Energy-conservation constraints on cosmic string loop production and distribution functions

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A network of cosmic strings would lead to gravitational waves which may be detected by pulsar timing or future interferometers. The details of the gravitational wave signal depend on the distribution of cosmic string loops, which are produced by intercommutations from the scaling network of long strings. We analyze the limits imposed by energy conservation, i.e., by the fact that the total amount of string flowing into loops cannot exceed the amount leaving the long strings. We show that some recent suggestions for the cosmic string loop production rate and distribution are ruled out by these limits. As a result, gravitational waves based on such suggestions, in particular "model 3" used in LIGO data analysis, are not to be expected.

I. INTRODUCTION

The Universe may contain a network of cosmic strings formed at a symmetry breaking transition in the early Universe or by brane inflation in string theory. (For reviews, see Refs. [1,2].) In the simplest cases, which we will discuss here, strings have neither ends nor vertices, so the network (in a spatially infinite universe) consists of infinite strings and closed loops.

The strings are continually intercommuting, so loops may break off of infinite strings or rejoin to them, and loops may also fragment or join with each other. However, the net effect is a production of loops, so string energy leaves the infinite string network and flows into the loop distribution. Loops then oscillate, emitting gravitational waves and eventually decaying. These processes allow for the cosmic string network to reach a scaling regime, in which all linear measures evolve (on average) in a way proportional to the cosmic time t. The energy density of the scaling network evolves as radiation in the radiation era and matter in the matter era, so the string network is always a subdominant component and does not cause the problems that monopoles would.

Gravitational waves are the leading way to look for a cosmic string network [3–12]. The observable gravitational waves come mostly from the loop distribution, and therefore it is of great importance to understand this distribution. Many distributions have been inferred from simulations or proposed on theoretical grounds. Here, we discuss some important constraints on the rate of loop production and the resulting distribution of loops, arising from the fact that the energy in loops comes from energy originally in the long string network, so energy conservation couples the loop production rate to the loss of energy in the long strings.

In the next section, we discuss the definition of the long string network and the loop production and distribution functions. In Sec. III, we derive and apply the constraints resulting from energy conservation in the production of loops and compare with specific models of loop production. In Sec. IV, we point out that the problem is more general than a conflict of numerical values but applies to any attempt to derive a rapidly diverging loop production function from a simulation, and in Sec. V, we point out that these constraints apply also to certain loop distribution functions. We conclude in Sec. VI.

A dictionary for translating between the parameters and functions used here and those in some other papers is given in Appendix A.

II. LOOPS AND LONG STRINGS

The separation between loops and infinite strings is not completely straightforward because a sufficiently long loop

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cannot be cleanly distinguished from an infinite string. Loops much larger than the horizon are continually reconnecting to infinite strings and breaking off from them again. A very long loop consists of many causally disconnected segments, and the dynamics of each segment may connect the loop to an infinite string [13], so the typical lifetime of such a loop between intercommutations drops inversely with the loop length.

Loops much smaller than the horizon, however, are very unlikely to join with other strings because these loops are much smaller than the distance between strings, which grows with the expansion of the Universe. Small loops, once formed, may fragment into smaller loops, but simulations show that this process does not continue indefinitely but rather yields a distribution of non-self-intersecting loops.[1] It is thus possible to make a reasonably clear distinction between loops, meaning small loops on non-self-intersecting trajectories and that we do not expect to rejoin larger structures, and long strings, in which we include both superhorizon loops and strings that really are infinite.

Simulations, of course, have no infinite strings. Simulators generally use periodic boundary conditions, meaning that all strings are in loops. Typically, all strings that cross the horizon are part of a single large loop that crosses through the periodic boundary conditions many times. Again it is possible to distinguish small loops from long strings, meaning loops above a certain size. In our simulations [15,16], we define loops existing at a certain time as closed strings of any length that will not self-intersect or rejoin in the future,[2] but the exact definition will not be important here, especially as we will mainly be concerned with loops far below the horizon size.

We will describe loops at time t by a loop distribution function, $n(l, t)$, that gives the density of loops per unit volume per unit loop length existing at time t. We will describe loop production by a function $f(l, t)$ giving the number of loops produced per unit time per unit volume per

unit loop length. Loops in self-intersecting trajectories are excluded from both of these functions. We also exclude loops that will join to long strings or other loops, but this is of little consequence for loops much smaller than the typical interstring distance because it is very unlikely that they will find any other string to join.

All lengths here are invariant; i.e., a loop of length l has energy μl, where μ is the energy per unit length (tension) of the string, and we work in units where the speed of light is set to 1. The energy density in long strings [i.e., everything that is not counted in $n(l, t)$] will be denoted ρ_∞.

III. ENERGY CONSERVATION

The breaking off of loops conserves energy, so that the total invariant length of string before and after an intercommutation is the same.[3] This leads to a constraint [17] because the energy flowing into loops must flow out of long strings. The long-string energy density also decreases due to dilution of strings and redshifting of the string velocity due to the expansion of the Universe. The resulting evolution equation for the energy density of long strings is

$$\frac{d\rho_\infty}{dt} = -2H(1 + \langle v_\infty^2 \rangle)\rho_\infty - \mu \int_0^\infty lf(l, t)dl, \quad (1)$$

where H is the Hubble constant and $\langle v_\infty^2 \rangle$ is the rms average velocity of the long strings. Equation (1) constrains the total rate of loop production.

In this paper, the parameter l refers to the invariant length of the loop at the time of production, i.e., its total energy divided by μ. Some of this energy is in the overall kinetic energy of the loop (with respect to the Hubble flow). If the loop is long-lived compared to the Hubble time, this kinetic energy will be lost to redshifting, so what matters is the rest energy [16]. For very short loops, which will be of most concern to us here, l is the natural variable.[4]

No model of the string network is necessary for Eq. (1), but we can go further if we assume that the network is in a scaling regime in a cosmological era in which the scale factor $a \propto t^\nu$ so that $\nu = 1/2$ in the radiation era and $2/3$ in the matter era. In that case, we define a scaling measure of the loop length, $x = l/t$, and define $n(x) = t^4 n(l, t)$ to be the number of loops per unit x in volume t^3, $f(x) = t^5 f(l, t)$ to be the number of loops per unit x produced in time t in volume t^3, and the "interstring distance" $\gamma = \sqrt{\mu/\rho_\infty}/t$. In a scaling regime, γ is constant, and $n(x)$ and $f(x)$ depend only on x and not on t. In that description, Eq. (1) becomes

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[1]Fragmentation is less common than one might at first think. Loops are formed by the combination of right-moving and left-moving excitations on the string. When a loop forms, many small excitations have already passed through each other without forming loops, so they will not do so on future oscillations. Others have not yet passed and may thus form a smaller loop in the first oscillation, but this loop forms with no causal dependence on the fact that it is part of a larger loop at the time of formation. The only small loops that form because of being on a larger loop are those which include at least one of the kinks arising from the larger loop's formation [14]. Simulations show this phenomenon to be quite rare.

[2]After a loop has oscillated three times, we remove it from the simulation, so we would miss rejoinings after that stage. But we have experimented with allowing many more oscillations before removal, and this makes no significant difference to any quantity reported by the simulation. We run long enough beyond the reported simulation ending time to allow loops of up to half the horizon size to undergo the necessary number of oscillations to be correctly classified as loops.

[3]We neglect a tiny amount of particle radiation here. Taking account of it would only strengthen our conclusions.

[4]If we let m be the loop rest energy, the total rest mass appearing in loops is $\int_0^\infty mf(m, t)dm$, which is less than $\mu \int_0^\infty lf(l, t)dl$, leading to a stronger constraint on $f(m, t)$ than on $f(l, t)$.

$$\int_0^\infty x f(x) dx = \frac{2}{\gamma^2} (1 - \nu(1 + \langle v_\infty^2 \rangle)) \equiv B. \qquad (2)$$

Any proposed scaling loop production function $f(x)$ must obey Eq. (2). In our simulations[5] [15] (values from other groups are very similar), we find $\gamma = 0.30$ and $\langle v_\infty^2 \rangle = 0.40$ in the radiation era and $\gamma = 0.51$ and $\langle v_\infty^2 \rangle = 0.35$ in the matter era, so[6]

$$B \approx \begin{cases} 6.7 & \text{radiation,} \\ 0.77 & \text{matter.} \end{cases} \qquad (3)$$

Reference [18] discusses loop production functions (further analyzed as part of Ref. [19]) which grow rapidly toward small scales until they are cut off at some value x_c, which is intended to represent the effect of gravitational smoothing on long strings. Specifically, they consider the possibility that[7]

$$f(x) = cx^{-\beta}\Theta(x - x_c), \qquad (4)$$

with $\beta > 2$. Integrating Eq. (4) gives

$$\int_0^\infty x f(x) dx = \frac{c}{(\beta - 2)x_c^{\beta-2}}. \qquad (5)$$

Following Ref. [20], Refs. [18,19] say that we should take

$$x_c = \Upsilon(G\mu)^{4-\beta}, \qquad (6)$$

with $\Upsilon \sim 20$, and suggest that we choose β and c to match the results of Ref. [21]. Thus, $\beta = 2.6$ in the radiation era and 2.41 in the matter era.[8] With $G\mu = 10^{-7}$ as suggested by Ref. [19], Eq. (6) gives $x_c \approx 3 \times 10^{-9}$

(radiation), 1×10^{-10} (matter). To find a corresponding value of c, we use Eq. (2.22) of Ref. [19],[9] which in our notation gives

$$n(x) = \frac{c}{\beta - \beta_{\text{crit}}} x^{-\beta}, \qquad (7)$$

where $\beta_{\text{crit}} = 4 - 3\nu = 5/2$ (radiation), 2 (matter). Our $n(x)$ corresponds to

$$(1 - \nu)^4 \frac{S(\alpha)}{\alpha} = (1 - \nu)^4 C_0 \alpha^{-p-1} \qquad (8)$$

in the notation of Ref. [21], where $C_0 = 0.21$ (radiation), 0.09 (matter). In our notation,

$$n(x) = (1 - \nu)^{4-\beta} C_0 x^{-\beta}. \qquad (9)$$

Setting Eqs. (7) and (9) equal gives

$$c = (\beta - \beta_{\text{crit}}) C_0 (1 - \nu)^{4-\beta}, \qquad (10)$$

and the numbers above give $c = 0.008$ in the radiation era and 0.006 in the matter era.

Putting these values in Eq. (5) gives

$$\int_0^\infty x f(x) dx \approx \begin{cases} 2000 & \text{radiation,} \\ 200 & \text{matter,} \end{cases} \qquad (11)$$

larger than the values in Eq. (3) by a factor more than 200 in both cases. In fact, the situation is much worse than that because nonobservation of gravitational waves limits $G\mu$ to be no more than of order 10^{-11} [7,9,10,22], and then the discrepancy is 6×10^5 in the radiation era and 9×10^4 in the matter era.

Thus, it is impossible to have the loop production function of Eq. (4) with parameters at all similar to those used by Refs. [18,21] and discussed in Ref. [19].

IV. NETWORKS WITHOUT GRAVITATIONAL SMOOTHING

Suppose we had a huge computer and could run large simulations (without gravitational effects) for as long as we wished. We could continue deep into the scaling regime[10] and discover the scaling loop production function. What could it be? Suppose it were a power law for small x. There would be no gravitational cutoff. So, we would have just $f(x) = cx^{-\beta}$, and if $\beta > 2$, $\int x f(x) dx$ would diverge at

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[5]The definition of loops and loop production used in our simulations is exactly as described above. However, it would be difficult to report long string statistics in a way which depends on the future evolution of the string. Instead, we report our ρ_∞ and $\langle v_\infty^2 \rangle$ including all string that has not been identified as being in non-self-intersecting loops. Some string in loops may later rejoin, and some string is in loops that we have not yet identified. However, we can recognize both of these phenomena later. The maximum error they could have introduced is less than 1%.

[6]Reference [15] defined scaling quantities in terms of the horizon distance, and consequently the B found there was larger by factor $(1 - \nu)^{-3}$. See Appendix A.

[7]Equation (2.15) of Ref. [19] includes a second term representing reduced but nonzero production of loops at scales below x_c. Including it would increase the energy flow into loops and so make the conflict here worse.

[8]These are the central values given in Ref. [21]. For other possibilities, see Appendix B.

[9]Since we are matching Ref. [21], in which there is no gravitational backreaction, we do not need the more general form of Eq. (2.17) of Ref. [19].

[10]Without gravitational effects, we cannot have true scaling because energy will collect in tiny loops. But we would expect scaling in the long string network and in loops above some continually decreasing lower limit size.

$x = 0$. Such a scenario would violate *any* energy conservation bound, so no such result is possible. We cannot evade this conclusion by proposing cutoffs due to gravitational scales because the hypothesized simulation does not include gravitation.

Our hypothetical simulation would have to give some $f(x)$ that obeyed energy conservation and so did not rise too quickly at small scales. It would give an $n(x)$ that would go as $x^{-\beta_{\text{crit}}}$ at small x. If we had such a simulation, we could then apply gravitational effects to give an updated $n(x)$, which would then go as $x^{-\beta_{\text{crit}}}$ down to some x where gravitational effects became important and then fall below that line.

But we do not have such a simulation. Instead, we must make do by extrapolation from smaller simulations. In Ref. [21], the authors fit $n(x)$ to a power law over the range in which they felt $n(x)$ was accurately determined. What can we do with this information? It would be an error to extrapolate this power law and conclude that in a much larger simulation it would continue forever, because we know that is impossible. Neither would it make sense to extrapolate the power law until some gravitational cutoff, because the simulation does not include gravitation. We know that in the nongravitational world, there will eventually be some new behavior, but we do not know what it is. Since it is wrong to extrapolate the power law form of $n(x)$ in the simulated world, it would be wrong to extrapolate in the real world. Thus, it does not make sense to use the power law $n(x)$ from a simulation to derive $n(x)$ in the real universe at any smaller x than those for which the simulation finds scaling behavior.

V. CONSTRAINTS ON THE LOOP DISTRIBUTION

The argument above constrains not only the loop production function but also the loop distribution. The number of loops can be found by integrating the production function, accounting for the decrease in loop size due to gravitational backreaction. In a scaling regime [16],

$$n(x) = \frac{\int_x^\infty (x' + \Gamma G\mu)^{\beta_{\text{crit}}-1} f(x') dx'}{(x + \Gamma G\mu)^{\beta_{\text{crit}}}}. \quad (12)$$

For $x \gg \Gamma G\mu$,

$$n(x) = \frac{\int_x^\infty x'^{\beta_{\text{crit}}-1} f(x') dx'}{x^{\beta_{\text{crit}}}}. \quad (13)$$

If the integral in the numerator does not depend on the lower limit as $x \to 0$, we find $n(x) \sim x^{-\beta_{\text{crit}}}$. If $n(x)$ diverges more rapidly than this as x decreases, the divergence must come partly from the numerator. The only way to have $n(x) \sim x^{-\beta}$ with $\beta > \beta_{\text{crit}}$ is to have $f(x) \sim x^{-\beta}$. In other words, a distribution $n(x) \sim x^{-\beta_{\text{crit}}}$ may arise from loops produced at earlier times, but $n(x)$ may only diverge more

rapidly than this if the tiny loops in question were produced very recently by a similarly diverging production function. But no argument based on simulation could support such a production function.

More directly, any scaling loop production function must obey Eq. (3) to conserve energy, so the integral in that equation must converge. In a simulation that does not include gravitational radiation effects, the relationship between $f(x)$ and $n(x)$ is given by Eq. (13). Since $\beta_{\text{crit}} \geq 2$, the integral in the numerator of Eq. (13) must also converge even if x is taken to 0. Thus, for small enough x, $n(x) \sim x^{-\beta_{\text{crit}}}$ and cannot diverge any faster.

Thus, no simulation can find $n(x) \sim x^{-\beta}$ with $\beta > \beta_{\text{crit}}$ for arbitrarily small x. The loop distribution suggested in Ref. [18] with $\beta = 2.6$ in the radiation era[11] cannot be supported by the simulations of Ref. [21]. Therefore, there is no reason to use gravitational wave predictions based on this spectrum, in particular "model 3" of Ref. [9].

This criticism does not affect "model 1" and "model 2" of Ref. [9]. Both of these models involve a loop production function which is peaked at a certain range of scales not depending on any gravitational cutoff. In such a model, Eq. (5) gives some finite number independent of x_c, and the only issue is that that number should agree with Eq. (2). In model 1, loops are all produced at the same scale (relative to the age of the Universe), and the production rate is adjusted to make Eq. (2) hold. Model 2 takes the loop density from Ref. [16], which is based on the production function found in Ref. [15], and we checked in Ref. [15] that indeed the $f(x)$ found there obeys Eq. (2).

VI. CONCLUSION

To predict observable signals, such as gravitational waves, from a cosmic string network requires knowledge of the distribution of loops at times when the signals may be emitted. To obtain that knowledge, we use simulations, but we cannot simulate the cosmologically necessary range of scales, so we must extrapolate from simulations. However, it does not make sense to use loop production functions that do not conserve energy nor to use loop distributions that can result only from such unrealistic production functions.

Of particular concern are loop production functions of the form $cx^{-\beta}$ with $\beta > 2$. If not cut off, such a function leads to an infinite flow of energy into loops. A cutoff will make the flow finite, but the actual gravitationally based cutoffs proposed for this purpose yield an energy flow much larger than is available from the scaling network of long strings. With modern limits on $G\mu$, the discrepancy is more than 10^5. This is much too large to be explained by

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[11]Note that $n(x) \sim x^{-2.5}$ is within the error bars of Ref. [21], and if that is the true shape of $n(x)$, there is no problem with energy conservation. But the conclusion in that case is very different. We can get $n(x) \sim x^{-2.5}$ from a wide range of loop production functions, even a δ function [1,16,19].

any effects such as small-scale structure or field-theoretic excitations on long strings. Thus, loop production functions of this form, and loop distribution functions arising from them, should not be used to calculate observable effects.

APPENDIX A: DICTIONARY OF NOTATIONS

Different papers use different symbols to denote the same concept, and furthermore even when the concept is the same, some papers use the cosmic time t to define scaling units, while others use the horizon distance $d_h = t/(1 - \nu)$. As a result, powers of $1 - \nu$ are needed to convert between values given in the different papers. Table I lists the notations and the conversion factors for this paper and several recent works.

APPENDIX B: RANGE OF EXPONENTS IN $n(x)$

Reference [21] gave error bars on the possible exponents derived from their simulation. In our notation,

$$\beta = \begin{cases} 2.60^{+0.21}_{-0.15} & \text{radiation} \\ 2.41^{+0.08}_{-0.07} & \text{matter.} \end{cases} \quad \text{(B1)}$$

In the main text, we considered only the central values of this parameter; here, we will consider whether other possibilities for the exponent will allow these distributions to escape the bounds above.

First, consider the radiation era. The error bars above allow the possibility that $\beta = 2.5$. If that is correct, $n(x)$ could arise from a wide range of distribution functions including those discussed in Refs. [15,16]. We cannot then include gravitational effects without knowing more about the loop production function. In particular, such an $n(x)$ from simulations cannot be used as evidence for a diverging loop distribution in the real Universe with gravitation.

Another possibility is that $\beta = 5/2 + \epsilon$ with $\epsilon \ll 1$. From Eq. (10), it appears that c would be very small and so the energy conservation bounds could be obeyed. However, in such a regime, we should consider the loop production function more carefully. Without gravity, the relationship between $n(x)$ and $f(x)$ in the radiation era is [15]

$$n(x) = x^{-5/2} \int_x^\infty x'^{3/2} f(x') dx' \quad \text{(B2)}$$

in our notation. As we discussed in Sec. II above, loops larger than the horizon rarely survive and should not be counted in $f(x)$. Thus, Eq. (7) must be modified. For $1 > x > x_c$, we have [19]

$$n(x) = c x^{-5/2} \int_x^1 x'^{3/2-\beta} dx' = \frac{c}{\epsilon} [x^{-\beta} - x^{-5/2}]. \quad \text{(B3)}$$

When we match this to Eq. (9), we find

$$c = 2^{\beta-4} \frac{\epsilon C_0}{1 - x^\epsilon}. \quad \text{(B4)}$$

This depends on the value of x used to determine c. To attempt to comply with energy conservation bounds, we would like to make c the smallest possible. Since c is an increasing function of x, we will use the smallest x in the range used in Ref. [21] to determine β, which is about 5×10^{-3}.

As we decrease ϵ, c will decrease, but never below its $\epsilon \to 0$ limit, $-C_0/(2\sqrt{2} \ln x)$. With $x = 5 \times 10^{-3}$ and

TABLE I. Notations used here and in some recent papers. The quantities in the different columns in each row are the same when evaluated using the conventions and notation of the papers listed.

This paper	BOS [15]	RSB [21]	ARSS [19]	
μ	μ	U	U	String tension
x	$(1-\nu)^{-1}x$	$(1-\nu)^{-1}\alpha$	γ	Loop length
γ	$(1-\nu)^{-1}\gamma$		γ_∞	Interstring distance
$f(x)$	$(1-\nu)^5 f(x)$		$t^5 \mathcal{P}$	Loop creation rate
$n(x)$	$(1-\nu)^4 n(x)$	$(1-\nu)^4 \mathcal{S}(\alpha)/\alpha$	$t^4 \mathcal{F}$	Loop distribution
β		$p + 1$	$3 - 2\chi$	Exponent in loop creation/distribution
B	$(1-\nu)^3 \mathcal{P}$			Energy flow into loops

$C_0 = 0.21$, we find[12] $c = 0.014$. Putting this in Eq. (5) with $\beta = 2.5$ gives[13] $\int_0^\infty x f(x) dx \approx 1000$ for $G\mu = 10^{-7}$, still many times larger than the value in Eq. (3).

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[12]The reason this is larger than the 0.008 that we found above is that taking account of the finite range of x where $f(x)$ contributes to the $n(x)$ found in Ref. [21] is more important than reducing ϵ to any (positive) value.

[13]The upper limit on x never makes any significant difference in Eq. (5) in the radiation era.

In the matter era, the situation is different because the power of x that multiplies $f(x)$ in the energy flow is the same one that multiplies $f(x)$ in the calculation of $n(x)$. If we make β small enough, it is indeed possible to obey the energy conservation constraint. However, the requisite β is about 2.15 if we take $G\mu = 10^{-7}$ or 2.10 if we take $G\mu = 10^{-11}$. (Including only a finite range of x makes little difference here.) These β lie far outside the range given by Eq. (B1).

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