CP asymmetry in $\tau \to K_S \pi \nu_\tau$ decays within the standard model and beyond

Feng-Zhi Chen,[*] Xin-Qiang Li,[†] Ya-Dong Yang,[‡] and Xin Zhang[§]
Institute of Particle Physics and Key Laboratory of Quark and Lepton Physics (MOE), Central China Normal University, Wuhan, Hubei 430079, China

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Motivated by the 2.8σ discrepancy observed between the *BABAR* measurement and the standard model prediction of the *CP* asymmetry in $\tau \to K_S \pi \nu_\tau$ decays, as well as the prospects of future measurements at Belle II, we revisit this observable in this paper. Firstly, we reproduce the known *CP* asymmetry due to $K^0 - \bar{K}^0$ mixing by means of the reciprocal basis, which is convenient when a $K_{S(L)}$ is involved in the final state. As the $K\pi$ tensor form factor plays a crucial role in generating a nonzero direct *CP* asymmetry that can arise only from the interference of vector and tensor operators, we then present a dispersive representation of this form factor, with its phase obtained in the context of chiral theory with resonances, which fulfills the requirements of unitarity and analyticity. Finally, the $\tau \to K_S \pi \nu_\tau$ decays are analyzed both within a model-independent low-energy effective theory framework and in a scalar leptoquark scenario. It is observed that the *CP* anomaly can be accommodated in the model-independent framework, even at the 1σ level, together with the constraint from the branching ratio of $\tau^- \to K_S \pi^- \nu_\tau$ decay; it can be, however, marginally reconciled only at the 2σ level, due to the specific relation between the scalar and tensor operators in the scalar leptoquark scenario. Once the combined constraints from the branching ratio and the decay spectrum of this decay are taken into account, these possibilities are, however, both excluded, even without exploiting further the stronger bounds from the (semi)leptonic kaon decays under the assumption of lepton-flavor universality, as well as from the neutron electric dipole moment and $D - \bar{D}$ mixing under the assumption of $SU(2)$ invariance of the weak interactions.

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I. INTRODUCTION

As the Kobayashi-Maskawa ansatz [1] for *CP* violation in the quark sector of the standard model (SM) is far too small to explain the observed baryon asymmetry of the Universe [2–5], we need to look for other sources of *CP* violation in different ways. This makes the *CP*-violating observables particularly interesting probes of new physics (NP) beyond the SM. In this respect, the hadronic decays of the τ lepton, besides serving as a clean laboratory for testing various low-energy aspects of the strong interaction [6,7], may also allow us to explore nonstandard *CP*-violating interactions [8–10].

In this paper, we shall focus on the *CP* asymmetry in $\tau \to K_S \pi \nu_\tau$ decays. After the initial null results from CLEO [11] and Belle [12], a nonzero *CP* asymmetry was reported for the first time by the *BABAR* Collaboration, with the result given by [13]

$$A_Q = \frac{\Gamma(\tau^+ \to [\pi^+\pi^-]_{``K_S"}\pi^+\bar{\nu}_\tau) - \Gamma(\tau^- \to [\pi^+\pi^-]_{``K_S"}\pi^-\nu_\tau)}{\Gamma(\tau^+ \to [\pi^+\pi^-]_{``K_S"}\pi^+\bar{\nu}_\tau) + \Gamma(\tau^- \to [\pi^+\pi^-]_{``K_S"}\pi^-\nu_\tau)} = (-0.36 \pm 0.23 \pm 0.11)\%, \tag{1.1}$$

where the first uncertainty is statistical and the second systematic. The subscript "K_S" indicates that the intermediate K_S is reconstructed in terms of a $\pi^+\pi^-$ final state with invariant mass around M_K and at a decay time close to the K_S lifetime. Within the SM, as there is no direct *CP* violation in hadronic τ decays at the tree level in weak interaction,[1] this asymmetry arises solely from the *CP* violation in K^0-$\bar{K}^0$ mixing [15,16], and is calculated to be [17,18]

$$A_{CP}^{\rm SM} = \frac{\int_{t_1}^{t_2} dt[\Gamma(K^0(t) \to \pi\pi) - \Gamma(\bar{K}^0(t) \to \pi\pi)]}{\int_{t_1}^{t_2} dt[\Gamma(K^0(t) \to \pi\pi) + \Gamma(\bar{K}^0(t) \to \pi\pi)]} \approx (3.32 \pm 0.06) \times 10^{-3}, \tag{1.2}$$

[*]ggchan@mails.ccnu.edu.cn
[†]xqli@mail.ccnu.edu.cn
[‡]yangyd@mail.ccnu.edu.cn
[§]xinzhang@mail.ccnu.edu.cn

[1]The direct *CP* asymmetry generated by the second-order weak interaction is estimated to be of order 10^{-12}, and can be therefore neglected safely [14].

where $K^0(t)$ [$\bar{K}^0(t)$] denotes the time-evolved state identified at time $t = 0$ as a pure K^0 [$\bar{K}^0$], and the second line is obtained after neglecting the small correction from direct CP violation in $K \to \pi^+\pi^-$ decays and when $t_1 \ll \tau_S$ and $\tau_S \ll t_2 \ll \tau_L$ [$\tau_{S(L)}$ being the $K_{S(L)}$ lifetime]. Such a CP asymmetry was first predicted by Bigi and Sanda [17] but with a sign mistake [18]. As emphasized by Grossman and Nir [18] (see also Ref. [19]), in the calculation of this CP asymmetry, the interference between the amplitudes of intermediate K_S and K_L is as important as the pure K_S amplitude, and hence the measured CP asymmetry depends sensitively on the decay time interval over which it is integrated. After taking into account the $K_S \to \pi^+\pi^-$ decay-time dependence of the event selection efficiency, the *BABAR* Collaboration obtained a multiplicative correction factor, 1.08 ± 0.01, for the CP asymmetry, with the resulting experimental data given by Eq. (1.1) and the corresponding SM prediction changed to $A_{CP}^{\rm SM} = (0.36 \pm 0.01)\%$ [13]. Thus, a 2.8σ discrepancy is observed between the *BABAR* measurement and the SM prediction and, if confirmed with a higher precision by Belle and/or Belle II [20], would be a clear NP signal.

Such a CP anomaly, together with the prospects of future measurements at Belle II [20], has motivated several studies of possible direct CP asymmetries in $\tau \to K_S\pi\nu_\tau$ decays due to nonstandard interactions [21–26]. As argued in Refs. [21,24], due to the lack of a relative strong phase, an explanation with scalar operators is already excluded,[2] and only the interference of vector and tensor operators can provide a possible strong phase difference, leaving new tensor interactions as the only potential NP explanation. Here, whether the tensor interaction is admissible to account for the anomaly or not depends crucially on the $K\pi$ tensor form factor. In Refs. [21,28,29], the tensor form factor was assumed to be a real constant, which is motivated by the analysis of $K_{\ell 3}$ ($K \to \pi\ell\nu_\ell$ with $\ell = e, \mu$) data [16], and the relative strong phase, being now just the phase of the vector form factor, was found to be large enough to produce a sizable CP asymmetry. This assumption was, however, pointed out to be incorrect by Cirigliano, Crivellin, and Hoferichter [24]. They demonstrated that, as the same spin-1 resonances contributing to the vector form factor will equivalently contribute to the tensor one, the crucial interference between vector and tensor phases is suppressed by at least 2 orders of magnitude due to Watson's final-state interaction theorem [30], and the amount of CP asymmetry that a tensor operator can produce is, therefore, strongly suppressed [24]. Such a conclusion is, however, based on the assumption that the inelastic contributions to the phases of vector and tensor form factors are of similar size but potentially opposite in sign [24].

In order to obtain sensible constraints on nonstandard interactions from $\tau \to K_S\pi\nu_\tau$ decays, the exact distributions of the $K\pi$ form factors, including both their moduli and phases, as a function of $s = q^2$, the invariant mass squared of the $K\pi$ final state, are needed. For the vector and scalar form factors, either the Breit-Wigner parametrizations [31–34] or the dispersive representations [35–44] can be used, with the relevant parameters determined via a successful fit to the measured $K\pi$ invariant mass spectrum [31]. For the tensor form factor, however, there exists no experimental data that can guide us to construct it, and we have to rely on theory. While a q^2-independent tensor form factor or its normalization [45] can be derived from the leading-order chiral perturbation theory (χPT) [46–49] with tensor sources [50,51], we have to get its q^2 dependence by solving numerically the dispersion relation [25,52], with its phase obtained in the context of chiral theory with resonances (RχT) [53,54]. It should be mentioned that the tensor form factor given in Ref. [52] is derived at the lowest chiral order [being $\mathcal{O}(p^4)$ in the chiral counting [50]] of RχT and fails to satisfy the unitarity requirement, which could be compensated by including the contributions from the next-to-leading-order (NLO) χPT Lagrangian with tensor sources. Although the spin-1 resonances can be described equivalently by vector or antisymmetric tensor fields [53,54], it will be shown that the former is more convenient in describing the interactions of tensor currents with the resonances. The unitarity property will also be satisfied automatically when the NLO [being $\mathcal{O}(p^6)$ in the chiral counting [50]] terms with the model II prescription [54] are properly taken into account. In this paper, motivated by these two observations and following Refs. [25,52], we shall present an alternative dispersive representation of the tensor form factor, with its phase obtained in the context of RχT, which fulfills the requirements of unitarity and analyticity.

Taking as inputs the three-times-subtracted (for the vector form factor) [37,38] and the coupled-channel (for the scalar form factor) [39–41] dispersive representations, together with our result of the tensor form factor, we shall then analyze the $\tau \to K_S\pi\nu_\tau$ decays both within a model-independent low-energy effective theory framework and in a scalar leptoquark (LQ) scenario [55]. It will be shown that the CP anomaly can be accommodated in the model-independent framework, even at the 1σ level, together with the constraint from the branching ratio of $\tau^- \to K_S\pi^-\nu_\tau$ decay. In the LQ scenario, however, this anomaly can be marginally reconciled only at the 2σ level, due to the specific relation between the scalar and tensor operators. Once the combined constraints from the branching ratio and the decay spectrum of this decay are taken into account,

[2] Although the interference of vector and scalar operators could still contribute to the CP asymmetry due to long-distance QED corrections [27], the scalar contribution is strongly suppressed and will be of little phenomenological relevance when the constraint from the $\tau \to K_S\pi\nu_\tau$ branching ratio is taken into account [24].

these possibilities are, however, both excluded, even without exploiting further the stronger bounds from the (semi) leptonic kaon decays [56] under the assumption of lepton-flavor universality, as well as from the neutron electric dipole moment (EDM) and D-$\bar{D}$ mixing under the assumption of $SU(2)$ invariance of the weak interactions [24]. It is therefore quite difficult to explain such a CP anomaly within the frameworks considered here.

Our paper is organized as follows: In Sec. II, we recalculate the CP asymmetry due to K^0-$\bar{K}^0$ mixing by means of the reciprocal basis, and reproduce the result given in Ref. [18]. In Sec. III, the $\tau \to K_S\pi\nu_\tau$ decays are analyzed both within the model-independent framework and in the scalar LQ scenario. Section IV is devoted to the calculation of the $K\pi$ tensor form factor in the context of χPT with tensor sources and RχT with the spin-1 resonances described by the conventional vector fields. Numerical results and discussions are then presented in Sec. V. Our conclusions are finally made in Sec. VI. For convenience, all the input parameters used throughout this paper are collected in the Appendix.

II. CP ASYMMETRY IN $\tau \to K_S\pi\nu_\tau$ DECAYS WITHIN THE SM

Before discussing the CP asymmetry in $\tau \to K_S\pi\nu_\tau$ decays within the SM, one should first notice that the τ^+ (τ^-) decay produces initially a K^0 ($\bar{K}^0$) state due to the $\Delta S = \Delta Q$ rule, and the relevant Feynman diagrams at the tree level in weak interaction are shown in Fig. 1. As the involved CKM matrix element V_{us} is real and the strong phase must be the same in these two CP-related processes, the transition amplitudes within the SM should satisfy

$$\mathcal{A}(\tau^+ \to K^0\pi^+\bar{\nu}_\tau) = \mathcal{A}(\tau^- \to \bar{K}^0\pi^-\nu_\tau). \tag{2.1}$$

Due to the K^0-$\bar{K}^0$ mixing, on the other hand, the experimentally reconstructed kaons are the mass ($|K_S\rangle$ and $|K_L\rangle$) rather than the flavor ($|K^0\rangle$ and $|\bar{K}^0\rangle$) eigenstates, which, in the absence of CP violation in the system, are related to each other via [57]

$$|K_{S,L}\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle \pm e^{i\zeta}|\bar{K}^0\rangle), \tag{2.2}$$

where ζ is the spurious phase brought about by the CP transformation, $\mathcal{CP}|K^0\rangle = e^{i\zeta}|\bar{K}^0\rangle$ [57]. Then, one can get

$$\Gamma(\tau^+ \to K_{S,L}\pi^+\bar{\nu}_\tau) = \frac{1}{2}\Gamma(\tau^+ \to K^0\pi^+\bar{\nu}_\tau),$$
$$\Gamma(\tau^- \to K_{S,L}\pi^-\nu_\tau) = \frac{1}{2}\Gamma(\tau^- \to \bar{K}^0\pi^-\nu_\tau), \tag{2.3}$$

which, when taken together with Eq. (2.1), would indicate that there exists no CP asymmetry in $\tau \to K_S\pi\nu_\tau$ decays within the SM. Furthermore, the contribution from second-order weak interaction is estimated to be of order 10^{-12}, and can be therefore neglected safely [14].

Once the CP violation in K^0-$\bar{K}^0$ mixing [15,16] is included, however, a nonzero CP asymmetry would appear in $\tau \to K_S\pi\nu_\tau$ decays, as elaborated in Refs. [17,18]. In order to see this clearly, we shall follow the convention specified in Ref. [57] and recalculate this asymmetry by means of the reciprocal basis [57–64]. In the presence of CP violation but with CPT invariance still assumed, the two mass eigenkets are now given by [57]

$$|K_{S,L}\rangle = p|K^0\rangle \pm q|\bar{K}^0\rangle, \tag{2.4}$$

with the normalization $|p|^2 + |q|^2 = 1$, and the corresponding mass eigenbras read [57]

$$\langle\tilde{K}_{S,L}| = \frac{1}{2}(p^{-1}\langle K^0| \pm q^{-1}\langle\bar{K}^0|), \tag{2.5}$$

which form the so-called reciprocal basis ($\langle\tilde{K}_S|$, $\langle\tilde{K}_L|$) that is featured by both the orthornormality and the completeness conditions [57]:

$$\langle\tilde{K}_S|K_S\rangle = \langle\tilde{K}_L|K_L\rangle = 1, \qquad \langle\tilde{K}_S|K_L\rangle = \langle\tilde{K}_L|K_S\rangle = 0,$$
$$|K_S\rangle\langle\tilde{K}_S| + |K_L\rangle\langle\tilde{K}_L| = 1. \tag{2.6}$$

Then, the time-evolution operator for the K^0-$\bar{K}^0$ system is determined by

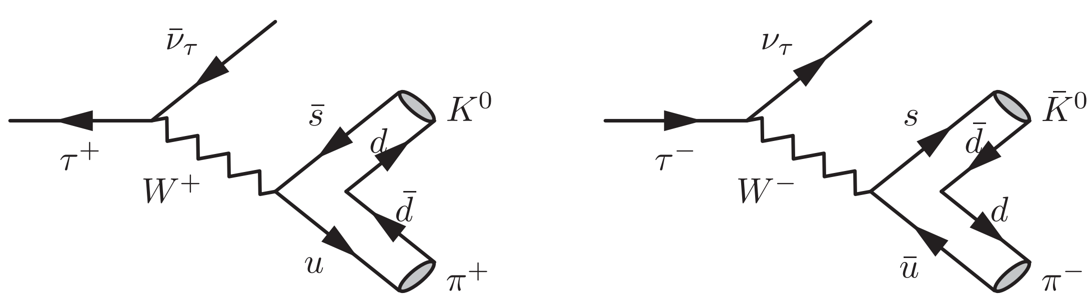

FIG. 1. Tree-level Feynman diagrams for the decay $\tau^+ \to K^0\pi^+\bar{\nu}_\tau$ (left) as well as its CP-conjugated mode $\tau^- \to \bar{K}^0\pi^-\nu_\tau$ (right) within the SM.

$$\exp(-i\boldsymbol{H}t) = e^{-i\mu_S t}|K_S\rangle\langle\tilde{K}_S| + e^{-i\mu_L t}|K_L\rangle\langle\tilde{K}_L|, \tag{2.7}$$

where $\mu_S = M_S - i/2\Gamma_S$ and $\mu_L = M_L - i/2\Gamma_L$ are the two eigenvalues of the 2×2 effective Hamiltonian $\boldsymbol{H}$ used to describe the K^0-$\bar{K}^0$ mixing.

The intermediate K_S in $\tau \to K_S\pi\nu_\tau$ decays is not directly observed in experiment, but reconstructed via a $\pi^+\pi^-$ final state with $M_{\pi\pi} \approx M_K$ and a time difference $t \approx \tau_S$ between the τ and the K decay [18]. However, as the CP symmetry is violated in K^0-$\bar{K}^0$ mixing [15,16], the final state $\pi^+\pi^-$ can be obtained not only from K_S but also from K_L, for long decay times of kaons. As a consequence, the complete amplitude for the process $\tau^+ \to [\pi^+\pi^-]\pi^+\bar{\nu}_\tau$ involves the amplitude for the initial τ^+ decay into the intermediate state $K_{S,L}\pi^+\bar{\nu}_\tau$, the time-evolution amplitude for this state, and finally the amplitude for the decay into $[\pi^+\pi^-]\pi^+\bar{\nu}_\tau$. Suppressing the reference to $\pi^+\bar{\nu}_\tau$, we can therefore write the complete amplitude as [64]

$$\begin{aligned}\mathcal{A}(\tau^+ \to K_{S,L} \to \pi^+\pi^-) &= \langle\pi^+\pi^-|T|K_S\rangle e^{-i\mu_S t}\langle\tilde{K}_S|T|\tau^+\rangle + \langle\pi^+\pi^-|T|K_L\rangle e^{-i\mu_L t}\langle\tilde{K}_L|T|\tau^+\rangle \\ &= \frac{1}{2p}\langle\pi^+\pi^-|T|K_S\rangle e^{-i\mu_S t}\langle K^0|T|\tau^+\rangle + \frac{1}{2p}\langle\pi^+\pi^-|T|K_L\rangle e^{-i\mu_L t}\langle K^0|T|\tau^+\rangle,\end{aligned} \tag{2.8}$$

in which Eq. (2.5) and the $\Delta S = \Delta Q$ rule have been used to obtain the second line. Then, the complete time-dependent decay width for $\tau^+ \to [\pi^+\pi^-]\pi^+\bar{\nu}_\tau$ can be written as[3]

$$\begin{aligned}\Gamma(\tau^+ \to [\pi^+\pi^-]\pi^+\bar{\nu}_\tau) &= |\langle K^0|T|\tau^+\rangle|^2\frac{|\langle\pi^+\pi^-|T|K_S\rangle|^2}{4|p|^2} \\ &\quad\times[e^{-\Gamma_S t} + |\eta_{+-}|^2 e^{-\Gamma_L t} + 2|\eta_{+-}|e^{-\Gamma t}\cos(\phi_{+-} - \Delta m t)] \qquad (2.9)\\ &= \Gamma(\tau^+ \to K^0)\Gamma(K^0(t) \to \pi^+\pi^-), \qquad (2.10)\end{aligned}$$

where $\Gamma = \frac{\Gamma_L+\Gamma_S}{2}$ and $\Delta m = M_L - M_S$ denote respectively the average width and the mass difference of the $K^0 - \bar{K}^0$ system, while the CP-violating amplitude ratio η_{+-} is defined as

$$\eta_{+-} = \frac{\langle\pi^+\pi^-|T|K_L\rangle}{\langle\pi^+\pi^-|T|K_S\rangle}, \tag{2.11}$$

with $|\eta_{+-}| = (2.232 \pm 0.011)\times 10^{-3}$ and $\phi_{+-} = (43.51 \pm 0.05)^\circ$ [16]. From Eq. (2.9) and the corresponding decay width for the CP-conjugated process $\tau^- \to [\pi^+\pi^-]\pi^-\nu_\tau$ [obtained from Eq. (2.9) by replacing p and $+2|\eta_{+-}|$ with q and $-2|\eta_{+-}|$, respectively], one can see that, for the sum of the two decay widths, both the interference (the last) and the pure K_L (the second) term are suppressed compared to the pure K_S (the first term in the bracket) contribution. For the difference of the two decay widths, however, the interference between the amplitudes of K_S and K_L is found to be as important as the pure K_S amplitude [18].

The time dependence of the decay width in Eq. (2.9) makes the measurement of the CP asymmetry in $\tau \to K_S\pi\nu_\tau$ decays sensitive to the experimental cuts [18]: one has to take into account not only the efficiency as a function of the kaon decay time, but also the kaon energy in the laboratory frame to account for the time dilation. Parametrizing all these experiment-dependent effects by a function $F(t)$ [18], one can write the total CP asymmetry as [21]

$$A_{CP}(t_1,t_2) = \frac{\Gamma_{\tau^+}\int_{t_1}^{t_2} dtF(t)\Gamma(K^0(t)\to\pi^+\pi^-) - \Gamma_{\tau^-}\int_{t_1}^{t_2} dtF(t)\Gamma(\bar{K}^0(t)\to\pi^+\pi^-)}{\Gamma_{\tau^+}\int_{t_1}^{t_2} dtF(t)\Gamma(K^0(t)\to\pi^+\pi^-) + \Gamma_{\tau^-}\int_{t_1}^{t_2} dtF(t)\Gamma(\bar{K}^0(t)\to\pi^+\pi^-)} = \frac{A_{CP}^\tau + A_{CP}^K(t_1,t_2)}{1 + A_{CP}^\tau A_{CP}^K(t_1,t_2)}, \tag{2.12}$$

where $\Gamma_{\tau^\pm} = \Gamma(\tau^\pm \to K^0(\bar{K}^0)\pi^\pm\bar{\nu}_\tau(\nu_\tau))$ instead of $\Gamma(\tau^\pm \to K_S\pi^\pm\bar{\nu}_\tau(\nu_\tau))$ as defined in Ref. [21], while A_{CP}^τ and $A_{CP}^K(t_1,t_2)$ are defined, respectively, as

[3]Our expression of the decay width is slightly different from that given in Ref. [18], because the latter corresponds to the time-dependent decay width of K^0 decaying into the 2π system with isospin $I = 0$, which contains both the $\pi^+\pi^-$ and $\pi^0\pi^0$ components, $\langle 2\pi, I=0| = \sqrt{\frac{2}{3}}\langle\pi^+\pi^-| - \sqrt{\frac{1}{3}}\langle\pi^0\pi^0|$. As the intermediate K_S is reconstructed via the $\pi^+\pi^-$ final state in experiment [13], one has to use η_{+-} instead of $\epsilon = \frac{\langle 2\pi,I=0|T|K_L\rangle}{\langle 2\pi,I=0|T|K_S\rangle}$.

$$A_{CP}^{\tau} \equiv \frac{\Gamma(\tau^+ \to K^0 \pi^+ \bar{\nu}_\tau) - \Gamma(\tau^- \to \bar{K}^0 \pi^- \nu_\tau)}{\Gamma(\tau^+ \to K^0 \pi^+ \bar{\nu}_\tau) + \Gamma(\tau^- \to \bar{K}^0 \pi^- \nu_\tau)}, \tag{2.13}$$

$$A_{CP}^{K}(t_1, t_2) \equiv \frac{\int_{t_1}^{t_2} dt F(t) [\Gamma(K^0(t) \to \pi^+\pi^-) - \Gamma(\bar{K}^0(t) \to \pi^+\pi^-)]}{\int_{t_1}^{t_2} dt F(t) [\Gamma(K^0(t) \to \pi^+\pi^-) + \Gamma(\bar{K}^0(t) \to \pi^+\pi^-)]}. \tag{2.14}$$

Here A_{CP}^{τ} denotes the direct CP asymmetry induced by potential NP dynamics, while $A_{CP}^{K}(t_1, t_2)$ represents the indirect CP asymmetry originating from the K^0-$\bar{K}^0$ mixing. Within the SM, $\Gamma_{\tau^+} = \Gamma_{\tau^-}$, implying that $A_{CP}^{\tau} = 0$, and only $A_{CP}^{K}(t_1, t_2)$ makes a nonzero contribution [17,18].

Plugging into Eq. (2.14) the expressions of the time-dependent decay widths $\Gamma(K^0(t) \to \pi^+\pi^-)$ and $\Gamma(\bar{K}^0(t) \to \pi^+\pi^-)$ [see Eqs. (2.9) and (2.10)] and neglecting all the terms suppressed by $\mathcal{O}(|\eta_{+-}|^2)$, one can finally reproduce the result given in Ref. [18],[4]

$$A_{CP}^{\rm SM} \approx +2\Re e(\epsilon) = 3.32 \times 10^{-3} \quad \text{for } t_1 \ll \Gamma_S^{-1} \quad \text{and} \quad \Gamma_S^{-1} \ll t_2 \ll \Gamma_L^{-1}, \tag{2.15}$$

in which the approximations with $|\eta_{+-}| \approx \frac{2\Re e(\epsilon)}{\sqrt{2}}$, $\phi_{+-} \approx 45°$, $\Gamma \approx \frac{\Gamma_S}{2}$, and $\Delta m \approx \frac{\Gamma_S}{2}$ [63], as well as a particular efficiency function $F(t)$ [18],

$$F(t) = \begin{cases} 1, & t_1 < t < t_2 \\ 0, & \text{otherwise} \end{cases}, \tag{2.16}$$

have been used. The SM CP asymmetry in Eq. (2.15), after being multiplied by the correction factor 1.08 ± 0.01 [13], is then changed to be $(0.36 \pm 0.01)\%$, as obtained by the *BABAR* Collaboration [13].

III. $\tau \to K_S \pi \nu_\tau$ DECAYS IN THE PRESENCE OF NP DYNAMICS

When NP dynamics beyond the SM are present, a nonzero direct CP asymmetry A_{CP}^{τ} can exist and hence contributes to the total CP asymmetry $A_{CP}(t_1, t_2)$. As neither the pseudoscalar nor the axial-vector interaction can produce the $K\pi$ final state due to the parity conservation in strong interaction, and the scalar interaction cannot create a nonzero direct CP asymmetry due to the lack of a relative strong phase, we are left only with the tensor interaction as a possible mechanism [21,24]. In this section, we shall firstly start with a model-independent low-energy effective Lagrangian that contains all the potential NP operators contributing to the $\tau \to K_S \pi \nu_\tau$ decays, and analyze the tensor operator contribution to A_{CP}^{τ}. Then, we shall discuss A_{CP}^{τ} in a scalar LQ scenario, which also contains the relevant operators.

A. Model-independent analysis

For the strangeness-changing hadronic τ decays, the most general model-independent effective Lagrangian at the characteristic scale $\mu_\tau = m_\tau$ can be written as [24,25,65][5]

$$\begin{aligned}
\mathcal{L}_{\rm eff} = &-\frac{G_F V_{us}}{\sqrt{2}} \{ (1+\epsilon_L) \bar{\tau}\gamma_\mu (1-\gamma_5)\nu_\tau \cdot \bar{u}\gamma^\mu (1-\gamma_5) s + \epsilon_R \bar{\tau}\gamma_\mu (1-\gamma_5)\nu_\tau \cdot \bar{u}\gamma^\mu (1+\gamma_5) s \\
&+ \bar{\tau}(1-\gamma_5)\nu_\tau \cdot \bar{u}[\epsilon_S - \epsilon_P \gamma_5] s + \epsilon_T \bar{\tau}\sigma_{\mu\nu}(1-\gamma_5)\nu_\tau \cdot \bar{u}\sigma^{\mu\nu}(1-\gamma_5) s \} + \text{H.c.} \\
= &-\frac{G_F V_{us}}{\sqrt{2}} (1+\epsilon_L+\epsilon_R) \{ \bar{\tau}\gamma_\mu (1-\gamma_5)\nu_\tau \cdot \bar{u}[\gamma^\mu - (1-2\hat{\epsilon}_R)\gamma^\mu\gamma_5] s \\
&+ \bar{\tau}(1-\gamma_5)\nu_\tau \cdot \bar{u}[\hat{\epsilon}_S - \hat{\epsilon}_P \gamma_5] s + 2\hat{\epsilon}_T \bar{\tau}\sigma_{\mu\nu}(1-\gamma_5)\nu_\tau \cdot \bar{u}\sigma^{\mu\nu} s \} + \text{H.c.},
\end{aligned} \tag{3.1}$$

where G_F is the Fermi constant, and $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$. The effective couplings ϵ_i parametrize the nonstandard contributions and can be generally complex, with the SM case recovered when all $\epsilon_i = 0$. We have also introduced the notations $\hat{\epsilon}_i = \epsilon_i/(1+\epsilon_L+\epsilon_R)$ for $i = R, S, P, T$, with the corresponding quark currents possessing definite parities and being therefore convenient to describe the vacuum to $K\pi$ matrix elements due to the parity conservation [25]. Here we have assumed Lorentz and $SU(3)_C \times U(1)_{\rm em}$ invariance, as well as the absence of light nonstandard

[4]If the K_L contributions to the decay width in Eq. (2.9) were neglected, on the other hand, one would obtain a result which is of the same magnitude but opposite in sign with the prediction made in Ref. [17].

[5]This is adopted from the most general flavor-dependent low-energy effective Lagrangian governing the semileptonic $d^j \to u^i \ell \nu_\ell$ transitions, which can be found, e.g., in Refs. [66–68]. It should be noted that, once the lepton-flavor universality is assumed, the effective couplings ϵ_i in Eq. (3.1) would also receive the constraints from (semi)leptonic kaon [56] and hyperon [69] decays.

particles when constructing $\mathcal{L}_{\rm eff}$.[6] Right-handed and wrong-flavor neutrino contributions have also been neglected in Eq. (3.1), because they do not interfere with the SM amplitudes.

Starting with Eq. (3.1) and working in the rest frame of the τ lepton, one can then obtain the differential decay width of the decay $\tau^- \to \bar{K}^0\pi^-\nu_\tau$ [24,25],

$$\frac{d\Gamma(\tau^- \to \bar{K}^0\pi^-\nu_\tau)}{ds} = \frac{G_F^2|F_+(0)V_{us}|^2 m_\tau^3 S_{\rm EW}}{384\pi^3 s}|1+\epsilon_L+\epsilon_R|^2\left(1-\frac{s}{m_\tau^2}\right)^2 \lambda^{1/2}(s, M_K^2, M_\pi^2) \\ \times [X_{VA} + \Re e\hat{\epsilon}_S X_S + \Re e\hat{\epsilon}_T X_{\Re eT} + \Im m\hat{\epsilon}_T X_{\Im mT} + |\hat{\epsilon}_S|^2 X_{S^2} + |\hat{\epsilon}_T|^2 X_{T^2}], \tag{3.2}$$

where $s = (p_K + p_\pi)^2$, and $\lambda(s, M_K^2, M_\pi^2) = [s-(M_K+M_\pi)^2][s-(M_K-M_\pi)^2]$. The product $|F_+(0)V_{us}| = 0.21654(41)$ is determined most precisely from the analysis of semileptonic kaon decays [73,74]. $S_{\rm EW} = 1.0201(3)$ encodes the short-distance electroweak correction [75] and is simply written as an overall constant [25]. We have also introduced the following quantities:

$$X_{VA} = \frac{1}{2s^2}\left[3|\tilde{F}_0(s)|^2\Delta_{K\pi}^2 + |\tilde{F}_+(s)|^2\left(1+\frac{2s}{m_\tau^2}\right)\lambda(s, M_K^2, M_\pi^2)\right], \tag{3.3}$$

$$X_S = \frac{3}{sm_\tau}|\tilde{F}_0(s)|^2\frac{\Delta_{K\pi}^2}{m_s - m_u}, \tag{3.4}$$

$$X_{\Re eT} = -\frac{6}{sm_\tau}\left|\frac{F_T(0)}{F_+(0)}\right||\tilde{F}_T(s)||\tilde{F}_+(s)|\cos[\delta_T(s)-\delta_+(s)]\lambda(s, M_K^2, M_\pi^2), \tag{3.5}$$

$$X_{\Im mT} = -\frac{6}{sm_\tau}\left|\frac{F_T(0)}{F_+(0)}\right||\tilde{F}_T(s)||\tilde{F}_+(s)|\sin[\delta_T(s)-\delta_+(s)]\lambda(s, M_K^2, M_\pi^2), \tag{3.6}$$

$$X_{S^2} = \frac{3}{2m_\tau^2}|\tilde{F}_0(s)|^2\frac{\Delta_{K\pi}^2}{(m_s - m_u)^2}, \tag{3.7}$$

$$X_{T^2} = \frac{4}{s}\left|\frac{F_T(0)}{F_+(0)}\right|^2|\tilde{F}_T(s)|^2\left(1+\frac{s}{2m_\tau^2}\right)\lambda(s, M_K^2, M_\pi^2). \tag{3.8}$$

Here we have split $F_i(s) = F_i(0)\tilde{F}_i(s)$ (with $i = +, 0, T$ corresponding to the vector, scalar, and tensor form factors, respectively) into $F_i(0)$ (form factors at zero momentum transfer) and $\tilde{F}_i(s)$ (the corresponding normalized form factors), with $F_i(s)$ defined, respectively, as [25]

$$\langle\bar{K}^0(p_K)\pi^-(p_\pi)|\bar{s}\gamma^\mu u|0\rangle = \left[(p_K - p_\pi)^\mu - \frac{\Delta_{K\pi}}{s}q^\mu\right]F_+(s) + \frac{\Delta_{K\pi}}{s}q^\mu F_0(s), \tag{3.9}$$

$$\langle\bar{K}^0(p_K)\pi^-(p_\pi)|\bar{s}u|0\rangle = \frac{\Delta_{K\pi}}{m_s - m_u}F_0(s), \tag{3.10}$$

$$\langle\bar{K}^0(p_K)\pi^-(p_\pi)|\bar{s}\sigma^{\mu\nu}u|0\rangle = iF_T(s)(p_K^\mu p_\pi^\nu - p_K^\nu p_\pi^\mu), \tag{3.11}$$

where $q^\mu = (p_K + p_\pi)^\mu$, $\Delta_{K\pi} = M_K^2 - M_\pi^2$, and the equation of motion has been used in Eq. (3.10).

The differential decay rate of the CP-conjugated process $\tau^+ \to K^0\pi^+\bar{\nu}_\tau$ is obtained from Eq. (3.2) by changing the sign of the term $\Im m\hat{\epsilon}_T$, which implies that only this term contributes to the direct CP asymmetry. From the definition

[6] One should keep in mind that, unless some NP between $\mu_\tau = m_\tau$ and the electroweak scale $v = 246$ GeV is assumed, the low-energy effective Lagrangian given by Eq. (3.1) comes generally from an $SU(2)$-invariant form [70–72]. This implies that the effective tensor operator contributing to $\tau \to K_S\pi\nu_\tau$ decays is also constrained by other processes—for example, by the neutron EDM and D-$\bar{D}$ mixing [24].

of Eq. (2.13), the direct *CP* asymmetry due to nonstandard tensor interaction can finally be written as [24,25]

$$
\begin{aligned}
A_{CP}^{\tau} = {}& \frac{2\Im m\hat{\epsilon}_T G_F^2 |F_+(0)V_{us}|^2 S_{\mathrm{EW}}}{256\pi^3 m_\tau^2 \Gamma(\tau \to K_S\pi\nu_\tau)} \left|\frac{F_T(0)}{F_+(0)}\right| \\
& \times \int_{s_{K\pi}}^{m_\tau^2} ds \frac{\lambda^{3/2}(s, M_K^2, M_\pi^2)(m_\tau^2 - s)^2}{s^2} \\
& \times |\tilde{F}_+(s)||\tilde{F}_T(s)| \sin[\delta_T(s) - \delta_+(s)], \qquad (3.12)
\end{aligned}
$$

where $s_{K\pi} = (M_K + M_\pi)^2$, and $\delta_T(s)$ and $\delta_+(s)$ are the strong phases of the tensor and vector form factors, respectively. The decay width $\Gamma(\tau^- \to K_S\pi^-\nu_\tau)$, as well as the branching ratio $B(\tau^- \to K_S\pi^-\nu_\tau) = \Gamma(\tau^- \to K_S\pi^-\nu_\tau)/\Gamma_\tau$, with Γ_τ being the total decay width of the τ^- lepton, are obtained by integrating Eq. (3.2) over s from $s_{K\pi}$ to m_τ^2.

B. Analysis in the scalar LQ scenario

In this section, we study the direct *CP* asymmetry in the scalar LQ scenario [55], which was proposed by Bauer and Neubert to address the $R_{D^{(*)}}$, R_K, and $(g-2)_\mu$ anomalies, and can also generate the required tensor operator. In this scenario, only a single TeV-scale scalar LQ ϕ, transforming as $(\mathbf{3}, \mathbf{1}, -\frac{1}{3})$ under the SM gauge group, is added to the SM particle content, and its couplings to fermions are described by the Lagrangian [55]

$$
L_{\mathrm{int}}^{\phi} = \bar{Q}_L^c \boldsymbol{\lambda}^L i\tau_2 L\phi^* + \bar{u}_R^c \boldsymbol{\lambda}^R \ell_R \phi^* + \mathrm{H.c.}, \qquad (3.13)
$$

where $\boldsymbol{\lambda}^{L,R}$ are the Yukawa coupling matrices in flavor space, Q_L and L denote the left-handed quark and lepton doublet, while u_R and ℓ_R are the right-handed up-type quark and lepton singlet, with $\psi^c = C\bar{\psi}^T$ and $\bar{\psi}^c = \psi^T C$ ($C = i\gamma^2\gamma^0$) being the charge-conjugated spinors.

With the SM as well as the tree-level ϕ-mediated contributions included, the resulting effective Hamiltonian governing the $\tau \to K_S\pi\nu_\tau$ decays can be written as

$$
\begin{aligned}
H_{\mathrm{eff}} = {}& \frac{G_F V_{us}}{\sqrt{2}} \{[1 + C_V(\mu_\phi)]\bar{\tau}\gamma^\mu(1-\gamma_5)\nu_\tau \cdot \bar{u}\gamma_\mu(1-\gamma_5)s \\
& + C_S(\mu_\phi)\bar{\tau}(1-\gamma_5)\nu_\tau \cdot \bar{u}(1-\gamma_5)s \\
& + C_T(\mu_\phi)\bar{\tau}\sigma^{\mu\nu}(1-\gamma_5)\nu_\tau \cdot \bar{u}\sigma_{\mu\nu}(1-\gamma_5)s\} \\
& + \mathrm{H.c.}, \qquad (3.14)
\end{aligned}
$$

where $C_V(\mu_\phi)$, $C_S(\mu_\phi)$, and $C_T(\mu_\phi)$ denote the Wilson coefficients of the corresponding operators at the matching scale $\mu_\phi = M_\phi$ and are given, respectively, as [55,76]

$$
\begin{aligned}
C_V(\mu_\phi) &= \frac{\lambda_{u\tau}^{L*}\lambda_{s\nu_\tau}^{L}}{4\sqrt{2}G_F V_{us} M_\phi^2}, \\
C_S(\mu_\phi) &= -4C_T(\mu_\phi) = -\frac{\lambda_{u\tau}^{R*}\lambda_{s\nu_\tau}^{L}}{4\sqrt{2}G_F V_{us} M_\phi^2}, \qquad (3.15)
\end{aligned}
$$

in which all the couplings $\lambda_{ij}^{L,R}$ are generally complex, with i and j denoting the flavors of quarks and leptons, respectively. In order to resum the potentially large logarithmic effects, these Wilson coefficients should be evolved down to the characteristic scale $\mu_\tau = m_\tau$. The vector current is conserved, and hence the corresponding Wilson coefficient is scale independent, while the evolution of the scalar (C_S) and tensor (C_T) ones at the leading logarithmic approximation can be written schematically as [77]

$$
C_{S,T}(\mu_\tau) = R_{S,T}(\mu_\tau, \mu_\phi) C_{S,T}(\mu_\phi), \qquad (3.16)
$$

with the corresponding evolution functions $R_{S,T}(\mu_\tau, \mu_\phi)$ given by

$$
R_{S,T}(\mu_\tau, \mu_\phi) \equiv \left[\frac{\alpha_s(m_b)}{\alpha_s(\mu_\tau)}\right]^{\frac{\gamma_{S,T}}{2\beta_0^{(4)}}} \left[\frac{\alpha_s(m_t)}{\alpha_s(m_b)}\right]^{\frac{\gamma_{S,T}}{2\beta_0^{(5)}}} \left[\frac{\alpha_s(\mu_\phi)}{\alpha_s(m_t)}\right]^{\frac{\gamma_{S,T}}{2\beta_0^{(6)}}}, \qquad (3.17)
$$

where $\beta_0^{(n_f)} = 11 - 2n_f/3$ is the leading-order coefficient of the QCD beta function, with n_f being the number of active quark flavors, and $\gamma_S = -8$ [78] and $\gamma_T = 8/3$ [79] are the leading-order anomalous dimensions of the scalar and tensor currents, respectively.

Matching the relevant terms of the effective Hamiltonian [Eq. (3.14)] onto that of the model-independent effective Lagrangian [Eq. (3.1)] at the same scale $\mu_\tau = m_\tau$, we get

$$
\begin{aligned}
&\epsilon_L = C_V(\mu_\tau), \qquad \epsilon_R = 0, \\
&\epsilon_S = \epsilon_P = C_S(\mu_\tau), \qquad \epsilon_T = C_T(\mu_\tau), \\
&\hat{\epsilon}_R = 0, \qquad \hat{\epsilon}_T = \hat{C}_T \equiv \frac{C_T(\mu_\tau)}{1 + C_V(\mu_\tau)}, \\
&\hat{\epsilon}_S = \hat{\epsilon}_P = \hat{C}_S \equiv \frac{C_S(\mu_\tau)}{1 + C_V(\mu_\tau)} = -4\frac{R_S(\mu_\tau, \mu_\phi)}{R_T(\mu_\tau, \mu_\phi)}\hat{C}_T. \qquad (3.18)
\end{aligned}
$$

Due to the specific relation $C_S(\mu_\phi) = -4C_T(\mu_\phi)$ at the matching scale $\mu_\phi = M_\phi$, we are actually left with only one effective coupling $\hat{C}_T$ in the scalar LQ scenario. This feature makes a sensitive difference compared to the model-independent case, as will be discussed later.

IV. FORM FACTORS IN $\tau \to K_S\pi\nu_\tau$ DECAYS

To set bounds on the nonstandard interactions, one needs to have a precise knowledge of the $K\pi$ form factors introduced in Eqs. (3.9)–(3.11). To this end, one of the most appropriate approaches is the dispersive representation of these form factors, which warrants the properties of unitarity and analyticity. In this section, we shall first recapitulate the $K\pi$ vector and scalar form factors, and then present a calculation of the tensor form factor in the context of χPT with tensor sources and RχT with both $K^*(892)$ and $K^*(1410)$ included.

A. Brief review of the vector and scalar form factors

For the normalized vector form factor $\tilde{F}_+(s)$, we shall adopt the optimal three-times-subtracted dispersion relation [37,38]

$$\tilde{F}_+(s) = \exp\left\{\lambda'_+ \frac{s}{M_{\pi^-}^2} + \frac{1}{2}(\lambda''_+ - \lambda'^2_+)\frac{s^2}{M_{\pi^-}^4} + \frac{s^3}{\pi}\int_{s_{K\pi}}^{s_{\rm cut}} ds' \frac{\delta_+(s')}{(s')^3(s'-s-i\epsilon)}\right\}, \tag{4.1}$$

where one subtraction constant is fixed by $F_+(0) = 1$, while the other two, λ'_+ and λ''_+, describe the slope and curvature of $\tilde{F}_+(s)$, respectively, when performing its Taylor expansion around $s = 0$, and hence encode the dominant low-energy behavior of $\tilde{F}_+(s)$. As the calculation of λ'_+ and λ''_+ requires the perfect knowledge of the form-factor phase $\delta_+(s)$ up to infinity, which is unrealistic, it becomes more suitable to treat them as free parameters that capture our ignorance of the higher-energy part of the dispersion integral [37,38]. The constants λ'_+ and λ''_+ can then be determined by fitting to the experimental data [37,38,42].[7] With such a procedure, the subtraction terms cannot cancel perfectly the polynomial terms coming from the dispersion integral, and the use of the three-times-subtracted dispersion representation would thus spoil the asymptotic behavior of the vector form factor in the limit $s \to \infty$ [81–83]. This deficiency is, however, considered to be acceptable, because the vector form factor is employed only up to about $\sqrt{s} \simeq 1.7$ GeV, which is still in the resonance region [37,38]. The three-times-subtracted dispersion representation of $\tilde{F}_+(s)$ has also been checked explicitly to be a decreasing function of s within the entire range applied, which renders this approach credible [38].

The procedure proposed in Refs. [37,38] is featured by the fact that the subtraction terms reduce the sensitivity of the dispersion integral to the higher-energy contribution, with the associated constants being less model dependent, and the impact of our ignorance of the form-factor phase $\delta_+(s)$ at relatively higher energies turns out to be very small. This makes it reasonable to determine the form-factor phase $\delta_+(s)$ in the context of $R\chi$T with the two vector resonances $K^*(892)$ and $K^*(1410)$ included [35–38]. Notice that, in the elastic region below roughly 1.2 GeV, the phase $\delta_+(s)$ equals the scattering phase $\delta_1^{1/2}(s)$ of the $K\pi$ system with spin 1 and isospin 1/2, as required by Watson's theorem [30]. The cutoff $s_{\rm cut}$ introduced in Eq. (4.1) is used to quantify the suppression of the higher-energy part of the integral, and the stability of the numerical results has been checked by varying $s_{\rm cut}$ in the range $m_\tau < \sqrt{s_{\rm cut}} < \infty$ [37,38].

Detailed information on the $K\pi$ vector form factor can also be obtained from the measured $\tau \to K_S\pi\nu_\tau$ spectrum [31]. This is, however, possible only for its modulus but not for its phase, as the extraction of the latter requires a fit function that preserves the analytic structure of the form factor. Indeed, the phase fitted via a superposition of Breit-Wigner functions with complex coefficients cannot be physical, as it does not vanish at threshold and violates Watson's theorem long before the resonance $K^*(1410)$ starts to play an effect [24]. Thus, one cannot rely on the formalism developed in Ref. [31] to study the CP asymmetry in $\tau \to K_S\pi\nu_\tau$ decays.

For the scalar form factor, a thorough description that takes into account analyticity, unitarity, and the large-N_C limit of QCD, as well as the couplings to $K\eta$ and $K\eta'$ channels has been presented in Ref. [40] and later updated in Refs. [41,84,85]. Here we shall employ such a coupled-channel dispersive representation, with the relevant numerical tables obtained via a combined analysis of the $\tau^- \to K_S\pi^-\nu_\tau$ and $\tau^- \to K^-\eta\nu_\tau$ decays [42].[8]

B. Calculation of the $K\pi$ tensor form factor

Unlike for the vector and scalar form factors, there exist no experimental data to guide us to construct the tensor form factor, and we have to rely on theory to perform this task. In this section, following Refs. [25,52], we present a new calculation of the $K\pi$ tensor form factor.

1. Result at the lowest chiral order of χPT with tensor sources

When the external tensor field $\bar{t}^{\mu\nu} = \sum_{a=0}^{8} \frac{\lambda_a}{2}\bar{t}_a^{\mu\nu}$, with $\lambda_0 = \sqrt{2/3}\,\mathbf{1}_{3\times3}$, and $\lambda_{1,\ldots,8}$ being the eight Gell-Mann matrices, is switched on, the lowest-order [$\mathcal{O}(p^4)$ in the chiral counting] χPT Lagrangian can be written as [50,51]

$$\mathcal{L}_4^{\chi\rm PT} = \Lambda_1\langle t_+^{\mu\nu} f_{+\mu\nu}\rangle - i\Lambda_2\langle t_+^{\mu\nu} u_\mu u_\nu\rangle + \Lambda_3\langle t_+^{\mu\nu} t^+_{\mu\nu}\rangle + \Lambda_4\langle t_+^{\mu\nu}\rangle^2, \tag{4.2}$$

where $\langle\cdots\rangle$ denotes the trace in flavor space and, among the four operators, only the one with the coefficient Λ_2 contributes to the $\tau \to K_S\pi\nu_\tau$ decays. The building blocks $t_+^{\mu\nu} = u^\dagger t^{\mu\nu} u^\dagger + u t^{\mu\nu\dagger} u$ and $u_\mu = i[u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger]$ are

[7] If the phase $\delta_+(s)$ were exactly known, these two constants could also be determined by the spectral sum rules dictated by the asymptotic behavior of $\tilde{F}_+(s)$ [80], but in this case the three-times-subtracted dispersion representation given by Eq. (4.1) would reduce to the standard once-subtracted version.

[8] We thank Pablo Roig for providing us with these necessary numerical tables obtained in Ref. [42].

built out of the unitary nonlinear representation of the pseudo-Goldstone fields, $u(\phi^a) = \exp\left(\frac{i}{2F_\pi}\phi^a\lambda^a\right)$ [86,87], where $\phi^a = (\pi, K, \eta)$, $F_\pi = 92.3(1)$ MeV is the physical pion decay constant [16], and l_μ and r_μ are the left- and right-handed sources, respectively. The chiral tensor sources $t^{\mu\nu}$ and $t^{\mu\nu\dagger}$ are related to $\bar{t}^{\mu\nu}$ via [50]

$$\bar{t}^{\mu\nu} = P_L^{\mu\nu\alpha\beta} t_{\alpha\beta} + P_R^{\mu\nu\alpha\beta} t^\dagger_{\alpha\beta}, \qquad t^{\mu\nu} = P_L^{\mu\nu\alpha\beta}\bar{t}_{\alpha\beta}, \tag{4.3}$$

in which $P_R^{\mu\nu\alpha\beta} = \frac{1}{4}(g^{\mu\alpha}g^{\nu\beta} - g^{\mu\beta}g^{\nu\alpha} + i\epsilon^{\mu\nu\alpha\beta})$, with the convention $\epsilon^{0123} = +1$ for the Levi-Civita tensor $\epsilon^{\mu\nu\alpha\beta}$, and the algebraic identity $\sigma^{\mu\nu}\gamma_5 = \frac{i}{2}\epsilon^{\mu\nu\alpha\beta}\sigma_{\alpha\beta}$ has been used to get the relation $P_L^{\mu\nu\alpha\beta} = (P_R^{\mu\nu\alpha\beta})^\dagger$.

Taking the functional derivative of Eq. (4.2) with respect to the tensor source $\bar{t}^{\mu\nu}$, with all the other external sources set to zero, expanding $u(\phi^a)$ in powers of ϕ^a, and then taking the suitable hadronic matrix element, one can finally get [25,45,52]

$$\left\langle \bar{K}^0(p_K)\pi^-(p_\pi)\left|\frac{\delta L_4^{\chi\mathrm{PT}}}{\delta\bar{t}_{\mu\nu}}\right|0\right\rangle = i\frac{\Lambda_2}{F_\pi^2}(p_K^\mu p_\pi^\nu - p_K^\nu p_\pi^\mu). \tag{4.4}$$

This, together with Eq. (3.11), fixes the normalization $F_T(0) = \frac{\Lambda_2}{F_\pi^2}$ at the lowest chiral order in χPT. Although the low-energy constant Λ_2 cannot be determined from the χPT itself, its value can be inferred either from other low-energy constants using the short-distance constraint in Eq. (4.13) (see the next section for more details) or from the lattice result for the normalization $F_T(0)$ [88]. Here we shall resort to the lattice result, $F_T(0) = 0.417(15)$ [88], to determine $\Lambda_2 = 11.1 \pm 0.4$ MeV, which is consistent within errors with that quoted in Refs. [16,52,89,90] for the $\pi\pi$ channel.[9] This value of Λ_2 will be used in our numerical analysis.

2. *Including the spin-1 resonances in the context of* RχT

As the invariant mass squared s in $\tau \to K_S\pi\nu_\tau$ decays varies from the $K\pi$ threshold $s_{K\pi}$ up to m_τ^2, contributions to the form factors from light resonances, giving therefore the s dependence of these form factors, should also be included for a refined analysis. As the spin-1 resonances can be described equivalently by vector or antisymmetric tensor fields [53,54], the same resonances that contribute to $F_+(s)$ will also appear in $F_T(s)$. To discuss the chiral couplings of these resonances to the pseudo-Goldstone fields in the presence of tensor currents, we shall use the more conventional vector representation of these spin-1 degrees of freedom, named the model **II** prescription in Ref. [54].

[9] When comparing the values of Λ_2 quoted in Refs. [16,52,89,90], one should keep in mind the different conventions used for Λ_2. Our convention is consistent with that used in Refs. [50,52].

Explicitly, the RχT Lagrangian that is linear in the octet vector field $\hat{V}_\mu$ and contains the couplings to the tensor source at the lowest chiral order can be constructed as [54]

$$\begin{aligned}\mathcal{L}_{\mathbf{II}} = {} & \mathcal{L}_{\mathrm{kin}}(\hat{V}_\mu) - \frac{1}{2\sqrt{2}}(f_V\langle\hat{V}_{\mu\nu}f_+^{\mu\nu}\rangle + ig_V\langle\hat{V}_{\mu\nu}[u^\mu, u^\nu]\rangle) \\ & - f_V^T\langle\hat{V}_{\mu\nu}t_+^{\mu\nu}\rangle,\end{aligned} \tag{4.5}$$

with the kinetic spin-1 part given by [54]

$$\mathcal{L}_{\mathrm{kin}}(\hat{V}_\mu) = -\frac{1}{4}\langle\hat{V}_{\mu\nu}\hat{V}^{\mu\nu} - 2M_V^2\hat{V}_\mu\hat{V}^\mu\rangle, \tag{4.6}$$

where $\hat{V}_{\mu\nu} = \nabla_\mu\hat{V}_\nu - \nabla_\nu\hat{V}_\mu$, with the covariant derivative defined by $\nabla_\mu\hat{V}_\nu = \partial_\mu\hat{V}_\nu + [\Gamma_\mu, \hat{V}_\nu]$ and $\Gamma_\mu = \frac{1}{2}[u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_\mu)u^\dagger]$. Here, $f_+^{\mu\nu} = uF_L^{\mu\nu}u^\dagger + u^\dagger F_R^{\mu\nu}u$ is expressed in terms of the field strength tensors $F_L^{\mu\nu} = \partial^\mu l^\nu - \partial^\nu l^\mu - i[l^\mu, l^\nu]$ and $F_R^{\mu\nu} = \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu]$, which are associated with the non-Abelian external fields l_μ and r_μ, respectively. The last term in Eq. (4.5) is added to describe the interactions between spin-1 vector resonances and external tensor fields. The three couplings f_V, g_V, and f_V^T are all real and given, respectively, as [54]

$$\begin{aligned} & f_V = \frac{F_V}{M_V} = \frac{\sqrt{2}F_\pi}{M_V}, \qquad g_V = \frac{G_V}{M_V} = \frac{F_\pi}{\sqrt{2}M_V}, \\ & \langle 0|\bar{u}(0)\sigma_{\mu\nu}s(0)|V(p)\rangle = if_V^T(\epsilon_\mu p_\nu - \epsilon_\nu p_\mu),\end{aligned} \tag{4.7}$$

where the first two result from the equivalence of the model **I** and **II** prescriptions for the spin-1 resonances [54], while f_V^T is determined from the one-particle to vacuum matrix element [91].

With Eq. (4.5) in hand, the effective action $S_{\mathbf{II}}$ for a single vector meson exchange can then be written as [54]

$$S_{\mathbf{II}} = \frac{1}{2}\int dxdy\langle J_{\mathbf{II}}^{\mu\nu}(x)\Delta^{\mathbf{II}}_{\mu\nu,\rho\sigma}(x-y)J_{\mathbf{II}}^{\rho\sigma}(y)\rangle, \tag{4.8}$$

where the antisymmetric current $J_{\mathbf{II}}^{\mu\nu}$ and the resonance propagator $\Delta^{\mathbf{II}}_{\mu\nu,\rho\sigma}$ are defined, respectively, by [54]

$$J_{\mathbf{II}}^{\mu\nu} = \frac{1}{2\sqrt{2}}(f_Vf_+^{\mu\nu} + ig_V[u^\mu, u^\nu]) + f_V^Tt_+^{\mu\nu}, \tag{4.9}$$

$$\begin{aligned}\Delta^{\mathbf{II}}_{\mu\nu,\rho\sigma}(x-y) = {} & \int\frac{d^4k}{(2\pi)^4}\frac{e^{-ik\cdot(x-y)}}{k^2 - M_V^2 + i\epsilon} \\ & \times[g_{\mu\rho}k_\nu k_\sigma - g_{\mu\sigma}k_\nu k_\rho - (\mu\leftrightarrow\nu)].\end{aligned} \tag{4.10}$$

From the effective action $S_{\mathbf{II}}$ given by Eq. (4.8), one can easily derive the resonance contribution to the $K\pi$ tensor form factor due to the exchange of the lightest vector resonance $K^*(892)$ [25,52]. After including also the

FIG. 2. Feynman diagrams contributing to $F_T(s)$ at the lowest chiral order of χPT (left) and from the lightest vector resonance [$K^*(892)$] exchange in the context of RχT (right). The crossed circles denote the insertion of a tensor current, and the blob represents the interaction vertex of a vector resonance (double line) with two pseudoscalar mesons (dashed line).

lowest-order χPT contribution given by Eq. (4.4), one then obtains the $K\pi$ tensor form factor

$$F_T(s) = \frac{\Lambda_2}{F_\pi^2}\left[1 + \frac{\sqrt{2}f_V^T g_V}{\Lambda_2}\frac{s}{M_{K^*}^2 - s}\right], \qquad (4.11)$$

with the corresponding Feynman diagrams in the context of χPT and RχT depicted in Fig. 2.

In the large-N_C limit within RχT, although an infinite tower of resonances with the same quantum numbers as that of $K^*(892)$ should in principle be included in the computation of resonance-exchange amplitudes, it turns out that only the second state $K^*(1410)$ will actually play a crucial role in the inelastic region [92]. Accordingly, Eq. (4.11) should be changed to

$$F_T(s) = \frac{\Lambda_2}{F_\pi^2}\left[1 + \frac{\sqrt{2}f_V^T g_V}{\Lambda_2}\frac{s}{M_{K^*}^2 - s} + \frac{\sqrt{2}f_V^{T\prime} g_V^\prime}{\Lambda_2}\frac{s}{M_{K^{*\prime}}^2 - s}\right], \qquad (4.12)$$

where $g_V^\prime$ and $f_V^{T\prime}$ are the counterparts of the corresponding unprimed couplings introduced in Eq. (4.5). We require further that $F_T(s)$ decrease at least as $1/s$ when $s \to \infty$ [81–83], resulting therefore in the short-distance constraint

$$f_V^T g_V + f_V^{T\prime} g_V^\prime = \frac{\Lambda_2}{\sqrt{2}}. \qquad (4.13)$$

As the resonance exchange amplitudes are dominated by the first pole, one can then determine Λ_2 from Eq. (4.13) in terms of the known values of f_V^T and g_V. Taking as inputs $F_\pi = 92.3$ MeV and $M_V = 770$ MeV, we get $\Lambda_2 \simeq \sqrt{2}f_V^T g_V \simeq \sqrt{2}F_\pi^2/M_V \simeq 15.6$ MeV, which is compatible with the lattice result $\Lambda_2 = 11.1 \pm 0.4$ MeV [88].

Analogous to the case for the vector form factor with the same two resonances included [37], the tensor form factor given by Eq. (4.12) can also be rewritten as

$$F_T(s) = \frac{\Lambda_2}{F_\pi^2}\left[\frac{M_{K^*}^2 + \beta s}{M_{K^*}^2 - s} - \frac{\beta s}{M_{K^{*\prime}}^2 - s}\right], \qquad (4.14)$$

where the mixing parameter, $\beta = -\sqrt{2}f_V^{T\prime} g_V^\prime/\Lambda_2 = \sqrt{2}f_V^T g_V/\Lambda_2 - 1$, is introduced to characterize the relative weight of the two resonances, and plays the same role as the parameter γ does for the vector form factor [37]. Although the parameter β cannot be determined directly from data for the moment, we can estimate it from the fitted value of γ with a three-times-subtracted dispersion representation of the vector form factor [37,38,42]. To this end, one needs to first find out the relation between the RχT couplings $f_V^{T\prime}$ and $f_V^\prime \simeq \sqrt{2}F_V^\prime$. The large-$N_C$ asymptotic analysis of $\langle VV\rangle$, $\langle TT\rangle$, and $\langle VT\rangle$ correlators suggests that a pattern with possible alternation in sign,

$$\xi_n = \frac{f_{Vn}^T}{f_{Vn}} = (-1)^n\frac{1}{\sqrt{2}}, \qquad (4.15)$$

exists for the whole $J^{PC} = 1^{--}$ excited states [91]. While ξ_{K^*} is now confirmed to be positive [93–97], the sign of $\xi_{K^{*\prime}}$ cannot be determined yet. Keeping both of these two possibilities, one can then derive the relation[10]

$$\frac{\beta}{\gamma} = \frac{-\sqrt{2}f_V^{T\prime} g_V^\prime/\Lambda_2}{-F_V^\prime G_V^\prime/F_\pi^2} = (-1)^n\frac{\sqrt{2}F_\pi^2}{M_{V^\prime}\Lambda_2}, \qquad (4.16)$$

where $M_{V^\prime} \simeq M_{\rho^\prime} = 1440$ MeV in the limit of $SU(3)$ flavor symmetry. Thus, together with $\Lambda_2 = 11.1$ MeV and $F_\pi = 92.3$ MeV, one can express the parameter β in terms of γ via

$$\beta \simeq \pm 0.75\gamma, \qquad (4.17)$$

where both positive and negative signs of $\xi_{K^{*\prime}}$ will be considered in this paper. Intriguingly, our estimate, $\beta \simeq -0.75\gamma$, gives also a support for the assumption made in Ref. [24] that the inelastic contributions to the phases of vector and tensor form factors are of similar size but potentially opposite in sign.

As in the case for the vector form factor [35,37], the denominator in Eq. (4.14) should be modified by including the energy-dependent width $\gamma_n(s)$ (proportional to the imaginary part of the one-loop contribution in the context of χPT [35,48,49]) and also by shifting the pole mass (due to the real part of the loop contribution) of the resonances, as required by analyticity [52]. After factoring out the normalization $F_T(0)$ at $s = 0$, one arrives at the reduced tensor form factor $\tilde{F}_T(s) \equiv F_T(s)/F_T(0)$, which is now given explicitly as

$$\tilde{F}_T(s) = \frac{m_{K^*}^2 - \kappa_{K^*}\tilde{H}_{K\pi}(0) + \beta s}{D(m_{K^*}, \gamma_{K^*})} - \frac{\beta s}{D(m_{K^{*\prime}}, \gamma_{K^{*\prime}})}, \qquad (4.18)$$

[10]Here we have used the relation $g_V^\prime = G_V^\prime/M_{V^\prime}$, which also results from the equivalence of models **I** and **II** for the spin-1 degrees of freedom [54].

with the normalization $F_T(0)$ and the denominator $D(m_n, \gamma_n)$ defined, respectively, by

$$F_T(0) = \frac{\Lambda_2}{F_\pi^2} \frac{m_{K^*}^2}{m_{K^*}^2 - \kappa_{K^*} \tilde{H}_{K\pi}(0)}, \tag{4.19}$$

$$D(m_n, \gamma_n) \equiv m_n^2 - s - \kappa_n \Re e \tilde{H}_{K\pi}(s) - i m_n \gamma_n(s). \tag{4.20}$$

Here the parameters m_n and γ_n denote the unphysical mass and width, respectively, to be distinguished from the physical mass M_n and width Γ_n that are obtained from the pole position in the complex s plane [35,37]. Explicit expressions for the one-loop function $\tilde{H}_{K\pi}(s)$, the energy-dependent width $\gamma_n(s)$, and the dimensional constant κ_n can be found in Refs. [35,37].

Our result of $\tilde{F}_T(s)$ given by Eq. (4.18) is quite similar to that of the reduced vector form factor $\tilde{F}_+(s)$ obtained in Ref. [37], except for the different normalization factors, as well as the different relative weight parameters introduced to characterize the inelastic contributions. In the elastic region below roughly 1.2 GeV [37], one needs to set $\beta = \gamma = 0$ and hence obtains $\tilde{F}_T(s) = \tilde{F}_+(s)$, which then implies that $\delta_T(s) = \delta_+(s)$, as required by the unitarity relation and the fact that the $K^*(892)$ resonance is described equivalently by a vector or an antisymmetric tensor field [53,54]. Furthermore, according to Watson's theorem [30], the phases of both $F_+(s)$ and $F_T(s)$ in the elastic region should coincide with the P-wave $K\pi$ phase shift $\delta_1^{1/2}(s)$. In such a case, no direct CP asymmetry in $\tau \to K_S \pi \nu_\tau$ decays will be predicted due to the lack of strong phase differences between vector and tensor form factors [24]. Beyond the elastic region, however, a nonzero strong phase difference can be generated due to the different relative weight parameters in these two form factors, as will be shown in the next section.

3. Dispersive representation of the tensor form factor

In order to connect all the information on the form factors inferred from χPT at low energies, from the resonance dynamics in the intermediate energy region [$\mathcal{O}(1\ \text{GeV})$], as well as from the short-distance QCD properties in the asymptotic regime [81–83], one can resort to the dispersive representation of the form factors, which fulfills the analyticity and unitarity requirements [98–100] and, at the same time, suppresses the less-known higher-energy contributions [37,38].

In the elastic region below roughly 1.2 GeV, the dispersion relation for the vector and tensor form factors admits the well-known Omnès solution [24]

$$F_{+,ela}(s) = F_+(0)\Omega(s), \quad F_{T,ela}(s) = F_T(0)\Omega(s), \tag{4.21}$$

with the Omnès factor [101] given by

$$\Omega(s) = \exp\left[\frac{s}{\pi}\int_{s_{K\pi}}^{\infty} ds' \frac{\delta_1^{1/2}(s')}{s'(s' - s - i\epsilon)}\right], \tag{4.22}$$

where the relation $\delta_T(s) = \delta_+(s) = \delta_1^{1/2}(s)$ in the elastic region has been used. As Watson's final-state interaction theorem is no longer valid starting from the threshold of inelastic states (most notably $K\pi\pi$ [24]), one must find a sensible way to determine the strong phase difference in the inelastic region, so as to predict a nonzero direct CP asymmetry in $\tau \to K_S \pi \nu_\tau$ decays. In this regard, our expression of $\tilde{F}_T(s)$ given by Eq. (4.18) and that of $\tilde{F}_+(s)$ given by Eq. (4.1) in Ref. [37] are advantageous, because they remain valid even beyond the elastic approximation, and the two form-factor phases can be calculated from the relations [35,37,42]

$$\tan\delta_T(s) = \frac{\Im m[\tilde{F}_T(s)]}{\Re e[\tilde{F}_T(s)]}, \quad \tan\delta_+(s) = \frac{\Im m[\tilde{F}_+(s)]}{\Re e[\tilde{F}_+(s)]}, \tag{4.23}$$

in which the inelastic effects are indicated by the mixing parameters β and γ, respectively.

It should be noted that the form-factor phases given by Eq. (4.23) are valid only in the τ-decay region $s_{K\pi} < s < m_\tau^2$. For the higher-energy region, these phases become generally unknown, but should be guided smoothly to π (modulo 2π) according to the asymptotic behavior of the form factors at large s [81–83]. Our ignorance of the form-factor phases at relatively higher energies also makes the numerical implementation of the dispersive integrals sensitive to the choice of the cutoff s_{cut}. For the vector form factor, once the three-times-subtracted dispersion representation [see Eqs. (4.1)] is adopted, the impact of this deficiency would be marginal, implying that the higher-energy contribution is well suppressed [37,38,42]. For example, an input with a larger error band, $\delta_+(s) = \pi \pm \pi$, at $s \geq s_{\text{cut}}$ has been assumed in Ref. [80], but the use of a three-times-subtracted dispersion relation makes the integrand converge rapidly, and hence the result becomes almost insensitive to this large error assignment. Furthermore, the stability of the fit results has been checked explicitly by varying the cutoff s_{cut} in a wide range $m_\tau < \sqrt{s_{\text{cut}}} < \infty$ [37,38]. It has also been demonstrated that the choice $s_{\text{cut}} = 4\ \text{GeV}^2$ is preferred, because such a cutoff, on the one hand, is large enough not to spoil the *priori* infinite interval of the dispersive integral and to avoid the spurious singularity effect generated at $s = s_{\text{cut}}$ and, on the other hand, is low enough to give a good description of the form-factor phase within the interval considered [102]. Due to the lack of precise low-energy information on the tensor interaction, however, one cannot apply these strategies to the tensor form factor [25,52]. Consequently, we shall simply use the once-subtracted dispersive representation [24,25,52]

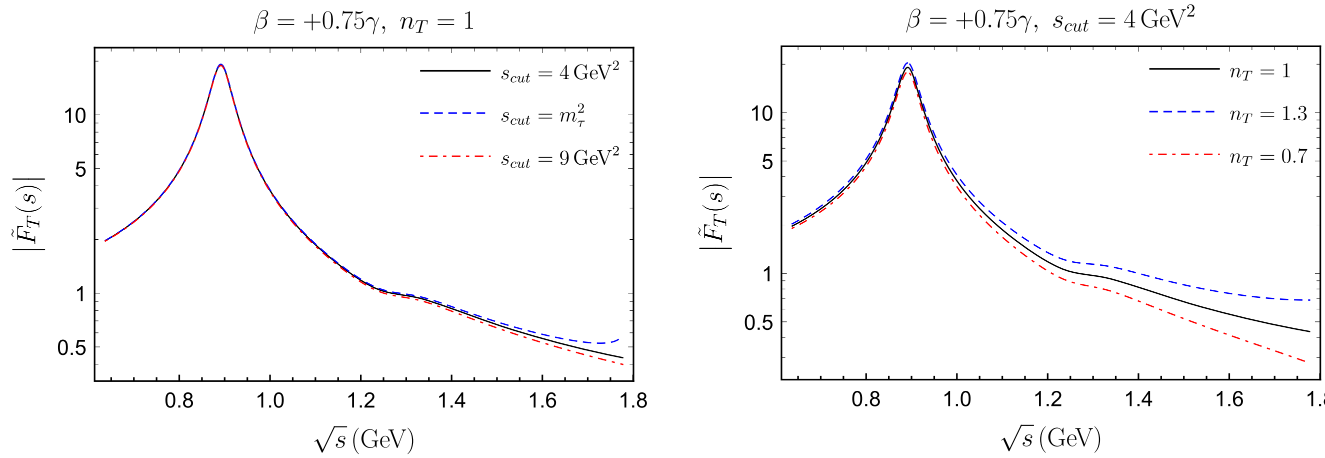

FIG. 3. Dependence of the modulus of the normalized tensor form factor on s_{cut} with fixed $n_T = 1$ (left) and on n_T with fixed $s_{\mathrm{cut}} = 4\ \mathrm{GeV}^2$ (right), in the $\beta = +0.75\gamma$ case.

$$\tilde{F}_T(s) = \exp\left\{\frac{s}{\pi}\int_{s_{K\pi}}^{\infty} ds' \frac{\delta_T(s')}{s'(s' - s - i\epsilon)}\right\}, \tag{4.24}$$

together with the following simple model for the phase $\delta_T(s)$ [103]:

$$\delta_T(s) = \begin{cases} \arctan[\frac{\Im m\tilde{F}_T(s)}{\Re e\tilde{F}_T(s)}], & s_{K\pi} < s < s_{\mathrm{cut}} \\ n_T\pi, & s \geq s_{\mathrm{cut}} \end{cases}, \tag{4.25}$$

where the phase is now made explicit even in the inelastic region, instead of the assumed relation $\delta_T(s) = -\delta_+(s)$ in the same region [24]. We have also introduced the quantity n_T, with its deviation from unity, to account for our estimate of the uncertainty resulting from the higher-energy contributions. In addition, the default choice with $s_{\mathrm{cut}} = 4\ \mathrm{GeV}^2$ and $\delta_T(s) = \pi$ for $s > s_{\mathrm{cut}}$ will be assumed in our numerical analysis.

To estimate the systematic uncertainty associated with our model for the tensor form factor, we proceed as follows[11]: by fixing $n_T = 1$ to see the sensitivity of the modulus of the normalized tensor form factor with respect to s_{cut}, with the three choices $s_{\mathrm{cut}} = m_\tau^2$, 4, and 9 GeV^2, and by fixing $s_{\mathrm{cut}} = 4\ \mathrm{GeV}^2$ to see the sensitivity with respect to n_T, with the three choices $n_T = 1$, 1.3, and 0.7. Our numerical results with $\beta = +0.75\gamma$ (the case with $\beta = -0.75\gamma$ is quite similar) are shown in Fig. 3, from which it can be seen that, in our model, the modulus of the normalized tensor form factor is almost insensitive to the choice of the cutoff s_{cut} when fixing $n_T = 1$, while it becomes rather sensitive to the choice of n_T when fixing $s_{\mathrm{cut}} = 4\ \mathrm{GeV}^2$, especially in the higher-energy region. This implies that the once-subtracted dispersive representation is not optimal, as is generally expected. But the lack of data sensitive to the tensor form factor makes it impossible to increase the number of subtractions for the moment.

To see the behaviors of the vector and tensor form factors both in the elastic and in the inelastic region, we show their moduli and phases as well as the ones predicted by the Omnès factor $\Omega(s)$ in Figs. 4 and 5, corresponding, respectively, to the two different choices given by Eq. (4.17). As the cutoff s_{cut} has been fixed at $s_{\mathrm{cut}} = 4\ \mathrm{GeV}^2$, we consider the uncertainties of the form-factor phases only from the input parameters. From Figs. 4 and 5, one can see that both the moduli and the phases of the normalized form factors are consistent with the ones obtained from $\Omega(s)$ in the energy region up to about 1.2 GeV, which is roughly the threshold of the inelastic region. The deviations from the ones predicted by $\Omega(s)$ in the higher-energy region, on the other hand, serve as an indication of the size of the inelastic contribution from the second resonance [24]. It is also observed that, unlike in the case of the vector form factor, the modulus of the tensor form factor is almost unaffected by the inelastic effect, and is therefore similar to that obtained with $\Omega(s)$. The inelastic effects on the form-factor phases are, however, rather significant, and a strong phase difference in the inelastic region is indeed obtained, especially in the $\beta = -0.75\gamma$ case. This is welcome for resolving the CP anomaly observed in $\tau \to K_S\pi\nu_\tau$ decays, as will be discussed in the next section.

V. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we discuss the numerical effects of the two NP scenarios introduced in Sec. III on the branching ratio $\mathcal{B}(\tau^- \to K_S\pi^-\nu_\tau)$ and the CP asymmetry $A_{CP}(\tau \to K_S\pi\nu_\tau)$. For each observable, the theoretical uncertainties are obtained by varying each input parameter within the corresponding range and then adding the individual errors in quadrature [104–107].

[11]These results are shown only for the purpose of making a comparison with that given in Ref. [25], and will not be considered in the subsequent numerical analysis.

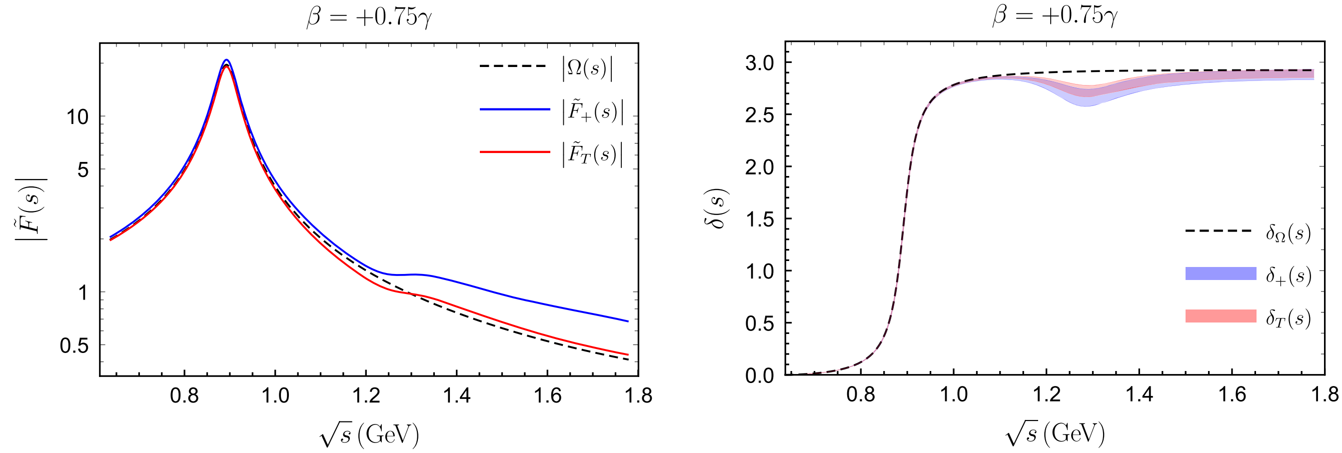

FIG. 4. Energy dependence of the moduli (left) and phases (right) of the normalized form factors, compared with the ones predicted by the Omnès factor $\Omega(s)$, with the bands resulting from the uncertainties of the input parameters, in the $\beta = +0.75\gamma$ case.

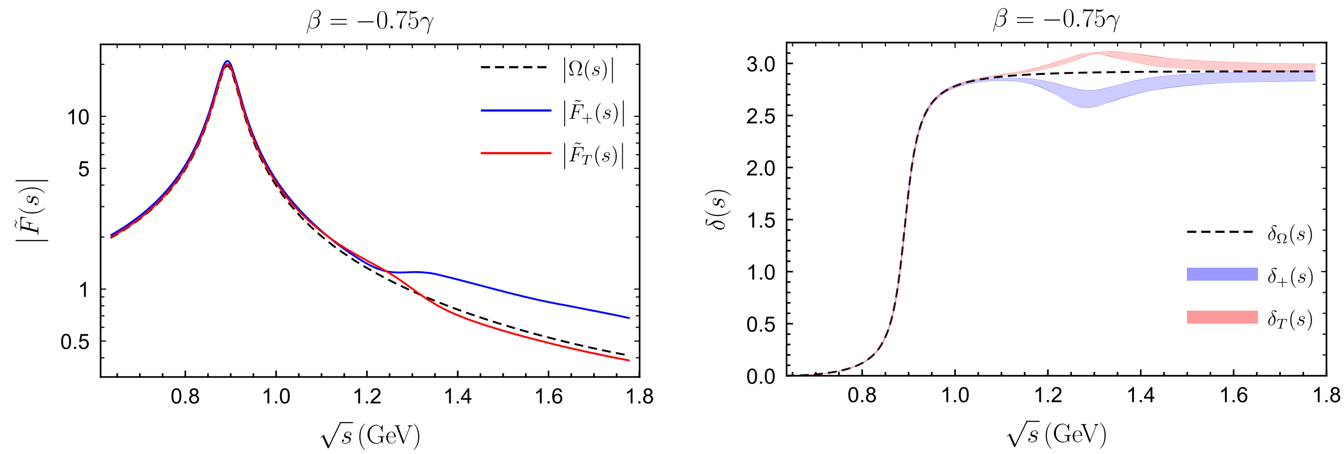

FIG. 5. Same as in Fig. 4, but in the $\beta = -0.75\gamma$ case.

A. Results in the model-independent framework

As the $\tau^- \to \bar{K}^0 \pi^- \nu_\tau$ decay width, which is obtained by integrating Eq. (3.2) over the invariant mass squared s within the kinematic regime $s_{K\pi} \le s \le m_\tau^2$, depends on the nonstandard scalar and tensor interactions, it could also set bounds on these effective couplings [21–26]. In order to enhance the sensitivity to these nonstandard interactions, we introduce the observable [25]

$$\Delta \equiv \frac{\Gamma - \Gamma_0}{\Gamma_0} = a \Re e \hat{\epsilon}_S + b \Re e \hat{\epsilon}_T + c \Im m \hat{\epsilon}_T + d |\hat{\epsilon}_S|^2 + e |\hat{\epsilon}_T|^2, \tag{5.1}$$

which is defined as the relative shift induced by these interactions, with Γ and Γ_0 standing for the $\tau \to K_S \pi \nu_\tau$ decay widths with and without these nonstandard contributions, respectively. The coefficients a, b, c, d, and e are calculated to be

$$a \in [0.27, 0.34], \qquad d \in [0.84, 1.12], \tag{5.2}$$

$$b \in [-4.46, -4.02], \qquad c \in [-0.005, 0.015],$$
$$e \in [6.0, 7.4], \quad \text{for } \beta = +0.75\gamma, \tag{5.3}$$

$$b \in [-4.68, -4.24], \qquad c \in [0.026, 0.046],$$
$$e \in [6.8, 8.3], \quad \text{for } \beta = -0.75\gamma. \tag{5.4}$$

It can be seen that our values of the coefficients a and d, characterizing, respectively, the linear and the quadratic terms of the nonstandard scalar contributions, are consistent with those of α and γ obtained in Ref. [25], while the values of the tensor coefficients are quite different due to the different forms of the $K\pi$ tensor form factor used. The numerical difference between scalar (a and d) and tensor (b and e) coefficients by about 1 order of magnitude implies a slightly larger sensitivity to the tensor than to the scalar contribution, as noted already in Ref. [25]. It is also observed that, although the real part of the interference between vector and tensor contributions is of similar magnitude to the pure tensor term, the imaginary part of the interference is almost negligible for both the $\beta = +0.75\gamma$ and $\beta = -0.75\gamma$ cases. This can be understood from the fact that the real and the imaginary part of this interference term are proportional to $\Re e[F_T(s)F_+(s)^*]$ and $\Im m[F_T(s)F_+(s)^*]$ [see Eqs. (3.5) and (3.6)], which in the elastic region are reduced to $\sim|F_T(s)||F_+(s)|$ and ~ 0, respectively, with $|F_T(s)||F_+(s)|$ being of similar size as $|F_T(s)|^2$ [25]. However, since only the imaginary part contributes to the direct CP asymmetry, its nonzero value is crucial in determining the observable $A_{CP}(\tau \to K_S\pi\nu_\tau)$.

For the CP asymmetry $A_{CP}(\tau \to K_S\pi\nu_\tau)$, the following subtle points should be clarified [13,26]. As the signal channel $\tau^- \to \pi^- K_S(\geq 0\pi^0)\nu_\tau$ (C1) is contaminated by the two background channels $\tau^- \to K^- K_S(\geq 0\pi^0)\nu_\tau$ (C2) and $\tau^- \to \pi^- K^0 \bar{K}^0 \nu_\tau$ (C3), the decay-rate asymmetry, $\mathcal{A} = (-0.27 \pm 0.18 \pm 0.08)\%$, measured by the *BABAR* Collaboration [13], is actually related to the signal asymmetry A_1 as well as the two background asymmetries A_2 and A_3 via [13]

$$\mathcal{A} = \frac{f_1 A_1 + f_2 A_2 + f_3 A_3}{f_1 + f_2 + f_3} = \frac{f_1 - f_2}{f_1 + f_2 + f_3} A_Q, \qquad (5.5)$$

where f_1, f_2, and f_3 denote the fractions of the channels C1, C2, and C3, respectively, in the total selected sample, with the corresponding numbers given in Table I of Ref. [13]. Within the SM, $A_1 = -A_2$ because the K_S state is produced via a $\bar{K}^0$ in channel C1 but via a K^0 in channel C2, and $A_3 = 0$ because of the cancellation between the CP asymmetries due to the K^0 and $\bar{K}^0$ states in channel C3. To extract the CP asymmetry A_Q given by Eq. (1.1) from the measured decay-rate asymmetry $\mathcal{A}$, these relations between A_1, A_2, and A_3 have been assumed by the *BABAR* Collaboration [13], as given by the second line in Eq. (5.5). In the presence of NP contributions, however, $A_1 \neq -A_2$ in general, and any theoretical prediction should be, therefore, compared with the measured quantity $\mathcal{A}$, instead of A_Q [26]. Assuming the NP contribution affects only the channel C1, and we can then write the three CP asymmetries as [26]

$$\begin{aligned} A_1 &= A_1^{\rm SM} + A_1^{\rm NP} = A_{CP}^{\rm SM} + A_1^{\rm NP}, \\ A_2 &= A_2^{\rm SM} = -A_1^{\rm SM} = -A_{CP}^{\rm SM}, \\ A_3 &= A_3^{\rm SM} = 0, \end{aligned} \qquad (5.6)$$

where the SM prediction $A_{CP}^{\rm SM} = (0.36 \pm 0.01)\%$ is obtained after taking into account the $K_S \to \pi^+\pi^-$ decay-time dependence of the event selection efficiency [13]. Combining $A_{CP}^{\rm SM}$ with the measured decay-rate asymmetry $\mathcal{A} = (-0.27 \pm 0.18 \pm 0.08)\%$ [13], we can therefore obtain the constraint on $A_1^{\rm NP}$, and then on the nonstandard tensor coupling.

We now apply the observable Δ to put constraints on the nonstandard scalar and tensor interactions. Since the effective couplings $\hat{\epsilon}_S$ and $\hat{\epsilon}_T$ are both considered to be complex, there are four degrees of freedom, $\Re e\hat{\epsilon}_S$, $\Im m\hat{\epsilon}_S$, $\Re e\hat{\epsilon}_T$, and $\Im m\hat{\epsilon}_T$, at our disposal. Combining our prediction for the branching ratio,

$$\mathcal{B}(\tau^- \to K_S\pi^-\nu_\tau)_{\rm SM} = (0.421 \pm 0.022)\%, \qquad (5.7)$$

with the experimental result measured by the Belle Collaboration [108],

$$\begin{aligned} &\mathcal{B}(\tau^- \to K_S\pi^-\nu_\tau)_{\rm Exp} \\ &\quad = (0.416 \pm 0.001({\rm stat.}) \pm 0.008({\rm syst.}))\%, \end{aligned} \qquad (5.8)$$

we obtain the allowed regions, $\Delta \in [-0.07, 0.05]$ and $\Delta \in [-0.12, 0.10]$, by varying both the theoretical and experimental uncertainties at 1σ and 2σ, respectively. To set bounds on one of the couplings $\hat{\epsilon}_S$ and $\hat{\epsilon}_T$, we shall assume the other to be zero, and our final results are shown in Fig. 6 for both the $\beta = +0.75\gamma$ and $\beta = -0.75\gamma$ cases. It can be clearly seen that, under the constraint from the observable Δ, the allowed region of $\hat{\epsilon}_S$ is larger than that of $\hat{\epsilon}_T$, which is consistent with our previous observation that a slightly larger sensitivity to the tensor than to the scalar contribution is preferred by the branching ratio. It is also observed that, while the imaginary parts of the allowed regions of both $\hat{\epsilon}_S$ and $\hat{\epsilon}_T$ are nearly symmetric about the axes, the real parts are not, but with the preference $\Re e\hat{\epsilon}_S < 0$ and $\Re e\hat{\epsilon}_T > 0$.

As only the interference between vector and tensor operators can provide a potential NP explanation of the CP anomaly observed in $\tau \to K_S\pi\nu_\tau$ decays [21,24], we now focus on the tensor coupling $\hat{\epsilon}_T$. To check if the region of $\hat{\epsilon}_T$ allowed by the branching ratio is compatible with that required by the CP asymmetry, we now add the constraint from the measured decay-rate asymmetry $\mathcal{A}$ by the *BABAR* Collaboration [13], and our final results are shown in Fig. 7 for both the $\beta = +0.75\gamma$ and $\beta = -0.75\gamma$ cases. One can see that, in both of these two cases, there exist common regions of the tensor coupling $\hat{\epsilon}_T$ that can accommodate both the branching ratio $\mathcal{B}(\tau^- \to K_S\pi^-\nu_\tau)$ and the CP asymmetry $A_{CP}(\tau \to K_S\pi\nu_\tau)$ simultaneously, even at the 1σ level. It is also observed that the $\beta = -0.75\gamma$ case is even preferred, in which a larger allowed region of $\hat{\epsilon}_T$ is obtained due to the slightly larger phase difference between the vector and tensor form factors, as mentioned already in the last section.

B. Results in the scalar LQ scenario

In the scalar LQ scenario, due to the specific relation $C_S(\mu_\phi) = -4C_T(\mu_\phi)$ at the matching scale $\mu_\phi = M_\phi$, we are actually left with only one effective coupling $\hat{C}_T$, and a more severe constraint on it is therefore expected than in the model-independent case.

Referring to Eq. (3.18), and by fixing $M_\phi = 1$ TeV and $\mu_\tau = m_\tau$, one obtains $\hat{C}_S \simeq -9.84\hat{C}_T$ at the μ_τ scale.

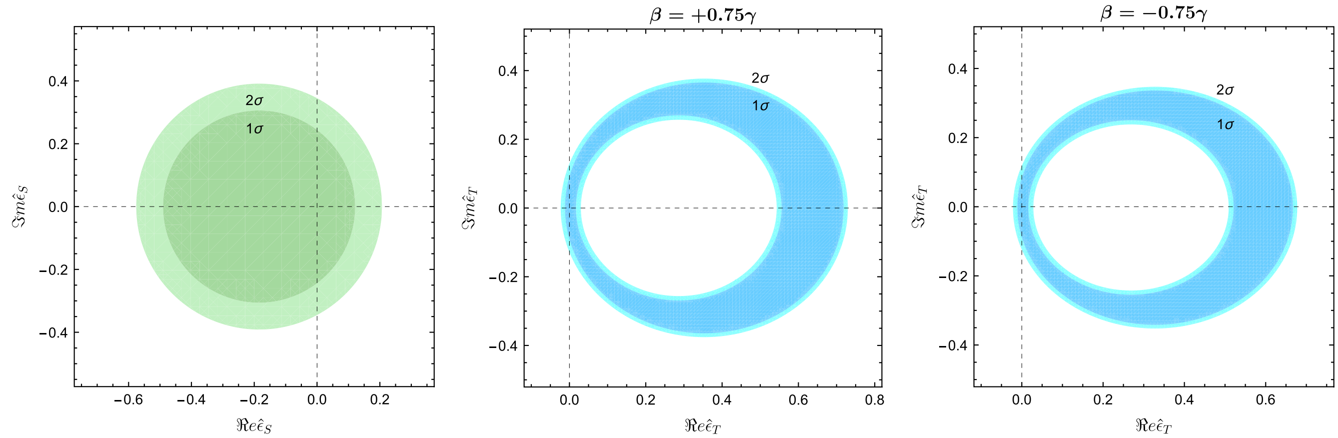

FIG. 6. Constraints on $\hat{\epsilon}_S$ for $\hat{\epsilon}_T = 0$ (left), as well as on $\hat{\epsilon}_T$ for $\hat{\epsilon}_S = 0$ in both the $\beta = +0.75\gamma$ (middle) and $\beta = -0.75\gamma$ (right) cases, from the branching ratio $\mathcal{B}(\tau^- \to K_S \pi^- \nu_\tau)$ by varying it at both 1σ and 2σ intervals.

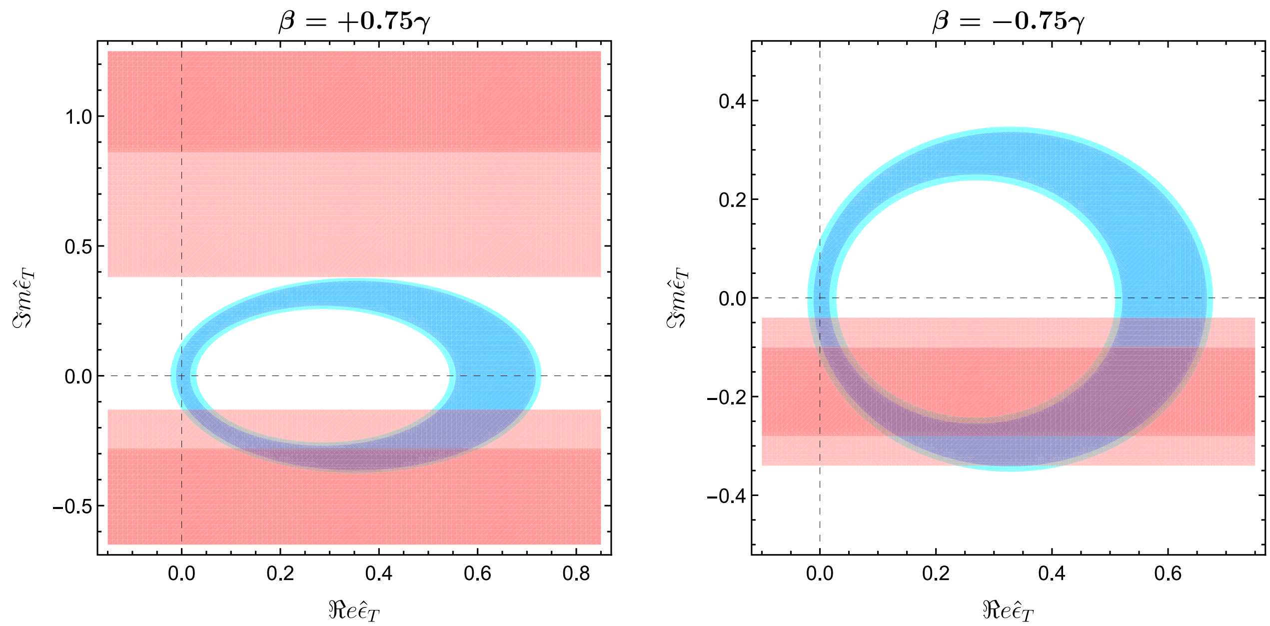

FIG. 7. Constraints on $\hat{\epsilon}_T$ from the branching ratio $\mathcal{B}(\tau^- \to K_S \pi^- \nu_\tau)$ (blue and cyan regions obtained at 1σ and 2σ, respectively) as well as the decay-rate asymmetry $\mathcal{A}$ (red and pink regions at 1σ and 2σ, respectively), in both the $\beta = +0.75\gamma$ (left) and $\beta = -0.75\gamma$ (right) cases.

This implies that the scalar contribution is enhanced relative to that from the tensor operator in such a scenario. Under the constraints from the branching ratio $\mathcal{B}(\tau^- \to K_S \pi^- \nu_\tau)$ and the *CP* asymmetry $A_{CP}(\tau \to K_S \pi \nu_\tau)$, our final allowed regions of $\hat{C}_T$ are shown in Fig. 8. One can see that there is no common region allowed simultaneously by these two observables at the 1σ level, and only a small region is allowed in the $\beta = -0.75\gamma$ case at the 2σ level. This implies that the scalar LQ scenario can hardly account for the observed *CP* anomaly under the constraint from the measured branching ratio, except for the marginal region obtained in the $\beta = -0.75\gamma$ case at the 2σ level.

C. Constraints from other observables and processes

It should be noted that in both of the two scenarios discussed above, we have assumed that the nonstandard scalar and tensor operators contribute only to the $\tau \to K_S \pi \nu_\tau$ decays, and only the branching ratio $\mathcal{B}(\tau^- \to K_S \pi^- \nu_\tau)$ and the *CP* asymmetry $A_{CP}(\tau \to K_S \pi \nu_\tau)$ have been considered to constrain the corresponding effective couplings. We now discuss the constraints on these nonstandard interactions from other observables and processes.

As pointed out already in Refs. [25,109], the $\tau^- \to K_S \pi^- \nu_\tau$ decay spectrum measured by the Belle Collaboration [31] can also provide very complementary constraints on these nonstandard interactions. Under the

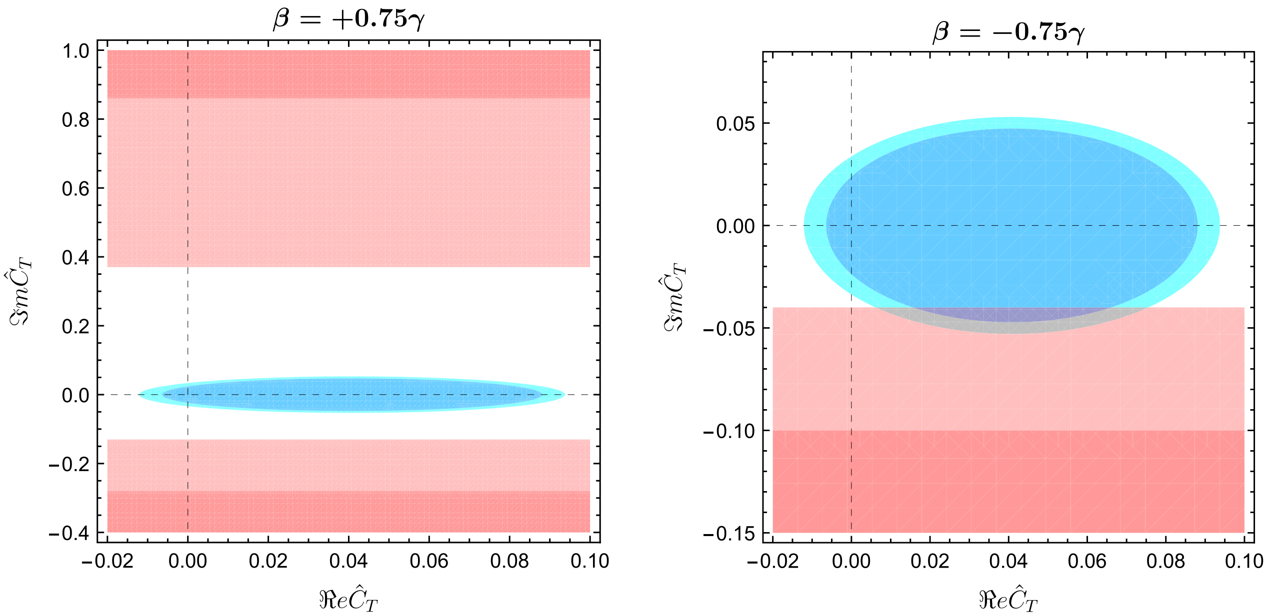

FIG. 8. Constraints on $\hat{C}_T$ in the scalar LQ scenario. The other captions are the same as in Fig. 7.

combined constraints from the branching ratio and the decay spectrum of this decay, the best fit values $\hat{\epsilon}_S = (1.3 \pm 0.9) \times 10^{-2}$ and $\hat{\epsilon}_T = (0.7 \pm 1.0) \times 10^{-2}$ have been obtained in Ref. [25]. Assuming that $\Re e\hat{\epsilon}_{S(T)} \sim \Im m\hat{\epsilon}_{S(T)} \sim \hat{\epsilon}_{S(T)}$,[12] one can see that these values are smaller by at least 1 order of magnitude than our results obtained under the constraint from only the branching ratio (see Fig. 6). This implies that, once the combined constraints from the branching ratio and the decay spectrum are taken into account, the allowed value of the tensor coupling $\hat{\epsilon}_T$ will be insufficient to explain the *CP* anomaly, which demands that $\Im m\hat{\epsilon}_T$ shoxauld be of the order $\mathcal{O}(10^{-1})$ at least (see Figs. 7 and 8) [24].

If the lepton-flavor university (LFU) is further assumed,[13] the effective operators given by Eq. (3.1), but with the τ lepton replaced by the electron and muon flavors, would also contribute to other strangeness-changing processes. In this case, our bounds on the nonstandard scalar and tensor couplings would be not competitive with that obtained from the (semi)leptonic kaon [56] and hyperon [69] decays, due to the larger systematic theory uncertainty inherent to the current framework for hadronic τ decays, especially in the inelastic region. For example, the global fit results, $\hat{\epsilon}_S = (-3.9 \pm 4.9) \times 10^{-4}$ and $\hat{\epsilon}_T = (0.5 \pm 5.2) \times 10^{-3}$, from the (semi)leptonic kaon decays [56], are already much stronger than our bounds shown in Figs. 7 and 8.

It is also noted that, unless some NP between the electroweak and the low-energy scale is assumed, the effective Lagrangian specified by Eq. (3.1) comes generally from an $SU(2)$-invariant form [70–72]. Thus, the demand of $SU(2)$ invariance of the weak interactions naturally relates the tensor operator relevant for $\tau \to K_S\pi\nu_\tau$ to the neutral-current tensor operator relevant for the neutron EDM and the D-$\bar{D}$ mixing, as pointed out already in Ref. [24]. This also brings the tensor coupling required by the *CP* asymmetry $A_{CP}(\tau \to K_S\pi\nu_\tau)$ to be already in conflict with the bounds from these two observables, leading to the claim that it is extremely difficult to explain the *CP* anomaly in terms of ultraviolet complete NP scenarios [24].

Based on the above observations, we conclude therefore that it is quite difficult to explain the *CP* anomaly within the two frameworks considered here, as claimed already in Refs. [24,25].

VI. CONCLUSION

In this paper, motivated by the 2.8σ discrepancy observed between the *BABAR* measurement and the SM prediction of the *CP* asymmetry in $\tau \to K_S\pi\nu_\tau$ decays, as well as the prospects of future measurements at Belle II, we have studied this observable within the model-independent low-energy effective theory framework and in the scalar LQ scenario, both of which contain a nonstandard tensor operator that is necessary to produce a nonvanishing direct *CP* asymmetry in the decays considered. Our main conclusions are summarized as follows:

[12]The couplings $\hat{\epsilon}_S$ and $\hat{\epsilon}_T$ are assumed to be real in Ref. [25]. As the modulus of the tensor form factor is almost unaffected by the inelastic effect and is quite consistent with that obtained with the Omnès factor $\Omega(s)$ (see Figs. 4 and 5), similar numerical results as that obtained in Ref. [25] are expected, even though a different phase, $\delta_T(s) = \delta_+(s)$ in Ref. [25] versus Eq. (4.25) in this work, has been adopted for the tensor form factor.

[13]Although the LFU is hinted to be violated by the current data on B-meson decays (see Refs. [110–113] and references therein for recent reviews), there exists up to now no compelling evidence for its violation in the strangeness sector [16]. Actually, the charged-current anomalies observed in semileptonic B decays become already the least compelling hints for the LFU violation by the latest Belle data [114].

(1) By employing the reciprocal basis, which is found to be most convenient when a K_S or K_L is involved in the final state, we have reproduced the known *CP* asymmetry due to K^0-$\bar{K}^0$ mixing, as predicted first by Bigi and Sanda [17] but with a sign mistake, and then corrected by Grossman and Nir [18].

(2) As the $K\pi$ tensor form factor plays a crucial role in generating a nonzero direct *CP* asymmetry that can arise only from the interference of vector and tensor operators, we have presented a new calculation of this form factor in the context of χPT with tensor sources and RχT with both $K^*(892)$ and $K^*(1410)$ included. For these spin-1 vector resonances, we have used the more conventional vector representation instead of the description based on antisymmetric tensor fields. A once-subtracted dispersive representation of this form factor has also been presented, which naturally fulfills the requirements of unitarity and analyticity. Furthermore, our estimate of the relation between the two weight parameters, $\beta \simeq -0.75\gamma$, gives a support for the assumption made in Ref. [24] that the inelastic contributions to the phases of vector and tensor form factors are of similar size but potentially opposite in sign.

(3) Adopting the three-times-subtracted (for the vector form factor) and the coupled-channel (for the scalar form factor) dispersive representations, together with our result of the tensor form factor, we have performed a detailed analysis of the $\tau \to K_S \pi \nu_\tau$ decays within the two scenarios mentioned above. It is observed that the *CP* anomaly can be accommodated in the model-independent framework, even at the 1σ level, together with the constraint from the branching ratio of $\tau^- \to K_S \pi^- \nu_\tau$ decay. In the LQ scenario, however, this anomaly can be marginally reconciled only at the 2σ level, due to the specific relation between the scalar and tensor operators. Once the combined constraints from the branching ratio and the decay spectrum of this decay are taken into account, these two possibilities are, however, both excluded, even without exploiting further the stronger bounds from the (semi)leptonic kaon decays [56] under the assumption of lepton-flavor universality, as well as from the neutron EDM and D-$\bar{D}$ mixing under the assumption of $SU(2)$ invariance of the weak interactions [24]. It is therefore difficult to explain such a *CP* anomaly within the frameworks considered here.

As both the theoretical predictions and the experimental measurements are still plagued by large uncertainties, more refined studies, especially the information on the $K\pi$ tensor form factor in the inelastic region as well as the dedicated measurements of $\tau \to K_S \pi \nu_\tau$ decays from the Belle II Collaboration [20], are expected.

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APPENDIX: INPUT PARAMETERS

In this Appendix, for convenience, we collect in Table I all the input parameters used throughout this paper. For further details, the readers are referred to the original references.

TABLE I. Summary of the input parameters used throughout this paper.

QCD and electroweak parameters [16]

$G_F[10^{-5}\ \mathrm{GeV}^{-2}]$	$\alpha_s(M_Z)$	m_t[GeV]	m_b[GeV]	F_π[MeV]	F_K[MeV]
1.1663787(6)	0.1181(11)	173.1	4.18	92.3(1)	$1.198F_\pi$

Particle masses and τ lifetime [16]

m_τ[MeV]	M_{K^0}[MeV]	M_{π^-}[MeV]	$\tau_\tau[10^{-15}\ \mathrm{s}]$
1776.86	497.61	139.57	290.3

Parameters in the $K\pi$ vector form factor with $s_{\mathrm{cut}} = 4\ \mathrm{GeV}^2$ [37]

m_{K^*}[MeV]	γ_{K^*}[MeV]	$m_{K^{*\prime}}$[MeV]	$\gamma_{K^{*\prime}}$[MeV]	γ
943.41 ± 0.59	66.72 ± 0.87	1374 ± 45	240 ± 131	-0.039 ± 0.020
M_{K^*}[MeV]		λ'_+		λ''_+
892.01 ± 0.92		$(24.66 \pm 0.77) \times 10^{-3}$		$(11.99 \pm 0.20) \times 10^{-4}$

CP-violating parameters as well as the measured decay-rate asymmetry

$\lvert\eta_{+-}\rvert \times 10^3$ [16]	ϕ_{+-} [16]	$\Re e(\epsilon) \times 10^3$ [16]	A_{CP}^{SM} [13]	$\mathcal{A}$ [13]
2.232 ± 0.011	$(43.51 \pm 0.05)°$	1.66 ± 0.02	$(0.36 \pm 0.01)\%$	$(-0.27 \pm 0.18 \pm 0.08)\%$

Other input parameters

$M_{V'}$[MeV] [39]	Λ_2[MeV] [88]	S_{EW} [75]	$\lvert V_{us}F_+(0)\rvert$ [74]	$\mathcal{B}(\tau^- \to K_S\pi^-\nu_\tau)$ [108]
1440	11.1(4)	1.0201(3)	0.21654(41)	$(0.416 \pm 0.001 \pm 0.008)\%$

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