Thick brane in reduced Horndeski theory

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Horndeski theory is the most general scalar-tensor theory retaining second-order field equations, although the action includes higher-order terms. This is achieved by a special choice of coupling constants. In this paper, we investigate the thick brane system in reduced Horndeski theory, especially the effects of the nonminimal derivative coupling and the cubic Galileon term on thick brane. First, the equations of motion are presented and a set of analytic background solutions are obtained. Then, the effects of these two terms on the thick brane are analyzed. It is found that the background solutions become asymmetric with the presence of the cubic Galileon term. However, the energy density of the thick brane is still symmetric with or without this term and splits with the presence of the nonminimal derivative coupling. The stability of the thick brane under tensor perturbation is also considered. It is shown that the tachyon is absent and the graviton zero mode can be localized on the brane. The localized graviton zero mode recovers the four-dimensional Newtonian potential and the presence of the nonminimal derivative coupling results in a splitting of its wave function. Besides, the presence of the cubic Galileon term results in the asymmetry of the effective potential and zero mode of the graviton. The correction of the massive graviton Kaluza-Klein modes to the Newtonian potential is also analyzed briefly.

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I. INTRODUCTION

It is well known that general relativity is just an effective gravitational theory at low energy and at the scale of the Solar System because of its nonrenormalization and is incapable of explaining dark matter and dark energy. Thus, it needs to be modified at high energy and galactic scale, such as adding higher-order curvature terms and introducing extra scalar fields. In this paper, we mainly focus on the most general scalar-tensor gravitational theory, i.e., Horndeski theory [1], which maintains the second-order equations of motion (for a recent review, see Refs. [2,3] and references therein). Besides, this theory was generalized in the teleparallel gravity framework recently; see [4] for more details. The action is

$$S = \int d^4x\sqrt{-g}\left(\sum_{i=2}^{5} L_i\right), \tag{1}$$

where

$$L_2 = K(\phi, X), \tag{2}$$

$$L_3 = -G_3(\phi, X)\Box\phi, \tag{3}$$

$$L_4 = G_4(\phi, X)R + G_{4,X}(\phi, X)[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)], \tag{4}$$

$$\begin{aligned} L_5 = {} & G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5,X}(\phi, X)[(\Box\phi)^3 \\ & - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) \\ & + 2(\nabla^\mu\nabla_\alpha\phi)(\nabla^\alpha\nabla_\beta\phi)(\nabla^\beta\nabla_\mu\phi)]. \end{aligned} \tag{5}$$

Here $\Box\phi = \nabla_\alpha\nabla^\alpha\phi$, $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$, K, G_i ($i = 3, 4, 5$) are functions of the scalar field ϕ and its kinetic term $X = -\nabla_\mu\phi\nabla^\mu\phi/2$, and $G_{i,X}(\phi, X) = \partial G_i(\phi, X)/\partial X$.

However, it is difficult to investigate the entire theory at once with the general functions K, G_3, G_4, and G_5. In this paper, we mainly focus on a simplified model; namely, G_5 is free of X. Besides, to reduce to general relativity at low energy scale, $G_4 = \frac{M_{\rm Pl}^2}{2}$. Then, the above action can be simplified as

$$\begin{aligned} S = \int d^4x\sqrt{-g}\Big[& \frac{M_{\rm Pl}^2}{2}R + G_5(\phi)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ & - G_3(\phi, X)\Box\phi + K(\phi, X)\Big]. \end{aligned} \tag{6}$$

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After integrating by parts and absorbing $G_5(\phi)$ into the scalar field ϕ, the above action can always be transformed as

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - bG_{\mu\nu}\nabla^\mu\tilde{\phi}\nabla^\nu\tilde{\phi} - \tilde{G}_3(\tilde{\phi},\tilde{X})\Box\tilde{\phi} + \tilde{K}(\tilde{\phi},\tilde{X})\right], \quad (7)$$

where $M_{\rm Pl}$ is the Planck scale, the second term is the so-called nonminimal derivative coupling [5], and the third term is the so-called kinetic gravity braiding [6]. Actually, $\kappa_1 R g_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi$ and $\kappa_2 R_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi$ are the only two terms which are necessary to be considered in scalar-curvature coupling theory [7], and it was shown in Ref. [8] that the equations of motion of the field $g_{\mu\nu}$ and ϕ reduce to second order if $\kappa = \kappa_2 = -2\kappa_1$, which leads to the nonminimal derivative coupling between the Einstein tensor and scalar field, $\kappa G_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi$.

Theories of the type (7) have been extensively investigated in different contexts. For instance, the conditions of existing a stable Einstein static universe under both scalar and tensor perturbations were investigated in Refs. [9,10]. The quasinormal modes of asymptotic anti–de Sitter (AdS) black holes were explored in Ref. [11]. In Ref. [12], the authors investigated thermodynamic properties of a new class of black holes in these theories and showed that this class of black holes presents rich thermodynamic behaviors and critical phenomena. In Ref. [13], the authors calculated the holographic complexity of AdS black holes in these theories and showed that the action growth for planar and spherical topologies satisfies Lloyd's bound. Besides, the nonminimal derivative coupling can result in the present cosmic acceleration, and provide an inflationary mechanism [7,14–19]. For a recent review, see Ref. [20]. Besides, since the kinetic gravity braiding would result in the deviations of the stress tensor from the perfect-fluid form, it was investigated extensively in cosmology, such as providing a new class of models for dark energy [6], driving inflation [21], constructing a nonsingular bouncing cosmology [22], and so on. In the present paper, we consider the simplest form $G_3(\phi, X)\Box^{(5)}\phi = C(\phi)X\Box^{(5)}\phi$, which is called the cubic Galileon term, and investigate its effect on thick brane.

On the other hand, inspired by string theory, braneworld models have drawn much attention in recent years. There are mainly two kinds of braneworld models: the thin braneworld model and thick braneworld model. The most famous thin braneworld models are the Randall-Sundrum (RS)-type models [23,24] and their extensions [25–28]. These models were extensively investigated in the last decades because of their advantages in solving some long-existing problems, such as the gauge hierarchy problem, the fermion mass hierarchy, the cosmological constant problem, and so on. The thick braneworld model was first introduced in Ref. [29] and further developed in Refs. [30,31]. In thick brane models, the size of the extra dimension is usually infinity, and the brane is generated dynamically instead of introduced by hand and the energy density of the brane is replaced with a smooth function along the extra dimension [29–40] instead of a delta function in the thin braneworld models. For a brief review, see Ref. [41]. Except for the above advantages, the thick branewolrd models also have the following promising features: (1) the effective low energy theory includes a localized massless graviton which can be used to produce the four-dimensional Newtonian potential [30,31,42–53]; (2) the massless scalar graviton decouples from the brane system, which avoids the fifth force [42,54–56]; (3) the localization of the fermion field and its chirality can be guaranteed by introducing an interaction with the background scalar field [57–60]; (4) the gauge field also can be localized on the brane if a mass term is added [61–63]; (5) the charge universality of the gauge boson can also be obtained via the Dvali-Shifman mechanism [64–68]. Thus, the thick braneworld model is a kind of candidate model producing both standard model and Newtonian gravity, which are two key elements forming our four-dimensional world.

Although the nonminimal derivative coupling has been considered in a different context, such as cosmology and black hole, the effects of this term on thick brane are still unclear in the literature. Recently, the thick brane under nonminimal derivative coupled gravity has been studied in Ref. [69]. However, the background solutions of the thick brane were only solved numerically and approximately and the equation of motion of the tensor perturbation $\bar{h}_{\mu\nu}$ given in Ref. [69] is not correct because the factor of $\bar{h}_{\mu\nu}$ should be eliminated for the flat brane after inserting the background equations of motion. Besides, the presence of the cubic Galileon term $C(\phi)X\Box^{(5)}\phi$ may break the symmetry of the action under $\phi \to -\phi$; it is interesting to investigate whether the thick brane model can be constructed within this term and what the effects are of this term on thick brane and its stability under tensor perturbation. These are motivations of our present work.

In Sec. II, we present the action of the thick brane system and derive the equations of motion. In Sec. III, a set of analytic solutions are obtained and the stability and canonical normalization of the scalar field are considered. In Sec. IV, the stability of the brane system under tensor perturbation, the localization of the graviton zero mode, and a brief analysis about the correction of the graviton Kaluza-Klein (KK) modes to the Newtonian potential are investigated. Section V comes with the conclusion.

II. FIELD EQUATIONS

In this paper, we study the thick brane system with the following action:

$$S = \int d^5x\sqrt{-g}\left[\frac{1}{2}R - bG_{MN}\nabla^M\phi\nabla^N\phi - G_3(\phi,X)\Box^{(5)}\phi + K(\phi,X)\right]. \tag{8}$$

Here we have set the five-dimensional Planck mass $M_5 = 1$. Varying the above action with respect to the metric g_{MN}, we obtain the equations of motion

$$G_{MN} = T_{MN} + 2b\Theta_{MN}, \tag{9}$$

where $\Box^{(5)} \equiv g^{MN}\nabla_M\nabla_N$,

$$T_{MN} = K_X\nabla_M\phi\nabla_N\phi + g_{MN}K - (\nabla_M G_3\nabla_N\phi + \nabla_N G_3\nabla_M\phi) + g_{MN}\nabla_Q G_3\nabla^Q\phi - G_{3X}\nabla_M\phi\nabla_N\phi\Box^{(5)}\phi, \tag{10}$$

and

$$\Theta_{MN} = -\frac{1}{2}\nabla_M\phi\nabla_N\phi R + 2\nabla_K\phi\nabla_{(M}\phi R^K_{N)} - \frac{1}{2}(\nabla\phi)^2 G_{MN} + \nabla^K\phi\nabla^L\phi R_{MKNL} + \nabla_M\nabla^K\phi\nabla_N\nabla_K\phi - \nabla_M\nabla_N\phi\Box^{(5)}\phi + g_{MN}\left[-\frac{1}{2}\nabla^K\nabla^L\phi\nabla_K\nabla_L\phi + \frac{1}{2}(\Box^{(5)}\phi)^2 - \nabla_K\phi\nabla_L\phi R^{KL}\right]. \tag{11}$$

The equation of motion of the scalar field can be obtained by varying the action (8) with respect to ϕ,

$$\nabla_M[(K_X - G_{3\phi} - G_{3X}\Box^{(5)}\phi)\nabla^M\phi - G_{3X}\nabla^M X] + 2bG^{MN}\nabla_M\nabla_N\phi + K_\phi - G_{3\phi}\Box^{(5)}\phi = 0. \tag{12}$$

More explicitly, the scalar field equation can be written as

$$K_X\Box^{(5)}\phi - K_{XX}(\nabla^M\nabla^N\phi)(\nabla_M\phi\nabla_N\phi) - 2K_{\phi X}X + 2bG^{MN}\nabla_M\nabla_N\phi + K_\phi - 2(G_{3\phi} - G_{3\phi X}X)\Box^{(5)}\phi + G_{3X}[(\nabla^M\nabla^N\phi)(\nabla_M\nabla_N\phi) - (\Box^{(5)}\phi)^2 + R_{MN}\nabla^M\phi\nabla^N\phi] + 2G_{3\phi X}(\nabla_M\nabla_N\phi)(\nabla^M\phi\nabla^N\phi) + 2G_{3\phi\phi}X - G_{3XX}(\nabla^M\nabla^K\phi - g^{MK}\Box^{(5)}\phi)(\nabla_M\nabla^N\phi)\nabla_N\phi\nabla_K\phi = 0, \tag{13}$$

which remains second order.

III. BRANEWORLD MODEL

In general, the line element of a static flat braneworld can be assumed as

$$d^2s = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \tag{14}$$

with y being the extra coordinate. The warp factor A and scalar field ϕ are independent of the brane coordinates, i.e., $A = A(y)$ and $\phi = \phi(y)$. In this paper, we assume $G_3(\phi,X) = C(\phi)X$ and $K(\phi,X) = L(\phi)X - V(\phi)$. Then, Eqs. (9) and (12) with the ansatz (14) can be reduced to

$$A''(6 - 15b\phi'^2) + A'^2(12 - 30b\phi'^2) - 12bA'\phi'\phi'' + \phi'^4C_\phi + 2C\phi'^2\phi'' + L\phi'^2 + 2V = 0, \tag{15}$$

$$12A'^2 - \phi'^2(54bA'^2 - 8A'C\phi' + \phi'^2C_\phi + L) + 2V = 0, \tag{16}$$

$$\phi''(60bA'^2 - 16A'C\phi' + 4\phi'^2C_\phi + 2L) + \phi'^4C_{\phi\phi} + \phi'(96bA'^3 + 8A'L + 48bA'A'') + \phi'^2(-8A''C - 32A'^2C + L_\phi) - 2V_\phi = 0, \tag{17}$$

where the prime denotes the derivative with respect to the extra dimension y, and $L_\phi = \frac{\partial L(\phi)}{\partial\phi}$, $V_\phi = \frac{\partial V(\phi)}{\partial\phi}$, $C_\phi = \frac{\partial C(\phi)}{\partial\phi}$, $C_{\phi\phi} = \frac{\partial^2 L(\phi)}{\partial\phi^2}$. In general, the spacetime of the thick brane is asymptotically anti–de Sitter far away form the brane, which requires $\phi'(y\to\pm\infty)\to 0$ and $A'(y\to\pm\infty)\to$ constant. Without loss of generality, we assume

$$\phi(y) = \phi_0\tanh^{-1}(\sinh(ky)), \tag{18}$$

$$A(y) = \ln[\mathrm{sech}(ky)]. \tag{19}$$

Besides, to investigate the effects of the cubic Galileon term $C(\phi)X\Box^{(5)}\phi$, we assume

$$C(\phi) = c_0 + c_1\tan(\phi/\phi_0), \tag{20}$$

where the first term breaks the symmetry of the action (8) under $\phi\to-\phi$ and the second term remains the symmetry. Now, the system can be solved as

$$L(\phi(y)) = -\frac{1}{2\phi_0^2}\mathrm{sech}^2(ky)[\cosh(2ky)(6bk^2\phi_0^2 + 4c_1k^2\phi_0^3 - 3) + 6c_0k^2\phi_0^3\sinh(ky) + 9bk^2\phi_0^2 - 2c_1k^2\phi_0^3 - 3], \tag{21}$$

$$V(\phi(y)) = \frac{k^2}{4}\mathrm{sech}^4(ky)[\cosh(2ky)(48bk^2\phi_0^2 + 5c_1k^2\phi_0^3 + 3) + 10c_0k^2\phi_0^3\sinh(ky) - 3\cosh(4ky) - 63bk^2\phi_0^2 - 5c_1k^2\phi_0^3 + 6]. \tag{22}$$

The above two functions can also be expressed in terms of the scalar field ϕ as

$$L(\phi) = -\frac{3}{4}k^2(b-2c_1\phi_0)\cos\left(\frac{2\phi}{\phi_0}\right) - \frac{3}{2}c_0k^2\phi_0\sin\left(\frac{2\phi}{\phi_0}\right) - \frac{27bk^2}{4} - \frac{5}{2}c_1k^2\phi_0 + \frac{3}{\phi_0^2}, \tag{23}$$

$$V(\phi) = \frac{k^2}{4}\cos^4\left(\frac{\phi}{\phi_0}\right)\left[+10c0k^2\phi_0^3\tan\left(\frac{\phi}{\phi_0}\right) + 2\tan^2\left(\frac{\phi}{\phi_0}\right)(48bk^2\phi_0^2+5c_1k^2\phi_0^3-9) - 24\tan^4\left(\frac{\phi}{\phi_0}\right) - 15bk^2\phi_0^2+6\right]. \tag{24}$$

Figure 1 shows that both functions are periodic in ϕ and asymmetric under $\phi\to-\phi$ due to the presence of $c_0X\Box^{(5)}\phi$. However, the energy density of the thick brane is symmetric with or without this term; see Fig. 2. The reason is that the behavior of the energy density is only determined by the warp factor and scalar field, which can be easily seen from the definition of the energy density $\rho\equiv T_0{}^0 = G_0{}^0 - 2b\Theta_0{}^0$. Besides, from the expression of the energy density

$$\rho = \frac{3}{2}A''(5b\phi'^2-2)+3A'^2(5b\phi'^2-2)+6bA'\phi'\phi'', \tag{25}$$

which is independent of $C(\phi)$, we can conclude that the cubic Galileon term has no effect on the energy density of the thick brane. However, the nonminimal derivative coupling would affect the energy density, causing the splitting of the brane; see Fig. 2.

IV. TENSOR PERTURBATION

In general, the tensor, vector, and scalar perturbations are decoupled from each other. Thus, they can be investigated individually. The metric under the tensor perturbation can be written as

$$ds^2 = e^{2A(y)}(\eta_{\mu\nu}+h_{\mu\nu})dx^\mu dx^\nu + dy^2, \tag{26}$$

where $h_{\mu\nu}$ represents the transverse and traceless (TT) tensor perturbation, i.e., $\eta^{\mu\alpha}\partial_\alpha h_{\mu\nu}=0$ and $h\equiv\eta^{\mu\nu}h_{\mu\nu}=0$. Then, the perturbation equation can be calculated as

$$\delta G_{\mu\nu} = e^{2A}h_{\mu\nu}(K-G_3\Box^{(5)}\phi) + e^{2A}G_3\phi' h'_{\mu\nu} + 2b\delta\Theta_{\mu\nu}, \tag{27}$$

where

$$\begin{aligned}\delta\Theta_{\mu\nu} = &-\frac{1}{2}\phi'^2\delta G_{\mu\nu} - e^{-2A}h^{\rho\sigma}\nabla_\nu\nabla_\rho\phi\nabla_\mu\nabla_\sigma\phi + 2(\nabla_\mu\nabla^\sigma\phi)\left(\phi'A'e^{2A}h_{\nu\sigma}+\frac{1}{2}\phi'e^{2A}h'_{\nu\sigma}\right) + e^{-2A}h^{\rho\sigma}\nabla_\rho\nabla_\sigma\phi\nabla_\mu\nabla_\nu\phi\\ &-\Box^{(5)}\phi\left(\phi'A'e^{2A}h_{\mu\nu}+\frac{1}{2}\phi'e^{2A}h'_{\mu\nu}\right) + \phi'^2\left(-A''e^{2A}h_{\mu\nu} - A'^2e^{2A}h_{\mu\nu} - A'e^{2A}h'_{\mu\nu} - \frac{1}{2}e^{2A}h''_{\mu\nu}\right)\\ &+e^{2A}h_{\mu\nu}\left(\frac{1}{2}\nabla^T\nabla^L\phi\nabla_T\nabla_L\phi + \frac{1}{2}\Box^{(5)}\phi\Box^{(5)}\phi - R_{KL}\nabla^K\phi\nabla^L\phi\right) + e^{2A}\eta_{\mu\nu}\left(e^{-2A}h^{\rho\sigma}(\nabla_\sigma\nabla^\tau\phi + \nabla_\sigma\phi'\nabla_\rho\phi')\right.\\ &\left.-(\nabla^\rho\nabla^\sigma\phi)\left(\phi'A'e^{2A}h_{\rho\sigma}+\frac{1}{2}\phi'e^{2A}h'_{\rho\sigma}\right) - e^{-2A}h^{\rho\sigma}\nabla_\rho\nabla_\sigma\phi\Box^{(5)}\phi\right).\end{aligned} \tag{28}$$

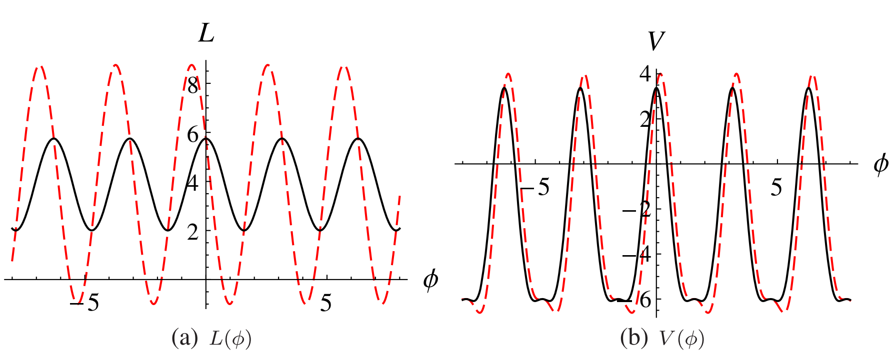


FIG. 1. Plots of the brane solutions $F(\phi)$ and $V(\phi)$. The parameters are set to $k=\phi_0=c_1=1$, $c_0=0$ for black lines, $c_0=3$ for red dashed lines.

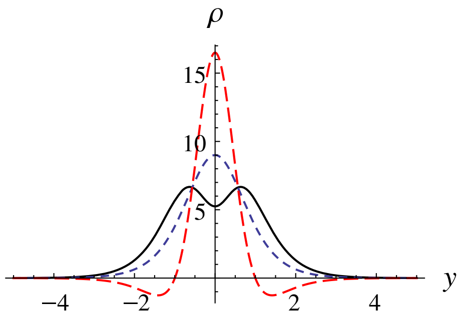


FIG. 2. Plots the energy density of the thick brane. The parameters are set to $k=\phi_0=1$, $b=-1$ for the red long dashed cure, $b=0$ for the blue short dashed curve, and $b=\frac{1}{2}$ for the black curve.

Considering the TT conditions, the above equation can be reduced to

$$T(y)h''_{\mu\nu}+B(y)h'_{\mu\nu}+\mathrm{e}^{-2A}\Box^{(4)}h_{\mu\nu}=0, \tag{29}$$

where $\Box^{(4)}\equiv\eta^{\mu\nu}\partial_\mu\partial_\nu$, and

$$T(y)=\frac{1-b\phi'^2}{1+b\phi'^2}, \tag{30}$$

$$B(y)=\frac{4A'-4b\phi'^2A'-2b\phi'\phi''-2G_3\phi'}{1+b\phi'^2}. \tag{31}$$

After a coordinate transformation $dy=\mathrm{e}^A dz$, Eq. (29) becomes

$$N(z)\partial_z^2h_{\mu\nu}+P(z)\partial_zh_{\mu\nu}+\Box^{(4)}h_{\mu\nu}=0, \tag{32}$$

where

$$N(z)=\frac{1-b\mathrm{e}^{-2A}(\partial_z\phi)^2}{1+b\mathrm{e}^{-2A}(\partial_z\phi)^2}, \tag{33}$$

$$P(z)=\frac{1}{1+b\mathrm{e}^{-2A}(\partial_z\phi)^2}(3\partial_zA-b\mathrm{e}^{-2A}(\partial_zA)(\partial_z\phi)^2 - 2b\mathrm{e}^{-2A}(\partial_z\phi)(\partial_z^2\phi)-2G_3(\partial_z\phi)). \tag{34}$$

With a further coordinate transformation $dz=\sqrt{N}dw$, Eq. (32) can be transformed as

$$\partial_w^2h_{\mu\nu}+\left(\frac{P}{\sqrt{N}}-\frac{\partial_wN}{2N}\right)\partial_wh_{\mu\nu}+\Box^{(4)}h_{\mu\nu}=0. \tag{35}$$

Considering the decomposition $h_{\mu\nu}(x,w)=\varepsilon_{\mu\nu}(x)\mathrm{e}^{-ipx}H(w)$ with $p^2=-m^2$, this equation simplifies to

$$\partial_w^2H+Q(w)\partial_wH+m^2H=0, \tag{36}$$

where $Q(w)=(\frac{P}{\sqrt{N}}-\frac{\partial_wN}{2N})$. Then, by redefining $H(w)=G(w)\tilde{H}(w)$ with $G(w)=\exp(-\frac{1}{2}\int Q(w)dw)$, we can obtain a Schrödinger-like equation,

$$-\partial_w^2\tilde{H}+U(w)\tilde{H}=m^2\tilde{H}, \tag{37}$$

where $U(w)=(\frac{1}{2}\partial_wQ+\frac{1}{4}Q^2)$. The above equation can be factorized as

$$\left(\partial_w+\frac{Q}{2}\right)\left(-\partial_w+\frac{Q}{2}\right)\tilde{H}=m^2\tilde{H}, \tag{38}$$

which indicates there is no tachyon state, i.e., $m^2\geq0$. Thus, the brane is stable under the tensor perturbation.

By setting $m=0$, the graviton zero mode can be solved from Eq. (38),

$$\tilde{H}_0=N_0\exp\left(\frac{1}{2}\int Qdw\right)=N_0\exp\left(\frac{1}{2}\int\frac{Q}{\sqrt{N}}dz\right), \tag{39}$$

where N_0 is a normalization constant. The normalization condition of the graviton zero mode is

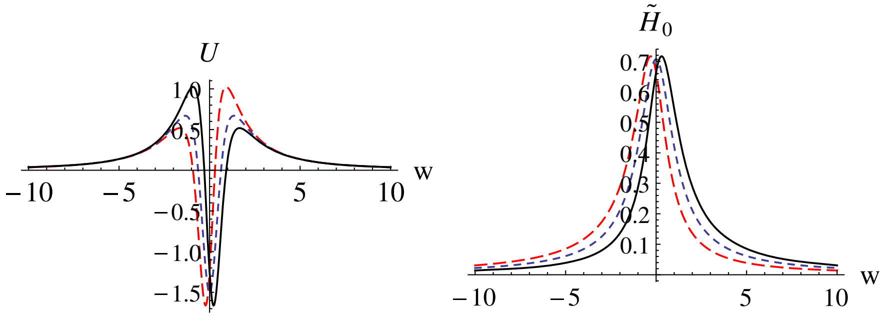
$$\int\tilde{H}_0^2(w)dw=\int\tilde{H}_0^2(z)\frac{dz}{\sqrt{N(z)}}<\infty. \tag{40}$$

A. Localization of the graviton zero mode

Considering Eq. (20) and the background solutions (19) and (18), the graviton zero mode can be solved as

$$\begin{aligned}\tilde{H}_0(z)=&\sqrt[4]{(k^2z^2+1)^{\frac{c_1\phi_0}{b}-5}(k^2z^2+\theta_+)(k^2z^2+\theta_-)^{1-\frac{c_1\phi_0}{b}}}\\&\times\exp\left[c_0\left(\frac{\phi_0}{2b}\tan^{-1}(kz)\right.\right.\\&\left.\left.-\frac{\phi_0}{2b\sqrt{\theta_-}}\tan^{-1}(kz/\sqrt{\theta_-})\right)\right],\end{aligned} \tag{41}$$

where $\theta_\pm\equiv1\pm bk^2\phi_0^2$. It is obvious that the second term of $\tilde{H}_0(z)$ is asymmetric about the extra dimension z, which comes from the cubic Galileon term $c_0X\Box^{(5)}\phi$. Because $\tan^{-1}(kz)\to\pm\frac{\pi}{2}$ in the large z region, the asymmetric part of $\tilde{H}_0(z)$ approaches a constant. Thus, the asymptotic behavior of the graviton zero mode is determined by the symmetric part. It can be easily shown that $\tilde{H}_0(\pm\infty)\to(1/k|z|)^{3/2}$, which is independent of parameters b, c_0, and c_1. Namely, the nonminimal derivative coupling and the cubic Galileon term will not affect the asymptotic property of the graviton zero mode. What is more, the graviton zero mode can always be localized on the brane because $\tilde{H}_0^2(z)/\sqrt{N(z)}$ falls off faster than $1/z$.



(a) $U(w)$ (b) $\tilde{H}_0(w)$

FIG. 3. The effective potential of the graviton and the wave function of the graviton zero mode for varying c_0. In this figure $c_0 = 1$ (red long dashed cures), $c_0 = 0$ (blue short dashed curves), and $c_0 = -1$ (black curves). The other parameters are set to $c_1 = 0$, $b = 0$, and $k = 1$.

To ensure the coordinate transformation $dw = dz/\sqrt{N}$ is well defined, $N(z)$ should be positive, namely,

$$\frac{k^2(z^2 - b\phi_0^2) + 1}{k^2(z^2 + b\phi_0^2) + 1} > 0, \tag{42}$$

which gives $-\frac{1}{k^2\phi_0^2} < b < \frac{1}{k^2\phi_0^2}$ for $z \in (-\infty, +\infty)$.

Figures 3–5 show the effects of the nonminimal derivative coupling and the cubic Galileon term on the graviton zero mode and its effective potential. It is obvious that the effective potential becomes asymmetric with nonzero c_0. Besides, a double well appears in the symmetric gravitational potential ($c_0 = 0$) with $|b|$ increasing or with c_1 decreasing, which suggests the splitting of the graviton zero mode; see Figs. 4 and 5.

In the following, as an example, we investigate the effects of the nonminimal derivative coupling on the graviton zero mode and its effective potential explicitly. The same procedure is suitable for the cubic Galileon term.

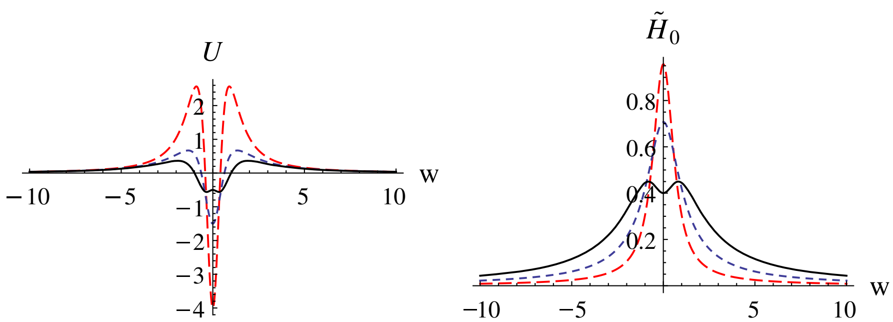
Since, it is difficult to obtain an analytic relation for $z(w)$ [70], the graviton zero mode and its effective potential can only be expressed analytically in terms of z. However, some behaviors of $\tilde{H}_0(w)$ and $U(w)$ at $w = 0$ can be obtained with the relations $\partial_w^2 \tilde{H}_0(w) = \frac{1}{2}\partial_z N(z)\partial_z \tilde{H}_0(z) + N(z)\partial_z^2 \tilde{H}_0(z)$, $\partial_w U(w) = \sqrt{N(z)}\partial_z U(z)$, and $\partial_w^2 U(w) = \frac{1}{2}\partial_z N(z)\partial_z U(z) + N(z)\partial_z^2 U(z)$. Setting $c_0 = c_1 = 0$, we have

$$\partial_w^2 U(0) = \frac{k^4}{2(bk^2\phi_0^2 + 1)^4}(55b^4k^8\phi_0^8 + 40b^3k^6\phi_0^6 - 102b^2k^4\phi_0^4 - 24bk^2\phi_0^2 + 27), \tag{43}$$

$$U(0) = \frac{k^2(5b^2k^4\phi_0^4 - 3)}{2(bk^2\phi_0^2 + 1)^2}, \tag{44}$$

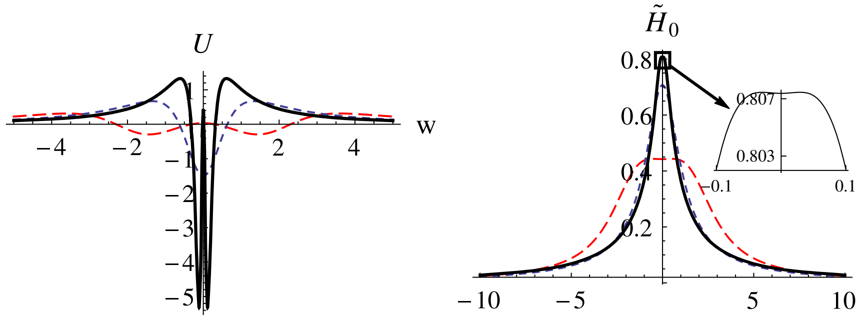
$$\partial_w^2 \tilde{H}_0(0) = -\frac{k^2(bk^2\phi_0^2 - 1)(5b^2k^4\phi_0^4 - 3)}{2(bk^2\phi_0^2 + 1)(1 - b^2k^4\phi_0^4)^{3/4}}. \tag{45}$$

From Eq. (43), $\partial_w^2 U(0) < 0$ when $-\frac{1}{k^2\phi_0^2} < b < -\frac{0.63}{k^2\phi_0^2}$ and $\frac{0.47}{k^2\phi_0^2} < b < \frac{1}{k^2\phi_0^2}$, which indicates a double-well potential appears. From Eqs. (44) and (45), $\partial_w^2 \tilde{H}_0(0) > 0$ and $U(0) > 0$ when $-\frac{1}{k^2\phi_0^2} < b < -\frac{0.77}{k^2\phi_0^2}$ and $\frac{0.77}{k^2\phi_0^2} < b < \frac{1}{k^2\phi_0^2}$, which indicates the splitting of the graviton zero mode. In the same way, setting $c_0 = 0$ and $b = 0$, we can obtain that a



(a) $U(w)$ (b) $\tilde{H}_0(w)$

FIG. 4. The effective potential of the graviton and the wave function of the graviton zero mode for varying c_1. In this figure $c_1 = 5$ (red long dashed cures), $c_1 = 0$ (blue short dashed curves), and $c_1 = -5$ (black curves). The other parameters are set to $c_0 = 0$, $b = 0$, and $k = 1$.



(a) $U(w)$ (b) $\tilde{H}_0(w)$

FIG. 5. The effective potential of the graviton and the wave function of the graviton zero mode for varying b. In this figure $b = 0.8$ (red long dashed cures), $b = 0$ (blue short dashed curves), and $b = -0.78$ (black curves). The other parameters are set to $c_0 = 0$, $c_1 = 0$, and $k = 1$.

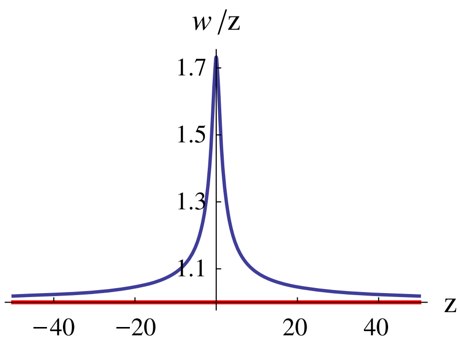


FIG. 6. The blue line represents $\frac{w(z)}{z}$, and the red thick line represents $\frac{w(z)}{z} = 1$. The parameters are set to be $b = \frac{1}{2}$, $k = 1$, and $\phi_0 = 1$.

double-well potential appears when $-\frac{16.34}{k^2\phi_0^3} < c_1 < -\frac{1.65}{k^2\phi_0^3}$ and the graviton zero mode splits when $c_1 < -\frac{3}{k^2\phi_0^3}$.

B. Correction to the Newtonian potential

Except for the localized graviton zero mode, there are a lot of continuous massive KK gravitons. These KK modes may lead a correction to the Newtonian potential. In the following, we give a brief analysis about this. The integrand of $w = \int \frac{1}{\sqrt{N(z)}} dz$ can be calculated as

$$w(z) = -\sqrt{-\theta_+/k^2} E\left(\sin^{-1}\left(z\sqrt{-k^2/\theta_-}\right)|\theta_-/\theta_+\right), \quad (46)$$

where E stands for an elliptic integral function. Figure 6 shows that $w \simeq z$ in the large z region. Thus, it is well approximated to investigate the asymptotic behavior of the effective potential of the graviton in the z coordinate. To obtain an approximate analytic expression for $U(z \to \pm\infty)$, we consider the thin brane limit, i.e., $k \gg 1$. In this limit, the parameter b should be small. Then, the effective potential $U(z)$ can be expanded in terms of b as

$$\begin{aligned} U(z) &= \frac{1}{4(k^2z^2+1)^4}[k^6\phi_0^6(c_0+c_1kz)^2 + 2k^4\phi_0^3(k^2z^2+1)(7c_0kz + c_1(6k^2z^2-1)) + 3(5k^2z^2-2)(k^3z^2+k)^2] \\ &\quad + \frac{k^4\phi_0^2}{2(k^2z^2+1)^4}[k^2\phi_0^3(-7c_0kz - 6c_1k^2z^2 + c_1) - 3k^2z^2(7k^2z^2+5) + 6]b + \mathcal{O}(b^2) \\ &= \frac{1}{\frac{4}{k^8z^6} + \frac{16}{k^6z^4} + \frac{24}{k^4z^2} + \frac{16}{k^2} + 4z^2}\left(\frac{c_0^2k^4\phi_0^6 - 2c_1k^2\phi_0^3 - 6}{k^6z^6} + \frac{2c_0c_1k^4\phi_0^6 + 14c_0k^2\phi_0^3}{k^5z^5} + \frac{14c_0\phi_0^3}{kz^3}\right. \\ &\quad \left. + \frac{c_1^2k^4\phi_0^6 + 10c_1k^2\phi_0^3 + 3}{k^4z^4} + \frac{12c_1k^2\phi_0^3 + 24}{k^2z^2} + 15\right) + \frac{1}{\frac{2}{k^8z^4} + \frac{8}{k^6z^2} + \frac{12}{k^4} + \frac{8z^2}{k^2} + 2z^4}\left(\frac{c_1k^2\phi_0^5 + 6\phi_0^2}{k^4z^4}\right. \\ &\quad \left. - \frac{7c_0\phi_0^5}{kz^3} - \frac{6c_1k^2\phi_0^5 + 15\phi_0^2}{k^2z^2} - 21\phi_0^2\right)b + \mathcal{O}(b^2). \end{aligned} \quad (47)$$

It is obvious that $U(z) \sim \frac{15}{4z^2}$ as $|z| \gg 1$, which has the particular form $\alpha(\alpha+1)/z^2$ and is independent of b, c_0, and c_1. Thus, the nonminimal derivative coupling and the cubic Galileon term will not affect the asymptotic behavior of the potential. Then, the KK modes for small masses on the brane obey the relation $\psi_m(0) \sim m^{\alpha-1}$ shown in Ref. [30], and the correction to the Newtonian potential between two massive objects at a distance of r is

$\Delta V(r) \propto 1/r^{2\alpha}$. For our potential, $\alpha = 3/2$, this leads to $|\psi_m(0)|^2 \sim m$. Thus the correction to the Newtonian potential is $\Delta V(r) \propto 1/r^3$.

V. CONCLUSIONS AND DISCUSSION

In this paper, we investigated thick brane system in reduced Horndeski theory, especially the effects of the nonminimal derivative coupling and the cubic Galileon term on the thick brane model. A set of analytic solutions for the brane system were obtained. The presence of the cubic Galileon term causes the asymmetry of the functions $F(\phi)$ and $V(\phi)$ under $\phi \to -\phi$. However, the energy density of the thick brane is always symmetric with or without this term, because it is only determined by the warp factor and scalar field. Besides, the nonminimal derivative coupling results in a splitting of the brane with b increasing.

For tensor perturbation, a Schrödinger-like equation of the graviton was obtained and its Hamiltonian can be factorized, which ensures the stability of the tensor perturbation of the brane system. The effective potential and wave function of the graviton become asymmetric under $y \to -y$ with the presence of the cubic Galileon term. A double-well structure shows up in the effective potential for $-\frac{1}{k^2\phi_0^2} < b < -\frac{0.63}{k^2\phi_0^2}$ and $\frac{0.47}{k^2\phi_0^2} < b < \frac{1}{k^2\phi_0^2}$ or $-\frac{16.34}{k^2\phi_0^2} < c_1 < -\frac{1.65}{k^2\phi_0^2}$. The wave function splits for $-\frac{1}{k^2\phi_0^2} < b < -\frac{0.77}{k^2\phi_0^2}$ and $\frac{0.77}{k^2\phi_0^2} < b < \frac{1}{k^2\phi_0^2}$ or $c_1 < -\frac{3}{k^2\phi_0^2}$. Besides, the graviton zero mode always can be localized on the brane, which ensures the Newtonian gravity at low energy. Except for the localized graviton zero mode, there are a lot of continuous KK modes that decouple from the brane system. Even so, a sufficient number of massive continuous KK modes produces an observable correction to the Newtonian potential. We gave a brief analysis about it and found that their corrections are $\Delta V(r) \propto 1/r^3$, which is independent of c_0, c_1, and b. The effects of the nonminimal derivative coupling and the cubic Galileon term on the scalar perturbation and the correction of their KK modes to the Newtonian potential are also interesting problems. These are left for our future works.

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