Gravitational alternatives to dark matter with tensor mode speed equaling the speed of light

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A number of theories of gravity have been proposed as proxies for dark matter in the regime of galaxies and cosmology. The recent observations of gravitational waves (GW170817) from the merger of two neutron stars, followed by an electromagnetic counterpart (GRB170817a) have placed stringent constraints on the difference between the speeds of gravity and light, severely restricting the phenomenological viability of such theories. We revisit the impact of these observations on the tensor-vector-scalar paradigm of relativistic modified Newtonian dynamics (MOND) and demonstrate the existence of a previously unknown class of this paradigm where the speed of gravity always equals the speed of light. We show that this holds without altering the usual (bimetric) MOND phenomenology in galaxies.

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I. INTRODUCTION

In the absence of the direct detection of a particle with the right properties to account for the entirety of dark matter, it remains a possibility that the effects attributed to dark matter represent a shortcoming in our understanding of the nature of gravity, that is, general relativity (GR) may not describe gravity correctly in all curvature regimes. Attempts to account for this [1–5] introduced additional fields into the gravitational sector whose influence on the visible matter produces dark-matter-like effects. In all metric theories of gravity including the ones with additional fields, spacetime is a dynamical entity leading to the generation and propagation of gravitational waves. Any additional fields coupled nontrivially to the spacetime curvature, however, generally lead to gravitational-wave speeds different than that in GR.

Gravitational waves (GWs) from the merger of a binary neutron star system have been observed by the Advanced Laser Interferometer Gravitational-wave Observatory (aLIGO) and the VIRGO interferometer [6]. Within seconds of this event (GW170817) being detected, a gamma-ray burst was independently observed from the same location [7,8]. Given the high likelihood that these represent signals from the same event, the specific small time difference—given the large distance from the location of emission (the galaxy NGC 4993)—implies that (in units where the speed of light is unity), the speed of propagation of GWs c_T obeys

$$|c_T^2 - 1| \lesssim 10^{-15}. \tag{1}$$

This is a remarkably stringent constraint[1] and has excluded many modified theories of gravity proposed in order to explain the phenomenon of dark energy [9–13]. Equally important is the impact of these observations on gravitational theories functioning as effective dark matter proxies. This stringent constraint was also used in Ref. [14] to place constraints on the Einstein-Aether theory [15].

Early evidence for dark matter came in the form of observations of the motion of stars within galaxies [16], where it was found that stars toward the outer regions of galaxies had orbital velocities significantly higher than expected due to the Newtonian gravitational field produced by visible matter. In 1983, Milgrom showed [17] that this motion of stars could instead result from a modification to the inertia/dynamics of stars at low Newtonian accelerations. Shortly afterwards it was found that these same effects could alternatively result from a nonlinear modification to the Poisson equation of Newtonian gravity [18]. These models are referred to as modified Newtonian dynamics (MOND).

A great deal of work has gone into deducing the astrophysical consequences of MOND, and whether MOND is consistent with data [19–29]. The inherently

[1]This constraint may be seen as an optimistic bound. It assumes the standard model of gamma-ray burst emissions with time delays of the order of 10 seconds between the emission of gravitational waves and photons. More exotic scenarios with 100–1000 s delays may alter the order of magnitude of this effect but constraints on the difference between the speeds of gravity and light will likely remain stringent. We thank an anonymous referee for this clarification.

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nonrelativistic nature of this modification renders it difficult to test as its realm of validity is unclear. As a result, there have been a number of proposals for relativistic theories that yield MONDian behavior on galactic scales [4,5,30–36]. Constructing a relativistic MOND theory presents a particular challenge as a great deal of contemporary evidence for dark matter arises from cosmological scales, particularly via the cosmic microwave background (CMB). Interestingly, the cold nature of dark matter has been thoroughly tested with the result [37–40] that any new gravitational degree of freedom that functions as a proxy for dark matter, must lead to a background cosmology that is very close to the ΛCDM model.

Perhaps the most widely known relativistic theory leading to MOND-like behavior is the Bekenstein-Sanders tensor-vector-scalar (TeVeS) theory [1,2] which depends on a metric $\hat{g}_{\mu\nu}$, a unit timelike vector field A_μ, and a scalar field ϕ. All types of matter are taken to couple universally to the metric $g_{\mu\nu}$ via

$$g_{\mu\nu} = e^{-2\phi}\hat{g}_{\mu\nu} - 2\sinh(2\phi)A_\mu A_\nu, \tag{2}$$

and as such the Einstein equivalence principle is obeyed.

Since its inception, there have been many studies of TeVeS theory in various situations: spherically symmetric solutions such as black holes [2,41–43] and neutron stars [44,45], the stability of spherically symmetric perturbations [46] and gravitational collapse [43], Hamiltonian analysis and the nonexistence of ghosts [47], parametrized post-Newtonian constraints [41,48] and Solar System saddle points [23,49–51], gravitational lensing [52–61], superluminality [62], gravitational redshift [63] and gravitational waves [64–68]. The cosmology of TeVeS theory was extensively investigated in Refs. [69–72]. It is reasonable to conclude that the versions of TeVeS that have been studied so far yield good fits to CMB data only following the introduction of additional gravitating matter, such as sterile neutrinos.

Due to the algebraic relation (2), the presence of two metrics does not imply the existence of additional spin-2 modes beyond those present in single-metric theories of gravity; however, the presence of dynamical fields ϕ and A_μ in the theory implies the existence of additional spin-1 and spin-0 gravitational waves, leading to a total of six propagating degrees of freedom in the gravitational sector [65]. Here we focus only on the spin-2 (tensor mode) gravitational waves as this is the type observed by LIGO [6].

The speed of the tensor mode GWs in TeVeS theory is in general different than the speed of light [65], and it is then natural to ask what is the status of the TeVeS paradigm after GW170817. Using a variety of methods, a number of articles [66–68] have tackled this question. The authors of Ref. [66] compared the Shapiro time delay of gravitational versus electromagnetic waves, as they pass through the potential wells of galaxies, proposed earlier as a generic test of TeVeS theory [64]. Such a test is superior to testing the propagation speed on a Friedman-Robertson-Walker (FRW) background considered in Refs. [9–12,14] in the case of other theories. The delay was calculated there by comparing the geodesics of $\hat{g}_{\mu\nu}$ to the geodesics of $g_{\mu\nu}$; however, as the metric is not an observable the generality of their result is unclear. For instance, the authors of Ref. [73] reformulated TeVeS theory using a single metric ($g_{\mu\nu}$) so that no geodesic comparisons are possible in that formulation.[2] A different method is necessary.

In Refs. [67,68] the speeds of all six polarizations of GWs present in TeVeS theory [65] were considered on a Minkowski background and after imposing Eq. (1), the analysis of the remaining parameter space led to the conclusion that TeVeS theory is ruled out.

In this article we investigate the propagation of GWs on perturbed FRW spacetimes, which includes the Shapiro time delay effect. We find that the original TeVeS theory [2] and its generalization [3,74] are ruled out by the GW170817/GRB170817a events, in agreement with previous studies [66–68]. We present, however, the existence of a previously unknown class of relativistic MOND theories also based on the tensor-(timelike)vector-scalar paradigm, where the speed of gravity always equals the speed of light while retaining the effective bimetric description leading to the usual MOND phenomenology in galaxies.

The article is organized as follows. In Sec. II we present the fundamentals of TeVeS theory in order to pave the way for the study of gravitational waves on arbitrary backgrounds. We do so in Sec. III in steps. First we consider GWs propagating on a FRW background in Sec. III A and show how a specific choice of functions leads to a new theory with $c_T = 1$. We repeat the calculation in Sec. III B where we consider the effects of inhomogeneities and show that the same functions again lead to $c_T = 1$. We explain that this phenomenon holds on arbitrary backgrounds in Sec. III C. We discuss our result and conclude in Sec. IV.

II. RUDIMENTS OF TeVeS THEORY

A slight generalization of TeVeS theory is given by the following action [3,74] which depends on the three above fields and the two auxiliary fields λ_A and μ:

$$\hat{S} = \frac{1}{16\pi G}\int d^4x\sqrt{-\hat{g}}[\hat{R} - \hat{K} + \lambda_A(A^\rho A_\rho + 1) - \mu\hat{g}^{\alpha\beta}\hat{\nabla}_\alpha\phi\hat{\nabla}_\beta\phi - \hat{V}(\mu)] + S_{\mathrm{M}}[g]. \tag{3}$$

[2]Other single-metric theories such as the Horndeski theory studied in Refs. [9–13] and Einstein-Aether theory studied in Ref. [14] may also yield a Shapiro time delay different than the one in GR. However, a proper study of such theories exhibiting screening mechanisms, including TeVeS theory, warrants a more precise determination of the screening effects inside the screening radius, introducing further complications and making the problem even more difficult to tackle.

Here G is the bare gravitational constant, $\hat{g}$ and $\hat{R}$ are the determinant and scalar curvature of $\hat{g}_{\mu\nu}$ respectively, $\hat{V}$ is a free function of μ, $S_M[g]$ is the action for all matter fields, and $\hat{K} = \hat{K}^{\mu\nu\alpha\beta}\hat{\nabla}_\mu A_\nu \hat{\nabla}_\alpha A_\beta$ is obtained using

$$\hat{K}^{\mu\nu\alpha\beta} = c_1 \hat{g}^{\mu\alpha}\hat{g}^{\nu\beta} + c_2 \hat{g}^{\mu\nu}\hat{g}^{\alpha\beta} + c_3 \hat{g}^{\mu\beta}\hat{g}^{\nu\alpha} + c_4 \hat{g}^{\nu\beta}A^\mu A^\alpha. \tag{4}$$

The indices of A_μ are always raised using $\hat{g}^{\mu\nu}$, the inverse metric of $\hat{g}_{\mu\nu}$ (i.e., $\hat{g}^{\mu\rho}\hat{g}_{\rho\nu} = \delta^\mu{}_\nu$). We emphasize that in contrast to Refs. [3,74] we allow here the c_I ($I = 1\ldots4$) to be functions of the scalar field ϕ and this turns out to be very important when analyzing the speed of GWs. The original TeVeS theory is obtained when $c_I = \{2K_B - \frac{1}{4}, -\frac{1}{2}, -2K_B + \frac{3}{4}, K_B - \frac{1}{4}\}$, for a constant K_B [74]. For notational compactness we define $c_{IJ\ldots} \equiv c_I + c_J + \cdots$

The emergence of MOND behavior in the quasistatic weak-field limit in the constant c_I case was analyzed extensively in Ref. [74]. We revisit that analysis here in order to show that it remains unchanged even when c_I are functions of ϕ. In particular, one expands the scalar field as $\phi = \phi_0 + \varphi$ where ϕ_0 is a constant and φ is time independent. The quasistatic metric is such that $\hat{g}_{00} = -e^{-2\phi_0}(1 - 2\hat{\Psi})$ and $\hat{g}_{ij} = e^{2\phi_0}(1 - 2\hat{\Phi})\gamma_{ij}$. In this coordinate system the vector field has components $A_0 = -e^{-\phi_0}(1 + \hat{\Psi})$ and $A_i = 0$. Using the metric transformation (2) we find the components of the metric $g_{\mu\nu}$ so that

$$ds^2 = -(1 + 2\Psi)dt^2 + (1 - 2\Phi)\gamma_{ij}dx^i dx^j \tag{5}$$

where

$$\hat{\Psi} = \Psi - \varphi, \qquad \hat{\Phi} = \Phi - \varphi. \tag{6}$$

With this ansatz, the vector field equations are identically satisfied while the Einstein and scalar field equations reduce to

$$\vec{\nabla}^2\hat{\Psi} = \frac{8\pi G}{2 - c_1 + c_4}\rho, \tag{7}$$

$$\vec{\nabla}_i(\mu\vec{\nabla}^i\varphi) = 8\pi G\rho, \tag{8}$$

$$\hat{\Phi} = \hat{\Psi}, \tag{9}$$

where ρ is the matter-energy density and the c_I's are evaluated at $\phi = \phi_0$ in Eq. (7). The nondynamical field μ is obtained via a constraint equation found from the action upon variation with respect to μ and this equation depends on the form of $\hat{V}(\mu)$. Not all functions $\hat{V}(\mu)$ lead to either Newtonian or MONDian limiting behaviors, and the ones that do must have appropriate properties (discussed in Ref. [74]).

III. TENSOR MODE PROPAGATION IN TeVeS THEORY AND GENERALIZATIONS

A. Tensor mode propagation on FRW backgrounds

In order to determine the speed of propagation of GWs, we need the tensor mode equation on an FRW background. We assume a metric $g_{\mu\nu}$ such that

$$ds^2 = -dt^2 + a^2(\gamma_{ij} + \chi_{ij})dx^i dx^j, \tag{10}$$

where a is the scale factor, γ_{ij} is the spatial metric of constant curvature κ, and χ_{ij} is the tensor mode GW which is traceless ($\gamma^{ij}\chi_{ij} = 0$) and transverse ($\vec{\nabla}_i\chi^i{}_j = 0$, where $\vec{\nabla}_i$ is the spatial covariant derivative compatible with γ_{ij}). As we are only interested in the tensor mode, we set the perturbations of ϕ and A_μ to zero so that $\phi = \bar{\phi}(t)$ and $A_0 = -e^{-\bar{\phi}}$, with $A_i = 0$. The perturbed Einstein equations for the tensor mode were obtained for constant c_I in Ref. [3]. In the case where $c_I = c_I(\phi)$, an additional term is present such that

$$\begin{aligned} &e^{2\bar{\phi}}(1 - c_{13})[\ddot{\chi}^i{}_j + (3H + 4\dot{\bar{\phi}})\dot{\chi}^i{}_j] - e^{2\bar{\phi}}\frac{dc_{13}}{d\phi}\dot{\bar{\phi}}\dot{\chi}^i{}_j \\ &\quad - \frac{1}{a^2}e^{-2\bar{\phi}}(\vec{\nabla}^2 - 2\kappa)\chi^i{}_j = 16\pi G e^{-2\bar{\phi}}\Sigma^{(g)i}{}_j, \end{aligned} \tag{11}$$

where $\Sigma^{(g)i}{}_j$ is a traceless source term due to matter. The only difference from the constant c_I case is the appearance of the $\frac{dc_{13}}{d\phi}$ term multiplying $\dot{\chi}^i{}_j$.

Now in the original and generalized TeVeS theories it is clear that the speed of propagation of the tensor mode is given by

$$c_T^2 = \frac{e^{-4\bar{\phi}}}{1 - c_{13}}. \tag{12}$$

Thus, in general c_T^2 will differ from unity, putting this theory in conflict with the observations that require $c_T^2 \approx 1$ unless some mechanism forces $\bar{\phi}$ to have an approximately constant value at very low redshift and equal to $\bar{\phi} = -\frac{1}{4}\ln(1 - c_{13})$. This is highly unlikely, and even though it is possible we show below that the Shapiro time delay rules this case out.

If c_I are functions of ϕ, however, there seems to be enough freedom to change this fact. In particular, the unique choice of

$$c_{13}(\phi) = 1 - e^{-4\phi} \tag{13}$$

transforms Eq. (11) into

$$\ddot{\chi}^i{}_j + 3H\dot{\chi}^i{}_j - \frac{1}{a^2}(\vec{\nabla}^2 - 2\kappa)\chi^i{}_j = 16\pi G\Sigma^{(g)i}{}_j, \tag{14}$$

which is identical to the tensor mode equation in GR, and thus with this choice $c_T^2 = 1$.

B. Beyond FRW

We have shown above that the choice (13) leads to GW tensor mode propagation as in GR while maintaining MONDian behavior. When gravitational and electromagnetic waves pass through potential wells generated by matter, however, they incur an additional (Shapiro) time delay, and (as proposed in Ref. [64]) this may be used to put strong constraints on such theories. We thus examine whether the condition (13) is sufficient to ensure tensor mode propagation with $c_T^2 = 1$ even when including the effect of inhomogeneities. We note that the effect of inhomogeneities on the speed of gravity in some other theories of gravity has studied in Refs. [10,13,75]. In these situations the physical metric $g_{\mu\nu}$ takes the form

$$ds^2 = -(1+2\Psi)dt^2 + a^2(1-2\Phi)(\gamma_{ij}+\chi_{ij})dx^i dx^j, \tag{15}$$

where the hierarchy $\chi_{ij} \ll \Phi, \Psi \sim 10^{-5}$ has been assumed. Furthermore, TeVeS theory's scalar field ϕ takes the form $\phi = \bar{\phi} + \varphi$ (with $\varphi \lesssim \Phi, \Psi$), while the vector field has components $A_0 = -e^{-\bar{\phi}}(1+\hat{\Psi})$ and $A_i = -ae^{\bar{\phi}}\vec{\nabla}_i\alpha$. The expressions (6) relate the potentials between the two frames. Given Eq. (15), the metric $\hat{g}_{\mu\nu}$ will not be in diagonal form but will contain terms coming from the vector perturbation α. In general, the potentials are assumed to be space and time dependent.

Defining $\mathcal{T}^{ik}{}_j = \vec{\nabla}^i\chi^k{}_j + \vec{\nabla}_j\chi^{ki} - \frac{2}{3}\vec{\nabla}^l\chi^k{}_l\delta^i{}_j$, after a lengthy and tedious calculation the tensor mode equation for χ_{ij} is found to be

$$\begin{aligned}
&e^{2\bar{\phi}}\left[(1-c_{13})(1-2\hat{\Psi}) - \frac{dc_{13}}{d\phi}\varphi\right]\ddot{\chi}^i{}_j + e^{2\bar{\phi}}\mathcal{A}\dot{\chi}^i{}_j - \left\{\frac{1}{a^2}e^{-2\bar{\phi}}[(1+2\hat{\Phi})(\vec{\nabla}^2-2\kappa) + \vec{\nabla}_k(\hat{\Psi}-\hat{\Phi})\vec{\nabla}^k] - \frac{1}{a}\mathcal{B}_k\vec{\nabla}^k\right\}\chi^i{}_j \\
&\quad + \frac{1}{a^2}e^{-2\bar{\phi}}(1+2\hat{\Phi})\left(\vec{\nabla}^i\vec{\nabla}_k\chi^k{}_j + \vec{\nabla}_j\vec{\nabla}^k\chi^i{}_k - \frac{2}{3}\vec{\nabla}^l\vec{\nabla}_k\chi^k{}_l\delta^i{}_j\right) + \frac{1}{a^2}e^{-2\bar{\phi}}\left\{\vec{\nabla}_k(\hat{\Psi}-\hat{\Phi})\mathcal{T}^{ik}{}_j\right. \\
&\quad \left. - 2\left[\vec{\nabla}_k\vec{\nabla}_j(\hat{\Phi}-\hat{\Psi})\chi^{ik} - \frac{1}{3}\vec{\nabla}_k\vec{\nabla}_l(\hat{\Phi}-\hat{\Psi})\chi^{lk}\delta^i{}_j\right]\right\} + \frac{1}{a}\mathcal{C}^i{}_j = 16\pi G e^{-2\bar{\phi}}(1-2\varphi)\Sigma^{(g)i}{}_j
\end{aligned} \tag{16}$$

where the terms $\mathcal{A}$, $\mathcal{B}_i$, and $\mathcal{C}^i{}_j$ are shown in the Appendix for clarity of presentation. Allowing all potentials as well as φ and α to vanish reduces Eq. (16) to Eq. (11).

Consider first the reduction of Eq. (16) to quasistatic backgrounds (also ignoring the source term) for the fine-tuned case where $c_{13} = 1 - e^{-4\phi_0}$ (so that $c_T^2 = 1$ on the background). We obtain this by setting $a = 1$, $\bar{\phi} = \phi_0$, and $\hat{\Psi} = \hat{\Phi}$ from Eq. (9). Then, $\mathcal{A}$, $\mathcal{B}_k$, and $\mathcal{C}^i{}_j$ all vanish. In addition, considering LIGO wavelengths ~1000 km which are far smaller than the scale of the potential wells, we may drop the terms containing derivatives of Ψ and φ, i.e., $\partial\Phi \ll \partial\chi$, even with $\chi \ll \Phi$ [13]. Finally, by further imposing the gauge condition $\vec{\nabla}_i\chi^i{}_j = 0$, Eq. (16) leads to

$$(1-2\hat{\Phi})\ddot{\chi}^i{}_j - (1+2\hat{\Phi})\vec{\nabla}^2\chi^i{}_j = 0. \tag{17}$$

Thus, in this case we expect a Shapiro time delay dictated by $\hat{\Phi}$, the potential formed by baryons alone. This is not the same as Φ which is the potential seen by photons; hence, this fine-tuned case is ruled out by the analysis of Ref. [66].

Let us turn now to the case where Eq. (13) holds so that $c_T^2 = 1$ on a FRW background. Imposing Eq. (13), we find $\mathcal{B}_k = 0$ and $\mathcal{C}^i{}_j = 0$. Further, by using Eq. (6) we find $e^{2\bar{\phi}}\mathcal{A} = e^{-2\bar{\phi}}[3H(1-2\Psi-2\varphi) - \dot{\Psi} - 3\dot{\Phi}]$, and after choosing the gauge condition $\vec{\nabla}_i\chi^i{}_j = \vec{\nabla}_i(\Phi-\Psi)\chi^i{}_j$, Eq. (16) turns into

$$\begin{aligned}
&(1-2\Psi)[\ddot{\chi}^i{}_j + (3H - \dot{\Psi} - 3\dot{\Phi})\dot{\chi}^i{}_j] \\
&\quad - \frac{1}{a^2}(1+2\Phi)[(\vec{\nabla}^2 - 2\kappa)\chi^i{}_j + \vec{\nabla}_j\vec{\nabla}^k(\Phi-\Psi)\chi^i{}_k \\
&\quad - \vec{\nabla}^i\vec{\nabla}_k(\Phi-\Psi)\chi^k{}_j - \vec{\nabla}_k(\Phi-\Psi)\vec{\nabla}^k\chi^i{}_j] = 16\pi G\Sigma^{(g)i}{}_j,
\end{aligned} \tag{18}$$

which is the same equation as in GR. Thus, with the choice (13) the tensor mode propagates at the speed of light even when including inhomogeneities and gives the same Shapiro time delay as for photons.

C. Tensor mode propagation on general backgrounds

Our results in Sec. III B are not an accident. To gain further insight as to why this behavior emerges, we consider the single-metric (physically equivalent) formulation of TeVeS theory. As shown in Ref. [73], one introduces a new field $B_\mu = A_\mu$ which leads to $B^\mu = e^{-2\phi}A^\mu$, and writes the Lagrange constraint as $g^{\mu\nu}B_\mu B_\nu \equiv B^2 = -e^{-2\phi}$. This enables us to solve for ϕ and thus remove both ϕ and $\hat{g}_{\mu\nu}$ from the action. The d.o.f. remain unchanged as now B_μ contains four d.o.f. rather than the three of A_μ. The action

$S[g, B, \mu]$ of this physically equivalent vector-tensor formulation is

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}[R - K - U] + S_m[g], \quad (19)$$

where $U = \hat{V}(\mu)/B^2$ and K is given by

$$K = (d_1 - d_3)F^{\mu\nu}F_{\mu\nu} + d_{13}M^{\mu\nu}M_{\mu\nu} + d_2J^2 + d_4J^\nu J_\nu + \frac{1}{2}d_5J^\mu\nabla_\mu B^2 + \frac{d_6}{4}(\nabla B^2)^2 + \frac{d_7}{2}QJ + \frac{d_8}{4}Q^2, \quad (20)$$

and we have also defined $F_{\mu\nu} = 2\nabla_{[\mu}B_{\nu]}$, $M_{\mu\nu} = 2\nabla_{(\mu}B_{\nu)}$, $J = \nabla_\mu B^\mu$, $J_\mu = B^\alpha\nabla_\alpha B_\mu$, and $Q = B^\alpha\nabla_\alpha B^2$. The functions for the generalized TeVeS theory d_I $(I = 1\ldots8)$ may be found in the appendix of Ref. [74].

Some of the d_I coefficients depend on μ so that MOND behavior may emerge upon choosing an appropriate $\hat{V}$. Allowing for a general dependence $d_I(B^2, \mu)$ in Eq. (19) represents a slight generalization of Eq. (3). Interestingly, the dynamical tendency towards B_μ having a nonvanishing norm in this picture arises from the presence of inverse powers of the norm B^2 in the Lagrangian, rather than via a Lagrangian constraint as in Eq. (3). In this formulation, the modification to the speed of propagation of GWs is entirely due to the coupling of gravity to the field B_μ through that field's kinetic term. A straightforward way to see this is by considering the case where B^μ is hypersurface orthogonal, in which case we can decompose the metric $g_{\mu\nu}$ as $g_{\mu\nu} = h_{\mu\nu} - n_\mu n_\nu$, where $n_\mu \equiv B_\mu/\sqrt{-B^2} = N\nabla_\mu t$ for some global time function t and $h_{\mu\nu}$ $(h_{\mu\nu}n^\nu = 0)$ is the spatial metric on surfaces of constant time. Then,

$$K = -d_{13}B^2\mathcal{K}^{\mu\nu}\mathcal{K}_{\mu\nu} - d_2B^2\mathcal{K}^2 + \ldots, \quad (21)$$

where we have defined the extrinsic curvature tensor $\mathcal{K}_{\mu\nu} \equiv \frac{1}{2}\mathcal{L}_n h_{\mu\nu}$, and … denotes terms of linear order or lower in $\mathcal{K}_{\mu\nu}$. As tensor mode perturbations reside in "trace-free" small perturbations to $h_{\mu\nu}$, only the first term in Eq. (21) (schematically of the form $\sim d_{13}B^2\dot{h}^{\mu\nu}\dot{h}_{\mu\nu}$) will affect the speed of the tensor mode. There will be no deviation from general relativity if

$$d_{13} = 0 \Rightarrow d_1 = -d_3. \quad (22)$$

The transformation of Eq. (3) into Eq. (19) gives $d_{13} = \frac{1-c_{13}}{B^6} - \frac{1}{B^2}$, so that $d_{13} = 0$ iff $c_{13} = 1 - B^4 = 1 - e^{-4\phi}$, which is the condition (13).

IV. DISCUSSION AND CONCLUSIONS

We have demonstrated the existence of a generic class of relativistic theories of MOND based on the tensor-(timelike)vector-scalar paradigm which retain the property that GWs in this class propagate as in general relativity. The original TeVeS theory is not part of this class and therefore not consistent with gravitational-wave constraints. Hence, actions of the form (19) are sufficiently general that they encompass both phenomenologically viable and nonviable models. Viable models are those for which $d_3 = -d_1$ so that the $M_{\mu\nu}$ term is absent while all remaining d_I's can in general be functions of both B^2 and μ. However, not all such viable actions lead to MOND behavior, but specific functional forms of d_I do. Indeed, it is possible to sufficiently simplify the viable subset of Eq. (19) while retaining a MOND limit and at the same time obtaining a realistic cosmology. It is expected that doing so will lead to a theory in harmony with the CMB observations. This is beyond the scope of this article and will be investigated elsewhere. The health of the remaining subclass of theories, e.g., the absence of ghosts, tachyons, and gradient instabilities is also another important topic to be investigated elsewhere.

The viable theory presented here has in general six types of GW polarizations. The speed of the other four modes depends on the d_I coefficients and the form of U. It would be interesting in a future work to determine the propagation of these other modes as well as their generation by astrophysical sources in more specific setups, particularly since it has been conjectured that they will not be generated in the regime of large potential gradients [76].

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APPENDIX: COEFFICIENTS OF THE GRAVITATIONAL-WAVE EQUATION

Defining $\mathcal{E}^i{}_j = \vec{\nabla}^i\alpha\vec{\nabla}_k\dot{\chi}^k{}_j - \frac{1}{3}\vec{\nabla}^l\alpha\vec{\nabla}_k\dot{\chi}^k{}_l\delta^i{}_j$, the tensors $\mathcal{A}$, $\mathcal{B}_i$, and $\mathcal{C}^i{}_j$ are given by

$$\mathcal{A} = (1-c_{13})[(3H+4\dot{\bar{\phi}})(1-2\hat{\Psi}) - \dot{\hat{\Psi}} - 3\dot{\hat{\Phi}}] - \frac{dc_{13}}{d\phi}[(3H+4\dot{\bar{\phi}})\varphi + \dot{\bar{\phi}}(1-2\hat{\Psi}) + \dot{\varphi}] - \frac{d^2c_{13}}{d\phi^2}\dot{\bar{\phi}}\varphi$$
$$+\frac{1}{a}e^{-2\bar{\phi}}(1-e^{4\bar{\phi}}+e^{4\bar{\phi}}c_{13})\vec{\nabla}^2\alpha - 2(e^{4\bar{\phi}}-1-e^{4\bar{\phi}}c_{13})\vec{\nabla}_k\alpha\vec{\nabla}^k, \tag{A1}$$

$$\mathcal{B}_i = -(e^{4\bar{\phi}}-1-e^{4\bar{\phi}}c_{13})\vec{\nabla}_i\dot{\alpha} - 2\left\{[e^{4\bar{\phi}}(1-c_{13})-1]H + \left[3e^{4\bar{\phi}}(1-c_{13}) - 1 - \frac{1}{2}e^{4\bar{\phi}}\frac{dc_{13}}{d\phi}\right]\dot{\bar{\phi}}\right\}\vec{\nabla}_i\alpha, \tag{A2}$$

$$\mathcal{C}^i{}_j = (e^{4\bar{\phi}}-1-e^{4\bar{\phi}}c_{13})\left\{\vec{\nabla}_k\vec{\nabla}_j\alpha\dot{\chi}^{ik} + \vec{\nabla}_k\vec{\nabla}^i\alpha\dot{\chi}^k{}_j - \frac{2}{3}\vec{\nabla}_k\vec{\nabla}_l\alpha\dot{\chi}^{lk}\delta^i{}_j + 2\left(\vec{\nabla}_k\vec{\nabla}_j\dot{\alpha}\chi^{ik} - \frac{1}{3}\vec{\nabla}_k\vec{\nabla}_l\dot{\alpha}\chi^{lk}\delta^i{}_j\right)\right.$$
$$\left. + \vec{\nabla}_k\dot{\alpha}\mathcal{T}^{ik}{}_j + \vec{\nabla}_k\alpha\dot{\mathcal{T}}^{ik}{}_j\right\} + \left(e^{4\bar{\phi}} - 1 - \frac{1}{2}e^{4\bar{\phi}}c_{13}\right)\mathcal{E}^i{}_j - \frac{1}{2}c_{13}\mathcal{E}_j{}^i + 2\left\{[e^{4\bar{\phi}}(1-c_{13})-1]H\right.$$
$$\left. + \left[3e^{4\bar{\phi}}(1-c_{13}) - 1 - \frac{1}{2}e^{4\bar{\phi}}\frac{dc_{13}}{d\phi}\right]\dot{\bar{\phi}}\right\}\left[\vec{\nabla}_k\alpha\mathcal{T}^{ik}{}_j + 2\left(\vec{\nabla}_k\vec{\nabla}_j\alpha\chi^{ik} - \frac{1}{3}\vec{\nabla}_k\vec{\nabla}_l\alpha\chi^{lk}\delta^i{}_j\right)\right]. \tag{A3}$$

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