Electroweak and QCD corrections to Z and W pole observables in the standard model EFT

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We compute the next-to-leading order QCD and electroweak corrections to Z and W pole observables using the dimension-6 Standard Model effective field theory (SMEFT) and present numerical results that can easily be included in global fitting programs. Limits on SMEFT coefficient functions are presented at leading order and at next-to-leading order under several assumptions.

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I. INTRODUCTION

The LHC experiments provide strong evidence that the $SU(3)\times SU(2)\times U(1)$ Standard Model (SM) gauge theory describes physics at the electroweak scale [1]. To date, there is no evidence of new interactions or high mass particles. Taken together, these features suggest that the weak scale can be described by an effective field theory (SMEFT) having the SM as its low energy limit. The SMEFT is defined by an infinite tower of dimension-6 and higher operators, involving only the SM particles and assumes that the Higgs boson is part of an $SU(2)$ doublet [2]. The effects of the higher dimension operators are suppressed by powers of a high scale, Λ, and we assume that the most numerically relevant operators are those of dimension-6. All possible new physics phenomena are contained in the coefficient functions.

Numerous studies have been performed extracting limits on the coefficients of dimension-6 operators from global fits to Higgs measurements, vector boson pair production, electroweak measurements at the Z and W poles, top quark measurements, and low energy data [3–10]. Typically, these fits use the most accurately known SM predictions, while the SMEFT effects are treated at lowest order (LO). A program of calculations has begun to treat the SMEFT contributions at next-to-leading order (NLO), for both the QCD and electroweak (EW) contributions. The SMEFT QCD corrections to gauge boson pair production [11,12] and top quark production and decay [13–16] are known. The electroweak SMEFT corrections to Higgs decays to $b\bar{b}$ [17–19], $\gamma\gamma$ [20–23], $Z\gamma$ [24,25], ZZ [24], and WW [20] have also been computed, along with partial corrections to the Drell Yan process [26]. The SMEFT NLO corrections to Z pole decays at NLO are also only partially known [27–29].

In this work, we take a major step by computing the NLO EW and QCD corrections in the SMEFT to Z and W pole observables. We assume flavor universality and use the Warsaw basis [30,31]. We are particularly interested in the numerical effects of the NLO corrections on the global fits. In Sec. II, we review the basics of the SMEFT theory and in Sec. III, we describe our NLO calculations. Our results are given in Sec. IV and Appendix, which contains numerical expressions for the Z and W pole observables, as well as limits on the SMEFT coefficients at LO and NLO. Section V contains some conclusions and a discussion of the implications of our results for global fits.

II. SMEFT BASICS

The SMEFT parametrizes new physics through an expansion in higher dimensional operators,

$$\mathcal{L}=\mathcal{L}_{\rm SM}+\Sigma_{k=5}^{\infty}\Sigma_{i=1}^{n}\frac{\mathcal{C}_i^k}{\Lambda^{k-4}}O_i^k, \qquad (1)$$

where the $SU(3)\times SU(2)_L\times U(1)_Y$ invariant dimension-k operators are constructed from SM fields and all of the effects of the beyond the SM (BSM) physics reside in the coefficient functions, $\mathcal{C}_i^k$. We assume all coefficients are real and do not consider the effects of CP violation. We use the Warsaw basis [30,31] and at tree level (neglecting flavor) there are 10 dimension-6 operators contributing to the Z and W pole observables of our study. These operators are listed in Table I, where ϕ is the $SU(2)_L$ doublet, τ^a are the Pauli matrices, $D_\mu=\partial_\mu+ig_sT^AG_\mu^A+ig_2\frac{\tau^a}{2}W_\mu^a+ig_1YB_\mu$, $q^T=(u_L,d_L)$, $l^T=(\nu_L,e_L)$, $W_{\mu\nu}^a=\partial_\mu W_\nu^a-\partial_\nu W_\mu^a-g_2\epsilon^{abc}W_\mu^bW_\nu^c$, $\phi^\dagger i\overleftrightarrow{D}_\mu\phi=i\phi^\dagger(D_\mu\phi)-i(D_\mu\phi)^\dagger\phi$, and $\phi^\dagger i\overleftrightarrow{D}_\mu^a\phi=i\phi^\dagger\tau^aD_\mu\phi-i(D_\mu\phi)^\dagger\tau^a\phi$.

TABLE I. Dimension-6 operators contributing to the Z and W pole observables of this study at tree level.

$\mathcal{O}_{ll}$	$(\bar{l}\gamma_\mu l)(\bar{l}\gamma^\mu l)$	$\mathcal{O}_{\phi WB}$	$(\phi^\dagger\tau^a\phi)W^a_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{\phi D}$	$(\phi^\dagger D^\mu\phi)^*(\phi^\dagger D_\mu\phi)$
$\mathcal{O}_{\phi e}$	$(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)(\bar{e}_R\gamma^\mu e_R)$	$\mathcal{O}_{\phi u}$	$(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)(\bar{u}_R\gamma^\mu u_R)$	$\mathcal{O}_{\phi d}$	$(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)(\bar{d}_R\gamma^\mu d_R)$
$\mathcal{O}^{(3)}_{\phi q}$	$(\phi^\dagger i\overleftrightarrow{D}^a_\mu\phi)(\bar{q}\tau^a\gamma^\mu q)$	$\mathcal{O}^{(1)}_{\phi q}$	$(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)(\bar{q}\tau^a\gamma^\mu q)$	$\mathcal{O}^{(3)}_{\phi l}$	$(\phi^\dagger i\overleftrightarrow{D}^a_\mu\phi)(\bar{l}\tau^a\gamma^\mu l)$
$\mathcal{O}^{(1)}_{\phi l}$	$(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)(\bar{l}\tau^a\gamma^\mu l)$				

At NLO, there are 22 additional operators that contribute:

$$\mathcal{O}_{ed},\quad \mathcal{O}_{ee},\quad \mathcal{O}_{eu},\quad \mathcal{O}_{lu},\quad \mathcal{O}_{ld},\quad \mathcal{O}_{le},\quad \mathcal{O}^{(1)}_{lq},\quad \mathcal{O}^{(3)}_{lq},\quad \mathcal{O}_{\phi B},\quad \mathcal{O}_{\phi W},\quad \mathcal{O}_{\Box},$$
$$\mathcal{O}_{qe},\quad \mathcal{O}_{uB},\quad \mathcal{O}_{uW},\quad \mathcal{O}_{W},\quad \mathcal{O}^{(1)}_{qd},\quad \mathcal{O}^{(3)}_{qq},\quad \mathcal{O}^{(1)}_{qq},\quad \mathcal{O}^{(1)}_{qu},\quad \mathcal{O}^{(1)}_{ud},\quad \mathcal{O}_{uu},\quad \mathcal{O}_{dd}. \tag{2}$$

Definitions for these operators can be found in Refs. [30,31]. We use the Feynman rules in R_ξ gauge from Ref. [32].

The SMEFT interactions cause the gauge field kinetic energies to have noncanonical normalizations and following Ref. [32], we define "barred" fields and couplings,

$$\begin{aligned}
\bar{W}^a_\mu &\equiv (1-\mathcal{C}_{\phi W}v^2/\Lambda^2)W^a_\mu\\
\bar{B}_\mu &\equiv (1-\mathcal{C}_{\phi B}v^2/\Lambda^2)B_\mu\\
\bar{g}_2 &\equiv (1+\mathcal{C}_{\phi W}v^2/\Lambda^2)g_2\\
\bar{g}_1 &\equiv (1+\mathcal{C}_{\phi B}v^2/\Lambda^2)g_1,
\end{aligned} \tag{3}$$

such that $\bar{W}_\mu\bar{g}_2 = W_\mu g_2$ and $\bar{B}_\mu\bar{g}_1 = B_\mu g_1$. The barred fields have canonically normalized kinetic energy interactions. The masses of the W and Z fields to $\mathcal{O}(\frac{1}{\Lambda^2})$ are [32,33],

$$\begin{aligned}
M_W^2 &= \frac{\bar{g}_2^2 v^2}{4},\\
M_Z^2 &= \frac{(\bar{g}_1^2+\bar{g}_2^2)v^2}{4} + \frac{v^4}{\Lambda^2}\left(\frac{1}{8}(\bar{g}_1^2+\bar{g}_2^2)\mathcal{C}_{\phi D} + \frac{1}{2}\bar{g}_1\bar{g}_2\mathcal{C}_{\phi WB}\right).
\end{aligned} \tag{4}$$

Dimension-6 4-fermion operators give contributions to the decay of the μ, changing the relation between the vacuum expectation value (vev), v, and the Fermi constant G_μ,

$$G_\mu \equiv \frac{1}{\sqrt{2}v^2} - \frac{1}{\sqrt{2}\Lambda^2}\mathcal{C}_{ll} + \frac{\sqrt{2}}{\Lambda^2}\mathcal{C}^{(3)}_{\phi l}. \tag{5}$$

The tree level SMEFT couplings of fermions to the Z and W are given in terms of our input parameters (α, M_Z, G_μ),

$$\begin{aligned}
L \equiv\ & 2M_Z\sqrt{\sqrt{2}G_\mu}Z_\mu[g_L^{Zq}+\delta g_L^{Zq}]\bar{q}\gamma_\mu q + 2M_Z\sqrt{\sqrt{2}G_\mu}Z_\mu[g_R^{Zu}+\delta g_R^{Zu}]\bar{u}_R\gamma_\mu u_R\\
&+ 2M_Z\sqrt{\sqrt{2}G_\mu}Z_\mu[g_R^{Zd}+\delta g_R^{Zd}]\bar{d}_R\gamma_\mu d_R + 2M_Z\sqrt{\sqrt{2}G_\mu}Z_\mu[g_L^{Zl}+\delta g_L^{Zl}]\bar{l}\gamma_\mu l\\
&+ 2M_Z\sqrt{\sqrt{2}G_\mu}Z_\mu[g_R^{Ze}+\delta g_R^{Ze}]\bar{e}_R\gamma_\mu e_R + 2M_Z\sqrt{\sqrt{2}G_\mu}(\delta g_R^{Z\nu})\bar{\nu}_R\gamma_\mu\nu_R\\
&+ \frac{\bar{g}_2}{\sqrt{2}}\{W_\mu[(1+\delta g_L^{Wq})\bar{u}_L\gamma_\mu d_L + (\delta g_R^{Wq})\bar{u}_R\gamma_\mu d_R]\\
&+ W_\mu[(1+\delta g_L^{Wl})\bar{\nu}_L\gamma_\mu e_L + (\delta g_R^{W\nu})\bar{\nu}_R\gamma_\mu e_R] + \text{H.c.}\}.
\end{aligned} \tag{6}$$

We assume all couplings are flavor independent and we neglect Cabbibo-Kobayashi-Maskawa (CKM) mixing. The weak coupling in Eq. (6) is evaluated using the LO SM relation and Eq. (7) serves as the definition of s_W^2,

$$\begin{aligned}
\bar{g}_2^2 &= 2\sqrt{2}G_\mu M_Z^2\left(1+\sqrt{1-\frac{4\pi\alpha}{\sqrt{2}G_\mu M_Z^2}}\right).\\
s_W^2 &\equiv \frac{4\pi\alpha}{\bar{g}_2^2}.
\end{aligned} \tag{7}$$

Since we are working to $\mathcal{O}(\frac{v^2}{\Lambda^2})$, we omit dipole type operators that do not interfere with the SM contributions to Z and W pole observables. Similarly, the contributions from right-handed W couplings and the right-handed $Z\bar{\nu}\nu$ interaction do not contribute to our study. The tree level couplings are,

$$g_R^{Zf} = -s_W^2 Q_f \quad \text{and} \quad g_L^{Zf} = T_3^f - s_W^2 Q_f \tag{8}$$

with $T_3^f = \pm\frac{1}{2}$. $SU(2)$ invariance implies,

$$\begin{aligned}\delta g_L^{Wq} &= \delta g_L^{Zu} - \delta g_L^{Zd}\\ \delta g_L^{Wl} &= \delta g_L^{Z\nu} - \delta g_L^{Ze}.\end{aligned} \tag{9}$$

The SMEFT contributions to the effective couplings are listed in Table II [34].

III. W AND Z POLE OBSERVABLES TO NLO

The observables we consider are

$$M_W,\quad \Gamma_W,\quad \Gamma_Z,\quad \sigma_h,\quad R_l,\quad A_{l,FB},\quad R_b,\quad R_c,\quad A_{FB,b},\quad A_{FB,c},\quad A_b,\quad A_c,\quad A_l. \tag{10}$$

The SM results for these observables are quite precisely known, and as a by product of our study we recover the known NLO QCD and NLO EW results as a check of our calculation [35].

The next-to-leading order contributions to Z and W pole observables require the calculation of one loop virtual diagrams in the SMEFT and in most cases, the contribution also of real photon and gluon emission diagrams. Since the SMEFT theory is renormalizable order by order in the (v^2/Λ^2) expansion, we retain only terms of $\mathcal{O}(v^2/\Lambda^2)$. The one-loop SMEFT calculations contain both tree level and one-loop contributions from the dimension-6 operators, along with the full electroweak and QCD one-loop SM amplitudes. Sample diagrams contributing to the Z decay widths at NLO are shown in Fig. 1. For W decays, there are also dipole-like $\gamma(g)Wf\bar{f}$ contact interactions that we include. (The corresponding $\gamma(g)Zf\bar{f}$ operators first arise at dimension-8.) Since we concentrate on the Z pole physics, we calculate the cross sections, $e^+e^- \to$ hadrons, using the narrow width approximation:

$$\sigma^0_{\text{had}} = \sum_{f=u,d,s,c,b} \frac{12\pi}{M_Z^2}\frac{\Gamma_e\Gamma_f}{\Gamma_Z^2}. \tag{11}$$

TABLE II. Anomalous fermion couplings at LO in the Warsaw [31] basis.

	Warsaw Basis
δg_L^{Zu}	$-\frac{v^2}{2\Lambda^2}(\mathcal{C}_{\phi q}^{(1)} - \mathcal{C}_{\phi q}^{(3)}) + \frac{1}{2}\delta g_Z + \frac{2}{3}(\delta s_W^2 - s_W^2\delta g_Z)$
δg_L^{Zd}	$-\frac{v^2}{2\Lambda^2}(\mathcal{C}_{\phi q}^{(1)} + \mathcal{C}_{\phi q}^{(3)}) - \frac{1}{2}\delta g_Z - \frac{1}{3}(\delta s_W^2 - s_W^2\delta g_Z)$
$\delta g_L^{Z\nu}$	$-\frac{v^2}{2\Lambda^2}(\mathcal{C}_{\phi l}^{(1)} - \mathcal{C}_{\phi l}^{(3)}) + \frac{1}{2}\delta g_Z$
δg_L^{Ze}	$-\frac{v^2}{2\Lambda^2}(\mathcal{C}_{\phi l}^{(1)} + \mathcal{C}_{\phi l}^{(3)}) - \frac{1}{2}\delta g_Z - (\delta s_W^2 - s_W^2\delta g_Z)$
δg_R^{Zu}	$-\frac{v^2}{2\Lambda^2}\mathcal{C}_{\phi u} + \frac{2}{3}(\delta s_W^2 - s_W^2\delta g_Z)$
δg_R^{Zd}	$-\frac{v^2}{2\Lambda^2}\mathcal{C}_{\phi d} - \frac{1}{3}(\delta s_W^2 - s_W^2\delta g_Z)$
δg_R^{Ze}	$-\frac{v^2}{2\Lambda^2}\mathcal{C}_{\phi e} - (\delta s_W^2 - s_W^2\delta g_Z)$
δg_L^{Wq}	$\frac{v^2}{\Lambda^2}\mathcal{C}_{\phi q}^{(3)} + c_W^2\delta g_Z + \delta s_W^2$
δg_L^{Wl}	$\frac{v^2}{\Lambda^2}\mathcal{C}_{\phi l}^{(3)} + c_W^2\delta g_Z + \delta s_W^2$
δg_Z	$-\frac{v^2}{\Lambda^2}(\delta v + \frac{1}{4}\mathcal{C}_{\phi D})$
δv	$\mathcal{C}_{\phi l}^{(3)} - \frac{1}{2}\mathcal{C}_{ll}$
δs_W^2	$-\frac{v^2}{\Lambda^2}\frac{s_W c_W}{c_W^2 - s_W^2}[2 s_W c_W(\delta v + \frac{1}{4}\mathcal{C}_{\phi D}) + \mathcal{C}_{\phi WB}]$

Corrections to this formula are of higher order and we do not include them [36]. Nonresonant contributions, such as photon exchange, box diagrams and 4-fermions interactions [34], are also not included because they do not contribute to the observables on the Z pole to $\mathcal{O}(\frac{1}{\Lambda^2})$.

We employ a modified on-shell (OS) scheme, where the SM parameters are renormalized in the OS scheme. The effective field theory coefficients of the dimension-6 operators are treated as $\overline{\text{MS}}$ parameters and the poles of the one-loop coefficients $\mathcal{C}_i$ are known from Refs. [33,37,38],

$$\mathcal{C}_i(\mu) = \mathcal{C}_{0,i} - \frac{1}{2\hat{\epsilon}}\frac{1}{16\pi^2}\gamma_{ij}\mathcal{C}_j, \tag{12}$$

where μ is the renormalization scale, γ_{ij} is the one-loop anomalous dimension,

$$\mu\frac{d\mathcal{C}_i}{d\mu} = \frac{1}{16\pi^2}\gamma_{ij}\mathcal{C}_j, \tag{13}$$

and $\hat{\epsilon}^{-1} \equiv \epsilon^{-1} - \gamma_E + \log(4\pi)$.

The renormalized SM gauge boson masses are,

$$M_V^2 = M_{0,V}^2 - \Pi_{VV}(M_V^2), \tag{14}$$

where $\Pi_{VV}(M_V^2)$ is the one-loop correction to the 2-point function for Z or W computed on-shell and tree level quantities are denoted with the subscript 0 in this section. The gauge boson 2- point functions in the SMEFT can be found analytically in Refs. [39,40]. The one-loop relation between the vacuum expectation value and the Fermi constant is,

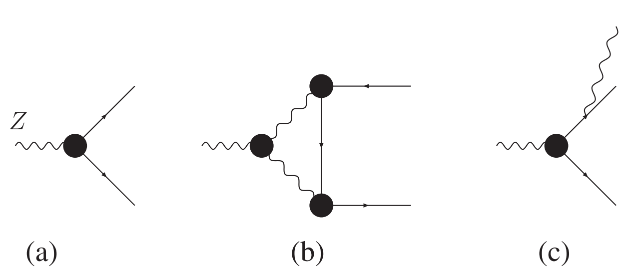

FIG. 1. Sample electroweak diagrams contributing to $Z \to f\bar{f}$ at NLO in the SMEFT: (a) Tree level SMEFT diagram, (b) virtual SMEFT diagram, and (c) real photon emission in the SMEFT. The circles represent potential insertions of dimension-6 SMEFT operators.

$$G_\mu + \frac{\mathcal{C}_{ll}}{\sqrt{2}\Lambda^2} - \sqrt{2}\frac{\mathcal{C}_{\phi l}^{(3)}}{\Lambda^2} \equiv \frac{1}{\sqrt{2}v_0^2}(1+\Delta r), \qquad (15)$$

where v_0 is the unrenormalized minimum of the potential and Δr is obtained from the one-loop corrections to μ decay. Complete analytic expressions for Δr in both the SM and the SMEFT at dimension-6 are given in Ref. [24]. Finally, the on-shell renormalization of α is extracted from the renormalization of the $\bar{l}l\gamma$ vertex.

We obtain the relevant amplitudes for the virtual contributions using FEYNARTS [41] with a model file generated by FEYNRULES [42] and the Feynman rules of Ref. [32]. Then we use FEYNCALC [43,44] to manipulate and reduce the integrals and LOOPTOOLS [45] for the numerical evaluation.

The Z decays to charged fermions receive contributions from one-loop virtual diagrams and from real photon emission that are separately IR divergent and we regulate these divergences with a photon mass. Since we only consider the inclusive quantities of Eq. (10), the photon mass dependence cancels after integration over the photon phase space and there is no need for a photon energy cut. The complicated form of the SMEFT vertices makes direct integration of the phase space difficult, so we use the method of Ref. [46], where the integration over the photon phase space is replaced with a loop integration. This is possible after we use the identity,

$$2i\pi\delta(p^2-m^2) = \frac{1}{p^2-m^2+i0} - \frac{1}{p^2-m^2-i0}. \qquad (16)$$

After making this replacement, we treat the momenta of the outgoing particles as internal loop momenta, the integration over the phase space becomes an integration over the loop momenta and we can use the IBP relations to reduce the loop integrals to known master integrals. In the case of $Z\to f\bar{f}\gamma$, the integrals are 2-point 2-loop integrals, for which a generic basis of master integrals is known [47,48] and the reduction can be done using FIRE [49]. This is identical to the technique we applied in the calculation of the real contributions to $H\to W^+W^-\gamma$ in Ref. [20].

IV. RESULTS

We take as our physical input parameters,

$$\begin{aligned} G_\mu &= 1.1663787(6)\times 10^{-5}\ \text{GeV}^{-2} \\ M_Z &= 91.1876 \pm .0021\ \text{GeV} \\ \frac{1}{\alpha} &= 137.035999139(31) \\ \Delta\alpha_{\text{had}}^{(5)} &= 0.02764 \pm 0.00009 \\ \alpha_s(M_Z) &= 0.1181 \pm 0.0011 \\ M_H &= 125.10 \pm 0.14\ \text{GeV} \\ M_t &= 172.9 \pm 0.5\ \text{GeV}. \end{aligned}$$

The lowest order SMEFT contributions to Z pole observables, $\mathcal{O}_i$ are well known. We write the SMEFT predictions for the observables as,

$$\begin{aligned} O_i^{\text{SMEFT,LO}} &= O_i^{\text{SM}} + \delta O_i^{\text{LO}} \\ O_i^{\text{SMEFT,NLO}} &= O_i^{\text{SM}} + \delta O_i^{\text{NLO}}, \end{aligned} \qquad (17)$$

and we present our results numerically. In Table III, we summarize the current state of the SM theory and the experimental results. The theory errors include the parametric uncertainties on M_t and M_H [36]. In evaluating O_i^{SM} in Eq. (17), we always use the most accurately calculated value given in Table III.

We do not include the effective weak leptonic mixing angle in our fit since it can be directly derived from other observables, but present it here for completeness.

$$\begin{aligned} \delta\sin^2\theta_{l,\text{eff}}^{\text{LO}} &= \frac{v^2}{\Lambda^2}\{-0.28785\mathcal{C}_{\phi e} - 0.21215\mathcal{C}_{\phi l}^{(1)} + 0.36851\mathcal{C}_{\phi l}^{(3)} - 0.29033\mathcal{C}_{ll} + 0.14517\mathcal{C}_{\phi D} + 0.71015\mathcal{C}_{\phi WB}\} \\ \delta\sin^2\theta_{l,\text{eff}}^{\text{NLO}} &= \frac{v^2}{\Lambda^2}\{-0.2726\mathcal{C}_{\phi e} - 0.23666\mathcal{C}_{\phi l}^{(1)} + 0.42246\mathcal{C}_{\phi l}^{(3)} - 0.31904\mathcal{C}_{ll} + 0.16629\mathcal{C}_{\phi D} + 0.77518\mathcal{C}_{\phi WB} \\ &\quad - 0.00036\mathcal{C}_{ed} - 0.00100\mathcal{C}_{ee} + 0.00677\mathcal{C}_{eu} + 0.00161\mathcal{C}_{\phi d} + 0.01033\mathcal{C}_{\phi q}^{(1)} \\ &\quad - 0.00871\mathcal{C}_{\phi q}^{(3)} - 0.01424\mathcal{C}_{\phi u} - 0.00028\mathcal{C}_{ld} - 0.00064\mathcal{C}_{le} - 0.00401\mathcal{C}_{lq}^{(1)} \\ &\quad - 0.00106\mathcal{C}_{lq}^{(3)} + 0.00531\mathcal{C}_{lu} + 0.00032\mathcal{C}_{\phi B} + 0.00004\mathcal{C}_{\phi\Box} + 0.00032\mathcal{C}_{\phi W} \\ &\quad - 0.00512\mathcal{C}_{qe} + 0.01087\mathcal{C}_{uB} + 0.00917\mathcal{C}_{uW} + 0.00053\mathcal{C}_{W}\}. \end{aligned} \qquad (18)$$

The NLO corrections to $\sin^2\theta_{l,\text{eff}}$ change the numerical effects of the coefficients appearing at tree level by $\mathcal{O}(5\text{–}10\%)$, and introduce dependencies on other coefficients.

For the W mass and total width, we find the predictions,

$$
\begin{aligned}
\delta M_W^{\mathrm{LO}} &= \frac{v^2}{\Lambda^2}\{-29.827\mathcal{C}_{\phi l}^{(3)} + 14.914\mathcal{C}_{ll} - 27.691\mathcal{C}_{\phi D} - 57.479\mathcal{C}_{\phi WB}\} \\
\delta M_W^{\mathrm{NLO}} &= \frac{v^2}{\Lambda^2}\{-35.666\mathcal{C}_{\phi l}^{(3)} + 17.243\mathcal{C}_{ll} - 30.272\mathcal{C}_{\phi D} - 64.019\mathcal{C}_{\phi WB} - 0.137\mathcal{C}_{\phi d} - 0.137\mathcal{C}_{\phi e} - 0.166\mathcal{C}_{\phi l}^{(1)} - 2.032\mathcal{C}_{\phi q}^{(1)} \\
&\quad + 1.409\mathcal{C}_{\phi q}^{(3)} + 2.684\mathcal{C}_{\phi u} + 0.438\mathcal{C}_{lq}^{(3)} - 0.027\mathcal{C}_{\phi B} - 0.033\mathcal{C}_{\phi\Box} - 0.035\mathcal{C}_{\phi W} - 0.902\mathcal{C}_{uB} - 0.239\mathcal{C}_{uW} - 0.15\mathcal{C}_{W}\} \\
\delta\Gamma_W^{\mathrm{LO}} &= \frac{v^2}{\Lambda^2}\{-5.092\mathcal{C}_{\phi l}^{(3)} + 2.784\mathcal{C}_{\phi q}^{(3)} + 3.242\mathcal{C}_{ll} - 2.143\mathcal{C}_{\phi D} - 4.448\mathcal{C}_{\phi WB}\} \\
\delta\Gamma_W^{\mathrm{NLO}} &= \frac{v^2}{\Lambda^2}\{-5.597\mathcal{C}_{\phi l}^{(3)} + 3.019\mathcal{C}_{\phi q}^{(3)} + 3.361\mathcal{C}_{ll} - 2.276\mathcal{C}_{\phi D} - 4.811\mathcal{C}_{\phi WB} - 0.01\mathcal{C}_{\phi d} - 0.01\mathcal{C}_{\phi e} - 0.017\mathcal{C}_{\phi l}^{(1)} - 0.153\mathcal{C}_{\phi q}^{(1)} \\
&\quad + 0.203\mathcal{C}_{\phi u} + 0.048\mathcal{C}_{lq}^{(3)} - 0.002\mathcal{C}_{\phi B} - 0.003\mathcal{C}_{\phi\Box} - 0.004\mathcal{C}_{\phi W} - 0.03\mathcal{C}_{qq}^{(1)} - 0.094\mathcal{C}_{qq}^{(3)} - 0.068\mathcal{C}_{uB} \\
&\quad - 0.014\mathcal{C}_{uW} - 0.013\mathcal{C}_{W}\}. \qquad (19)
\end{aligned}
$$

It is interesting to note that some of the contributions to the W mass and width change by more than 10% when going from LO to NLO in the SMEFT. The NLO SMEFT contributions to the other observables of Eq. (10) given in Sec. IV.

We fit to the experimental data given in Table III, (omitting $\sin^2\theta_{\mathrm{eff}}$) since it can be directly derived from other observables). The most accurate SM predictions are given in the right-hand column and we use these values in our fits, as opposed to the LO or NLO SM contributions directly calculated. The pole observables we consider are [60–62]:

$$M_W, \quad \Gamma_W, \quad \Gamma_Z, \quad \sigma_h, \quad R_l, \quad A_{l,FB}, \quad R_b, \quad R_c, \quad A_{FB,b}, \quad A_{FB,c}, \quad A_b, \quad A_c, \quad A_l, \qquad (20)$$

where we assume lepton universality and the experimental correlations can be found in Ref. [53]. We include the measurements of A_l from LEP and SLD as separate data points.

The χ^2 is computed from,

$$\chi^2 = \Sigma_{i,j}(O_i^{\mathrm{exp}} - O_i^{\mathrm{SMEFT}})\sigma_{ij}^{-2}(O_j^{\mathrm{exp}} - O_j^{\mathrm{SMEFT}}). \qquad (21)$$

TABLE III. Experimental results and SM predictions for W and Z pole observables, assuming lepton universality The theory includes the full set of 2-loop contributions for the Z pole observables, along with higher order corrections when known. When not specified, the numbers are taken from Table 10.5 of the electroweak review of Ref. [50]. The theory predictions are computed using the formulas in the indicated references and our input parameters, and the theory errors include the parametric uncertainties on M_t and M_H [36], along with the estimated theory uncertainties described in the respective papers.

Measurement	Experiment	"Best" theory
Γ_Z(GeV)	2.4952 ± 0.0023	2.4945 ± 0.0006 [36,51,52]
σ_h(nb)	41.540 ± 0.037	41.491 ± 0.008 [36,51,52]
R_l	20.767 ± 0.025 [53]	20.749 ± 0.009 [36,51,52]
R_b	0.21629 ± 0.00066	0.21586 ± 0.0001 [36,51,52]
R_c	0.1721 ± 0.0030	0.17221 ± 0.00005 [36,51,52]
A_l	0.1465 ± 0.0033 [53]	0.1472 ± 0.0004 [36,54]
A_c	0.670 ± 0.027	0.6679 ± 0.0002 [36,54]
A_b	0.923 ± 0.020	0.92699 ± 0.00006 [36,54,55]
$A_{l,FB}$	0.0171 ± 0.0010	0.0162 ± 0.0001 [36,54]
$A_{b,FB}$	0.0992 ± 0.0016	0.1023 ± 0.0003 [36,54,55]
$A_{c,FB}$	0.0707 ± 0.0035	0.0737 ± 0.0003 [36,54]
$A_l(SLD)$	0.1513 ± 0.0021 [53]	0.1472 ± 0.0004 [36,54]
$\sin^2\theta_{l,\mathrm{eff}}$	0.23179 ± 0.00035 [56]	0.23150 ± 0.00006 [36,54,55]
M_W(GeV)	80.379 ± 0.012 [50]	80.359 ± 0.006 [57,58]
Γ_W(GeV)	2.085 ± 0.042 [50]	2.0904 ± 0.0003 [59]

Using the LO SMEFT expressions for the observables and taking $\Lambda = 1$ TeV, we find,[1]

$$\chi^2_{\rm LO} = \chi^2_{\rm SM} + 32\mathcal{C}_{\phi d} + 105\mathcal{C}_{\phi e} - 445\mathcal{C}^{(1)}_{\phi l} + 639\mathcal{C}^{(3)}_{\phi l} - 49\mathcal{C}^{(1)}_{\phi q} - 60\mathcal{C}^{(3)}_{\phi q} - 11\mathcal{C}_{\phi u} - 424\mathcal{C}_{ll} + 491\mathcal{C}_{\phi D} + 1114\mathcal{C}_{\phi WB} + \vec{\mathcal{C}}^T_{\rm LO} M_{\rm LO} \vec{\mathcal{C}}_{\rm LO} \tag{22}$$

where

$$\vec{\mathcal{C}}^T_{\rm LO} = (\mathcal{C}_{ll}, \mathcal{C}_{\phi WB}, \mathcal{C}_{\phi u}, \mathcal{C}^{(3)}_{\phi q}, \mathcal{C}^{(1)}_{\phi q}, \mathcal{C}^{(3)}_{\phi l}, \mathcal{C}^{(1)}_{\phi l}, \mathcal{C}_{\phi e}, \mathcal{C}_{\phi D}, \mathcal{C}_{\phi d}) \tag{23}$$

and we find $\chi^2_{\rm SM} \sim 13.42$. The symmetric matrix $M_{\rm LO}$ is,

$$M_{\rm LO} = \begin{pmatrix} 25279 & -108322 & 1799 & 14960 & 4513 & -71171 & 27975 & 16835 & -37889 & -831 \\ & 148456 & -851 & -11405 & -3882 & 165479 & -51962 & -55619 & 102746 & -629 \\ & & 574 & 6873 & 1615 & -7314 & -6662 & 3620 & -899 & -697 \\ & & & 24474 & 13826 & -54867 & -45834 & 27540 & -7486 & -5161 \\ & & & & 3097 & -15840 & -12754 & 8236 & -2257 & -1569 \\ & & & & & 70369 & 18870 & -56402 & 60803 & 4835 \\ & & & & & & 58121 & -44390 & -13987 & 5382 \\ & & & & & & & 31734 & -8417 & -2193 \\ & & & & & & & & 21176 & 415 \\ & & & & & & & & & 318 \end{pmatrix}. \tag{24}$$

Using the NLO SMEFT expressions we find $\chi^2_{\rm NLO}$, (for $\Lambda = 1$ TeV),

$$\begin{aligned}\chi^2_{\rm NLO} = \chi^2_{\rm SM} &- 403\mathcal{C}_{ll} + 1070\mathcal{C}_{\phi WB} - 53\mathcal{C}_{\phi u} - 93\mathcal{C}^{(3)}_{\phi q} - 19\mathcal{C}^{(1)}_{\phi q} + 667\mathcal{C}^{(3)}_{\phi l} - 402\mathcal{C}^{(1)}_{\phi l} + 176\mathcal{C}_{\phi e} + 503\mathcal{C}_{\phi D} + 27\mathcal{C}_{\phi d} \\ &- 1.48\mathcal{C}^{(1)}_{qq} + 0.55\mathcal{C}_{\phi \Box} + 0.62\mathcal{C}_{\phi W} + 0.48\mathcal{C}_{\phi B} + 6.55\mathcal{C}_{uW} + 15\mathcal{C}_{uB} + 0.23\mathcal{C}_{ed} + 0.063\mathcal{C}_{dd} + 0.56\mathcal{C}_{ee} + 1.40\mathcal{C}^{(3)}_{qq} \\ &+ 2.38\mathcal{C}_{W} + 0.53\mathcal{C}_{uu} - 0.54\mathcal{C}^{(1)}_{ud} + 1.05\mathcal{C}^{(1)}_{qu} - 4.88\mathcal{C}^{(3)}_{lq} + 2.8\mathcal{C}_{qe} + 0.34\mathcal{C}^{(1)}_{qd} + 9.8\mathcal{C}_{lu} - 0.32\mathcal{C}_{le} - 0.49\mathcal{C}_{ld} \\ &- 3.8\mathcal{C}_{eu} - 7.4\mathcal{C}^{(1)}_{lq} + \vec{\mathcal{C}}^T_{\rm NLO} M_{\rm NLO} \vec{\mathcal{C}}_{\rm NLO},\end{aligned} \tag{25}$$

where,

$$\begin{aligned}\vec{\mathcal{C}}^T_{\rm NLO} = (&\mathcal{C}_{ll}, \mathcal{C}_{\phi WB}, \mathcal{C}_{\phi u}, \mathcal{C}^{(3)}_{\phi q}, \mathcal{C}^{(1)}_{\phi q}, \mathcal{C}^{(3)}_{\phi l}, \mathcal{C}^{(1)}_{\phi l}, \mathcal{C}_{\phi e}, \mathcal{C}_{\phi D}, \mathcal{C}_{\phi d}, \\ &\mathcal{C}_{ed}, \mathcal{C}_{ee}, \mathcal{C}_{eu}, \mathcal{C}_{lu}, \mathcal{C}_{ld}, \mathcal{C}_{le}, \mathcal{C}^{(1)}_{lq}, \mathcal{C}^{(3)}_{lq}, \mathcal{C}_{\phi B}, \mathcal{C}_{\phi W}, \mathcal{C}_{\phi \Box}, \\ &\mathcal{C}_{qe}, \mathcal{C}_{uB}, \mathcal{C}_{uW}, \mathcal{C}_{W}, \mathcal{C}^{(1)}_{qd}, \mathcal{C}^{(3)}_{qq}, \mathcal{C}^{(1)}_{qq}, \mathcal{C}^{(1)}_{qu}, \mathcal{C}^{(1)}_{ud}, \mathcal{C}_{uu}, \mathcal{C}_{dd}),\end{aligned} \tag{26}$$

where the numerical form of $M_{\rm NLO}$ is given in the Supplemental Material [65]. At NLO, the χ^2 now depends on 32 coefficients, and the effects of the coefficients appearing at LO have shifted by 5%–10%. The (relatively) large shift of the coefficients of $\mathcal{C}_{\phi u}$ and $\mathcal{C}^{(1,3)}_{\phi Q}$ are due to the top quark loop.

To study the numerical importance of the NLO effects, we begin by keeping only one coefficient nonzero at a time. We find the 95% confidence level regions at LO and NLO shown in Tables IV and V. The largest effect of the NLO corrections is on the coefficient of $\mathcal{C}_{\phi u}$. The relatively large allowed values for $\mathcal{C}_{\phi d}$ are the result of the discrepancy in the measured value $A_{FB,b}$ from the SM prediction.

[1] Our results are consistent with SMEFT fits to purely LEP observables using slightly different sets of inputs [9,60–62].

TABLE IV. 95% confidence level allowed ranges for single parameter fit to coefficients contributing to the lowest order predictions. The scale Λ is taken to be 1 TeV.

Coefficient	LO	NLO
$\mathcal{C}_{ll}$	$[-0.0039, 0.021]$	$[-0.0044, 0.019]$
$\mathcal{C}_{\phi WB}$	$[-0.0088, 0.0013]$	$[-0.0079, 0.0016]$
$\mathcal{C}_{\phi u}$	$[-0.072, 0.091]$	$[-0.035, 0.084]$
$\mathcal{C}^{(3)}_{\phi q}$	$[-0.011, 0.014]$	$[-0.010, 0.014]$
$\mathcal{C}^{(1)}_{\phi q}$	$[-0.027, 0.043]$	$[-0.031, 0.036]$
$\mathcal{C}^{(3)}_{\phi l}$	$[-0.012, 0.0029]$	$[-0.010, 0.0028]$
$\mathcal{C}^{(1)}_{\phi l}$	$[-0.0043, 0.012]$	$[-0.0047, 0.012]$
$\mathcal{C}_{\phi e}$	$[-0.013, 0.0094]$	$[-0.013, 0.0080]$
$\mathcal{C}_{\phi D}$	$[-0.025, 0.0019]$	$[-0.023, 0.0023]$
$\mathcal{C}_{\phi d}$	$[-0.16, 0.060]$	$[-0.13, 0.063]$

At lowest order, the $\chi^2_{\rm LO}$ is sensitive to 8 combinations of operators, implying that there are 2 blind directions [62–64]. These 8 combinations can be thought of as the combinations of operators contributing to δg_L^{Zu}, δg_L^{Zd}, δg_L^{Ze}, $\delta g_L^{Z\nu}$, δg_R^{Zu}, δg_R^{Zd}, δg_R^{Ze}, and M_W. Because of the $SU(2)$ symmetry of Eq. (9), at LO there is no additional information from Γ_W. Since our study includes only 14 data points, we clearly cannot fit to all of the SMEFT coefficients appearing at one loop At NLO, the fit is sensitive to only 10 combinations of operators. The additional information can be thought of as coming from δg_L^{Zb} and Γ_W where the top quark makes significant contributions. Since there are 32 coefficients that contribute to the NLO fit to the electroweak observables, resolving these 22 blind directions requires input from other processes and/or assumptions about which operators can be safely neglected.

We chose to perform our fits setting $\mathcal{C}_{\phi e} = 0$ and $\mathcal{C}^{(3)}_{\phi q} = 0$, along with setting all of the operators that first appear at NLO to 0. We then marginalize over the remaining operators to study the numerical impacts of the NLO contributions. These results are shown in Table VI. We see that the effects of the NLO corrections can be significant, although the numerical results are sensitive to which operators are set to 0. Our results suggest that including the NLO corrections in the global fits (where the complete set of operators can potentially be bounded) may be important.

TABLE VI. 95% confidence level allowed ranges for fit to coefficients marginalizing over the other 7 operators we are considering. The coefficients of all operators not listed in the table are set to 0. The scale Λ is taken to be 1 TeV.

Coefficient	LO	NLO
$\mathcal{C}_{\phi D}$	$[-0.034, 0.041]$	$[-0.039, 0.051]$
$\mathcal{C}_{\phi WB}$	$[-0.080, 0.0021]$	$[-0.098, 0.012]$
$\mathcal{C}_{\phi d}$	$[-0.81, -0.093]$	$[-1.07, -0.03]$
$\mathcal{C}^{(3)}_{\phi l}$	$[-0.025, 0.12]$	$[-0.039, 0.16]$
$\mathcal{C}_{\phi u}$	$[-0.12, 0.37]$	$[-0.21, 0.41]$
$\mathcal{C}^{(1)}_{\phi l}$	$[-0.0086, 0.036]$	$[-0.0072, 0.037]$
$\mathcal{C}_{ll}$	$[-0.085, 0.035]$	$[-0.087, 0.033]$
$\mathcal{C}^{(1)}_{\phi q}$	$[-0.060, 0.076]$	$[-0.095, 0.075]$

As another way of examining the impact of the NLO contributions, we consider the oblique parameters. The tree level SMEFT contributions are,

$$\alpha\Delta S = 4c_W s_W \frac{v^2}{\Lambda^2}\mathcal{C}_{\phi WB}$$
$$\alpha\Delta T = -\frac{v^2}{2\Lambda^2}\mathcal{C}_{\phi D}. \tag{27}$$

For the NLO oblique parameter fit, we set all coefficients to 0, except $\mathcal{C}_{\phi WB}$ and $\mathcal{C}_{\phi D}$. The resulting limits are shown in Fig. 2, (where what we are really plotting are the limits on the coefficients from a 2 parameter fit to our observables). In this example, the effect of the NLO SMEFT corrections is small. At NLO, new coefficients can influence the oblique parameters, and the complete one-loop SMEFT result is given in Ref. [39].

Our calculation includes only the resonant Z and W contributions to the precision electroweak observables. In

TABLE V. 95% confidence level allowed ranges for single parameter fit to coefficients not contributing to the lowest order predictions. The scale Λ is taken to be 1 TeV.

Coefficient	NLO	Coefficient	NLO	Coefficient	NLO
$\mathcal{C}_{W}$	$[-4.8, 0.48]$	$\mathcal{C}_{uu}$	$[-1.1, 0.99]$	$\mathcal{C}_{uW}$	$[-0.78, 0.29]$
$\mathcal{C}_{uB}$	$[-0.57, 0.11]$	$\mathcal{C}^{(1)}_{qu}$	$[-2.2, 1.3]$	$\mathcal{C}^{(3)}_{qq}$	$[-0.32, 0.29]$
$\mathcal{C}^{(1)}_{qq}$	$[-0.93, 1.49]$	$\mathcal{C}_{qe}$	$[-0.75, 0.48]$	$\mathcal{C}^{(1)}_{qd}$	$[-9.8, 5.0]$
$\mathcal{C}_{\phi\Box}$	$[-22, 1.9]$	$\mathcal{C}_{\phi W}$	$[-17, 2.2]$	$\mathcal{C}_{\phi B}$	$[-19, 3.3]$
$\mathcal{C}_{lu}$	$[-0.49, 0.19]$	$\mathcal{C}^{(3)}_{lq}$	$[-0.32, 0.57]$	$\mathcal{C}^{(1)}_{lq}$	$[-0.25, 0.66]$
$\mathcal{C}_{le}$	$[-5.3, 11]$	$\mathcal{C}_{ld}$	$[-3.8, 8.7]$	$\mathcal{C}_{dd}$	$[-51, 26]$
$\mathcal{C}_{ed}$	$[-12, 6.7]$	$\mathcal{C}_{ee}$	$[-3.9, 2.4]$	$\mathcal{C}_{eu}$	$[-0.36, 0.58]$
$\mathcal{C}^{(1)}_{ud}$	$[-3.0, 5.6]$				

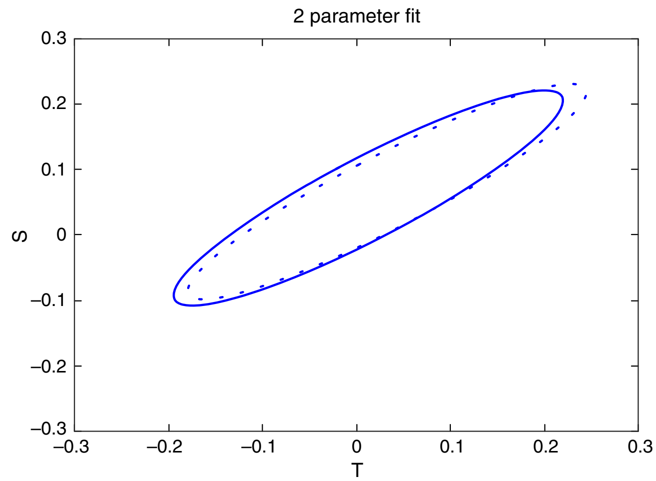

FIG. 2. 95% CL limits from a 2 parameter fit to $\mathcal{C}_{\phi WB}$ and $\mathcal{C}_{\phi D}$, setting all other coefficients to 0. The scale $\Lambda = 1$ TeV.The solid line is the result of the LO fit, while the dotted line is the NLO fit to the electroweak parameters of this study.

the SMEFT, there are tree level nonresonant contributions due to 4-fermion operators that can complicate the experimental extraction of the widths from the data. The size of these effects has been estimated in Ref. [34]. For example,

$$\delta\Gamma(Z \to \text{hadrons}) \sim 0.6\ \text{MeV} C_{4f}\left(\frac{1\ \text{TeV}}{\Lambda}\right)^2, \quad (28)$$

where C_{4f} is a generic 4-fermion operator. These effects could potentially be similar in size to the electroweak corrections we have computed. There are also off-shell corrections proportional to $\frac{q^2}{\Lambda^2}$, where q^2 is the momentum running through the Z boson propagator. The experimental cuts [53] were designed to extract predominantly the on-shell Z events, so although the size of these effects is expected to be small a detailed theoretical study would be needed to quantify them.

V. CONCLUSIONS

We have computed the NLO electroweak and QCD corrections to the SMEFT predictions for the precision electroweak observables. Our results are presented in a numerical form that can easily be incorporated in the global fitting programs. We also present numerical results for the LO and NLO χ^2 that can be customized for the reader's use. Our studies suggest that the NLO SMEFT corrections may have a sizable effect on the global fits. Numerical results for the SMEFT NLO expressions for the observables considered here, along with the $\chi^2_{\text{LO}}, \chi^2_{\text{NLO}}$ and the matrix M_{NLO}, are posted at https://quark.phy.bnl.gov/Digital_Data_Archive/dawson/ewpo_19.

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APPENDIX: OBSERVABLES TO LO AND NLO IN THE SMEFT

In this Appendix, we report the contributions to the observables of Table II using the definitions of Eq. (17). The contributions to the Z width are,

$$\delta\Gamma(Z \to \nu\bar{\nu})^{\text{LO}} = \frac{v^2}{\Lambda^2}\{-0.3318\mathcal{C}^{(1)}_{\phi l} + 0.1659\mathcal{C}_{ll} - 0.0829\mathcal{C}_{\phi D}\}$$

$$\begin{aligned}\delta\Gamma(Z \to \nu\bar{\nu})^{\text{NLO}} = \frac{v^2}{\Lambda^2}\{&-0.3446\mathcal{C}^{(1)}_{\phi l} + 0.1640\mathcal{C}_{ll} - 0.0853\mathcal{C}_{\phi D} - 0.0003\mathcal{C}_{\phi d} - 0.0003\mathcal{C}_{\phi e} - 0.0018\mathcal{C}^{(3)}_{\phi l} - 0.0073\mathcal{C}^{(1)}_{\phi q}\\ &+ 0.0054\mathcal{C}^{(3)}_{\phi q} + 0.0083\mathcal{C}_{\phi u} - 0.0004\mathcal{C}_{ld} - 0.0004\mathcal{C}_{le} - 0.0061\mathcal{C}^{(1)}_{lq} - 0.0061\mathcal{C}^{(3)}_{lq} + 0.008\mathcal{C}_{lu}\\ &- 0.0002\mathcal{C}_{\phi\Box} - 0.0001\mathcal{C}_{\phi W} + 0.0063\mathcal{C}_{\phi WB} + 0.0001\mathcal{C}_{uW} - 0.0001\mathcal{C}_{W}\}\end{aligned}$$

$$\delta\Gamma(Z \to l^+l^-)^{\text{LO}} = \frac{v^2}{\Lambda^2}\{-0.1408\mathcal{C}_{\phi e} + 0.191\mathcal{C}^{(1)}_{\phi l} - 0.037\mathcal{C}^{(3)}_{\phi l} + 0.114\mathcal{C}_{ll} - 0.057\mathcal{C}_{\phi D} - 0.0713\mathcal{C}_{\phi WB}\}$$

$$\begin{aligned}\delta\Gamma(Z \to l^+l^-)^{\text{NLO}} = \frac{v^2}{\Lambda^2}\{&-0.1597\mathcal{C}_{\phi e} + 0.1834\mathcal{C}^{(1)}_{\phi l} - 0.0221\mathcal{C}^{(3)}_{\phi l} + 0.0985\mathcal{C}_{ll} - 0.0508\mathcal{C}_{\phi D} - 0.0349\mathcal{C}_{\phi WB} - 0.0001\mathcal{C}_{\phi W}\\ &- 0.0002\mathcal{C}_{ed} - 0.0005\mathcal{C}_{ee} + 0.0035\mathcal{C}_{eu} - 0.0002\mathcal{C}_{\phi d} - 0.0042\mathcal{C}^{(1)}_{\phi q} + 0.0032\mathcal{C}^{(3)}_{\phi q} + 0.0049\mathcal{C}_{\phi u}\\ &+ 0.0002\mathcal{C}_{ld} + 0.0001\mathcal{C}_{le} + 0.0034\mathcal{C}^{(1)}_{lq} - 0.0031\mathcal{C}^{(3)}_{lq} - 0.0045\mathcal{C}_{lu} - 0.0001\mathcal{C}_{\phi\Box} - 0.0027\mathcal{C}_{qe}\\ &- 0.0007\mathcal{C}_{uB} - 0.0007\mathcal{C}_{uW} - 0.0001\mathcal{C}_{W}\}\end{aligned}$$

$$\begin{aligned}
\delta\Gamma(Z\to u\bar{u})^{\rm LO} &= \delta\Gamma(Z\to c\bar{c})^{\rm LO}\\
&= \frac{v^2}{\Lambda^2}\{-0.9261\mathcal{C}_{\phi l}^{(3)} - 0.7138\mathcal{C}_{\phi q}^{(1)} + 0.7138\mathcal{C}_{\phi q}^{(3)} + 0.2815\mathcal{C}_{\phi u} + 0.4631\mathcal{C}_{ll} - 0.2315\mathcal{C}_{\phi D} - 0.4093\mathcal{C}_{\phi WB}\}\\
\delta\Gamma(Z\to u\bar{u})^{\rm NLO} &= \delta\Gamma(Z\to c\bar{c})^{\rm NLO}\\
&= \frac{v^2}{\Lambda^2}\{-0.9697\mathcal{C}_{\phi l}^{(3)} - 0.7619\mathcal{C}_{\phi q}^{(1)} + 0.7537\mathcal{C}_{\phi q}^{(3)} + 0.3539\mathcal{C}_{\phi u} + 0.4673\mathcal{C}_{ll} - 0.2421\mathcal{C}_{\phi D} - 0.4049\mathcal{C}_{\phi WB}\\
&\quad + 0.0004\mathcal{C}_{eu} - 0.0013\mathcal{C}_{\phi d} - 0.0013\mathcal{C}_{\phi e} - 0.0022\mathcal{C}_{\phi l}^{(1)} - 0.0009\mathcal{C}_{lq}^{(1)} + 0.0099\mathcal{C}_{lq}^{(3)} + 0.0004\mathcal{C}_{lu}\\
&\quad - 0.0002\mathcal{C}_{\phi B} - 0.0003\mathcal{C}_{\phi\Box} - 0.0004\mathcal{C}_{\phi W} - 0.0009\mathcal{C}_{qd}^{(1)} - 0.0009\mathcal{C}_{qe} - 0.0448\mathcal{C}_{qq}^{(1)} - 0.0629\mathcal{C}_{qq}^{(3)}\\
&\quad + 0.0223\mathcal{C}_{qu}^{(1)} - 0.006\mathcal{C}_{uB} + 0.0004\mathcal{C}_{ud}^{(1)} - 0.0221\mathcal{C}_{uu} - 0.0049\mathcal{C}_{uW} - 0.0005\mathcal{C}_{W}\}\\
\delta\Gamma(Z\to d\bar{d})^{\rm LO} &= \delta\Gamma(Z\to s\bar{s})^{\rm LO}\\
&= \frac{v^2}{\Lambda^2}\{-0.1408\mathcal{C}_{\phi d} - 1.0299\mathcal{C}_{\phi l}^{(3)} + 0.8545\mathcal{C}_{\phi q}^{(1)} + 0.8545\mathcal{C}_{\phi q}^{(3)} + 0.5149\mathcal{C}_{ll} - 0.2575\mathcal{C}_{\phi D} - 0.3379\mathcal{C}_{\phi WB}\}\\
\delta\Gamma(Z\to d\bar{d})^{\rm NLO} &= \delta\Gamma(Z\to s\bar{s})^{\rm NLO}\\
&= \frac{v^2}{\Lambda^2}\{-0.1671\mathcal{C}_{\phi d} - 1.1143\mathcal{C}_{\phi l}^{(3)} + 0.8889\mathcal{C}_{\phi q}^{(1)} + 0.9222\mathcal{C}_{\phi q}^{(3)} + 0.5371\mathcal{C}_{ll} - 0.2779\mathcal{C}_{\phi D} - 0.361\mathcal{C}_{\phi WB}\\
&\quad - 0.0004\mathcal{C}_{dd} - 0.0002\mathcal{C}_{ed} - 0.0013\mathcal{C}_{\phi e} - 0.0024\mathcal{C}_{\phi l}^{(1)} + 0.0255\mathcal{C}_{\phi u} - 0.0002\mathcal{C}_{ld} + 0.0011\mathcal{C}_{lq}^{(1)}\\
&\quad + 0.011\mathcal{C}_{lq}^{(3)} - 0.0002\mathcal{C}_{\phi B} - 0.0004\mathcal{C}_{\phi\Box} - 0.0004\mathcal{C}_{\phi W} - 0.0016\mathcal{C}_{qd}^{(1)} + 0.0011\mathcal{C}_{qe} + 0.0293\mathcal{C}_{qq}^{(1)}\\
&\quad - 0.0118\mathcal{C}_{qq}^{(3)} - 0.0205\mathcal{C}_{qu}^{(1)} - 0.0053\mathcal{C}_{uB} + 0.0035\mathcal{C}_{ud}^{(1)} - 0.0041\mathcal{C}_{uW} - 0.0005\mathcal{C}_{W}\}\\
\delta\Gamma(Z\to b\bar{b})^{\rm LO} &= \frac{v^2}{\Lambda^2}\{-0.1400\mathcal{C}_{\phi d} - 1.0242\mathcal{C}_{\phi l}^{(3)} + 0.8498\mathcal{C}_{\phi q}^{(1)} + 0.8498\mathcal{C}_{\phi q}^{(3)} + 0.5121\mathcal{C}_{ll} - 0.2561\mathcal{C}_{\phi D} - 0.3361\mathcal{C}_{\phi WB}\}\\
\delta\Gamma(Z\to b\bar{b})^{\rm NLO} &= \frac{v^2}{\Lambda^2}\{-0.1662\mathcal{C}_{\phi d} - 1.0751\mathcal{C}_{\phi l}^{(3)} + 0.8795\mathcal{C}_{\phi q}^{(1)} + 0.8861\mathcal{C}_{\phi q}^{(3)} + 0.5177\mathcal{C}_{ll} - 0.268\mathcal{C}_{\phi D} - 0.3441\mathcal{C}_{\phi WB}\\
&\quad - 0.0004\mathcal{C}_{dd} - 0.0002\mathcal{C}_{ed} - 0.0013\mathcal{C}_{\phi e} - 0.0023\mathcal{C}_{\phi l}^{(1)} + 0.0222\mathcal{C}_{\phi u} - 0.0002\mathcal{C}_{ld} + 0.0011\mathcal{C}_{lq}^{(1)}\\
&\quad + 0.0109\mathcal{C}_{lq}^{(3)} - 0.0002\mathcal{C}_{\phi B} - 0.0004\mathcal{C}_{\phi\Box} - 0.0004\mathcal{C}_{\phi W} - 0.0016\mathcal{C}_{qd}^{(1)} + 0.0011\mathcal{C}_{qe} + 0.0292\mathcal{C}_{qq}^{(1)}\\
&\quad - 0.0117\mathcal{C}_{qq}^{(3)} - 0.0204\mathcal{C}_{qu}^{(1)} - 0.0069\mathcal{C}_{uB} + 0.0035\mathcal{C}_{ud}^{(1)} - 0.0168\mathcal{C}_{uW} - 0.0027\mathcal{C}_{W}\}.
\end{aligned}$$

The SMEFT contributions to the total Z width are,

$$\begin{aligned}
\delta\Gamma_Z^{\rm LO} &= \frac{v^2}{\Lambda^2}\{-0.4223\mathcal{C}_{\phi d} - 0.4223\mathcal{C}_{\phi e} - 0.4223\mathcal{C}_{\phi l}^{(1)} - 5.053\mathcal{C}_{\phi l}^{(3)} + 1.1361\mathcal{C}_{\phi q}^{(1)} + 3.9911\mathcal{C}_{\phi q}^{(3)} + 0.5631\mathcal{C}_{\phi u} + 3.3106\mathcal{C}_{ll}\\
&\quad - 1.6553\mathcal{C}_{\phi D} - 2.0463\mathcal{C}_{\phi WB}\}\\
\delta\Gamma_Z^{\rm NLO} &= \frac{v^2}{\Lambda^2}\{-0.5044\mathcal{C}_{\phi d} - 0.4864\mathcal{C}_{\phi e} - 0.4948\mathcal{C}_{\phi l}^{(1)} - 5.315\mathcal{C}_{\phi l}^{(3)} + 1.0991\mathcal{C}_{\phi q}^{(1)} + 4.2636\mathcal{C}_{\phi q}^{(3)} + 0.8207\mathcal{C}_{\phi u} + 3.3141\mathcal{C}_{ll}\\
&\quad - 1.7165\mathcal{C}_{\phi D} - 1.9615\mathcal{C}_{\phi WB} - 0.0013\mathcal{C}_{dd} - 0.0011\mathcal{C}_{ed} - 0.0016\mathcal{C}_{ee} + 0.0113\mathcal{C}_{eu} - 0.0011\mathcal{C}_{ld} - 0.0011\mathcal{C}_{le}\\
&\quad - 0.0065\mathcal{C}_{lq}^{(1)} + 0.025\mathcal{C}_{lq}^{(3)} + 0.0113\mathcal{C}_{lu} - 0.0014\mathcal{C}_{\phi B} - 0.0028\mathcal{C}_{\phi\Box} - 0.0026\mathcal{C}_{\phi W} - 0.0065\mathcal{C}_{qd}^{(1)} - 0.0065\mathcal{C}_{qe}\\
&\quad - 0.0017\mathcal{C}_{qq}^{(1)} - 0.161\mathcal{C}_{qq}^{(3)} - 0.0168\mathcal{C}_{qu}^{(1)} - 0.0318\mathcal{C}_{uB} + 0.0113\mathcal{C}_{ud}^{(1)} - 0.0443\mathcal{C}_{uu} - 0.0365\mathcal{C}_{uW} - 0.0054\mathcal{C}_{W}\}.
\end{aligned}$$

The ratios are defined to be

$$R_l = \frac{\Sigma_q\Gamma(Z\to q\bar{q})}{\Gamma(Z\to ll)} \qquad R_c = \frac{\Gamma(Z\to u\bar{u})}{\Sigma_q\Gamma(Z\to q\bar{q})} \qquad R_b = \frac{\Gamma(Z\to d\bar{d})}{\Sigma_q\Gamma(Z\to q\bar{q})},$$

and the SMEFT contributions are,

$$\begin{aligned}
\delta R_l^{\rm LO} &= \frac{v^2}{\Lambda^2}\{-4.978\mathcal{C}_{\phi d}+33.673\mathcal{C}_{\phi e}-45.688\mathcal{C}_{\phi l}^{(1)}-49.393\mathcal{C}_{\phi l}^{(3)}+13.39\mathcal{C}_{\phi q}^{(1)}+47.041\mathcal{C}_{\phi q}^{(3)}+6.637\mathcal{C}_{\phi u}+1.853\mathcal{C}_{ll}\\
&\quad -0.926\mathcal{C}_{\phi D}-4.532\mathcal{C}_{\phi WB}\}\\
\delta R_l^{\rm NLO} &= \frac{v^2}{\Lambda^2}\{-5.926\mathcal{C}_{\phi d}+39.228\mathcal{C}_{\phi e}-45.528\mathcal{C}_{\phi l}^{(1)}-56.848\mathcal{C}_{\phi l}^{(3)}+14.509\mathcal{C}_{\phi q}^{(1)}+49.697\mathcal{C}_{\phi q}^{(3)}+8.104\mathcal{C}_{\phi u}+5.621\mathcal{C}_{ll}\\
&\quad -2.962\mathcal{C}_{\phi D}-13.432\mathcal{C}_{\phi WB}-0.015\mathcal{C}_{dd}+0.038\mathcal{C}_{ed}+0.124\mathcal{C}_{ee}-0.834\mathcal{C}_{eu}-0.063\mathcal{C}_{ld}-0.012\mathcal{C}_{le}-0.795\mathcal{C}_{lq}^{(1)}\\
&\quad +1.362\mathcal{C}_{lq}^{(3)}+1.083\mathcal{C}_{lu}-0.004\mathcal{C}_{\phi B}-0.002\mathcal{C}_{\phi\Box}-0.005\mathcal{C}_{\phi W}-0.077\mathcal{C}_{qd}^{(1)}+0.654\mathcal{C}_{qe}-0.020\mathcal{C}_{qq}^{(1)}-1.898\mathcal{C}_{qq}^{(3)}\\
&\quad -0.198\mathcal{C}_{qu}^{(1)}-0.168\mathcal{C}_{uB}+0.133\mathcal{C}_{ud}^{(1)}-0.522\mathcal{C}_{uu}-0.254\mathcal{C}_{uW}-0.037\mathcal{C}_{W}\}\\
\delta R_c^{\rm LO} &= \frac{v^2}{\Lambda^2}\{0.0421\mathcal{C}_{\phi d}-0.0449\mathcal{C}_{\phi l}^{(3)}-0.5279\mathcal{C}_{\phi q}^{(1)}+0.0164\mathcal{C}_{\phi q}^{(3)}+0.1073\mathcal{C}_{\phi u}+0.0224\mathcal{C}_{ll}-0.0112\mathcal{C}_{\phi D}-0.0549\mathcal{C}_{\phi WB}\}\\
\delta R_c^{\rm NLO} &= \frac{v^2}{\Lambda^2}\{0.0490\mathcal{C}_{\phi d}-0.0383\mathcal{C}_{\phi l}^{(3)}-0.5499\mathcal{C}_{\phi q}^{(1)}+0.0138\mathcal{C}_{\phi q}^{(3)}+0.1263\mathcal{C}_{\phi u}+0.0184\mathcal{C}_{ll}-0.0097\mathcal{C}_{\phi D}-0.0469\mathcal{C}_{\phi WB}\\
&\quad +0.0001\mathcal{C}_{dd}+0.0001\mathcal{C}_{ed}+0.0001\mathcal{C}_{eu}-0.0001\mathcal{C}_{\phi e}-0.0001\mathcal{C}_{\phi l}^{(1)}+0.0001\mathcal{C}_{ld}-0.0007\mathcal{C}_{lq}^{(1)}+0.0005\mathcal{C}_{lq}^{(3)}\\
&\quad +0.0001\mathcal{C}_{lu}+0.0001\mathcal{C}_{qd}^{(1)}-0.0007\mathcal{C}_{qe}-0.0258\mathcal{C}_{qq}^{(1)}-0.0205\mathcal{C}_{qq}^{(3)}+0.0146\mathcal{C}_{qu}^{(1)}-0.0005\mathcal{C}_{uB}-0.0009\mathcal{C}_{ud}^{(1)}\\
&\quad -0.0084\mathcal{C}_{uu}+0.0006\mathcal{C}_{uW}+0.0002\mathcal{C}_{W}\}\\
\delta R_b^{\rm LO} &= \frac{v^2}{\Lambda^2}\{-0.02808\mathcal{C}_{\phi d}+0.02993\mathcal{C}_{\phi l}^{(3)}+0.3519\mathcal{C}_{\phi q}^{(1)}-0.01094\mathcal{C}_{\phi q}^{(3)}-0.07156\mathcal{C}_{\phi u}-0.01497\mathcal{C}_{ll}+0.00748\mathcal{C}_{\phi D}\\
&\quad +0.03661\mathcal{C}_{\phi WB}\}\\
\delta R_b^{\rm NLO} &= \frac{v^2}{\Lambda^2}\{-0.03300\mathcal{C}_{\phi d}+0.03298\mathcal{C}_{\phi l}^{(3)}+0.36473\mathcal{C}_{\phi q}^{(1)}-0.01690\mathcal{C}_{\phi q}^{(3)}-0.08461\mathcal{C}_{\phi u}-0.01596\mathcal{C}_{ll}+0.00835\mathcal{C}_{\phi D}\\
&\quad +0.03495\mathcal{C}_{\phi WB}-0.00008\mathcal{C}_{dd}-0.00004\mathcal{C}_{ed}-0.00009\mathcal{C}_{eu}+0.00007\mathcal{C}_{\phi e}+0.00010\mathcal{C}_{\phi l}^{(1)}-0.00004\mathcal{C}_{ld}\\
&\quad +0.00044\mathcal{C}_{lq}^{(1)}-0.00034\mathcal{C}_{lq}^{(3)}-0.00009\mathcal{C}_{lu}+0.00001\mathcal{C}_{\phi B}+0.00001\mathcal{C}_{\phi W}-0.00008\mathcal{C}_{qd}^{(1)}+0.00044\mathcal{C}_{qe}\\
&\quad +0.01717\mathcal{C}_{qq}^{(1)}+0.01367\mathcal{C}_{qq}^{(3)}-0.00972\mathcal{C}_{qu}^{(1)}-0.00028\mathcal{C}_{uB}+0.00060\mathcal{C}_{ud}^{(1)}+0.00562\mathcal{C}_{uu}-0.00534\mathcal{C}_{uW}\\
&\quad -0.00098\mathcal{C}_{W}\}.\\
\sigma_h^{\rm LO} &= \frac{v^2}{\Lambda^2}\{4.05\mathcal{C}_{\phi d}-55.524\mathcal{C}_{\phi e}+109.235\mathcal{C}_{\phi l}^{(1)}+32.796\mathcal{C}_{\phi l}^{(3)}-10.896\mathcal{C}_{\phi q}^{(1)}-38.278\mathcal{C}_{\phi q}^{(3)}-5.4\mathcal{C}_{\phi u}+4.319\mathcal{C}_{ll}\\
&\quad -2.16\mathcal{C}_{\phi D}-10.565\mathcal{C}_{\phi WB}\}\\
\sigma_h^{\rm NLO} &= \frac{v^2}{\Lambda^2}\{4.692\mathcal{C}_{\phi d}-62.858\mathcal{C}_{\phi e}+106.834\mathcal{C}_{\phi l}^{(1)}+40.991\mathcal{C}_{\phi l}^{(3)}-11.605\mathcal{C}_{\phi q}^{(1)}-39.315\mathcal{C}_{\phi q}^{(3)}-6.315\mathcal{C}_{\phi u}-1.355\mathcal{C}_{ll}\\
&\quad +0.829\mathcal{C}_{\phi D}+3.311\mathcal{C}_{\phi WB}+0.012\mathcal{C}_{dd}-0.068\mathcal{C}_{ed}-0.205\mathcal{C}_{ee}+1.382\mathcal{C}_{eu}+0.142\mathcal{C}_{ld}+0.063\mathcal{C}_{le}+1.945\mathcal{C}_{lq}^{(1)}\\
&\quad -1.101\mathcal{C}_{lq}^{(3)}-2.598\mathcal{C}_{lu}-0.001\mathcal{C}_{\phi B}+0.002\mathcal{C}_{\phi\Box}+0.063\mathcal{C}_{qd}^{(1)}-1.064\mathcal{C}_{qe}+0.017\mathcal{C}_{qq}^{(1)}+1.544\mathcal{C}_{qq}^{(3)}+0.161\mathcal{C}_{qu}^{(1)}\\
&\quad -0.009\mathcal{C}_{uB}-0.108\mathcal{C}_{ud}^{(1)}+0.424\mathcal{C}_{uu}+0.067\mathcal{C}_{uW}+0.027\mathcal{C}_{W}\}
\end{aligned}$$

The asymmetries are defined as,

$$A_l=\frac{\Gamma(Z\to e_L^+e_L^-)-\Gamma(Z\to e_R^+e_R^-)}{\Gamma(Z\to e^+e^-)}\qquad A_c=\frac{\Gamma(Z\to u_L\bar{u}_L)-\Gamma(Z\to u_R\bar{u}_R)}{\Gamma(Z\to u\bar{u})}\qquad A_b=\frac{\Gamma(Z\to d_L\bar{d}_L)-\Gamma(Z\to d_R\bar{d}_R)}{\Gamma(Z\to d\bar{d})},$$

and the SMEFT contributions are,

$$\delta A_l^{\mathrm{LO}} = \frac{v^2}{\Lambda^2}\{2.1503\mathcal{C}_{\phi e} + 1.5848\mathcal{C}_{\phi l}^{(1)} - 2.7529\mathcal{C}_{\phi l}^{(3)} + 2.1689\mathcal{C}_{ll} - 1.0844\mathcal{C}_{\phi D} - 5.305\mathcal{C}_{\phi WB}\}$$

$$\begin{aligned}\delta A_l^{\mathrm{NLO}} = \frac{v^2}{\Lambda^2}\{&+2.1666\mathcal{C}_{\phi e} + 1.8745\mathcal{C}_{\phi l}^{(1)} - 3.3587\mathcal{C}_{\phi l}^{(3)} + 2.5342\mathcal{C}_{ll} - 1.3250\mathcal{C}_{\phi D} - 6.1599\mathcal{C}_{\phi WB} + 0.0027\mathcal{C}_{ed} + 0.0076\mathcal{C}_{ee}\\ &- 0.0518\mathcal{C}_{eu} - 0.0128\mathcal{C}_{\phi d} - 0.0867\mathcal{C}_{\phi q}^{(1)} + 0.0719\mathcal{C}_{\phi q}^{(3)} + 0.1190\mathcal{C}_{\phi u} + 0.0021\mathcal{C}_{ld} + 0.0049\mathcal{C}_{le} + 0.0307\mathcal{C}_{lq}^{(1)}\\ &+ 0.0098\mathcal{C}_{lq}^{(3)} - 0.0406\mathcal{C}_{lu} - 0.0026\mathcal{C}_{\phi B} - 0.0004\mathcal{C}_{\phi\Box} - 0.0026\mathcal{C}_{\phi W} + 0.0392\mathcal{C}_{qe} - 0.0866\mathcal{C}_{uB}\\ &- 0.0711\mathcal{C}_{uW} - 0.0046\mathcal{C}_{W}\}\end{aligned}$$

$$\delta A_c^{\mathrm{LO}} = \frac{v^2}{\Lambda^2}\{-1.779\mathcal{C}_{\phi l}^{(3)} - 0.65\mathcal{C}_{\phi q}^{(1)} + 0.65\mathcal{C}_{\phi q}^{(3)} - 1.648\mathcal{C}_{\phi u} + 0.889\mathcal{C}_{ll} - 0.445\mathcal{C}_{\phi D} - 2.175\mathcal{C}_{\phi WB}\}$$

$$\begin{aligned}\delta A_c^{\mathrm{NLO}} = \frac{v^2}{\Lambda^2}\{&-2.295\mathcal{C}_{\phi l}^{(3)} - 0.867\mathcal{C}_{\phi q}^{(1)} + 0.856\mathcal{C}_{\phi q}^{(3)} - 1.798\mathcal{C}_{\phi u} + 1.110\mathcal{C}_{ll} - 0.581\mathcal{C}_{\phi D} - 2.699\mathcal{C}_{\phi WB} - 0.002\mathcal{C}_{eu}\\ &- 0.006\mathcal{C}_{\phi d} - 0.006\mathcal{C}_{\phi e} - 0.007\mathcal{C}_{\phi l}^{(1)} - 0.001\mathcal{C}_{lq}^{(1)} + 0.025\mathcal{C}_{lq}^{(3)} - 0.002\mathcal{C}_{lu} - 0.001\mathcal{C}_{\phi B} - 0.001\mathcal{C}_{\phi W}\\ &- 0.001\mathcal{C}_{qd}^{(1)} - 0.001\mathcal{C}_{qe} - 0.045\mathcal{C}_{qq}^{(1)} - 0.063\mathcal{C}_{qq}^{(3)} - 0.014\mathcal{C}_{qu}^{(1)} - 0.037\mathcal{C}_{uB} - 0.002\mathcal{C}_{ud}^{(1)} + 0.13\mathcal{C}_{uu}\\ &- 0.03\mathcal{C}_{uW} - 0.002\mathcal{C}_{W}\}\end{aligned}$$

$$\delta A_b^{\mathrm{LO}} = \frac{v^2}{\Lambda^2}\{0.727\mathcal{C}_{\phi d} - 0.328\mathcal{C}_{\phi l}^{(3)} + 0.12\mathcal{C}_{\phi q}^{(1)} + 0.12\mathcal{C}_{\phi q}^{(3)} + 0.164\mathcal{C}_{ll} - 0.082\mathcal{C}_{\phi D} - 0.401\mathcal{C}_{\phi WB}\}$$

$$\begin{aligned}\delta A_b^{\mathrm{NLO}} = \frac{v^2}{\Lambda^2}\{&+0.842\mathcal{C}_{\phi d} - 0.424\mathcal{C}_{\phi l}^{(3)} + 0.149\mathcal{C}_{\phi q}^{(1)} + 0.157\mathcal{C}_{\phi q}^{(3)} + 0.205\mathcal{C}_{ll} - 0.107\mathcal{C}_{\phi D} - 0.501\mathcal{C}_{\phi WB} + 0.002\mathcal{C}_{dd}\\ &+ 0.001\mathcal{C}_{ed} - 0.001\mathcal{C}_{\phi e} - 0.001\mathcal{C}_{\phi l}^{(1)} + 0.009\mathcal{C}_{\phi u} + 0.001\mathcal{C}_{ld} + 0.005\mathcal{C}_{lq}^{(3)} + 0.014\mathcal{C}_{qd}^{(1)} + 0.005\mathcal{C}_{qq}^{(1)}\\ &- 0.002\mathcal{C}_{qq}^{(3)} - 0.003\mathcal{C}_{qu}^{(1)} - 0.007\mathcal{C}_{uB} - 0.018\mathcal{C}_{ud}^{(1)} - 0.007\mathcal{C}_{uW} - 0.001\mathcal{C}_{W}.\}\end{aligned}$$

Finally, the forward backward asymmetries are defined as

$$A_{FB,i} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \tag{A1}$$

where defining θ to be the angle between the incoming l^- and the outgoing $\bar{f}_i$, σ_F has θ between $(0, \frac{\pi}{2})$ and σ_B has θ between $(\frac{\pi}{2}, \pi)$. The SMEFT results are,

$$A_{FB,l}^{\mathrm{LO}} = \frac{v^2}{\Lambda^2}\{0.9547\mathcal{C}_{\phi e} + 0.7037\mathcal{C}_{\phi l}^{(1)} - 1.2223\mathcal{C}_{\phi l}^{(3)} + 0.9630\mathcal{C}_{ll} - 0.4815\mathcal{C}_{\phi D} - 2.3555\mathcal{C}_{\phi WB}\}$$

$$\begin{aligned}A_{FB,l}^{\mathrm{NLO}} = \frac{v^2}{\Lambda^2}\{&+0.4783\mathcal{C}_{\phi e} + 0.4138\mathcal{C}_{\phi l}^{(1)} - 0.7414\mathcal{C}_{\phi l}^{(3)} + 0.5594\mathcal{C}_{ll} - 0.2925\mathcal{C}_{\phi D} - 1.3598\mathcal{C}_{\phi WB} + 0.0006\mathcal{C}_{ed} + 0.0017\mathcal{C}_{ee}\\ &- 0.0114\mathcal{C}_{eu} - 0.0028\mathcal{C}_{\phi d} - 0.0191\mathcal{C}_{\phi q}^{(1)} + 0.0159\mathcal{C}_{\phi q}^{(3)} + 0.0263\mathcal{C}_{\phi u} + 0.0005\mathcal{C}_{ld} + 0.0011\mathcal{C}_{le} + 0.0068\mathcal{C}_{lq}^{(1)}\\ &+ 0.0022\mathcal{C}_{lq}^{(3)} - 0.009\mathcal{C}_{lu} - 0.0006\mathcal{C}_{\phi B} - 0.0001\mathcal{C}_{\phi\Box} - 0.0006\mathcal{C}_{\phi W} + 0.0086\mathcal{C}_{qe} - 0.0191\mathcal{C}_{uB} - 0.0157\mathcal{C}_{uW}\\ &- 0.0010\mathcal{C}_{W}\}\end{aligned}$$

$$\begin{aligned}A_{FB,c}^{\mathrm{LO}} = \frac{v^2}{\Lambda^2}\{&1.1785\mathcal{C}_{\phi e} + 0.8686\mathcal{C}_{\phi l}^{(1)} - 1.9036\mathcal{C}_{\phi l}^{(3)} - 0.1443\mathcal{C}_{\phi q}^{(1)} + 0.1443\mathcal{C}_{\phi q}^{(3)} - 0.3658\mathcal{C}_{\phi u} + 1.3861\mathcal{C}_{ll} - 0.693\mathcal{C}_{\phi D}\\ &- 3.3903\mathcal{C}_{\phi WB}\}\end{aligned}$$

$$A_{FB,c}^{\rm NLO} = \frac{v^2}{\Lambda^2}\{+1.0846\mathcal{C}_{\phi e} + 0.9381\mathcal{C}_{\phi l}^{(1)} - 1.9356\mathcal{C}_{\phi l}^{(3)} - 0.1391\mathcal{C}_{\phi q}^{(1)} + 0.1305\mathcal{C}_{\phi q}^{(3)} - 0.1388\mathcal{C}_{\phi u} + 1.3918\mathcal{C}_{ll} - 0.7278\mathcal{C}_{\phi D}$$
$$- 3.3833\mathcal{C}_{\phi WB} + 0.0014\mathcal{C}_{ed} + 0.0038\mathcal{C}_{ee} - 0.0262\mathcal{C}_{eu} - 0.007\mathcal{C}_{\phi d} + 0.0011\mathcal{C}_{ld} + 0.0024\mathcal{C}_{le} + 0.0153\mathcal{C}_{lq}^{(1)}$$
$$+ 0.0076\mathcal{C}_{lq}^{(3)} - 0.0206\mathcal{C}_{lu} - 0.0014\mathcal{C}_{\phi B} - 0.0002\mathcal{C}_{\phi\Box} - 0.0014\mathcal{C}_{\phi W} - 0.0001\mathcal{C}_{qd}^{(1)} + 0.0195\mathcal{C}_{qe} - 0.0050\mathcal{C}_{qq}^{(1)}$$
$$- 0.0070\mathcal{C}_{qq}^{(3)} - 0.0016\mathcal{C}_{qu}^{(1)} - 0.0475\mathcal{C}_{uB} - 0.0002\mathcal{C}_{ud}^{(1)} + 0.0143\mathcal{C}_{uu} - 0.0389\mathcal{C}_{uW} - 0.0025\mathcal{C}_{W}\}$$

$$A_{FB,b}^{\rm LO} = \frac{v^2}{\Lambda^2}\{0.1615\mathcal{C}_{\phi d} + 1.5275\mathcal{C}_{\phi e} + 1.1258\mathcal{C}_{\phi l}^{(1)} - 2.0284\mathcal{C}_{\phi l}^{(3)} + 0.0266\mathcal{C}_{\phi q}^{(1)} + 0.0266\mathcal{C}_{\phi q}^{(3)} + 1.5771\mathcal{C}_{ll} - 0.7886\mathcal{C}_{\phi D}$$
$$- 3.8576\mathcal{C}_{\phi WB}\}$$

$$A_{FB,b}^{\rm NLO} = \frac{v^2}{\Lambda^2}\{+0.0840\mathcal{C}_{\phi d} + 1.5062\mathcal{C}_{\phi e} + 1.3031\mathcal{C}_{\phi l}^{(1)} - 2.3819\mathcal{C}_{\phi l}^{(3)} - 0.0439\mathcal{C}_{\phi q}^{(1)} + 0.0673\mathcal{C}_{\phi q}^{(3)} + 1.7845\mathcal{C}_{ll} - 0.9331\mathcal{C}_{\phi D}$$
$$- 4.3379\mathcal{C}_{\phi WB} + 0.0002\mathcal{C}_{dd} + 0.0020\mathcal{C}_{ed} + 0.0053\mathcal{C}_{ee} - 0.0360\mathcal{C}_{eu} + 0.0838\mathcal{C}_{\phi u} + 0.0016\mathcal{C}_{ld} + 0.0034\mathcal{C}_{le}$$
$$+ 0.0214\mathcal{C}_{lq}^{(1)} + 0.0073\mathcal{C}_{lq}^{(3)} - 0.0283\mathcal{C}_{lu} - 0.0018\mathcal{C}_{\phi B} - 0.0003\mathcal{C}_{\phi\Box} - 0.0018\mathcal{C}_{\phi W} + 0.0015\mathcal{C}_{qd}^{(1)} + 0.0273\mathcal{C}_{qe}$$
$$+ 0.0005\mathcal{C}_{qq}^{(1)} - 0.0002\mathcal{C}_{qq}^{(3)} - 0.0003\mathcal{C}_{qu}^{(1)} - 0.0610\mathcal{C}_{uB} - 0.0020\mathcal{C}_{ud}^{(1)} - 0.0502\mathcal{C}_{uW} - 0.0033\mathcal{C}_{W}\}.$$

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[65] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevD.101.013001 which contains text expressions for the LO and NLO observables described in the text.