Dynamic nature of the anomalous magnetic moment of an electron in a constant magnetic field in topologically massive two-dimensional electrodynamics

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An analytical expression is obtained for the anomalous magnetic moment of an electron in a constant magnetic field in topologically massive two-dimensional electrodynamics. In the limiting case of a relatively weak magnetic field, asymptotic formulas are found, which specify the dependence of the anomalous magnetic moment on the Chern-Simons parameter and dynamic parameter of synchrotron radiation. The conditions are identified for the applicability of the computations of the anomalous magnetic moment of an electron, which have previously been made based on the calculation of the vertex function in two-dimensional electrodynamics with the Chern-Simons term.

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I. INTRODUCTION

A number of fundamental theoretical studies and experimental results have attracted special attention to low-dimensional models of the quantum field theory. These investigations involve discovery of the integer [1] and fractional quantum Hall effect, explained within the two-dimensional anionic model [2–5]; the appearance of works in which it is shown that a possibility exists to develop in $(2+1)$-dimensional space a gauge-invariant theory with a massive gauge field due to adding the topological Chern-Simons term to the Lagrangian of the interaction between gauge fields and fields of matter [6,7]; and the discovery of quasiplanar structures in high-temperature superconductors [8,9], production of graphene [10–12], and other low-dimensional structures [13].

Studies show that the finite mass of the gauge field initiates screening of both electrical and magnetic fields [14–17], making possible the attraction of similar charges [18,19] and a number of other unusual effects [20].

Previous studies [21,22] showed that if in the original Lagrangian of two-dimensional electrodynamics with massive fermion there is no Chern-Simons term, it is generated dynamically by means of the one-loop contribution to the antisymmetric part of the photon polarization operator, both at the finite temperature and density and in the external magnetic field [23–26]. An induced Chern-Simons term in the polarization operator has the following structure [14,21]:

$$\Pi^A_{\mu\nu}(q) = i\varepsilon_{\mu\nu\alpha}q^\alpha\Pi^A(q^2),$$

where $\varepsilon_{\mu\nu\alpha}$ is the absolutely antisymmetric 3rd rank unit pseudotensor, and the induced Chern-Simons mass is described by the formula

$$\Theta_{ind} = \lim_{q\to 0}\Pi^A(q^2).$$

Here, the higher orders of perturbation theory do not contribute to value Θ_{ind} [27,28].

The study of radiation effects in two-dimensional field theories at the finite temperature and nonzero chemical potential shows that a Chern-Simons term may play a role as a regulator of infrared divergences [6]. For example, the one-loop mass operator of QED_{2+1} without the Chern-Simons term on the mass surface comprises infrared divergence. To eliminate this, a charge screening effect is taken into consideration [29]. At the same time, the mass operator in topologically massive two-dimensional electrodynamics at the nonzero temperature and density is finite even in a one-loop approximation.

A one-loop radiative energy shift of an electron in the charge-symmetric plasma, as well as at a nonzero chemical potential, in two-dimensional electrodynamics with the Chern-Simons term has been analyzed in the papers [30,31]. The ground state electron mass shift in a constant magnetic field has been calculated within the framework of topologically massive two-dimensional electrodynamics and in the case of planar charged fermion in QED_{2+1} without the Chern-Simons term in Refs [32,33], respectively.

Alongside the effects of finite temperature and nonzero chemical potential, of interest is the study of the spin effects in QED_{2+1} associated with the electron anomalous magnetic moment (AMM). It relates to, specifically, the

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emergence of experimental data on the measurement of the interaction energy of the AMM of an electron with the external magnetic field in graphene [34,35] and possible applications in physics of planar high-temperature superconductors for the purposes of explaining the parity violation in the presence of the external magnetic field [36–38].

The first calculations of electron AMM in topologically massive QED_{2+1} with the Chern-Simons term were made based on the vertex function in the fieldfree case, that is, without taking into account the influence of the external magnetic field [39–43]. In Ref. [43], calculations of the finite temperature contribution to the electron AMM were also carried out.

For the purposes of analyzing the experimental results given in [34,35], in Ref. [44] presented the calculation of AMM of an electron in the context of pseudo-QED_{2+1} in the approximation, which is linear throughout the external field, that is, with no regard the dynamic nature of the electron AMM [45–50].

It should be noted that in works [36–38] devoted to the examination of the electron AMM in the P-even two-dimensional model of quantum electrodynamics, a spectral representation of the photon propagator was used for eliminating the infrared divergence of the vertex function in a constant magnetic field.

In Ref. [51], a complete description of the electron stationary states in a magnetic field was conducted in two-dimensional electrodynamics with the doubled fermion representation. This result is used in [51] to compute the radiative shift of the electron ground state energy and electron AMM in the magnetized plasma of topologically massive two-dimensional electrodynamics in a relatively weak magnetic field. The AMM of the excited states of the electron in a constant magnetic field in QED_{2+1} without the Chern-Simons term was investigated in Ref. [52].

In the present work the AMM of the excited states of an electron is calculated in a constant magnetic field in a topologically massive QED_{2+1} with 2×2 matrices [20]. In Sec. II, a method is suggested for deriving the interaction energy of the electron AMM with the external magnetic field from the radiative shift of the electron mass in two-dimensional electrodynamics, which is odd under P and T transformation. An exact expression is obtained for the AMM of the electron moving in a constant magnetic field in topologically massive two-dimensional electrodynamics. A model of QED_{2+1} with the doubled fermion representation has also been considered, as in the case in which algebra of Dirac matrices is described using Pauli matrices [25,53,54]. In Sec. III, the asymptotic formulas for the AMM of an electron are obtained as a function of magnetic field strength, electron energy, and the Chern-Simons parameter.

The calculation of AMM in the P-odd theory, carried out in this paper, has led to completely new, not only quantitative, but also qualitative results on the role of the magnetic field and the Chern-Simons parameter in the study of the interaction energy of an electron AMM with an external magnetic field in two-dimensional electrodynamics. A discussion of these results is carried out in Sec. IV.

II. AMM OF EXCITED STATES OF AN ELECTRON IN TWO-DIMENSIONAL ELECTRODYNAMICS WITH CHERN-SIMONS TERM

The Lagrangian of two-dimensional electrodynamics with the Chern-Simons term is described by formula [15,20]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(\hat{p} + e\hat{A} - m)\psi + \frac{1}{4}\Theta\varepsilon^{\mu\nu\lambda}F_{\mu\nu}A_\lambda - \frac{1}{2\varsigma}(\partial_\mu A^\mu)^2. \tag{2.1}$$

Here, $\psi = \binom{\psi_1}{\psi_2}$—is the two-component spinor, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the gauge field tensor, ς is the gauge-fixing parameter, m is the mass of the electron, $-e < 0$ is the electron charge, $\varepsilon^{\mu\nu\lambda}$ is the completely antisymmetric third rank unit pseudotensor, $\varepsilon^{012} = 1$, Θ is the Chern-Simons term, metric tensor $g^{\mu\nu} = \mathrm{diag}(1, -1, -1)$.

The vector potential of an external magnetic field in the Landau gauge is given by the formula

$$A^\mu_{\mathrm{ext}} = (0, 0, xH).$$

For two-dimensional gamma-matrices, we use the Dirac representation, in which

$$\gamma^0 = \sigma_3, \qquad \gamma^1 = i\sigma_1, \qquad \gamma^2 = i\sigma_2, \tag{2.2}$$

where σ_k ($k = 1, 2, 3$) is the Pauli matrices.

Matrices γ^μ meet the following relations:

$$\gamma^\mu\gamma^\nu = g^{\mu\nu} - i\varepsilon^{\mu\nu\lambda}\gamma_\lambda, \qquad Sp(\gamma^\mu\gamma^\nu\gamma^\rho) = -2i\varepsilon^{\mu\nu\rho}. \tag{2.3}$$

We note that, as opposed to QED_{3+1}, it is impossible to build a dual tensor of the field in $(2+1)$-dimensional models of the field theory, since there is no 4th rank tensor $\varepsilon_{\mu\nu\alpha\beta}$, and a trace of an odd number of two-dimensional gamma-matrices is different from zero.

We also provide a brief account following [20,55]. In the rest frame of an electron without the magnetic field, the Dirac equation can be transformed into an equation for the spin operator $\frac{1}{2}\sigma_3$

$$\sigma_3\Psi = \zeta\Psi, \tag{2.4}$$

where $\zeta = 2s = \pm 1$. But the spin is a pseudoscalar with respect to the Lorenz group $SO(2,1)$ in $2+1$ dimensions, and so a Lorentz boost leaves it unchanged. Thus, Eq. (2.4) gives the spin of a particle in an arbitrary frame. It should also be emphasized that the magnetic field in QED_{2+1} is not a pseudovector, as in QED_{3+1}, but a pseudoscalar [20].

In the one-loop approximation, mass operator and the radiative energy shift of an electron in a constant magnetic field are described by the formulas [56,57]

$$\Sigma(x,x') = -ie^2\gamma^\mu S_c(H,x,x')\gamma^\nu D_{\mu\nu}(x-x'), \qquad (2.5)$$

$$\Delta E_n = \int d^3x d^3x' \bar{\Psi}_{q\zeta}(x)\Sigma(x,x')\Psi_{q\zeta}(x'), \qquad (2.6)$$

where

$$S_c(H,x,x') = -\frac{1}{2\pi i}\int_{-\infty}^{\infty} d\omega \exp[i\omega(t-t')] \times \sum_{s,\varepsilon=\pm 1} \frac{\Psi_s^{(\varepsilon)}(\vec{x})\bar{\Psi}_s^{(\varepsilon)}(\vec{x'})}{\omega+\varepsilon E_s(1-i\delta)} \qquad (2.7)$$

is a causal Green function of the electron in a constant magnetic field [57,58], and the photon propagator in the Landau gauge is determined by the formula [14]

$$D_{\mu\nu}(p) = -\frac{i}{p^2-\Theta^2+i0} \times \left[g_{\mu\nu} + i\theta\varepsilon_{\mu\nu\lambda}\frac{p^\lambda}{p^2+i0} - \frac{p_\mu p_\nu}{p^2+i0}\right]. \qquad (2.8)$$

The summation in formula (2.7) made through all quantum numbers $\{s\}$ of positive-frequency ($\varepsilon = +1$) and negative-frequency ($\varepsilon = -1$) stationary states of an electron, $\Psi_s^\varepsilon(\vec{x})$ is the coordinate part of the solution to the Dirac equation in a constant magnetic field in QED_{2+1}, E_s is the energy of the electron stationary states.

The solution of the Dirac equation in the gauge $A_\mu = (0,0,xH)$ and in representation (2.2) for gamma-matrices was obtained, for example, in works [25,53,59]. In Ref. [25], the method of eigenfunctions in an external electromagnetic field is used, which was developed in works [60–62]. The electron energy level in a magnetic field in $(2+1)$-dimensional QED and the Dirac equation normalized positive-frequency solutions are described by formulas [25]:

$$\Psi_s(x,y,t) = \frac{\exp(-iE_nt+iyp_y)}{\sqrt{2E_n}}\begin{pmatrix} u_n(\eta)\sqrt{E_n+m} \\ u_{n-1}(\eta)\sqrt{E_n-m}\end{pmatrix}, \quad \{s\} = (p_y, n, \zeta), \qquad (2.9)$$

$$E_n = \sqrt{m^2+2eHn}, \qquad n = 0,1,2\ldots \qquad (2.10)$$

Here, $u_n(\eta)$ is the Hermite function [56],

$$u_n(\eta) = \frac{(eH)^{\frac{1}{4}}}{[2^n n! \pi^{\frac{1}{2}}]^{\frac{1}{2}}}\exp\left(-\frac{\eta^2}{2}\right)H_n(\eta), \qquad (2.11)$$

and argument of Hermite polynomials

$$\eta = \sqrt{eH}\left(x+\frac{p_y}{eH}\right). \qquad (2.12)$$

If the two-dimensional space is presented as embedded into the ordinary three-dimensional space, and $A_\mu = (0,0,xH)$, the projection of the magnetic field is $H_z = -F_{12} = -H$, that is, the direction of the magnetic field is against that of the OZ axis. On the other hand, eigenvectors $\psi_1 = \binom{1}{0}$ and $\psi_2 = \binom{0}{1}$, which correspond to eigenvalues $\zeta = \pm 1$ of matrix σ_3, describe the states of the electron, the spin that is oriented along the positive (ψ_1) or negative (ψ_2) direction of the OZ axis.

In using the results of work [25] and retaining conventional for the QED_{3+1} physical interpretation, we will say that the projection of spin in the state with $u_n(\eta)$ in formula (2.9) to the direction of the magnetic field is $-\frac{1}{2}$, i.e., spin quantum number $\zeta = -1$, and in the state with $u_{n-1}(\eta)$, on the contrary, $\zeta = +1$. Such an interpretation is in agreement with the formula for the main quantum number n, which specifies the spectrum of the electron in QED_{2+1} [25,60]:

$$n = k - \frac{\zeta}{2}\mathrm{sign}(eH) - \frac{1}{2}, \qquad k = 0,1,2\ldots$$

where $H > 0$.

Indeed, $k = 0, e < 0, H > 0$ and $\zeta = -1$, correspond to the ground state of an electron in this formula. Thus, after expansion of the two component wave function by eigenfunctions of matrix σ_3, formula (2.9) may be presented in the form that is most convenient for further computations:

$$\Psi_{p_y,n,\zeta}(x,y,t) = \frac{\exp(-iE_nt+iyp_y)}{\sqrt{2E_n}}\left[D_{-1}\begin{pmatrix}1\\0\end{pmatrix}u_n(\eta)\sqrt{E_n+m} + D_1\begin{pmatrix}0\\1\end{pmatrix}u_{n-1}(\eta)\sqrt{E_n-m}\right] = \frac{\exp(-iE_nt+iyp_y)}{\sqrt{2E_n}}\begin{pmatrix}D_{-1}u_n(\eta)\sqrt{E_n+m}\\ D_1u_{n-1}(\eta)\sqrt{E_n-m}\end{pmatrix}. \qquad (2.13)$$

Here, when $\zeta = +1$, it is necessary to set $D_1 = 1, D_{-1} = 0$, when $\zeta = -1$, on the contrary, $D_1 = 0, D_{-1} = 1$, and coefficients D_1 and D_{-1} meet the normalization requirement

$$D_{-1}^2 + D_1^2 = 1.$$

Using (2.2), (2.7), (2.9), (2.13), and the method of calculation suggested in works [63,64], electronic Green's function (2.7) in QED_{2+1} in a constant magnetic field shall be presented as follows:

$$\begin{aligned} S_c(H,x,x') &= \exp\left[-i\frac{eH}{2}(y-y')(x+x')\right] \\ &\quad\times\int\frac{d\vec{k}}{(2\pi)^2}\exp[i\vec{k}(\vec{x}-\vec{x}')]S(\vec{k}), \\ S(\vec{k}) &= -i\int_0^\infty ds\exp\left[-is\left(m^2-k_0^2+\vec{k}^2\frac{\tan y}{y}-i0\right)\right] \\ &\quad\times\left\{(m+ik^0)+\gamma^0(k^0+im\tan y)-\frac{\vec{k}\vec{\gamma}}{\cos^2 y}\right\}. \end{aligned} \tag{2.14}$$

This result is the same as the result of works [24,25,53], in which the method of Schwinger was used [65]. It is interesting to compare (2.14) with the respective result of works [51–53,66], where 4-dimensional representation is applied for gamma-matrices, in which:

$$\gamma^0 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}, \qquad \gamma^{1,2} = \begin{pmatrix} i\sigma_{1,2} & 0 \\ 0 & -i\sigma_{1,2} \end{pmatrix}. \tag{2.15}$$

In work [51], using the solutions (2.19) and (2.20) of Dirac equations in a constant magnetic field, in which the spin properties of an electron are taken into account using operator (2.17) of spin projection to the direction of the magnetic field for propagator (2.7) the result has been obtained described by formula (2.19) from work [52]. It should be noted that formula (2.19) from work [52] overlaps with the result (2.14) of the present work, if in (2.19) gamma-matrices (2.15) are replaced with the two-dimensional gamma-matrices (2.2).

As is known, the AMM of the lepton in a standard Weinberg-Salam-Glashow model is defined by that part of the radiative mass shift in the external magnetic field, which explicitly depends on the projection of spin to the direction of the magnetic field and, at the same time, is a true scalar [56,57,60,65,67,68]. QED_{3+1} is the theory, which is invariant relative to spatial reflections. Therefore, the energy of interaction between the AMM and the magnetic field is P-even and contain only one term with the spin and magnetic field correlation. This term is proportional to the bilinear combination $(D_{-1}D_{-1} - D_1D_1)$ of spin coefficients, which is -1 in the case $\zeta = 1 (D_1 = 1, D_{-1} = 0)$ and equal to $+1$ for $\zeta = -1 (D_1 = 0, D_{-1} = 1)$ and, accordingly, this part of the interaction energy reverses its sign, depending on the orientation of the electron spin [56].

In the Lorentz-invariant notation, the AMM of a lepton is determined by those terms in the radiative mass shift that are proportional to the value of $s_\mu(\tilde{F})^{\mu\nu}p_\nu$, where s_μ is the particle polarization 4-vector, $(\tilde{F})^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ is the dual tensor of the external field, and p_ν is the particle momentum 4-vector [60,68].

Breakdown of spatial parity in weak interactions results in emergence of the second spin summand in the radiative mass shift of the Dirac neutrino in a constant magnetic field. This summand is proportional to the value $s_\mu\tilde{F}^{\mu\alpha}F_{\alpha\beta}p^\beta$ [57], which is a pseudoscalar with no contribution to the neutrino AMM. In $(2+1)$-dimensional models of the quantum field theory, as mentioned previously, it is impossible to identify a dual tensor of the field, and spatial parity is broken.

Consequently, the methods used in the Weinberg-Salam-Glashow model for determining the AMM of the electron from the spin terms in the radiation mass shift in a constant magnetic field should be developed for the case of two-dimensional electrodynamics. The calculation of the AMM of an electron in QED_{2+1} with the Chern-Simons term, which is a theory with broken spatial parity, will begin with the determination of that part of the radiation mass shift, which obviously depends of the electron spin. A method shall further be given that allows division of the obtained expression into scalar and pseudoscalar parts. Of interest will be a summand that is a true scalar, which defines the energy of the interaction of the AMM of the electron with the external magnetic field.

First, we consider the transformational properties of the classical action of the theory with the Lagrangian (2.1) under the parity and time-reversal transformations [20,36,53,59,66]:

$$\begin{aligned} P\colon\ x &\to \bar{x}(x_0,-x,y), \\ (A_0,A_1,A_2)(x) &\to (A_0,-A_1,A_2)(\bar{x}), \\ \psi(x) &\to \gamma^1\psi(\bar{x}), \end{aligned} \tag{2.16}$$

$$\begin{aligned} T\colon\ t \to -t, &\qquad x \to (-t,\vec{x}), \\ \psi(x) \to \gamma^2\psi(-t,\vec{x}), &\qquad \vec{A} \to -\vec{A}(-t,\vec{x}). \end{aligned} \tag{2.17}$$

We see that the classic action in the case of a massless fermion in QED_{2+1} without a Chern-Simons member is invariant with respect to the inversion operation (2.16), while in the case of a massive fermion, when $m \neq 0$, the fermion mass term $m\bar{\psi}\psi$ in formula (2.1) with two-dimensional gamma-matrices (2.2) is odd under P- and T- transformation:

$$P\colon\ \bar{\psi}\psi \to -\bar{\psi}\psi, \qquad T\colon\ \bar{\psi}\psi \to -\bar{\psi}\psi. \tag{2.18}$$

At the same time, the mass term and Chern-Simons term in formula (2.1) have the same properties under the time-reversal and parity transformations. Consider also the four-component fermions in $(2+1)$-dimensions, connected with a four-dimensional reducible representation (2.15) of Dirac's matrices. Considering a four-component spinor $\psi = \binom{\psi_1}{\psi_2}$, the four-component mass term $m\bar{\psi}\psi$ may be represented in the form

$$m\bar{\psi}\psi = m\psi_1^+\sigma_3\psi_1 - m\psi_2^+\sigma_3\psi_2.$$

In the case of inversion, which is similarly defined for all spinors, the parity transformation becomes

$$P: \psi_1 \to \sigma_1\psi_1, \quad \psi_2 \to \sigma_1\psi_2, \quad m\bar{\psi}\psi \to m\bar{\psi}\psi. \tag{2.19}$$

and therefore, the mass term in the theory with a four-component spinor is a P-even value, while the Chern-Simons term is a P-odd value.

As a result, as opposed to QED_{3+1}, in both models of two-dimensional electrodynamics, the spin part of the radiative mass shift of an electron in a constant magnetic field is defined as a sum of the scalar and pseudoscalar summand.

The noninvariance of the Chern-Simons term under parity transformation is directly manifested in the tensor structure of the photon propagator in (2.8), where the second summand in parentheses, as opposed to the first and the third summands, is antisymmetric toward permutation $\mu \leftrightarrow \nu$.

It should also be noted that, as shown in works [43,51], the last summand in propagator (2.8) does not contribute to the electron AMM and shall not be considered henceforth.

For calculation of the radiative shift of an electron's energy, which is determined by formula (2.6), taking into consideration (2.5), (2.8), (2.13), and (2.14), we use the method proposed in [51,52].

Using Schwinger parametrization

$$\frac{1}{p_0^2 - \vec{p}^2 - \Theta^2 + i0} = -i\int_0^\infty ds_2 \exp[is_2(p_0^2 - \vec{p}^2 - \Theta^2 + i0)], \tag{2.20}$$

integration over space and time variables performed by the formula [52]

$$\begin{aligned}\int d^3x d^3x' \exp\Big[&-i(p+k)(x-x') - i\frac{eH}{2}(y-y')(x+x') \\ &- iE_n(t-t') + i(p_y - p'_y)\Big] u_n(\eta)u_m(\eta') \\ = (2\pi)^2 &LT\delta(p_0 + k_0 - E_n)\frac{2}{eH} \\ &\times (-1)^m \exp[i(n-m)\chi] I_{n,m}\left(\frac{2\kappa^2}{eH}\right),\end{aligned} \tag{2.21}$$

where $p^\mu = (p^0, \vec{p})$ is 4-momentum of a virtual photon; $\vec{\kappa} = \vec{p} + \vec{k}$, T is the interaction time, which is assumed to be a unity thereafter; L is the length of periodicity in direction of axis OY; δ-function of Dirac $\delta(p^0 + k^0 - E_n)$ expresses the energy conservation, $\chi = \frac{\pi}{2} - \phi$, $\phi = \arctan\frac{\kappa_2}{\kappa_1}$; and the Laguerre function $I_{n,m}(\tau)$ is connected to the Laguerre polynomial $L_n^{n-m}(\tau)$ by relation [56]

$$I_{n,m}(\tau) = \sqrt{\frac{m!}{n!}}\exp\left(-\frac{\tau}{2}\right)\tau^{\frac{n-m}{2}}L_m^{n-m}(\tau), \qquad \tau = \frac{2\vec{\kappa}^2}{eH}. \tag{2.22}$$

We performed integration over variable k^0 using the δ-function, and integrals over variable p^0 are Gaussian.

Next, we move from integrating over a variable $\vec{p}$ to integrating over a variable $\vec{\kappa} = \vec{p} + \vec{k}$:

$$d\vec{k}d\vec{p} = d\vec{k}d\vec{\kappa} = kdkd\alpha\kappa d\kappa\psi \to 2\pi kdk\kappa d\kappa d\psi, \tag{2.23}$$

where $\psi = \phi - \alpha$, $\psi \in [0, 2\pi]$, α and ϕ are the polar angles of vectors $\vec{k}$ and $\vec{\kappa}$.

The integration over variable ψ gives the Bessel functions of zero and the first order of the real argument

$$b = 2s_2k\kappa$$

and further integration over variable k is performed by means of the Weber's formula [69]

$$\int_0^\infty \exp[-px^2]x^{\nu+1}J_\nu(cx)dx = \frac{c^\nu}{(2p)^{\nu+1}}\exp\left[-\frac{c^2}{4p}\right], \qquad p > 0, c > 0, \Re\nu > -1. \tag{2.24}$$

We will carry out a separate study of the contributions to the magnetic moment of an electron due to summands in the photon propagator (2.8), which are proportional to the mass Θ of the gauge field and to the metric tensor $g^{\mu\nu}$, respectively, and replace the variables s_1 and s_2 with u and y according to the formulas

$$u = \frac{s_1}{s_1 + s_2}, \qquad y = u(s_1 + s_2),$$
$$0 \le u \le 1, \qquad 0 \le y < \infty, \qquad ds_1ds_2 = \frac{y}{u^2}dudy. \tag{2.25}$$

Finally, the following representation, which is precise in the one-loop approximation, shall be obtained for the part of the radiation shift in the energy of the two-dimensional electron in QED_{2+1} with the Chern-Simons term, which depends on the spin quantum number:

$$\Delta E_n^\zeta = \Delta E_n^\zeta(g^{\mu\nu}) + \Delta E_n^\zeta(\Theta), \tag{2.26}$$

$$\begin{pmatrix} \Delta E_n^{\zeta}(g^{\mu\nu}) \\ \Delta E_n^{\zeta}(\Theta) \end{pmatrix} = i(-1)^{n+1} \exp\left[i\frac{\pi}{4}\right] \frac{\zeta e^2}{(2\pi)^2 E_n} \int_0^1 du \int_0^\infty \sqrt{\frac{\pi u}{y}} dy \frac{y}{u^2}$$
$$\times \int_0^\infty \kappa d\kappa \exp[-iE_n^2 uy + iy2eHn](-i) \frac{\lambda u}{2y(1-u)\tan eHy}$$
$$\times \exp\left[-i\frac{\lambda}{eH}\kappa^2 - iy\frac{1-u}{u}\Theta^2\right] \begin{pmatrix} F_1 \\ F_2 \end{pmatrix}, \tag{2.27}$$

where the following notations are adopted:

$$F_1 = -m^2\left\{I_{n,n}\left[(2-u)\frac{e^{iz}}{\cos z} + 2u\frac{e^{-iz}}{\cos z}\right] + I_{n-1,n-1}\left[(2-u)\frac{e^{-iz}}{\cos z} + 2u\frac{e^{iz}}{\cos z}\right]\right\}$$
$$- mE_n\left\{I_{n,n}\left[(2-u)\frac{e^{iz}}{\cos z} + 2u\frac{e^{-iz}}{\cos z}\right] - I_{n-1,n-1}\left[(2-u)\frac{e^{-iz}}{\cos z} + 2u\frac{e^{iz}}{\cos z}\right]\right\}$$
$$+ 2eHn(1-u)[(I_{n,n} + I_{n-1,n-1}) - 3i\tan z(I_{n,n} - I_{n-1,n-1})], \tag{2.28}$$

$$F_2 = \frac{2}{\Theta}\left(1 - \exp\left[iy\frac{1-u}{u}\Theta^2\right]\right)R_2,$$
$$R_2 = \frac{m}{\cos z}\left[u^2E_n^2 + i\frac{u}{2y}\right][I_{n,n}e^{-iz} + I_{n-1,n-1}e^{iz}] + \frac{E}{\cos z}\left[u^2E_n^2 + i\frac{u}{2y}\right][I_{n,n}e^{-iz} - I_{n-1,n-1}e^{iz}]$$
$$+ \frac{1}{\cos^2 z}[E_n(I_{n,n} - I_{n-1,n-1}) + m(I_{n,n} + I_{n-1,n-1})]\left[i\frac{\lambda u}{y(1-u)\tan z} + \frac{\kappa^2}{eH}\frac{\lambda^2 u}{y(1-u)\tan z}\right]$$
$$- (2eHn)\frac{uE_n}{\cos z}[I_{n,n}e^{-iz} - I_{n-1,n-1}e^{iz}], \tag{2.29}$$

$$\lambda = \frac{\tan z}{1 + \frac{u}{1-u}\frac{\tan z}{z}}, \qquad z = eHy. \tag{2.30}$$

Thus, to compute the AMM of an electron, it is necessary to obtain, using formula (2.27), the electron radiative mass shift, which is a Lorentz invariant, and present it as a sum of the scalar and pseudoscalar values:

$$\Delta m_\zeta = \frac{E_n}{m}\Delta E_n^\zeta = \Delta m_\zeta^{(s)} + \Delta m_\zeta^{(ps)}. \tag{2.31}$$

Then, the electron AMM shall be defined by value $\Delta m_\zeta^{(s)}$ in (2.31) according to formula [57,60,68]

$$\Delta\mu = -\frac{m}{E_n}\frac{\Re(\Delta m_\zeta^{(s)})}{\zeta H}. \tag{2.32}$$

After integrating over the variable κ in formulas (2.27)–(2.29), the problem posed received an unexpected solution.

Integration over the variable κ is carried out using the formulas:

$$J_{n,n}^{(1)} = \int_0^\infty \exp\left[-\frac{1}{eH}(1+i\lambda)t\right] L_n\frac{2t}{eH}dt$$
$$= (-1)^n eH \exp[-i2n\arctan\lambda]\frac{\exp[-i\arctan\lambda]}{\sqrt{1+\lambda^2}}, \tag{2.33}$$

$$J_{n,n}^{(2)} = \int_0^\infty \exp\left[-\frac{1}{eH}(1+i\lambda)t\right] L_n\frac{2t}{eH}tdt$$
$$= (-1)^n eH \exp[-i2n\arctan\lambda]$$
$$\times \frac{2n+1-i\lambda}{\sqrt{1+\lambda^2}}\frac{\exp[-i\arctan\lambda]}{\sqrt{1+\lambda^2}}. \tag{2.34}$$

When integrating over κ in formula (2.27) with the integrand, which determines the contribution coming from the $g^{\mu\nu}$ term in the photon propagator (2.8), only integrals $J_{n,n}^{(1)}$ and $J_{n-1,n-1}^{(1)}$ are used.

In this case, expression (2.27) is a linear combination of different pairs of terms, each of which is determined by one of the integrals

$$\begin{pmatrix} K_1 \\ K_2 \end{pmatrix} = \int_0^\infty \kappa d\kappa [I_{n,n}(t)\exp[i(\pm\alpha z - \arctan\lambda)] \pm I_{n-1,n-1}(t)\exp[i(\pm\alpha z - \arctan\lambda)]] = (-1)^n \frac{eH}{\sqrt{1+\lambda^2}} \exp[-i2n\arctan\lambda] \times \begin{pmatrix} i\sin(\pm\alpha z - \arctan\lambda) \\ \cos(\pm\alpha z - \arctan\lambda) \end{pmatrix}, \tag{2.35}$$

where $t = \frac{2\kappa^2}{eH}$, $\alpha = 0, 1$.

Of the two types of such summands, proportional to $\sin(\pm\alpha z - \arctan\lambda)$ and $\cos(\pm\alpha z - \arctan\lambda)$, respectively, contribution to the AMM is made only by the first type of summands, which are odd functions of the magnetic field strength.

For example, in formula (2.28), only the first term, proportional to m^2, gives a contribution to the AMM.

The contribution from the $\varepsilon^{\mu\nu\lambda}$ term in the propagator in (2.8) to the magnetic moment is determined by formula (2.27) with a function F_2.

As compared with the first case, a new structure arises here, connected with integrals (2.34), and only the first summand and the third term in formula (2.29) with a multiplier $m[I_{n,n} + I_{n-1,n-1}]$ contribute to the electron AMM.

As a result, the magnitude

$$\Delta m^s_\zeta = \Delta m^s_\zeta(g^{\mu\nu}) + \Delta m^s_\zeta(\Theta), \tag{2.36}$$

which determines in the one-loop approximation the AMM of an electron in a constant magnetic field in the two-dimensional quantum electrodynamics with a Chern-Simons term, is defined by the formula:

$$\begin{pmatrix} \Delta m^s_\zeta(g^{\mu\nu}) \\ \Delta m^s_\zeta(\Theta) \end{pmatrix} = -\zeta \frac{me^2\exp[i\frac{\pi}{4}]}{16\cdot 2\pi^{\frac{3}{2}}} \times \int_0^1 \frac{du}{\sqrt{u}} \int_0^\infty \frac{dy}{\sqrt{y}} \exp[-i\phi] \begin{pmatrix} \Omega_1 \\ \Omega_2, \end{pmatrix} \tag{2.37}$$

$$\Omega_1 = \frac{2-u+2u\exp[-2iz]}{1-u+u\exp[-iz]\frac{\sin z}{z}} - \frac{\exp[2i\arctan\lambda][2u+(2-u)\exp[-2iz]]}{1-u+u\exp[-iz]\frac{\sin z}{z}}, \tag{2.38}$$

$$\Omega_2 = -i\frac{\lambda}{\sqrt{1-\lambda^2}(1-u)\sin z}\frac{4}{m\Theta}\left(1-\exp\left[iy\Theta^2\frac{1-u}{u}\right]\right) \times \left[-i\left(m^2u^2 + i\frac{u}{2y}\right)\sin(z+\arctan\lambda) + \frac{\lambda^2 u}{y(1-u)\sin z}\left(1+\frac{1-\lambda^2}{(1+\lambda)^{\frac{3}{2}}}\right)\right], \tag{2.39}$$

$$\phi = m^2uy + y\frac{1-u}{u}\Theta^2 + 2n\arctan\lambda - 2eHny(1-u). \tag{2.40}$$

It should be noted that formulas (2.37)–(2.39) for the excited states of an electron, as in ordinary QED_{3+1}, do not contain divergences and are finite in the entire range of changes in the magnetic field.

The results (2.37)–(2.40) is obtained in a completely different model [20], compared with the works [51] and [52], which considers only the model of $(2+1)$-QED with a double fermion representation. We also calculate the AMM of the electron in a constant magnetic field in QED_{2+1} with the Chern-Simons term in a model with the doubled fermion representation. Computation was made by formulas (2.5)–(2.8) and (2.15). Following [51], we require that the solution of the Dirac equation be an eigenfunction of the Hamiltonian from Dirac equation, operator $\hat{p}_y = -i\frac{\partial}{\partial y}$ of the projection of momentum to the OY axis, a spin operator $\hat{A} = i\gamma^0\gamma^1\gamma^2$, and is determined by the formula

$$\Psi_{\varepsilon=+1} = \frac{(eH)^{\frac{1}{4}}}{\sqrt{2E_n}}\exp[-iE_nt + iyp_y]\left[\begin{pmatrix} \sqrt{E_n+m}u_{n-1} \\ \sqrt{E_n-m}u_n \\ 0 \\ 0 \end{pmatrix} D_1 + \begin{pmatrix} 0 \\ 0 \\ \sqrt{E_n-m}u_{n-1} \\ \sqrt{E_n+m}u_n \end{pmatrix} D_{-1}\right]. \tag{2.41}$$

Where, with $\zeta = +1$ (spin is oriented along the field direction), it is necessary to set $D_1 = 1, D_{-1} = 0$, and when $\zeta = -1$ (spin is oriented against the field direction), $D_1 = 0, D_{-1} = 1$ to the contrary.

The study of the AMM for the excited states of an electron in this model is similar to that shown above, so the details of the calculations are omitted here.

As a result, the summand $\Delta m_\zeta(g^{\mu\nu})$, as opposed to the considered above case of the QED_{2+1} with gamma-matrices (2.2), does not contain a pseudoscalar component and is defined by the formulas (2.37)–(2.38) of the present work and (2.21)–(2.22) of Ref. [52]. For the second term, $\Delta m_\zeta(\Theta)$, before integrating over the κ variable, we obtained the result, which is described by formulas (2.27) and (2.29), in which the following replacement should be made

$$F_2 \to F_2' = \frac{2}{\Theta}\left(1 - \exp\left[iy\frac{1-u}{u}\Theta^2\right]\right)\left\{[I_{n,n}\exp[-iz] - I_{n-1,n-1}\exp[iz]]\frac{E_n^2 u^2 + i\frac{u}{2y}}{\cos z}\right.$$
$$\left. + \frac{I_{n,n} - I_{n-1,n-1}}{\cos^2 z}\left(i\frac{\lambda u}{y(1-u)\tan z} + \frac{\kappa^2}{eH}\frac{\lambda^2 u}{y(1-u)\tan z}\right) + 2\sqrt{2eHn}\frac{\kappa}{eH}\frac{\lambda u}{y(1-u)}I_{n,n-1}\right\}. \quad (2.42)$$

The integral over the variable κ is calculated using formulas (2.33)–(2.35). As a result, the value $\Re\Delta m_\zeta(\Theta)$ is a pseudoscalar and does not contribute to the electronic AMM.

Hence, in the Chern-Simons QED_{2+1} with a doubled fermion representation, only the term with $g^{\mu\nu}$ in the photon propagator contributes to the electron AMM.

The expression for the mass shift of the electron ground state may be obtained from formulas (2.5)–(2.8) and (2.13)–(2.14), taking into account that

$$\bar{\psi}_0\gamma^\mu\left[m + ik^0\tan z + \gamma^0(k^0 + im\tan z) - \frac{\vec{k}\vec{\gamma}}{\cos^2 z}\right]\begin{pmatrix}\gamma_\mu \\ \varepsilon_{\mu\nu\lambda}\gamma^\nu p^\lambda\end{pmatrix}\psi_0$$
$$= 2m + p_0 + i\tan z(2m - 3p_0)$$
$$- \frac{2i}{\cos z}(p_0^2\exp[-iz]) + \frac{\vec{p}\vec{k}}{\cos z}, \quad (2.43)$$

where $\Psi_0 = \binom{1}{0}$. As a result, we arrive at the next representation for the ground state electron mass shift

$$\begin{pmatrix}\Delta E_0(g_{\mu\nu}) \\ \Delta E_0(\Theta)\end{pmatrix} = \frac{e^2}{8\pi^{\frac{3}{2}}}\int_0^1 \frac{du}{\sqrt{u}}\int_0^\infty \frac{dy}{\sqrt{y}} \times \exp[-m^2 uy]\begin{pmatrix}A_1 \\ A_2\end{pmatrix}, \quad (2.44)$$

$$A_1 = \frac{\exp[-\nu][2 - u + 2u\exp[-2eHy]]}{F} - u - 2,$$
$$\nu = \frac{y(1-u)\Theta^2}{u}, \qquad F = 1 - u + u\exp[-eHy]\frac{shz}{z},$$
$$A_2 = \frac{\exp[-\nu] - 1}{\nu}(1-u)\frac{\Theta}{m} \times \left\{\exp[-2eHy]\left[1 + \frac{2}{F} - 2m^2 uy\right] - (3 - 2m^2 uy)\right\}. \quad (2.45)$$

It should be noted that as opposed to the case of the excited states of an electron, in formulas (2.44)–(2.45), the mass shift was renormalized through subtraction of a divergent term $\Delta E_0(H \to 0)$, and result (2.44)–(2.45) is the same as the respective results given in Refs. [32,51].

III. THE AMM OF AN ELECTRON IN QED_{2+1} WITH THE CHERN-SIMONS TERM: CONTRIBUTION FROM THE $\varepsilon^{\mu\nu\lambda}$ TERM IN THE PHOTON PROPAGATOR

First, we consider the case of a weak magnetic field and a nonrelativistic electron, when the following conditions are satisfied

$$\beta = \frac{H}{H_0} \ll 1, \qquad n \ll \beta^{-1}, \quad (3.1)$$

where, $H_0 = \frac{m^2c^3}{e\hbar} = 4.41 \times 10^{13}G$—is the Schwinger's critical field. The electron mass shift $\Delta m_\zeta^s(g_{\mu\nu})$ in this case are described by the asymptotic formulas (4.6) and (3.7) of the work [52]:

$$\Re(\Delta m_\zeta^{(s)}(g_{\mu\nu})) = \begin{pmatrix} -\zeta\frac{e^2\beta}{8\pi}\left[\frac{3}{2} + \ln\left(\frac{\beta}{2}\right)\right], \rho = 0, \beta \ll 1, n \ll \beta^{-1}, \\ \zeta\frac{e^2\beta}{16\pi}\left[3 - 3\rho - \left(2 - \frac{3\rho^2}{2}\right)\ln\frac{\rho+2}{\rho}\right], \beta \ll 2\rho\left(1 - \frac{\rho}{2}\right)\end{pmatrix}. \quad (3.2)$$

Here, the corresponding asymptotics will be obtained for the quantity $\Delta m_\zeta^{(s)}(\Theta)$ in formulas (2.37) and (2.39), which describes the contribution of the term with $\varepsilon^{\mu\nu\lambda}$ in the photon propagator (2.8) to the AMM of an electron. Taking into account (3.1) and (2.30), the exponent in formula (2.37) can be represented in the form

$$-i\phi \simeq -\frac{t}{\beta}F(u,\rho),$$
$$F(u,\rho) = u + \left(\frac{1}{u} - 1\right)\rho^2, \qquad t = eHy, \quad (3.3)$$

where, without restricting the generality, it is assumed that $\rho = \frac{\Theta}{m} < 2$. We note that function $F(u,\rho)$ of variable $u \in [0,1]$ takes the values from 1 at $u = 1$ until $+\infty$ at $u \to +0$ for all $\rho \neq 0$, and at the critical point $u_0 = \rho = \frac{\Theta}{m}$, which is a minimum point of the function $F(u,\rho)$ on the interval [0,1], the following inequality is true

$$F(u_0,\rho) = 2\rho\left(1 - \frac{\rho}{2}\right) > 0.$$

This means that in the limiting case,

$$\frac{F(u_0,\rho)}{\beta} \gg 1, \tag{3.4}$$

i.e., if the field parameter β is small compared with the Chern-Simons parameter, then in the multiplier Ω_2 in formulas (2.37) and (2.39), the main contribution to the AMM provides the domain $t = eHy \ll 1$.

In this limit, we have

$$\Omega_2 = \frac{eHu}{m\Theta}\left[6 - 5u - i(4u - 2u^2)\frac{t}{\beta}\right] \times \left(1 - \exp\left[i\frac{ty(1-u)\rho^2}{\beta u}\right]\right). \tag{3.5}$$

Further integration over variable t is carried out taking into account the infinitesimal imaginary part of the electron mass in the causal propagator [69]:

$$\lim_{\delta\to+0}\int_0^\infty t^{\mu-1}\exp[-\delta t]\begin{pmatrix}\sin\alpha t\\ \cos\alpha t\end{pmatrix}dt = \frac{\Gamma(\mu)}{\alpha^\mu}\begin{pmatrix}\sin\frac{\pi\mu}{2}, \Re\mu > -1\\ \cos\frac{\pi\mu}{2}, \Re\mu > 0\end{pmatrix}. \tag{3.6}$$

After integration over variable t, in the leading order in the small parameter $\frac{\beta}{\rho}$ the contribution to the mass shift $\Delta m_\zeta^{(s)}$, coming only from the $\varepsilon^{\mu\nu\lambda}$ in the photon propagator, takes the form

$$\Re(\Delta m_\zeta^{(s)}(\Theta)) = \frac{\zeta e^2}{4\pi}\frac{eH}{4m\Theta}\int_0^1 udu \times \left[\frac{5u-6}{(u^2+(1-u)\rho^2)^{\frac{1}{2}}} + \frac{u^2(2-u)}{(u^2+(1-u)\rho^2)^{\frac{3}{2}}} - \frac{4(u-1)}{u}\right], \quad \beta \ll F(u_0,\rho). \tag{3.7}$$

It follows from (3.7) that the radiative mass shift in the case (3.1) is defined by the formula

$$\Re(\Delta m_\zeta^{(s)}(\Theta)) = \frac{\zeta e^2}{4\pi}\frac{eH}{4m^2}\left[2 - \rho\ln\frac{\rho+2}{\rho}\right], \quad \beta \ll \rho. \tag{3.8}$$

Further, let us consider the case of a weak magnetic field and ultrarelativistic values of energy of the electron, when the following condition is justified.

$$\beta \ll 1, \qquad p_\perp = \sqrt{2eHn} \gg m. \tag{3.9}$$

In this area, the movement of electrons obeys the quasi-classical laws. Maintaining the first two expansion terms in formula (2.30) in the significant region,

$$\arctan\lambda \simeq t(1-u) + \frac{t^3u(1-u)^2}{3}$$

we find the following Lorentz-invariant expression for the part of the electron mass shift defined by the interaction energy of the AMM with the external magnetic field:

$$\Delta m_\zeta^{(s)}(\Theta) = \frac{\zeta e^2 H \exp[i\frac{\pi}{4}]}{16\pi^2\rho H_0} \times \int_0^1 du\sqrt{z_0}\{(6-5u)[F(z) - F(z_0)] + 2(2-u)z_0[F'(z) - F'(z_0)]\}. \tag{3.10}$$

Here, the function $F(z)$ is determined by the integral

$$F(z) = \int_0^\infty \frac{d\tau}{\sqrt{\tau}}\exp\left[-i\left(\tau z + \frac{\tau^3}{3}\right)\right], \tag{3.11}$$

$F'(z) = \frac{dF}{dz}$, where the following notations have been adopted:

$$z = z_0\left[1 + \rho^2\frac{1-u}{u^2}\right], \qquad z_0 = \left(\frac{u}{\chi(1-u)}\right)^{\frac{2}{3}}.$$

It should be emphasized that the integrand in (3.10) depends only on the dimensionless Chern-Simons parameter and on the invariant dynamic parameter of synchrotron radiation

$$\chi = \frac{e}{m^3}\sqrt{-(F_{\mu\nu}p^\nu)^2} = \frac{p_\perp}{m}\frac{H}{H_0}, \tag{3.12}$$

and also that result (3.10) is obtained supposing that alongside with (3.9), the condition is met

$$\chi \ll \rho.$$

It immediately follows from Lagrangian (2.1) that if the parameter Θ is assumed to be zero, then we get the case of massive two-dimensional electrodynamics without the Chern-Simons term, in which the one-loop contribution to the AMM was considered in [52]. Although the radiative corrections initiate the generation of the Chern-Simons member, it does not contribute to the AMM of an electron in the one-loop approximation.

Taking into account the conditions for the validity of the results of (3.8) and (3.10), it is of interest to study the asymptotics of the value $\Re\Delta m_\zeta^{(s)}(\Theta)$ in the limiting case when $\frac{\rho}{\chi} \ll 1$.

We shall consider here the case in which the non-relativistic electron moves in a weak magnetic field

$$\rho \ll \beta \ll 1, \tag{3.13}$$

which we think is particularly interesting.

The integration region for variable u in formulas (2.37) and (2.39) is divided into two parts. In the first area $u \in [0, u_0]$, in the second $u \in [u_0, 1]$, where the parameter u_0 meets the following condition:

$$\rho, \beta \ll u_0 \ll 1.$$

Then, in the first region, the integrand in formula (2.37) for $\Delta m_\zeta^{(s)}(\Theta)$ shall be expanded, except for the respective exponential factor, into variable u series, since $u \leq u_0 \ll 1$, and in the second area, where $u \geq u_0 \gg \rho$, β, the main contribution to the integral is given by the region of integration, with the variable $t \ll 1$, and we expand the integrand in a series in the variable t. Further, similarly to work [52], the integrals with respect to the variable t are taken according to the formulas [69]:

$$\int_0^\infty t^{\mu-1} \sin at \begin{pmatrix} \sin bt \\ \cos bt \end{pmatrix} dt = \frac{\Gamma(\mu)}{2} \begin{pmatrix} \cos\frac{\pi\mu}{2}(|b-a|^{-\mu} - (b+a)^{-\mu}), a > b, -2 < \Re\mu < 1 \\ \sin\frac{\pi\mu}{2}((a+b)^{-\mu} + |a-b|^{-\mu}\sin(a-b)), |\Re\mu| < 1 \end{pmatrix}. \tag{3.14}$$

After integration over the variable t, we get

$$\Re(\Delta m_\zeta^{(s)}(\Theta)) = \zeta \frac{me^2}{16\pi^{\frac{3}{2}}}[T_1 + T_2], \tag{3.15}$$

where

$$\begin{aligned} T_1 &= \sqrt{\pi}\frac{2\beta}{m\rho} \\ &\times \int_{u_0}^1 udu\left[\frac{6-5u}{(u^2+\rho^2(1-u))^{\frac{1}{2}}} - \frac{u^2(2-u)}{(u^2+\rho^2(1-u))^{\frac{3}{2}}}\right] + \cdots, \\ T_2 &= \frac{2\sqrt{\pi}}{m\rho}\int_0^{u_0} du\left\{\left[\frac{u^2}{\sqrt{u^2+2\beta u+\rho^2}} + 3\sqrt{u^2+2\beta u+\rho^2}\right]\right. \\ &\left. - l\left[\frac{u^2}{\sqrt{u^2-2\beta u+\rho^2}} + 3\sqrt{u^2-2\beta u+\rho^2}\right]\right\} + \cdots, \end{aligned} \tag{3.16}$$

$$l = \begin{pmatrix} 1, u \in [0, u_1] \cup [u_2, u_0], u_1 = \beta - \sqrt{\beta^2 - \rho^2}, \\ 0, u \in [u_1, u_2], u_2 = \beta + \sqrt{\beta^2 - \rho^2} \end{pmatrix}, \tag{3.17}$$

and the dots in (3.16) correspond to terms of a higher order of smallness.

As a result, in the limiting case (3.13), the radiative mass shift and the corresponding AMM of the electron, coming from the $\varepsilon^{\mu\nu\lambda}$ term in the photon propagator, are defined by the formulas

$$\Re(\Delta m_\zeta^{(s)}(\Theta)) = -\zeta\frac{e^2}{4\pi}\rho\ln(2\beta), \qquad \rho \ll \beta \ll 1, \tag{3.18}$$

$$\frac{\Delta\mu(\Theta)}{\mu_B} = \frac{e^2}{2\pi m}\frac{\rho}{\beta}\ln(2\beta). \tag{3.19}$$

Following from formulas (3.18)–(3.19), the contribution to the AMM coming from the $\varepsilon^{\mu\nu\lambda}$ term in the photon propagator, tends toward zero when $\rho \to 0$.

IV. CONCLUSION

In the one-loop approximation, an exact expression is obtained, described by the formulas (2.37)–(2.39), for that part of the radiative mass shift of the excited states of an electron in a constant magnetic field in QED_{2+1} with the Chern-Simons term, which determines the interaction energy of the AMM with the external magnetic field

$$\Delta E_n(\zeta) = -\zeta H \Delta\mu.$$

In deriving formulas (2.37)–(2.39), the Dirac representation (2.2) has been used for two-dimensional gamma matrices.

It was mentioned above that opposed to standard QED_{3+1}, massive QED_{2+1} both with and without the Chern-Simons term is the theory with broken P- and T-parity.

As a result, the electron radiative mass shift in QED_{2+1}, which is the Lorentz invariant, contains two types of terms with a correlation of electron spin and magnetic field, one of which is a pseudo-scalar and does not contribute to the AMM. The second term is a true scalar. It defines the interaction energy of the AMM with the external magnetic field.

It should be noted that such a property is inherent in both the electron mass shifts $\Delta m_\zeta(g_{\mu\nu})$ due to the term with $g_{\mu\nu}$ in the photon propagator and the value $\Delta m_\zeta(\Theta)$ associated with $\varepsilon_{\mu\nu\lambda}$ in the propagator (2.8).

The AMM of the excited states of an electron in a constant magnetic field in Chern-Simons QED_{2+1} with a doubled representation of fermions has also been studied. In this theory, the mass term in (2.1) is P-even with respect to reflection of one of the spatial coordinates; however, the presence of the Chern-Simons term in Lagrangian (2.1) breaks parity in this model as well. It has been shown that the entire part of the radiation mass shift of an electron, which depends on the orientation of the electron spin and is defined by the value $\Delta m_\zeta(g_{\mu\nu})$, is a scalar value and is the same as result (2.37)–(2.38). At the same time, the value

$\Delta m_\zeta(\Theta)$ is a pseudoscalar and does not contribute to the electron AMM.

Thus, in the theory with the doubled fermion representation, the energy of the interaction between the AMM of the electron and magnetic field in topologically massive two-dimensional electrodynamics is fully defined by the value $\Delta m_\zeta(g_{\mu\nu})$ in formula (2.37).

Another line of investigations, intersecting with the results of this paper, is connected to the problem of infrared divergences in two-dimensional models of quantum field theory in the study of radiative and spin effects. In [43], on the basis of computing the vertex function of the topologically massive two-dimensional electrodynamics, a conclusion is made that the presence of the Chern-Simons term in (2.1) results in regularization of infrared divergence of the AMM, and at the same time, the contribution of the term with $\varepsilon_{\mu\nu\lambda}$ in the photon propagator, when $\rho \to 0$ tends toward some finite value, is different from zero.

However, a detailed analysis of the AMM of an electron in QED_{2+1} with a Chern-Simons term in a constant magnetic field, carried out in this work, shows that in the nonrelativistic approximation, the electron AMM, according to formulas (3.2), (3.8), and (3.19), has the following asymptotics:

$$\frac{\Delta\mu(g_{\mu\nu})}{\mu_B} = -\frac{e^2}{8\pi m}\left[-(3-3\rho) + \left(2-\frac{3}{2}\rho^2\right)\ln\frac{\rho+2}{\rho}\right],\quad \beta \ll \rho,\quad \beta \ll 1, \tag{4.1}$$

$$\frac{\Delta\mu(g_{\mu\nu})}{\mu_B} = \frac{e^2}{4\pi m}\left[\frac{3}{2} + \ln\frac{\beta}{2}\right],\qquad \rho = 0,\qquad \beta \ll 1, \tag{4.2}$$

$$\frac{\Delta\mu(\Theta)}{\mu_B} = -\frac{e^2}{8\pi m}\left[2-\rho\ln\frac{\rho+2}{\rho}\right],\quad \beta \ll \rho,\quad \beta \ll 1, \tag{4.3}$$

$$\frac{\Delta\mu(\Theta)}{\mu_B} = \frac{e^2}{2\pi m}\frac{\rho}{\beta}\ln 2\beta,\quad \rho \ll \beta \ll 1,\quad \ln\frac{1}{2\beta} \gg 1. \tag{4.4}$$

The comparison shows that formulas (4.1) and (4.3) have the form coinciding with the corresponding results (16) and (24) of the paper [43], which are obtained on the basis of the calculation of the vertex function. However, a fundamental difference between these results is that the conditions of applicability of formulas (16) and (24) were not studied in work [43]. As a result of this, in [43], it is concluded that the contribution to the AMM of an electron described by the formula (16) of this work contains an infrared logarithmic divergence when $\rho \to 0$, which is eliminated by the Chern-Simons term in the Lagrangian (2.1).

As for formula (24), in [43] it is concluded that the contribution to the AMM, given by the term with $\varepsilon_{\mu\nu\lambda}$ in the photon propagator, tends to a finite nonzero value at $\rho \to 0$. Our results (4.1) and (4.3) show that limiting transition $\rho \to 0$ in these formulas is conceptually impossible, and the behavior of the AMM of the nonrelativistic electron in the limiting case $\rho \to 0$ is described by formulas (4.2) and (4.4). Here, as follows from formula (4.2), the external magnetic field regulates infrared divergence, and at $\rho = 0$, the energy of the interaction between the electron AMM and the external magnetic field is proportional to the value $\beta \ln \beta$. Thus, the external magnetic field plays the role of the infrared regulator in the calculation of the AMM in QED_{2+1}.

Alongside this, according to result (4.4), the one-loop contribution to the electron AMM, coming from the term with $\varepsilon_{\mu\nu\lambda}$ in the photon propagator, tends to zero in proportion to the parameter $\frac{\rho}{\beta} \ll 1$ and not to the finite result following from formula (24) of [43].

We conclude our analysis of the AMM of an electron in a topologically massive two-dimensional electrodynamics with two observations. First, unlike QED_{3+1}, in two-dimensional electrodynamics, there is no free movement along the field, and the electron energy spectrum in a magnetic field is completely discrete. Second, as follows from formula (17) in work [43], the magnetic form factor in QED_{2+1} contains logarithmical infrared divergence at $\rho = 0$. Therefore, it should be expected that the simultaneous consideration of these factors is necessary for correct calculation of the AMM based on the vertex function in QED_{2+1}, both with and without the Chern-Simons term.

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