Comment on "Generalized near horizon extreme binary black hole geometry"

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It is demonstrated that the near–horizon geometry of two extreme Kerr black holes of equal mass, which are held finite distance apart by a massless strut, introduced recently in [Phys. Rev. D **100**, 044033 (2019)], is a particular member of the near horizon Kerr–Bolt class.

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The near horizon geometries of the extreme black holes in diverse dimensions attracted recently considerable attention (for a review see [1]). In this context, the key feature is the enhanced isometry group which involves the conformal factor $SL(2,R)$. Denoting the temporal, radial, and azimuthal coordinates by t, r, and ϕ_i, $i=1,\ldots,n$, one can represent the $SL(2,R)$–transformations in the form

$$t'=t+\alpha;$$
$$t'=t+\beta t,\qquad r'=r-\beta r;$$
$$t'=t+\left(t^2+\frac{1}{r^2}\right)\gamma,\qquad r'=r-2tr\gamma,\qquad \phi_i'=\phi_i-\frac{2}{r}\gamma,\tag{1}$$

where the infinitesimal parameters α, β, and γ correspond to the time translation, dilatation and special conformal transformation, respectively. The latitudinal coordinates remain inert under the action of the conformal group. The transformations (1) proved crucial for various physical applications including the Kerr/CFT–correspondence [2], the construction of novel superconformal mechanics models [3,4], and the study of new integrable systems associated with black hole backgrounds [5,6].

Typically, a near horizon geometry is constructed by implementing a specific limit to a given extreme black hole configuration [1]. Yet, one can turn the logic around and use the conformal invariants associated with (1)

$$r^2dt^2-\frac{dr^2}{r^2},\qquad rdt+d\phi_i,\qquad d\phi_i-d\phi_j,\tag{2}$$

so as to build a Ricci–flat metric. It suffices to consider the most general quadratic form built in terms of (2) and involving arbitrary coefficient functions depending on the latitudinal coordinates only, and to impose the Einstein equations. In four dimensions, the analysis can be carried out in full generality [7] which reproduces the near horizon Kerr–Bolt metric [8]

$$ds^2=a(\theta)\left(-r^2dt^2+\frac{dr^2}{r^2}+d\theta^2\right)+b(\theta)(rdt+d\phi)^2,$$
$$a(\theta)=L_1(1+\cos^2\theta)+L_2\cos\theta,\quad b(\theta)=\frac{(4L_1^2-L_2^2)\sin^2\theta}{a(\theta)},\tag{3}$$

where θ is the latitudinal angular variable and L_1, L_2 are free parameters.[1] Thus, the $SL(2,R)$ invariance alone allows one to unambiguously fix the NUT–charge extension of the $4d$ near horizon extreme Kerr (NHEK) geometry [9].

In a very recent work [10] (see also [11]), the near–horizon geometry of two extreme Kerr black holes of equal mass, which are held finite distance apart by a massless strut, was introduced

$$ds^2=\Gamma(\Theta)\Bigg[-R^2dT^2+\frac{dR^2}{R^2}+d\Theta^2+\Lambda^2(\Theta)\left(d\Phi+\frac{\sqrt{11}-\sqrt{3}}{2}RdT\right)^2\Bigg],$$
$$\Gamma(\Theta)=\frac{2(3\sqrt{33}-13)\cos\Theta+(15-\sqrt{33})(3+\cos2\Theta)}{16},$$
$$\Gamma(\Theta)\Lambda^2(\Theta)=\frac{256\sin^2\Theta}{4(-59+11\sqrt{33})\cos\Theta+(93-13\sqrt{33})(3+\cos2\Theta)}.\tag{4}$$

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[1] In order to guarantee the Lorentzian signature, one has to demand $4L_1^2>L_2^2$.

Because the parent metric [12] was rather complicated, the authors of [10] fixed free parameters of the original binary system so as to implement the near horizon limit in the most tractable way. The line element (4) was interpreted as describing the NHEK black hole pierced by the cosmic string/conical singularity, which extends all the way from the horizon to infinity. It was called the pierced–NHEK geometry [10].

It was also argued in [10] that (4) has the $SL(2,R)\times U(1)$ isometry group. Yet, if (3) were the most general metric constructed from the $SL(2,R)$ invariants, (4) would belong to that class. Let us demonstrate that this is indeed the case.

Taking into account the elementary relations $3+\cos 2\Theta = 2(1+\cos^2\Theta)$, $\frac{93-13\sqrt{33}}{15-\sqrt{33}} - \frac{2(-59+11\sqrt{33})}{3\sqrt{33}-13} = 0$, redefining the azimuthal angular variable $\frac{2\Phi}{\sqrt{11-\sqrt{3}}} = \tilde{\Phi}$, and identifying the coordinates $(T,R,\Theta,\tilde{\Phi})$ with (t,r,θ,ϕ) above, one finds that (4) is a particular member of the two-parameter family (3) which occurs at

$$L_1 = \frac{15-\sqrt{33}}{8}, \qquad L_2 = \frac{3\sqrt{33}-13}{8}. \qquad (5)$$

Thus, we have demonstrated that the pierced–NHEK geometry introduced recently in [10] is a particular member of the well-known near horizon extreme Kerr–Bolt class [8].

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[1] H. K. Kunduri and J. Lucietti, Classification of near–horizon geometries of extremal black holes, Living Rev. Relativity **16**, 8 (2013).

[2] M. Guica, T. Hartman, W. Song, and A. Strominger, The Kerr/CFT correspondence, Phys. Rev. D **80**, 124008 (2009).

[3] A. Galajinsky, $N=4$ superconformal mechanics from the $su(2)$ perspective, J. High Energy Phys. 02 (2015) 091.

[4] A. Galajinsky, Couplings in $D(2,1;\alpha)$ superconformal mechanics from the $SU(2)$ perspective, J. High Energy Phys. 03 (2017) 054.

[5] T. Hakobyan, A. Nersessian, and M. M. Sheikh–Jabbari, Near horizon extremal Myers-Perry black holes and integrability of associated conformal mechanics, Phys. Lett. B **772**, 586 (2017).

[6] H. Demirchian, A. Nersessian, S. Sadeghian, and M. M. Sheikh-Jabbari, Integrability of geodesics in near-horizon extremal geometries: Case of Myers-Perry black holes in arbitrary dimensions, Phys. Rev. D **97**, 104004 (2018).

[7] A. Galajinsky and K. Orekhov, On the near horizon rotating black hole geometries with NUT charges, Eur. Phys. J. C **76**, 477 (2016).

[8] A. M. Ghezelbash, Kerr–Bolt spacetimes and Kerr/CFT correspondence, Mod. Phys. Lett. A **27**, 1250046 (2012).

[9] J. M. Bardeen and G. T. Horowitz, The extreme Kerr throat geometry: A vacuum analog of $AdS_2\times S^2$, Phys. Rev. D **60**, 104030 (1999).

[10] J. Ciafre, S. Hadar, E. Rickenbach, and M. J. Rodriguez, Generalized near horizon extreme binary black hole geometry, Phys. Rev. D **100**, 044033 (2019).

[11] J. Ciafre and M. J. Rodriguez, A near horizon extreme binary black hole geometry, Eur. Phys. J. C **79**, 754 (2019).

[12] V. S. Manko and E. Ruiz, On a simple representation of the Kinnersley-Chitre metric, Prog. Theor. Phys. **125**, 1241 (2011).