Equivalent dual theories for 3D $\mathcal{N} = 2$ supergravity

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$\mathcal{N} = 2$ three-dimensional supergravity with internal R-symmetry generators can be understood as a two-dimensional chiral Wess-Zumino-Witten model. In this paper, we present the reduced phase space description of the theory, which turns out to be a flat limit of a generalized Liouville theory, up to zero modes. The reduced phase space description can also be explained as a gauged chiral Wess-Zumino-Witten model. We show that both these descriptions possess identical gauge and global (quantum $\mathcal{N} = 2$ super BMS_3) symmetries.

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I. INTRODUCTION AND SUMMARY

There is a connection between $D + 1$-dimensional diffeomorphism invariant theories and D-dimensional field theories. The details of this duality strongly depend on the precise form of boundary conditions on various fields. One of the simplest contexts where this has been studied is $2 + 1$-dimensional gravity theories. It is a well-known fact that three-dimensional gravity can be described by a two-dimensional field theory. 3D gravity solutions with nontrivial topology correspond to stress-energy tensors of a dual two-dimensional theory. This duality is best understood in the Chern-Simons formulation of 3D gravity [1,2]. The reduced dual theory in this case is, in general, a (chiral) Wess-Zumino-Witten (WZW) model[3], defined on a closed spatial section, and is obtained by solving part of the constraints in the Chern-Simons theory[4–6]. Such reductions have been mostly performed for asymptotically anti–de Sitter 3D gravity [7–15], where the dual 2D theory is a conformal field theory with infinite-dimensional symmetry. In this paper, we are interested in the dual of asymptotically flat 3D (super)gravity. In particular, ordinary asymptotically flat 3D gravity can be understood as a ISO(2,1) Chern-Simons gauge theory with a flat boundary condition at null infinity, where the Chern-Simons level k is identified with Newton's constant. Here, the spatial section is a plane, and the choice of boundary conditions is crucial in determining the dual theory. The reduction of ISO(2,1) Chern-Simons (CS) to the WZW model was first studied in Ref. [16]. An alternate route has been taken in Ref. [17], where the dual WZW model has been constructed for flat ordinary 3D gravity.[1] In Ref. [17], other than ISO(2,1) gauge algebra, the boundary conditions suitable for flat asymptotics (at null infinity) have been applied for the gauge field. As a result, the dual chiral WZW model, when is gauged, shows invariance under infinite-dimensional quantum BMS_3 algebra, which is the asymptotic symmetry of flat 3D gravity. The analysis was further extended for the minimal $\mathcal{N} = 1$ supergravity theory in Ref. [26], higher spin gravity [27], and recently, for the $\mathcal{N} = 2$ case, with (out) internal R symmetry in Ref. [28].

In Refs. [17,26], it was further shown that, for the pure and $\mathcal{N} = 1$ 3D supergravity theory, the asymptotic boundary conditions lead to a reduced phase space description as a flat limit of (super)Liouville theory at null infinity (up to zero modes). In view of CS-WZW duality, we can understand this result as due to the fact that the asymptotic conditions are strong enough to enforce the Hamiltonian reduction from SL(2,R)-WZW to Liouville theory [29–31]. Another way of looking at it would be to recall that the dual chiral WZW model shows further gauge invariance. It was described in Refs. [32–36] that particular subsectors of symmetry can be gauged without introducing any anomaly to the system. The gauged chiral WZW model then can be shown to be equivalent to the flat Liouville description. The gauging is identical to imposing first-class constraints to the WZW model that arises due to the asymptotic boundary condition.

The current paper should be considered as a follow-up of our recent work [28], where we have constructed the dual chiral WZW model for $\mathcal{N} = 2$ 3D supergravity with internal R symmetry. Here, we present the reduced phase

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[1]Higher spin and supersymmetric generalizations were performed in Refs. [18–25].

space description of the theory and study its properties. The reduced phase space turns out to be a flat limit of a generalized super-Liouville-type theory and is identical to the dual chiral WZW model constructed in Ref. [28], when appropriately gauged. Finally, we present the gauge invariance of the reduced system, which is the same as the residual gauge invariance of the gauged chiral WZW model.

The paper is organized as follows: In Sec. II, we briefly present the $\mathcal{N}=2$ 3D supergravity with internal R symmetry and its asymptotic boundary condition at null infinity that reproduces the infinite-dimensional quantum BMS_3 symmetry. In Sec. III, we write down the equivalent chiral WZW model that describes the dynamics of the theory and present its symmetries with minimal required details. Then we present the gauged version of the theory. Section IV contains the main result of this paper, where we present the phase space description of the dual theory and show its equivalence with the gauged chiral WZW model. Section V points out an interesting outlook of this work. Our conventions and some computational details have been presented in the Appendixes.

II. 3-DIMENSIONAL $\mathcal{N}=2$ SUPERGRAVITY AND ITS ASYMPTOTIC SYMMETRY

There are two different versions of $\mathcal{N}=2$ super-Poincaré algebra known in the literature [37]. One of them, commonly known as $\mathcal{N}=(1,1)$, contains two supercharges but no internal R symmetry. The other one, known as $\mathcal{N}=(2,0)$ super-Poincaré algebra, is more interesting, as it allows the two supercharges to transform under an internal R symmetry. The algebra can be presented as

$$[J_a, J_b] = \epsilon_{abc}J^c, \qquad [J_a, P_b] = \epsilon_{abc}P^c,$$
$$[J_a, Q^i_\alpha] = \frac{1}{2}(\Gamma_a)^\beta_\alpha Q^i_\beta, \qquad [Q^i_\alpha, T] = \epsilon^{ij}Q^j_\alpha,$$
$$\{Q^i_\alpha, Q^j_\beta\} = -\frac{1}{2}\delta^{ij}(C\Gamma^a)_{\alpha\beta}P_a + C_{\alpha\beta}\epsilon^{ij}Z. \tag{2.1}$$

Here, J_a and P_a $(a=0,1,2)$ are the Poincaré generators and $\mathcal{Q}^i_\alpha$, two distinct $(i=1,2)$ two-component $(\alpha=+1,-1)$ spinors, are the two fermionic generators of the algebra. These fermionic generators transform under a spinor representation of an internal R-symmetry generator T. The above algebra has a nondegenerate invariant bilinear form only in the presence of a central term Z [37]. Our conventions are presented in Appendix A. In this paper, we shall work with a 3D supergravity theory invariant under the above symmetry. In the CS formulation, 3D (super)gravity theory can be represented as

$$I[A] = \frac{k}{4\pi}\int_M \left\langle A, dA + \frac{2}{3}A^2\right\rangle. \tag{2.2}$$

Here, the gauge field A is regarded as a Lie-algebra-valued one form, and $\langle\rangle$ represents a metric in the field space that one obtains by construing a nondegenerate invariant bilinear form on the Lie algebra space. k is the level for the theory, and we express $A = A^a_\mu T_a \mathrm{d}x^\mu$, where $\{T_a\}$ is a particular basis of the superalgebra. The equation of motion is given as

$$F \equiv dA + A\wedge A = 0. \tag{2.3}$$

For our purpose, the gauge group is $\mathcal{N}=(2,0)$ super-Poincaré groups. The 3-manifold will be one with a *boundary,* and we shall identify the level k with Newton's constant as $k=\frac{1}{4G}$. The basis elements $\{T_a\}$ are J_a, P_a, Q^i_α, T, and Z. Using the supertrace elements, we get the corresponding supergravity action as

$$I^{(2,0)}_{\mu,\bar{\mu},\gamma} = \frac{k}{4\pi}\int [2e^a\hat{R}_a + \mu L(\hat{\omega}_a) - \bar{\Psi}^i_\beta\nabla\Psi^\beta_i - 2BdC + \bar{\mu}BdB],$$
$$A = e^aP_a + \hat{\omega}^aJ_a + \psi^\alpha_i Q^i_\alpha + BT + CZ, \tag{2.4}$$

where $\hat{\omega}^a = \omega^a + \gamma e^a$, for some constant γ and $\bar{\Psi}^i_\beta$, is the Majorana conjugate gravitino. The $\mathcal{N}=2$ supergravity theory of Refs. [23,24] is recovered in the $\mu=\bar{\mu}=\gamma=0$ limit. The curvature two-form $\hat{R}_a$, the Lorentz Chern-Simons three-form $L(\hat{\omega}_a)$, and the covariant derivative of the gravitino of Eq. (2.4) can, respectively, be defined as

$$\hat{R}_a = d\hat{\omega}_a + \frac{1}{2}\epsilon_{abc}\hat{\omega}^b\hat{\omega}^c,$$
$$L(\hat{\omega}_a) = \hat{\omega^a}d\hat{\omega}_a + \frac{1}{3}\epsilon^{abc}\hat{\omega}_a\hat{\omega}_b\hat{\omega}_c,$$
$$\nabla\Psi^\beta_i = d\Psi^\beta_i + \frac{1}{2}\hat{\omega}^a\Psi^\delta_i(\Gamma^a)^\beta_\delta + B\Psi^\beta_j\epsilon^{ij}. \tag{2.5}$$

Since the CS theory is a gauge theory, the equation of motion (2.3) implies that locally the solution of a CS field is pure gauge $A = G^{-1}dG$, where G is a local group element. Defining $\hat{\omega} = \frac{1}{2}\hat{\omega}^a\Gamma_a$, $e = \frac{1}{2}e^a\Gamma_a$, $\mathcal{G}^1 = \frac{1}{2}(\Psi_1 - i\Psi_2)$, and $\mathcal{G}^2 = \frac{1}{2}(\Psi_1 + i\Psi_2)$, the on-shell configuration for various fields of Eq. (2.4) can be written as

$$\hat{\omega} = \Lambda^{-1}d\Lambda, \qquad B = d\tilde{B},$$
$$\mathcal{G}_1 = e^{-i\tilde{B}}\Lambda^{-1}d\eta_1, \qquad \mathcal{G}_2 = e^{i\tilde{B}}\Lambda^{-1}d\eta_2,$$
$$C = -i(\bar{\eta}_{1\alpha}d\eta^\alpha_2 - \bar{\eta}_{2\alpha}d\eta^\alpha_1 + d\tilde{C}),$$
$$e = -\Lambda^{-1}\left[\frac{1}{2}\left(\eta_1 d\bar{\eta}_2 - \frac{1}{2}\eta_1 d\bar{\eta}_2\mathbf{I}\right) + \frac{1}{2}\left(\eta_2 d\bar{\eta}_1 - \frac{1}{2}\eta_2 d\bar{\eta}_1\mathbf{I}\right) + db\right]\Lambda. \tag{2.6}$$

Here, Λ is an arbitrary $SL(2,R)$ group element of unit determinant, and Γ_a is the generator of $SL(2,R)$. B and C are $SL(2,R)$ scalars, η_i, $i=1,2$ are Grassmann-valued $SL(2,R)$ spinors, and b is a traceless 2×2 matrix. All of these are local functions of three space time coordinates

u, ϕ, r. Since we are dealing with a gauge theory, we further choose a (radial) gauge condition $\partial_\phi A_r = 0$, and, hence, the group element splits as $G(u,\phi,r) = g(u,\phi)h(u,r)$. Thus, the gauge fields have the following form:

$$A = h^{-1}(a+d)h,$$
$$a = g^{-1}dg = a_u(u,\phi)du + a_\phi(u,\phi)d\phi.$$

We further choose that asymptotically $h = e^{-rP_0}$ and, hence, $\dot{h}(u,r) = \frac{\partial h(u,r)}{\partial u} = 0$ at the boundary. The advantage of this gauge choice is that the dependence in the radial coordinate is completely absorbed by the group element h. Hence, the boundary can be assumed to be uniquely located at any arbitrary fixed value of $r = r_0$, in particular, to infinity. Thus, the boundary describes a two-dimensional timelike surface with the topology of a cylinder. The radial gauge condition makes the above solutions of various field parameters decompose as

$$\begin{aligned}
\Lambda &= \lambda(u,\phi)\zeta(u,r),\\
\tilde{B} &= a(u,\phi) + \tilde{a}(u,r),\\
\tilde{C} &= c(u,\phi) + \tilde{c}(u,r) + \bar{d}_2\lambda\tilde{d}_1 - \bar{d}_1\lambda\tilde{d}_2,\\
\eta_1 &= e^{ia}(\lambda\tilde{d}_1(u,r) + d_1(u,\phi)),\\
\eta_2 &= e^{-ia}(\lambda\tilde{d}_2(u,r) + d_2(u,\phi)),\\
b &= \lambda E(u,r)\lambda^{-1} - \frac{1}{2}(d_1\bar{\tilde{d}}_2\lambda^{-1} - d_1\bar{\tilde{d}}_2\lambda^{-1}\mathbf{I})\\
&\quad - \frac{1}{2}(d_2\bar{\tilde{d}}_1\lambda^{-1} - d_2\bar{\tilde{d}}_1\lambda^{-1}\mathbf{I}) + F(u,\phi), \qquad (2.7)
\end{aligned}$$

where $\dot{\zeta}(u,r_0) = \dot{\tilde{a}}(u,r_0) = \dot{\tilde{c}}(u,r_0) = \dot{\tilde{d}}_1(u,r_0) = \dot{\tilde{d}}_2(u,r_0) = \dot{E}(u,r_0) = 0$. Thus, even on shell, the system contains arbitrary local functions λ, F, a, c, d_1, and d_2 of time u (and ϕ) as residual degrees of freedom of the gauge system.

In Ref. [23] for $\mathcal{N}=2$ supergravity, the asymptotic fall of condition on the r-independent part of the gauge field was given as

$$\begin{aligned}
a = \sqrt{2}\Big[&J_1 + \frac{\pi}{k}\left(\mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2\right)J_0 + \frac{\pi}{k}\left(\mathcal{J} + \frac{2\pi}{k}\tau\mathcal{Z}\right)P_0\\
&- \frac{\pi}{k}\psi_i Q^i_+ - \frac{2\pi}{k}\mathcal{Z}T - \frac{2\pi}{k}\tau Z\Big]d\phi\\
&+ \left[\sqrt{2}P_1 + \frac{8\pi}{k}\mathcal{Z}Z + \frac{\pi}{k}\left(\mathcal{P} - \frac{4\pi}{k}\mathcal{Z}^2\right)P_0\right]du. \qquad (2.8)
\end{aligned}$$

Here, $\mathcal{P}$, $\mathcal{J}$, $\mathcal{Z}$, τ, and ψ_i are functions of u and ϕ only. These are the residual degrees of freedom that correspond to λ, F, a, c, d_1, and d_2 as introduced above in Eq. (2.7). We do not consider the holonomy terms, and, hence, the resulting action principle at the boundary captures only the asymptotic symmetries of the original gravitational theory.

III. $\mathcal{N}=2$ SUPER-POINCARÉ WESS-ZUMINO-WITTEN MODEL AND ITS SYMMETRIES

In this section, we shall write down the dual WZW model that describes the dynamics of the above theory (2.4). For this purpose, notice that the asymptotic gauge field (2.8) is highly constrained. First, its u and ϕ components are related as

$$\begin{aligned}
&e^a_u = \omega^a_\phi, \qquad \omega^a_u = 0, \qquad \psi^\pm_{Iu} = 0,\\
&B_u = 0, \qquad -4B_\phi = C_u. \qquad (3.1)
\end{aligned}$$

The u component of the gauge field (2.8) is further constrained as

$$\begin{aligned}
&\hat{\omega}^1_\phi = \sqrt{2}; \qquad \omega^2_\phi = 0;\\
&\psi^{1+}_\phi = \psi^{2+}_\phi = 0; \qquad e^1_\phi = e^2_\phi = 0. \qquad (3.2)
\end{aligned}$$

As the gauge field does not vanish at the boundary, for a well-defined variational principle, we need the surface term to vanish in the action. At the boundary, the surface term looks like

$$\begin{aligned}
I_{\text{surf}} &= -\frac{k}{2\pi}\int dud\phi\langle A_u, \delta\tilde{A}\rangle_{r_0\to\infty}\\
&= -\frac{k}{4\pi}\int_{\partial M} dud\phi[\omega^a_\phi\omega_{a\phi} + 4B^2_\phi]_{r_0\to\infty}, \qquad (3.3)
\end{aligned}$$

where the ϕ- total derivative has been set to zero as ϕ is a compact direction. Using the field parameters as defined in Eq. (2.4), the supertrace elements of the algebra,[2] and the configuration (2.7), the total on-shell action can be expressed as

$$\begin{aligned}
I_{(2,0)} = \frac{k}{4\pi}\Big\{\int dud\phi\mathrm{Tr}\Big[&2\mu\lambda^{-1}\lambda'\lambda^{-1}\dot{\lambda} - 2(\bar{d}_1{}'\dot{d}_2 + \bar{d}_2{}'\dot{d}_1)\\
&- 2ia'(\bar{d}_1\dot{d}_2 - \bar{d}_2\dot{d}_1 + \dot{\lambda}\lambda^{-1}(d_2\bar{d}_1 - d_1\bar{d}_2))\\
&- 4(a')^2 - 4\dot{\lambda}\lambda^{-1}\Big(\frac{1}{2}(d_1\bar{d}_2' - d_1\bar{d}_2{}'\mathbf{I})\\
&+ \frac{1}{2}(d_2\bar{d}_1' - d_2\bar{d}_1{}'\mathbf{I}) + F'\Big) - 2(\lambda^{-1}\lambda')^2 + 2i\dot{a}c' + \bar{\mu}a'\dot{a}\Big]\\
&+ \frac{2\mu}{3}\int \mathrm{Tr}[(d\Lambda\Lambda^{-1})^3]\Big\}. \qquad (3.4)
\end{aligned}$$

One can convince himself that the above action describes a chiral WZW model with gauge group $SL(2,R)$. We refer the readers to Ref. [28] for the detailed computations required to arrive at the above result. The system shows gauge invariance under the following (infinitesimal) gauge transformation:

[2]Supertrace elements were computed in Ref. [28] and are given as

$$\begin{aligned}
&\langle J_a, P_b\rangle = \eta_{ab}, \qquad \langle J_a, J_b\rangle = \mu\eta_{ab}, \qquad \langle Q^I_\alpha, Q^J_\beta\rangle = \delta^{IJ}C_{\alpha\beta},\\
&\langle T, Z\rangle = -1, \qquad \langle T, T\rangle = \bar{\mu}.
\end{aligned}$$

$$\delta\lambda = \beta\lambda, \qquad \delta d_i = \beta d_i, \qquad \delta F = [\beta, F], \quad (3.5)$$

where the transformation parameter

$$\beta = \begin{pmatrix} \beta_1 & \beta_2 \\ \beta_3 & \beta_4 \end{pmatrix} \quad (3.6)$$

is a function of u and other fields do not transform.

A. Global symmetries of the chiral WZW model

The WZW model of Eq. (3.4) is invariant under a set of global symmetries. As shown in Ref. [28], various fields change in a coordinate (u,ϕ)-dependent transformation under these symmetries as

$$\begin{aligned} & a \to a + A(\phi); \qquad c \to c - 4iuA'; \qquad d_1 \to e^{-iA} d_1; \\ & d_2 \to e^{iA} d_2, \qquad c \to c + \mathcal{C}(\phi), \qquad \lambda \to \lambda\theta^{-1}(\phi); \\ & F \to F + u\lambda(\theta^{-1}\theta')\lambda^{-1}, \qquad F \to F + \lambda N(\phi)\lambda^{-1}, \\ & d_1 \to d_1 + \lambda D_1(\phi); \qquad c \to c + \bar{D}_1(\phi)\lambda^{-1} d_2; \\ & F \to F - \frac{1}{2} d_2 \bar{D}_1(\phi)\lambda^{-1}, \qquad d_2 \to d_2 + \lambda D_2(\phi); \\ & c \to c - \bar{D}_2(\phi)\lambda^{-1} d_1; \qquad F \to F - \frac{1}{2} d_1 \bar{D}_2(\phi)\lambda^{-1}. \quad (3.7) \end{aligned}$$

In each of the above expressions, the fields that are not written remain unchanged under that corresponding symmetry transformation. Symmetries are generated by scalar parameters $A(\phi)$ and $\mathcal{C}(\phi)$, matrix-valued parameters $\theta(\phi)$ and $N(\phi)$, and spinor parameters $D_1(\phi)$ and $D_2(\phi)$. These parameters are independent of u, and, thus, they represent global symmetry transformations.

The conserved currents corresponding to the above symmetries have also been constructed in Ref. [28]. Below, we present those currents:

$$\begin{aligned} J^\mu_A &= \delta^\mu_0 \frac{k}{4\pi} \mathrm{Tr}[2\bar{\mu}a' + 2ic' - 8ua'' \\ &\quad + 2i(\bar{d}_2{}' d_1 - \bar{d}_1{}' d_2 - ia'(\bar{d}_2 d_1 + \bar{d}_1 d_2))]A \\ &= \delta^\mu_0[(-Q^A)(-A)], \\ J^\mu_C &= \delta^\mu_0 \frac{k}{4\pi} \mathrm{Tr}[2ia'\mathcal{C}] = \delta^\mu_0[Q_C(-i\mathcal{C})], \qquad Q_C = -\frac{ka'}{2\pi}, \\ J^\mu_\Theta &= \delta^\mu_0 \frac{k}{2\pi} \mathrm{Tr}[\{\lambda^{-1}\hat{\alpha}\lambda + 2u(\lambda^{-1}\lambda')' - 2\mu\lambda^{-1}\lambda'\}\Theta] \\ &= \delta^\mu_0 2\mathrm{Tr}[Q^J_a \Theta^a], \\ J^\mu_N &= \delta^\mu_0 \frac{k}{4\pi} \mathrm{Tr}[-4\lambda^{-1}\lambda' N] = \delta^\mu_0 2\mathrm{Tr}[Q^P_a(-N^a)], \\ J^\mu_{D_2} &= \delta^\mu_0 \left(-\frac{k}{\pi}\right) \mathrm{Tr}[(\bar{d}_1{}'\lambda + ia'\bar{d}_1\lambda)D_2] = \delta^\mu_0 \mathrm{Tr}[Q^{G_2}_\alpha D^\alpha_2], \\ J^\mu_{D_1} &= \delta^\mu_0 \left(-\frac{k}{\pi}\right) \mathrm{Tr}[(\bar{d}_2{}'\lambda - ia'\bar{d}_2\lambda)D_1] \\ &= \delta^\mu_0 \mathrm{Tr}[Q^{G_1}_\alpha D^\alpha_1], \quad (3.8) \end{aligned}$$

where $N(\phi)$ and $\Theta(\phi)$ are infinitesimal $SL(2,\mathbb{R})$ matrices which can be further expanded in the basis of Γ matrices as $N(\phi) = N^a(\phi)\Gamma_a$ and $\Theta(\phi) = \Theta^a(\phi)\Gamma_a$. Other parameters have also been considered as infinitesimal. It can be checked that the above currents satisfy the following current algebra:

$$\begin{aligned} \{Q^P_a(\phi), Q^P_b(\phi')\}_{DB} &= \{Q^P_a(\phi), Q^A(\phi')\}_{DB} \\ &= \{Q^P_a(\phi), Q^C(\phi')\}_{DB} = 0, \\ \{Q^P_a(\phi), Q^{G_1}_\alpha(\phi')\}_{DB} &= \{Q^P_a(\phi), Q^{G_2}_\alpha(\phi')\}_{DB} = 0, \\ \{Q^P_a(\phi), Q^J_b(\phi')\}_{DB} &= \{Q^J_a(\phi), Q^P_b(\phi')\}_{DB} \\ &= \epsilon_{abc} Q^P_c(\phi)\delta(\phi-\phi') \\ &\quad - \frac{k}{2\pi}\eta_{ab}\partial_\phi\delta(\phi-\phi'), \\ \{Q^J_a(\phi), Q^J_b(\phi')\}_{DB} &= \epsilon_{abc} Q^J_c(\phi)\delta(\phi-\phi') \\ &\quad + \mu\frac{k}{2\pi}\eta_{ab}\partial_\phi\delta(\phi-\phi'), \\ \{Q^J_a(\phi), Q^A(\phi')\}_{DB} &= \{Q^J_a(\phi), Q^C(\phi')\}_{DB} = 0, \\ \{Q^{G_1}_\alpha(\phi), Q^J_a(\phi')\}_{DB} &= -\frac{1}{2}(\Gamma_a)^\beta_\alpha Q^{G_1}_\beta(\phi)\delta(\phi-\phi'), \\ \{Q^{G_2}_\alpha(\phi), Q^J_a(\phi')\}_{DB} &= -\frac{1}{2}(\Gamma_a)^\beta_\alpha Q^{G_2}_\beta(\phi)\delta(\phi-\phi'), \\ \{Q^C(\phi), Q^C(\phi')\}_{DB} &= \{Q^C(\phi), Q^{G_1}_\alpha(\phi')\}_{DB} \\ &= \{Q^C(\phi), Q^{G_2}_\alpha(\phi')\}_{DB} = 0, \\ \{Q^C(\phi), Q^A(\phi')\}_{DB} &= \frac{k}{2\pi}\partial_\phi\delta(\phi-\phi'), \\ \{Q^A(\phi), Q^A(\phi')\}_{DB} &= \frac{k}{2\pi}\bar{\mu}\partial_\phi\delta(\phi-\phi'), \\ \{Q^{G_1}_\alpha(\phi), Q^A(\phi')\}_{DB} &= -iQ^{G_1}_\alpha(\phi)\delta(\phi-\phi'), \\ \{Q^{G_2}_\alpha(\phi), Q^A(\phi')\}_{DB} &= iQ^{G_2}_\alpha(\phi)\delta(\phi-\phi'), \\ \{Q^{G_1}_\alpha(\phi), Q^{G_2}_\beta(\phi')\}_{DB} &= -(C\Gamma^a)_{\alpha\beta} Q^P_a\delta(\phi-\phi') \\ &\quad - \frac{k}{\pi}C_{\alpha\beta}\partial_\phi\delta(\phi-\phi') \\ &\quad + ia'\frac{k}{\pi}C_{\alpha\beta}\delta(\phi-\phi'). \quad (3.9) \end{aligned}$$

Next, we notice that the constraints of Eq. (3.2) further imply that the canonical current generators are constrained as

$$\begin{aligned} & Q^P_0 = \sqrt{2}\frac{k}{2\pi}, \qquad Q^J_0 = -\sqrt{2}\frac{\mu k}{2\pi}, \\ & Q^1_+ = 0, \qquad Q^2_+ = 0, \\ & Q^P_2 = 0, \qquad Q^J_2 = 0, \quad (3.10) \end{aligned}$$

where

$$\begin{aligned} Q^1_\alpha(\phi) &= \frac{1}{2}(Q^{G_1}_\alpha(\phi) + Q^{G_2}_\alpha(\phi)), \\ Q^2_\alpha(\phi) &= \frac{1}{2i}(Q^{G_1}_\alpha(\phi) - Q^{G_2}_\alpha(\phi)), \qquad \alpha = \pm. \end{aligned}$$

The above relations can be expressed as constraints on various fields of the WZW model (3.4). Thus, we see that, when the asymptotic boundary condition of Eq. (2.8) is explored to its full capacity, the theory gets more constrained. The first four of Eqs. (3.10) are first-class constraints, and they will produce a gauge invariance for the system. To understand the proper symmetry structure of the theory, we shall need gauge-invariant canonical symmetry generators. Hence, we implement a modified Sugawara construction to define these gauge invariant currents as

$$\mathcal{H}=H+\partial_\phi Q_2^P;\qquad \mathcal{J}=J-\partial_\phi Q_2^J;$$
$$\hat{\mathcal{G}}^I=\mathcal{G}^I+\partial_\phi Q_+^I,\ Q^A,\ Q^C, \tag{3.11}$$

where we have defined

$$\begin{aligned}
H&=\frac{\pi}{k}Q_a^PQ_a^P+4\frac{\pi}{k}Q^CQ^C,\\
J&=-\mu\frac{\pi}{k}Q_a^PQ_a^P-2\frac{\pi}{k}Q_a^JQ_a^P+\frac{\pi}{k}C_{\alpha\beta}Q_\alpha^{G_1}Q_\beta^{G_2}\\
&\quad+2\frac{\pi}{k}Q^AQ^C-\bar{\mu}\frac{\pi}{k}Q^CQ^C,\\
\mathcal{G}^1&=\frac{\pi}{k}(Q_2^PQ_+^{G_1}+\sqrt{2}Q_0^PQ_-^{G_1})+2i\frac{\pi}{k}Q_+^{G_1}Q^C,\\
\mathcal{G}^2&=\frac{\pi}{k}(Q_2^PQ_+^{G_2}+\sqrt{2}Q_0^PQ_-^{G_2})-2i\frac{\pi}{k}Q_+^{G_1}Q^C.
\end{aligned}\tag{3.12}$$

It is easy to see that the above conserved charges are close to following generalized quantum super BMS_3 algebra on the constrained surface as

$$\begin{aligned}
\{\mathcal{J}(\phi),\mathcal{J}(\phi')\}_{DB}&=(\mathcal{J}(\phi)+\mathcal{J}(\phi'))\partial_\phi\delta(\phi-\phi')\\
&\quad-\mu\frac{k}{2\pi}\partial_\phi^3\delta(\phi-\phi'),\\
\{\mathcal{H}(\phi),\mathcal{J}(\phi')\}_{DB}&=(\mathcal{H}(\phi)+\mathcal{H}(\phi'))\partial_\phi\delta(\phi-\phi')\\
&\quad-\frac{k}{2\pi}\partial_\phi^3\delta(\phi-\phi'),\\
\{\tilde{\mathcal{H}}(\phi),\tilde{\mathcal{H}}(\phi')\}_{DB}&=0,\\
\{\mathcal{H}(\phi),Q^A(\phi')\}_{DB}&=4Q^C(\phi)\partial_\phi\delta(\phi-\phi'),\\
\{\mathcal{J}(\phi),Q^A(\phi')\}_{DB}&=Q^A(\phi)\partial_\phi\delta(\phi-\phi'),\\
\{\mathcal{J}(\phi),Q^C(\phi')\}_{DB}&=Q^C(\phi)\partial_\phi\delta(\phi-\phi'),\\
\{\mathcal{J}(\phi),Q^A(\phi')\}_{DB}&=Q^A(\phi)\partial_\phi\delta(\phi-\phi'),\\
\{Q^C(\phi),Q_A(\phi')\}_{DB}&=\frac{k}{2\pi}\partial_\phi\delta(\phi-\phi'),\\
\{Q^A(\phi),Q^A(\phi')\}_{DB}&=\frac{k}{2\pi}\bar{\mu}\partial_\phi\delta(\phi-\phi'),\\
\{\mathcal{J}(\phi),\hat{\mathcal{G}}^i(\phi')\}_{DB}&=\left(\hat{\mathcal{G}}^i(\phi)+\frac{1}{2}\hat{\mathcal{G}}^i(\phi')\right)\partial_\phi\delta(\phi-\phi')\\
&\quad(i=1,2),\\
\{\mathcal{H}(\phi),\hat{\mathcal{G}}^i(\phi')\}_{DB}&=0\quad(i=1,2),\\
\{\hat{\mathcal{G}}^1(\phi),Q^A(\phi')\}_{DB}&=-i\hat{\mathcal{G}}^1(\phi)\delta(\phi-\phi'),\\
\{\hat{\mathcal{G}}^2(\phi),Q^A(\phi')\}_{DB}&=i\hat{\mathcal{G}}^2(\phi)\delta(\phi-\phi'),\\
\{\hat{\mathcal{G}}^1(\phi),\hat{\mathcal{G}}^2(\phi')\}_{DB}&=\mathcal{H}(\phi)\delta(\phi-\phi')-\frac{k}{\pi}\partial_\phi^2\delta(\phi-\phi')\\
&\quad-2i(Q^C(\phi)+Q^C(\phi'))\delta'(\phi-\phi').
\end{aligned}\tag{3.13}$$

Let us next gauge the symmetries introduced at the beginning of Sec. III A in Eq. (3.7). As we have already mentioned, there are four first-class constraints as noted in Eq. (3.10), and they will produce four gauge symmetries to the system. Thus, it is clear to see that the last four transformations of Eq. (3.7) can be gauged; i.e., the transformation parameters can be made a local function of u as well. Below, we present the gauged version of the chiral WZW model (3.4).

B. Gauging the chiral WZW model

Imposing the first-class constraints of Eq. (3.2) on on-shell gauge field parameters implies the following relations:

$$(\lambda^{-1}\lambda')^1=\sqrt{2},\qquad\left(\lambda^{-1}\frac{\hat{\alpha}}{2}\lambda\right)^1=0,$$
$$(\lambda^{-1}d_1'+ia'\lambda^{-1}d_1)^-=(\lambda^{-1}d_2'-ia'\lambda^{-1}d_2)^-=0.$$

The above relations can be equivalently recast in terms of global symmetry currents as given in Eq. (3.10). Here, we are setting a part of the currents to a constant or zero value that comes from symmetry transformations involving bosonic symmetry transformation parameter N, Θ along Γ_0 and fermionic parameters $[\bar{D}_1]_+=[\bar{D}_2]_+=0$. To gauge the corresponding symmetries, one needs four "gauge fields" corresponding to four constrained currents J. Since the currents are nontrivial along u directions, only the u component of the gauge fields will appear in the modified action. In general, for gauging a global symmetry, we need to replace the ordinary derivatives on various fields by the corresponding covariant ones. For the WZW model on a Lie group G, only special subgroups of G can be gauged, as, otherwise, the WZW term makes it anomalous. The detailed procedure of gauging the WZW model has been greatly described in a seminal paper [36], where it has been noted that only subgroups generated by root vectors associated with positive and negative roots can be gauged. This implies the subgroup elements must be nilpotent matrices. A similar strategy has already been implemented in Refs. [17,26] for gauging $SL(2,R)$ chiral WZW models with(out) minimal supersymmetric extensions. We shall follow the procedure of Ref. [14], which is also similar in spirit. In this case, we introduce four Lagrange multipliers for gauging the constrained (only first-class ones) currents.

Since the constrained currents are along Γ_0, the above criteria is satisfied. Using these multipliers, we write down an improved action, where the improvement term is local. Furthermore, the transformations of the Lagrange multipliers are derived by demanding that the full improved action is invariant under the above-mentioned gauging of symmetries. The improved action looks like

$$I[\lambda, c, a, F, d_1, d_2, \Psi, A_\mu] = I[\lambda, c, a, F, d_1, d_2] + I_g, \qquad (3.14)$$

where $I[\lambda, C, F, d_1, d_2]$ is as given in Eq. (3.4) and

$$I_g = \frac{k}{\pi}\int dud\phi \mathrm{Tr}\left[A_u\left(\lambda^{-1}\frac{\hat{\alpha}}{2}\lambda\right) + \tilde{A}_u\lambda^{-1}\lambda' - \mu_M\tilde{A}_u + \frac{1}{2}\left[\lambda^{-1}(d_1' + ia'd_1)\right]\bar{\Psi}_2 + \frac{1}{2}\left[\lambda^{-1}(d_2' - ia'd_2)\right]\bar{\Psi}_1\right]. \qquad (3.15)$$

Here, I_g is a local function of "gauge fields" A, $\tilde{A}$, and $\bar{\Psi}_i$, $i = 1, 2$. A_u and $\tilde{A}_u$ are along Γ_0 and $[\bar{\Psi}_1]_+ = 0 = [\bar{\Psi}_2]_+$. Furthermore, we have chosen $\mu_M \coloneqq \tilde{\mu}\Gamma_1$,[3] where $\tilde{\mu}$ is an arbitrary constant, to be able to set the currents to the required constant value. It can indeed be checked that the above modified action (3.14) is invariant under the following four gauge transformations:

$$T_1:\ \delta_N F = \lambda N\lambda^{-1}, \quad \delta_N\lambda = \delta_N d_1 = \delta_N d_2 = \delta_N c = \delta_N a = 0, \quad \delta_N\Psi_1 = \delta_N\Psi_2 = \delta_N A_u = 0, \quad \delta_N\tilde{A}_u = (\dot{N} + [A_u, N]); \qquad (3.16)$$

$$T_2:\ \delta_\Theta\lambda = -\lambda\Theta, \quad \delta_\Theta F = u\lambda\Theta^{-1}\Theta'\lambda^{-1}, \quad \delta_\Theta d_1 = \delta_\Theta d_2 = \delta_N c = \delta_N a = 0, \quad \delta_\Theta A_u = -(\dot{\Theta} + [A_u, \Theta]), \quad \delta_\Theta\Psi_1 = \delta_\Theta\Psi_2 = 0, \quad \delta_\Theta\tilde{A}_u = u(\dot{\Theta}' + [A_u, \Theta']) + \frac{\mu}{2}\dot{\Theta} - [\tilde{A}_u, \Theta]; \qquad (3.17)$$

$$T_3:\ \delta_{D_1}\lambda = \delta_{D_1}d_2 = \delta_N a = 0, \quad \delta_{D_1}d_1 = \lambda D_1, \quad \delta_{D_1}C = \bar{D}_1\lambda^{-1}d_2, \quad \delta_{D_1}F = -\frac{1}{2}d_2\bar{D}_1\lambda^{-1}, \quad \delta_{D_1}A_u = \delta_{D_1}\tilde{A}_u = \delta_{D_1}\Psi_2 = 0, \quad \delta_{D_1}\Psi_1 = -\partial_\mu\bar{D}_1; \qquad (3.18)$$

$$T_4:\delta_{D_2}\lambda = \delta_{D_2}d_1 = \delta_N a = 0, \quad \delta_{D_2}d_2 = \lambda D_2, \quad \delta_{D_2}C = -\bar{D}_2\lambda^{-1}d_1, \quad \delta_{D_2}F = -\frac{1}{2}d_1\bar{D}_2\lambda^{-1}, \quad \delta_{D_2}A_u = \delta_{D_2}\tilde{A}_u = \delta_{D_2}\Psi_1 = 0, \quad \delta D_2\Psi_2 = -\partial_\mu\bar{D}_2. \qquad (3.19)$$

[3] Γ_1 is also a nilpotent matrix.

Note that all the parameters of the transformation mentioned here depend on both (u, ϕ) and, as said earlier, Θ, N are along Γ_0 and $[\bar{D}_1]_+ = [\bar{D}_2]_+ = 0$. The equations of motion of the four nondynamical Lagrange multipliers A_u, $\tilde{A}_u$, $\bar{\Psi}_1$, and $\bar{\Psi}_2$ rightly reproduce back the constrained relations as given in the beginning of this section if one chooses $\tilde{\mu} = \frac{1}{\sqrt{2}}$. Thus, we conclude that Eq. (3.14) represents the gauged version of chiral WZW model (3.4), where we have gauged the specific part of global symmetries whose corresponding currents give first-class constraints. The gauge symmetry along Γ_2 is still present. In the next section, we shall write down the reduced phase space description for the WZW model (3.4) and show that the reduced action is an equivalent description of the above gauged chiral WZW model of (3.14). We shall also comment on the equivalence of the residual symmetries of the two descriptions.

IV. LIOUVILLE-LIKE THEORY

In this section, we present the reduced phase space description of the chiral WZW model of Eq. (3.4). For this purpose, a particular decomposition of the fields, known as Gauss decomposition,[4] is useful. The procedure is to expand the fields in the Chevalley-Serre basis of the corresponding gauge group, i.e., $SL(2, R)$ for the present case. Our conventions are listed in Appendix A. The decomposition is

$$\lambda = e^{\sigma\Gamma_1/2}e^{-\varphi\Gamma_2/2}e^{\tau\Gamma_0}, \quad F = -\left(\frac{\eta}{2}\Gamma_0 + \frac{\theta}{2}\Gamma_2 + \frac{\zeta}{2}\Gamma_1\right), \qquad (4.1)$$

where σ, φ, τ, η, θ, and ξ are scalar fields and are functions of both u and ϕ. The Gaussian decomposition is useful, as in this decomposition the 3D bulk part of the WZW model (3.4) simplifies to a total derivative term as[5]

$$\frac{2}{3}\mathrm{Tr}[(d\Lambda\Lambda^{-1})^3] = drdud\phi \quad \epsilon^{\nu\gamma\delta}\partial_\nu(e^{-\varphi}\partial_\gamma\tau\partial_\delta\sigma).$$

Thus, using Stoke's formula, the bulk term can be reduced to a two-dimensional integral. This makes further computations technically simple. Two product operators that are mostly used are given as

$$\lambda^{-1}\lambda' = \begin{pmatrix} -\sigma'\tau e^{-\phi} - \phi'/2 & -\sqrt{2}\sigma'\tau^2 e^{-\phi} + \sqrt{2}\tau' - \sqrt{2}\tau\phi' \\ \frac{\sigma'}{\sqrt{2}}e^{-\phi} & \tau\sigma' e^{-\phi} + \phi'/2 \end{pmatrix} \qquad (4.2)$$

[4] Important aspects of Gauss decomposition are discussed in Ref. [38].

[5] Here, we have assumed the same notation for component fields, but they have a dependence on all three directions, r, u, and ϕ.

and

$$\dot{\lambda}\lambda^{-1} = \begin{pmatrix} -\frac{\dot{\phi}}{2}-\sigma\dot{\tau}e^{-\phi} & \sqrt{2}\dot{\tau}e^{-\phi} \\ \frac{\dot{\sigma}}{\sqrt{2}}-\frac{\sigma}{\sqrt{2}}\dot{\phi}-\frac{\sigma^2}{\sqrt{2}}\dot{\tau}e^{-\phi} & \frac{\dot{\phi}}{2}+\sigma\dot{\tau}e^{-\phi} \end{pmatrix}. \quad (4.3)$$

The first-class constraints can also be recast in terms of these newly defined fields. Let us first look at the Q_0^P condition. It reduces as

$$Q_0^P = \frac{\sqrt{2}k}{2\pi} \Rightarrow \sigma' = \sqrt{2}e^{\varphi}. \quad (4.4)$$

Next, we look at two fermionic current constraints. They are given as

$$\begin{aligned} Q_+^1 = 0 \Rightarrow & \left[-d_1^{-\prime} + d_1^{+\prime}\frac{\sigma}{\sqrt{2}} - d_2^{-\prime} + \frac{\sigma}{\sqrt{2}}d_2^{+\prime}\right] \\ & + (ia')\left[-d_1^- + d_1^+\frac{\sigma}{\sqrt{2}} + d_2^- - \frac{\sigma}{\sqrt{2}}d_2^+\right] = 0, \\ Q_+^2 = 0 \Rightarrow & \left[d_1^{-\prime} - d_1^{+\prime}\frac{\sigma}{\sqrt{2}} - d_2^{-\prime} + \frac{\sigma}{\sqrt{2}}d_2^{+\prime}\right] \\ & + (ia')\left[d_1^- - d_1^+\frac{\sigma}{\sqrt{2}} + d_2^- - \frac{\sigma}{\sqrt{2}}d_2^+\right] = 0. \quad (4.5) \end{aligned}$$

Redefining new fermionic parameters as $d_1 = e^{ia}d_1^N$ and $d_2 = e^{-ia}d_2^N$, the above two conditions can be written in a compact form as

$$d_1^{N-\prime} = \frac{\sigma}{\sqrt{2}}d_1^{N+\prime}, \qquad d_2^{N-\prime} = \frac{\sigma}{\sqrt{2}}d_2^{N+\prime}. \quad (4.6)$$

Finally, we look at the Q_0^J constraint. The reduced constraint looks like

$$\begin{aligned} Q_0^J = & -\frac{\sqrt{2}\mu k}{2\pi} \\ \Rightarrow & 2\mu(\sqrt{2}e^{\varphi} - \sigma') + 2u(\sigma'' - \varphi'\sigma') - \eta'\sigma^2 - 2\sigma\theta' + 2\zeta' \\ & + (ia')[2\sigma d_2^+ d_1^- - 2\sqrt{2}d_2^- d_1^- - \sqrt{2}\sigma^2 d_2^+ d_1^+ + 2\sigma d_2^- d_1^+] \\ & + (\sigma)(d_2^+ d_1^{-\prime} + d_2^- d_1^{+\prime} + d_1^+ d_2^{-\prime} + d_1^- d_2^{+\prime}) \\ & + (\sqrt{2})(-d_2^- d_1^{-\prime} - d_1^- d_2^{-\prime}) - \left(\frac{\sigma^2}{\sqrt{2}}\right)(d_2^+ d_1^{+\prime} + d_1^+ d_2^{+\prime}) = 0. \end{aligned}$$

Using the redefined fermions and the last three current constraints, the above condition simplifies as

$$-\eta'\sigma^2 - 2\sigma\theta' + 2\zeta' = 0. \quad (4.7)$$

Equations (4.4), (4.6), and (4.7) represents the first-class constraints in terms of the new fields.

A. The reduced action

We present the action by computing various terms of Eq. (3.4) in terms of the above newly defined fields and reducing it further by using the constraint relations of the last section as[6]

$$\begin{aligned} I = \frac{k}{4\pi}\int dud\phi[& \mu\varphi'\dot{\varphi} + \xi'\dot{\varphi} - \varphi'^2 - 2i\dot{a}'D + \bar{\mu}a'\dot{a} \\ & - 4(a')^2 + 2(\dot{\chi}_1\chi_2 + \dot{\chi}_2\chi_1)], \end{aligned} \quad (4.8)$$

where the redefined fields are

$$\begin{aligned} & \xi = 2(\theta + \sigma\eta) + \bar{d}_2 d_1, \qquad \chi_i = e^{\phi/2}d_i^+, \\ & D = c + (d_2^+ d_1^- - d_1^+ d_2^-) - \sqrt{2}\sigma d_2^+ d_1^+. \end{aligned}$$

Equation (4.8) is a flat limit of a super-Liouville action with two supercharges and two internal R-symmetry fields. This is a generalized version of the flat limit of super-Liouville actions presented in Refs. [17,26]. To understand the connection with Liouville, we refer readers to Appendix C. The above action (4.8) is equivalent to the gauged chiral WZW model of Eq. (3.14). Solving the algebraic equation of motions of the Lagrange multipliers and putting it back in Eq. (3.14) will exactly give us Eq. (4.8), when expressed in terms of Gauss variables.

Finally, we present a realization of super BMS_3 generators of Eq. (3.11) in terms of Liouville fields. With straight algebra and the use of (4.4), (4.6), and (4.7), they can be found as

$$\begin{aligned} \mathcal{H} = & \frac{k}{4\pi}[\varphi'^2 - 2\varphi'' + 4(a')^2], \\ \mathcal{J} = & \frac{k}{4\pi}[\xi'\varphi' - \xi'' + u(2\varphi''' - 2\varphi'\varphi'' - 8a''a') \\ & + 2(\chi_1'\chi_2 + \chi_2'\chi_1) + 2ia'D' + \bar{\mu}(a')^2 - 12\mu(a')^2] + \mu\mathcal{H}, \\ \hat{\mathcal{G}}^1 = & \frac{k}{\pi}\left(\left(\frac{\varphi'}{2} + \imath a'\right)\chi_2 - \chi_2'\right), \\ \hat{\mathcal{G}}^2 = & \frac{k}{\pi}\left(\left(\frac{\varphi'}{2} - \imath a'\right)\chi_1 - \chi_1'\right), \\ Q^A = & -\frac{k}{4\pi}\mathrm{Tr}[2\bar{\mu}a' + 2iD' - 8ua'' + 4i\chi_2\chi_1], \\ Q^C = & -\frac{ka'}{2\pi}. \end{aligned} \quad (4.9)$$

It can be checked that the above generators constitute a set of global symmetries of the reduced action (4.8). To obtain the symmetry transformations, we first find the canonical conjugate momenta of fields of Eq. (4.8). They are given as

[6]Look at Appendix B for some details.

$$p_\varphi = \frac{k}{4\pi}(\xi' + \mu\varphi'), \quad p_\xi = \frac{k}{4\pi}\varphi', \quad p_a = \frac{k}{4\pi}(\bar{\mu}a' + 2iD'),$$
$$p_D = \frac{k}{4\pi}2ia', \quad p_{\chi_1} = \frac{k}{4\pi}4\chi_2, \quad p_{\chi_2} = \frac{k}{4\pi}4\chi_1. \tag{4.10}$$

To obtain the variation of various fields, we use the Hamiltonian formulation. The variation of fields can be computed from their Poisson brackets with the global charge as

$$-\delta A = \{A, Q\}, \tag{4.11}$$

where

$$Q = \int_0^{2\pi} d\phi [\mathcal{H}T + \mathcal{J}Y + Q^A B + Q^C K + 2\mathcal{G}^1\epsilon_1 + 2\mathcal{G}^2\epsilon_2] \tag{4.12}$$

and T, Y, B, K, ϵ_1, and ϵ_2 are ϕ-dependent symmetry transformation parameters. The transformations of various fields are then given as

$$-\delta\varphi = Y\varphi' + Y', \tag{4.13}$$

$$-\delta\xi = 2f\varphi' + \xi' Y + 2f' - 4\epsilon_1\chi_2 - 4\epsilon_2\chi_1, \tag{4.14}$$

$$-\delta a = a'Y - B, \tag{4.15}$$

$$-\delta D = D'Y - 4ia'T + 4iuB' + iK - 4iua'Y' - 4\epsilon_1\chi_2 + 4\epsilon_2\chi_1 - ia'\bar{\mu}Y + i\bar{\mu}B, \tag{4.16}$$

$$-\delta\chi_1 = -\chi_1' Y - \frac{1}{2}Y'\chi_1 + 2\left(\frac{\varphi'}{2} + ia'\right)\epsilon_1 + 2\epsilon_1' - i\chi_1 B, \tag{4.17}$$

$$-\delta\chi_2 = -\chi_2' Y - \frac{1}{2}\chi_2 Y' + 2\left(\frac{\varphi'}{2} - ia'\right)\epsilon_2 + 2\epsilon_2' + i\chi_2 B, \tag{4.18}$$

where

$$f = T(\phi) + \mu Y(\phi) + uY'. \tag{4.19}$$

It can be checked that the above transformations are the global (u-independent) symmetry of the reduced action (4.8), as expected. The algebra of the corresponding Noether charges is again super BMS_3 of Eq. (3.13). The system is also invariant under transformations

$$\delta\varphi = F_1(u), \qquad \delta\xi = F_2(u), \qquad \delta D = F_3(u).$$

Let us briefly elaborate on the source of these local u-dependent symmetries: The symmetry transformation of ξ and D is an artifact of the form of the reduced action, as it involves only the ϕ derivative of these fields. The symmetry transformation of φ is related to the gauge invariance of Eq. (3.4), as given in Eq. (3.5). For the Gauss decomposed fields, Eq. (3.5) implies the following transformation:

$$\delta\varphi = -2\beta_1 - \sqrt{2}\beta_2\sigma, \qquad \delta\theta = \sqrt{2}\beta_2\zeta - \sqrt{2}\beta_3\eta,$$
$$\delta\eta = (\beta_1 - \beta_4)\eta - \sqrt{2}\beta_2\theta,$$
$$\delta\sigma = \sqrt{2}\beta_3 + \beta_4\sigma - \beta_1\sigma - \beta_2\frac{\sigma^2}{\sqrt{2}},$$
$$\delta d_i^+ = \beta_1 d_i^+ + \beta_2 d_i^-, \qquad \delta d_i^- = \beta_3 d_i^+ + \beta_4 d_i^- \tag{4.20}$$

with $\beta_1 + \beta_4 = 0$. Hence, we can decompose the β matrix in the basis of $SL(2,R)$ generators, like field F. Here, β_1, β_2, and β_3 are three independent transformation parameters that depend on u. For the reduced fields of Eq. (4.8), we get

$$\delta\varphi = -2\beta_1 - \sqrt{2}\beta_2\sigma, \quad \delta\xi = 2\sqrt{2}\beta_2\zeta - 2\sqrt{2}\sigma\beta_2\theta - \sqrt{2}\beta_2\eta\sigma^2, \tag{4.21}$$

$$\delta D = 2\beta_2 d_2^- d_1^- - \sqrt{2}\sigma\{\beta_2 d_2^- d_1^+ + \beta_2 d_2^+ d_1^-\} + \beta_2\sigma^2 d_2^+ d_1^+, \tag{4.22}$$

$$\delta\chi_i = -\frac{\beta_2}{\sqrt{2}}\sigma e^{\varphi/2} d_i^+ + e^{\varphi/2}\beta_2 d_i^-. \tag{4.23}$$

It can be checked that the above transformations, in the presence of all three parameters β_1, β_2, and β_3 are not symmetries of Eq. (4.8). Instead, the transformation of φ by turning on only β_1 is a symmetry of the action. Turning on β_1 implies gauge transformation along Γ_2, which is the only non-nilpotent generator of $SL(2,R)$. This is the residual gauge symmetry of the gauged chiral WZW model of Eq. (3.14). Thus, we see that both the reduced phase space Lagrangian (4.8) and the gauged chiral WZW model (3.14) preserve identical global and gauge symmetry.

V. OUTLOOK

In this paper, we have presented three equivalent descriptions of the $\mathcal{N} = 2$ three-dimensional supergravity theory. The first description in terms of a chiral WZW model was derived in Ref. [28], whereas the other two equivalent descriptions in terms of a gauged version of the chiral WZW model and a flat limit of generalized super-Liouville theory have been derived in this paper. All these theories are invariant under the most generic quantum $\mathcal{N} = 2$ super BMS_3 symmetry constructed in Ref. [28] at null infinity.

One interesting point to note here is that the Liouville theory can also be viewed as a free field theory under proper Bäclund transformations [39]. In Refs. [40,41], we have presented a free field realization of BMS_3 algebra and its supersymmetric and higher spin generalizations. It would be nice to find a connection between these two realizations. We hope to report on this in the future.

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APPENDIX A: CONVENTIONS AND IDENTITIES

In this appendix, we shall present our conventions. The tangent space metric η_{ab}, $a=0,\ 1,\ 2$, is flat and off diagonal, given as

$$\eta_{ab}=\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}.$$

The space time coordinates are u,ϕ,r with the positive orientation in the bulk being $dud\phi dr$. Accordingly, the Levi-Civita symbol is chosen such that $\epsilon_{012}=1$.

The three-dimensional Dirac matrices satisfy the usual commutation relation $\{\Gamma_a,\Gamma_b\}=2\eta_{ab}$. They also satisfy the following useful identities:

$$\Gamma_a\Gamma_b=\epsilon_{abc}\Gamma^c+\eta_{ab}\mathbb{I},\qquad (\Gamma^a)^\alpha_\beta(\Gamma_a)^\gamma_\delta=2\delta^\alpha_\delta\delta^\gamma_\beta-\delta^\alpha_\beta\delta^\gamma_\delta.$$

The explicit forms of the Dirac matrices are chosen as

$$\Gamma_0=\sqrt{2}\begin{pmatrix}0&1\\0&0\end{pmatrix},\quad \Gamma_1=\sqrt{2}\begin{pmatrix}0&0\\1&0\end{pmatrix},\quad \Gamma_2=\begin{pmatrix}1&0\\0&-1\end{pmatrix}. \tag{A1}$$

All spinors in this work are Majorana, and our convention for the Majorana conjugate of the fermions is given as

$$\bar\psi_{\alpha i}=\psi^\beta_i C_{\beta\alpha},\qquad C_{\alpha\beta}=\epsilon_{\alpha\beta}=C^{\alpha\beta}=\begin{pmatrix}0&1\\-1&0\end{pmatrix}.$$

Here, $i=1,\ 2$ is the internal index, and $C_{\alpha\beta}$ is the charge conjugation matrix that satisfies

$$C^T=-C,\qquad C\Gamma_aC^{-1}=-(\Gamma_a)^T,\qquad C_{\alpha\beta}C_{\beta\gamma}=-\delta_{\alpha\gamma}.$$

For any traceless 2×2 matrix A, it can be shown that $C_{\alpha\beta}A^\beta_\gamma=(C\Gamma^a)_{\alpha\gamma}\mathrm{Tr}[\Gamma_aA]$.

Other useful identities are

$$\zeta\bar\eta=-\frac{1}{2}\bar\eta\zeta\mathbb{1}-\frac{1}{2}(\bar\eta\Gamma^a\zeta)\Gamma_a,\qquad \bar\psi\Gamma_a\eta=\bar\eta\Gamma_a\psi,$$
$$\bar\psi\Gamma_a\epsilon=-\bar\epsilon\Gamma_a\psi, \tag{A2}$$

where ζ,ψ, and η are Grassmannian one-forms, while ϵ is a Grassmann parameter.

The generators of $sL(2,R)$ are considered as $\frac{\Gamma_i}{2}$. Furthermore, in the Chevalley-Serre basis, they are given as

$$E_+=\begin{pmatrix}0&1\\0&0\end{pmatrix},\qquad E_-=\begin{pmatrix}0&0\\1&0\end{pmatrix},\qquad H=\begin{pmatrix}1&0\\0&-1\end{pmatrix}. \tag{A3}$$

Thus, $E_\pm$ corresponds to the positive (negative) root of the Cartan subalgebra, and they are nilpotent.

APPENDIX B: TERMS IN REDUCED ACTION

For writing the phase space reduced action, we need to reduce various terms of Eq. (3.4) in terms of Gauss decomposed fields and further use the first-class constraints relations of (4.4), (4.6), and (4.7). Below, we note the simplified forms of various terms:

bosonic terms:

$$\mathrm{Tr}[2\mu\lambda^{-1}\lambda'\lambda^{-1}\dot\lambda]=\mu\varphi'\dot\varphi+2\mu(\dot\sigma\tau'e^{-\varphi})+2\mu(\sigma'\dot\tau e^{-\varphi}), \tag{B1}$$

$$\mathrm{Tr}[-4\dot\lambda\lambda^{-1}F']=-4\left[-\frac{\dot\varphi\theta'}{2}-\sigma\dot\tau e^{-\varphi}\theta'+\dot\tau\zeta'e^{-\varphi}+\frac{\dot\sigma\eta'}{2}-\frac{\sigma\dot\varphi\eta'}{2}-\frac{\sigma^2\dot\tau e^{-\varphi}\eta'}{2}\right], \tag{B2}$$

$$\mathrm{Tr}[-2(\lambda^{-1}\lambda')^2]=-2\left(2\sigma'\tau'e^{-\varphi}+\frac{\varphi'^2}{2}\right), \tag{B3}$$

$$\frac{2\mu}{3}\int\mathrm{Tr}[(d\Lambda\Lambda^{-1})^3]=\int dud\phi(-2\mu(\dot\sigma\tau'e^{-\varphi})+2\mu(\sigma'\dot\tau e^{-\varphi})). \tag{B4}$$

The rest of the terms in the action are scalar $2i\dot aC'+\bar\mu a'\dot a-4(a')^2$, and they will remain as it is. Next, we look at the fermionic terms:

$$\mathrm{Tr}[-2(\bar d_1'\dot d_2+\bar d_2'\dot d_1)]=2d_1^-{}'\dot d_2^+-2d_1^+{}'\dot d_2^-+2d_2^-{}'\dot d_1^+-2d_2^+{}'\dot d_1^-, \tag{B5}$$

$$\mathrm{Tr}[-2ia'(\bar d_{1\alpha}\dot d_2-\bar d_{2\alpha}\dot d_1)]=2ia'd_1^-\dot d_2^+-2ia'd_1^+\dot d_2^--2ia'd_2^-\dot d_1^++2ia'd_2^+\dot d_1^-, \tag{B6}$$

$$\mathrm{Tr}[-2ia'\dot{\lambda}\lambda^{-1}(d_2\bar{d}_1 - d_1\bar{d}_2)] = (-2ia')[-2\sqrt{2}\dot{\tau}e^{-\varphi}d_2^-d_1^- + \sqrt{2}\dot{\sigma}d_2^+d_1^+ - \sqrt{2}\sigma\dot{\varphi}d_2^+d_1^+ - \sqrt{2}\sigma^2\dot{\tau}e^{-\varphi}d_2^+d_1^+ - \dot{\varphi}d_1^-d_2^+ - \dot{\varphi}d_1^+d_2^- - 2\sigma\dot{\tau}e^{-\varphi}d_1^-d_2^+ - 2\sigma\dot{\tau}e^{-\varphi}d_1^+d_2^-], \tag{B7}$$

$$\mathrm{Tr}\left[-4\dot{\lambda}\lambda^{-1}\frac{1}{2}d_1\bar{d}_2'\right] = -2\left[\frac{\dot{\varphi}}{2}d_1^+d_2^{-\prime} + \sigma\dot{\tau}e^{-\varphi}d_1^+d_2^{-\prime} - \sqrt{2}\dot{\tau}e^{-\varphi}d_1^-d_2^{-\prime} + \frac{\dot{\sigma}}{\sqrt{2}}d_1^+d_2^{+\prime} - \frac{\sigma}{\sqrt{2}}\dot{\varphi}d_1^+d_2^{+\prime} - \frac{\sigma^2}{\sqrt{2}}\dot{\tau}e^{-\varphi}d_1^+d_2^{+\prime} + \frac{\dot{\varphi}}{2}d_1^-d_2^{+\prime} + \sigma\dot{\tau}e^{-\varphi}d_1^-d_2^{+\prime}\right], \tag{B8}$$

$$\mathrm{Tr}\left[-4\dot{\lambda}\lambda^{-1}\frac{1}{2}d_2\bar{d}_1'\right] = -2\left[\frac{\dot{\varphi}}{2}d_2^+d_1^{-\prime} + \sigma\dot{\tau}e^{-\varphi}d_2^+d_1^{-\prime} - \sqrt{2}\dot{\tau}e^{-\varphi}d_2^-d_1^{-\prime} + \frac{\dot{\sigma}}{\sqrt{2}}d_2^+d_1^{+\prime} - \frac{\sigma}{\sqrt{2}}\dot{\varphi}d_2^+d_1^{+\prime} - \frac{\sigma^2}{\sqrt{2}}\dot{\tau}e^{-\varphi}d_2^+d_1^{+\prime} + \frac{\dot{\varphi}}{2}d_2^-d_1^{+\prime} + \sigma\dot{\tau}e^{-\varphi}d_2^-d_1^{+\prime}\right]. \tag{B9}$$

The trace terms are trivially zero. We can further write all these terms in terms of newly defined fermions d_1^N and d_2^N. Finally, combining them and using the constraints, the reduced action looks like

$$\begin{aligned} I = \frac{k}{4\pi}\int dud\phi[&\mu\varphi'\dot{\varphi} + 2\dot{\varphi}\theta' + 4\sigma\dot{\tau}e^{-\varphi}\theta' - 4\dot{\tau}\zeta'e^{-\varphi} - 2\dot{\sigma}\eta' + 2\sigma\dot{\varphi}\eta' + 2\sigma^2\dot{\tau}e^{-\varphi}\eta' \\ &- \varphi'^2 + 2i\dot{a}C' + \bar{\mu}a'\dot{a} - 4(a')^2 + \dot{\varphi}d_2^{N+}(d_1^{N-})' - \sqrt{2}\dot{\sigma}d_2^{N+}(d_1^{N+})' - \dot{\varphi}d_2^{N-}(d_1^{N+})' \\ &+ \dot{\varphi}d_1^{N+}(d_2^{N-})' - \sqrt{2}\dot{\sigma}d_1^{N+}(d_2^{N+})' - \dot{\varphi}d_1^{N-}(d_2^{N+})' + \sqrt{2}\sigma i\dot{a}(d_1^{N+})'d_2^{N+} \\ &+ \sqrt{2}\sigma(d_1^{N+})'(\dot{d_2^{N+}}) - 2i\dot{a}(d_1^{N+})'d_2^{N-} - 2(d_1^{N+})'(\dot{d_2^{N-}}) - \sqrt{2}\sigma i\dot{a}(d_2^{N+})'d_1^{N+} \\ &+ \sqrt{2}\sigma(d_2^{N+})'(\dot{d_1^{N+}}) + 2i\dot{a}(d_2^{N+})'d_1^{N-} - 2(d_2^{N+})'(\dot{d_1^{N-}})], \end{aligned} \tag{B10}$$

and using reduction relations this can be further simplified as

$$\begin{aligned} I = \frac{k}{4\pi}\int dud\phi[&\mu\varphi'\dot{\varphi} + \xi'\dot{\varphi} - \varphi'^2 + 2i\dot{a}C' + \bar{\mu}a'\dot{a} - 4(a')^2 - \sqrt{2}\dot{\sigma}d_2^{N+}(d_1^{N+})' \\ &- \sqrt{2}\dot{\sigma}d_1^{N+}(d_2^{N+})' + \sqrt{2}\sigma i\dot{a}(d_1^{N+})'d_2^{N+} \\ &+ \sqrt{2}\sigma(d_1^{N+})'(\dot{d_2^{N+}}) - 2i\dot{a}(d_1^{N+})'d_2^{N-} - 2(d_1^{N+})'(\dot{d_2^{N-}}) - \sqrt{2}\sigma i\dot{a}(d_2^{N+})'d_1^{N+} \\ &+ \sqrt{2}\sigma(d_2^{N+})'(\dot{d_1^{N+}}) + 2i\dot{a}(d_2^{N+})'d_1^{N-} - 2(d_2^{N+})'(\dot{d_1^{N-}})], \end{aligned} \tag{B11}$$

where $\xi = 2(\theta + \sigma\eta) + (d_1^{N+}d_2^{N-} + d_2^{N+}d_1^{N-})$. Now notice one relation $d_1^{Ni}d_2^{Nj} = d_1^i d_2^j$; i.e., the product of redefined fermions is the same as old ones. Next, to reduce the above action further, we look at terms with $\dot{a}$:

$$I_{\dot{a}} = \left(\frac{k}{4\pi}\right)\int dud\phi[\sqrt{2}\sigma i\dot{a}(d_1^{N+})'d_2^{N+} - \sqrt{2}\sigma i\dot{a}(d_2^{N+})'d_1^{N+} - 2i\dot{a}(d_1^{N+})'d_2^{N-} + 2i\dot{a}(d_2^{N+})'d_1^{N-}]. \tag{B12}$$

Up to total derivatives in ϕ and using the reduction conditions, we get

$$I_{\dot{a}} = -2i\left(\frac{k}{4\pi}\right)\int dud\phi[-2\dot{a}e^{\phi}d_2^{N+}d_1^{N+} + (\dot{a})'((d_2^{N+}d_1^{N-} - d_1^{N+}d_2^{N-}) - \sqrt{2}\sigma d_2^{N+}d_1^{N+})]. \tag{B13}$$

Notice further that from all the above terms N can be omitted. Furthermore, the second term can be absorbed in the redefinition of c of Eq. (3.4). Next, we look at two fermion terms without $\dot{a}$ in Eq. (B11). There are six such terms given as

$$\begin{aligned} I_{FF} = \left(\frac{k}{4\pi}\right)\int dud\phi[&-\sqrt{2}\dot{\sigma}d_2^{N+}(d_1^{N+})' - \sqrt{2}\dot{\sigma}d_1^{N+}(d_2^{N+})' \\ &+ \sqrt{2}\sigma(d_1^{N+})'(\dot{d_2^{N+}}) + \sqrt{2}\sigma(d_2^{N+})'(\dot{d_1^{N+}}) - 2(d_1^{N+})'(\dot{d_2^{N-}}) - 2(d_2^{N+})'(\dot{d_1^{N-}})]. \end{aligned} \tag{B14}$$

The last two terms of the above expression are the same as the third and fourth terms up to total ϕ, u derivatives. Thus, we get

$$I_{FF}=\left(\frac{k}{4\pi}\right)\int dud\phi[-\sqrt{2}\dot{\sigma}d_2^{N+}(d_1^{N+})'-\sqrt{2}\dot{\sigma}d_1^{N+}(d_2^{N+})'+2\sqrt{2}\sigma(d_1^{N+})'(\dot{d_2^{N+}})+2\sqrt{2}\sigma(d_2^{N+})'(\dot{d_1^{N+}})]. \tag{B15}$$

The above four terms can be further simplified up to total derivatives and reduction conditions as

$$I_{FF}=\left(\frac{k}{4\pi}\right)\int dud\phi[2e^{\phi}(\dot{d}_1^{N+}d_2^{N+}+\dot{d}_2^{N+}d_1^{N+})]. \tag{B16}$$

Let us now put the above equation (B16) with the first term of (B13), and we get

$$I=2\left(\frac{k}{4\pi}\right)\int dud\phi[e^{\phi}(\dot{d}_1^{N+}d_2^{N+}+\dot{d}_2^{N+}d_1^{N+})+2i\dot{a}e^{\phi}d_2^{N+}d_1^{N+}]=2\left(\frac{k}{4\pi}\right)\int dud\phi e^{\phi}(\dot{d}_1^{+}d_2^{+}+\dot{d}_2^{+}d_1^{+}). \tag{B17}$$

Finally, we redefine $\chi_i=e^{\phi/2}d_i^+$, $i=1$, 2, and get

$$I=2\left(\frac{k}{4\pi}\right)\int dud\phi(\dot{\chi}_1\chi_2+\dot{\chi}_2\chi_1). \tag{B18}$$

Combining all the terms, we get the reduced action as in Eq. (4.8).

APPENDIX C: FLAT LIMIT OF LIOUVILLE THEORY AND ITS EQUIVALENT DESCRIPTIONS

In this appendix, we shall present some equivalent descriptions of the Liouville theory in the "flat" limit. We shall mostly follow Refs. [39,42]. A classical Liouville theory describes the dynamics of a two-dimensional scalar field ϕ such that, when a two-dimensional metric is scaled by $e^{2\phi}$, the transformed metric has constant curvature R. The quantum Liouville action is given as

$$S_L=\int d^2x\sqrt{|g|}\left(-\frac{1}{2}g^{ab}\partial_a\phi\partial_b\phi+R\phi\frac{\gamma^2+4}{2\gamma}+\frac{\tilde{\mu}}{2\gamma^2}e^{\gamma\phi}\right). \tag{C1}$$

This is an interacting theory with γ and $\tilde{\mu}$ being constants. The above action in the Hamiltonian form (that contains only one time derivative of the field) on the Minkowskian cylinder (hence, $R=0$) with time coordinate time u, compact angular coordinate θ,[7] and metric $\eta_{\mu\nu}=$ diagonal $(-1,l^2)$ can be expressed as

$$S_L=\int dud\theta\left(\pi\dot{\phi}-\frac{\pi^2}{2}-\frac{\phi'^2}{2l^2}-\frac{\tilde{\mu}}{2\gamma^2}e^{\gamma\phi}\right), \tag{C2}$$

where π is conjugate momenta, $\dot{\phi}$ represents the u derivative, and ϕ' represents the θ derivative. The action is invariant under two-dimensional conformal transformations.

[7]With respect to light cone coordinates of flat Minkowski space, $x_{\pm}=\frac{u}{l}\pm\theta$.

We are interested in a large l limit of this theory such that

$$\phi=l\Phi,\qquad \pi=\frac{\Pi}{l},\qquad \beta=\gamma l,\qquad \nu=\tilde{\mu}l^2$$

are fixed. The action in this limit looks like

$$S_{FL}=\int dud\theta\left(\Pi\dot{\Phi}-\frac{\Phi'^2}{2}-\frac{\nu}{2\beta^2}e^{\beta\phi}\right). \tag{C3}$$

This is the flat limit of Liouville that preserves BMS_3 symmetry with zero $\mathcal{J}-\mathcal{J}$ central extension. One important point to note that this is a first-order action, and it does not have a second-order counterpart. There are two equivalent descriptions of the same theory that we shall list below. The first one is a free field realization given by

$$S_{FL}=\int dud\theta\left(\pi_\psi\dot{\psi}-\frac{\pi_\psi^2}{2}+\frac{d}{du}\left(\Phi\psi'-\frac{\sqrt{\nu}}{\beta}e^{\beta\Phi/2}\psi\right)\right), \tag{C4}$$

where the fields are related by Bäcklund transformations:

$$\Pi=\psi'-\frac{\sqrt{\nu}}{2}e^{\beta\Phi/2}\psi,\qquad \pi_\psi=\Phi'+\frac{\sqrt{\nu}}{\beta}e^{\beta\Phi/2}.$$

The second realization is given as

$$S_{FL}=\frac{k}{4\pi}\int dud\theta(\xi'\dot{\varphi}-\varphi'^2), \tag{C5}$$

where field transformations are

$$\beta\Pi=\xi'-(\log\sigma)'\xi,\qquad \beta\Phi=2\varphi-2\log\sigma-\log\frac{8}{\nu},$$

with $\beta^2=32\pi G$ and $\sigma'=\sqrt{2}e^{\varphi}$. The second description arises as the reduced phase space description of the $SL(2,R)$ chiral WZW model with appropriately constrained (due to specific asymptotic boundary conditions of fields at null infinity) global currents.

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