First law of entanglement entropy in flat-space holography

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According to flat/Bondi-Metzner-Sachs invariant field theories (BMSFT) correspondence, asymptotically flat spacetimes in $(d+1)$ dimensions are dual to d-dimensional BMSFTs. In this duality, similar to the Ryu-Takayanagi proposal in the AdS/CFT correspondence, the entanglement entropy of subsystems in the field theory side is given by the area of some particular surfaces in the gravity side. In this paper we find the holographic counterpart of the first law of entanglement entropy (FLEE) in a two-dimensional BMSFT. We show that FLEE for the BMSFT perturbed states, which are descried by three-dimensional flat-space cosmology, corresponds to the integral of a particular one-form on a closed curve. This curve consists of a BMSFT interval and also null and spacelike geodesics in the bulk gravitational theory. The exterior derivative of this form is 0 when it is calculated for the flat-space cosmology. However, for a generic perturbation of three-dimensional global Minkowski spacetime, the exterior derivative of the one-form yields the Einstein equation. This is the first step for constructing bulk geometry by using FLEE in the flat/BMSFT correspondence.

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I. INTRODUCTION

Flat/Bondi-Metzner-Sachs invariant field theories (BMSFT) is an extension of AdS/CFT correspondence to non-anti–de Sitter (AdS) geometries. According to this duality quantum gravity in the asymptotically flat spacetimes in $(d+1)$ dimensions can be described by a d-dimensional field theory that is Bondi-Metzner-Sachs (BMS) invariant [1,2]. In the gravity side, BMS symmetry is the asymptotic symmetry of asymptotically flat spacetimes at null infinity [3,4]. In the field theory side, the global part of BMS algebra is given by ultrarelativistic contraction of conformal algebra. Thus one can interpret the flat-space limit (zero cosmological constant limit) in the gravity side as the ultrarelativistic limit of CFT in the boundary theory [2]. In this view, one can study flat/BMSFT by starting from AdS/CFT and taking a limit, the flat-space limit in the bulk and the ultrarelativistic limit in the boundary.

BMS symmetry as the asymptotic symmetry is infinite dimensional in three and four dimensions [5–7]. Hence one may expect to find some universal aspects for two- and three-dimensional BMSFTs. This situation is very similar to the two-dimensional conformal field theories (CFTs); their infinite-dimensional symmetry is used to predict the structure of correlation functions as well as entanglement entropy of subsystems. Similarly, the entanglement entropy formula for some particular intervals in BMSFT_2 has been introduced in [8] by just using the infinite symmetry of two-dimensional BMSFTs and then studied more carefully in [9–15].

In the context of AdS/CFT correspondence, the entanglement entropy of CFT subsystems has a holographic description. According to the Ryu-Takayanagi proposal, this entropy is proportional to the area of a bulk surface that has the minimum area among the surfaces connected to the boundary subsystem [16,17]. A similar proposal for the BMSFT entanglement entropy has been introduced in [12]. Accordingly, the BMSFT entanglement entropy can be given by the area of particular surfaces. These surfaces are not connected directly to the boundary of the subsystem but there are null rays that connect them to null infinity where the subsystem is supposed to live. The corresponding surface and null rays, and the subsystem together, construct a closed surface.

Another interesting problem that was studied in the context of AdS/CFT is the holographic description of the first law of the entanglement entropy (FLEE). It was shown in [18,19] that writing both sides of the FLEE in terms of corresponding bulk parameters finally yields linearized Einstein equations. In other words, the FLEE as a constraint in the boundary theory reduces to a constraint on the bulk geometry that is exactly the Einstein equation. If this connection is an intrinsic property of gauge/gravity

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dualities, one can use entanglement entropy and its first law in an arbitrary field theory to find a dual gravitational geometry.

In this paper we study the proposal of [18,19] in the context of $\mathrm{flat}_3/\mathrm{BMSFT}_2$ correspondence. We start from FLEE and use flat/BMSFT correspondence to write it in terms of components of the asymptotically flat bulk metric. We focus on the BMSFT states; their gravitational dual is flat-space cosmology (FSC) [20–23]. It is shown that both sides of the FLEE formula can be written in terms of the integral of a one-form over curves consisting of BMSFT interval and the null and the spacelike geodesics introduced in [12]. These curves construct a closed curve; thus one can use Stokes's theorem to write integrals as the integral of the external derivative of the one-form over the surface bounded by the curves. For the metric of the flat-space cosmology, the exterior derivative of this form is 0. For a generic metric that satisfies the BMS boundary condition (see, e.g., [24]), the exterior derivative of the one-form results in the Einstein equation. Our work is not only the first-step generalization of the proposal of [18,19] for the flat-space holography but also shows that the flat/BMSFT correspondence studied in several previous works (see references in [25]) is a worthwhile duality.

In Sec. II we review the proposal of [19] in the context of AdS/CFT. In Sec. III after briefly reviewing the flat/BMSFT correspondence and holographic description of BMSFT entanglement entropy, we write the FLEE in terms of the bulk metric and deduce the Einstein equation.

II. LINEAR BULK EQUATION FROM THE FLEE IN ADS/CFT

A. Entanglement entropy and its first law

For a quantum field theory state $|\psi\rangle$, the density matrix is

$$\rho = |\psi\rangle\langle\psi|. \tag{2.1}$$

If we decompose a spatial (time constant) slice Σ to two subsystems B and $\bar{B}$ ($\Sigma = B \cup \bar{B}$), then the density matrix associated to B can be obtained from ρ by tracing out the degrees of freedom of the complement subsystem $\bar{B}$ as

$$\rho_B = \mathrm{tr}_{\bar{B}}\rho. \tag{2.2}$$

The entanglement entropy of subsystems B is the von Neumann entropy associated to the density matrix ρ_B,

$$S_B = -\mathrm{tr}(\rho_B \ln \rho_B). \tag{2.3}$$

For a small perturbation $|\psi(\varepsilon)\rangle$ to the initial state $|\psi(0)\rangle$ of the whole system, the FLEE is

$$\delta S_B = \frac{d}{d\varepsilon} S_B = \frac{d}{d\varepsilon}\langle H_B\rangle = \frac{d}{d\varepsilon}\mathrm{tr}(H_B\rho_B) \equiv \delta E_B, \tag{2.4}$$

where H_B is the modular Hamiltonian that is independent of perturbation and defined through

$$H_B = -\ln \rho_B(\epsilon = 0). \tag{2.5}$$

Formula (2.4) is a quantum generalization of the first law of thermodynamics. This formula holds for any arbitrary small perturbation of quantum state and for any subsystem B.

Mostly, it is difficult to compute the modular Hamiltonian H_B and its associated density matrix ρ_B. However, for the cases that H_B is a local operator, one may find a unitary transformation (and hence reversible, which acts also on the coordinates) that maps ρ_B to a thermal density matrix. Hence the resultant entropy is a thermal one (see [26]). If we denote the unitary transformation by U and the final thermal density matrix by $\rho_{\mathcal{H}}$, then

$$\rho_B = U\rho_{\mathcal{H}}U^{-1}. \tag{2.6}$$

It is not difficult to check that the thermal entropy given by

$$S_{TH} = -\mathrm{tr}(\rho_{\mathcal{H}} \ln \rho_{\mathcal{H}}) \tag{2.7}$$

is the same as the entanglement entropy (2.3). Since $\rho_{\mathcal{H}}$ is thermal, it can be written as[1]

$$\rho_{\mathcal{H}} = \frac{e^{-H_{\mathcal{H}}}}{\mathrm{tr}(e^{-H_{\mathcal{H}}})}, \tag{2.8}$$

where $H_{\mathcal{H}}$ is the associated charge of the symmetry generator ξ. ξ is called modular flow and generates translation along the thermal circle of the transformed coordinates. Thus, first one can apply this unitary transformation and calculate the thermal entropy with the help of $H_{\mathcal{H}}$ and then through the inverse unitary transformation (2.6) calculate the density matrix ρ_B (or equivalently modular Hamiltonian H_B). Moreover, it is clear that H_B is the conserved charge of ξ up to an additive constant. This constant can be ignored when the variation of the modular Hamiltonian in the FLEE is considered. In the rest of this paper we mostly use modular flow instead of modular Hamiltonian.

B. Holographic FLEE in AdS/CFT

Formula (2.4) holds for small perturbations in any quantum field theory. One may ask about the holographic counterpart of this formula for the field theories that have holographic duals. The first step is applying the FLEE for the CFTs and wondering about the holographic formula in the dual AdS geometry in the context of AdS/CFT. It was shown in [18,19] that the FLEE for a CFT yields the linearized equations of motion in the AdS gravity side. In this subsection we review the derivation.

Take a d-dimensional CFT on Minkowski spacetime $\mathbb{R}^{1,d-1}$. The dual $(d+1)$-dimensional holographic dual consists of the asymptotically AdS spacetimes. For the

[1]We have absorbed a factor of 2π into the definition of $H_{\mathcal{H}}$.

vacuum state the dual spacetime is pure AdS whose metric g^0_{ab} in the Feffermann-Graham coordinates reads

$$ds^2 = \frac{\ell^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2). \tag{2.9}$$

We consider a spacial time slice Σ of d-dimensional Minkowski space and divide it into two regions B and $\bar{B}$ ($\Sigma = B \cup \bar{B}$). Let B be a $(d-1)$-dimensional ball with radius R.

In order to find δE_B in (2.4), we need to calculate the vacuum expectation value of the modular Hamiltonian. The modular Hamiltonian for this ball-shaped region is calculated in [26] as follows,

$$H_B = 2\pi \int_B \mathrm{d}^{d-1}x \frac{R^2 - \delta_{ij}(x^i - x^i_0)(x^j - x^j_0)}{2R} T^{tt}(x), \tag{2.10}$$

where x^i_0 are the coordinates of the center of the ball B and $T^{\mu\nu}$ is the stress tensor of CFT. We use the convention $x^\mu = (t, x^i)$. Hence the FLEE (2.4) can be written as

$$\delta S_B = 2\pi \int_B \mathrm{d}^{d-1}x \frac{R^2 - \delta_{ij}(x^i - x^i_0)(x^j - x^j_0)}{2R} \delta\langle T^{tt}(x)\rangle. \tag{2.11}$$

Now we use holography to calculate δS_B. When the CFT vacuum state $|\Psi(0)\rangle$ is perturbed to the state $|\Psi(\varepsilon)\rangle$, in the dual gravitational theory, the metric of the dual AdS spacetime is perturbed as

$$ds^2 = \frac{\ell^2}{z^2}((\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu + dz^2), \tag{2.12}$$

where $h_{\mu\nu}$ are infinitesimal. By means of the Ryu-Takayanagi formula [16,17] we can write

$$S_B = S_{HEE} = \frac{A_{\tilde{B}}}{4G}, \tag{2.13}$$

where $A_{\tilde{B}}$ is the minimal area of the codimension two surface $\tilde{B}$ in the bulk AdS space, which is homologous to B and given by

$$A_{\tilde{B}} = \int_{\tilde{B}} \mathrm{d}^{d-1}\sigma \sqrt{\det(\gamma_{AB})}. \tag{2.14}$$

Here γ_{AB} is the induced metric on $\tilde{B}$.

Let us illustrate the holographic counterpart of δS_B and δE_B, respectively, as $\delta S^{\mathrm{grav}}_B$ and $\delta E^{\mathrm{grav}}_B$. It was shown in [18,19] that they are given as follows in terms of bulk perturbed metric h_{ij},

$$\delta S^{\mathrm{grav}}_B = \frac{\ell^{d-3}}{8GR} \int_{\tilde{B}} \mathrm{d}^{d-1}x (R^2\delta^{ij} - (x^i x^j)) h_{ij}(x, z), \tag{2.15}$$

$$\delta E^{\mathrm{grav}}_B = \frac{\ell^{d-3}d}{16GR} \int_B \mathrm{d}^{d-1}x (R^2 - (\vec{x} - \vec{x}_0)^2)\delta^{ij} h_{ij}(x, z = 0). \tag{2.16}$$

Thus the FLEE formula (2.4) is written as

$$\int_{\tilde{B}} \mathrm{d}^{d-1}x (R^2\delta^{ij} - (x^i x^j)) h_{ij} = \frac{d}{2} \int_B \mathrm{d}^{d-1}x (R^2 - (\vec{x} - \vec{x}_0)^2)\delta^{ij} h_{ij}. \tag{2.17}$$

This is a nonlocal equation that is correct for any ball-shaped region with arbitrary radius R and center coordinate $\{x^i_0\}$. Thus one may think about a local equation that is equivalent to (2.17). In order to find this local constraint, we look for a form χ such that

$$\int_B \chi = \delta E^{\mathrm{grav}}_B, \qquad \int_{\tilde{B}} \chi = \delta S^{\mathrm{grav}}_B. \tag{2.18}$$

If such a form χ exists, using (2.4) we can write

$$\delta S^{\mathrm{grav}}_B - \delta E^{\mathrm{grav}}_B = 0 = \int_{\tilde{B}} \chi - \int_B \chi = \int_{B\cup\tilde{B}} \chi = \int_\Pi \mathrm{d}\chi, \tag{2.19}$$

where Π is the hypersurface bounded by B and $\tilde{B}$ ($B \cup \tilde{B} = \partial\Pi$) and located at $t = t_0$. For the asymptotically AdS spacetimes, χ is given by [19]

$$\chi = -\frac{1}{16\pi G}[\delta(\nabla^a \xi^b \epsilon_{ab}) + \xi^b \epsilon_{ab}(\nabla_c h^{ac} - \nabla^a h^c_c)], \tag{2.20}$$

where ξ^a is the bulk modular flow

$$\xi = -\frac{2\pi}{R}(t - t_0)[z\partial_z + (x^i - x^i_0)\partial_i] + \frac{\pi}{R}[R^2 - z^2 - (x^i - x^i_0)^2 - (t - t_0)^2]\partial_t. \tag{2.21}$$

For this form, the exterior derivative is given by

$$\mathrm{d}\chi = -\frac{1}{8\pi G}\xi^a \delta G_{ab} \epsilon^b, \tag{2.22}$$

where δG_{ab} are linearized Einstein equations around AdS spacetimes,

$$\delta G_{ab} = -\frac{1}{2}\nabla_b\nabla_a h^c{}_c + \frac{1}{2}\nabla_c\nabla_a h_b{}^c + \frac{1}{2}\nabla_c\nabla_b h_a{}^c - \frac{1}{2}\nabla_c\nabla^c h_{ab} - \frac{1}{2}g_{ab}\nabla_d\nabla_c h^{cd} + \frac{1}{2}g_{ab}\nabla_d\nabla^d h^c{}_c - \frac{2\Lambda}{d-1}\left(h_{ab} - \frac{1}{2}g_{ab}h^c{}_c\right), \tag{2.23}$$

and ϵ^b is related to volume form as follows:

$$\epsilon^a = g^{ab}\frac{1}{d!}\bar{\epsilon}_{bi_2\ldots i_{d+1}}\sqrt{-g}\mathrm{d}x^{i_2}\wedge\ldots\wedge\mathrm{d}x^{i_{d+1}}. \tag{2.24}$$

Moreover, the exterior derivative is 0 on the boundary.

From (2.19) and (2.22) it is obvious that the holographic interpretation of the first law of entanglement entropy leads to

$$\int_\Pi \xi^a \delta G_{ab}\epsilon^b = 0. \tag{2.25}$$

Using the fact that only the t component of ξ^a is nonvanishing on Π and also FLEE is valid for all of the ball-shaped regions with arbitrary R, from (2.25) one can deduce that [27]

$$\delta G_{tt} = 0. \tag{2.26}$$

In the above derivation, B was a constant time slice in the boundary. Thus for a constant time slice or rest frame of references, we can deduce the tt component of the linearized Einstein equation. Repeating the same argument for the ball-shaped regions in the arbitrary frame of references we can find $\delta G_{\mu\nu} = 0$, where μ and ν are directions of the field theory. Moreover, from the fact that the exterior derivative of χ is 0 on the boundary we can deduce that $\delta G_{z\mu} = 0$ and $\delta G_{zz} = 0$ on the boundary or $z = 0$. Thus all components of the linearized Einstein equation are 0 at $z = 0$. One can use this result as the initial condition and using the Bianchi identity prove that $\delta G_{z\mu}$ and δG_{zz} are 0 everywhere [28].

We see that the gravitational interpretation of the FLEE in CFTs leads to the linearized equations of motion of the dual AdS gravity. In the next section we apply the above procedure for asymptotically flat spacetimes in the context of flat/BMSFT correspondence.

III. HOLOGRAPHIC FLEE IN FLAT/BMSFT CORRESPONDENCE

A. Flat/BMSFT correspondence

Asymptotic symmetries of the asymptotically AdS spacetimes in $(d+1)$ dimensions are the same as local symmetries of the d-dimensional CFTs. One may expect such an equivalence between the gravity solutions and their dual field theory for the non-AdS spacetimes. Asymptotically AdS spacetimes are solutions of Einstein gravity with negative cosmological constant. Taking the flat space limit that is equivalent to the zero cosmological constant limit results in asymptotically flat spacetimes. Although this limit is not well defined for the asymptotically AdS spacetimes written in the Fefferman-Graham coordinate, it is possible to find appropriate coordinates with a well-defined flat-space limit [29,30]. A relevant question is finding a counterpart for the flat-space limit of the gravity theory in the field theory side. To answer this question one needs to study the asymptotic symmetry of the asymptotically flat spacetimes. This study has been done in [3] for the four-dimensional spacetimes and in [4] for the three-dimensional spacetimes. More recent studies show that for the four-dimensional cases the asymptotic symmetry algebra at null infinity is the semidirect sum of infinite-dimensional local conformal symmetry algebra on a two-sphere and the Abelian ideal algebra of supertranslations [6]. This algebra is known as bms_4. Such an infinite-dimensional locally well-defined symmetry algebra also exists at null infinity of three-dimensional asymptotically flat spacetimes [5]. This algebra is called bms_3.

The observation of [2] is that the bms_3 is isomorphic to an infinite-dimensional algebra in two dimensions, which is given by ultrarelativistic contraction of conformal algebra. Thus it was proposed in [2] that the holographic dual of asymptotically flat spacetimes in $(d+1)$ dimensions is field theories in d dimensions that have BMS symmetry. We call these BMS invariant field theories BMSFT and the correspondence between them and asymptotically flat spacetimes flat/BMSFT.

To be more precise, let us consider Einstein-Hilbert action with negative cosmological constant in three dimensions,

$$S = \frac{1}{16\pi G}\int \mathrm{d}^3x\sqrt{-g}\left(R + \frac{4}{\ell^2}\right). \tag{3.1}$$

An appropriate coordinate with well-defined flat space limit is BMS gauge [29]

$$ds^2 = \left(-\frac{r^2}{\ell^2} + \mathcal{M}\right)du^2 - 2dudr + 2\mathcal{N}dud\phi + r^2d\phi^2, \tag{3.2}$$

where $\mathcal{M}$ and $\mathcal{N}$ are functions of u and ϕ and are constrained by using the equations of motion as

$$\partial_u\mathcal{M} = \frac{2}{\ell^2}\partial_\phi\mathcal{N}, \qquad 2\partial_u\mathcal{N} = \partial_\phi\mathcal{M}. \tag{3.3}$$

The asymptotic symmetry algebra is exactly the conformal algebra in two dimensions,

$$[\mathcal{L}_m, \mathcal{L}_n] = (m-n)\mathcal{L}_{m+n},$$
$$[\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] = (m-n)\bar{\mathcal{L}}_{m+n},$$
$$[\mathcal{L}_m, \bar{\mathcal{L}}_n] = 0, \quad m, n \in \mathbb{Z}. \tag{3.4}$$

The algebra of conserved charges is centrally extended with central charges $c = \bar{c} = 3\ell/2G$.

Taking the flat-space limit from metric (3.2) yields asymptotically flat spacetimes with metric

$$ds^2 = Mdu^2 - 2dudr + 2Ndud\phi + r^2 d\phi^2, \tag{3.5}$$

where M and N are functions of u and ϕ and they satisfy

$$\partial_u M = 0, \qquad 2\partial_u N = \partial_\phi M. \tag{3.6}$$

The asymptotic symmetry algebra at null infinity is infinite-dimensional bms_3 algebra [5],

$$\begin{aligned}
&[L_m, L_n] = (m-n)L_{m+n},\\
&[L_m, M_n] = (m-n)M_{m+n},\\
&[M_m, M_n] = 0, \quad m, n \in \mathbb{Z}.
\end{aligned} \tag{3.7}$$

The algebra of conserved charges is also centrally extended.

The generators of bms_3 can be obtained by taking the flat-space limit from the generators of conformal algebra [29],

$$L_m = \lim_{\frac{G}{\ell}\to 0}(\mathcal{L}_m - \bar{\mathcal{L}}_{-m}), \qquad M_m = \frac{G}{\ell}\lim_{\frac{G}{\ell}\to 0}(\mathcal{L}_m + \bar{\mathcal{L}}_{-m}). \tag{3.8}$$

It was argued in [2] that the limit (3.8) that is taken in the gravity side corresponds to the ultrarelativistic limit in the field theory side. In the rest of this paper by BMSFT_2 we mean a field theory that has the symmetry algebra (3.7).

From BMSFT_3 we mean a field theory with the following symmetry algebra

$$\begin{aligned}
&[L_m, L_n] = (m-n)L_{m+n},\\
&[\bar{L}_m, \bar{L}_n] = (m-n)\bar{L}_{m+n},\\
&[L_m, \bar{L}_n] = 0,\\
&[L_l, M_{m,n}] = \left(\frac{l+1}{2} - m\right)M_{m+l,n},\\
&[\bar{L}_l, M_{m,n}] = \left(\frac{l+1}{2} - n\right)M_{m,n+l}, \quad m, n, l \in \mathbb{Z}.
\end{aligned} \tag{3.9}$$

This algebra is called bms_4 and is the asymptotic symmetry of the four-dimensional asymptotically flat spacetimes at null infinity [6]. L_m and $\bar{L}_m$ are generators of super-rotations and $M_{m,n}$ are generators of supertranslations. The Poincare subalgebra is generated by

$$\{L_{-1}, L_0, L_{+1}, \bar{L}_{-1}, \bar{L}_0, \bar{L}_{+1}, M_{0,0}, M_{0,+1}, M_{+1,0}, M_{+1,+1}\}. \tag{3.10}$$

B. Holographic entanglement entropy in flat/BMSFT

Similar to other field theories, it is possible to define entanglement entropy for the subsystems of BMSFT. The infinite-dimensional symmetry of BMSFTs admits to finding universal formulas for the entanglement entropy of subregions [8]. Moreover, using the flat/BMSFT correspondence one can find a holographic description for the BMSFT entanglement entropy. Recently, a prescription (similar to the Ryu-Takayanagi's proposal for the CFT entanglement entropy [16,17]) was proposed for the BMSFT entanglememnt entropy [12] that relates it to the area of some particular curves into the bulk flat spacetimes. According to [12], the entanglement entropy of subregion B of BMSFT_2 is given by

$$S_{HEE} = \frac{\mathrm{Length}(\gamma)}{4G} = \frac{\mathrm{Length}(\gamma \cup \gamma_+ \cup \gamma_-)}{4G}, \tag{3.11}$$

where γ is a spacelike geodesic and γ_+ and γ_- are null rays from $\partial\gamma$ to ∂B.

The most generic solution of Einstein gravity with zero cosmological constant in three dimensions is given by (3.5). In the rest of this paper we consider an interval B in the BMSFT that is determined by $-\frac{l_u}{2} < u < \frac{l_u}{2}$ and $-\frac{l_\phi}{2} < \phi < \frac{l_\phi}{2}$, where l_u and l_ϕ are constants. Among the various values of functions M and N in (3.5), the following three metrics are of more interest:

(1) Null-orbifold or Poincaré patch with metric ($M = N = 0$ in (3.5))

$$ds^2 = -2dudr + r^2 d\phi^2. \tag{3.12}$$

In this case the bulk modular flow is

$$\begin{aligned}
\xi^{\mathrm{bulk}} = -\frac{\pi}{2l_\phi}\Bigg[&\left(l_\phi^2 - 4\phi^2 + \frac{8(ul_\phi - l_u\phi)}{rl_\phi}\right)\partial_\phi\\
&+\left(l_u l_\phi + \frac{4l_u}{l_\phi}\phi^2 - 8u\phi\right)\partial_u\\
&+\left(\frac{8l_u}{l_\phi} + 8r\phi\right)\partial_r\Bigg].
\end{aligned} \tag{3.13}$$

Here γ is given by

$$r = -\frac{l_u}{l_\phi \phi}, \qquad u = \frac{l_u l_\phi}{8\phi} + \frac{l_u \phi}{2l_\phi}. \tag{3.14}$$

By using the coordinate transformations

$$\begin{aligned}
t &= \frac{l_\phi}{4}r + \frac{2}{l_\phi}u + \frac{1}{l_\phi}r\phi^2,\\
x &= \frac{l_u}{l_\phi} + r\phi,\\
y &= \frac{l_\phi}{4}r - \frac{2}{l_\phi}u - \frac{1}{l_\phi}r\phi^2,
\end{aligned} \tag{3.15}$$

we can change the metric of the null orbifold to the Cartesian coordinate

$$ds^2 = -dt^2 + dx^2 + dy^2. \tag{3.16}$$

In these coordinates the bulk modular flow is given by

$$\xi^{\rm bulk} = -2\pi(x\partial_t + t\partial_x), \tag{3.17}$$

and geodesics are

$$\gamma\colon\ x = t = 0, \qquad -\frac{l_u}{l_\phi} \le y \le +\frac{l_u}{l_\phi}, \tag{3.18}$$

$$\gamma_+\colon\ x = t, \qquad y = -\frac{l_u}{l_\phi}, \tag{3.19}$$

$$\gamma_-\colon\ x = -t, \qquad y = +\frac{l_u}{l_\phi}. \tag{3.20}$$

(2) Global Minkowski with metric [$M = -1$ and $N = 0$ in (3.5)]

$$ds^2 = -du^2 - 2dudr + r^2 d\phi^2. \tag{3.21}$$

The bulk modular flow is

$$\begin{aligned}\xi^{\rm bulk} = {}& \pi\csc\frac{l_\phi}{2}\left(2\left(\cos\frac{l_\phi}{2} - \cos\phi\right)\right.\\ &\left.+\frac{1}{r}\left(l_u\sin\phi\cot\frac{l_\phi}{2} - 2u\cos\phi\right)\right)\partial_\phi,\\ &+\pi\csc\frac{l_\phi}{2}\left(-l_u\csc\frac{l_\phi}{2} + l_u\cos\phi\cot\frac{l_\phi}{2}\right.\\ &\left.+2u\sin\phi\right)\partial_u,\\ &-\pi\csc\frac{l_\phi}{2}\left(l_u\cos\phi\cot\frac{l_\phi}{2} + 2(r+u)\sin\phi\right)\partial_r,\end{aligned} \tag{3.22}$$

where γ is given by[2]

$$r = -\frac{l_u\csc\frac{l_\phi}{2}}{2\sin\phi}, \qquad u = -\frac{l_u}{2}\cot\frac{l_\phi}{2}\cot\phi - r. \tag{3.23}$$

Using coordinate transformation [31]

$$\begin{aligned} t &= (r+u)\csc\frac{l_\phi}{2} - r\cos\phi\cot\frac{l_\phi}{2},\\ x &= r\sin\phi + \frac{l_u}{2}\csc\frac{l_\phi}{2},\\ y &= r\cos\phi\csc\frac{l_\phi}{2} - (r+u)\cot\frac{l_\phi}{2},\end{aligned} \tag{3.24}$$

we have

$$ds^2 = -dt^2 + dx^2 + dy^2. \tag{3.25}$$

In these Cartesian coordinates the bulk modular flow is the same as (3.17) and geodesics are

$$\gamma\colon\ x = 0 = t, \qquad -\frac{l_u}{2}\cot\frac{l_\phi}{2} \le y \le +\frac{l_u}{2}\cot\frac{l_\phi}{2}, \tag{3.26}$$

$$\gamma_+\colon\ x = t, \qquad y = -\frac{l_u}{2}\cot\frac{l_\phi}{2}, \tag{3.27}$$

$$\gamma_-\colon\ x = -t, \qquad y = +\frac{l_u}{2}\cot\frac{l_\phi}{2}. \tag{3.28}$$

(3) FSC with metric ($M = m$ and $N = j$)

$$ds^2 = mdu^2 - 2dudr + 2jdud\phi + r^2 d\phi^2, \tag{3.29}$$

where m and j are constants. It has a cosmological horizon at radius $r_c = \frac{j}{\sqrt{m}}$. FSC is a shift-boost orbifold of Minkowski spacetime [21] and can be brought into the Cartesian coordinate locally by using the following transformation:

$$\begin{aligned} r &= \sqrt{m(t^2 - x^2) + r_c^2},\\ \phi &= -\frac{1}{\sqrt{m}}\log\frac{\sqrt{m}(t-x)}{r + r_c},\\ u &= \frac{1}{m}(r - \sqrt{m}y - \sqrt{m}r_c\phi).\end{aligned} \tag{3.30}$$

Both the null-orbifold and global Minkowski correspond to the BMSFT states that are nonthermal but for the null orbifold, BMSFT is on a plane and for the global Minkowski the corresponding BMSFT is on the cylinder. FSC (3.29) corresponds to the BMSFT thermal states. The holographic entanglement entropy of interval B is given by

$$S = \frac{1}{2G}\left[\frac{\pi}{\beta_\phi}\left(l_u + \frac{\beta_u}{\beta_\phi}\ell_\phi\right)\coth\left(\frac{\pi\ell_\phi}{\beta_\phi}\right) - \frac{\beta_u}{\beta_\phi}\right], \tag{3.31}$$

where

$$\beta_\phi = \frac{2\pi}{\sqrt{m}}, \qquad \frac{\beta_u}{\beta_\phi} = \frac{j}{m}. \tag{3.32}$$

[2] We assume that $l_\phi < \pi$.

C. Holographic FLEE

In this section we consider the BMSFT dual to the global Minkowski. The starting point is FLEE formula (2.4), which is written in the field theory side. We want to use $\mathrm{Flat}_3/\mathrm{BMSFT}_2$ to write both sides of this formula in the gravity side. BMSFT lives on a cylinder with coordinates (u, ϕ) and interval B is given by $-\frac{l_u}{2} < u - u_0 < \frac{l_u}{2}$ and $-\frac{l_\phi}{2} < \phi - \phi_0 < \frac{l_\phi}{2}$, where l_u, l_ϕ, u_0, and ϕ_0 are constants.

Let us start from the right-hand side of (2.4). In order to calculate the expectation value of the modular Hamiltonian, we use the fact that up to an additive constant, the modular Hamiltonian H_B is the same as the conserved charge of the modular flow ξ. If we show the stress tensor of BMSFT by T_{ab}, the corresponding charge of ξ can be calculated on a spacelike surface Σ with metric σ_{ab} as [32]

$$Q_\xi = \int_\Sigma d\sigma \sqrt{\det(\sigma_{ab})} n_a \xi^b T^a_b, \tag{3.33}$$

where σ is the coordinate on the surface Σ and n^a is the unit timelike vector normal to Σ. The most challenging problem in the flat-space holography is the definition of Σ. In the AdS/CFT correspondence, Σ is a spacelike surface on the conformal boundary of the asymptotically AdS spacetimes. However, such a definition for conformal infinity of asymptotically flat spacetimes is not appropriate in the flat-space holography. In the previous works [30,33–38], in the flat-space holography, Σ has been defined by using the corresponding surface of asymptotically AdS spacetimes; their flat-space limit yields the asymptotically flat metric. To be precise, let us consider the AdS_3 metric written in the BMS coordinate,

$$ds^2 = -\left(1 + \frac{r^2}{\ell^2}\right) du^2 - 2du d\phi + r^2 d\phi^2, \tag{3.34}$$

where ℓ is the radius of AdS space. At fixed but large r we can write

$$ds_B^2 = \frac{r^2}{\ell^2}(-du^2 + \ell^2 d\phi^2) + \mathcal{O}(r^0). \tag{3.35}$$

Thus we can write the metric of the conformal boundary as

$$ds_{CB}^2 = -du^2 + \ell^2 d\phi^2. \tag{3.36}$$

In the AdS/CFT correspondence, the metric of Σ in (3.33) is given by using (3.36). The new point in all of the papers [30] and [33–38] is that (3.36) is also appropriate for writing the metric of Σ in the $\ell \to \infty$ limit. The proposal of [30] for the definition of Σ is that we use a metric similar to (3.36) but replace ℓ with three-dimensional Newton constant G. In this paper we employ this definition of Σ. Since we want to study the FLEE in a BMSFT that is the holographic dual of the global Minkowski, the metric of bulk spacetime is given by (3.21), which is the $\ell \to \infty$ limit of (3.34). Thus we choose Σ as a spacelike subspace of a space that is determined by metric

$$ds_{CB}^2 = -du^2 + G^2 d\phi^2. \tag{3.37}$$

It proves convenient to first make a coordinate transformation as

$$w = u - u_0 - \frac{l_u}{2} \frac{\sin(\phi - \phi_0)}{\sin\frac{l_\phi}{2}}. \tag{3.38}$$

In this coordinate, the interval is defined as $-\frac{l_\phi}{2} < \phi - \phi_0 < \frac{l_\phi}{2}$. Moreover, by taking the $r \to \infty$ limit from (3.22), we can find the BMSFT modular flow on the internal $(w = 0)$ as

$$\xi^w = 0, \qquad \xi^\phi = \frac{2\pi}{\sin\frac{l_\phi}{2}} \left(\cos\frac{l_\phi}{2} - \cos(\phi - \phi_0)\right). \tag{3.39}$$

If we determine Σ as $w = 0, -\frac{l_\phi}{2} < \phi - \phi_0 < \frac{l_\phi}{2}$ then using (3.37) and (3.39) we find

$$\delta E_B = \delta\langle H_B\rangle = \frac{2\pi G}{\sin\frac{l_\phi}{2}} \int_{\phi_0 - \frac{l_\phi}{2}}^{\phi_0 + \frac{l_\phi}{2}} d\phi \left(\cos\frac{l_\phi}{2} - \cos(\phi - \phi_0)\right) \delta\langle T^w_\phi\rangle. \tag{3.40}$$

Hence we can write the right-hand side of (2.4) in terms of the BMSFT stress tensor by using flat-space holography.

In order to calculate the left-hand side of (2.4) holographically, we perturb the metric of global coordinate (3.21) as

$$ds^2 = (-1 + h_{uu}) du^2 - 2du dr + 2h_{u\phi} du d\phi + r^2 d\phi^2. \tag{3.41}$$

We consider the case in which h_{uu} and $h_{u\phi}$ are constants. With this choice (3.41) is similar to flat-space cosmology (3.29). For writing (3.41) we do not use equations of motion. The fixed components of the metric have been determined by using boundary conditions that are necessary to have BMS symmetry at null infinity (see, e.g., [24]). In other words, the fact that the dual theory is BMSFT imposes (3.41) for the form of the metric. This is similar to choosing the Fefferman-Graham coordinate in the context of AdS/CFT correspondence. Line element (3.41) is not the generic one that fulfils the BMS boundary conditions. In order to simplify equations we have fixed some components. However, our argument in the rest of the paper can be generalized to more generic cases.

Since h_{uu} and $h_{u\phi}$ are infinitesimal constants, we can use (3.31) to calculate δS. We find

$$\delta S = \frac{1}{4G}\left[2\left(-1+\frac{l_\phi}{2}\cot\frac{l_\phi}{2}\right)h_{u\phi} + \frac{l_u}{2}\left(\cot\frac{l_\phi}{2} - \frac{l_\phi}{2\sin^2\frac{l_\phi}{2}}\right)h_{uu}\right]. \quad (3.42)$$

Using (3.40) and (3.42), we can write the FLEE as

$$\int_{\phi_0-\frac{l_\phi}{2}}^{\phi_0+\frac{l_\phi}{2}} d\phi\left(\cos\frac{l_\phi}{2} - \cos(\phi-\phi_0)\right)\delta\langle T^w_\phi\rangle = \frac{\sin\frac{l_\phi}{2}}{8\pi G^2}\left[2\left(-1+\frac{l_\phi}{2}\cot\frac{l_\phi}{2}\right)h_{u\phi} + \frac{l_u}{2}\left(\cot\frac{l_\phi}{2} - \frac{l_\phi}{2\sin^2\frac{l_\phi}{2}}\right)h_{uu}\right]. \quad (3.43)$$

This formula is valid for all intervals determined by l_ϕ, l_u, and (u_0, ϕ_0). For a very small interval that is given by $l_\phi \to 0$, $l_u \to 0$, but $\frac{l_u}{l_\phi} =$ fixed, the expectation value of the stress tensor can be considered as a function of the center of the interval. Since the center of the interval is an arbitrary point, using (3.43) we find

$$\delta\langle T^w_\phi\rangle = \frac{1}{8\pi G^2}\left(h_{u\phi} + \frac{l_u}{2}\frac{\cos\phi}{\sin\frac{l_\phi}{2}}h_{uu}\right). \quad (3.44)$$

Putting (3.44) into (3.40), we find δE_B as

$$\delta E_B = \frac{1}{4G\sin\frac{l_\phi}{2}}\int_{\phi_0-\frac{l_\phi}{2}}^{\phi_0+\frac{l_\phi}{2}} d\phi\left(\cos\frac{l_\phi}{2} - \cos(\phi-\phi_0)\right) \times\left(h_{u\phi} + \frac{l_u}{2}\frac{\cos\phi}{\sin\frac{l_\phi}{2}}h_{uu}\right). \quad (3.45)$$

The interesting point is that both of δS_B and δE_B given by (3.42) and (3.45) are written as the integral of a specific one-form χ. Precisely, we can write[3]

$$\delta E = \int_B \chi, \qquad \delta S = \int_{\gamma_-\cup\gamma\cup\gamma_+}\chi, \quad (3.46)$$

where

$$\chi = \frac{1}{16\pi G}\epsilon_{\mu\nu\alpha}\left[\xi^\nu\nabla^\mu h - \xi^\nu\nabla_\sigma h^{\mu\sigma} + \xi_\sigma\nabla^\nu h^{\mu\sigma} + \frac{1}{2}h\nabla^\nu\xi^\mu + \frac{1}{2}h^{\nu\sigma}(\nabla^\mu\xi_\sigma - \nabla_\sigma\xi^\mu)\right]dx^\alpha. \quad (3.47)$$

ξ is the bulk modular flow (3.22), $h = h^\nu_\mu$, and $\epsilon_{\mu\nu\alpha}$ is the completely antisymmetric tensor with component $\epsilon_{012} = \sqrt{|g_0|}$ where g_0 is the determinant of global Minkowski (3.21). Thus the FLEE formula (2.4) for BMSFT can be written as

$$\int_B \chi - \int_{\gamma_-\cup\gamma\cup\gamma_+}\chi = 0. \quad (3.48)$$

Curves B and $\gamma_-\cup\gamma\cup\gamma_+$ construct a closed path. Hence, we can write (3.48) as

$$\int_\Pi d\chi = 0, \quad (3.49)$$

where $d\chi$ is the exterior derivative of χ and Π is any surface bounded by $B\cup\gamma_-\cup\gamma\cup\gamma_+$. Since Π is any bounded surface, from (3.49) one may expect that

$$d\chi = 0. \quad (3.50)$$

It is not difficult to check that (3.50) is satisfied for the perturbed metric given by (3.41). In fact, the metric (3.41) with constant h_{uu} and $h_{u\phi}$ is a solution of the Einstein equation.

Let us consider a case in which h_{uu} and $h_{u\phi}$ are arbitrary functions of u and ϕ. Now we have

$$d\chi = \frac{1}{16Gr^2}(d\chi_{ru}dr\wedge du + d\chi_{u\phi}du\wedge d\phi), \quad (3.51)$$

where

$$d\chi_{ru} = (\partial_\phi h_{uu} - 2\partial_u h_{u\phi}) \times\left(l_u\cot\frac{l_\phi}{2}\cos\theta - l_u\csc\frac{l_\phi}{2} + 2u\sin\theta\right), \quad (3.52)$$

and

$$d\chi_{u\phi} = r\left\{(\partial_\phi h_{uu} - 2\partial_u h_{u\phi})\left[-\cot\frac{l_\phi}{2}\left(2r + l_u\csc\frac{l_\phi}{2}\sin\theta\right) + 2\cos\theta\csc\frac{l_\phi}{2}(r+u)\right]\right\} + r\csc\frac{l_\phi}{2}\partial_\phi(\partial_\phi h_{uu} - 2\partial_u h_{u\phi})\left(l_u\cot\frac{l_\phi}{2}\cos\theta - l_u\csc\frac{l_\phi}{2} + 2u\sin\theta\right) + r^2\csc\frac{l_\phi}{2}\partial_u h_{uu}\left(l_u\cot\frac{l_\phi}{2}\cos\theta - l_u\csc\frac{l_\phi}{2} + 2u\sin\theta\right). \quad (3.53)$$

Thus using (3.50), (3.52) and (3.53) we find that

[3] In the global Minkowski coordinate, γ_+ consists of two null curves connected at $r = 0$ [12]. Since χ is singular at $r = 0$, we use contour $r = \epsilon$ in the calculation of $\int_{\gamma_+}\chi$ and after integration take $\epsilon \to 0$.

$$\partial_\phi h_{uu} = 2\partial_u h_{u\phi}, \qquad \partial_u h_{uu} = 0. \tag{3.54}$$

These are the relations that one can conclude from the Einstein equation for the metric (3.41).

IV. SUMMARY AND CONCLUSION

In this paper we studied another aspect of flat/BMSFT that was previously introduced in the context of AdS/CFT. We wrote the FLEE of BMSFT_2 in terms of three-dimensional asymptotically flat metrics. The steps are analogue to those that are used in the context of AdS/CFT correspondence. We rewrite both sides of the FLEE (2.4) by using corresponding bulk parameters. δS_B in (2.4) is the variation of entanglement entropy with respect to the state by which the system is described. Using the proposal of [12] one can write this variation as the variation of length of some spatial curves in the bulk geometry. δE_B in the right-hand side of the FLEE (2.4) is variation of the expectation value of the modular Hamiltonian. For calculating this quantity, we used the fact that the modular Hamiltonian is the conserved charge of modular flow up to an additive constant that can be ignored in the variation. BMSFT conserved charges are given by using the stress tensor. Using the flat/BMSFT dictionary we relate the calculation of the conserved charges to a bulk calculation similar to the Brown-York proposal [32]. The key point in this calculation is the definition of the spatial surface over which the integration is performed. In the AdS/CFT correspondence this surface is given by using the conformal boundary of asymptotically AdS spacetimes. In this case we do not use the standard definition of conformal boundary. Our proposal is that this surface for the flat spacetimes is the same as that for the asymptotically AdS case whose flat-space limit yields the asymptotically flat spacetimes [30]. This proposal works again in this problem similar to all previous works [33–38]; however, a thorough investigation is necessary, which we hope to do in our future studies.

In this paper we assumed that the perturbed state in the field theory side corresponds to a metric similar to the flat-space cosmology [20–23] in the bulk theory. Hence, the gravitational counterpart of the FLEE was the exterior derivative of a one-form that is 0 for the flat-space cosmology. The exterior derivative of this form for a generic metric that satisfies the BMS boundary condition results in Einstein equations for undermined components of the metric. This is a good hint that the holographic FLEE is the Einstein equation in the flat/BMSFT correspondence.

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Note added.—Recently, Ref. [39] was posted whose results overlap with ours.

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