χ_{c1} decays into a pseudoscalar meson and a vector-vector molecule

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We evaluate ratios of the χ_{c1} decay rates to η (η', K^-) and one of the $f_0(1370)$, $f_0(1710)$, $f_2(1270)$, $f'_2(1525)$, $K_2^*(1430)$ resonances, which in the local hidden gauge approach are dynamically generated from the vector-vector interaction. With the simple assumption that the χ_{c1} is a singlet of SU(3), and the input from the study of these resonances as vector-vector molecular states, we describe the experimental ratio $\mathcal{B}(\chi_{c1} \to \eta f_2(1270))/\mathcal{B}(\chi_{c1} \to \eta' f'_2(1525))$ and make predictions for six more ratios that can be tested in future experiments.

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I. INTRODUCTION

The topic of hadronic resonances is capturing the attention of hadron physics and gradually the field is piling up more evidence that some hadronic states stand for a molecular interpretation in terms of more elementary hadrons. Recent reviews on the topic can be seen in [1,2].

A good laboratory to see such states in the light quark sector is the decay of charmonium states into three lighter mesons. Indeed the $c\bar{c}$ states can be considered as an SU(3) singlet, in the same way that an $s\bar{s}$ state is an isospin singlet. With the SU(3) matrices for the mesons one can construct SU(3) invariants, which provide trios of mesons with a certain weight. If a given resonance is formed from the interaction of pairs of mesons, the picture to produce them is to produce first these mesons and then let them interact through final state interaction where the resonances will be formed. This picture has been thoroughly used. Indeed, in [3] the $J/\psi \to \phi(\omega) f_0(980) (f_0(500))$ decays were studied producing $\phi(\omega)$ and $K\bar{K}$, $\pi\pi$, $\eta\eta$ pairs and allowing the $K\bar{K}$, $\pi\pi$, $\eta\eta$ pairs to interact to produce the $f_0(500)$, $f_0(980)$ states, which are well described within the chiral unitary approach as dynamically generated from the interactions of these pairs of mesons [4–7]. A different formalism of the same problem, although equivalent, is used in [8], which is then followed in [9]. The same J/ψ decay into three mesons is considered in [10] with $J/\psi \to VPP$, but letting the VP pairs interact to form an axial-vector meson. A mixture of the SU(3) trace $\langle VPP \rangle$ and $\langle V \rangle \langle PP \rangle$ was used and it was shown to correspond to the same structures developed in [3] and [8] and there it was used to describe successfully the BESIII data on the $J/\psi \to \eta(\eta') h_1(1380)$ decay [11]. The $\langle VPP \rangle$ term was shown to be the one corresponding to the dominant one in [3,8].

The dominance of the three meson matrix trace seems to be quite general in these processes since in the study of the $\chi_{c1} \to \eta \pi^+ \pi^-$ reaction leading to the formation of the $f_0(500)$, $f_0(980)$ and $a_0(980)$ resonances [12], the $\langle PPP \rangle$ production structure was found largely dominant and the final state interaction of the different PP pairs led to the production of these scalar resonances [13,14].

We have described processes that proceed as a first step from the VPP and the PPP production. The processes proceeding from VVV production were studied in [15] in the $J/\psi \to \phi(\omega) f_2(1270)$ and related reactions. In this case one of the vectors $\phi(\omega)$ acts as a spectator and the remaining VV pair produces the $f_2(1275)$, $f'_2(1525)$ resonances, which according to [16,17] are produced mostly from the $\rho\rho$ and $K^*\bar{K}^*$ interaction, respectively.[1]

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[1]The ability of the methods used in Refs. [16,17] to obtain the tensor states has been questioned in [18] and [19]. In [20] and [21], a thorough discussion of these works has been done, showing that the methods proposed cannot be extrapolated to the energies where the tensor states appear. At the same time an improved method is proposed that provides couplings of the resonances to the coupled channels practically identical to those of [16,17] which we use here.

A fresh look to this latter problem from the new perspective of $\langle VVV\rangle$ and $\langle VV\rangle\langle V\rangle$ production vertices, including more experimental data, has been given in [22], where a detailed discussion on molecular states is made in the Introduction.

As to processes proceeding with VVP production we can have two cases. One of them corresponds to having a V as spectator and VP interacting producing an axial vector meson. An example of this is the χ_{c1} decay to $\phi h_1(1380)$ [23], which has been described successfully along the lines described above with the $\langle VVP\rangle$ structure [24]. There is only one more case to be studied which corresponds to the same structure but now the P will be a spectator and the VV will interact to form the vector-vector molecular states of [16,17], $f_2(1270)$, $f'_2(1525)$, $f_0(1370)$, $f_0(1710)$, $K_2^*(1430)$. We undertake this work here and compare the results with the few data available on η (η', $\bar{K}$) production together with one of these resonances.

II. FORMALISM

We study reactions that can be compared with present results in the PDG [25]. This includes χ_{c1} decays with η, η' production and some of the f_0, f_2 resonances, but we shall make predictions for related decays which are likely to be measured in the near future. As advanced in the Introduction, we introduce the $\langle VVP\rangle$ and $\langle VV\rangle\langle P\rangle$ structures in the primary step, where V, P are the SU(3) vector and pseudoscalar matrices (corresponding to $q\bar{q}$) given by

$$V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}, \quad (1)$$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta' \end{pmatrix}, \quad (2)$$

where we have assumed the η, η' mixing according to [26]. The algebra for $\langle VVP\rangle$ is trivial, and isolating the terms containing η, η' or K^- we find the production structures:

$$1)\ \eta:\ \frac{\eta}{\sqrt{3}}\{\rho^0\rho^0 + \rho^+\rho^- + \rho^-\rho^+ + \omega\omega - \phi\phi\}; \quad (3)$$

$$2)\ \eta':\ \frac{\eta'}{\sqrt{6}}\{\rho^0\rho^0 + \rho^+\rho^- + \rho^-\rho^+ + \omega\omega + 3K^{*+}K^{*-} + 3K^{*0}\bar{K}^{*0} + 2\phi\phi\}; \quad (4)$$

$$3)\ K^-:\ K^-\left\{\left(\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}}\right)K^{*+} + \rho^+K^{*0} + K^{*+}\phi\right\}. \quad (5)$$

In order to connect with the isospin formalism of [16,17] we write the isospin states with the unitary normalization with our phase convention, $(-\rho^+, \rho^0, \rho^-)$, (K^{*+}, K^{*0}), $(\bar{K}^{*0}, -K^{*-})$, as

$$|\rho\rho, I=0\rangle = -\frac{1}{\sqrt{6}}|\rho^0\rho^0 + \rho^+\rho^- + \rho^-\rho^+\rangle, \quad (6)$$

$$|\omega\omega, I=0\rangle = \frac{1}{\sqrt{2}}|\omega\omega\rangle, \quad (7)$$

$$|\phi\phi, I=0\rangle = \frac{1}{\sqrt{2}}|\phi\phi\rangle, \quad (8)$$

$$\left|\rho K, I=\frac{1}{2}, I_3=\frac{1}{2}\right\rangle = -\sqrt{\frac{2}{3}}\rho^+K^{*0} - \sqrt{\frac{1}{3}}\rho^0K^{*+}. \quad (9)$$

Taking into account the symmetry factor $n!$ for production of n identical particles we find the weights for primary production of the different components as

$$h_{\rho\rho}^{(\eta)} = -\sqrt{\frac{1}{2}};\quad h_{\omega\omega}^{(\eta)} = \sqrt{\frac{2}{3}};\quad h_{\phi\phi}^{(\eta)} = -\sqrt{\frac{2}{3}};$$
$$h_{K^*\bar{K}^*}^{(\eta)} = 0;\quad h_{\rho\rho}^{(\eta')} = -\frac{1}{2};\quad h_{\omega\omega}^{(\eta')} = \sqrt{\frac{1}{3}};$$
$$h_{\phi\phi}^{(\eta')} = \frac{2}{\sqrt{3}};\quad h_{K^*\bar{K}^*}^{(\eta')} = -\frac{\sqrt{3}}{2};\quad h_{\rho K^*}^{(K^-)} = -\sqrt{\frac{3}{2}};$$
$$h_{\omega K^*}^{(K^-)} = \sqrt{\frac{1}{2}};\quad h_{\phi K^*}^{(K^-)} = 1. \quad (10)$$

In addition we will also consider the $\langle VV\rangle\langle P\rangle$ structure, which could mix with the expected dominant $\langle VVP\rangle$ one with some small admixture. Now we have

$$\langle VV\rangle\langle P\rangle = \left(\frac{\eta}{\sqrt{3}} + \frac{4\eta'}{\sqrt{6}}\right)(\rho^0\rho^0 + \rho^+\rho^- + \rho^-\rho^+ + \omega\omega + \phi\phi + 2K^{*+}K^{*-} + 2K^{*0}\bar{K}^{*0}). \quad (11)$$

We see that there is no contribution for K^- production with this term and we find the weights for η, η' production as

$$h'^{(\eta)}_{\rho\rho} = h^{(\eta)}_{\rho\rho};\quad h'^{(\eta)}_{\omega\omega} = h^{(\eta)}_{\omega\omega};\quad h'^{(\eta)}_{\phi\phi} = -h^{(\eta)}_{\phi\phi};$$
$$h'^{(\eta)}_{K^*\bar{K}^*} = -\sqrt{\frac{2}{3}};\quad h'^{(\eta')}_{\rho\rho} = 4h^{(\eta')}_{\rho\rho};\quad h'^{(\eta')}_{\omega\omega} = 4h^{(\eta')}_{\omega\omega};$$
$$h'^{(\eta')}_{\phi\phi} = 2h^{(\eta')}_{\phi\phi};\quad h'^{(\eta')}_{K^*\bar{K}^*} = \frac{8}{3}h^{(\eta')}_{K^*\bar{K}^*}. \quad (12)$$

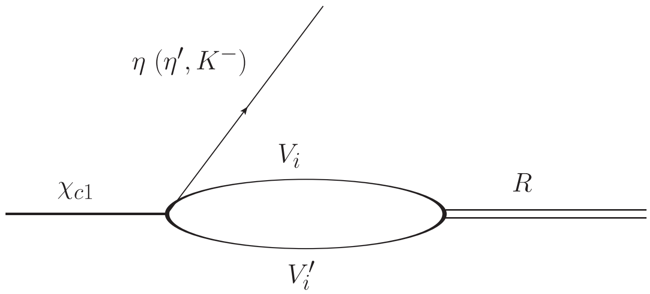

FIG. 1. Mechanism for χ_{c1} decay into $\eta(\eta', K^-)$ and the resonance R, via intermediate V_i, V_i' production and coupling of these states to the resonance.

We will assume a structure

$$\langle VVP\rangle + \beta\langle VV\rangle\langle P\rangle, \tag{13}$$

and, hence, in order to take into account the second term of Eq. (13) one simply has to substitute

$$h_i^{(\eta)} \to h_i^{(\eta)} + \beta h_i^{\prime(\eta)}; \qquad h_i^{(\eta')} \to h_i^{(\eta')} + \beta h_i^{\prime(\eta')}. \tag{14}$$

Note that there is no contribution to K^- production from the $\langle VV\rangle\langle P\rangle$ structure. The final state interaction is taken into account as described diagrammatically in Fig. 1.

Analytically the mechanism of Fig. 1 leads to the χ_{c1} decay amplitude

$$t_R^{\prime(\eta)} = \sum_i h_i^{(\eta)} G_i(M_R) g_{R,i}, \tag{15}$$

and the same for η', K^- production, where $h_i^{(\eta)}$ are the weights given in Eq. (10), $G_i(M_R)$ is the loop function for the pair of intermediate vectors, and $g_{R,i}$ are the couplings of the resonance R to the different V_iV_i' intermediate channels. All these magnitudes are evaluated in [16,17] and for completeness they are given here in Table I. When considering the combination of Eq. (13) we shall make the replacement of Eq. (14).

At this point it is worth refreshing what has been learned since the original papers in the VV interaction of Refs. [16,17]. In these works the vector exchange as a source of interaction was used, in addition to a contact term, following the local hidden gauge approach [27–30]. An approximation was done in [16,17] consisting on eliminating the q^2 dependence in the $[q^2 - m_V^2]^{-1}$ propagator of the vector exchange term. In Ref. [18] relativistic corrections were done in addition, and the q^2 dependence of the vector exchange propagators was kept, but another approximation was done, the "on shell" approximation assuming that the nonexchanged vector particles were on shell, $p_i^2 = m_V^2$, which is certainly not the case for external bound states and particles in the loops. As a consequence of this, the potential and the amplitudes developed singularities in the bound region which were found unphysical in [20] and [19]. Hopes of a solution using the N/D method and dispersion relations methods were raised in [18,19], and in [19] a detailed discussion of the method was made. Yet, the approximations done there were shown to lead to problems, since in [21] the method was shown not to converge below the energy where the "on-shell" singularity appeared in [18].

From these discussions two good points appeared:

(a) All methods give the same results at low energies, for instance for predictions of the $f_0(1370)$ state as a $\rho\rho$ bound state.

(b) At lower energies below the "on-shell" singularity of Ref [18] the methods could not be trusted and the "on-shell" factorization should not be used. This is the case of the $f_2(1270)$, whose mass falls below the singular "on-shell" energy.

The need for an approach that does not rely at all on the "on-shell" factorization became apparent and this step was given in [20]. In this latter work the loop functions of the problem using the contact term and the vector exchanged terms, described in Fig. 2, were exactly evaluated, keeping the full relativistic propagators in all the loops, and an effective potential was defined

$$V_{\rm eff} = V_{\rm c} + \tilde{V}_{\rm ex} \tag{16}$$

with

$$\tilde{V}_{\rm ex} = V_{\rm ex}(-M_\rho^2) G_{\rho,\rm eff} \tag{17}$$

where $V_{\rm ex}$ is the static vector exchange potential (for ρ exchange in this case) of Ref. [16] and $G_{\rho,\rm eff} = t_{\rm ex}/G(s)$ with $t_{\rm ex}$, the exact amplitude for the loop function of the exchange term of the diagram of Fig. 2(b), and $G(s)$ the ordinary $\rho\rho$ loop function (including the convolution due to the ρ mass distribution). This was done in a way such that $V_{\rm eff} G V_{\rm eff}$ gives the full set of amplitudes of Fig. 2 [including also Fig. 2(d)], and then the T matrix was obtained by means of this effective potential as

$$T = [1 - V_{\rm eff} G]^{-1} V_{\rm eff}. \tag{18}$$

The method overcomes the difficulties of all previous methods and shows that there are no singularities when the propagators are taken exactly inside the loop, and poles both for the f_0 and f_2 resonances are found.

What is of relevance for the present work is that this method provides a potential $V_{\rm eff}$, a bit smaller than the one used in [16], which is natural since it incorporates the q^2 dependence of the vector exchange, neglected in [16]. Yet, the effective methods require a final regularization of the loops which is done with a cutoff in the three momentum, and the freedom is taken to use this cutoff (always in a natural range of energies) to fine tune the mass of a resonance in this case $f_2(1270)$ of the $\rho\rho$ interaction.

TABLE I. Table of g, G values of the different channels and different resonances, like in [15] (including the $f_0(1370) f_0(1710)$) and errors).

		$\rho\rho$	$K^*\bar{K}^*$	$\omega\omega$	$\phi\phi$	$\rho\bar{K}^*$	$\omega\bar{K}^{*0}$	$\phi\bar{K}^{*0}$
$f_2(1270)$	g_i(MeV)	10551	4771	−503	−771	0	0	0
	error of g_i(%)	4	3	22	22	0	0	0
	$G_i(\times 10^{-3})$	−4.74	−3.00	−4.97	0.475	0	0	0
	error of G_i(%)	10	29	42	220	0	0	0
$f'_2(1525)$	g_i(MeV)	−2611	9692	−2707	−4611	0	0	0
	error of g_i(%)	12	6	2	2	0	0	0
	$G_i(\times 10^{-3})$	−8.67	−4.98	−9.63	−0.710	0	0	0
	error of G_i(%)	6	17	19	141	0	0	0
$\bar{K}_2^{*0}(1430)$	g_i(MeV)	0	0	0	0	10613	2273	−2906
	error of g_i(%)	0	0	0	0	3	5	5
	$G_i(\times 10^{-3})$	0	0	0	0	−6.41	−5.94	−2.70
	error of G_i(%)	0	0	0	0	12	19	43
$f_0(1370)$	g_i(MeV)	(7913.62, −1048.34)	(1209.79, −414.83)	(−39.29, 30.62)	(12.10, 23.68)	0	0	0
	error of g_i(%)	4	3	22	22	0	0	0
	$G_i(\times 10^{-3})$	(−7.69, 1.72)	(−4.13, 0.26)	(−8.97, 0.87)	(−0.63, 0.14)	0	0	0
	error of G_i(%)	10	29	42	220	0	0	0
$f_0(1710)$	g_i(MeV)	(−1029.91, 1086.85)	(7126.63, 94.28)	(−1763.75, 108.81)	(−2493.76, 204.50)	0	0	0
	error of g_i(%)	12	6	2	2	0	0	0
	$G_i(\times 10^{-3})$	(−9.70, 6.18)	(−7.68, 0.58)	(−10.85, 8.19)	(−2.16, 0.13)	0	0	0
	error of G_i(%)	6	17	19	141	0	0	0

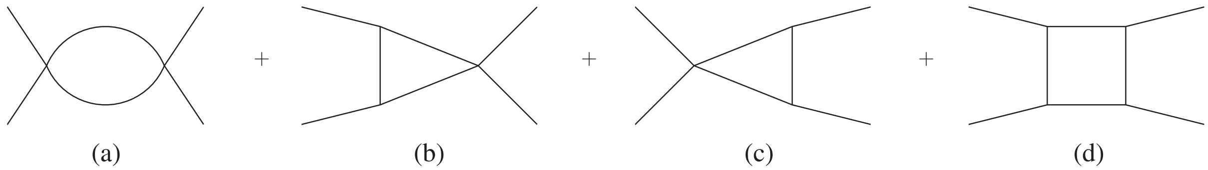

FIG. 2. Diagrams appearing at one-loop level with the contact and ρ exchange terms [20].

When this is done, what was found in [20] was that the couplings of the $f_2(1270)$ to $\rho\rho$ differed in less than 10% from the one obtained in [16] and in a much smaller fraction for the $f_0(1370)$. This remarkable agreement in spite of the nontrivial improvements of the $V_{\rm eff}$ method were attributed in [20] to the memory of the systems on the Weinberg compositeness condition [31], which links the coupling to the binding for small binding energies, plus the fact that even if apparently the $f_2(1270)$ resonance is quite bound, it is not so much when one considers the mass distributions of the two ρ mesons.

In view of the facts discussed above, we use the couplings obtained in [16,17], yet, with generous errors that allow us to find final results for the problems at work of the χ_{c1} decays, and estimate reasonable uncertainties. Advancing results, we should also note that the largest uncertainties do not come from the uncertainties of the couplings but from the unknown β parameter that we shall discuss below. Thus, the estimations of errors due to the uncertainties of the couplings of Table I is more than sufficient for the present work.

We still have to make some considerations concerning the spin of the VV states and angular momentum conservation of the production vertex. The χ_{c1} has $J^{PC} = 1^{++}$, the $\eta(\eta')0^{-+}$ and the vector-vector states that we consider are 0^{++}, 2^{++}. C-parity is conserved but in order to conserve parity and total angular momentum we need a P-wave. Since we require S-wave for the propagation of the two vectors, the momentum involved must be the one of the pseudoscalar. Thus

$$t_R^{(\eta)} = t_R'^{(\eta)} \vec{\epsilon}_{\chi_{c1}} \cdot \vec{p}_\eta, \tag{19}$$

and similarly for the other cases, where

$$|\vec{p}_\eta| = \frac{\lambda^{1/2}(M_{\chi_{c1}}^2, M_\eta^2, M_R^2)}{2M_{\chi_{c1}}}. \tag{20}$$

In addition we should take into account that the $V_i V_i'$ pair is produced with spin 0 or 2 for which the proper amplitudes are provided in [16,17]. However, their consideration only introduces spin factors after summing over spins, and since we only compare rates for VV' states with the same spin, $f_0(1370)$ with $f_0(1710)$, $f_2(1270)$ with $f_2'(1525)$ etc., it is unnecessary to consider these extra factors. Note that we have studied the flavor dependence of the production vertex to relate the different components, but the production rate could be different for the production of VV' pairs with different spin. The P-wave structure of Eq. (19) is important since it introduces a $\vec{p}_\eta^{\,2}(\vec{p}_{\eta'}^{\,2}, \vec{p}_{K^-}^{\,2})$ factor in $|t_R|^2$ and this factor is quite different for η or η' for instance.

The decay width is given by

$$\Gamma_R^{(\eta)} = \frac{1}{8\pi} \frac{1}{M_{\chi_{c1}}^2} \mathcal{C} |t_R'^{(\eta)}|^2 p_\eta^2 p_\eta, \tag{21}$$

with p_η given by Eq. (20), and similarly for the other cases. In Eq. (21) $\mathcal{C}$ is a global normalization constant, which cancels in the ratios that we discuss.

III. RESULTS

In the PDG we find the following branching ratios

$$\mathcal{B}(\chi_{c1} \to \eta f_2(1270)) = (6.7 \pm 1.1) \times 10^{-4}; \tag{22}$$

$$\mathcal{B}(\chi_{c1} \to \eta' f_0(1710)) = (7^{+7}_{-5}) \times 10^{-5}; \tag{23}$$

$$\mathcal{B}(\chi_{c1} \to \eta' f_2'(1525)) = (9 \pm 6) \times 10^{-5}. \tag{24}$$

We can only test the ratio

$$R_1 = \frac{\mathcal{B}(\chi_{c1} \to \eta f_2(1270))}{\mathcal{B}(\chi_{c1} \to \eta' f_2'(1525))} = [3.7 - 26] \quad \text{(centroid at 7.4)}. \tag{25}$$

Unfortunately, the errors in the measurements are very large, but even then we can test our formalism, which has no free parameters for this ratio if we take first $\beta = 0$. For this evaluation it is also important to consider the theoretical errors. We take the same errors in g_i, G_i discussed in [15] and shown in Table I. To calculate the errors in R_1 we generate random numbers for g_i, G_i within the restricted range and evaluate R_1 each time. After several runs we evaluate the average value of R_1 and its dispersion. We find

$$R_1^{th}(\beta = 0) = [1.1 \pm 0.3], \tag{26}$$

which is still lower than Eq. (25), indicating that some admixture of the $\langle VV \rangle \langle P \rangle$ structure is needed.

Before we proceed to include the contribution of the new term it is interesting to make the following observation. In [17] it was found (see Table I) that the $f_0(1370)$ and $f_2(1270)$ couple mostly to $\rho\rho$ but very weakly to $K^* \bar{K}^*$, while $f_0(1710)$ and $f_2'(1525)$ couple mostly to $K^* \bar{K}^*$. If we look at the weights $h^{(\eta)}$ in Eq. (10) we see that $h_{K^* \bar{K}^*}^{(\eta)} = 0$. The $K^* \bar{K}^*$ pair is not primary produced with the

η and hence the $f_2(1710)$ and $f_2'(1525)$ resonance would not be produced. This is not the case for η' since in Eq. (10) we find that $h^{(\eta')}_{K^*\bar{K}^*} = \frac{-\sqrt{3}}{2}$ and the $f_0(1710)$ and $f_2'(1525)$ can be produced, and are indeed observed. The fact that in the PDG we do not find branching ratios for $\eta f_0(1710)$, $\eta f_2'(1525)$ would then find a natural explanation in the dominance of the $\langle VVP \rangle$ structure. Note that if we consider the $\langle VV \rangle \langle P \rangle$ structure, the coefficient $h'^{(\eta)}_{K^*\bar{K}^*}$ is no longer zero. The possibly very small $\eta f_0(1710)$, $\eta f_2'(1525)$ rates would point out to a small admixture of the $\langle VV \rangle \langle P \rangle$ term.

We can improve the ratio of Eq. (26) by taking a value of

$$\beta = -0.28, \tag{27}$$

and we find

$$R_1^{th}(\beta = -0.28) = 7.1. \tag{28}$$

It is easy to make one estimate of this value of β based on the dominance of the $\rho\rho$ channel in the $f_2(1270)$ and the $K^*\bar{K}^*$ channel in the $f_2'(1525)$. By using the values $h^{(\eta)}_{\rho\rho}$, $h'^{(\eta)}_{\rho\rho}$ on one side and $h^{(\eta')}_{K^*\bar{K}^*}$, $h'^{(\eta')}_{K^*\bar{K}^*}$ we find the extra factor for the ratio R_1^{th} of Eq. (26),

$$\left(\frac{1+\beta}{1+\frac{8}{3}\beta}\right)^2, \tag{29}$$

which for $\beta = -0.267$ renders R_1^{th} to the value of 7.1 in agreement with the results of Eq. (28).

With the caveat of large uncertainties in the predictions, given the wide range of Eq. (25), we can make predictions for other ratios using the value of β in the Eq. (27) and also the value $\beta = 0$ in parenthesis

$$R_2 = \frac{\mathcal{B}[\chi_{c1} \to \eta f_2'(1525)]}{\mathcal{B}[\chi_{c1} \to \eta f_2(1270)]} = 0.16(3.8 \times 10^{-3});$$

$$R_3 = \frac{\mathcal{B}[\chi_{c1} \to \eta f_0(1710)]}{\mathcal{B}[\chi_{c1} \to \eta f_0(1370)]} = 0.060(2.3 \times 10^{-2});$$

$$R_4 = \frac{\mathcal{B}[\chi_{c1} \to \eta' f_0(1370)]}{\mathcal{B}[\chi_{c1} \to \eta' f_0(1710)]} = 0.060(0.54);$$

$$R_5 = \frac{\mathcal{B}[\chi_{c1} \to \eta' f_2(1270)]}{\mathcal{B}[\chi_{c1} \to \eta' f_2'(1525)]} = 4.6 \times 10^{-4}(0.81);$$

$$R_6 = \frac{\mathcal{B}[\chi_{c1} \to K^- K_2^{*+}(1430)]}{\mathcal{B}[\chi_{c1} \to \eta f_2(1270)]} = 10.3(4.2);$$

$$R_7 = \frac{\mathcal{B}[\chi_{c1} \to \eta f_0(1370)]}{\mathcal{B}[\chi_{c1} \to \eta' f_0(1710)]} = 10.9(1.1), \tag{30}$$

and we associate an error of about 30% to all these ratios for a given value of β.

We can see that R_1 is very sensitive to the value of β, but some of the other ratios are not so sensitive. The experimental situation should improve in the future and one can make the predictions more accurate. To facilitate the comparison of our predictions with future measurements, we present in Fig. 3 the results of the different ratios as a function of β.

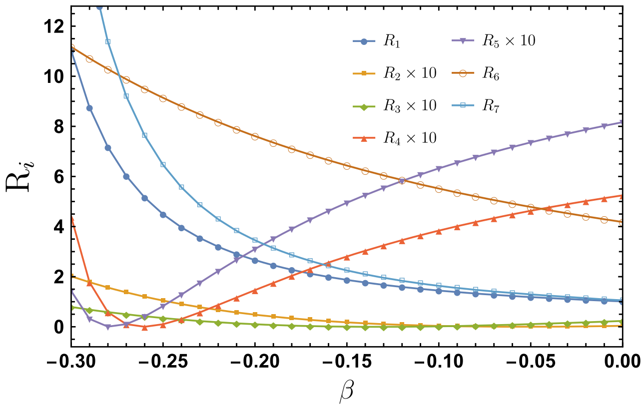

FIG. 3. Ratio R_i as a function of β.

The fact that the parameter β is negative makes the results more sensitive to the value of β, in particular for values around -0.3. We also find that the behavior with β is different for different ratios, some increase with β and others decrease. It would be most convenient to have some other ratios measured to pin down the value of β with some precision and make our predictions more constrained. Only then shall we be able to appreciate the predictive power of the theory. We hope that the experimental situation is improved in the coming years and the present work should be a motivation to carry out these measurements.

IV. CONCLUSIONS

We have studied reactions of χ_{c1} decay going to η (η', K^-) and one of the resonances $f_0(1370)$, $f_2(1270)$, $f_0(1710)$, $f_2'(1525)$ and $K_2^*(1430)$. For this we have assumed that these resonances are dynamically generated from the vector-vector interaction which was studied in detail in [16,17] and more recently in [20]. Then the mechanism of production proceeds via a primary production of a VVP structure with $P = \eta, \eta', K^-$ and the VV interact later to produce the given resonance. Based on the simple assumption that a $c\bar{c}$ state is a singlet of $SU(3)$, in the same way as an $s\bar{s}$ state is a singlet of isospin, we have constructed invariants with the $SU(3)$ matrices for vectors V and pseudoscalars P. The study of previous related processes indicated the dominance of the $\langle VVP \rangle$ structure in the production vertex, with a possible admixture of $\langle VV \rangle \langle P \rangle$. With the limited experimental information that we have at our disposal we could confirm this hypothesis and reproduced the ratio $R_1 = \mathcal{B}(\chi_{c1} \to \eta f_2(1270))/\mathcal{B}(\chi_{c1} \to \eta' f_2'(1525))$ which unfortunately has still large errors. We could then determine six more ratios which are predictions of the model, some of which are more stable under the changes within the experimental uncertainties of the R_1 ratio. We hope that in the near future some of these ratios are measured to test the predictions of the model and the underlying hypothesis that the resonances considered are indeed dynamically generated from the VV interaction.

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