Canonical deformation of $N=2$ AdS_4 supergravity

Dragoljub Gočanin* and Voja Radovanović†
University of Belgrade, Faculty of Physics Studentski trg 12, 11000 Beograd, Serbia

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It is well known that one can define a consistent theory of extended, $N = 2$ anti-de Sitter (AdS) supergravity (SUGRA) in $D = 4$. Besides the standard gravitational part (including a negative cosmological constant), this theory involves a single $U(1)$ gauge field and a pair of Majorana vector spinors that can be combined to form a pair of Dirac vector spinors (charged spin-3/2 gravitini). The action for $N = 2$ AdS_4 SUGRA is invariant under diffeomorphisms, $SO(1,3) \times U(1)$ gauge transformations, and under local complex supersymmetry. We present a geometric action that involves two "inhomogeneous" parts: an orthosymplectic OSp(4|2) gauge-invariant action quadratic in the gauge field strength and a supplementary term invariant under the purely bosonic $SO(2,3) \times U(1) \sim Sp(4) \times SO(2)$ sector of OSp(4|2), which needs to be added for consistency. This action reduces to $N = 2$ AdS_4 SUGRA after suitable gauge fixing, for which we use a constrained auxiliary field in the manner of Stelle and West. Canonical (θ-constant) deformation is performed by using the Seiberg-Witten approach to noncommutative (NC) gauge field theory with Moyal star product. The NC-deformed action is expanded in powers of the deformation parameter $\theta^{\mu\nu}$, up to the first order. We show that $N = 2$ AdS_4 SUGRA has nonvanishing linear NC correction in the physical gauge, originating from the additional, purely bosonic action term. For comparison, simple $N = 1$ Poincaré SUGRA can be obtained in the same manner from an OSp(4|1) gauge-invariant action (without introducing additional terms). The first nonvanishing NC correction is quadratic in the deformation parameter $\theta^{\mu\nu}$ and therefore exceedingly difficult to calculate. Under Wigner-Inönü contraction, $N = 2$ AdS superalgebra reduces to $N = 2$ Poincaré superalgebra, and it is not clear whether this relation holds after canonical NC deformation. We present the linear NC correction to $N = 2$ AdS_4 SUGRA explicitly and discuss its low-energy limit and what remains of it after Wigner-Inönü contraction.

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I. INTRODUCTION

In our quest for the theory of "quantum gravity," we must be prepared to go beyond some deeply rooted assumptions on which we are accustomed; in particular, at very short distances (very high energies), we might have to abandon the notion of a continuous space-time and the associated mathematical concept of a smooth manifold that describes it. One distinguished approach to the problem is *noncommutative (NC) field theory*—a theory of relativistic fields on *noncommutative space-time*, based on the method of deformation quantization by the NC star product [1–3]. One speaks of a deformation of an object/structure whenever there is a family of similar objects/structures of which the "distortion" from the original, "undeformed" one can be somehow parametrized. In physics, this so-called *deformation parameter* appears as some fundamental constant of nature that measures the deviation from the classical (i.e., undeformed) theory. This way of "quantizing" space-time is essentially different from the standard quantum field theory (QFT) quantization procedure for matter fields. Different space-time dimensions (the usual $3+1$) are regarded as being mutually "incompatible," in the sense that there exists a lower bound for the product of uncertainties $\Delta x^\mu \Delta x^\nu$ for a pair of two different coordinates. To capture this "pointlessness" of space-time, one introduces an abstract algebra of NC coordinates as a deformation of the classical structure. These NC coordinates, denoted by $\hat{x}^\mu$, satisfy some nontrivial commutation relations, and so it is no longer the case that $\hat{x}^\mu \hat{x}^\nu = \hat{x}^\nu \hat{x}^\mu$. Abandoning this basic property of space-time leads to various new physical effects that were not present in theories based on classical space-time. The simplest case of noncommutativity is the so-called *canonical* (*or* θ-constant) *noncommutativity*,

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu} \sim \Lambda_{NC}^2, \tag{1.1}$$

*dgocanin@ipb.ac.rs
†rvoja@ipb.ac.rs

where $\theta^{\mu\nu}$ are components of a *constant* antisymmetric matrix and $\Lambda_{\rm NC}$ is the length scale at which NC effects become relevant. The deformation parameter is a fundamental constant, like the Planck length or the speed of light.

Instead of deforming the abstract algebra of coordinates, one can take an alternative but equivalent approach in which noncommutativity appears in the form of NC products of functions (fields) of *commutative* variables (coordinates). These products are called *star products* ($\star$-products). In particular, to introduce canonical noncommutativity, we use the Moyal $\star$-product,

$$(\hat{f}\star\hat{g})(x)=e^{\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial x^{\mu}}\frac{\partial}{\partial y^{\nu}}}f(x)g(y)|_{y\to x}. \qquad (1.2)$$

The leading term in the expansion of the exponential is the ordinary commutative product of functions, and the higher-order terms represent nonclassical NC corrections.

To date, we still lack direct physical evidence of supersymmetry (SUSY), at least in its simplest form. Nevertheless, its beneficial influence on high-energy physics (improved renormalizability in QFT and a natural resolution of the hierarchy problem), along with its mathematical consistency and unification power [especially the unification of gravity and the Standard Model within supergravity (SUGRA) and an ultimate unification scheme such as superstring theory], motivate us to seriously consider SUSY as an integral part of our description of nature. Since the pioneering work of Freedman *et al.* [4,5], and Deser and Zumino [6], the theory of supergravity has become a well-developed field of research. SUGRA provides a natural unification of gravity with other fields by imposing the gauge principle on SUSY, the associated gauge field being the spin-3/2 gravitino field described by a Majorana vector spinor. It was demonstrated in Refs. [7,8] that one can have a consistent theory of extended $N=2$ AdS_4 SUGRA with a complex [$U(1)$-charged] gravitino field. In this paper, we propose a geometric way of obtaining $N=2$ AdS_4 SUGRA action and perform its NC deformation. The obtained NC correction can be regarded as a low-energy signature of the underlying theory of quantum gravity. We calculate the NC correction explicitly and discuss some of its properties.

The results of this paper amount to a supersymmetric extension of the theory of NC gravity of which the various aspects have been treated extensively in the literature [9–23]. In particular, an approach based on NC-deformed anti-de Sitter (AdS) gauge group $SO(2,3)$ was developed in Refs. [24–27], building on the results of MacDowell and Mansouri [28], Towsend [29], Stelle and West [30], Chamseddine and Mukhanov [31,32], and Wilczek [33]. One starts with a classical (undeformed) action invariant under $SO(2,3)$ gauge transformations. To relate AdS gauge theory with general relativity (GR), original $SO(2,3)$ gauge symmetry has to be broken to $SO(1,3)$, by gauge fixing. For that matter, a constrained auxiliary field is introduced, as in Ref. [30], to define the physical gauge. Spin-connection and vierbein are treated on equal footing, as components of the general $SO(2,3)$ gauge field. The $SO(2,3)$ gauge-invariant action is deformed by introducing Moyal $\star$-product and expanded in powers of $\theta^{\mu\nu}$ via the Seiberg-Witten (SW) map [34–37]. After symmetry breaking, one obtains NC corrections to classical gravity, invariant under $SO(1,3)$ gauge transformations. The first-order NC correction vanishes, as confirmed by Ref. [38]. The second-order NC correction to GR is found explicitly, and deformed equations of motion are analyzed. It is argued that the apparent breaking of diffeomorphism invariance stems from the fact that, by introducing the *canonical* anticommutation relations between space-time coordinates (1.1), we implicitly set ourselves in a preferred coordinate system—the Fermi inertial frame along a geodesic [39–41].

Similarly, one can establish NC SUGRA by gauging an appropriate supergroup [42–50] and subsequently performing canonical deformation. Since pure gravity can be obtained by gauging AdS group $SO(2,3)$, orthosymplectic supergroup OSp(4|1) comes as a natural choice for pure $N=1$ Poincaré SUGRA. The bosonic sector of $\mathfrak{osp}(4|1)$ superalgebra—symplectic algebra $\mathfrak{sp}(4)$—is isomorphic to AdS algebra $\mathfrak{so}(2,3)$ that reduces to Poincaré algebra by Wigner-Inönü contraction [51]. The subject of NC SUGRA has been treated in Refs. [52,53]. Classical action for OSp(4|1) SUGRA presented in Ref. [53] is manifestly invariant under OSp(4|1) gauge transformations, and we will use it as a motivation. However, obtaining explicit NC deformation of this action is exceedingly difficult because the first nonvanishing NC correction is quadratic in $\theta^{\mu\nu}$. Taking a lesson from Refs. [54–56] that inclusion of Dirac spinors coupled to $U(1)$ gauge field produces (much simpler) linear NC correction, we will make a transition to OSp(4|2) SUGRA that involves a pair of Majorana spinors that can be mixed into a pair of charged spin-3/2 gravitini coupled to $U(1)$ gauge field. We present a geometric action that consists of two "inhomogeneous" parts: an OSp(4|2) gauge-invariant action quadratic in gauge field strength and a supplementary action, invariant under the purely bosonic $SO(2,3)\times U(1)$ sector of OSp(4|2), that has to be included in order to obtain complete $N=2$ AdS_4 SUGRA at the classical level; this additional bosonic term produces a nontrivial linear NC correction to $N=2$ AdS_4 SUGRA, after deformation.

In Sec. II, we introduce undeformed geometric action for OSp(4|2) SUGRA and make comparison with the similar action for OSp(4|1) SUGRA. In Sec. III, we perform NC deformation by using the Seiberg-Witten approach and study the first-order NC correction to $N=2$ AdS_4 SUGRA. Section IV contains discussion and proposals for further investigation. Appendixes A and B contain supplementary material.

II. CLASSICAL ORTHOSYMPLECTIC SUGRA

We consider two *classical* (undeformed) SUGRA models based on the orthosymplectic OSp$(4|N)$ gauge group: the simple $N = 1$ AdS$_4$ SUGRA, describing pure supergravity with the negative cosmological constant, and the extended $N = 2$ AdS$_4$ SUGRA that involves also a pair of charged gravitini fields coupled to $U(1)$ gauge field. We focus our attentions on the latter ($N = 2$), since the former ($N = 1$) has been treated extensively in Ref. [53], including its NC deformation, and we discuss it just for comparison. Some significant differences of the two models in question have been manifested already at the level of their classical actions, and this reflects drastically on the structure of their NC corrections after deformation.

A. Classical OSp(4|2) SUGRA

Orthosymplectic group OSp(4|2) has 19 generators, and they are of two kinds: bosonic and fermionic. Ten bosonic generators $\hat{M}_{AB} = -\hat{M}_{BA}$ ($A, B = 0, 1, 2, 3, 5$) span AdS Lie algebra $\mathfrak{so}(2,3)$ (symmetry algebra of AdS$_4$),

$$[\hat{M}_{AB}, \hat{M}_{CD}] = i(\eta_{AD}\hat{M}_{BC} + \eta_{BC}\hat{M}_{AD} - \eta_{AC}\hat{M}_{BD} - \eta_{BD}\hat{M}_{AC}), \tag{2.1}$$

where η_{AB} is a flat five-dimensional (5D) metric with signature $(+,-,-,-,+)$. By splitting this set of generators into six $\hat{M}_{ab}$ AdS rotation generators ($a, b = 0, 1, 2, 3$) and four AdS translation generators $\hat{M}_{a5}$, we can recast $\mathfrak{so}(2,3)$ algebra in a more explicit form:

$$\begin{aligned}
&[\hat{M}_{a5}, \hat{M}_{b5}] = -i\hat{M}_{ab},\\
&[\hat{M}_{ab}, \hat{M}_{c5}] = i(\eta_{bc}\hat{M}_{a5} - \eta_{ac}\hat{M}_{b5}),\\
&[\hat{M}_{ab}, \hat{M}_{cd}] = i(\eta_{ad}\hat{M}_{bc} + \eta_{bc}\hat{M}_{ad} - \eta_{ac}\hat{M}_{bd} - \eta_{bd}\hat{M}_{ac}).
\end{aligned} \tag{2.2}$$

If we introduce a new set of generators $(\hat{\mathcal{M}}_{ab}, \hat{\mathcal{P}}_a)$ defined by $\hat{\mathcal{M}}_{ab} \coloneqq \hat{M}_{ab}$ and $\hat{\mathcal{P}}_a \coloneqq l^{-1}\hat{M}_{a5} = \alpha\hat{M}_{a5}$, where l is a length scale related to AdS radius and $\alpha = l^{-1}$ (we will use both parameters in the following formulas), the algebra (2.2) transforms into

$$\begin{aligned}
&[\hat{\mathcal{P}}_a, \hat{\mathcal{P}}_b] = -i\alpha^2\hat{\mathcal{M}}_{ab},\\
&[\hat{\mathcal{M}}_{ab}, \hat{\mathcal{P}}_c] = i(\eta_{bc}\hat{\mathcal{P}}_a - \eta_{ac}\hat{\mathcal{P}}_b),\\
&[\hat{\mathcal{M}}_{ab}, \hat{\mathcal{M}}_{cd}] = i(\eta_{ad}\hat{\mathcal{M}}_{bc} + \eta_{bc}\hat{\mathcal{M}}_{ad} - \eta_{ac}\hat{\mathcal{M}}_{bd} - \eta_{bd}\hat{\mathcal{M}}_{ac}).
\end{aligned} \tag{2.3}$$

In the limit $\alpha \to 0$ (or $l \to \infty$), AdS algebra reduces to Poincaré algebra; in particular, we obtain $[\hat{\mathcal{P}}_a, \hat{\mathcal{P}}_b] = 0$ with all other commutators left unchanged. This is a famous example of the Wigner-Inönü (WI) contraction, the contraction parameter being α (or l). This Lie-algebra contraction (or deformation) can be extended to AdS superalgebra, and we will be interested, later on, in its effect on the NC correction of $N = 2$ AdS$_4$ SUGRA.

A representation of the AdS sector of $\mathfrak{osp}(4|1)$ superalgebra can be obtained by using 5D gamma matrices Γ_A satisfying Clifford algebra $\{\Gamma_A, \Gamma_B\} = 2\eta_{AB}$; the AdS generators $\hat{M}_{AB}$ are represented by 6×6 *supermatrices*, which reduce to 4×4 matrices $M_{AB} = \frac{i}{4}[\Gamma_A, \Gamma_B]$ in the AdS subspace; see Appendix A. One choice of Γ matrices is $\Gamma_A = (i\gamma_a\gamma_5, \gamma_5)$, where γ_a are the usual four-dimensional γ matrices. In this particular representation, the components of M_{AB} are given by $M_{ab} = \frac{i}{4}[\gamma_a, \gamma_b] = \frac{1}{2}\sigma_{ab}$ and $M_{a5} = -\frac{1}{2}\gamma_a$.

The ten AdS bosonic generators M_{AB} are accompanied by eight independent fermionic generators $\hat{Q}^I_\alpha$, with spinor index $\alpha = 1, 2, 3, 4$ and $SO(2)$ index $I = 1, 2$, comprising a pair of Majorana spinors, and one additional bosonic generator $\hat{T}$ related to $SO(2)\sim U(1)$ extension. Together, they satisfy $\mathfrak{osp}(4|2)$ superalgebra (consistency requires that fermionic generators $\hat{Q}^I_\alpha$ transform as components of an AdS Majorana spinor),

$$\begin{aligned}
[\hat{M}_{AB}, \hat{M}_{CD}] &= i(\eta_{AD}\hat{M}_{BC} + \eta_{BC}\hat{M}_{AD} - \eta_{AC}\hat{M}_{BD} - \eta_{BD}\hat{M}_{AC}),\\
[\hat{M}_{AB}, \hat{Q}^I_\alpha] &= -(M_{AB})_\alpha{}^\beta\hat{Q}^I_\beta,\\
\{\hat{Q}^I_\alpha, \hat{Q}^J_\beta\} &= -2\delta^{IJ}(M^{AB}C^{-1})_{\alpha\beta}\hat{M}_{AB} - i\varepsilon^{IJ}C_{\alpha\beta}\hat{T},\\
[\hat{T}, \hat{Q}^I_\alpha] &= -i\varepsilon^{IJ}\hat{Q}^I_\alpha,
\end{aligned} \tag{2.4}$$

with antisymmetric tensor ε^{IJ}, $\varepsilon^{12} = 1$. Matrix C^{-1} is the inverse of the charge-conjugation matrix (spinor metric) for which we use the following representation given in terms of Pauli matrices: $C = -\sigma^3 \otimes i\sigma^2$ and $C_{\alpha\beta} = -C_{\beta\alpha}$. Numerically, we have $C^{-1} = -C$, but the index structure of the two is different since $C_{\alpha\gamma}(C^{-1})^{\gamma\beta} = \delta^\beta_\alpha$. More visually,

$$C = \left(\begin{array}{cc|cc} 0 & -1 & \multicolumn{2}{c}{\multirow{2}{*}{$0_{2\times 2}$}} \\ 1 & 0 & & \\ \hline \multicolumn{2}{c|}{\multirow{2}{*}{$0_{2\times 2}$}} & 0 & 1 \\ & & -1 & 0 \end{array}\right). \tag{2.5}$$

An explicit matrix representation of $\mathfrak{osp}(4|2)$ superalgebra is given in Appendix A.

By introducing a new set of (rescaled) generators $\{\hat{\mathcal{M}}_{ab} \coloneqq \hat{M}_{ab}, \hat{\mathcal{P}}_a \coloneqq \alpha\hat{M}_{a5}, \hat{\mathcal{Q}}^I_\alpha \coloneqq \sqrt{\alpha}\hat{Q}^I_\alpha, \hat{\mathcal{T}} \coloneqq \alpha\hat{T}\}$, we can recast the $\mathfrak{osp}(4|2)$ superalgebra (2.4) into the following form:

$$
\begin{aligned}
[\hat{\mathcal{P}}_a,\hat{\mathcal{P}}_b] &= -i\alpha^2\hat{\mathcal{M}}_{ab},\\
[\hat{\mathcal{M}}_{ab},\hat{\mathcal{P}}_c] &= i(\eta_{bc}\hat{\mathcal{P}}_a-\eta_{ac}\hat{\mathcal{P}}_b),\\
[\hat{\mathcal{M}}_{ab},\hat{\mathcal{M}}_{cd}] &= i(\eta_{ad}\hat{\mathcal{M}}_{bc}+\eta_{bc}\hat{\mathcal{M}}_{ad}-\eta_{ac}\hat{\mathcal{M}}_{bd}-\eta_{bd}\hat{\mathcal{M}}_{ac}),\\
[\hat{\mathcal{P}}_a,\hat{\mathcal{Q}}^I_\alpha] &= -\alpha(M_{a5})_\alpha{}^\beta\hat{\mathcal{Q}}^I_\beta,\\
[\hat{\mathcal{M}}_{ab},\hat{\mathcal{Q}}^I_\alpha] &= -(M_{ab})_\alpha{}^\beta\hat{\mathcal{Q}}^I_\beta,\\
[\hat{\mathcal{T}},\hat{\mathcal{Q}}^I_\alpha] &= -i\varepsilon^{IJ}\hat{\mathcal{Q}}^I_\alpha,\\
\{\hat{\mathcal{Q}}^I_\alpha,\hat{\mathcal{Q}}^J_\beta\} &= -2\delta^{IJ}\alpha(M^{ab}C^{-1})_{\alpha\beta}\hat{\mathcal{M}}_{ab}\\
&\quad -2\delta^{IJ}(M^{a5}C^{-1})_{\alpha\beta}\hat{\mathcal{P}}_a-i\varepsilon^{IJ}C_{\alpha\beta}\hat{\mathcal{T}}. \qquad (2.6)
\end{aligned}
$$

Under WI contraction $\alpha\to 0$, it reduces to $N=2$ Poincaré superalgebra.

Orthosymplectic supergroup $\mathrm{OSp}(2n|m)$ (the symplectic sector is always even dimensional) consists of those supermatrices U that preserve the graded metric

$$
G=\left(\begin{array}{c|c}\Sigma_{\alpha\beta} & 0_{2n\times m}\\ \hline 0_{m\times 2n} & \Delta_{ij}\end{array}\right), \qquad (2.7)
$$

with some real $2n\times 2n$ matrix $\Sigma_{\alpha\beta}=-\Sigma_{\beta\alpha}$ and some real $m\times m$ matrix $\Delta_{ij}=\Delta_{ji}$. Considering only infinitesimal transformations $U=1+\epsilon M$, generated by some $\mathfrak{osp}(2n|m)$-valued supermatrix

$$
M=\left(\begin{array}{c|c}A & B\\ \hline C & D\end{array}\right) \qquad (2.8)
$$

(bosonic blocks $A_{2n\times 2n}$ and $D_{m\times m}$ have ordinary *commuting* entries, and fermionic blocks $B_{2n\times m}$ and $C_{m\times 2n}$ have *Grassmann-valued* entries), the defining relation becomes

$$
M^{ST}G+GM=0. \qquad (2.9)
$$

The supertranspose, super-Hermitian adjoint and supertrace are defined by imposing the standard rules $(MN)^{ST}=N^{ST}M^{ST}$, $(MN)^\dagger=N^\dagger M^\dagger$ and $S\mathrm{Tr}(MN)=S\mathrm{Tr}(NM)$,

$$
M^{ST}=\left(\begin{array}{c|c}A^T & C^T\\ \hline -B^T & D^T\end{array}\right),\qquad M^\dagger=\left(\begin{array}{c|c}A^\dagger & C^\dagger\\ \hline B^\dagger & D^\dagger\end{array}\right),
$$
$$
\mathrm{STr}(M)=\mathrm{Tr}(A)-\mathrm{Tr}(D). \qquad (2.10)
$$

Now, the key observation is that a pair of Majorana fields χ^I_μ (describing a pair of neutral spin-3/2 gravitini) constitute the fermionic sector of the $\mathfrak{osp}(4|2)$ connection supermatrix Ω_μ. We can expand this superconnection over the basis $\{\hat{M}_{ab},\hat{M}_{a5},\hat{Q}^I_\alpha,\hat{T}\}$ with the corresponding gauge fields $\{\omega_\mu{}^{ab},\omega_\mu{}^{a5},\bar{\chi}^I_\mu,A_\mu\}$, as

$$
\Omega_\mu=\frac{1}{2}\omega_\mu{}^{ab}\hat{M}_{ab}+\omega_\mu{}^{a5}\hat{M}_{a5}+(\bar{\chi}^I_\mu)^\alpha\hat{Q}^I_\alpha+A_\mu\hat{T}=\left(\begin{array}{c|cc}\omega_\mu & \chi^1_\mu & \chi^2_\mu\\ \hline \bar{\chi}^1_\mu & 0 & iA_\mu\\ \bar{\chi}^2_\mu & -iA_\mu & 0\end{array}\right), \qquad (2.11)
$$

where we have $\mathfrak{so}(2,3)$ gauge field $\omega_\mu=\frac{1}{2}\omega_\mu{}^{AB}M_{AB}=\frac{1}{2}\omega_\mu{}^{ab}M_{ab}+\omega_\mu{}^{a5}M_{a5}=\frac{1}{4}\omega_\mu{}^{ab}\sigma_{ab}-\frac{1}{2}\omega_\mu{}^{a5}\gamma_a$, a pair of Majorana vector spinors χ^I_μ with components $(\chi^I_\mu)_\alpha$, and their Dirac-adjoints $\bar{\chi}^I_\mu=-(\chi^I_\mu)^TC^{-1}$ with components $(\bar{\chi}^I_\mu)^\alpha=-(\chi^I_\mu)_\beta(C^{-1})^{\beta\alpha}$ ($\alpha=1,2,3,4$).

Equivalently, we can expand Ω_μ over the rescaled basis $\{\hat{\mathcal{M}}_{ab},\hat{\mathcal{P}}_a,\hat{\mathcal{Q}}^I_\alpha,\hat{\mathcal{T}}\}$, but with a different set of gauge fields $\{\omega_\mu{}^{ab},e^a_\mu\coloneqq\frac{1}{\alpha}\omega_\mu{}^{a5},\bar{\psi}^I_\mu\coloneqq\frac{1}{\sqrt{\alpha}}\bar{\chi}^I_\mu,\mathcal{A}_\mu\coloneqq\frac{1}{\alpha}A_\mu\}$, as

$$
\Omega_\mu=\frac{1}{2}\omega_\mu{}^{ab}\hat{\mathcal{M}}_{ab}+\frac{1}{\alpha}\omega_\mu{}^{a5}\hat{\mathcal{P}}_a+\frac{1}{\sqrt{\alpha}}\bar{\chi}^\alpha_\mu\hat{\mathcal{Q}}_\alpha+\frac{1}{\alpha}A_\mu\hat{\mathcal{T}}=\left(\begin{array}{c|cc}\omega_\mu & \sqrt{\alpha}\psi^1_\mu & \sqrt{\alpha}\psi^2_\mu\\ \hline \sqrt{\alpha}\bar{\psi}^1_\mu & 0 & i\alpha\mathcal{A}_\mu\\ \sqrt{\alpha}\bar{\psi}^2_\mu & -i\alpha\mathcal{A}_\mu & 0\end{array}\right), \qquad (2.12)
$$

where we again have $\mathfrak{so}(2,3)$ gauge field $\omega_\mu=\frac{1}{4}\omega_\mu{}^{ab}\sigma_{ab}-\frac{\alpha}{2}e^a_\mu\gamma_a$, two independent Majorana spinors ψ^I_μ, and (dimensionless) $U(1)$ vector potential $\mathcal{A}_\mu$. We will use this particular representation because it makes WI contraction more transparent.

The two Majorana spinors, ψ^1_μ and ψ^2_μ, can be combined into an $SO(2)$ doublet,

$$
\Psi_\mu=\begin{pmatrix}\psi^1_\mu\\ \psi^2_\mu\end{pmatrix}. \qquad (2.13)
$$

It can be readily confirmed that the gauge supermatrix (2.12) satisfies the defining relation for the elements of $\mathfrak{osp}(4|2)$ superalgebra [C is the charge-conjugation matrix (2.5)],

$$\Omega_\mu^{ST}\left(\begin{array}{c|cc} C & 0 & 0 \\ \hline 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right)+\left(\begin{array}{c|cc} C & 0 & 0 \\ \hline 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right)\Omega_\mu = 0. \tag{2.14}$$

The field strength associated with AdS gauge field ω_μ is

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu\omega_\nu - \partial_\nu\omega_\mu - i[\omega_\mu,\omega_\nu] \\ &= (R_{\mu\nu}{}^{ab} - (\omega_\mu{}^{a5}\omega_\nu{}^{b5} - \omega_\mu{}^{b5}\omega_\nu{}^{a5}))\frac{\sigma_{ab}}{4} \\ &\quad - F_{\mu\nu}{}^{a5}\frac{\gamma_a}{2}, \end{aligned} \tag{2.15}$$

with [note that D^L_μ stands for the Lorentz $SO(1,3)$ covariant derivative]

$$R_{\mu\nu}{}^{ab} = \partial_\mu\omega_\nu{}^{ab} - \partial_\nu\omega_\mu{}^{ab} + \omega_\mu{}^a{}_c\omega_\nu{}^{cb} - \omega_\nu{}^a{}_c\omega_\mu{}^{cb}, \tag{2.16}$$

$$F_{\mu\nu}{}^{a5} = D^L_\mu\omega_\nu{}^{a5} - D^L_\nu\omega_\mu{}^{a5}. \tag{2.17}$$

It was shown in the 1970s [28–30] that one can relate AdS gauge field theory to gravity (GR in the first-order formalism) by identifying $\omega_\mu{}^{ab}$ with the Lorentz spin connection and $\omega_\mu{}^{a5}$ with the rescaled vierbein field αe^a_μ; vierbein is related to the metric tensor by $\eta_{ab}e^a_\mu e^b_\nu = g_{\mu\nu}$ and $e=\det(e^a_\mu)=\sqrt{-g}$. The geometric meaning of this relation is thoroughly explained in Ref. [57]. By extension, $R_{\mu\nu}{}^{ab}$ can be identified with the curvature tensor, and $F_{\mu\nu}{}^{a5}$ can be identified with rescaled torsion $\alpha T_{\mu\nu}{}^a$. Therefore, in the AdS setting, we have a natural unification of the vierbein and spin connection as components of a general $SO(2,3)$ gauge field; each transforms as a gauge field and stands on equal footing. To establish this identification, one has to break the original AdS gauge symmetry to the Lorentz $SO(1,3)$ gauge symmetry by introducing an auxiliary field $\phi=\phi^A\Gamma_A$ [30]. This field transforms in the adjoint representation of $SO(2,3)$, and it is constrained by $\eta_{AB}\phi^A\phi^B = l^2$. We can now start with an action quadratic in AdS gauge field strength, originally suggested by MacDowell and Mansouri [28], invariant under $SO(2,3)$ gauge transformations,

$$S_{\mathrm{AdS}} = \frac{il}{64\pi G_N}\mathrm{Tr}\int \mathrm{d}^4x\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\phi, \tag{2.18}$$

where we have the AdS covariant derivative in the adjoint representation,

$$D_\mu\phi = \partial_\mu\phi - i[\omega_\mu,\phi]. \tag{2.19}$$

We choose the *physical gauge* by setting $\phi^a=0$ and $\phi^5=l$ and thus obtain

$$S_{\mathrm{AdS}}|_{\mathrm{g.f.}} = -\frac{1}{16\pi G_N}\int \mathrm{d}^4x\bigg(e(R(e,\omega)-6/l^2) + \frac{l^2}{16}R_{\mu\nu}{}^{ab}R_{\rho\sigma}{}^{cd}\varepsilon^{\mu\nu\rho\sigma}\varepsilon_{abcd}\bigg), \tag{2.20}$$

which is the standard GR action (written in the first-order formalism) involving the Einstein-Hilbert term, negative cosmological constant $\Lambda=-3/l^2=-3\alpha^2$, and the topological Gauss-Bonnet term that can be omitted.

Therefore, we can write the $SO(2,3)$ field strength as

$$F_{\mu\nu} = \frac{1}{4}(R_{\mu\nu}{}^{ab} - \alpha^2(e^a_\mu e^b_\nu - e^b_\mu e^a_\nu))\sigma_{ab} - \frac{\alpha}{2}T_{\mu\nu}{}^a\gamma_a, \tag{2.21}$$

and we see that the vierbein and torsion terms vanish under WI contraction.

By generalization, we introduce $\mathrm{OSp}(4|2)$ field strength $\mathbb{F}_{\mu\nu}$ associated with the superconnection Ω_μ,

$$\begin{aligned} \mathbb{F}_{\mu\nu} &= \partial_\mu\Omega_\nu - \partial_\nu\Omega_\mu - i[\Omega_\mu,\Omega_\nu] \\ &= \left(\begin{array}{c|cc} \tilde{F}_{\mu\nu} & \sqrt{\alpha}(\mathcal{D}_\mu\psi^1_\nu - \mathcal{D}_\nu\psi^1_\mu) & \sqrt{\alpha}(\mathcal{D}_\mu\psi^2_\nu - \mathcal{D}_\nu\psi^2_\mu) \\ \hline \sqrt{\alpha}(\mathcal{D}_\mu\bar{\psi}^1_\nu - \mathcal{D}_\nu\bar{\psi}^1_\mu) & 0 & i\alpha\tilde{\mathcal{F}}_{\mu\nu} \\ \sqrt{\alpha}(\mathcal{D}_\mu\bar{\psi}^2_\nu - \mathcal{D}_\nu\bar{\psi}^2_\mu) & -i\alpha\tilde{\mathcal{F}}_{\mu\nu} & 0 \end{array}\right), \end{aligned} \tag{2.22}$$

with extended AdS field strength $\tilde{F}_{\mu\nu}$ (summation over $I=1,2$ is implied)

$$\begin{aligned} \tilde{F}_{\mu\nu} &= F_{\mu\nu} - i\alpha(\psi^I_\mu\bar{\psi}^I_\nu - \psi^I_\nu\bar{\psi}^I_\mu) \\ &= \frac{1}{4}\tilde{R}_{\mu\nu}{}^{mn}\sigma_{mn} - \frac{\alpha}{2}\tilde{T}_{\mu\nu}{}^m\gamma_m, \end{aligned} \tag{2.23}$$

involving extended curvature tensor $\tilde{R}_{\mu\nu}{}^{mn}$ and extended torsion $\tilde{T}_{\mu\nu}{}^m$, given by

$$\tilde{R}_{\mu\nu}{}^{mn} \coloneqq R_{\mu\nu}{}^{mn} - \alpha^2(e^m_\mu e^n_\nu - e^n_\mu e^m_\nu) - i\alpha(\bar{\Psi}_\mu\sigma^{mn}\Psi_\nu), \tag{2.24}$$

$$\tilde{T}_{\mu\nu}{}^m \coloneqq T_{\mu\nu}{}^m + i(\bar{\Psi}_\mu\gamma^m\Psi_\nu). \tag{2.25}$$

Electromagnetic field strength is also modified by a bilinear current term $\mathcal{J}_{(e)}$,

$$\tilde{\mathcal{F}}_{\mu\nu} \coloneqq \mathcal{F}_{\mu\nu} - \mathcal{J}_{(e)\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu - \bar{\Psi}_\mu i\sigma^2 \Psi_\nu. \quad (2.26)$$

Note that Pauli matrix $i\sigma^2$ mixes the two Majorana components in $\mathcal{J}_{(e)}$.

In the fermionic sector of $\mathbb{F}_{\mu\nu}$, we introduced

$$\mathcal{D}_\mu \psi^1_\nu \coloneqq D_\mu \psi^1_\nu + \alpha \mathcal{A}_\mu \psi^2_\nu, \quad (2.27)$$

$$\mathcal{D}_\mu \psi^2_\nu \coloneqq D_\mu \psi^2_\nu - \alpha \mathcal{A}_\mu \psi^1_\nu, \quad (2.28)$$

where D_μ stands for $SO(2,3)$ covariant derivative. The fact that Majorana spinors ψ^1_μ and ψ^2_μ are not charged is reflected in the manner in which they couple to the gauge field $\mathcal{A}_\mu$. Using them, we can define two charged Dirac vector spinors $\psi^\pm_\mu = \psi^1_\mu \pm i\psi^2_\mu$, related to each other by C conjugation, $\psi^-_\mu = \psi^{c+}_\mu = C\bar{\psi}^{+T}_\mu$, that do couple to $\mathcal{A}_\mu$ in the right way. Using the Pauli matrix $i\sigma^2$, we can unify (2.27) and (2.28) as

$$\begin{aligned} \mathcal{D}_\mu \Psi_\nu &= (D_\mu + \alpha \mathcal{A}_\mu i\sigma^2)\Psi_\nu \\ &= \left(D^L_\mu + \frac{i\alpha}{2}\gamma_\mu + \alpha \mathcal{A}_\mu i\sigma^2\right)\Psi_\nu. \end{aligned} \quad (2.29)$$

Now, consider an action, similar to the one defined in (2.18) for pure gravity, but now appropriately generalized to be invariant under extended OSp(4|2) gauge transformations,

$$S_{42} = \mathrm{STr}\int \mathrm{d}^4x \varepsilon^{\mu\nu\rho\sigma} \mathbb{F}_{\mu\nu}(a\mathbb{I}_{6\times 6} + b\Phi^2/l^2)\mathbb{F}_{\rho\sigma}\Phi. \quad (2.30)$$

The action is real, and we introduced a pair of free parameters, a and b, that will be fixed later. The first part of (2.30) is a MacDowell-Mansouri type of action, quadratic in OSp(4|2) gauge field strength, and the second part (b term) is necessary for having local SUSY after the symmetry breaking.

Generalized auxiliary field Φ is given by a supermatrix (we now have two Majorana spinors λ_1 and λ_2 and additional scalar fields π, m, and σ), see also Refs. [45–47,53],

$$\Phi = \left(\begin{array}{c|cc} \frac{1}{4}\pi + i\phi^a\gamma_a\gamma_5 + \phi^5\gamma_5 & \lambda_1 & \lambda_2 \\ \hline -\bar{\lambda}_1 & \pi - \sigma & m \\ -\bar{\lambda}_2 & m & \sigma \end{array}\right). \quad (2.31)$$

In the physical gauge, $\lambda_1 = \lambda_2 = \pi = \sigma = m = \phi^a = 0$, and $\phi^5 = l$, yielding

$$\Phi|_{\mathrm{g.f.}} = \left(\begin{array}{c|cc} l\gamma_5 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right). \quad (2.32)$$

Field strength $\mathbb{F}_{\mu\nu}$ and the auxiliary field Φ transform in the adjoint representation of OSp(4|2), with infinitesimal variations

$$\delta_\epsilon \mathbb{F}_{\mu\nu} = i[\epsilon, \mathbb{F}_{\mu\nu}], \qquad \delta_\epsilon \Phi = i[\epsilon, \Phi], \quad (2.33)$$

for some $\mathfrak{osp}(4|2)$-valued gauge parameter ϵ given by a supermatrix,

$$\epsilon = \left(\begin{array}{c|cc} \frac{1}{2}\epsilon^{AB}M_{AB} & \xi_1 & \xi_2 \\ \hline \bar{\xi}_1 & 0 & i\alpha \\ \bar{\xi}_2 & -i\alpha & 0 \end{array}\right). \quad (2.34)$$

From (2.33), the invariance of the action (2.30) follows immediately.

After the gauge fixing, field Φ^2/l^2 that appears in the second term of (2.30) becomes a projector that reduces any $\mathfrak{osp}(4|2)$ supermatrix to its $\mathfrak{so}(2,3)$ sector, and the classical OSp(4|2) gauge-invariant action (2.30) reduces to

$$\begin{aligned} S_{42}|_{\mathrm{g.f.}} = \int \mathrm{d}^4x \varepsilon^{\mu\nu\rho\sigma}\Bigg(&\frac{(a+b)il}{4}\tilde{R}_{\mu\nu}{}^{mn}\tilde{R}_{\rho\sigma}{}^{rs}\varepsilon_{mnrs} \\ &- 4a(\mathcal{D}_\mu\bar{\Psi}_\nu\gamma_5\mathcal{D}_\rho\Psi_\sigma)\Bigg). \end{aligned} \quad (2.35)$$

The term that is quadratic in the Lorentz $SO(1,3)$ covariant derivative D^L_μ can be transformed by partial integration,

$$\begin{aligned} &\int \mathrm{d}^4x \varepsilon^{\mu\nu\rho\sigma}(D^L_\mu\bar{\Psi}_\nu\gamma_5 D^L_\rho\Psi_\sigma) \\ &= \frac{1}{16}\int \mathrm{d}^4x \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu}{}^{mn}(\bar{\Psi}_\rho\sigma^{rs}\Psi_\sigma)\varepsilon_{mnrs}, \end{aligned} \quad (2.36)$$

where we invoked the commutator of two Lorentz covariant derivatives

$$i[D^L_\mu, D^L_\nu]\Psi_\sigma = \frac{1}{4}R_{\mu\nu}{}^{mn}\sigma_{mn}\Psi_\sigma. \quad (2.37)$$

A term of the same type appears in the first part of the action (2.35). These two contributions have to cancel each other in order to have SUSY, and this implies the constraint $b = -a/2$. Moreover, to obtain the correct normalization of the Einstein-Hilbert term, we set $a = il/32\pi G_N = il/4\kappa^2$, yielding

$$
\begin{aligned}
S_{42}|_{\mathrm{g.f.}} = -\frac{1}{2\kappa^2}\int \mathrm{d}^4x\bigg(& e(R(e,\omega)-6\alpha^2) \\
& +\frac{1}{16\alpha^2}R_{\mu\nu}{}^{mn}R_{\rho\sigma}{}^{rs}\varepsilon^{\mu\nu\rho\sigma}\varepsilon_{mnrs} \\
& +\varepsilon^{\mu\nu\rho\sigma}\bigg(2\bar{\Psi}_\mu\gamma_5\gamma_\nu(D_\rho+\alpha\mathcal{A}_\rho i\sigma^2)\Psi_\sigma \\
& +i\mathcal{F}_{\mu\nu}(\bar{\Psi}_\rho\gamma_5 i\sigma^2\Psi_\sigma) \\
& -\frac{i}{2}(\bar{\Psi}_\mu i\sigma^2\Psi_\nu)(\bar{\Psi}_\rho\gamma_5 i\sigma^2\Psi_\sigma)\bigg)\bigg). \qquad (2.38)
\end{aligned}
$$

However, this is *not* the full $N=2$ AdS$_4$ SUGRA action. The gravity part is correct (we can omit the topological Gauss-Bonnet term), and we also get the correct kinetic term for the gravitino doublet. There are also two bilinear source terms, electric and magnetic,

$$
\begin{aligned}
\mathcal{J}_{(e)\mu\nu} &\coloneqq \bar{\Psi}_\mu i\sigma^2\Psi_\nu, \\
\mathcal{J}^{\mu\nu}_{(m)} &\coloneqq \frac{i}{2e}\varepsilon^{\mu\nu\rho\sigma}(\bar{\Psi}_\rho\gamma_5 i\sigma^2\Psi_\sigma). \qquad (2.39)
\end{aligned}
$$

But we are missing the contribution from the $SO(2)$ part of the bosonic sector, in particular, the kinetic term for $U(1)$ gauge field $\mathcal{A}_\mu$. The reason for this defect can be traced back to the specific form that the auxiliary field assumes in the physical gauge $\Phi|_{\mathrm{g.f.}}$ (2.32); it completely annihilates the $SO(2)$ sector of any $\mathfrak{osp}(4|2)$ supermatrix. To restore the missing terms, we must introduce an additional action, supplementing (2.30). In Ref. [55], following the approach of Ref. [58], we defined a classical action invariant under $SO(2,3)\times U(1)$ gauge transformations [locally isomorphic to the bosonic sector of OSp$(4|2)$] that involves an *additional auxiliary field* $f=\frac{1}{2}f^{AB}M_{AB}$. Its role is to produce the canonical kinetic term for $U(1)$ gauge field in the absence of the Hodge dual operator (this is, of course, the crucial point; we are trying to construct a purely geometrical action that does not involve the metric tensor $g_{\mu\nu}$ explicitly). This auxiliary field f is a $U(1)$-neutral 0-form that takes values in $\mathfrak{so}(2,3)$ algebra, and it transforms in the adjoint representation of $SO(2,3)$.

The way to proceed is to employ this auxiliary field method to include the modified $U(1)$ field strength $\tilde{\mathcal{F}}_{\mu\nu}$ defined in (2.26). However, there seems to be no way to construct an OSp$(4|2)$ gauge-invariant action that is compatible with this procedure. Therefore, we will use an action, analogous to the one in Ref. [55], invariant under the purely bosonic $SO(2,3)\times U(1)$ sector of OSp$(4|2)$, involving the bosonic field strength $\tilde{f}_{\mu\nu}\coloneqq\tilde{F}_{\mu\nu}+\kappa^{-1}\tilde{\mathcal{F}}_{\mu\nu}=\tilde{F}_{\mu\nu}+\kappa^{-1}(\mathcal{F}_{\mu\nu}-\mathcal{J}_{(e)\mu\nu})$ of $SO(2,3)\times U(1)$. The action is given by

$$
\begin{aligned}
S_A = \mathrm{Tr}\int \mathrm{d}^4x\varepsilon^{\mu\nu\rho\sigma}(& cf\tilde{f}_{\mu\nu}D_\rho\phi D_\sigma\phi\phi \\
& +df^2D_\mu\phi D_\nu\phi D_\rho\phi D_\sigma\phi\phi)+\mathrm{c.c.} \qquad (2.40)
\end{aligned}
$$

Note that, by doing this, we lose the complete OSp$(4|2)$ gauge invariance of the undeformed action before the symmetry breaking. Nevertheless, we will obtain the correct action for $N=2$ SdS_4 SUGRA in the physical gauge, and this is the only requirement that has to be satisfied in order to perform NC deformation.

After calculating traces (see Appendix B), we obtain

$$
\begin{aligned}
S_A = \int \mathrm{d}^4x\varepsilon^{\mu\nu\rho\sigma}\bigg(&\frac{ic}{2}f^{AB}F_{\mu\nu}{}^{CD}(D_\rho\phi)^E(D_\sigma\phi)^F \\
&\times\phi^G(\eta_{FG}\varepsilon_{ABCDE}+2\eta_{AD}\varepsilon_{BCEFG}) \\
&+c\kappa^{-1}f^{AB}\tilde{\mathcal{F}}_{\mu\nu}(D_\rho\phi)^E(D_\sigma\phi)^F\phi^G\varepsilon_{ABEFG} \\
&-\frac{id}{2}f^{AB}f_{AB}(D_\mu\phi)^E(D_\nu\phi)^F(D_\rho\phi)^G(D_\sigma\phi)^H \\
&\times\phi^R\varepsilon_{EFGHR}\bigg)+\mathrm{c.c.} \qquad (2.41)
\end{aligned}
$$

We conclude that parameter c must be real; otherwise, the second term, involving $\tilde{\mathcal{F}}_{\mu\nu}$, would be purely imaginary and would not contribute (and this term is the one that we need to include). Therefore, assuming real c, the first term (involving gravitational quantities like curvature tensor and torsion) becomes purely imaginary and vanishes after adding its complex conjugate (c.c.). Also, d must be purely imaginary for the procedure to work.

Gauge fixing yields

$$
S_A|_{\mathrm{g.f.}} = \int \mathrm{d}^4xe(-8lc\kappa^{-1}f^{ab}\tilde{\mathcal{F}}_{\mu\nu}e^\mu_ae^\nu_b+24ildf^{AB}f_{AB}). \qquad (2.42)
$$

By varying this gauge-fixed action over f^{ab} and f^{a5} independently, we obtain algebraic equations of motion (EoM) for the components f_{ab} and f_{a5} of the auxiliary field f, respectively, and they are given by

$$
f_{ab} = -\frac{ic}{6\kappa d}\tilde{\mathcal{F}}_{\mu\nu}e^\mu_ae^\nu_b, \qquad f_{a5}=0. \qquad (2.43)
$$

Inserting them back into the action (2.42), we obtain

$$
S_A|_{\mathrm{g.f.}} = \frac{2ilc^2}{3\kappa^2d}\int \mathrm{d}^4xe\tilde{\mathcal{F}}^2. \qquad (2.44)
$$

To get the consistent normalization, we set the prefactor to $(8\kappa^2)^{-1}$, yielding another constraint $16ilc^2=3d$ for the parameters c and d. To make the connection with the results of Ref. [55], we take $c=1/32l$ and $d=i/192l$, implying

$$
f_{ab} = -\kappa^{-1}\tilde{\mathcal{F}}_{\mu\nu}e^\mu_ae^\nu_b. \qquad (2.45)
$$

Therefore, after imposing the physical gauge, the original bosonic action (2.40), invariant under $SO(2,3)\times U(1)$ gauge transformations, reduces to the $SO(1,3)\times U(1)$

gauge-invariant action containing the canonical kinetic term for $U(1)$ gauge field $\mathcal{A}_\mu$ in curved space-time and two additional terms involving gravitino current $\mathcal{J}_{(e)\mu\nu} = \bar{\Psi}_\mu i\sigma^2\Psi_\nu$,

$$S_A|_{\text{g.f.}} = \frac{1}{4\kappa^2}\int d^4xe\tilde{\mathcal{F}}^2 = \frac{1}{4\kappa^2}\int d^4xe(\mathcal{F}^2 - 2\mathcal{F}\cdot\mathcal{J}_{(e)} + \mathcal{J}_{(e)}^2). \quad (2.46)$$

This is exactly the piece that was missing in (2.38). With this result in hand, we have the complete classical $N=2$ AdS$_4$ SUGRA action [43,44],

$$(S_{42}+S_A)|_{\text{g.f.}} = -\frac{1}{2\kappa^2}\int d^4xe\Big(R-6\alpha^2 + 2e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar{\Psi}_\mu\gamma_5\gamma_\nu(D_\rho+\alpha\mathcal{A}_\rho i\sigma^2)\Psi_\sigma + 2\mathcal{F}\cdot\mathcal{J}_{(m)} - \mathcal{J}_{(e)}\cdot\mathcal{J}_{(m)} - \frac{1}{4}(\mathcal{F}^2+\mathcal{J}_{(e)}^2-2\mathcal{F}\cdot\mathcal{J}_{(e)})\Big). \quad (2.47)$$

The most important characteristics of this SUGRA model are the negative cosmological constant $\Lambda = -3\alpha^2 = -3/l^2$ and the fact that $U(1)$ coupling strength is equal to the WI contraction parameter α. Under WI contraction ($\alpha\to 0$), the $N=2$ AdS$_4$ SUGRA action consistently reduces to the $N=2$ Poincaré SUGRA action.

In terms of charged Dirac vector spinors $\psi_\mu^\pm = \psi_\mu^1 \pm i\psi_\mu^2$ (actually, we can use only one of them since they are related to each other by C conjugation), the action becomes

$$(S_{42}+S_A)|_{\text{g.f.}} = -\frac{1}{2\kappa^2}\int d^4xe\Big(R(e,\omega)-6\alpha^2 + 2e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu^+\gamma_5\gamma_\nu(D_\rho - i\alpha\mathcal{A}_\rho)\psi_\sigma^+ + 2\mathcal{F}\cdot\mathcal{J}_{(m)}^+ - \mathcal{J}_{(e)}^+\cdot\mathcal{J}_{(m)}^+ - \frac{1}{4}(\mathcal{F}^2 - 2\mathcal{F}\cdot\mathcal{J}_{(e)}^+ + (\mathcal{J}_{(e)}^+)^2)\Big), \quad (2.48)$$

with $\mathcal{J}_{(e)}^+ = \frac{1}{2i}(\bar{\psi}_\mu^+\psi_\nu^+ - \bar{\psi}_\nu^+\psi_\mu^+)$ and $\mathcal{J}_{(m)}^+ = \frac{1}{4e}(\bar{\psi}_\mu^+\gamma_5\psi_\nu^+ - \bar{\psi}_\nu^+\gamma_5\psi_\mu^+)$.

For later purposes, we note that action (2.48) contains a masslike term for the charged gravitino (we absorbed the parameter κ^{-1} into ψ_μ^+ to obtain the canonical dimensions),

$$i\alpha\int d^4xe\bar{\psi}_\mu^+\sigma^{\mu\nu}\psi_\nu^+, \quad (2.49)$$

with masslike parameter equal to the WI contraction parameter.

B. OSp(4|1) SUGRA

The OSp(4|1) supergroup has 14 generators: ten bosonic AdS generators $\hat{M}_{AB}$ and four fermionic generators $\hat{Q}_\alpha$ comprising a single Majorana spinor (describing a single neutral gravitino). The supermatrix for the OSp(4|1) gauge field Ω_μ is given by

$$\Omega_\mu = \left(\begin{array}{c|c} \omega_\mu & \sqrt{\alpha}\psi_\mu \\ \hline \sqrt{\alpha}\bar{\psi}_\mu & 0 \end{array}\right). \quad (2.50)$$

Consider the following action invariant under OSp(4|1) gauge transformations [52]:

$$S_{41} = \frac{il}{32\pi G_N}\mathrm{STr}\int d^4x\varepsilon^{\mu\nu\rho\sigma}\mathbb{F}_{\mu\nu}\left(\mathbb{I}_{5\times5} - \frac{1}{2l^2}\Phi^2\right)\mathbb{F}_{\rho\sigma}\Phi. \quad (2.51)$$

The auxiliary field is

$$\Phi = \left(\begin{array}{c|c} \frac{1}{4}\pi + i\phi^a\gamma_a\gamma_5 + \phi^5\gamma_5 & \lambda \\ \hline -\bar{\lambda} & \pi \end{array}\right) \overset{\text{g.f.}}{=} \left(\begin{array}{c|c} l\gamma_5 & 0 \\ \hline 0 & 0 \end{array}\right). \quad (2.52)$$

In the physical gauge, the OSp(4|1) gauge-invariant action (2.51) *exactly* reduces to the $N=1$ AdS$_4$ SUGRA action [43,44,53],

$$S_{41}|_{\text{g.f.}} = -\frac{1}{2\kappa^2}\int d^4x(e(R(e,\omega)-6/l^2) + 2\varepsilon^{\mu\nu\rho\sigma}(\bar{\psi}_\mu\gamma_5\gamma_\nu D_\rho\psi_\sigma)) = -\frac{1}{2\kappa^2}\int d^4xe(R(e,\omega)-6\alpha^2 + 2e^{-1}\varepsilon^{\mu\nu\rho\sigma}(\bar{\psi}_\mu\gamma_5\gamma_\nu D_\rho^L\psi_\sigma) - 2i\alpha(\bar{\psi}_\mu\sigma^{\mu\nu}\psi_\nu)). \quad (2.53)$$

It contains the Einstein-Hilbert term with the negative cosmological constant $\Lambda = -3/l^2$, the Rarita-Schwinger kinetic term for the *neutral* gravitino, and a masslike gravitino term that is needed in the presence of the cosmological constant to insure the invariance under local SUSY (the gravitino actually remains massless). The topological Gauss-Bonnet term is omitted. The cosmological constant and the masslike term vanish under WI contraction, yielding minimal $N=1$ Poincaré SUGRA. Note that we do not need additional action terms in (2.51) to obtain a consistent classical theory.

It is shown in Ref. [53] that linear (in $\theta^{\mu\nu}$) NC correction to (2.51) vanishes and that one has to calculate the second-order NC correction in order to see NC effects, which is exceedingly difficult. In the following section, we use the Seiberg-Witten approach to NC gauge field theories, to calculate linear NC correction to $N=2$ AdS$_4$ SUGRA, and conclude that *it is not equal to zero*. The nonvanishing part comes from the additional bosonic action, S_A.

III. NC DEFORMATION

Canonical deformation of the orthoymplectic action (2.30) is obtained by replacing ordinary commutative field multiplication with Moyal $\star$-product, yielding an NC action (denoted by $\star$) manifestly invariant under NC-deformed $\mathrm{OSp}(4|2)_\star$ gauge transformations,

$$S^\star_{42} = \frac{il}{32\pi G_N}\mathrm{STr}\int \mathrm{d}^4x\varepsilon^{\mu\nu\rho\sigma}\Big(\hat{\mathbb{F}}_{\mu\nu}\star\hat{\mathbb{F}}_{\rho\sigma}\star\hat{\Phi} - \frac{1}{2l^2}\hat{\mathbb{F}}_{\mu\nu}\star\hat{\Phi}\star\hat{\Phi}\star\hat{\mathbb{F}}_{\rho\sigma}\star\hat{\Phi}\Big). \tag{3.1}$$

Likewise, we have a canonically deformed version of the bosonic action (2.40) with $c = 1/32l$ and $d = i/192l$,

$$S^\star_A = \frac{1}{32l}\mathrm{Tr}\int \mathrm{d}^4x\varepsilon^{\mu\nu\rho\sigma}\Big(\hat{f}\star\hat{\bar{f}}_{\mu\nu}\star D_\rho\hat{\phi}\star D_\sigma\hat{\phi}\star\hat{\phi} + \frac{i}{6}\hat{f}\star\hat{f}\star D_\mu\hat{\phi}\star D_\nu\hat{\phi}\star D_\rho\hat{\phi}\star D_\sigma\hat{\phi}\star\hat{\phi}\Big) + \mathrm{c.c.} \tag{3.2}$$

We denote NC fields by a "hat" symbol. In the Seiberg-Witten approach [3,34–37], NC gauge field theory is completely defined by its commutative (classical) counterpart. For some non-Abelian gauge group $\mathcal{G}$ with generators T_A satisfying Lie-algebra relations $[T_A, T_B] = if_A{}^C{}_B T_C$, the commutator of two infinitesimal gauge transformations δ_{ϵ_1} and δ_{ϵ_2} closes in the algebra,

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] = \delta_{-i[\epsilon_1,\epsilon_2]}. \tag{3.3}$$

There is, however, difficulty, in general, concerning the closure axiom for NC gauge transformations. Namely, for a given pair of NC gauge parameters $\hat{\Lambda}_1$ and $\hat{\Lambda}_2$, we would like to find a third one, $\hat{\Lambda}_3$, such that

$$[\delta^\star_1 \stackrel{\star}{,} \delta^\star_2] = \delta^\star_3. \tag{3.4}$$

Now, if NC gauge parameter $\hat{\Lambda}$ is supposed to be Lie-algebra valued, $\hat{\Lambda}(x) = \hat{\Lambda}^A(x)T_A$, then, for some generic NC field $\hat{\Psi}$ that transforms in the fundamental representation of the gauge group (although the argument holds in any representation), we have

$$\begin{aligned}[\delta^\star_1 \stackrel{\star}{,} \delta^\star_2]\hat{\Psi} &= (\hat{\Lambda}_1\star\hat{\Lambda}_2 - \hat{\Lambda}_2\star\hat{\Lambda}_1)\star\hat{\Psi}\\ &= \frac{1}{2}([\hat{\Lambda}^A_1 \stackrel{\star}{,} \hat{\Lambda}^B_2]\{T_A, T_B\} + \{\hat{\Lambda}^A_1 \stackrel{\star}{,} \hat{\Lambda}^B_2\}[T_A, T_B])\star\hat{\Psi}\\ &= i\hat{\Lambda}_3\star\hat{\Psi} = \delta^\star_3\hat{\Psi}.\end{aligned} \tag{3.5}$$

The NC closure rule

$$[\delta^\star_{\hat{\Lambda}_1} \stackrel{\star}{,} \delta^\star_{\hat{\Lambda}_2}] = \delta^\star_{-i[\hat{\Lambda}_1 \stackrel{\star}{,} \hat{\Lambda}_2]} \tag{3.6}$$

consistently generalizes its commutative counterpart.

However, Eq. (3.5) implies that the commutator of two NC gauge transformations does not generally close in the Lie algebra, because anticommutator $\{T_A, T_B\}$ does not in general belong to this algebra [except for the $U(N)$ gauge group]. To overcome this difficulty, we will apply the universal enveloping algebra (UEA) approach. The enveloping algebra is "large enough" to ensure that closure property of NC gauge transformations holds, provided that NC gauge parameter $\hat{\Lambda}$ is UEA valued.

The NC covariant derivative (for a generic gauge group $\mathcal{G}$) in the fundamental representation is defined by

$$D_\mu\hat{\Psi} = \partial_\mu\hat{\Psi} - i\hat{V}_\mu\star\hat{\Psi}, \tag{3.7}$$

where $\hat{V}_\mu$ stands for the corresponding NC gauge field, and it transforms as

$$\delta^\star_\Lambda D_\mu\hat{\Psi} = i\hat{\Lambda}\star D_\mu\hat{\Psi}, \tag{3.8}$$

implying

$$\delta^\star_\Lambda\hat{V}_\mu = \partial_\mu\hat{\Lambda} + i[\hat{\Lambda} \stackrel{\star}{,} \hat{V}_\mu]. \tag{3.9}$$

Therefore, the NC gauge field must also be UEA valued, and it can be represented in its basis. But UEA has an infinite basis, and it seems that by invoking it we actually introduced an infinite number of new degrees of freedom (new fields) in the NC theory, rendering it unrealistic. This problem is resolved by the SW)\ map [34–37]. Essentially, we assume that classical gauge transformations induce the corresponding NC gauge transformations,

$$\delta^\star_\Lambda\hat{V}_\mu = \hat{V}_\mu(V_\mu + \delta_\epsilon V_\mu) - \hat{V}_\mu(V_\mu). \tag{3.10}$$

This allows us to represent every NC field as a perturbation series in powers of the deformation parameter $\theta^{\mu\nu}$ with expansion coefficients built out of fields from the undeformed theory, e.g., $\hat{\Lambda}_\epsilon = \epsilon + \hat{\Lambda}^{(1)} + \hat{\Lambda}^{(2)} + \cdots$. At zeroth order, NC fields reduce to their undeformed counterparts. For example, the NC gauge parameter and potential can be represented as

$$\hat{\Lambda}_\epsilon = \epsilon - \frac{1}{4}\theta^{\rho\sigma}\{V_\rho, \partial_\sigma\epsilon\} + \mathcal{O}(\theta^2), \tag{3.11}$$

$$\hat{V}_\mu = V_\mu - \frac{1}{4}\theta^{\rho\sigma}\{V_\rho, \partial_\sigma V_\mu + F_{\sigma\mu}\} + \mathcal{O}(\theta^2). \tag{3.12}$$

After these general considerations, we return to the NC action (3.1). Field strength $\hat{\mathbb{F}}_{\mu\nu}$ appearing in (3.1) is defined in terms of $\mathrm{OSp}(4,2)_\star$ gauge potential $\hat{\Omega}_\mu$ as

$$\hat{\mathbb{F}}_{\mu\nu} = \partial_\mu\hat{\Omega}_\nu - \partial_\nu\hat{\Omega}_\mu - i[\hat{\Omega}_\mu \stackrel{\star}{,} \hat{\Omega}_\nu]. \tag{3.13}$$

It transforms in the adjoint representation of the $\mathrm{OSp}(4,2)_\star$ supergroup as well as the NC auxiliary field $\hat{\Phi}$,

$$\delta^\star_\epsilon\hat{\mathbb{F}}_{\mu\nu} = i[\hat{\Lambda}_\epsilon \stackrel{\star}{,} \hat{\mathbb{F}}_{\mu\nu}], \qquad \delta^\star_\epsilon\hat{\Phi} = i[\hat{\Lambda}_\epsilon \stackrel{\star}{,} \hat{\Phi}]. \tag{3.14}$$

At this point, it would be tempting to proceed by directly imposing the physical gauge. However, this operation would not yield an action with an appropriate symmetry because gauge fixing does not commute with NC deformation. A bypass is provided by the SW expansion. Representing NC fields in terms of their undeformed counterparts, we obtain a perturbative expansion of $\mathrm{OSp}(4|2)_*$ gauge-invariant NC action (3.1) in powers of the deformation parameter $\theta^{\mu\nu}$. By construction, the SW map ensures invariance of the expanded action under ordinary OSp(4|2) gauge transformations, order by order.

We now present some relevant steps in the expansion procedure of the NC action (3.1). The goal is to calculate and analyze linear NC correction to the classical action (2.30). The SW expansions, up to the first order in the deformation parameter, of the NC auxiliary field $\hat{\Phi}$ and NC field strength $\hat{\mathbb{F}}_{\mu\nu}$ are given by

$$\hat{\Phi} = \Phi - \frac{1}{4}\theta^{\rho\sigma}\{\Omega_\rho, (\partial_\sigma + \hat{D}_\sigma)\Phi\} + \mathcal{O}(\theta^2), \tag{3.15}$$

$$\hat{\mathbb{F}}_{\mu\nu} = \mathbb{F}_{\mu\nu} - \frac{1}{4}\theta^{\rho\sigma}\{\Omega_\rho, (\partial_\sigma + \hat{D}_\sigma)\mathbb{F}_{\mu\nu}\} + \frac{1}{2}\theta^{\rho\sigma}\{\mathbb{F}_{\rho\mu}, \mathbb{F}_{\sigma\nu}\} + \mathcal{O}(\theta^2), \tag{3.16}$$

where $\hat{D}_\mu$ stands for the OSp(4|2) covariant derivative (associated to Ω_μ).

Generally, for a pair of NC fields $\hat{A}$ and $\hat{B}$, linear NC correction to their product is

$$(\hat{A} \star \hat{B})^{(1)} = \hat{A}^{(1)}B + A\hat{B}^{(1)} + \frac{i}{2}\theta^{\rho\sigma}\partial_\rho A\partial_\sigma B. \tag{3.17}$$

In particular, if both fields transform in the adjoint representation of $\mathrm{OSp}(4|2)_\star$, we have

$$(\hat{A} \star \hat{B})^{(1)} = -\frac{1}{4}\theta^{\rho\sigma}\{\Omega_\rho, (\partial_\sigma + \hat{D}_\sigma)AB\} + \frac{i}{2}\theta^{\rho\sigma}\hat{D}_\rho A\hat{D}_\sigma B + (\hat{A}^{(1)})'B + A(\hat{B}^{(1)})'. \tag{3.18}$$

For example, in (3.15) and (3.16), we have

$$(\hat{\Phi}^{(1)})' = 0, \tag{3.19}$$

$$(\hat{\mathbb{F}}^{(1)}_{\mu\nu})' = \frac{1}{2}\theta^{\rho\sigma}\{\mathbb{F}_{\rho\mu}, \mathbb{F}_{\sigma\nu}\}. \tag{3.20}$$

Successive application of the rule (3.18) gives us the first-order NC correction to the classical action (2.30):

$$S^{(1)}_{42} = \frac{il\theta^{\lambda\tau}}{32\pi G_N}\mathrm{STr}\int \mathrm{d}^4x\varepsilon^{\mu\nu\rho\sigma}\Big(-\frac{1}{4}\{\mathbb{F}_{\lambda\tau}, \mathbb{F}_{\mu\nu}\mathbb{F}_{\rho\sigma}\}\Phi + \frac{i}{2}\hat{D}_\lambda\mathbb{F}_{\mu\nu}\hat{D}_\tau\mathbb{F}_{\rho\sigma}\Phi + \frac{1}{2}\{\mathbb{F}_{\lambda\mu}, \mathbb{F}_{\tau\nu}\}\mathbb{F}_{\rho\sigma}\Phi + \frac{1}{2}\mathbb{F}_{\mu\nu}\{\mathbb{F}_{\lambda\rho}, \mathbb{F}_{\tau\sigma}\}\Phi$$
$$- \frac{1}{2l^2}\Big(-\frac{1}{4}\{\mathbb{F}_{\lambda\tau}, \mathbb{F}_{\mu\nu}\Phi^2\}\mathbb{F}_{\rho\sigma}\Phi + \frac{i}{2}\hat{D}_\lambda\mathbb{F}_{\mu\nu}\hat{D}_\tau\Phi^2\mathbb{F}_{\rho\sigma}\Phi + \frac{1}{2}\{\mathbb{F}_{\lambda\mu}, \mathbb{F}_{\tau\nu}\}\Phi^2\mathbb{F}_{\rho\sigma}\Phi + \frac{i}{4}\mathbb{F}_{\mu\nu}[\hat{D}_\lambda\Phi, \hat{D}_\tau\Phi]\mathbb{F}_{\rho\sigma}\Phi$$
$$+ \frac{i}{2}\mathbb{F}_{\mu\nu}\Phi^2\hat{D}_\lambda\mathbb{F}_{\rho\sigma}\hat{D}_\tau\Phi + \frac{1}{2}\mathbb{F}_{\mu\nu}\Phi^2\{\mathbb{F}_{\lambda\rho}, \mathbb{F}_{\tau\sigma}\}\Phi\Big)\Big). \tag{3.21}$$

The correction is real and invariant under OSp(4, 2) gauge transformations. However, a careful examination shows that after the gauge fixing (2.32) it vanishes completely,

$$S^{(1)}_{42}|_{\mathrm{g.f.}} = 0. \tag{3.22}$$

But we still have the additional NC action $S^\star_A$ invariant under purely bosonic NC-deformed $SO(2,3)_\star \times U(1)_\star$ gauge transformations. The only additional SW expansion we need is that of $\hat{f}$, and it is given by

$$\hat{f} = f - \frac{1}{4}\theta^{\rho\sigma}\{\Omega_\rho, (\partial_\sigma + D_\sigma)f\} + \mathcal{O}(\theta^2). \tag{3.23}$$

Before gauge fixing, the first-order term in the SW expansion of (3.2) is

$$S^{(1)}_A = S^{(1)}_{Af} + S^{(1)}_{Aff}$$
$$= -\frac{\theta^{\lambda\tau}}{64l}\mathrm{Tr}\int \mathrm{d}^4x\varepsilon^{\mu\nu\rho\sigma}\Big(-ifD_\lambda\tilde{f}_{\mu\nu}D_\tau(D_\rho\phi D_\sigma\phi\phi) + \frac{1}{2}\{\tilde{f}_{\lambda\tau}, f\}\tilde{f}_{\mu\nu}D_\rho\phi D_\sigma\phi\phi - f\{\tilde{f}_{\lambda\mu}, \tilde{f}_{\tau\nu}\}D_\rho\phi D_\sigma\phi\phi$$
$$- if\tilde{f}_{\mu\nu}D_\lambda(D_\rho\phi D_\sigma\phi)D_\tau\phi - if\tilde{f}_{\mu\nu}(D_\lambda D_\rho\phi)(D_\tau D_\sigma\phi)\phi - f\tilde{f}_{\mu\nu}\{\{\tilde{f}_{\lambda\rho}, D_\tau\phi\}, D_\sigma\phi\}\phi$$
$$+ \frac{i}{3!}\Big(\frac{1}{2}\{\tilde{f}_{\lambda\tau}, f^2\}D_\mu\phi D_\nu\phi D_\rho\phi D_\sigma\phi\phi - f^2\{[\{\tilde{f}_{\lambda\mu}, D_\tau\phi\}, D_\nu\phi], D_\rho\phi D_\sigma\phi\}\phi$$
$$- if^2(D_\lambda(D_\mu\phi D_\nu\phi D_\rho\phi D_\sigma\phi)D_\tau\phi + D_\lambda(D_\mu\phi D_\nu\phi D_\rho\phi)(D_\tau D_\sigma\phi)\phi$$
$$+ D_\lambda(D_\mu\phi D_\nu\phi)(D_\tau D_\rho\phi))D_\sigma\phi\phi + (D_\lambda D_\mu\phi)(D_\tau D_\nu\phi)D_\rho\phi D_\sigma\phi\phi)\Big)\Big) + \mathrm{c.c.}, \tag{3.24}$$

where we can distinguish the linear f part and the quadratic f^2 part, and all terms are manifestly $SO(2,3)\times U(1)$ invariant by the virtue of SW map.

After calculating traces and evaluating the gauge-fixed action $S_A^{(1)}|_{\mathrm{g.f.}}$ on the EoM of the components of the auxiliary field f [as it turns out, to obtain the first-order NC correction, we only need to insert *zeroth-order* (classical) EoM (2.43) in the gauge-fixed *first-order* NC action $S_A^{(1)}|_{\mathrm{g.f.}}$], we obtain

$$S_{A,EoM}^{(1)}|_{\mathrm{g.f.}}=\sum_{j=1}^{6}S_{A,EoMf.j}^{(1)}|_{\mathrm{g.f.}}+S_{A,EoMff}^{(1)}|_{\mathrm{g.f.}}, \tag{3.25}$$

with the individual terms

$$\begin{aligned}S_{A,EoMf.1}^{(1)}|_{\mathrm{g.f.}}=&-\frac{\theta^{\lambda\tau}}{64\kappa}\int\mathrm{d}^4xe\bigg\{\tilde{\mathcal{F}}^{\mu\nu}R_{\mu\nu ab}\bigg(R_{\lambda\tau}{}^{ab}-\frac{2}{l^2}e_\lambda^ae_\tau^b\bigg)\\&+\tilde{\mathcal{F}}^{\rho\sigma}e_\rho^ae_\sigma^b\bigg(R_{\mu\nu ab}R_{\lambda\tau}{}^{cd}e_c^\mu e_d^\nu-\frac{2}{l^2}R_{\lambda\tau ab}\bigg)+4\tilde{\mathcal{F}}^{\rho\mu}e_\rho^c\bigg(R_{\mu\nu ac}R_{\lambda\tau}{}^{ab}e_b^\nu+\frac{2}{l^2}R_{\mu\lambda ac}e_\tau^a\bigg)\\&+\tilde{\mathcal{F}}_{\rho\sigma}e_a^\rho e_b^\sigma R_{\lambda\tau}{}^{ab}\bigg(R_{\mu\nu}{}^{mn}e_m^\mu e_n^\nu-\frac{12}{l^2}\bigg)+\frac{2}{l^2}\tilde{\mathcal{F}}^{\mu\nu}T_{\lambda\tau}{}^a(T_{\mu\nu a}-2T_{\rho\nu m}e_a^\rho e_\mu^m)\\&-\frac{2}{l^2}\tilde{\mathcal{F}}_{\lambda\tau}\bigg(R_{\mu\nu}{}^{mn}e_m^\mu e_n^\nu-\frac{12}{l^2}-4\frac{l^2}{\kappa^2}\tilde{\mathcal{F}}^{\mu\nu}\tilde{\mathcal{F}}_{\mu\nu}\bigg)\bigg\},\end{aligned} \tag{3.26}$$

$$\begin{aligned}S_{A,EoMf.2}^{(1)}|_{\mathrm{g.f.}}=&-\frac{\theta^{\lambda\tau}}{8\kappa}\int\mathrm{d}^4xe\\&\times\bigg\{+(D_\lambda^LR_{\mu\nu}{}^{mc})(D_\tau^Le_\rho^r)e_c^\sigma(e_m^\nu(\tilde{\mathcal{F}}_\sigma{}^\mu e_r^\rho-\tilde{\mathcal{F}}_\sigma{}^\rho e_r^\mu)+\tilde{\mathcal{F}}_\sigma{}^\nu e_r^\mu e_m^\rho)\\&-\frac{1}{l^2}\tilde{\mathcal{F}}_\rho{}^\mu e_c^\rho(D_\lambda^LT_{\tau\mu}{}^c-e_{\lambda b}R_{\tau\mu}{}^{bc})-\frac{4}{l^2}\tilde{\mathcal{F}}_\nu{}^\mu(D_\lambda^Le_\rho^r)(D_\tau^Le_\mu^m)e_m^\nu e_r^\rho\\&-\frac{1}{l^2}(D_\lambda^Le_\rho^r)e_r^\rho(e_c^\nu\tilde{\mathcal{F}}_\tau{}^\mu T_{\mu\nu}{}^c-e_c^\nu\tilde{\mathcal{F}}_\nu{}^\mu T_{\mu\tau}{}^c)+\frac{1}{2l^2}T_{\lambda\tau}{}^rT_{\mu\nu}{}^c\tilde{\mathcal{F}}_\sigma{}^\nu e_c^\sigma e_r^\mu\\&+\frac{1}{l^2}(D_\lambda^Le_\rho^r)e_r^\nu(T_{\mu\nu}{}^m(\tilde{\mathcal{F}}_\tau{}^\mu e_m^\rho-\tilde{\mathcal{F}}_\tau{}^\rho e_m^\mu)+T_{\tau\nu}{}^m\tilde{\mathcal{F}}_\mu{}^\rho e_m^\mu)\\&+\frac{2}{l^2}\tilde{\mathcal{F}}_\sigma{}^\rho e_c^\sigma(D_\lambda^Le_\rho^r)(D_\tau^Le_\nu^c)e_r^\nu+\frac{1}{l^4}\tilde{\mathcal{F}}_{\lambda\tau}\bigg\},\end{aligned} \tag{3.27}$$

$$\begin{aligned}S_{A,EoMf.3}^{(1)}|_{\mathrm{g.f.}}=&\frac{\theta^{\lambda\tau}}{32\kappa}\int\mathrm{d}^4xe\bigg\{-\tilde{\mathcal{F}}^{\mu\nu}R_{\lambda\nu am}\bigg(R_{\tau\mu}{}^{am}-\frac{4}{l^2}e_\tau^ae_\mu^m\bigg)\\&+\tilde{\mathcal{F}}_{\rho\sigma}R_{\lambda\mu}{}^{am}R_{\tau\nu}{}^{bn}(e_a^\mu e_m^\nu e_b^\rho e_n^\sigma+e_a^\rho e_m^\sigma e_b^\mu e_n^\nu+2e_n^\rho e_m^\sigma(e_a^\mu e_b^\nu-e_a^\nu e_b^\mu))\\&-\frac{2}{l^2}e_n^\rho e_b^\sigma(2\tilde{\mathcal{F}}_{\lambda\rho}R_{\tau\sigma}{}^{bn}+\tilde{\mathcal{F}}_{\rho\sigma}R_{\lambda\tau}{}^{bn})+\frac{2}{l^2}\tilde{\mathcal{F}}^{\mu\nu}T_{\lambda\mu}{}^aT_{\tau\nu a}+\frac{8}{\kappa^2}\tilde{\mathcal{F}}^{\mu\nu}\tilde{\mathcal{F}}_{\lambda\mu}\tilde{\mathcal{F}}_{\tau\nu}\bigg\},\end{aligned} \tag{3.28}$$

$$\begin{aligned}(S_{A,EoMf.4}^{(1)}|_{\mathrm{g.f.}}+S_{A,EoMf.5}^{(1)}|_{\mathrm{g.f.}})=&-\frac{\theta^{\lambda\tau}}{32\kappa}\int\mathrm{d}^4xe\\&\times\bigg\{+R_{\mu\nu}{}^{ab}(D_\lambda^Le_\rho^m)(D_\tau^Le_{\sigma m})(\tilde{\mathcal{F}}^{\mu\nu}e_a^\rho e_b^\sigma+\tilde{\mathcal{F}}^{\rho\sigma}e_a^\mu e_b^\nu-4\tilde{\mathcal{F}}^{\mu\rho}e_a^\nu e_b^\sigma)\\&-\frac{1}{l^2}R_{\mu\nu}{}^{ab}(\tilde{\mathcal{F}}^{\mu\nu}e_{\lambda a}e_{\tau b}+\tilde{\mathcal{F}}_{\lambda\tau}e_a^\mu e_b^\nu-4\tilde{\mathcal{F}}^\mu{}_\lambda e_a^\nu e_{\tau b})-\frac{1}{l^2}\tilde{\mathcal{F}}^{\mu\nu}\eta_{mn}(D_\lambda^Le_\mu^m)(D_\tau^Le_\nu^n)\\&+\frac{2}{l^2}T_{\mu\nu}{}^a(D_\lambda^Le_\rho^b)(\tilde{\mathcal{F}}^{\mu\nu}(2e_a^\rho e_{\tau b}-e_b^\rho e_{\tau a})+2\tilde{\mathcal{F}}_\tau{}^\mu(e_a^\rho e_b^\nu-e_b^\rho e_a^\nu)\\&+2\tilde{\mathcal{F}}^{\rho\mu}(2e_a^\nu e_{\tau b}-e_b^\nu e_{\tau a})+2\tilde{\mathcal{F}}^\rho{}_\tau e_a^\mu e_b^\nu)-\frac{2}{l^2}\tilde{\mathcal{F}}^{\rho\mu}T_{\mu\nu c}T_{\lambda\tau}{}^de_\rho^ce_d^\nu\\&+\frac{4}{l^2}T_{\lambda\nu a}(D_\tau^Le_\rho^d)e_\sigma^a(\tilde{\mathcal{F}}^{\sigma\nu}e_d^\rho-\tilde{\mathcal{F}}^{\sigma\rho}e_d^\nu)-\frac{4}{l^2}R_{\lambda\nu a}{}^ce_\rho^a(\tilde{\mathcal{F}}^{\rho\nu}e_{\tau c}-\tilde{\mathcal{F}}^\rho{}_\tau e_c^\nu)-\frac{6}{l^4}\tilde{\mathcal{F}}_{\lambda\tau}\bigg\},\end{aligned} \tag{3.29}$$

$$
\begin{aligned}
S^{(1)}_{A,EoMf.6}|_{\text{g.f.}} = \frac{\theta^{\lambda\tau}}{32\kappa}\int \mathrm{d}^4xe\bigg\{&\tilde{\mathcal{F}}^{\rho\sigma}e^a_\rho e^b_\sigma e^\mu_m e^\nu_n (R_{\lambda\tau}{}^{mn}R_{\mu\nu ab} - 2R_{\lambda\mu}{}^{mn}R_{\tau\nu ab}) \\
&+\frac{8}{\kappa^2}(\tilde{\mathcal{F}}_{\lambda\tau}\tilde{\mathcal{F}}^2 - 2\tilde{\mathcal{F}}^{\mu\nu}\tilde{\mathcal{F}}_{\lambda\mu}\tilde{\mathcal{F}}_{\tau\nu}) + \frac{2}{l^2}\tilde{\mathcal{F}}^{\mu\nu}e^a_\mu e^b_\nu R_{\lambda\tau ab} + \frac{4}{l^2}\tilde{\mathcal{F}}_{\lambda\nu}\epsilon^\nu_a e^\rho_b R_{\tau\rho}{}^{ab} \\
&-\frac{4}{l^2}\tilde{\mathcal{F}}^{\mu\nu}e^a_\nu e^\rho_b(T_{\rho\mu a}T_{\lambda\tau}{}^b - T_{\mu\lambda a}T_{\tau\rho}{}^b - T_{\lambda\rho a}T_{\tau\mu}{}^b) - \frac{8}{l^4}\tilde{\mathcal{F}}_{\lambda\tau}\bigg\}, \qquad (3.30)
\end{aligned}
$$

and, finally, the f^2 term,

$$
S^{(1)}_{A,EoMff}|_{\text{g.f.}} = -\frac{\theta^{\lambda\tau}}{16\kappa^3}\int \mathrm{d}^4xe\tilde{\mathcal{F}}_{\lambda\tau}\tilde{\mathcal{F}}^2. \qquad (3.31)
$$

Action (3.25) represents the first-order NC correction to $N=2$ AdS_4 SUGRA. It involves various new couplings between $U(1)$ gauge field, gravity, and gravitini fields that appear due to space-time noncommutativity. As it stands, this action seems too complicated to be analyzed in its entirety. However, we can restrict ourselves to some particular domain of parameters and work with an approximated NC action. In particular, we will derive a low-energy approximation of (3.25), by taking into account terms at most quadratic in the partial derivative. Therefore, we include only terms linear in curvature and linear and quadratic in torsion. Additionally, we assume that spin connection $\omega_\mu{}^{ab}$ and the first-order derivatives of the vierbeins are of the same order. Note also that the torsion constraint $\tilde{T}_{\mu\nu}{}^a = 0$ (2.25) gives us $T_{\mu\nu}{}^a = -i\Psi_\mu\gamma^a\Psi_\nu$. These assumptions yield a very simple action,

$$
\begin{aligned}
S^{(1)}_{\text{low-energy}} &= -\frac{9\theta^{\mu\nu}}{16l^4\kappa}\int \mathrm{d}^4xe\tilde{\mathcal{F}}_{\mu\nu} \\
&= -\frac{9\theta^{\mu\nu}}{16l^4\kappa}\int \mathrm{d}^4xe(\mathcal{F}_{\mu\nu} - \bar{\Psi}_\mu i\sigma^2\Psi_\nu) \\
&= \frac{9\theta^{\mu\nu}}{16l^4\kappa}\int \mathrm{d}^4xe(\bar{\psi}^1_\mu\psi^2_\nu - \bar{\psi}^2_\mu\psi^1_\nu) + \text{surface term} \\
&= -\frac{9}{8l^4\kappa}\int \mathrm{d}^4xe(\bar{\psi}^+_\mu i\Theta^{\mu\nu}\psi^+_\nu) + \text{surface term}.
\end{aligned} \qquad (3.32)
$$

This masslike term for charged gravitino ψ^+_μ, minimally coupled to gravity, appears due to space-time noncommutativity and "renormalizes" the corresponding term (2.49) in the classical SUGRA action (2.48). If we again absorb κ^{-1} in ψ^+_μ to obtain the canonical dimensions, the masslike parameter is of the order of $l_P\Lambda^2_{\text{NC}}/l^4$, and it vanishes under WI contraction.

After WI contraction, the action (3.25) reduces to

$$
\begin{aligned}
S^{(1)}_A|_{\text{g.f.}} \overset{\text{WI}}{=} -\frac{\theta^{\lambda\tau}}{64\kappa}\int \mathrm{d}^4xe\bigg\{&\tilde{\mathcal{F}}^{\mu\nu}R_{\mu\nu\rho\sigma}R_{\lambda\tau}{}^{\rho\sigma} - \tilde{\mathcal{F}}^{\mu\nu}R_{\rho\sigma\mu\nu}R_{\lambda\tau}{}^{\rho\sigma} - 4\tilde{\mathcal{F}}^{\mu\rho}R_{\mu\nu\rho\sigma}R_{\lambda\tau}{}^{\nu\sigma} \\
&- 2\tilde{\mathcal{F}}^{\mu\nu}R_{\lambda\mu}{}^{\rho\sigma}R_{\tau\nu\rho\sigma} + 8\tilde{\mathcal{F}}_{\rho\sigma}R_{\lambda\mu}{}^{\mu\rho}R_{\tau\nu}{}^{\nu\sigma} + \tilde{\mathcal{F}}_{\mu\nu}R_{\lambda\tau}{}^{\mu\nu}R - \frac{4}{\kappa^2}\tilde{\mathcal{F}}_{\lambda\tau}\tilde{\mathcal{F}}^2 + \frac{16}{\kappa^2}\tilde{\mathcal{F}}^{\mu\nu}\tilde{\mathcal{F}}_{\lambda\mu}\tilde{\mathcal{F}}_{\tau\nu} \\
&+ 8(D^L_\lambda R_{\mu\nu}{}^{mc})(D^L_\tau e^r_\rho)e^\sigma_c(\tilde{\mathcal{F}}_\sigma{}^\mu e^\nu_m e^\rho_r - \tilde{\mathcal{F}}_\sigma{}^\rho e^\mu_r e^\nu_m + \tilde{\mathcal{F}}_\sigma{}^\nu e^\mu_r e^\rho_m) \\
&+ 2R_{\mu\nu}{}^{ab}\eta_{rs}(D^L_\lambda e^r_\rho)(D^L_\tau e^s_\sigma)(\tilde{\mathcal{F}}^{\mu\nu}e^\rho_a e^\sigma_b + \tilde{\mathcal{F}}^{\rho\sigma}e^\mu_a e^\nu_b - 4\tilde{\mathcal{F}}^{\mu\rho}e^\nu_a e^\sigma_b)\bigg\}. \qquad (3.33)
\end{aligned}
$$

At this point, we are confronted with an interesting question. The fact that $N=2$ AdS_4 superalgebra contracts to $N=2$ Poincaré superalgebra when $l\to\infty$ is consistently reflected on the level of classical (undeformed) action (2.47); classical $N=2$ AdS_4 SUGRA reduces to classical $N=2$ Poincaré SUGRA under WI contraction. However, it is not *a priori* clear whether this relation holds *after NC deformation*, that is, whether NC deformation and WI contraction actually commute. For that matter, one would have to explicitly compute the NC correction to classical $N=2$ Poincaré SUGRA and compare it to the action (3.33).

IV. CONCLUSIONS

Let us emphasize the main points of this paper and propose some further paths of investigation. At this stage, our primary goal was to obtain explicit NC correction to $N=2$ AdS SUGRA in $D=4$. We stared from a classical (undeformed) action (2.30) of the MacDowell-Mansouri type (already advocated in the literature), invariant under orthosymplectic OSp(4|2) gauge transformations. However, this action alone is not enough to obtain $N = 2$ AdS_4 SUGRA after fixing the gauge (for which we use a constrained auxiliary field). In particular, one has to add a supplementary action (2.40) endowed with $SO(2,3) \times U(1)$ gauge symmetry [bosonic sector of OSp(4|2)] that provides the missing terms in the classical SUGRA action [e.g., the kinetic term for the $U(1)$ gauge field]. Therefore, we have the following relation:

$$\underbrace{\begin{array}{ccc} (OSp(4|2)\ \text{invariant action}) & + & (SO(2,3)\times U(1)\ \text{invariant action}) \\ \Big\downarrow \text{g.f.} & & \Big\downarrow \text{g.f.} \\ (SO(1,3)\times U(1)\ \text{invariant action}) & + & (SO(1,3)\times U(1)\ \text{invariant action}) \end{array}}_{\text{N=2 AdS SUGRA in D=4}}$$

This situation seems curious considering that a similar OSp(4|1) gauge-invariant action (2.51) in the same gauge reduces to the complete $N = 1$ AdS_4 SUGRA action. We may conclude that extended $N > 1$ AdS_4 SUGRA cannot be obtained simply by gauging the corresponding orthosymplectic group OSp(4|N) and subsequently fixing the gauge. For $N > 2$, we would have to include an additional term similar to (2.40) that involves a non-Abelian Yang-Mills gauge field.

NC deformation is performed following the Seiberg-Witten approach to NC gauge field theory that involves universal enveloping algebra-valued gauge field and perturbative expansion of the NC-deformed action in powers of the deformation parameter $\theta^{\mu\nu}$. The expanded action possesses gauge symmetry of the corresponding classical action, order by order, and we focus only on the linear NC correction that remains after the gauge fixing. For the OSp(4|2) gauge-invariant part, linear NC correction vanishes. The reason why this result strikes us as curious is related to some previously establish facts about canonical NC deformation of the similar models. Namely, canonical deformation of pure gravity, regarded as a gauge theory of the $SO(2,3)$ group, leads to quadratic NC correction [25,26,38]. However, after including charged matter (Dirac spinors) coupled to the $U(1)$ gauge field, linear NC correction appears [54–56]. Since we can take a pair of Majorana vector spinors of OSp(4|2) SUGRA and form a pair of $U(1)$-charged Dirac vector spinors, related to each other by C conjugation, we expected to obtain a nonvanishing first-order NC correction from the OSp(4|2) action (3.1), as well.

However, the supplementary bosonic action provides a nontrivial linear NC correction that is calculated explicitly (3.25). It involves various new interaction terms that are present due to space-time noncommutativity. The full action is difficult to analyze, but we can restrict ourselves to the low-energy sector of the theory by taking into account only terms that are at most quadratic in partial derivatives. This leaves us with a single masslike term for a charged gravitino minimally coupled to gravity.

WI contraction eliminates many of these new interaction terms, but not all of them (3.33). The ones remaining may help us understand the relation between the canonical NC deformation and WI contraction, at least in this particular case. $N = 2$ AdS superalgebra reduces to $N = 2$ Poincaré superalgebra under WI contraction, and the same holds for their classical actions. Therefore, it may be the case that the same relation pertains even after canonical NC deformation. To confirm this assumption directly, we have to calculate linear NC correction to $N = 2$ Poincaré SUGRA and make the comparison.

Let us just mention that there are two additional terms with OSp(4|2) gauge symmetry that we could include. We denote them by S' and S'', and they are given by

$$S' = \frac{a'}{128\pi G_N l}\mathrm{STr}\int \mathrm{d}^4x\varepsilon^{\mu\nu\rho\sigma}\mathbb{F}_{\mu\nu}\hat{D}_\rho\Phi\hat{D}_\sigma\Phi\Phi + \text{c.c.},$$

$$S'' = -\frac{ia''}{128\pi G_N l^3}\mathrm{STr}\int \mathrm{d}^4x\varepsilon^{\mu\nu\rho\sigma}\hat{D}_\mu\Phi\hat{D}_\nu\Phi\hat{D}_\rho\Phi\hat{D}_\sigma\Phi\Phi,$$

with free dimensionless parameters a', a'', and OSp(4|2) covariant derivative $\hat{D}_\mu$. Their $SO(2,3)$ gauge-invariant counterparts were analyzed in Ref. [26]. After gauge fixing, they modify the coefficients in the classical action but do not introduce new terms. In particular, they give us a freedom to eliminate the cosmological constant in the classical action. NC deformation of S' and S'' will certainly change our final result, but their importance is not yet clear. Analysis of these additional NC corrections remains to be done.

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APPENDIX A: MATRIX REPRESENTATION OF *osp*(4|2) SUPERALGEBRA

Here, we present an explicit 6×6 matrix representation of OSp(4|2) generators $\{\hat{M}_{AB}, \hat{Q}^I_\alpha, \hat{T}\}$. The AdS generators are the same as for OSp(4|1), but for OSp(4|2), we have an additional set of fermionic generators (comprising another Majorana spinor) and additional bosonic generator $\hat{T}$ of $SO(2) \sim U(1)$.

The bosonic generators of OSp(4|2) are

$$\hat{M}_{AB} = \left(\begin{array}{cccc|cc} & & & & 0 & 0 \\ & M_{AB} & & & 0 & 0 \\ & & & & 0 & 0 \\ & & & & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \qquad \hat{T} = \left(\begin{array}{cccc|cc} & & & & 0 & 0 \\ & 0_{4\times 4} & & & 0 & 0 \\ & & & & 0 & 0 \\ & & & & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 & -i & 0 \end{array}\right) \tag{A1}$$

The imaginary unit in $\hat{T}$ is introduced for convenience. The fermionic generators of OSp(4|2) are

$$(\hat{Q}^1)_1 = \left(\begin{array}{cccc|cc} & & & & 0 & 0 \\ & 0_{4\times 4} & & & 1 & 0 \\ & & & & 0 & 0 \\ & & & & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right), \qquad (\hat{Q}^1)_2 = \left(\begin{array}{cccc|cc} & & & & -1 & 0 \\ & 0_{4\times 4} & & & 0 & 0 \\ & & & & 0 & 0 \\ & & & & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \tag{A2}$$

$$(\hat{Q}^1)_3 = \left(\begin{array}{cccc|cc} & & & & 0 & 0 \\ & 0_{4\times 4} & & & 0 & 0 \\ & & & & 0 & 0 \\ & & & & -1 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \qquad (\hat{Q}^1)_4 = \left(\begin{array}{cccc|cc} & & & & 0 & 0 \\ & 0_{4\times 4} & & & 0 & 0 \\ & & & & 1 & 0 \\ & & & & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \tag{A3}$$

The second set of fermionic generators $(\hat{Q}^2)_\alpha$ ($\alpha = 1, 2, 3, 4$) is obtained from the first one just by interchanging fifth and sixth columns and fifth and sixth rows. One can readily check that supermatrices (A1), (A2), and (A3), along with the second set of fermionic generators, satisfy the $\mathfrak{osp}(4|2)$ superalgebra (2.4).

APPENDIX B: USEFUL IDENTITIES

Some basic Fierz identities involving Majorana spinors ψ and χ are

$$\begin{aligned} \bar\psi\chi &= \bar\chi\psi = (\bar\psi\chi)^\dagger \\ \bar\psi\gamma_5\chi &= \bar\chi\gamma_5\psi = -(\bar\psi\gamma_5\chi)^\dagger \\ \bar\psi\gamma_a\gamma_5\chi &= \bar\chi\gamma_a\gamma_5\psi = (\bar\psi\gamma_a\gamma_5\chi)^\dagger \\ \bar\psi\gamma_a\chi &= -\bar\chi\gamma_a\chi = -(\bar\psi\gamma_a\chi)^\dagger \\ \bar\psi\sigma_{ab}\chi &= -\bar\chi\sigma_{ab}\psi = -(\bar\psi\sigma_{ab}\chi)^\dagger. \end{aligned} \tag{B1}$$

Also, we frequently use the following important identity, valid in four dimensions. For any pair of Majorana spinors, ψ and χ, we can expand $\psi\bar\chi$ in the Clifford algebra basis:

$$\begin{aligned} -4\psi\bar\chi &= (\bar\chi\psi)\mathbb{I}_4 + (\bar\chi\gamma^a\psi)\gamma_a + (\bar\chi\gamma_5\psi)\gamma_5 + (\bar\chi\gamma^a\gamma_5\psi)\gamma_5\gamma_a \\ &\quad + \frac{1}{2}(\bar\chi\sigma^{ab}\psi)\sigma_{ab}. \end{aligned} \tag{B2}$$

Some AdS algebra relations are[1]

$$\begin{aligned} [M_{AB}, M_{CD}] &= i(\eta_{AD}M_{BC} + \eta_{BC}M_{AD} - \eta_{AC}M_{BD} \\ &\quad - \eta_{BD}M_{AC}) \\ \{M_{AB}, M_{CD}\} &= \frac{i}{2}\epsilon_{ABCDE}\Gamma^E + \frac{1}{2}(\eta_{AC}\eta_{BD} - \eta_{AD}\eta_{BC}) \\ \{M_{AB}, \Gamma_C\} &= i\epsilon_{ABCDE}M^{DE} \\ [M_{AB}, \Gamma_C] &= i(\eta_{BC}\Gamma_A - \eta_{AC}\Gamma_B) \\ \Gamma_A^\dagger &= -\gamma_0\Gamma_A\gamma_0, \qquad M_{AB}^\dagger = \gamma_0 M_{AB}\gamma_0. \end{aligned} \tag{B3}$$

[1] $\epsilon^{01235} = +1, \epsilon^{0123} = +1$.

Some useful identities involving γ matrices and σ matrices are

$$\begin{aligned} \gamma_a\gamma_b &= \eta_{ab} - i\sigma_{ab} \\ \sigma_{ab}\gamma_c &= i\eta_{bc}\gamma_a - i\eta_{ac}\gamma_b + \varepsilon_{abcd}\gamma_5\gamma^d \\ \gamma_c\sigma_{ab} &= i\eta_{ac}\gamma_b - i\eta_{bc}\gamma_a + \varepsilon_{abcd}\gamma_5\gamma^d \\ \sigma_{ab}\gamma_5 &= \frac{i}{2}\varepsilon_{abcd}\sigma^{cd} \\ \sigma_{ab}\sigma_{cd} &= \eta_{ac}\eta_{bd} - \eta_{ad}\eta_{bc} + i\varepsilon_{abcd}\gamma_5 \\ &\quad + i(\eta_{ad}\sigma_{bc} + \eta_{bc}\sigma_{ad} - \eta_{ac}\sigma_{bd} - \eta_{bd}\sigma_{ac}). \end{aligned} \tag{B4}$$

Identities with traces are

$$\begin{aligned} \mathrm{Tr}(\Gamma_A\Gamma_B) &= 4\eta_{AB} \\ \mathrm{Tr}(\Gamma_A) &= \mathrm{Tr}(\Gamma_A\Gamma_B\Gamma_C) = 0 \\ \mathrm{Tr}(\Gamma_A\Gamma_B\Gamma_C\Gamma_D) &= 4(\eta_{AB}\eta_{CD} - \eta_{AC}\eta_{BD} + \eta_{AD}\eta_{CB}) \\ \mathrm{Tr}(\Gamma_A\Gamma_B\Gamma_C\Gamma_D\Gamma_E) &= -4i\epsilon_{ABCDE} \\ \mathrm{Tr}(M_{AB}M_{CD}\Gamma_E) &= i\epsilon_{ABCDE} \\ \mathrm{Tr}(M_{AB}M_{CD}) &= -\eta_{AD}\eta_{CB} + \eta_{AC}\eta_{BD} \\ \mathrm{Tr}(M_{AB}\Gamma_E\Gamma_F\Gamma_G) &= 2\varepsilon_{ABEFG} \\ \mathrm{Tr}(M_{AB}M_{CD}\Gamma_E\Gamma_F\Gamma_G) &= i\varepsilon_{ABCDE}\eta_{FG} - i\varepsilon_{ABCDF}\eta_{EG} \\ &\quad + i\varepsilon_{ABCDG}\eta_{EF} + i\varepsilon_{BCEFG}\eta_{AD} \\ &\quad + i\varepsilon_{ADEFG}\eta_{BC} - i\varepsilon_{BDEFG}\eta_{AC} \\ &\quad - i\varepsilon_{ACEFG}\eta_{BD}. \end{aligned} \tag{B5}$$

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