^2 + \|\eta_{u_R}-\eta_{\nu_{s,R}}\|^2 + \|\eta_{u_R}-\eta_{\nu_{s',R}}\|^2 + \|\eta_{u_R}-\eta_{\nu_{s'',R}}\|^2 + 2\|\eta_{d_R}-\eta_{\nu_{s,R}}\|^2 + 2\|\eta_{d_R}-\eta_{\nu_{s',R}}\|^2 + 2\|\eta_{d_R}-\eta_{\nu_{s'',R}}\|^2 + \|\eta_{\nu_{s,R}}-\eta_{\nu_{s',R}}\|^2 + \|\eta_{\nu_{s,R}}-\eta_{\nu_{s'',R}}\|^2 + \|\eta_{\nu_{s',R}}-\eta_{\nu_{s'',R}}\|^2\}\Big], \quad (6.13)$$

$$I_{C_3}^{(pm3)} = b_6 \exp\Big[-\frac{1}{6}\{2\|\eta_{Q_L}-\eta_{u_R}\|^2 + 2\|\eta_{Q_L}-\eta_{\ell_R}\|^2 + 2\|\eta_{Q_L}-\eta_{\nu_{s,R}}\|^2 + 2\|\eta_{Q_L}-\eta_{\nu_{s',R}}\|^2 + \|\eta_{u_R}-\eta_{\ell_R}\|^2 + \|\eta_{u_R}-\eta_{\nu_{s,R}}\|^2 + \|\eta_{u_R}-\eta_{\nu_{s',R}}\|^2 + \|\eta_{\ell_R}-\eta_{\nu_{s,R}}\|^2 + \|\eta_{\ell_R}-\eta_{\nu_{s',R}}\|^2 + \|\eta_{\nu_{s,R}}-\eta_{\nu_{s',R}}\|^2\}\Big], \quad (6.14)$$

$$I_{C_4}^{(nm3)} = b_6 \exp\Big[-\frac{1}{6}\{2\|\eta_{Q_L}-\eta_{d_R}\|^2 + 2\|\eta_{Q_L}-\eta_{\nu_{s,R}}\|^2 + 2\|\eta_{Q_L}-\eta_{\nu_{s',R}}\|^2 + 2\|\eta_{Q_L}-\eta_{\nu_{s'',R}}\|^2 + \|\eta_{d_R}-\eta_{\nu_{s,R}}\|^2 + \|\eta_{d_R}-\eta_{\nu_{s',R}}\|^2 + \|\eta_{d_R}-\eta_{\nu_{s'',R}}\|^2 + \|\eta_{\nu_{s,R}}-\eta_{\nu_{s',R}}\|^2 + \|\eta_{\nu_{s,R}}-\eta_{\nu_{s'',R}}\|^2 + \|\eta_{\nu_{s',R}}-\eta_{\nu_{s'',R}}\|^2\}\Big], \quad (6.15)$$

$$I_{C_5}^{(pm3,nm3)} = b_6 \exp\Big[-\frac{1}{6}\{\|\eta_{Q_L}-\eta_{L_{\ell,L}}\|^2 + \|\eta_{Q_L}-\eta_{u_R}\|^2 + \|\eta_{Q_L}-\eta_{d_R}\|^2 + \|\eta_{Q_L}-\eta_{\nu_{s,R}}\|^2 + \|\eta_{Q_L}-\eta_{\nu_{s',R}}\|^2 + \|\eta_{L_{\ell,L}}-\eta_{u_R}\|^2 + \|\eta_{L_{\ell,L}}-\eta_{d_R}\|^2 + \|\eta_{L_{\ell,L}}-\eta_{\nu_{s,R}}\|^2 + \|\eta_{L_{\ell,L}}-\eta_{\nu_{s',R}}\|^2 + \|\eta_{u_R}-\eta_{d_R}\|^2 + \|\eta_{u_R}-\eta_{\nu_{s,R}}\|^2 + \|\eta_{u_R}-\eta_{\nu_{s',R}}\|^2 + \|\eta_{d_R}-\eta_{\nu_{s,R}}\|^2 + \|\eta_{d_R}-\eta_{\nu_{s',R}}\|^2 + \|\eta_{\nu_{s,R}}-\eta_{\nu_{s',R}}\|^2\}\Big], \quad (6.16)$$

$$I_{C_6}^{(pm3,nm3)} = b_6 \exp\Big[-\frac{1}{6}\{\|\eta_{Q_L}-\eta_{L_{\ell,L}}\|^2 + 2\|\eta_{Q_L}-\eta_{u_R}\|^2 + \|\eta_{Q_L}-\eta_{\ell_R}\|^2 + \|\eta_{Q_L}-\eta_{\nu_{s,R}}\|^2 + 2\|\eta_{L_{\ell,R}}-\eta_{u_R}\|^2 + \|\eta_{L_{\ell,L}}-\eta_{\ell_R}\|^2 + \|\eta_{L_{\ell,L}}-\eta_{\nu_{s,R}}\|^2 + 2\|\eta_{u_R}-\eta_{\ell_R}\|^2 + 2\|\eta_{u_R}-\eta_{\nu_{s,R}}\|^2 + \|\eta_{\ell_R}-\eta_{\nu_{s,R}}\|^2\}\Big], \tag{6.17}$$

$$I_{C_7}^{(pm3,nm3)} = b_6 \exp\Big[-\frac{1}{6}\{3\|\eta_{Q_L}-\eta_{L_{\ell,L}}\|^2 + 3\|\eta_{Q_L}-\eta_{\nu_{s,R}}\|^2 + 3\|\eta_{Q_L}-\eta_{\nu_{s',R}}\|^2 + \|\eta_{L_{\ell,L}}-\eta_{\nu_{s,R}}\|^2 + \|\eta_{L_{\ell,L}}-\eta_{\nu_{s',R}}\|^2 + \|\eta_{\nu_{s,R}}-\eta_{\nu_{s',R}}\|^2\}\Big], \tag{6.18}$$

and

$$I_{C_8}^{(pm3,nm3)} = b_6 \exp\Big[-\frac{1}{6}\{4\|\eta_{Q_L}-\eta_{L_{\ell,L}}\|^2 + 2\|\eta_{Q_L}-\eta_{u_R}\|^2 + 2\|\eta_{Q_L}-\eta_{\nu_{s,R}}\|^2 + 2\|\eta_{L_{\ell,L}}-\eta_{u_R}\|^2 + 2\|\eta_{L_{\ell,L}}-\eta_{\nu_{s,R}}\|^2 + \|\eta_{u_R}-\eta_{\nu_{s,R}}\|^2\}\Big]. \tag{6.19}$$

Using these calculations and typical values of fermion separation distances obeying the constraints from nucleon decays discussed in Sec. III, we find that these $\Delta L = -3$ nucleon decays are strongly suppressed relative to nucleon decays mediated by four-fermion operators. Making reference to the comparison of rates in Eq. (2.32) and the illustrative numerical example in Eq. (2.34), we find that the difference $\langle S_{(6)}\rangle - \langle S_{(4)}\rangle$ is positive, adding to the suppression from the prefactor. The basic reason that the $\Delta L = -3$ decays to trilepton final states are strongly suppressed in this model, while $n-\bar{n}$ oscillations can occur at levels comparable to current limits, is BNV nucleon decays can be suppressed by making the separation between quark and lepton wave function centers sufficiently large. This does not suppress $n-\bar{n}$ oscillations but considerably suppresses these $\Delta L = -3$ decays, since they involve outgoing (anti)leptons. This reason also explains the suppression that we will find for the various types of BNV nucleon and dinucleon decays in the following sections.

Thus, we find that the resultant expected predictions for partial lifetimes for these $\Delta L = -3$ nucleon decays are compatible with existing experimental limits. These limits include $(\tau/B)_{p\to e^+xx} > 0.58\times 10^{30}$ yr [20], $(\tau/B)_{p\to\mu^+xx} > 0.58\times 10^{30}$ yr [20], and $(\tau/B)_{n\to xxx} > 0.58\times 10^{30}$ yr [22], where here x denotes an unobserved neutral, weakly interacting fermion with negligibly small mass that does not decay in the detector. Thus, for example, the lower limit on (τ/B) for the decay $p\to\ell^+xx$ applies to all of the decays $p\to\ell^+\bar{\nu}\bar{\nu}'$ (with $\Delta L = -3$), $p\to\ell^+\nu\bar{\nu}'$ (with $\Delta L = -1$), and $p\to\ell^+\nu\nu'$ (with $\Delta L = 1$) for $\ell^+ = e^+$ or μ^+, and similarly, the lower bound on $n\to xxx$ applies to all neutron decays to combinations of (anti)neutrinos with ΔL ranging from $\Delta L = -3$ to $\Delta L = +3$. Further searches for these and other types of nucleon decays are worthwhile (e.g., [60–62]). In addition to continued data taking at SuperKamiokande, future searches for nucleon decays are planned at HyperKamiokande [63] and in the liquid argon detector in DUNE (Deep Underground Neutrino Experiment) [64].

VII. $\Delta L = 1$ NUCLEON DECAYS TO TRILEPTONS

Here we study the $\Delta L = 1$ nucleon decays to trilepton final states (1.3) and (1.4). These decays are mediated by six-fermion operators, as was the case with the $\Delta L = -3$ nucleon decays to trilepton final states analyzed in Sec. VI. Our procedure for analyzing these decays is analogous to the procedure we used in Sec. VI. Indeed, there is a one-to-one correspondence between the operators here and a subset of the operators in that section, namely $\mathcal{O}_r^{(Nm3)}$ with $r = 1$, 3, 4, 6, 7, obtained by the replacement of an EW-singlet neutrino bilinear by one with each $\nu_{s,R}$ field replaced by $(\nu_{s,R})^c \equiv (\nu^c)_{s,L}$ (the charge conjugation reverses the chirality), i.e., by replacing $[\nu_{s,R}^T C\nu_{s',R}]$ by $[\nu_{s,L}^{c\,T} C\nu_{s',L}^c]$. We denote these with a superscript $(p1)$, $(n1)$, or $(p1, n1)$, corresponding to the decays (1.3) and (1.4) to which the operator contributes, where $p1$ stands for "proton decay to tripleptons, with ΔL equal to 1" and similarly for $n1$. The charge conjugation leaves the position of the fermion unchanged, so $\eta_{\nu_s,R} = \eta_{\nu_{s,L}^c}$. Consequently, the five classes to which the operators for the $\Delta L = 1$ nucleon decays to trileptons belong are in one-to-one correspondence with five of the seven classes to which the operators for the $\Delta L = -3$ nucleon decays to trileptons belong, and the corresponding integrals are equal:

$$C_1^{(p3)} \leftrightarrow C_1^{(p1)}, \qquad C_2^{(n3)} \leftrightarrow C_2^{(n1)}, \qquad C_3^{(p3)} \leftrightarrow C_3^{(p1)},$$
$$C_4^{(n3)} \leftrightarrow C_4^{(n1)}, \qquad C_5^{(p3,n3)} \leftrightarrow C_1^{(p1,n1)}, \qquad (7.1)$$

where here the symbol $\leftrightarrow$ means replacement of a $\nu\nu$ bilinear by a $\nu^c\nu^c$ bilinear. The integrals satisfy the equalities

$$I_{C_1}^{(p1)} = I_{C_1}^{(pm3)}, \qquad I_{C_2}^{(n1)} = I_{C_2}^{(nm3)}, \qquad I_{C_3}^{(p1)} = I_{C_3}^{(pm3)},$$
$$I_{C_4}^{(n1)} = I_{C_4}^{(nm3)}, \qquad I_{C_5}^{(p1,n1)} = I_{C_5}^{(pm3,nm3)}. \qquad (7.2)$$

Operators mediating these $\Delta L = 1$ dinucleon decays to trileptons are

$$\mathcal{O}_1^{(p1)} = \epsilon_{\alpha\beta\gamma}[u_R^{\alpha T}Cd_R^\beta][u_R^{\gamma T}C\ell_R][\nu_{s,L}^{cT}C\nu_{s',R}^c] \in C_1^{(p1)}, \quad (7.3)$$

$$\mathcal{O}_2^{(n1)} = \epsilon_{\alpha\beta\gamma}[u_R^{\alpha T}Cd_R^\beta][d_R^{\gamma T}C\nu_{s,R}][\nu_{s',L}^{cT}C\nu_{s'',L}^c] \in C_2^{(n1)}, \quad (7.4)$$

$$\mathcal{O}_3^{(p1)} = \epsilon_{ij}\epsilon_{\alpha\beta\gamma}[Q_L^{i\alpha T}CQ_L^{j\beta}][u_R^{\gamma T}C\ell_R][\nu_{s,L}^{cT}C\nu_{s',L}^c]$$
$$= 2\epsilon_{\alpha\beta\gamma}[u_L^{\alpha T}Cd_L^\beta][u_R^{\gamma T}C\ell_R][\nu_{s,L}^{cT}C\nu_{s',L}^c] \in C_3^{(p1)}, \quad (7.5)$$

$$\mathcal{O}_4^{(n1)} = \epsilon_{ij}\epsilon_{\alpha\beta\gamma}[Q_L^{i\alpha T}CQ_L^{j\beta}][d_R^{\gamma T}C\nu_{s,R}][\nu_{s',L}^{cT}C\nu_{s'',L}^c]$$
$$= 2\epsilon_{\alpha\beta\gamma}[u_L^{\alpha T}Cd_L^\beta][d_R^{\gamma T}C\nu_{s,R}][\nu_{s',L}^{cT}C\nu_{s'',L}^c] \in C_4^{(n1)}, \quad (7.6)$$

and

$$\mathcal{O}_5^{(p1,n1)} = \epsilon_{ij}\epsilon_{\alpha\beta\gamma}[Q_L^{i\alpha T}CL_{\ell,L}^j][u_R^{\beta T}Cd_R^\gamma][\nu_{s,L}^{cT}C\nu_{s',L}^c]$$
$$= \epsilon_{\alpha\beta\gamma}([u_L^{\alpha T}C\ell_L] - [d_L^{\alpha T}C\nu_{\ell,L}])[u_R^{\beta T}Cd_R^\gamma][\nu_{s,L}^{cT}C\nu_{s',L}^c]$$
$$\in C_5^{(p1,n1)}. \quad (7.7)$$

We summarize these classes in Table II. Owing to the equalities (7.2), our conclusions concerning upper bounds on the rates for these $\Delta L = 1$ nucleon decays to trilepton final states are the same as for the $\Delta L = -3$ nucleon decays to trileptons.

TABLE II. Structures of classes $C_k^{(N1)}$ of operators contributing to $\Delta L = 1$ nucleon decays to trileptons. The first column lists the class number; the second column lists the number N_d of $SU(2)_L$ doublets in the operators in this class; and the third column lists the structure of operators in the class. As in the text, we use the abbreviations $p1$ for $p \to \ell^+\nu\nu'$ and $n1$ for $n \to \bar{\nu}\nu'\nu''$. The abbreviations for fermion fields are the same as in Table I. The primes distinguishing different ν fields are suppressed in the notation.

Class $C_k^{(N1)}$	N_d	Structure
$C_1^{(p1)}$	0	$u^2 d\ell\bar{\nu}^2$
$C_2^{(n1)}$	0	$ud^2\nu\bar{\nu}^2$
$C_3^{(p1)}$	2	$Q^2u\ell\bar{\nu}^2$
$C_4^{(n1)}$	2	$Q^2d\nu\bar{\nu}^2$
$C_5^{(p1,n1)}$	2	$QLud\bar{\nu}^2$

VIII. $\Delta L = -2$ DINUCLEON DECAYS TO DILEPTONS: GENERAL OPERATOR ANALYSIS

In this section we carry out a general operator analysis of the $\Delta L = -2$ dinucleon decays to dileptons (1.5)–(1.8). In later sections, we shall use our results to obtain approximate estimates of expected rates for these decays in the extra-dimensional model. As is obvious from the selection rule $\Delta L = -2$ for these decays, they arise differently from the $\Delta B = -2$, $\Delta L = 0$ dinucleon-to-dilepton decays for which we set bounds in [43]. The process by which the $\Delta B = -2$, $\Delta L = 0$ dinucleon-to-dilepton decays occur involves a local six-fermion operator that mediates the $n - \bar{n}$ transition, in conjunction with $n\bar{n}$ annihilation leading to a virtual γ, Z, or $\bar{n}p$ annihilation leading to a virtual W^+. The virtual γ, Z, or W^+ then produce the final-state lepton-antilepton pairs, namely $\ell^+\ell^-$, $\nu_\ell\bar{\nu}_\ell$, and $\ell^+\nu_\ell$, respectively. Although the amplitudes involve eight external fermion lines, the lepton-antilepton operator product is bilocal with respect to the six-quark operator product (separated by a Euclidean distance $\sim 1/\mathrm{fm}$ for the γ, $\sim 1/m_Z$ and $\sim 1/m_W$ for the processes with a virtual Z and W^+, respectively; i.e., these $\Delta B = -2$, $\Delta L = 0$ amplitudes do not dominantly involve local eight-fermion operator products.

Proceeding with our analysis, we first discuss the general structure of an effective Lagrangian for the $\Delta B = -2$, $\Delta L = -2$ dinucleon-to-dilepton decays. For labeling purposes, we shall introduce the superscript NN', which takes on the respective values $(NN') = (pp)$ for $pp \to \ell^+\ell'^+$ decays, $(NN') = (np)$ for $np \to \ell^+\bar{\nu}$ decays, and $(NN') = (nn)$ for $nn \to \bar{\nu}\bar{\nu}'$ decays, with the dilepton final state kept implicit in the notation. This effective Lagrangian has the form

$$\mathcal{L}_{\mathrm{eff}}^{(NN')}(x) = \sum_r c_r^{(NN')}\mathcal{O}_r^{(NN')}(x) + \mathrm{H.c.}$$
$$= \sum_r \kappa_r^{(NN')}\int d^n y O_r^{(NN')}(x,y) + \mathrm{H.c.}$$
$$= \sum_r \kappa_r^{(NN')}U_r^{(NN')}(x)\int d^n y V_r^{(NN')}(y) + \mathrm{H.c.}$$
$$= \sum_r \kappa_r^{(NN')}I_r^{(NN')}U_r^{(NN')}(x) + \mathrm{H.c.}, \qquad (8.1)$$

where, in accord with the general notation (2.23),

$$I_r^{(NN')} = \int d^n y V_r^{(NN')}(y). \qquad (8.2)$$

Various sets of operators $O_r^{(NN)}$ yield the same integrals $I_r^{(NN)}$, so they can be organized into certain classes, as we will discuss below.

By the same logic as for the four-fermion operators contributing to individual nucleon decays and the six-quark operators contributing to $n-\bar{n}$ oscillations and dinucleon decays to mesonic final states, since existing limits imply that the mass scale characterizing the physics responsible for these dinucleon-to-dilepton decays must be large compared with the electroweak symmetry-breaking scale v, it follows that the eight-fermion operators $O_r^{(NN')}(x,y)$ must be singlets under the Standard-Model gauge group $G_{\rm SM}$.

Six of the eight fermions in these operators are quark fields. The color indices of the six quark fields, denoted as $\alpha, \beta, \gamma, \delta, \rho$, and σ, are coupled together to make an $\mathrm{SU}(3)_c$ singlet. This can be done in any of three ways, corresponding to the color tensors $(T_s)_{\alpha\beta\gamma\delta\rho\sigma}$ in Eq. (4.12), $(T_a)_{\alpha\beta\gamma\delta\rho\sigma}$ in Eq. (4.13), and

$$(T_{a3})_{\alpha\beta\gamma\delta\rho\sigma} = \epsilon_{\rho\alpha\beta}\epsilon_{\sigma\gamma\delta} - \epsilon_{\sigma\alpha\beta}\epsilon_{\rho\gamma\delta}. \tag{8.3}$$

Some properties of these tensors are reviewed in Appendix B. As discussed in [9], there are also color tensors related to these by redefinition of indices, such as $T_{a2'(saa)}$ and $T_{a2'(asa)}$ in Eqs. (3.4) and (3.5) of [9], but these will not be needed here.

The eight-fermion operators can be classified according to how many of the eight fermions are $\mathrm{SU}(2)_L$ nonsinglets; the possibilities are 0, 2, 4, 6, and 8. For operators containing a nonzero number (2, 4, 6, or 8) fermions in $\mathrm{SU}(2)_L$ nonsinglets, there are various ways to contract the $\mathrm{SU}(2)_L$ weak isospin indices. One way is to contract each pair of weak isospin-1/2 indices antisymmetrically to make singlets, using the ϵ_{ij} tensor for two $\mathrm{SU}(2)_L$ indices, and so forth for other $\mathrm{SU}(2)_L$ indices. Alternatively, one can combine pairs of weak isospin-1/2 fields symmetrically to make adjoint (i.e., weak isospin 1) representations of $\mathrm{SU}(2)_L$ and then contract these to obtain an $\mathrm{SU}(2)_L$ singlet. For example, starting with four weak isospin 1/2 representations with $\mathrm{SU}(2)_L$ indices $(i,j), (k,m)$, the (i,j) and (k,m) indices can each be combined symmetrically, and then the resulting two isovectors can be contracted to make an $\mathrm{SU}(2)_L$ singlet. This is done with the $\mathrm{SU}(2)_L$ tensor

$$(I_{ss})_{ijkm} \equiv (\epsilon_{ik}\epsilon_{jm} + \epsilon_{im}\epsilon_{jk}). \tag{8.4}$$

For operators with six fermions in $\mathrm{SU}(2)_L$ doublets, another relevant $\mathrm{SU}(2)_L$ tensor involves symmetric combinations of two pairs of isospin-1/2 representations combined with an antisymmetric combination of the third pair of isospin-1/2 representations, via the tensor

$$(I_{ssa})_{ijkmnp} \equiv (\epsilon_{ik}\epsilon_{jm} + \epsilon_{im}\epsilon_{jk})\epsilon_{np}, \tag{8.5}$$

where the subscript (ssa) refers to this symmetric-symmetric-antisymmetric structure of $\mathrm{SU}(2)_L$ contractions. Finally, one can also use a set of $\mathrm{SU}(2)_L$ contractions in which all pairs of isospin-1/2 representations are combined symmetrically. The $\mathrm{SU}(2)_L$ tensor that does this is

$$I_{sss} = \epsilon_{ik}(\epsilon_{jn}\epsilon_{mp} + \epsilon_{mn}\epsilon_{jp}) + \epsilon_{im}(\epsilon_{jn}\epsilon_{kp} + \epsilon_{kn}\epsilon_{jm}) + \epsilon_{jk}(\epsilon_{\rm in}\epsilon_{mp} + \epsilon_{mn}\epsilon_{ip}) + \epsilon_{jm}(\epsilon_{\rm in}\epsilon_{kp} + \epsilon_{kn}\epsilon_{ip}), \tag{8.6}$$

where the (sss) subscript refers to the threefold symmetric set of contractions.

Since there is a one-to-one correspondence between an operator $\mathcal{O}_r^{(NN')}$ in $\mathcal{L}_{\rm eff}^{(NN')}$ and an operator $O_r^{(NN')}$ in $\mathcal{L}_{{\rm eff},4+n}^{(NN')}$, one can use either of these for a structural analysis; we will use the $\mathcal{O}_r^{(NN')}$. We will determine a general set of classes of operators that yield the same integrals $I_r^{(NN')}$, as defined in Eq. (8.2). A given class typically contains several different individual operators. However, since it is the integrals $I_r^{(NN')}$ that control the contribution to the amplitude, the natural organization for our analysis is in terms of these classes, rather than the individual operators.

We proceed with the general structural analysis of the $\Delta L = -2$ dinucleon-to-dilepton decays. The eight fermions that comprise a given operator $\mathcal{O}_r^{(NN')}$ are comprised of six quarks and two leptons, namely $uud, uud, \ell^+, \ell'^+$, $uud, ddu, \ell^+\bar{\nu}$, and $ddu, ddu, \bar{\nu}, \bar{\nu}'$ for the decays (1.5), (1.7), and (1.8), respectively. As discussed above, the quarks can be chosen from the $\mathrm{SU}(2)_L$-doublet Q_L or the $\mathrm{SU}(2)_L$-singlets u_R and d_R, and the leptons can be chosen from the $\mathrm{SU}(2)_L$-doublets $L_{\ell,L}$ and $L_{\ell',L}$ and the $\mathrm{SU}(2)_L$-singlets ℓ_R, ℓ'_R, and $\nu_{s,R}$. We can abstractly represent a generic eight-fermion operator product $\mathcal{O}_r^{(NN')}$ as

$$\mathcal{O}^{(NN')} = Q_L^{n_Q} L_L^{n_L} u_R^{n_u} d_R^{n_d} \ell_R^{n_\ell} \nu_{s,R}^{n_{\nu_s}}, \tag{8.7}$$

where we have suppressed the arguments y and η in the fermion fields, have suppressed the difference between lepton fields with and without primes, and have left the chiralities of the fermions implicit in the exponents. The fact that the operator involves eight fermions is the condition

$$n_Q + n_L + n_u + n_d + n_\ell + n_{\nu_s} = 8. \tag{8.8}$$

The condition that the initial state is a dinucleon is that

$$n_Q + n_u + n_d = 2N_c = 6, \tag{8.9}$$

where $N_c = 3$ is the number of colors. With the color contractions discussed above, this condition is sufficient for the operator be an $\mathrm{SU}(3)_c$ singlet. The condition that the final state has $L = -2$, i.e., is comprised of two antileptons, is that

$$n_L + n_\ell + n_{\nu_s} = 2. \tag{8.10}$$

Note that only two of the three equations (8.8)–(8.10) are linearly independent. The requirement that $\mathcal{O}$ must be invariant under the SM gauge group implies that it must have zero weak hypercharge and that it must be a singlet under $\mathrm{SU}(2)_L$. The condition that it must have weak hypercharge $Y = 0$ is that $\sum_f n_f Y_f = 0$, or, explicitly,

$$n_Q\left(\frac{1}{3}\right)+n_L(-1)+n_u\left(\frac{4}{3}\right)+n_d\left(-\frac{2}{3}\right)+n_\ell(-2)=0. \tag{8.11}$$

The condition that the operator must be an $SU(2)_L$ singlet requires that the number of $SU(2)_L$ doublets must be even:

$$n_Q+n_L=(0,2,4,6,\text{ or }8). \tag{8.12}$$

Equations (8.8)–(8.12) comprise five linear equations, of which four are linearly independent, in the six (non-negative, integer) unknown numbers, n_Q, n_L, n_u, n_d, n_ℓ, and n_{ν_s}, with the constraint that each number must lie in the range [0, 8]. The solutions to these equations with the given constraint determine the general structures of the operators for dinucleon-to-dilepton decays with $\Delta L=-2$. We have obtained these solutions, which we list in Table III. The abbreviations used for the fermion fields are $Q=Q_L$, $L=L_L$, $u=u_R$, $d=d_R$, $\ell=\ell_R$, and $\nu=\nu_{s,R}$. The first column lists the class number; the second column lists the number of $SU(2)_L$ doublets, denoted N_d; and the third column lists the general structure. Primes distinguishing different lepton fields are suppressed in the notation. In checking candidate solutions of Eqs. (8.8)–(8.12), it is necessary to verify that they do not vanish identically because of combined $SU(3)_c$ and $SU(2)_L$ tensors. We find that one class with $N_d=6$, of the abstract form $Q^6\ell\nu$, contains no nonvanishing operators of our type. We denote a given class symbolically as $C_k^{(NN')}$. These contribute as follows:

TABLE III. Structures of classes $C_k^{(NN')}$ of operators contributing to dinucleon-to-dilepton decays with $\Delta L=-2$. The first column lists the class number; the second column lists the number of $SU(2)_L$ doublets in the operators in this class; and the third column lists the structure of operators in the class. The abbreviations in the superscripts on the classes are pp for $pp\to\ell^+\ell'^+$, np for $np\to\ell^+\bar\nu$, and nn for $nn\to\bar\nu\bar\nu'$. The abbreviations for fermion fields are the same as in Table I. The primes distinguishing different lepton fields are suppressed in the notation.

Class $C_k^{(NN')}$	N_d	Structure
$C_1^{(pp)}$	0	$u^4d^2\ell^2$
$C_2^{(np)}$	0	$u^3d^3\ell\nu$
$C_3^{(nn)}$	0	$u^2d^4\nu^2$
$C_4^{(pp)}$	2	$Q^2u^3d\ell^2$
$C_5^{(np)}$	2	$Q^2u^2d^2\ell\nu$
$C_6^{(nn)}$	2	$Q^2ud^3\nu^2$
$C_7^{(pp,np)}$	2	$QLu^3d^2\ell$
$C_8^{(np,nn)}$	2	$QLu^2d^3\nu$
$C_9^{(np)}$	2	$L^2u^3d^3$
$C_{10}^{(pp)}$	4	$Q^4u^2\ell^2$
$C_{11}^{(np)}$	4	$Q^4ud\ell\nu$
$C_{12}^{(nn)}$	4	$Q^4d^2\nu^2$
$C_{13}^{(pp,np)}$	4	$Q^3Lu^2d\ell$
$C_{14}^{(np,nn)}$	4	$Q^3Lud^2\nu$
$C_{15}^{(pp,np,nn)}$	4	$Q^2L^2u^2d^2$
$C_{16}^{(pp,np,nn)}$	6	Q^4L^2ud
$C_{17}^{(pp,np)}$	6	$Q^5Lu\ell$
$C_{18}^{(np,nn)}$	6	$Q^5Ld\nu$
$C_{19}^{(pp,np,nn)}$	8	Q^6L^2

$$pp\to\ell^+\ell'^+:\ C_k^{(NN')},\quad k=1,4,7,10,13,15,16,17,19, \tag{8.13}$$

$$np\to\ell^+\bar\nu:\ C_k^{(NN')},\quad k=2,5,7,8,9,11,13,14,15,16,17,18,19, \tag{8.14}$$

and

$$nn\to\bar\nu\bar\nu':\ C_k^{(NN')},\quad k=3,6,8,12,14,15,16,18,19. \tag{8.15}$$

As is evident in these lists, some classes of operators only contribute to one type of $\Delta L=-2$ dinucleon-to-dilepton decay, while others contribute to two or three of these decays. We will sometimes indicate this explicitly, writing, for example, $C_1^{(NN')}=C_1^{(pp)}$, $C_2^{(NN')}=C_2^{(np)}$, $C_3^{(NN')}=C_3^{(nn)}$, $C_7^{(NN')}=C_7^{(pp,np)}$, $C_8^{(NN')}=C_8^{(np,nn)}$, and $C_{15}^{(NN')}=C_{15}^{(pp,np,nn)}$, where abbreviations for superscripts are pp for the decays $pp\to\ell^+\ell'^+$, np for $np\to\ell^+\bar\nu$, and nn for $nn\to\bar\nu\bar\nu'$. For brevity, we will also sometimes suppress the superscript (NN') on $C_k^{(NN')}$, writing simply C_k, as in the notation $I_{C_k}^{(NN')}\equiv I_{C_k^{(NN')}}$.

The integrand function of a class of operators $C_k^{(NN')}$ in this table with a given set of exponents $(n_Q,n_L,n_u,n_d,n_\ell,n_\nu)$ is of the form

$$V_k^{(NN')}(y)=A^8\exp\left[-\sum_{\{f\}}n_f\|\eta-\eta_f\|^2\right]. \tag{8.16}$$

The integral of $V_k^{(NN')}(y)$ over the extra spatial coordinates is

$$I_{C_k}^{(NN')}=\int d^ny V_k^{(NN')}(y). \tag{8.17}$$

This gives

$$I_{C_k}^{(NN')} = b_8 \exp\left[-\frac{1}{8}\sum_{f,f';f\neq f',ord} n_f n_{f'} \|\eta_f - \eta_{f'}\|^2\right], \qquad (8.18)$$

where the sum is over all of the types of fermion fields in the operator product, in an ordered manner, as indicated in Eq. (A2). The prefactor $b_8 = (2^{1/2}\pi^{-3/2}\mu^3)^n$, from Eq. (2.29). As noted in connection with Eq. (A2), for an operator $O_k^{(NN')}$ containing N_f different types of fermion fields, the integral $I_{C_k}^{(NN')}$ depends on $\binom{N_f}{2}$ different separation distances $\|\eta_f - \eta_{f'}\|$.

As the $k = 8$ special case of Eq. (2.30), the coefficient $c_r^{(NN')}$ can be expressed as

$$c_r^{(NN')} = \kappa_r^{(NN')} I_r^{(NN')} = \frac{\bar{\kappa}_r^{(NN')}}{(M_{\rm BNV})^{8+3n}} b_8 e^{-S_r^{(NN')}}$$
$$= \frac{\bar{\kappa}_r^{(NN')}}{M_{\rm BNV}^8}\left(\frac{2^{1/2}\mu^3}{\pi^{3/2}M_{\rm BNV}^3}\right)^n e^{-S_r^{(NN')}}. \qquad (8.19)$$

Then the decay rate for one of the three dinucleon-to-dilepton decays (1.5)–(1.8) is

$$\Gamma_{NN'} = \left(\frac{1}{2m_N}\right) S\left(\frac{1}{M_{\rm BNV}^{16}}\right)\left(\frac{2}{\pi^3}\right)^n\left(\frac{\mu}{M_{\rm BNV}}\right)^{6n}$$
$$\times\left|\sum_r \bar{\kappa}_r^{(NN')} e^{-S_r^{(NN')}} \langle f.s.|\mathcal{O}_r^{(NN')}|NN'\rangle\right|^2 R_2, \qquad (8.20)$$

where S is a symmetry factor, $S = 1/2$ for decays with identical leptons in the final state and R_2 is the phase-space factor.

IX. $pp \to \ell^+\ell'^+$ DECAYS

In this section we apply our general analysis to the $\Delta L = -2$ dinucleon decays $pp \to \ell^+\ell'^+$ of Eq. (1.5), where ℓ and ℓ' can be e, μ, or τ, as allowed by phase space. Thus, these are the decays $pp \to (e^+e^+, \mu^+\mu^+, e^+\mu^+, e^+\tau^+$, or $\mu^+\tau^+)$. The $pp \to e^+e^+$ decay is related by crossing to hydrogen-antihydrogen transitions $(ep) \to (\bar{e}\,\bar{p})$ [65]. These decays are of particular interest because if an experiment were to observe any of them, this would be not only an observation of baryon number violation with $\Delta B = -2$, but also an observation of the violation of total lepton number by $\Delta L = -2$ [66]. In contrast, since an experiment does not observe any outgoing (anti)neutrino(s), the $\Delta L = -2$ decay $np \to \ell^+\bar{\nu}$ is experimentally indistinguishable from the $\Delta L = 0$ decay $np \to \ell^+\nu$. For the same reason, the $\Delta L = -2$ decay $nn \to \bar{\nu}\bar{\nu}'$, the $\Delta L = 0$ decay $nn \to \nu\bar{\nu}'$, and the $\Delta L = 2$ decay $nn \to \nu\nu'$ are all indistinguishable experimentally. Furthermore, an experiment cannot determine whether a final-state neutrino is an EW-doublet neutrino of some generation (ν_e, ν_μ, or ν_τ), or whether it is an EW-singlet, ν_s.

Because six-quark operators of the form $uuduud$ have nonzero charge ($Q_{em} = 2$), they cannot, by themselves, be a singlet under $G_{\rm SM}$. However, a subset of the six-quark operators is invariant under $SU(2)_L$. The fact that the six-quark parts of these operators are invariant under $SU(2)_L$ implies that the lepton bilinears must also be invariant under $SU(2)_L$, and this fixes them to be of the form $[\ell_R^T C \ell'_R]$. For this set we list the following operators, together with the class to which they belong, as defined in Table III:

$$\mathcal{O}_1^{(pp)} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}[u_R^{\alpha T} C u_R^\beta][u_R^{\gamma T} C u_R^\delta][d_R^{\rho T} C d_R^\sigma][\ell_R^T C \ell'_R] \in C_1^{(pp)}, \qquad (9.1)$$

$$\mathcal{O}_2^{(pp)} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}[u_R^{\alpha T} C d_R^\beta][u_R^{\gamma T} C d_R^\delta][u_R^{\rho T} C u_R^\sigma][\ell_R^T C \ell'_R] \in C_1^{(pp)}, \qquad (9.2)$$

$$\mathcal{O}_3^{(pp)} = (T_a)_{\alpha\beta\gamma\delta\rho\sigma}[u_R^{\alpha T} C d_R^\beta][u_R^{\gamma T} C d_R^\delta][u_R^{\rho T} C u_R^\sigma][\ell_R^T C \ell'_R] \in C_1^{(pp)}, \qquad (9.3)$$

$$\mathcal{O}_4^{(pp)} = \epsilon_{ij}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T} C Q_L^{j\beta}][u_R^{\gamma T} C d_R^\delta][u_R^{\rho T} C u_R^\sigma][\ell_R^T C \ell'_R] \in C_4^{(pp)}, \qquad (9.4)$$

$$\mathcal{O}_5^{(pp)} = \epsilon_{ij}\epsilon_{km}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C Q_L^{m\delta}][u_R^{\rho T} C u_R^\sigma][\ell_R^T C \ell'_R] \in C_{10}^{(pp)}, \qquad (9.5)$$

and

$$\mathcal{O}_6^{(pp)} = (I_{ss})_{ijkm}(T_s)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C Q_L^{m\delta}][u_R^{\rho T} C u_R^\sigma][\ell_R^T C \ell'_R] \in C_{10}^{(pp)}. \qquad (9.6)$$

The remark concerning linear (in)dependence of operators given above after Eq. (6.11) also applies here. There are also operators contributing to $pp \to \ell^+\ell'$ in which one or both of the lepton fields is (are) contained in $SU(2)_L$ doublets rather than being $SU(2)_L$-singlets. Although we have carried out an enumeration of these other operations, this enumeration is actually not necessary for our analysis. Instead, as before, the key observation is that the contribution of a given operator $\mathcal{O}_r^{(NN')}$ to the amplitude for the diproton-to-dilepton decay is determined by the integrand function (8.17), given in general by Eq. (8.2). Since there are substantially fewer classes of integrand functions, and hence integrals, than the total number of operators contributing to $pp \to \ell^+\ell'^+$, this simplifies the analysis.

Applying our general formula (8.18), we calculate the following integrals for the classes of operators contributing to $pp \to \ell^+\ell'^+$, as listed in Table III and Eq. (8.13). For the superscript (NN'), we list all of the $\Delta L = -2$ dinucleon decays to which the class contributes. In accord with our general formula (8.18), we calculate the integrals

$$I_{C_1}^{(pp)} = b_8 \exp\Big[-\frac{1}{8}\{8\|\eta_{u_R} - \eta_{d_R}\|^2 + 4\|\eta_{u_R} - \eta_{\ell_R}\|^2 + 4\|\eta_{u_R} - \eta_{\ell'_R}\|^2 + 2\|\eta_{d_R} - \eta_{\ell_R}\|^2 + 2\|\eta_{d_R} - \eta_{\ell'_R}\|^2 + \|\eta_{\ell_R} - \eta_{\ell'_R}\|^2\}\Big], \tag{9.7}$$

$$I_{C_4}^{(pp)} = b_8 \exp\Big[-\frac{1}{8}\{6\|\eta_{Q_L} - \eta_{u_R}\|^2 + 2\|\eta_{Q_L} - \eta_{d_R}\|^2 + 2\|\eta_{Q_L} - \eta_{\ell_R}\|^2 + 2\|\eta_{Q_L} - \eta_{\ell'_R}\|^2 + 3\|\eta_{u_R} - \eta_{d_R}\|^2 + 3\|\eta_{u_R} - \eta_{\ell_R}\|^2 + 3\|\eta_{u_R} - \eta_{\ell'_R}\|^2 + \|\eta_{d_R} - \eta_{\ell_R}\|^2 + \|\eta_{d_R} - \eta_{\ell'_R}\|^2 + \|\eta_{\ell_R} - \eta_{\ell'_R}\|^2\}\Big], \tag{9.8}$$

$$I_{C_7}^{(pp,np)} = b_8 \exp\Big[-\frac{1}{8}\{\|\eta_{Q_L} - \eta_{L_{\ell,L}}\|^2 + 3\|\eta_{Q_L} - \eta_{u_R}\|^2 + 2\|\eta_{Q_L} - \eta_{d_R}\|^2 + \|\eta_{Q_L} - \eta_{\ell'_R}\|^2 + 3\|\eta_{L_{\ell,L}} - \eta_{u_R}\|^2 + 2\|\eta_{L_{\ell,L}} - \eta_{d_R}\|^2 + \|\eta_{L_{\ell,L}} - \eta_{\ell'_R}\|^2 + 6\|\eta_{u_R} - \eta_{d_R}\|^2 + 3\|\eta_{u_R} - \eta_{\ell'_R}\|^2 + 2\|\eta_{d_R} - \eta_{\ell'_R}\|^2\}\Big], \tag{9.9}$$

$$I_{C_{10}}^{(pp)} = b_8 \exp\Big[-\frac{1}{8}\{8\|\eta_{Q_L} - \eta_{u_R}\|^2 + 4\|\eta_{Q_L} - \eta_{\ell_R}\|^2 + 4\|\eta_{Q_L} - \eta_{\ell'_R}\|^2 + 2\|\eta_{u_R} - \eta_{\ell_R}\|^2 + 2\|\eta_{u_R} - \eta_{\ell'_R}\|^2 + \|\eta_{\ell_R} - \eta_{\ell'_R}\|^2\}\Big], \tag{9.10}$$

$$I_{C_{13}}^{(pp,np)} = b_8 \exp\Big[-\frac{1}{8}\{3\|\eta_{Q_L} - \eta_{L_{\ell,L}}\|^2 + 6\|\eta_{Q_L} - \eta_{u_R}\|^2 + 3\|\eta_{Q_L} - \eta_{d_R}\|^2 + 3\|\eta_{Q_L} - \eta_{\ell'_R}\|^2 + 2\|\eta_{L_{\ell,L}} - \eta_{u_R}\|^2 + \|\eta_{L_{\ell,L}} - \eta_{d_R}\|^2 + \|\eta_{L_{\ell,L}} - \eta_{\ell'_R}\|^2 + 2\|\eta_{u_R} - \eta_{d_R}\|^2 + 2\|\eta_{u_R} - \eta_{\ell'_R}\|^2 + \|\eta_{d_R} - \eta_{\ell'_R}\|^2\}\Big], \tag{9.11}$$

$$I_{C_{15}}^{(pp,np,nn)} = b_8 \exp\Big[-\frac{1}{8}\{2\|\eta_{Q_L} - \eta_{L_{\ell,L}}\|^2 + 2\|\eta_{Q_L} - \eta_{L_{\ell',L}}\|^2 + 4\|\eta_{Q_L} - \eta_{u_R}\|^2 + 4\|\eta_{Q_L} - \eta_{d_R}\|^2 + \|\eta_{L_{\ell,L}} - \eta_{L_{\ell',L}}\|^2 + 2\|\eta_{L_{\ell,L}} - \eta_{u_R}\|^2 + 2\|\eta_{L_{\ell,L}} - \eta_{d_R}\|^2 + 2\|\eta_{L_{\ell',L}} - \eta_{u_R}\|^2 + 2\|\eta_{L_{\ell',L}} - \eta_{d_R}\|^2 + 4\|\eta_{u_R} - \eta_{d_R}\|^2\}\Big], \tag{9.12}$$

$$I_{C_{16}}^{(pp,np,nn)} = b_8 \exp\Big[-\frac{1}{8}\{4\|\eta_{Q_L} - \eta_{L_{\ell,L}}\|^2 + 4\|\eta_{Q_L} - \eta_{L_{\ell',L}}\|^2 + 4\|\eta_{Q_L} - \eta_{u_R}\|^2 + 4\|\eta_{Q_L} - \eta_{d_R}\|^2 + \|\eta_{L_{\ell,L}} - \eta_{L_{\ell',L}}\|^2 + \|\eta_{L_{\ell,L}} - \eta_{u_R}\|^2 + \|\eta_{L_{\ell,L}} - \eta_{d_R}\|^2 + \|\eta_{L_{\ell',L}} - \eta_{u_R}\|^2 + \|\eta_{L_{\ell',L}} - \eta_{d_R}\|^2 + \|\eta_{u_R} - \eta_{d_R}\|^2\}\Big], \tag{9.13}$$

$$I_{C_{17}}^{(pp,np)} = b_8 \exp\Big[-\frac{1}{8}\{5\|\eta_{Q_L} - \eta_{L_{\ell,L}}\|^2 + 5\|\eta_{Q_L} - \eta_{u_R}\|^2 + 5\|\eta_{Q_L} - \eta_{\ell'_R}\|^2 + \|\eta_{L_{\ell,L}} - \eta_{u_R}\|^2 + \|\eta_{L_{\ell,L}} - \eta_{\ell'_R}\|^2 + \|\eta_{u_R} - \eta_{\ell'_R}\|^2\}\Big], \tag{9.14}$$

and

$$I_{C_{19}}^{(pp,np,nn)} = b_8 \exp\Big[-\frac{1}{8}\{6\|\eta_{Q_L} - \eta_{L_{\ell,L}}\|^2 + 6\|\eta_{Q_L} - \eta_{L_{\ell',L}}\|^2 + \|\eta_{L_{\ell,L}} - \eta_{L_{\ell',L}}\|^2\}\Big]. \tag{9.15}$$

Next, we use the lower bounds on the distances separating the centers of fermion wave functions in the extra dimension that we inferred from lower bounds on partial lifetimes of proton decay modes. We substitute these lower bounds on separation distances into the integrals $I_n^{(pp)}$ and Eq. (8.20) to obtain upper bounds on the rates for the $pp \to \ell^+ \ell'^+$ decays. Using the lower bounds on the distances separating centers of fermion wave functions that we derived from limits on nucleon decay, we find that the resultant values of $(\tau/B)_{pp\to\ell^+\ell'^+} = (\Gamma_{pp\to\ell^+\ell'^+})^{-1}$ predicted by the extra-dimensional model are easily in agreement with current experimental lower bounds on these $\Delta L = -2$ dinucleon-to-dilepton decays. As embodied in Eqs. (2.33) and (2.35), this result follows because of the lower bounds on the exponent sums $S_r^{(pp)}$, together with the fact that the amplitude is much more highly suppressed, by the prefactor $1/M_{\rm BNV}^8$, as compared with the prefactor $1/M_{\rm BNV}^2$ that enters in the amplitude for $\Delta L = -1$ nucleon decays such as $p \to \ell^+ \pi^0$, where $M_{\rm BNV}$. The lower bounds (from the SK experiment) are [23]

$$(\tau/B)_{pp\to e^+e^+} > 4.2 \times 10^{33}\ \text{yr}, \tag{9.16}$$

$$(\tau/B)_{pp\to \mu^+\mu^+} > 4.4 \times 10^{33}\ \text{yr}, \tag{9.17}$$

and

$$(\tau/B)_{pp\to e^+\mu^+} > 4.4 \times 10^{33}\ \text{yr} \tag{9.18}$$

per ^{16}O nucleus in the water.

X. $np \to \ell^+ \bar{\nu}$ DECAYS

In this section we proceed to apply the same methods to set upper bounds on decay rates for the decays $np \to \ell^+\bar{\nu}$, where ℓ^+ can be e^+, μ^+, or τ^+ and $\bar{\nu}$ can be an electroweak-doublet antineutrino of any generation or an electroweak-singlet antineutrino. Several of the classes of integrals for $np \to \ell^+\bar{\nu}$ are the same as those for $pp \to \ell^+\nu$ decays, which we have already analyzed. These are the $C_k^{(NN')}$ with $k = 7, 13, 15, 16, 17, 19$. For the other classes, we calculate the integrals

$$I_{C_2}^{(np)} = b_8 \exp\Big[-\frac{1}{8}\{9\|\eta_{u_R}-\eta_{d_R}\|^2 + 3\|\eta_{u_R}-\eta_{\ell_R}\|^2 + 3\|\eta_{u_R}-\eta_{\nu_{s,R}}\|^2 + 3\|\eta_{d_R}-\eta_{\ell_R}\|^2 + 3\|\eta_{d_R}-\eta_{\nu_{s,R}}\|^2 + \|\eta_{\ell_R}-\eta_{\nu_{s,R}}\|^2\}\Big], \tag{10.1}$$

$$I_{C_5}^{(np)} = b_8 \exp\Big[-\frac{1}{8}\{4\|\eta_{Q_L}-\eta_{u_R}\|^2 + 4\|\eta_{Q_L}-\eta_{d_R}\|^2 + 2\|\eta_{Q_L}-\eta_{\ell_R}\|^2 + 2\|\eta_{Q_L}-\eta_{\nu_{s,R}}\|^2 + 4\|\eta_{u_R}-\eta_{d_R}\|^2 + 2\|\eta_{u_R}-\eta_{\ell_R}\|^2 + 2\|\eta_{u_R}-\eta_{\nu_{s,R}}\|^2 + 2\|\eta_{d_R}-\eta_{\ell_R}\|^2 + 2\|\eta_{d_R}-\eta_{\nu_{s,R}}\|^2 + \|\eta_{\ell_R}-\eta_{\nu_{s,R}}\|^2\}\Big], \tag{10.2}$$

$$I_{C_8}^{(np,nn)} = b_8 \exp\Big[-\frac{1}{8}\{\|\eta_{Q_L}-\eta_{L_{\ell,L}}\|^2 + 2\|\eta_{Q_L}-\eta_{u_R}\|^2 + 3\|\eta_{Q_L}-\eta_{d_R}\|^2 + \|\eta_{Q_L}-\eta_{\nu_{s,R}}\|^2 + 2\|\eta_{L_{\ell,L}}-\eta_{u_R}\|^2 + 3\|\eta_{L_{\ell,L}}-\eta_{d_R}\|^2 + \|\eta_{L_{\ell,L}}-\eta_{\nu_{s,R}}\|^2 + 6\|\eta_{u_R}-\eta_{d_R}\|^2 + 2\|\eta_{u_R}-\eta_{\nu_{s,R}}\|^2 + 3\|\eta_{d_R}-\eta_{\nu_{s,R}}\|^2\}\Big], \tag{10.3}$$

$$I_{C_9}^{(np)} = b_8 \exp\Big[-\frac{1}{8}\{\|\eta_{L_{\ell,L}}-\eta_{L_{\ell',L}}\|^2 + 3\|\eta_{L_{\ell,L}}-\eta_{u_R}\|^2 + 3\|\eta_{L_{\ell,L}}-\eta_{d_R}\|^2 + 3\|\eta_{L_{\ell',L}}-\eta_{u_R}\|^2 + 3\|\eta_{L_{\ell',L}}-\eta_{d_R}\|^2 + 9\|\eta_{u_R}-\eta_{d_R}\|^2\}\Big], \tag{10.4}$$

$$I_{C_{11}}^{(np)} = b_8 \exp\Big[-\frac{1}{8}\{4\|\eta_{Q_L}-\eta_{u_R}\|^2 + 4\|\eta_{Q_L}-\eta_{d_R}\|^2 + 4\|\eta_{Q_L}-\eta_{\ell_R}\|^2 + 4\|\eta_{Q_L}-\eta_{\nu_{s,R}}\|^2 + \|\eta_{u_R}-\eta_{d_R}\|^2 + \|\eta_{u_R}-\eta_{\ell_R}\|^2 + \|\eta_{u_R}-\eta_{\nu_{s,R}}\|^2 + \|\eta_{d_R}-\eta_{\ell_R}\|^2 + \|\eta_{d_R}-\eta_{\nu_{s,R}}\|^2 + \|\eta_{\ell_R}-\eta_{\nu_{s,R}}\|^2\}\Big], \tag{10.5}$$

$$I_{C_{14}}^{(np,nn)} = b_8 \exp\Big[-\frac{1}{8}\{3\|\eta_{Q_L}-\eta_{L_{\ell,L}}\|^2 + 3\|\eta_{Q_L}-\eta_{u_R}\|^2 + 6\|\eta_{Q_L}-\eta_{d_R}\|^2 + 3\|\eta_{Q_L}-\eta_{\nu_{s,R}}\|^2 + \|\eta_{L_{\ell,L}}-\eta_{u_R}\|^2 + 2\|\eta_{L_{\ell,L}}-\eta_{d_R}\|^2 + \|\eta_{L_{\ell,L}}-\eta_{\nu_{s,R}}\|^2 + 2\|\eta_{u_R}-\eta_{d_R}\|^2 + \|\eta_{u_R}-\eta_{\nu_{s,R}}\|^2 + 2\|\eta_{d_R}-\eta_{\nu_{s,R}}\|^2\}\Big], \tag{10.6}$$

and

$$I_{C_{18}}^{(np,nn)} = b_8 \exp\left[-\frac{1}{8}\{5\|\eta_{Q_L} - \eta_{L_{\ell,L}}\|^2 + 5\|\eta_{Q_L} - \eta_{d_R}\|^2 + 5\|\eta_{Q_L} - \eta_{\nu_{s,R}}\|^2 + \|\eta_{L_{\ell,L}} - \eta_{d_R}\|^2 + \|\eta_{L_{\ell,L}} - \eta_{\nu_{s,R}}\|^2 + \|\eta_{d_R} - \eta_{\nu_{s_R}}\|^2\}\right]. \tag{10.7}$$

Although it is not necessary for our analysis, one can construct explicit operators of each class, as we have done for the operators contributing to $pp \to \ell^+\ell'^+$. Some of these contribute to decays with EW-singlet antineutrinos, while others contribute to decays with EW-doublet antineutrinos, but since these decays are indistinguishable experimentally, we include all of these operators together. For example, there are several operators in which all fermions are $\mathrm{SU}(2)_L$ singlets:

$$\mathcal{O}_1^{(np)} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}[u_R^{\alpha T} C u_R^{\beta}][u_R^{\gamma T} C d_R^{\delta}][d_R^{\rho T} C d_R^{\sigma}][\ell_R^T C \nu_{s,R}] \in C_2^{(np)}, \tag{10.8}$$

$$\mathcal{O}_2^{(np)} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}[u_R^{\alpha T} C d_R^{\beta}][u_R^{\gamma T} C d_R^{\delta}][u_R^{\rho T} C d_R^{\sigma}][\ell_R^T C \nu_{s,R}] \in C_2^{(np)}, \tag{10.9}$$

$$\mathcal{O}_3^{(np)} = (T_a)_{\alpha\beta\gamma\delta\rho\sigma}[u_R^{\alpha T} C d_R^{\beta}][u_R^{\gamma T} C d_R^{\delta}][u_R^{\rho T} C d_R^{\sigma}][\ell_R^T C \nu_{s,R}] \in C_2^{(np)}, \tag{10.10}$$

$$\mathcal{O}_4^{(np)} = \epsilon_{ij}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T} C Q_L^{j\beta}][u_R^{\gamma T} C d_R^{\delta}][u_R^{\rho T} C d_R^{\sigma}][\ell_R^T C \nu_{s,R}] \in C_5^{(np)}, \tag{10.11}$$

$$\mathcal{O}_5^{(np)} = \epsilon_{ij}\epsilon_{km}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C Q_L^{m\delta}][u_R^{\rho T} C d_R^{\sigma}][\ell_R^T C \nu_{s,R}] \in C_{11}^{(np)}, \tag{10.12}$$

and

$$\mathcal{O}_6^{(np)} = (I_{ss})_{ijkm}(T_s)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C Q_L^{m\delta}][u_R^{\rho T} C d_R^{\sigma}][\ell_R^T C \nu_{s,R}] \in C_{11}^{(np)}. \tag{10.13}$$

There are also operators contributing to $np \to \ell^+\bar{\nu}$ in which one or both of the lepton fields is (are) contained in $\mathrm{SU}(2)_L$ doublets rather than being $\mathrm{SU}(2)_L$ singlets. We have constructed these explicitly, using the same methods that we used for the corresponding operators contributing to $pp \to \ell^+\ell'^+$.

Proceeding as in Sec. IX, we have calculated the resultant rates for the $\Delta L = -2$ decays $np \to \ell^+\bar{\nu}$. Using the lower bounds on distances between fermion wave function centers in the extra dimensions that we have derived in Sec. III, we find that the resultant lower bounds on the partial lifetimes are in agreement with the current experimental lower bounds on these decays. Furthermore, as noted earlier, since an experiment would not observe the outgoing antineutrino, it would not be able to distinguish the $\Delta L = -2$ decay $np \to \ell^+\bar{\nu}$ from the $\Delta L = 0$ decay $np \to \ell^+\nu$. As discussed in [43], the latter decay can occur via the combination of a six-quark BNV vertex with SM fermion processes and hence is generically much less suppressed than the $\Delta L = -2$ dinucleon-to-dilepton decays.

XI. $nn \to \bar{\nu}\bar{\nu}'$ AND $nn \to \nu\nu'$ DECAYS

In this section we consider the $\Delta L = -2$ dineutron decay $nn \to \bar{\nu}\bar{\nu}'$ and the corresponding $\Delta L = 2$ decay $nn \to \nu\nu'$. Of the classes of eight-fermion operators contributing to the $\Delta L = -2$ dineutron decay $nn \to \bar{\nu}\bar{\nu}'$, the six resultant $I_k^{(NN')}$ integrals have already been given above, namely those for $k = 8$, 14, 15, 16, 18, and 19. The remaining three integrals are for $k = 3$, 6, 12. We calculate the integrals

$$I_{C_3}^{(nn)} = b_8 \exp\left[-\frac{1}{8}\{8\|\eta_{u_R} - \eta_{d_R}\|^2 + 2\|\eta_{u_R} - \eta_{\nu_{s,R}}\|^2 + 2\|\eta_{u_R} - \eta_{\nu_{s',R}}\|^2 + 4\|\eta_{d_R} - \eta_{\nu_{s,R}}\|^2 + 4\|\eta_{d_R} - \eta_{\nu_{s',R}}\|^2 + \|\eta_{\nu_{s,R}} - \eta_{\nu_{s',R}}\|^2\}\right], \tag{11.1}$$

$$I_{C_6}^{(nn)} = b_8 \exp\left[-\frac{1}{8}\{2\|\eta_{Q_L}-\eta_{u_R}\|^2+6\|\eta_{Q_L}-\eta_{d_R}\|^2+2\|\eta_{Q_L}-\eta_{\nu_{s,R}}\|^2+2\|\eta_{Q_L}-\eta_{\nu_{s',R}}\|^2+3\|\eta_{u_R}-\eta_{d_R}\|^2 + \|\eta_{u_R}-\eta_{\nu_{s,R}}\|^2+\|\eta_{u_R}-\eta_{\nu_{s',R}}\|^2+3\|\eta_{d_R}-\eta_{\nu_{s,R}}\|^2+3\|\eta_{d_R}-\eta_{\nu_{s',R}}\|^2+\|\eta_{\nu_{s,R}}-\eta_{\nu_{s',R}}\|^2\}\right], \tag{11.2}$$

and

$$I_{C_{12}}^{(nn)} = b_8 \exp\left[-\frac{1}{8}\{8\|\eta_{Q_L}-\eta_{d_R}\|^2+4\|\eta_{Q_L}-\eta_{\nu_{s,R}}\|^2+4\|\eta_{Q_L}-\eta_{\nu_{s',R}}\|^2+2\|\eta_{d_R}-\eta_{\nu_{s,R}}\|^2 + 2\|\eta_{d_R}-\eta_{\nu_{s',R}}\|^2+\|\eta_{\nu_{s,R}}-\eta_{\nu_{s',R}}\|^2\}\right]. \tag{11.3}$$

Applying our lower bounds on the distances between centers of fermion wave functions in the extra dimension from Sec. III, we find that these $\Delta L = -2$ dinucleon decays are highly suppressed, similar to what we showed for the $pp \to \ell^+\ell'^+$ and $np \to \ell^+\bar{\nu}$ decays.

One can also consider the $\Delta L = 2$ dineutron-to-dilepton decays $nn \to \nu\nu'$ in Eq. (1.9). Given that $\nu_{s,R}$ is assigned lepton number $L = 1$, there is a corresponding charge-conjugate field, $(\nu_{s,R})^c = (\nu_s^c)_L$ with lepton number $L = -1$. The eight-fermion operators that contribute to the decays (1.9) are obtained from those for the decay $nn \to \bar{\nu}\bar{\nu}'$ by replacing the $[\nu_{s,R}^T C \nu_{s',R}]$ neutrino bilinear by $[(\nu_s^c)_L^T C (\nu_{s'}^c)_L]$. There are thus three classes of operators, which are the results of this change applied to the classes $C_k^{(nn)}$ with $k = 3, 6, 12$ for $nn \to \bar{\nu}\bar{\nu}'$ decays. Carrying out the resultant analysis, we reach the same conclusions as we did for the $\Delta L = -2$ dinucleon-to-dilepton decays concerning the highly suppressed rates.

A general comment concerning both of these $\Delta L = \pm 2$ dineutron decays is that since an experiment would not observe the outgoing (anti)neutrinos, it could not distinguish these decays from the $\Delta L = 0$ dineutron decays $nn \to \nu\bar{\nu}$ decays, which can occur via a six-quark BNV operator combined with SM processes and hence are generically much less suppressed than the $\Delta L = -2$ decays $nn \to \bar{\nu}\bar{\nu}'$ [43].

One also expects similar suppression in this extra-dimensional model for B- and L-violating decays involving trinucleon initial states, such as $ppp \to \ell^+\pi^+\pi^+$ and $ppn \to \ell^+\pi^+$, mediated by ten-fermion operators, or $ppp \to \ell^+\ell'^+\ell''^+$ and $ppn \to \ell^+\ell'^+\bar{\nu}$, mediated by 12-fermion operators. Recent experimental bounds on trinucleon decays include [67,68].

XII. CONCLUSIONS

In this paper we have studied several baryon-number-violating nucleon and dinucleon decays in a model with large extra dimensions, including (i) the $\Delta L = -3$ nucleon decays $p \to \ell^+\bar{\nu}\bar{\nu}'$ and $n \to \bar{\nu}\bar{\nu}'\bar{\nu}''$; (ii) the $\Delta L = 1$ nucleon decays $p \to \ell^+\nu\nu'$ and $n \to \bar{\nu}\nu'\nu''$; (iii) the $\Delta L = -2$ dinucleon decays $pp \to (e^+e^+, \mu^+\mu^+, e^+\mu^+, e^+\tau^+$, or $\mu^+\tau^+)$, $np \to \ell^+\bar{\nu}$, and $nn \to \bar{\nu}\bar{\nu}'$, where $\ell^+ = e^+, \mu^+$, or τ^+; and (iv) the $\Delta L = 2$ dineutron decays $nn \to \nu\nu'$. The decays of type (i) and (ii) are mediated by six-fermion operators, while the decays of type (iii) and (iv) are mediated by eight-fermion operators. Motivated by the earlier finding in Ref. [25] that, even with fermion wave function positions chosen so as to render the rates for baryon-number-violating nucleon decays much smaller than experimental limits, $n-\bar{n}$ oscillations could occur at rates comparable to experimental bounds, we have addressed the generalized question of whether nucleon and dinucleon decays to leptonic final states mediated by six-fermion and eight-fermion operators are sufficiently suppressed to agree with experimental bounds. To investigate this question, we have determined constraints on separations between wave functions in the extra dimensions from limits on the best constrained proton and bound neutron decay modes and then have applied these in analyses of relevant six-fermion and eight-fermion operators contributing to the decays (i)–(iv). From these analyses, we find that in this extra-dimensional model these decays are strongly suppressed, in accord with experimental limits. The reason that $n-\bar{n}$ oscillations can occur at a level comparable with current limits, while the decays (i)–(iv) are suppressed well below experimental limits on the respective modes can be traced to the fact that nucleon decays can be suppressed by making the separations between quark and lepton wave function centers sufficiently large. This procedure does not suppress $n-\bar{n}$ oscillations but considerably suppresses the baryon-number-violating decays of nucleons and dinucleons considered here. In addition to its phenomenological value, our analysis provides an interesting example of the application of low-energy effective field theory techniques to a problem involving several relevant mass scales. Here, these mass scales include the fermion wave function localization parameter μ, the overall mass scale of baryon number violation, $M_{\rm BNV}$, and the multiple inverse separation distances $\|y_{f_i} - y_{f_j}\|^{-1}$ between various fermion wave function centers in the extra dimensions.

ACKNOWLEDGMENTS

This research was supported in part by the National Science Foundation Grants No. NSF-PHY-1620628 and No. NSF-PHY-1915093 (R. S.).

APPENDIX A: SOME INTEGRALS

We record here some relevant formulas that are used for our calculations. First, with η a real variable and the (real) constants $a_i > 0$, $i = 1, \ldots, m$, we have

$$\int_{-\infty}^{\infty} d\eta \exp\left[-\sum_{i=1}^{m} a_i(\eta - \eta_{f_i})^2\right] = \left[\frac{\pi}{\sum_{i=1}^{m} a_i}\right]^{1/2} \exp\left[\frac{-\sum_{j,k=1;j<k}^{m} a_j a_k (\eta_{f_j} - \eta_{f_k})^2}{\sum_{s=1}^{m} a_s}\right]. \tag{A1}$$

The sum $\sum_{j,k=1;j<k}^{m} a_j a_k (\eta_{f_j} - \eta_{f_k})^2$ contains $\binom{m}{2}$ terms, where $\binom{m}{r} \equiv m!/[r!(m-r)!]$ is the binomial coefficient.

Second, now generalizing η to an n-dimensional vector $\eta \in \mathbb{R}^n$ with components η_j, $j = 1, \ldots, n$, and norm $\|\eta\| = [\sum_{j=1}^{n} \eta_j^2]^{1/2}$, and denoting $[\prod_{j=1}^{n} \int_{-\infty}^{\infty} d\eta_j] F(\eta) \equiv \int d^n\eta F(\eta)$, we have

$$\int d^n\eta \exp\left[-\sum_{i=1}^{m} a_i \|\eta - \eta_{f_i}\|^2\right] = \left[\frac{\pi}{\sum_{i=1}^{m} a_i}\right]^{n/2} \exp\left[\frac{-\sum_{j,k=1;j<k}^{m} a_j a_k \|\eta_{f_j} - \eta_{f_k}\|^2}{\sum_{s=1}^{m} a_s}\right]. \tag{A2}$$

Thus, for example, for $m = 3$,

$$\int d^n\eta \exp[-(a_1\|\eta - \eta_{f_1}\|^2 + a_2\|\eta - \eta_{f_2}\|^2 + a_3\|\eta - \eta_{f_3}\|^2)]$$
$$= \left[\frac{\pi}{a_1 + a_2 + a_3}\right]^{n/2} \exp\left[\frac{-(a_1 a_2\|\eta_{f_1} - \eta_{f_2}\|^2 + a_2 a_3\|\eta_{f_2} - \eta_{f_3}\|^2 + a_3 a_1\|\eta_{f_3} - \eta_{f_1}\|^2)}{a_1 + a_2 + a_3}\right]. \tag{A3}$$

APPENDIX B: PROPERTIES OF COLOR TENSORS

The tensors T_s and T_a in Eqs. (4.12) and (4.13) were defined and used in [8]; in [9] their properties were discussed further and a third type of color tensor, denoted T_{a3}, was defined and applied. In this Appendix we review the properties of these tensors. We use the notation (a, b) and $[a, b]$ to mean, respectively, symmetry and antisymmetry under the interchange $a \leftrightarrow b$, where a and b can be single $SU(3)_c$ indices or sets of indices. The tensor T_s has the properties

$$(T_s)_{\alpha\beta\gamma\delta\rho\sigma}\colon\ (\alpha,\beta), (\gamma,\delta), (\rho,\sigma), (\alpha\beta,\gamma\delta), (\gamma\delta,\rho\sigma), (\alpha\beta,\rho\sigma). \tag{B1}$$

Thus, in a contraction of T_s with a product of six fundamental ($\underline{3}$) representations of $SU(3)_c$, the first two pairs are each combined as $(3 \times 3)_s = 6$, i.e., in terms of Young tableaux, $(\square \times \square)_s = \square\square$; then the resultant two $\underline{6}$ representations are combined symmetrically as $(6 \times 6)_s = \bar{6}$, i.e., $(\square\square \times \square\square)_s = \overline{\square\square}$, and finally this $\bar{6}$ is combined with the 6 resulting from the third pair $(3 \times 3)_s = 6$ to make an $SU(3)_c$ singlet.

The tensor T_a has the properties

$$(T_a)_{\alpha\beta\gamma\delta\rho\sigma}\colon\ [\alpha,\beta], \quad [\gamma,\delta], \quad (\rho,\sigma), \quad (\alpha\beta,\gamma\delta). \tag{B2}$$

Hence, in a contraction of T_a with a product of six fundamental representations of $SU(3)_c$, the first two pairs are each combined as $(3 \times 3)_a = \bar{3}$, then the resultant two $\bar{3}$ representations are combined as $(\bar{3} \times \bar{3})_s = \bar{6}$, and finally, this is combined with the 6 from the $(\rho\sigma)$ combination to make an $SU(3)_c$ singlet. To indicate more explicitly these (anti)symmetry properties, Ref. [9] introduced the notation

$$(T_a)_{\alpha\beta\gamma\delta\rho\sigma} \equiv (T_{aas})_{\alpha\beta\gamma\delta\rho\sigma}, \tag{B3}$$

where the subscript (aas) refers to the antisymmetry on the first two pairs of color indices and symmetry on the last pair. In an obvious notation, there are two other related color tensors, T_{asa} and T_{saa}.

As noted in [9], there is a third way to couple six fundamental representations of $SU(3)_c$ together to make a singlet, namely to couple each pair antisymmetrically, via the tensor T_{a3} given in Eq. (8.3). This tensor was not needed in the analysis of $n - \bar{n}$ oscillations in [8] but did enter in the analysis of six-quark operators involving higher generations in [9]. It has the properties

$$(T_{a3})_{\alpha\beta\gamma\delta\rho\sigma}\colon\ [\alpha,\beta], \quad [\gamma,\delta], \quad [\rho,\sigma], \quad [\alpha\beta,\gamma\delta], \quad [\gamma\delta,\rho\sigma], \quad [\alpha\beta,\rho\sigma]. \tag{B4}$$

APPENDIX C: PHASE-SPACE FACTORS

For an initial state with invariant mass $\sqrt{s}$ decaying to an n-body final state $f.s.$, the phase-space factor is

$$\int dR_n = \frac{1}{(2\pi)^{3n-4}} \int \left[\prod_{i=1}^{n} \frac{d^3 p_i}{2E_i}\right] \delta^4\left(p - \left(\sum_{i=1}^{n} p_i\right)\right), \quad \text{(C1)}$$

where p is the four-momentum of the initial state and E_i and p_i denote the energies and four-momenta of the final-state particles, respectively. We define the Lorenz-invariant phase-space factor as

$$R_n = \int dR_n. \quad \text{(C2)}$$

We will only need R_2, which is

$$R_2 = \frac{1}{8\pi}[\lambda(1, \delta_1, \delta_2)]^{1/2}, \quad \text{(C3)}$$

where $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2(xy + yz + zx)$ and $\delta_i = m_i^2/s$. If m_i^2/s is zero or negligibly small for all particles i in the final state, then $R_2 = 1/(8\pi)$. If $\delta_1 = \delta_2 \equiv \delta$, then $R_2 = (8\pi)^{-1}\sqrt{1-4\delta}$.

APPENDIX D: OPERATORS CONTRIBUTING TO $pp \to \ell^+ \ell'^+$

Although our results in this paper depend only on the classes of operators $C_k^{(NN')}$ and the resultant integrals of fermion fields over the extra dimensions, $I_{C_k}^{(NN')} \equiv I_{C_k^{(NN')}}$, it is worthwhile, for illustrative purposes, to display various explicit operators that contribute to the $\Delta L = -2$ diproton decays $pp \to \ell^+ \ell'^+$. We have listed operators of this type in which all fermions are $SU(2)_L$ singlets in the text. Here we give operators contributing to $pp \to \ell^+ \ell'^+$ in which one or both of the lepton fields is (are) in $SU(2)_L$ doublets. As remarked after Eq. (6.11) in the text, since our analysis only depends on the classes of operators (defined by the integrals), which are manifestly independent, since they are comprised of different fermion fields, it is not necessary to work out all linear independence properties among these explicit operators.

Operators with one lepton field arising from an $SU(2)_L$ doublet and the other an $SU(2)_L$ singlet include the following. The first of these is

$$\mathcal{O}_7^{(pp,np)} = \epsilon_{ij}(T_s)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T} C L_{\ell,L}^j][u_R^{\beta T} C d_R^\gamma][u_R^{\delta T} C u_R^\rho][d_R^{\sigma T} C \ell'_R] \in C_7^{(pp,np)}. \quad \text{(D1)}$$

Carrying out the $SU(2)_L$ contractions in $\mathcal{O}_7^{(pp,np)}$ explicitly, one has

$$\mathcal{O}_7^{(pp,np)} = (T_s)_{\alpha\beta\gamma\delta\rho\sigma}([u_L^{\alpha T} C \ell_L] - [d_L^{\alpha T} C \nu_{\ell,L}])[u_R^{\beta T} C d_R^\gamma][u_R^{\delta T} C u_R^\rho][d_R^{\sigma T} C \ell'_R]. \quad \text{(D2)}$$

Of the two terms in Eq. (D2), the one containing the $[u_L^{\alpha T} C \ell_L]$ fermion bilinear contributes to $pp \to \ell^+ \ell'^+$, while the other term contributes to $np \to \ell'^+ \bar{\nu}_\ell$. Since it is straightforward to determine which dinucleon-to-dilepton decays each operator contributes to, we do not indicate this explicitly. Other operators include

$$\mathcal{O}_8^{(pp,np)} = \epsilon_{ij}(T_s)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T} C L_{\ell,L}^j][u_R^{\beta T} C d_R^\gamma][u_R^{\delta T} C d_R^\rho][u_R^{\sigma T} C \ell'_R] \in C_7^{(pp,np)}, \quad \text{(D3)}$$

$$\mathcal{O}_9^{(pp,np)} = \epsilon_{ij}(T_s)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T} C L_{\ell,L}^j][u_R^{\beta T} C u_R^\gamma][d_R^{\delta T} C d_R^\rho][u_R^{\sigma T} C \ell'_R] \in C_7^{(pp,np)}, \quad \text{(D4)}$$

$$\mathcal{O}_{10}^{(pp,np)} = \epsilon_{ij}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T} C L_{\ell,L}^j][u_R^{\beta T} C d_R^\gamma][u_R^{\delta T} C u_R^\rho][d_R^{\sigma T} C \ell'_R] \in C_7^{(pp,np)}, \quad \text{(D5)}$$

$$\mathcal{O}_{11}^{(pp,np)} = \epsilon_{ij}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T} C L_{\ell,L}^j][u_R^{\beta T} C d_R^\gamma][u_R^{\delta T} C d_R^\rho][u_R^{\sigma T} C \ell'_R] \in C_7^{(pp,np)}, \quad \text{(D6)}$$

$$\mathcal{O}_{12}^{(pp,np)} = \epsilon_{ij}\epsilon_{km}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C L_{\ell,L}^m][u_R^{\rho T} C u_R^\sigma][d_R^{\delta T} C \ell'_R] \in C_{13}^{(pp,np)}, \quad \text{(D7)}$$

$$\mathcal{O}_{13}^{(pp,np)} = \epsilon_{ij}\epsilon_{km}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C L_{\ell,L}^m][u_R^{\rho T} C d_R^\sigma][u_R^{\delta T} C \ell'_R] \in C_{13}^{(pp,np)}, \quad \text{(D8)}$$

$$\mathcal{O}_{14}^{(pp,np)} = \epsilon_{ij}\epsilon_{km}(T_{a3})_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C L_{\ell,L}^m][u_R^{\rho T} C d_R^\sigma][u_R^{\delta T} C \ell'_R] \in C_{13}^{(pp,np)}, \quad \text{(D9)}$$

$$\mathcal{O}_{15}^{(pp,np)} = \epsilon_{ij}\epsilon_{km}\epsilon_{np}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C Q_L^{m\delta}][Q_L^{n\rho T} C L_{\ell,L}^p][u_R^{\sigma T} C \ell'_R] \in C_{17}^{(pp,np)}, \quad \text{(D10)}$$

$$\mathcal{O}_{16}^{(pp,np)} = (I_{ss})_{ijkm}(T_s)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C L_{\ell,L}^m][u_R^{\rho T} C u_R^\sigma][d_R^{\delta T} C \ell'_R] \in C_{13}^{(pp,np)}, \quad \text{(D11)}$$

$$\mathcal{O}_{17}^{(pp,np)} = (I_{ss})_{ijkm}(T_s)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T} C Q_L^{j\beta}][Q_L^{k\gamma T} C L_{\ell,L}^m][u_R^{\rho T} C d_R^\sigma][u_R^{\delta T} C \ell'_R] \in C_{13}^{(pp,np)}, \quad \text{(D12)}$$

$$\mathcal{O}_{18}^{(pp,np,nn)} = \epsilon_{ij}\epsilon_{km}(T_s)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T}CL_{\ell,L}^{j}][Q_L^{k\beta T}CL_{\ell',L}^{m}][u_R^{\gamma T}Cu_R^{\delta}][d_R^{\rho T}Cd_R^{\sigma}] \in C_{15}^{(pp,np,nn)}, \tag{D13}$$

$$\mathcal{O}_{19}^{(pp,np,nn)} = \epsilon_{ij}\epsilon_{km}(T_s)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T}CL_{\ell,L}^{j}][Q_L^{k\beta T}CL_{\ell',L}^{m}][u_R^{\gamma T}Cd_R^{\delta}][u_R^{\rho T}Cd_R^{\sigma}] \in C_{15}^{(pp,np,nn)}, \tag{D14}$$

$$\mathcal{O}_{20}^{(pp,np,nn)} = \epsilon_{ij}\epsilon_{km}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\rho T}CL_{\ell,L}^{j}][Q_L^{k\sigma T}CL_{\ell',L}^{m}][u_R^{\alpha T}Cd_R^{\beta}][u_R^{\gamma T}Cd_R^{\delta}] \in C_{15}^{(pp,np,nn)}, \tag{D15}$$

$$\mathcal{O}_{21}^{(pp,np,nn)} = \epsilon_{ij}\epsilon_{km}\epsilon_{np}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T}CQ_L^{j\beta}][Q_L^{k\rho T}CL_{\ell,L}^{m}][Q_L^{n\sigma T}CL_{\ell',L}^{p}][u_R^{\gamma T}Cd_R^{\delta}] \in C_{16}^{(pp,np,nn)}, \tag{D16}$$

$$\mathcal{O}_{22}^{(pp,np,nn)} = (I_{ss})_{ijkm}(T_s)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T}CL_{\ell,L}^{j}][Q_L^{k\beta T}CL_{\ell',L}^{m}][u_R^{\gamma T}Cu_R^{\delta}][d_R^{\rho T}Cd_R^{\sigma}] \in C_{15}^{(pp,np,nn)}, \tag{D17}$$

$$\mathcal{O}_{23}^{(pp,np,nn)} = (I_{ss})_{ijkm}(T_s)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T}CL_{\ell,L}^{j}][Q_L^{k\beta T}CL_{\ell',L}^{m}][u_R^{\gamma T}Cd_R^{\delta}][u_R^{\rho T}Cd_R^{\sigma}] \in C_{15}^{(pp,np,nn)}, \tag{D18}$$

$$\mathcal{O}_{24}^{(pp,np)} = (I_{ssa})_{\rm ijkmnp}(T_s)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T}CQ_L^{j\beta}][Q_L^{k\gamma T}CQ_L^{m\delta}][Q_L^{n\sigma T}CL_{\ell,L}^{p}][u_R^{\sigma T}C\ell'_R] \in C_{17}^{(pp,np,nn)}, \tag{D19}$$

$$\mathcal{O}_{25}^{(pp,np,nn)} = (I_{ssa})_{\rm ijkmnp}(T_s)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T}CQ_L^{j\beta}][Q_L^{k\rho T}CL_{\ell,L}^{m}][Q_L^{n\sigma T}CL_{\ell',L}^{p}][u_R^{\gamma T}Cd_R^{\delta}] \in C_{16}^{(pp,np,nn)}, \tag{D20}$$

$$\mathcal{O}_{26}^{(pp,np,nn)} = (I_{ssa})_{\rm kmnpij}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T}CQ_L^{j\beta}][Q_L^{k\rho T}CL_{\ell,L}^{m}][Q_L^{n\sigma T}CL_{\ell',L}^{p}][u_R^{\gamma T}Cd_R^{\delta}] \in C_{16}^{(pp,np,nn)}, \tag{D21}$$

$$\mathcal{O}_{27}^{(pp,np,nn)} = (I_{sss})_{\rm ijkmnp}(T_s)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T}CQ_L^{j\beta}][Q_L^{k\rho T}CL_{\ell,L}^{m}][Q_L^{n\sigma T}CL_{\ell',L}^{p}][u_R^{\gamma T}Cd_R^{\delta}] \in C_{16}^{(pp,np,nn)}, \tag{D22}$$

and

$$\mathcal{O}_{28}^{(pp,np,nn)} = \epsilon_{ij}\epsilon_{km}\epsilon_{np}\epsilon_{st}(T_a)_{\alpha\beta\gamma\delta\rho\sigma}[Q_L^{i\alpha T}CQ_L^{j\beta}][Q_L^{k\gamma T}CQ_L^{m\delta}].[Q_L^{n\rho T}CL_{\tau,L}^{p}][Q_L^{s\sigma T}CL_{\ell,L}^{t}] \in C_{19}^{(pp,np,nn)}. \tag{D23}$$

---

[1] $\mathrm{SU}(2)_L$ instantons break B and L (while conserving $B-L$), as discussed in G. 't Hooft, Phys. Rev. Lett. **37**, 8 (1976); Phys. Rev. D **14**, 3432 (1976). However, at zero temperature and, more generally, at temperatures $T \ll v$, where $v = 246$ GeV is the scale of electroweak symmetry breaking, the effects of these instantons is exponentially suppressed and hence is negligibly small.

[2] A. D. Sakharov, Zh. Eksp. Teor. Fiz. Pis'ma **5**, 32 (1967) [JETP Lett. B **91**, 24 (1967)].

[3] Particle Data Group, Review of Particle Properties, http://pdg.lbl.gov.

[4] V. Kuzmin, Zh. Eksp. Theor. Fiz. Pis'ma **12**, 335 (1970) [JETP Lett. **12**, 228 (1970)].

[5] S. L. Glashow, Report No. HUTP-79/A059.

[6] R. N. Mohapatra and R. E. Marshak, Phys. Rev. Lett. **44**, 1316 (1980).

[7] L.-N. Chang and N.-P. Chang, Phys. Lett. **92B**, 103 (1980); Phys. Rev. Lett. **45**, 1540 (1980); T. K. Kuo and S. Love, Phys. Rev. Lett. **45**, 93 (1980); R. Cowsik and S. Nussinov, Phys. Lett. B **101**, 237 (1981).

[8] S. Rao and R. E. Shrock, Phys. Lett. **116B**, 238 (1982).

[9] S. Rao and R. E. Shrock, Nucl. Phys. **B232**, 143 (1984).

[10] M. Baldo-Ceolin *et al.*, Z. Phys. C **63**, 409 (1994).

[11] M. Takita *et al.* (Kamiokande Collaboration), Phys. Rev. D **34**, 902 (1986).

[12] C. Berger *et al.* (Fréjus Collaboration), Phys. Lett. B **240**, 237 (1990); **269**, 227 (1991).

[13] J. Chung *et al.* (Soudan Collaboration), Phys. Rev. D **66**, 032004 (2002).

[14] T. Araki *et al.* (KamLAND Collaboration), Phys. Rev. Lett. **96**, 101802 (2006).

[15] M. Litos *et al.* (SuperKamiokande Collaboration), Phys. Rev. Lett. **112**, 131803 (2014).

[16] K. Abe *et al.* (SuperKamiokande Collaboration), Phys. Rev. D **91**, 072006 (2015).

[17] J. Gustafson *et al.* (SuperKamiokande Collaboration), Phys. Rev. D **91**, 072009 (2015).

[18] B. Aharmim *et al.* (SNO Collaboration), Phys. Rev. D **96**, 092005 (2017).

[19] D. Bryman, Phys. Lett. B **733**, 190 (2014).

[20] V. Takhistov *et al.* (SuperKamiokande Collaboration), Phys. Rev. Lett. **113**, 101801 (2014).

[21] V. Takhistov *et al.* (SuperKamiokande Collaboration), Phys. Rev. Lett. **115**, 121803 (2015).

[22] T. Araki *et al.* (KamLAND Collaboration), Phys. Rev. Lett. **96**, 101802 (2006).

[23] S. Sussman *et al.* (SuperKamiokande Collaboration), arXiv:1811.12430.
[24] M. Anderson *et al.* (SNO Collaboration), Phys. Rev. D **99**, 032008 (2019).
[25] S. Nussinov and R. Shrock, Phys. Rev. Lett. **88**, 171601 (2002).
[26] N. Arkani-Hamed and M. Schmaltz, Phys. Rev. D **61**, 033005 (2000).
[27] E. A. Mirabelli and M. Schmaltz, Phys. Rev. D **61**, 113011 (2000).
[28] J. M. Arnold, B. Fornal, and M. B. Wise, Phys. Rev. D **87**, 075004 (2013).
[29] Y. Kamyshkov, Nucl. Phys. B, Proc. Suppl. **52A**, 263 (1997).
[30] R. N. Mohapatra, J. Phys. G **36**, 104006 (2009).
[31] D. C. Phillips *et al.*, Phys. Rep. **612**, 1 (2016).
[32] T. Kaluza, Sitzungsber. Preuss. Akad. Wiss., Berlin **1921**, 966 (1921); O. Klein, Z. Phys. **37**, 895 (1926).
[33] V. Rubakov and M. Shaposhnikov, Phys. Lett. **125B**, 136 (1983); D. Kaplan and M. Schmaltz, Phys. Lett. B **368** (1996) 44; G. Dvali and M. Shifman, Phys. Lett. B **475**, 295 (2000); Usp. Fiz. Nauk **171**, 913 (2001) [V. A. Rubakov, Sov. Phys. Usp. **44**, 871 (2001)].
[34] S. Nussinov and R. Shrock, Phys. Lett. B **526**, 137 (2002).
[35] Y. Grossman and G. Perez, Phys. Rev. D **67**, 015011 (2003).
[36] Z. Surujon, Phys. Rev. D **73**, 016008 (2006).
[37] A. Delgado, A. Pomerol, and M. Quiros, J. High Energy Phys. 01 (2000) 030.
[38] G. Barenboim, G. Branco, A. de Gouvêa, and M. Rebelo, Phys. Rev. D **64**, 073005 (2001).
[39] A different class of models features extra dimensions in which SM fields propagate but without strong localization of fermion wave functions [40]. In the model of [40], special symmetries specific to a particular compactification suppress both proton decay and $n-\bar{n}$ oscillations.
[40] T. Appelquist, H.-C. Cheng, and B. Dobrescu, Phys. Rev. D **64**, 035002 (2001); T. Appelquist, B. Dobrescu, E. Ponton, and H.-U. Yee, Phys. Rev. Lett. **87**, 181802 (2001).
[41] The models considered here, as well as those in [40], with SM fields propagating in the large extra dimensions, are to be contrasted with models in which only the gravitons propagate in these dimensions. Many papers have discussed the latter type of model, e.g., N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998); K. Dienes, E. Dudas, and T. Gherghetta, Phys. Lett. B **436**, 55 (1998); S. Nussinov and R. Shrock, Phys. Rev. D **59**, 105002 (1999).
[42] S. Girmohanta and R. Shrock, arXiv:1910.08356.
[43] S. Girmohanta and R. Shrock, Phys. Rev. D **100**, 115025 (2019).
[44] L. S. Littenberg and R. Shrock, Phys. Lett. B **491**, 285 (2000).
[45] H. Fusaoka and Y. Koide, Phys. Rev. D **57**, 3986 (1998).
[46] These values are quite close to the values $\|\eta_{Q_L}-\eta_{u_R}\|=4.87$ and $\|\eta_{Q_L}-\eta_{d_R}\|=4.74$ used in [27]. The resultant ratio $\|\eta_{Q_L}-\eta_{u_R}\|/\|\eta_{Q_L}-\eta_{d_R}\|=1.03$ is the same as we get.
[47] S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979); Phys. Rev. D **22**, 1694 (1980).
[48] F. Wilczek and A. Zee, Phys. Rev. Lett. **43**, 1571 (1979).
[49] H. A. Weldon and A. Zee, Nucl. Phys. **B173**, 269 (1980).
[50] L. F. Abbott and M. B. Wise, Phys. Rev. D **22**, 2208 (1980).
[51] S. Aoki *et al.* (JLQCD Collaboration), Phys. Rev. D **62**, 014506 (2000).
[52] Y. Aoki, E. Shintani, and A. Soni, Phys. Rev. D **89**, 014505 (2014); Y. Aoki, T. Izubuchi, E. Shintani, and A. Soni, Phys. Rev. D **96**, 014506 (2017).
[53] K. Abe *et al.* (SK Collaboration), Phys. Rev. D **95**, 012004 (2017).
[54] K. Abe *et al.* (SK Collaboration), Phys. Rev. D **96**, 012003 (2017).
[55] K. Abe *et al.* (SK Collaboration), Phys. Rev. Lett. **113**, 121802 (2014).
[56] C. McGrew *et al.* (IMB-3 Collaboration), Phys. Rev. D **59**, 052004 (1999).
[57] W.-S. Hou, M. Nagashima, and A. Soddu, Phys. Rev. D **72**, 095001 (2005).
[58] E. Rinaldi, S. Syritsyn, M. L. Wagman, M. I. Buchoff, C. Schroeder, and J. Wasem, Phys. Rev. D **99**, 074510 (2019).
[59] The limit from the KamLAND experiment on $n\to$ inv in [14] was based on a search for γ rays from the resultant ^{10}C nucleus in the liquid scintillator. This method is similar to the use of γ rays from nuclear decays to detect low-energy neutral current events; see, e.g., S. Nussinov and R. Shrock, Phys. Rev. Lett. **86**, 2223 (2001) [alternately, C. H. Albright, R. E. Shrock, and J. Smith, Phys. Lett. **90B**, 465 (1980)].
[60] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **109**, 091803 (2012).
[61] A. Kobach, Phys. Lett. B **758**, 455 (2016).
[62] J. Heeck and V. Takhistov, arXiv:1910.07647.
[63] K. Abe *et al.* (HyperKamiokande Working Group), arXiv:1109.3262; http://hyperk.org.
[64] B. Abi *et al.* (DUNE Collaboration), arXiv:1807.10334.
[65] L. Arnellos and W. J. Marciano, Phys. Rev. Lett. **48**, 1708 (1982).
[66] In referring to an experimental observation of the decay $pp\to\ell^+\ell'^+$, it is implicitly understood that the experiment can discriminate between a decay of this type and a $\Delta L=0$ decay of the form $pp\to\ell^+\ell'^+\chi^\dagger$, where χ is an undetected hypothetical light scalar boson with $L=-2$ (e.g., one type of Majoron). As in searches for neutrinoless double beta decay, this discrimination can be accomplished by using the fact that in the $pp\to\ell^+\ell'^+$ decay, up to small corrections due to Fermi motion, the ℓ^+ and ℓ'^+ would emerge back to back in the rest frame of the nucleus, whereas in contrast, in the $pp\to\ell^+\ell'^+\chi^\dagger$ decay, they would not emerge back to back, except for a set of measure zero in the final-state three-body phase space. A similar comment applies for discrimination against emission of multiple light scalars, some of which could carry nonzero lepton number.
[67] J. B. Albert *et al.* (EXO-200 Collaboration), Phys. Rev. D **97**, 072007 (2018).
[68] S. I. Alvis *et al.* (Majorana Collaboration), Phys. Rev. D **99**, 072004 (2019).