Revisiting scalar and tensor perturbations in a nonlocal gravity

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Nonlocal RT gravity is a successful modified gravity theory, which not only explains the late-time cosmic acceleration but also behaves well in the Solar System. Previous analysis generally assumes the auxiliary field S_i vanishes at the cosmic background. However, we find the background S_i is proportional to a^2 with the expansion of the Universe. Then, we discuss the influence of the nonzero background S_i on the cosmic background evolution and the scalar and tensor perturbations. We find the cosmic background evolution is independent of S_i, and the influence of the nonzero background S_i on the weak field limit at Solar System scales is negligible. For the tensor perturbation, we find the only possible observable effect is the influence of nonzero background S_i on the LIGO gravitational wave amplitude and also luminosity distance. Future high-redshift gravitational wave observations could be used to constrain the background value of S_i.

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I. INTRODUCTION

Nonlocal RT gravity is proposed by Ref. [1] to explain the late-time cosmic acceleration. Unlike the classical way of modifying gravity, Ref. [1] does not write down the Lagrangian but directly proposes the modified field equation

$$G_{\mu\nu} - \frac{m^2}{3}(g_{\mu\nu}\Box^{-1}R)^{\mathrm{T}} = \kappa T_{\mu\nu}, \tag{1.1}$$

where the constant $m = \mathcal{O}(H_0/c)$. The dimension of m is length^{-1}. Note that until now we still do not know what kind of the Lagrangian corresponds to the above field equation. For the cosmological applications of the nonlocal RT gravity, Refs. [1,2] analyzed the cosmological background evolution,and Refs. [3–5] gave detailed cosmological perturbation analyses with a modified CLASS Boltzmann code. The observational constraints obtained in Refs. [3–5] show that the nonlocal RT gravity and ΛCDM model perform equally well in cosmology. For the performance of the nonlocal RT gravity at Solar System scales, Refs. [3,6] show that the weak field limit of the nonlocal RT gravity gives the Poisson equation, $\Psi = \Phi$ and $G = \text{const.}$, which means this theory can explain the dynamics of the Solar System as general relativity does. This is an very important property, as Refs. [7,8] pointed out most of the nonlocal gravity theories cannot explain the dynamics of the Solar System, e.g., the original Deser-Woodard theory [9], nonlocal RR gravity [10], nonlocal Gauss-Bonnet gravity [11], and the scalar-tensor nonlocal gravity [12]. But note that the nonlocal RT gravity is not the only nonlocal theory that can explain the Solar System dynamics as discussed in Ref. [13].

To solve Eq. (1.1), one needs to introduce the auxiliary fields U and S_μ as we present in Sec. II. Previous works about the nonlocal RT gravity assumed $S_i = 0$ in the cosmic background (see Refs. [1–6] for examples). However, as shown in Sec. II, the value of the background S_i is proportional to a^2 with the expansion of the Universe. Thus, it is unreasonable to assume the background S_i equals 0 in the relevant analyses for the nonlocal RT gravity. This is our motivation to reanalyze the scalar and tensor perturbations of the nonlocal RT gravity with nonzero background S_i. The conventions of this paper are as follows: the greek indices run from 0 to 3, and the latin indices run from 1 to 3. All numerical calculations in this paper are performed in the SI units.

II. COSMIC BACKGROUND EVOLUTION

The localized form of Eq. (1.1) can be written as [3,7,14]

$$G_{\mu\nu} + \frac{m^2}{6}(2Ug_{\mu\nu} + \nabla_\mu S_\nu + \nabla_\nu S_\mu) = \kappa T_{\mu\nu}, \tag{2.1a}$$

$$\Box U = -R, \tag{2.1b}$$

$$\Box S_\mu + \nabla^\nu\nabla_\mu S_\nu = -2\partial_\mu U, \tag{2.1c}$$

where U and S_μ are the auxiliary fields. One can directly verify that energy and momentum conservation can be

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derived from the above equations. To be consistent with current observations [15] and the inflation theory [16], we assume the Universe is described by the flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a^2\mathrm{d}\mathbf{r}^2, \tag{2.2}$$

where $a = a(t)$. For the perfect fluid, we know the energy-momentum tensor $T^{\mu}{}_{\nu} = \mathrm{diag}(-\rho c^2, p, p, p)$. For the auxiliary fields, we assume $U = U_0(t)$ and

$$S_\mu = (c^2\mathcal{S}_0, a\mathcal{S}_1, a\mathcal{S}_1, a\mathcal{S}_1), \tag{2.3}$$

where $\mathcal{S}_0 = \mathcal{S}_0(t)$ and $\mathcal{S}_1 = \mathcal{S}_1(t)$. Here, we set $S_i = a\mathcal{S}_1$ because the Universe is isotropic. This is the core difference between our work and previous works (e.g., Refs. [1–6]) that assume $S_i = 0$.

Substituting the above assumptions into Eq. (2.1a), the $0i$ component gives

$$\dot{\mathcal{S}}_1 - H\mathcal{S}_1 = 0, \tag{2.4}$$

where $\dot{} \equiv \mathrm{d}/\mathrm{d}t$ and the Hubble parameter $H \equiv \dot{a}/a$. Integrating the above equation gives

$$\mathcal{S}_1(t) = l_1\frac{a(t)}{a_1}, \tag{2.5}$$

where a_1 can be regarded as the value of $a(t)$ at one specific time point and l_1 is the integral constant with dimension of length [see Eq. (3.3) for the dimension]. In principle, l_1/a_1 is just one parameter. However, to facilitate the dimensional analysis of the following calculations, we reserve these two parameters. This solution means $S_i \propto a^2$, i.e., the value of S_i increases as the Universe expands. In other words, $S_i = 0$ that was used in Refs. [1–6] is unstable. Taking into account the above solution of $\mathcal{S}_1$, Eq. (2.1) gives

$$\frac{m^2}{3}U_0 - \frac{m^2}{3}\dot{\mathcal{S}}_0 - \frac{3H^2}{c^2} = -\kappa\rho c^2, \tag{2.6a}$$

$$\frac{m^2}{3}U_0 - \frac{m^2}{3}H\mathcal{S}_0 - \frac{2\ddot{a}}{c^2 a} - \frac{H^2}{c^2} = \kappa p, \tag{2.6b}$$

$$\ddot{U}_0 + 3H\dot{U}_0 = 6\frac{\ddot{a}}{a} + 6H^2, \tag{2.6c}$$

$$\ddot{\mathcal{S}}_0 + 3H\dot{\mathcal{S}}_0 - 3H^2\mathcal{S}_0 = \dot{U}_0, \tag{2.6d}$$

which determine the evolution of the Universe. Equation (2.6) shows U_0 is dimensionless and the dimension of $\mathcal{S}_0$ is time. The surprising thing is that l_1 does not appear in Eq. (2.6), which means the cosmic background evolution in the nonlocal RT gravity is independent of S_i. In the following sections, we study the influence of the nonzero background S_i on the scalar and tensor perturbations.

III. SCALAR PERTURBATION

In this section, we analyze the scalar perturbation of the nonlocal RT gravity with nonzero background S_i. Especially, we focus on the Newtonian approximation. The perturbed metric can be written as

$$\mathrm{d}s^2 = -c^2(1 + 2\varepsilon\Phi/c^2)\mathrm{d}t^2 + a^2(1 - 2\varepsilon\Psi/c^2)\mathrm{d}\mathbf{r}^2, \tag{3.1}$$

where $\Phi = \Phi(\mathbf{r}, t)$ and $\Psi = \Psi(\mathbf{r}, t)$. Here and hereafter, we use ε to denote the first-order perturbation, and we set $\varepsilon = 1$ after the Taylor expansion. Note that the FLRW background is necessary to clarify the possible time-varying G [7,8]. For the matter, the only nonzero component of $T_{\mu\nu}$ at the first order is $T_{00} = \varepsilon\rho c^4$ [8]. For the auxiliary fields, we assume

$$U = U_0(t) + \varepsilon U_1(\mathbf{r}, t), \tag{3.2a}$$

$$S_0 = c^2\mathcal{S}_0(t) + \varepsilon c^2\xi_0(\mathbf{r}, t), \tag{3.2b}$$

$$S_i = \frac{l_1 a^2}{a_1} + \varepsilon a\xi_i(\mathbf{r}, t), \tag{3.2c}$$

and then the dimensions of ξ_0 and ξ_i are time and length, respectively.

Substituting the above assumptions into Eq. (2.1a), the $ij(i \neq j)$ component gives

$$\frac{\partial^2(\Psi - \Phi)}{\partial x^i \partial x^j} + \frac{m^2c^2a}{6}\frac{\partial}{\partial x^i}\left(\xi_j + \frac{2l_1 a}{a_1}\frac{\Psi}{c^2}\right) + \frac{m^2c^2a}{6}\frac{\partial}{\partial x^j}\left(\xi_i + \frac{2l_1 a}{a_1}\frac{\Psi}{c^2}\right) = 0. \tag{3.3}$$

Integrating the above equation gives

$$\xi_i = -\frac{2l_1 a}{a_1}\frac{\Psi}{c^2} + \frac{6}{m^2c^2a}\frac{\partial\zeta}{\partial x^i}, \tag{3.4}$$

$$\Psi = \Phi - 2\zeta, \tag{3.5}$$

where $\zeta = \zeta(\mathbf{r}, t)$ is an arbitrary function with dimension of the square of speed. Taking into account the above solutions, the ii component of Eq. (2.1a) gives

$$\frac{2}{a^2}\nabla^2\zeta + \frac{\gamma m}{3a}\tilde{\nabla}(2\zeta - \Phi) + \mathcal{O}(m^2\Phi) = 0, \tag{3.6}$$

and the 00 component gives

$$\frac{2}{a^2}\nabla^2(\Phi - 2\zeta) - \frac{\gamma m}{3a}\tilde{\nabla}\Phi + \mathcal{O}(m^2\Phi) = \kappa\rho c^4, \tag{3.7}$$

where the dimensionless parameter $\gamma \equiv aml_1/a_1$ and the differential operator $\tilde{\nabla} = \partial/\partial x + \partial/\partial y + \partial/\partial z$. In the next section, based on the gravitational wave (GW) observations, we show that it is reasonable to assume $|\gamma| \lesssim \mathcal{O}(1)$ at today. For the Newtonian approximation, we have $m \ll \partial/(a\partial x)$ [8]. Thus, the leading term of Eq. (3.6) gives $\nabla^2\zeta = 0$, and the leading term of Eq. (3.7) gives the Poisson equation if $\kappa = 8\pi G/c^4$. Without loss of generality, we can set $\zeta = 0$, which gives $\Psi = \Phi$ as observations required [8]. Note that a constant but nonzero ζ can be absorbed by rescaling the spatial coordinates. In summary, even considering the nonzero background S_i, the nonlocal RT gravity can still give the desired Poisson equation, $\Psi = \Phi$, and time-independent G in the weak field limit (see Refs. [7,8] for the observational constraints).

Reference [14] analyzes the spherically symmetric static solution of the nonlocal RT gravity with vanishing background S_i and finds the corrections to the Schwarzschild metric are of the form $1 + \mathcal{O}(m^2r^2)$ in the region $r \ll m^{-1}$. However, as we will see, the appearance of the background S_i would change this conclusion. In the region $r_S \ll r \ll m^{-1}$, omitting the $\mathcal{O}(m^2\Phi)$ terms in Eqs. (3.6) and (3.7), we obtain the solutions

$$\frac{\Phi}{c^2} = -\frac{r_S}{2r}\left[1 + \frac{\gamma m}{4}(x+y+z)\right], \tag{3.8}$$

$$\frac{\zeta}{c^2} = -\frac{r_S}{24r}\gamma m(x+y+z), \tag{3.9}$$

and then Eq. (3.5) gives

$$\frac{\Psi}{c^2} = -\frac{r_S}{2r}\left[1 + \frac{\gamma m}{12}(x+y+z)\right], \tag{3.10}$$

where $r_S = 2GM/c^2$ is the Schwarzschild radius. Thus, the leading term of the correction is $\mathcal{O}(mr)$ instead of $\mathcal{O}(m^2r^2)$. However, such a correction is still unobservable. In addition, the above solutions show that the spacetime around the point mass is not spherically symmetric if $S_i \neq 0$ in the background.

The above results are applicable to the Solar System and binary star systems but not to the cosmic large-scale structures, as we omit the $\mathcal{O}(m^2\Phi)$ term after Eq. (3.6). For the observational constraints involving the cosmological scalar perturbations, we should set l_1/a_1 as a parameter to fit the data. To do this, we need to modify the Einstein-Boltzmann solver as is done in Refs. [3–5]. And we would like to leave the work to the future. However, one important thing is worth mentioning here. Equations (3.2c) and (3.4) show the perturbation of S_i is proportional to $a^2\Psi$, which is similar to the behavior of the background S_i. But we do not think this growth will cause a fatal blow to the theory. The reason is $\delta S_i \propto a^2$ even if $l_1 = 0$ (see pages 11, 29, and 30 in Ref. [3]), and this case can indeed fit observations well [3–5]. Nonzero l_1 can change the value of δS_i only by the same order of magnitude if l_1 is not extremely large [17]. Therefore, it is reasonable to believe that observations allow the existence of nonzero l_1.

IV. TENSOR PERTURBATION

In this section, we analyze the GW propagation in the nonlocal RT gravity with nonzero background S_i. Since observations prefer pure tensor modes over pure vector or scalar modes (see GW170814 [18] and GW170817 [19] for examples), here we only consider the tensor modes. Without loss of generality, we assume the GW propagates in the z direction. The perturbed metric can be written as

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + g_{ij}\mathrm{d}x^i\mathrm{d}x^j, \tag{4.1a}$$

where

$$g_{ij} = a^2\begin{pmatrix} 1+\varepsilon h_+ & \varepsilon h_\times & 0 \\ \varepsilon h_\times & 1-\varepsilon h_+ & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{4.1b}$$

and $h_+ = h_+(z,t)$, $h_\times = h_\times(z,t)$. For the energy-momentum tensor, all the components vanish at $\mathcal{O}(\varepsilon)$ order. For the auxiliary fields, we also assume Eq. (3.2) but replace $\mathbf{r}$ with z.

Substituting the above assumptions into Eq. (2.1a), we obtain [20]

$$\frac{\partial^2 h}{\partial t^2} + (3+\alpha)H\frac{\partial h}{\partial t} - \frac{c^2}{a^2}\frac{\partial^2 h}{\partial z^2} + \frac{m^2c^2 l_1}{3a_1}\frac{\partial h}{\partial z} = 0, \tag{4.2}$$

where the dimensionless parameter $\alpha = -m^2c^2\mathcal{S}_0/(3H)$. Here, we omit the subscripts because h_+ and $h_\times$ satisfy the same evolution equation. Hereafter, we define the dimensionless constant $\gamma \equiv a_3ml_1/a_1$, where a_3 is the value of the scale factor today. Note that this definition is slightly different from the definition in the previous section. Using the Fourier transformation $h(z,t) = \int_{-\infty}^{+\infty}\hat{h}(k,t)e^{ikz}\mathrm{d}k$, we obtain

$$\ddot{\hat{h}} + (3+\alpha)H\dot{\hat{h}} + \left(\frac{c^2k^2}{a^2} + \frac{i\gamma kmc^2}{3a_3}\right)\hat{h} = 0. \tag{4.3}$$

If we ignore the $\dot{\hat{h}}$ term and assume $a = a_3$, then the solution of Eq. (4.3) is $\hat{h} = c_1e^{i\omega t}$, where c_1 is the integral constant and

$$\omega = \pm\sqrt{\frac{c^2k^2}{a_3^2} + \frac{i\gamma kmc^2}{3a_3}}. \tag{4.4}$$

Without loss of generality, in the following, we assume $k > 0$ and adopt the minus sign in Eq. (4.4), which corresponds to the GW propagating along the positive direction of the z axis. For the GWs detected by the ground-based detectors, we have $a_3/k \ll m^{-1}$; i.e., the wavelength is much shorter than the cosmic scale. This relation allows us to take Taylor expansion for Eq. (4.4), which gives

$$\omega = -\frac{ck}{a_3}\left(1 + \frac{i\gamma m a_3}{6k} + \frac{\gamma^2 a_3^2 m^2}{72k^2} + \cdots\right). \quad (4.5)$$

The γ^2 term affects the GW dispersion relation, and the GW velocity is

$$v = \left(1 + \frac{\gamma^2 a_3^2 m^2}{72k^2}\right)c. \quad (4.6)$$

For the LIGO GWs, the typical frequency is 100 Hz, and the typical wavelength $\lambda \approx 3 \times 10^6$ m, which gives the typical wave number $k/a_3 = 2\pi/\lambda \approx 2 \times 10^{-6}\ \mathrm{m}^{-1}$. For the parameter m, Ref. [1] gives $m \approx 0.67 H_0/c \approx 5 \times 10^{-27}\ \mathrm{m}^{-1}$. Thus, the typical value of $a_3^2 m^2/k^2$ is 6×10^{-42}. If γ is not extremely large, Eq. (4.6) with such tiny value only leaves negligible effects in the GW dispersion relation and absolute velocity measurements. For example, GW170817 and GRB 170817A give $v/c = 1 \pm \mathcal{O}(10^{-15})$ [21], which requires $\gamma^2 a_3^2 m^2/(72k^2) < 10^{-15}$, i.e., $\gamma < 10^{14}$. In other words, the modification that appears in Eq. (4.6) is unobservable with current observations if $\gamma = \mathcal{O}(1)$. The γ term affects the GW amplitude. This effect is independent of k, which is similar to the role of α in Eq. (4.3). Based on Eq. (4.5), we know that if the propagation time is comparable to $1/H_0$ then this effect is observable. One important thing worth mentioning is that Eq. (4.2) shows that the $\partial h/\partial z$ term could appear in the GW propagation equation, which extends the general propagation equation used in the previous works (see Refs. [22,23] for examples).

To quantify the impact of the $\partial h/\partial z$ term on the GW amplitude, we can no longer assume $a = a_3$ in Eq. (4.3). Here, we assume the GW signal was emitted at $t = t_2$ and $a(t_2) = a_2$ and was detected at $t = t_3$ and $a(t_3) = a_3$. The relation between the redshift and scale factor is $1 + z_{\mathrm{red}} = a_3/a_2$. Taking the coordinate transformation $\eta(t) = \int_{t_2}^{t} \frac{a_2}{a(t')} \mathrm{d}t'$, Eq. (4.3) can be written as

$$\hat{h}'' + (2+\alpha)\mathcal{H}\hat{h}' + \left(\frac{c^2k^2}{a_2^2} + \frac{i\gamma k m c^2 a^2}{3a_2^2 a_3}\right)\hat{h} = 0, \quad (4.7)$$

where $' \equiv \mathrm{d}/\mathrm{d}\eta$ and $\mathcal{H} \equiv a'/a$. Generally, η is called the conformal time. This transformation is used to eliminate the time dependence of the c^2k^2/a^2 term in Eq. (4.3). To eliminate the $\mathcal{H}\hat{h}'$ term in Eq. (4.7), we use the function transformation $\hat{h}(\eta) = f(\eta)\tilde{h}(\eta)$, where

$$f(\eta) = \exp\left(-\frac{1}{2}\int_0^{\eta}[2 + \alpha(\eta')]\mathcal{H}(\eta')\mathrm{d}\eta'\right), \quad (4.8)$$

and then we obtain

$$\tilde{h}'' + \left[\frac{c^2k^2}{a_2^2} + \frac{i\gamma k m c^2 a^2}{3a_2^2 a_3} + \mathcal{O}(\mathcal{H}^2)\right]\tilde{h} = 0. \quad (4.9)$$

The $\mathcal{O}(\mathcal{H}^2)$ term acts as the mass term in the general dispersion relation and is negligible for current observations [24]. Omitting the $\mathcal{O}(\mathcal{H}^2)$ term, the solution of Eq. (4.9) can be written as $\tilde{h}(\eta) = g(\eta) \cdot \exp(-ikc\eta/a_2)$, where

$$g(\eta) = \exp\left(\frac{\gamma c m}{6a_2 a_3}\int_0^{\eta} a^2(\eta')\mathrm{d}\eta'\right). \quad (4.10)$$

Then, the GW amplitude is proportional to $f(\eta) \cdot g(\eta)$. Hereafter, we denote $\eta_3 \equiv \eta(t_3)$. For the standard siren, the GW luminosity distance is inversely proportional to the amplitude [25]. In addition, the GW luminosity distance in general relativity satisfies $d_L^{(\mathrm{GR})} \propto 1/(fg)|_{\alpha,\gamma=0}$. Therefore, the ratio of the luminosity distance between the nonlocal RT gravity and general relativity is

$$\frac{d_L^{(\mathrm{RT})}}{d_L^{(\mathrm{GR})}} = \frac{f(\eta_3)\cdot g(\eta_3)|_{\alpha,\gamma=0}}{f(\eta_3)\cdot g(\eta_3)} = \exp\left(\int_0^{\eta_3}\left[\frac{\alpha(\eta')\mathcal{H}(\eta')}{2} - \frac{\gamma c m a^2(\eta')}{6a_2 a_3}\right]\mathrm{d}\eta'\right). \quad (4.11)$$

Transforming to the redshift, we obtain

$$\frac{d_L^{(\mathrm{RT})}(z_{\mathrm{red}})}{d_L^{(\mathrm{GR})}(z_{\mathrm{red}})} = \exp\left(\int_0^{z_{\mathrm{red}}}\left[\frac{\alpha(\tilde{z})}{2(1+\tilde{z})} - \frac{\gamma c m}{6H(\tilde{z})\cdot(1+\tilde{z})^2}\right]\mathrm{d}\tilde{z}\right). \quad (4.12)$$

If $\gamma = 0$, the above equation is equivalent to the result obtained in Ref. [26]. GW170817 [27] rules out the possibility of $|\gamma| \gg 1$. Future high-redshift GW observations [28–33] will provide tighter constraints on γ. Therefore, it is reasonable to assume $|\gamma| \lesssim \mathcal{O}(1)$ now.

The above discussion focuses on subhorizon modes. Here, we discuss the influence of the $\partial h/\partial z$ term on the evolution of superhorizon modes (primordial GWs), which is related to the early Universe (inflation). The starting point is Eq. (4.3). Cosmic microwave background (CMB) observations are the main method to detect primordial GWs [34]. The typical wave number observed through CMB measurements is $k/a_3 \approx H_0/c$ [35]. The ratio of the two terms appearing in Eq. (4.3) is

$$\frac{i\gamma kmc^2}{3a_3} \Big/ \frac{c^2k^2}{a^2} = \frac{i\gamma a_3 m}{3k}\cdot\frac{a^2}{a_3^2} \lesssim \frac{a_e^2}{a_3^2} \approx 10^{-64}, \tag{4.13}$$

where a_e is the scale factor at the end of inflation, and we estimate a_e/a_3 with $a_e/a_3 \approx 2.7\ \mathrm{K}/T_\mathrm{P}$, where T_P is the Planck temperature. Such a tiny value means the influence of the γ term on the GW dispersion relation is negligible in the superhorizon case. For a rough estimate of the influence on GW amplitude, we ignore the $\dot{\hat{h}}$ term in Eq. (4.3) and assume $a = a_e$, and then the solution of Eq. (4.3) is $\hat{h} = c_1 e^{i\omega t}$, where c_1 is the integral constant and

$$\begin{aligned}\omega &= -\sqrt{\frac{c^2k^2}{a_e^2} + \frac{i\gamma kmc^2}{3a_3}} \\ &= -\frac{ck}{a_e} - \frac{i\gamma cm}{6}\cdot\frac{a_e}{a_3} + \mathcal{O}\left(\frac{a_e^2}{a_3^2}\right).\end{aligned} \tag{4.14}$$

Therefore, the main factor induced by the γ term is

$$\exp\left(-i\frac{i\gamma cm}{6}\frac{a_e}{a_3}t\right) \approx \exp\left(0.1\gamma H_0 t\cdot\frac{a_e}{a_3}\right) \approx 1 + \gamma\cdot\mathcal{O}(10^{-89}), \tag{4.15}$$

where we assume $t \approx 10^5 t_\mathrm{P}$ and t_P is the Planck time. The above result shows that the influence of the γ term on the GW amplitude is also negligible in the superhorizon case. References [36,37] discuss the influence of nonzero α on the initial conditions of perturbations given by inflation. Figure 7 in Ref. [30] shows $|\alpha| \ll 1$ in the early Universe for the nonlocal RT gravity. These results indicate that the α term appearing in Eq. (4.3) is negligible in the early Universe. Our results show the γ term is also negligible. In summary, in the early Universe, the GW evolution in nonlocal RT gravity is the same as in general relativity.

V. CONCLUSIONS

In this paper, after realizing that the background value of the auxiliary field S_i increases with the expansion of the Universe, we reanalyze the scalar and tensor perturbations of the nonlocal RT gravity with nonzero background S_i. For the scalar perturbation, we find the leading term of the corrections to Φ and Ψ/Φ is of the order of $\mathcal{O}(mr)$ instead of $\mathcal{O}(m^2r^2)$ obtained in Ref. [14]. However, these corrections are still unobservable, and thus the nonlocal RT gravity can still recover all successes (Poisson equation, $\Psi = \Phi$, and $G = \text{const.}$) of general relativity in the Solar System as concluded in Refs. [3,6–8]. For the tensor perturbation, we find the $\partial h/\partial x^i$ term appears in the GW propagation equation, which extends the general propagation equation that used in Refs. [22,23]. Our calculations show that the influence of the $\partial h/\partial x^i$ term on the GW dispersion relation is negligible in both the early and late-time Universe, but the influences on the LIGO GW amplitude and also luminosity distance is observable if the dimensionless constant $\gamma = \mathcal{O}(1)$.

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