Corrections to the elastic proton-proton analyzing power parametrization at high energies

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The Polarized Atomic Hydrogen Gas Jet Target polarimeter (HJET) was designed to measure the absolute polarization of the proton beams at the Relativistic Heavy Ion Collider. In these measurements, the small scattering angle elastic pp single $A_N(t)$ and double $A_{NN}(t)$ spin analyzing powers can be precisely determined. The experimental accuracy achieved at HJET requires corrections to the $A_N(t)$ parametrization, conventionally used for such studies. In this paper, we evaluate the corrections to the analyzing powers due to (i) the differences between the electromagnetic and hadronic form factors and (ii) the m_p^2/s terms in the elastic spin-flip pp electromagnetic amplitude. The corresponding alterations of the evaluated hadronic spin-flip amplitudes are about the same as the experimental uncertainties of the HJET measurements. The proposed corrections may have implications for the elastic pp forward real-to-imaginary amplitude ratios ρ determined in unpolarized pp experiments.

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I. INTRODUCTION

The Polarized Atomic Hydrogen Gas Jet Target [1] polarimeter (HJET) is employed to measure the absolute polarization of proton beams at the Relativistic Heavy Ion Collider (RHIC). For that, the vertically polarized proton beam is elastically scattered at small angles (Fig. 1) on the vertically polarized target (the jet) with well-determined polarization $|P_j| = 0.957 \pm 0.001$, and the beam and jet spin correlated asymmetries of the recoil protons [Eq. (20)] are studied.

The major upgrade of HJET in 2015, along with the development of new methods in data analysis, allowed us to reduce the systematic uncertainties of the beam polarization measurements to a $\sigma_P^{\text{syst}}/P \lesssim 0.5\%$ [2] level. Such a small systematic uncertainty of measurements, combined with large statistics of approximately 2×10^9 elastic pp events per RHIC run accumulated in 2015 ($E_{\text{lab}} = 100$ GeV) and 2017 ($E_{\text{lab}} = 255$ GeV), allowed us to precisely measure the single A_{N} and double A_{NN} spin analyzing powers [3] in the Coulomb-nuclear interference (CNI) region.

Generally, $A_{\text{N}}(s,t)$ and $A_{\text{NN}}(s,t)$ are functions of the invariant variables s, center-of-mass energy squared, and t, 4-momentum transfer squared. An important part of the experimental study of the analyzing power is isolation of the hadronic spin-flip amplitudes. The theoretical basis for such studies was developed in Refs. [4,5]. An update [6] for the RHIC spin program provided a parametrization of $A_{\text{N}}(s,t)$, which was used in all previous experimental evaluations [7–9] of the hadronic spin-flip amplitudes in high energy near-forward elastic pp scattering.

Recently, it was pointed out [10] that analyzing power $A_N(t)$ given in Ref. [6] was derived with some simplifications, which might be essential for the experimental accuracy achieved at HJET: (i) it was implicitly assumed that the electromagnetic form factor is equal to the hadronic form factor $\exp(Bt/2)$, and (ii) the elastic pp electric form factor, G_E^{pp}, was approximated, $G_E^{pp} = G_E^2(t)$, by an electric form factor $G_E(t)$ determined in electron-proton

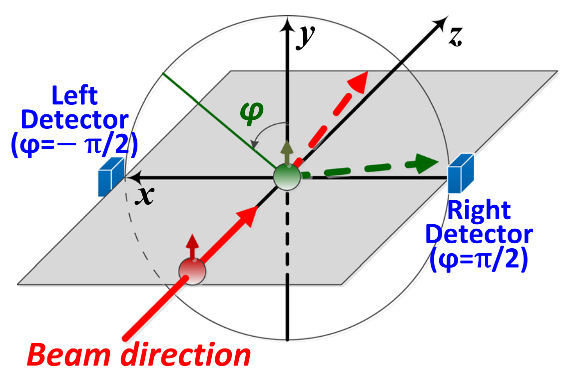

FIG. 1. A schematic view of the $p^\uparrow p^\uparrow$ spin correlated asymmetry measurements at HJET. The recoil protons are counted in left/right symmetric detectors. The beam moves along the z axis. The transverse polarization direction is along the y axis.

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scattering experiments. The absorptive corrections, due to the initial and final state inelastic hadronic interactions between the colliding protons [10], were not considered in Ref. [6].

After this paper was accepted for publication, a theoretical evaluation of the absorptive corrections in elastic pp scattering was given in Ref. [10]. These new results were not taken into account below.

Here, we analyze the effect of the possible corrections to the $A_{\mathrm{N}}(t)$ parametrization on the results of the recent HJET measurements. The evaluated alteration of the measured hadronic spin-flip amplitudes suggests including the corrections in the data analysis. Also, it was found that the discussed corrections may be important for experimental determination of the real-to-imaginary ratio ρ of the pp forward elastic scattering amplitude.

II. PARAMETRIZATION OF THE CNI ANALYZING POWERS AT HIGH ENERGIES

Elastic $p^\uparrow p^\uparrow$ scattering is described by five helicity amplitudes [6]

$$\begin{aligned}
\phi_1(s,t) &= \langle ++|M|++\rangle,\\
\phi_2(s,t) &= \langle ++|M|--\rangle,\\
\phi_3(s,t) &= \langle +-|M|+-\rangle,\\
\phi_4(s,t) &= \langle +-|M|-+\rangle,\\
\phi_5(s,t) &= \langle ++|M|+-\rangle. \qquad (1)
\end{aligned}$$

For scattering in the CNI region, the hadronic and electromagnetic components of the elastic pp amplitude should be explicitly indicated,

$$\phi_i = \phi_i^{\mathrm{h}} + \phi_i^{\mathrm{em}} \exp(i\delta_C). \qquad (2)$$

The Coulomb phase is approximately independent of helicity [5,11],

$$\delta_C = \alpha \ln \frac{-2}{t(B+8/\Lambda^2)} - \alpha\gamma \sim 0.02, \qquad (3)$$

where $\gamma = 0.5772$ is Euler's constant and $\Lambda^2 = 0.71\ \mathrm{GeV}^2$. For numerical estimates in Eq. (3) and below, we assume the HJET measurement values of s and t. The differential cross section slope $B(s)$ depends on energy as $B_0 + B_1 \ln s$ [12] and is about 11.5 GeV^{-2}. To the lowest order in α, the fine structure constant, the electromagnetic amplitudes were calculated in Ref. [5].

For very low t, the hadronic amplitude is dominated by the

$$\phi_+(s,t) = [\phi_1(s,t) + \phi_3(s,t)]/2 \qquad (4)$$

term. According to the optical theorem,

$$\mathrm{Im}\phi_+^{\mathrm{h}}(s,0) = \frac{\sigma_{\mathrm{tot}}(s)s}{8\pi}\sqrt{1-4m_p^2/s}, \qquad (5)$$

where m_p is proton mass and $\sigma_{\mathrm{tot}}(s)$ is the total pp cross section. Therefore, $\phi_+^{\mathrm{h}}(s,t)$ can be presented as

$$\phi_+^{\mathrm{h}}(s,t) = (\rho + i)\frac{\alpha s}{-t_c}(1-4m_p^2/s)^{1/2} e^{Bt/2}, \qquad (6)$$

where

$$\rho(s) = \mathrm{Re}\phi_+^{\mathrm{h}}(s,0)/\mathrm{Im}\phi_+^{\mathrm{h}}(s,0), \qquad (7)$$

$$t_c(s) = -8\pi\alpha/\sigma_{\mathrm{tot}}(s) \approx -1.84\times 10^{-3}\ \mathrm{GeV}^2, \qquad (8)$$

and $\exp(Bt/2)$ is the nuclear form factor.

Similarly, hadronic single and double spin-flip amplitudes may be parametrized by the dimensionless factors

$$r_5(s) = \frac{m_p \phi_5^{\mathrm{h}}}{\sqrt{-t}\,\mathrm{Im}\phi_+^{\mathrm{h}}} = R_5 + iI_5 \qquad (9)$$

and

$$r_2(s) = \frac{\phi_2^{\mathrm{h}}}{2\mathrm{Im}\phi_+^{\mathrm{h}}} = R_2 + iI_2, \qquad (10)$$

respectively.

Using the expression for the elastic pp cross section

$$\frac{d\sigma}{dt} = \frac{2\pi}{s(s-4m_p^2)}(|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 + 4|\phi_5|^2), \qquad (11)$$

the single spin analyzing power can be presented [6] as

$$\begin{aligned}
A_{\mathrm{N}}(t) &= \frac{-4\pi(d\sigma/dt)^{-1}}{s(s-4m_p^2)}\mathrm{Im}[\phi_5^*(\phi_1+\phi_2+\phi_3-\phi_4)]\\
&= \frac{\sqrt{-t}}{m_p}\frac{(t_c/t)f_{\mathrm{N}}^0 + f_{\mathrm{N}}^1}{f_{\mathrm{cs}}(t)}, \qquad (12)
\end{aligned}$$

where

$$f_{\mathrm{N}}^0(r_5) = \varkappa(1-\rho\delta_C) - 2(I_5 - \delta_C R_5), \qquad (13)$$

$$f_{\mathrm{N}}^1(r_5) = -2(R_5 - \rho I_5), \qquad (14)$$

$$f_{\mathrm{cs}}(t) = \left(\frac{t_c}{t}\right)^2 - 2(\rho+\delta_C)\frac{t_c}{t} + 1 + \rho^2. \qquad (15)$$

In Eq. (13), $\varkappa = \mu_p - 1 = 1.793$ is the proton's anomalous magnetic moment. The $A_N(t)$ dependence on r_5 appears in a linear function of t,

$$f_{\rm N}(t,r_5) = f_{\rm N}^0 + f_{\rm N}^1 t/t_c \approx \varkappa - 2I_5 - 2R_5 t/t_c, \qquad (16)$$

while the dependence on r_2 is negligible.

Similarly [6],

$$A_{\rm NN}(t) = \frac{4\pi(d\sigma/dt)^{-1}}{s(s-4m^2)}[2|\phi_5|^2 + {\rm Re}(\phi_1\phi_2^* - \phi_3\phi_4^*)] = \frac{(t_c/t)f_{\rm NN}^0 + f_{\rm NN}^1}{f_{\rm cs}(t)}, \qquad (17)$$

$$f_{\rm NN}^0(r_2) = -2(R_2 + \delta_C I_2), \qquad (18)$$

$$f_{\rm NN}^1(r_2) = 2I_2 + 2\rho R_2 - (\rho\varkappa - 4R_5)\frac{\varkappa t_c}{2m_p^2}. \qquad (19)$$

III. ANALYZING POWER MEASUREMENTS AT HJET

For elastic scattering of vertically polarized beam and target protons, the recoil proton azimuthal angle φ distribution is given [3] by

$$\frac{d^2\sigma}{dt d\varphi} = \frac{1}{2\pi}\frac{d\sigma}{dt} \times [1 + A_{\rm N}\sin\varphi(P_j + P_b) + (A_{\rm NN}\sin^2\varphi + A_{\rm SS}\cos^2\varphi)P_b P_j]. \qquad (20)$$

Here, φ used is defined in accordance with Fig. 1, and P_j and P_b are jet and beam polarizations, respectively.

For HJET detectors, $\sin\varphi = \pm 1$, and, thus, three spin correlated asymmetries $A_{\rm N}P_j$, $A_{\rm N}P_b$, and $A_{\rm NN}P_jP_b$ can be experimentally determined in the momentum transfer range $0.001 \lesssim -t \lesssim 0.020\ {\rm GeV}^2$. Consequently, one can derive the beam polarization P_b (the main purpose of HJET) as well as analyzing powers $A_{\rm N}(t)$ and $A_{\rm NN}(t)$.

The preliminary analysis of the HJET data acquired in RHIC runs 2015 and 2017 has been done using the analyzing power formulas of Ref. [6]. The values of $\sigma_{\rm tot}(s)$ and $\rho(s)$ were taken from Ref. [13] fit. The slope $B(s)$ was derived from Ref. [14]. Only for numerical estimates below, these preliminary results could be summarized as

Run 15 (100 GeV): $\sqrt{s} = 13.76$ GeV,
$\rho = -0.079$, $\sigma_{\rm tot} = 38.39$ mb, $B = 11.2 \pm 0.2\ {\rm GeV}^{-2}$,
$R_5 = (-15.5 \pm 0.9_{\rm stat} \pm 1.0_{\rm syst}) \times 10^{-3}$,
$I_5 = (-0.7 \pm 2.9_{\rm stat} \pm 3.5_{\rm syst}) \times 10^{-3}$,
$R_2 = (-3.65 \pm 0.28_{\rm stat}) \times 10^{-3}$,
$I_2 = (-0.10 \pm 0.12_{\rm stat}) \times 10^{-3}$.
Run 17 (255 GeV): $\sqrt{s} = 21.92$ GeV,
$\rho = -0.009$,
$\sigma_{\rm tot} = 39.19$ mb, $B = 11.6 \pm 0.2\ {\rm GeV}^{-2}$,
$R_5 = (-7.3 \pm 0.5_{\rm stat} \pm 0.8_{\rm syst}) \times 10^{-3}$,
$I_5 = (21.5 \pm 2.5_{\rm stat} \pm 2.5_{\rm syst}) \times 10^{-3}$,
$R_2 = (-2.15 \pm 0.20_{\rm stat}) \times 10^{-3}$,
$I_2 = (-0.35 \pm 0.07_{\rm stat}) \times 10^{-3}$.

For r_2, systematic errors are small in these measurements.

IV. CORRECTIONS TO THE ANALYZING POWERS

To calculate corrections to Eq. (12), it is convenient to use the scaled amplitudes

$$\varphi_i(s,t) = \phi_i(s,t)/{\rm Im}\phi_+^{\rm h}(s,t). \qquad (21)$$

Since a possible dependence of ρ, r_2, and r_5 on t may be neglected in the CNI region, the scaled hadronic amplitudes can be approximated by

$$\begin{aligned} \varphi_1^{\rm h} &= \varphi_3^{\rm h} = \rho(s) + i, \\ \varphi_2^{\rm h} &= 2r_2(s), \\ \varphi_4^{\rm h} &= r_4(s) \times (-t/m_p^2) \approx 0, \\ \varphi_5^{\rm h} &= r_5(s) \times \sqrt{-t}/m_p. \end{aligned} \qquad (22)$$

For the electromagnetic amplitudes, we should include the m_p^2/s corrections, which can be significant for $E_{\rm Lab} = 100$ GeV. Using the following expressions for the proton's electromagnetic form factors [6,15],

$$F_1 = \frac{G_E - G_M t/4m_p^2}{1 - t/4m_p^2}, \qquad \varkappa F_2 = \frac{G_M - G_E}{1 - t/4m_p^2}, \qquad (23)$$

and neglecting the t/s terms, one can derive from Ref. [5]

$$\begin{aligned} \varphi_1^{\rm em} &= \varphi_3^{\rm em} = \varphi_0^{\rm em} \times (1 - 2m_p^2/s)/\sqrt{1 - 4m_p^2/s}, \\ -\varphi_2^{\rm em} &= \varphi_4^{\rm em} = \varphi_0^{\rm em} F_\varkappa^2 \times (1 - 2m_p^2/s)/\sqrt{1 - 4m_p^2/s}, \\ \varphi_5^{\rm em} &= \varphi_0^{\rm em}\left(F_\varkappa - \frac{\sqrt{-t}}{2m_p}\frac{2m_p^2}{s - 4m_p^2}\right). \end{aligned} \qquad (24)$$

The following shorthand was used:

$$\varphi_0^{\rm em} = \frac{t_c(s)}{t} \times F_1^2(t)\exp(-Bt/2) \qquad (25)$$

$$F_1 = \frac{1 - \mu_p t/4m_p^2}{1 - t/4m_p^2} \times (1 + r_E^2 t/6), \qquad (26)$$

$$F_\varkappa = \frac{\sqrt{-t}}{2m_p}\frac{\varkappa F_2}{F_1} = \frac{\sqrt{-t}}{2m_p}\frac{\varkappa}{1 - \mu_p t/4m_p^2}. \qquad (27)$$

The proton's electric form factor $G_E(t)$ was approximated in (26) by the proton charge radius $r_E = \langle r_E^2\rangle^{1/2}$. In Eq. (27), we did not distinguish between proton electric r_E and magnetic r_M radii.

The electromagnetic and hadronic form factors difference can be realized by the substitution $t_c \to t_c + bt$, where

$$b/t_c = \frac{d}{dt}[F_1^2(t)e^{-Bt/2}]_{t=0}. \tag{28}$$

Since the electric form factor $G_E(t)$ in the dipole form [11,16]

$$G_D(t) = (1 - t/\Lambda^2)^{-2}, \qquad \Lambda^2 = 0.71\ \mathrm{GeV}^2 \tag{29}$$

was commonly used in the elastic pp data analysis, it is convenient to explicitly isolate the corresponding term b_D in (28)

$$b = b_D + b_{\mathrm{nf}}, \tag{30}$$

where

$$b_D/t_c = \frac{d}{dt}(G_D^2(t)e^{-Bt/2})|_{t=0} = \left(\frac{4}{\Lambda^2} - \frac{B}{2}\right). \tag{31}$$

For the RHIC beam energies,

$$100\ \mathrm{GeV}\colon\ b_D = (-0.06 \pm 0.19) \times 10^{-3}, \tag{32}$$

$$255\ \mathrm{GeV}\colon\ b_D = (+0.31 \pm 0.19) \times 10^{-3}. \tag{33}$$

The errors here correspond to the systematic uncertainties in the values of $B(s)$ [14].

For b_{nf}, one finds

$$b_{\mathrm{nf}}/t_c = r_E^2/3 - 4/\Lambda^2 - \varkappa/2m_p^2. \tag{34}$$

Currently, PDG [17] gives two values of proton charge radius

$$r_{ep} = 0.8751 \pm 0.0061\ \mathrm{fm}, \tag{35}$$

$$r_{\mu p} = 0.84086 \pm 0.00026 \pm 0.00029\ \mathrm{fm}, \tag{36}$$

obtained in three kinds of measurements: with atomic hydrogen, with electron scattering off hydrogen, and with muonic hydrogen. The discrepancy between the methods is not resolved yet. Assuming $r_E = 0.858 \pm 0.017$ fm, one obtains

$$b_{\mathrm{nf}} = (0.64 \pm 0.46) \times 10^{-3}. \tag{37}$$

Approximating $\varphi_0^{\mathrm{em}} = (t_c/t)e^{bt/t_c}$, one finds a correction to the denominator (15) of the analyzing power expressions

$$\begin{aligned} f_{\mathrm{cs}}(t,\rho) \to f_{\mathrm{cs}}(t, \rho - b + \varkappa^2 t_c/4m_p^2) \\ + b^2 - 2b\delta_C - 2\rho\varkappa^2 t_c/4m_p^2 + \ldots \\ \approx f_{\mathrm{cs}}(t, \rho - b_D - b_{\mathrm{cs}}). \end{aligned} \tag{38}$$

where

$$b_{\mathrm{cs}} = b_{\mathrm{nf}} - \varkappa^2 t_c/4m_p^2 = (2.3 \pm 0.5) \times 10^{-3}. \tag{39}$$

The term $\varkappa^2 t_c/4m_p^2$ here is due to the spin-flip amplitude φ_5^{em} contribution to $d\sigma/dt$.

The experimental determination of the real to imaginary ratio ρ at high energies is based on an analysis of the $d\sigma/dt(t) \propto f_{\mathrm{cs}}(t,\rho)$. The proton-proton electromagnetic form factor was approximated by $G_D^2(t)$ in almost all experimental studies of ρ. Therefore, a biased value of ρ was measured in these experiments, $\rho^{\mathrm{exp}} = \rho - b_{\mathrm{cs}}$. The bias is small compared to the uncertainty of measurements in any of the experiments listed in PDG, but it may be substantial for the global fit [18]. Since the values of ρ from the global fit are used in the analyzing power measurements, we should replace

$$\rho \to \rho + b_{\mathrm{cs}} \tag{40}$$

in (38) as well as in the expressions for f_{N}^0, f_{N}^1, and f_{NN}^1 above. Thus, the leading order corrections to the analyzing power $A_N(t)$ from Ref. [6] can be approximated in (12) as

$$f_{\mathrm{cs}}(t,\rho) \to f_{\mathrm{cs}}(t, \rho - b_D), \tag{41}$$

$$f_{\mathrm{N}}^0 \to f_{\mathrm{N}}^0 - 2m_p^2/s, \tag{42}$$

$$f_{\mathrm{N}}^1 \to f_{\mathrm{N}}^1 + \varkappa(b_D + b_{\mathrm{nf}} + b_\varkappa), \tag{43}$$

where

$$b_\varkappa = \mu_p t_c \left[\frac{1}{4m_p^2} + \frac{r_M^2 - r_E^2}{6\varkappa}\right] \approx (-1.4 \pm 0.7) \times 10^{-3} \tag{44}$$

reflects the spin-flip contribution [see Eq. (27)] to the electromagnetic form factor. The specified error is dominated by the experimental uncertainties in the value of proton magnetic radius $r_M = 0.851 \pm 0.026$ fm [19].

For $A_{\mathrm{NN}}(t)$, the corrections are small compared to uncertainties of the measurement at HJET. Also, we can neglect the correction to the Coulomb phase $\delta_C(r_E, B)$.

V. NUMERICAL ESTIMATES OF THE CORRECTIONS

The effect of the substitution (41) can be parametrized by the effective correction $b_D \Delta f_{\mathrm{N}}(t)$ to the linear function $f_{\mathrm{N}}(t)$:

$$\frac{1}{f_{\mathrm{cs}}(t, \rho - b_D)} = \frac{1 + b_D \Delta f_{\mathrm{N}}(t)}{f_{\mathrm{cs}}(t,\rho)}. \tag{45}$$

The dependence of $\Delta f_{\mathrm{N}}(t)$ on ρ and b_D can be neglected. The calculated value of this correction is shown in Fig. 2. For the HJET values of b_D given in Eqs. (32) and (33), the nonlinearity is not experimentally observable, and, thus, we can approximate

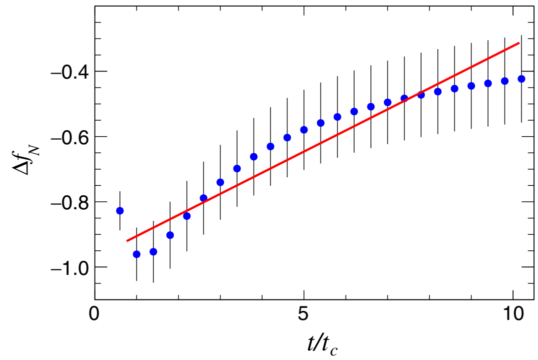

FIG. 2. Calculation of the correction function $\Delta f_{\mathrm{N}}(t)$ (blue points). The displayed error bars $\sigma_{0.1}(t)$ correspond to the HJET measurement statistical uncertainties if $b_D = 0.1$. The error dependence on b_D can be approximated by $\sigma(t, b_D) = \sigma_{0.1}(t) \times 0.1/b_D$. The red line is a linear fit.

$$\Delta f_{\mathrm{N}}(t) = c_0 + c_1 t/t_c \tag{46}$$

or, equivalently,

$$f_{\mathrm{N}}^i \to f_{\mathrm{N}}^i + c_i \varkappa b_D, \quad i = 0, 1. \tag{47}$$

Obviously, the values of c_0 and c_1 depend on the t-range and experimental uncertainties. The HJET data analysis leads to $c_0 \sim -1.0$ and $c_1 \sim 0.1$.

Combining (42), (43), and (47), we find the corrections to the measured hadronic form factors as follows:

$$\Delta I_5 = (\varkappa/2) \times c_0 b_D - m_p^2/s, \tag{48}$$

$$\Delta R_5 = (\varkappa/2) \times [(1 + c_1) b_D + b_{\mathrm{nf}} + b_\varkappa] + \rho \Delta I_5. \tag{49}$$

For HJET measurements, the calculation gives

$$\begin{aligned} 100\ \mathrm{GeV}:\ \Delta R_5 &= (-0.4 \pm 0.2_B \pm 0.4_{r_E} \pm 0.6_{r_M}) \times 10^{-3}, \\ \Delta I_5 &= (-4.6 \pm 0.2_B) \times 10^{-3}, \\ 255\ \mathrm{GeV}:\ \Delta R_5 &= (-0.4 \pm 0.2_B \pm 0.4_{r_E} \pm 0.6_{r_M}) \times 10^{-3}, \\ \Delta I_5 &= (-2.1 \pm 0.2_B) \times 10^{-3}. \end{aligned} \tag{50}$$

The errors here are due to uncertainties in values of B, r_E, and r_M. Each error is strongly correlated through Eqs. (50). The large corrections to I_5 are due to the term m_p^2/s in (48), which is 0.0047 for 100 GeV and 0.0018 for 255 GeV. Alterations of the measured r_5 are comparable with the experimental uncertainties (see Fig. 3) and, thus, should not be neglected.

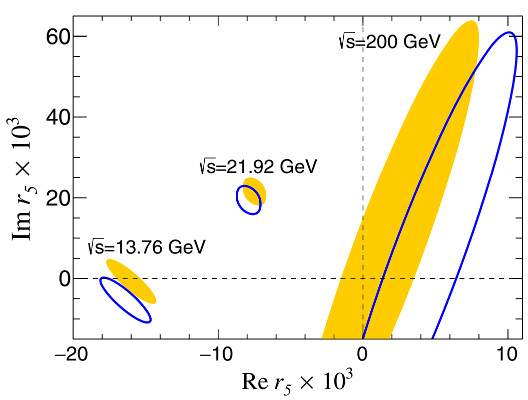

FIG. 3. $\Delta\chi^2 = 1$ correlation (stat + syst) contours for r_5 with (solid lines) and without (filled areas) corrections to A_{N}. The absorptive corrections are not included. To display the $\sqrt{s} = 200$ GeV contours, we used the data from Ref. [9].

VI. POSSIBLE EFFECT OF THE ABSORPTIVE CORRECTION

A dependence of the measured r_5 on the absorptive corrections could be readily estimated if the corresponding modification of the electromagnetic form factor $\mathcal{F}^{\mathrm{em}}(t)$ of an elastic pp amplitude can be approximated in the CNI region by a linear function of t,

$$\mathcal{F}^{\mathrm{em}}(t) \to \mathcal{F}^{\mathrm{em}}(t) \times [1 + a(s) t/t_c]. \tag{51}$$

Generally, $a(s)$ is spin dependent. It can be effected by the substitutions $b_{\mathrm{nf}} \to b_{\mathrm{nf}} + a_{\mathrm{nf}}$, $b_\varkappa \to b_\varkappa + a_{\mathrm{sf}} - a_{\mathrm{nf}}$, where $a_{\mathrm{nf}}(s)$ and $a_{\mathrm{sf}}(s)$ are absorptive corrections to nonflip and spin-flip amplitudes, respectively. The dominant absorptive corrections to r_5 and r_2 can be written as

$$\Delta_{\mathrm{a}} R_5 = a_{\mathrm{sf}} \varkappa/2, \qquad \Delta_{\mathrm{a}} I_5 = -a_{\mathrm{nf}} \delta_C \varkappa/2 \approx 0, \tag{52}$$

$$\Delta_{\mathrm{a}} R_2 = 0, \qquad \Delta_{\mathrm{a}} I_2 = a_{\mathrm{nf}} \frac{\varkappa^2 t_c}{4 m_p^2} \approx 0. \tag{53}$$

As it was underlined above, the correction, such as given in Eq. (51), does not modify $f_{\mathrm{cs}}(t, \rho)$ but specifies the systematic errors in the experimental determinations of ρ. In case of large corrections, say $|a_{\mathrm{nf}} + b_{\mathrm{cs}}| \gtrsim 0.003$, the results of all forward unpolarized proton-proton scattering measurements should be revised, and, consequently, a new global fit of $\rho(s)$ and $\sigma_{\mathrm{tot}}(s)$ should be carried out.

VII. CONCLUSIONS

In this paper, the corrections to the analyzing powers given in Ref. [6] were studied. For the experimental results already published, Eqs. (48) and (49) allows one to evaluate with sufficient accuracy the corrections to the measured single spin-flip amplitude parameter r_5.

The improved expressions for $A_{\mathrm{N}}(t)$ and $A_{\mathrm{NN}}(t)$ could be written in the same form as in Ref. [6] (if neglecting the absorptive correction terms $\Delta_{\mathrm{N}}^a = \Delta_{\mathrm{NN}}^a = 0$),

$$\frac{m_p}{\sqrt{-t}}A_{\rm N}(t) = \frac{[\varkappa'(1-\rho'\delta_C) - 2(I_5 - \delta_C R_5)]t_c'/t - 2(R_5 - \rho' I_5) + \Delta_{\rm N}^a \varkappa}{(t_c/t)^2 - 2(\tilde\rho + \delta_C)t_c/t + 1 + \tilde\rho^2}, \tag{54}$$

$$A_{\rm NN}(t) = \frac{-2(R_2 + \delta_C I_2)t_c'/t + 2(I_2 + \rho' R_2) - (\rho'\varkappa - 4R_5)\varkappa t_c/2m_p^2 + \Delta_{\rm NN}^a \varkappa t/m_p^2}{(t_c/t)^2 - 2(\tilde\rho + \delta_C)t_c/t + 1 + \tilde\rho^2}, \tag{55}$$

but with the following modification of some parameters:

$$t_c'/t = t_c/t + (r_E^2/3 - B/2 - \varkappa/2m_p^2)t_c, \tag{56}$$

$$\rho' = \rho + (r_E^2/3 - 4/\Lambda^2 - \varkappa/2m_p^2 - \varkappa^2/4m_p^2)t_c, \tag{57}$$

$$\tilde\rho = \rho - (4/\Lambda^2 - B/2)t_c, \tag{58}$$

$$\varkappa' = \varkappa/(1 - \mu_p t/4m_p^2) - 2m_p^2/(s - 4m_p^2). \tag{59}$$

The published HJET results [20] were obtained using Eqs. (54)–(59) without absorptive corrections.

The double spin-flip amplitude terms in (54) and the term $|\varphi_t^{\rm h}|^2$ in (55) were dropped off because they are negligible for the HJET experimental accuracy and, also, are comparable with the omitted corrections of order of $(m_p^2/s)^2$ and t/s. The $(m_p^2/s)^2$ corrections (24) to nonflip amplitudes $\varphi_{1,3}$ were neglected in Eq. (54). It should be noted that for experimental uncertainties similar to those at HJET these corrections become noticeable if $\sqrt{s} \lesssim 5$ GeV.

The absorptive corrections are currently undetermined [no values of $a(s)$ are published yet] but once calculated may be introduced by the following substitutions:

$$r_E^2/3 \to r_E^2/3 + a_{\rm nf}/t_c, \tag{60}$$

$$\Delta_{\rm N}^a = a_{\rm sf} - a_{\rm nf}, \tag{61}$$

$$\Delta_{\rm NN}^a = 4R_5(a_{\rm sf} - a_{\rm nf}) - \rho\varkappa(a_{\rm df} - a_{\rm nf}), \tag{62}$$

where $a_{\rm df}(s)$ is the absorptive correction (51) to the double spin-flip electromagnetic form factor. The absorptive corrections may also affect the results of determination of ρ in unpolarized elastic pp scattering.

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