Constraints from a large-N_c analysis on meson-baryon interactions at chiral order Q^3

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We consider the chiral Lagrangian for baryon fields with $J^P = \frac{1}{2}^+$ or $J^P = \frac{3}{2}^+$ quantum numbers as constructed from QCD with up, down and strange quarks. The specific class of counterterms that are of chiral order Q^3 and contribute to meson-baryon interactions at the two-body level is constructed. Altogether, we find 24 terms. In order to pave the way for realistic applications, we establish a set of 22 sum rules for the low-energy constants as they are implied by QCD in the large-N_c limit. Given such a constraint, there remain only two independent unknown parameters that need to be determined by either lattice QCD simulations or directly from experimental cross section measurements. At subleading order, we arrive at five parameters.

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I. INTRODUCTION

Still after many decades of vigorous studies, the outstanding challenge of modern physics is to establish a rigorous link of QCD to low-energy hadron physics as it is observed in the many experimental cross section measurements. After all, it is the only fundamental field theory there is that leads to the emergence of structure as a consequence of truly nonperturbative interactions in a quantum field theory. On the one hand, the dataset has been extended recently by LHCb, BES, COMPASS, and Belle with more and more exciting new phenomena; on the other hand, there is a huge dataset of pion- and photon-induced reactions in the resonance region which still today are not understood in terms of QCD dynamics [1,2]. Such reactions constitute the doorway to understanding nonperturbative QCD, like studies of the hydrogen atom paved the way to understanding QED.

While simulations of QCD on finite lattices made considerable progress in the last decade, it is still not feasible to derive cross sections systematically as measured in the laboratory in the resonance region of QCD. Thus, at present, it may be advantageous to resort to a well-established method of modern physics—to derive the implications of the fundamental theory by matching it to effective field theory approaches that are formulated in terms of the relevant degrees of freedom.

With the great advances of lattice QCD simulations, such an approach is going through a revolution at present, since the effective field theory can now be scrutinized systematically by QCD lattice data. In turn, the typically quite large set of low-energy constants can be derived from QCD prior to confronting the effective field theory with scattering data taken in the laboratory. This has been emphasized and illustrated recently in Ref. [3]. Some results for sets of low-energy constants have already been obtained from the masses of baryons and mesons in their ground states with $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ and $J^P = 0^-, 1^-$ quantum numbers [3–7].

Since the majority of available lattice data were taken at unphysical quark masses, it is mandatory to establish reliable tools to translate such data back to the physical case. We see the fact that lattice data are typically for unphysical hadrons so far as a fortunate circumstance, since this way information on QCD is provided that cannot be inferred from the PDG or any experimental cross section so easily. Moreover, the determination of large sets of low-energy constants from lattice data on the hadron ground-state masses at various unphysical quark masses appears to be much easier and better controlled compared to their extraction from the first few available phase shifts as computed on QCD lattices at unphysical quark masses.

Here we wish to emphasize that our strategy to pave the way toward the understanding of nonperturbative QCD relies heavily on our recent claim that the chiral Lagrangian properly formulated for the physics of up, down, and strange quarks can be successfully applied to low-energy QCD once it is set up in terms of on-shell meson and

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baryon masses. It was demonstrated that in that case, the size of the physical strange quark mass does not prohibit the application of the chiral Lagrangian. This is contrasted by the conventional χPT approach, in which bare masses are to be used inside any loop expression. Here any low-order application to the flavor $SU(3)$ case should be avoided, being of no physical significance.

The purpose of the current study is to further prepare the quantitative application of the chiral Lagrangian with three light flavors to meson-baryon scattering data. Our target is the set of counterterms that carry chiral order Q^3 and contribute to meson-baryon scattering at the two-body level. Such Q^3 counterterms play a decisive role in the chiral dynamics of the meson-baryon systems. As was pointed out already in Ref. [8], only in the presence of such terms may it be feasible to establish a universal set of Q^2 counterterms that describe pion, kaon, and antikaon nucleon scattering data. Though there is a plethora of works [9–20] that fit the Q^2 counterterms to pion-nucleon, kaon-nucleon, or antikaon-nucleon scattering, the only so far to take a universal approach is documented in Ref. [8]. In turn, there are various mutually noncompatible sets of the Q^2 counterterms available.

We would argue that there are also still some residual deficiencies in Ref. [8] which may hamper the direct use of the most comprehensive set of low-energy constants as extracted from the published lattice dataset on the baryon octet and decuplet masses in Ref. [5]. Most severe, we would argue, are the particularities of the unitarization schemes. Within the flavor $SU(3)$ framework, so far, all published works rely on neglect or improper treatment of left-hand branch points. Though we do not expect this to lead to huge qualitative issues, a quantitative and controlled study of, in particular, p-wave phase shifts should consider it in a reliable manner. We feel this to be an achievable request owing to the fact that such a scheme exists by now with Refs. [21–23]. So far, it was applied only to the flavor $SU(2)$ case with the πN and γN channels.

Within a flavor $SU(3)$ context, such Q^3 terms were first used in Ref. [8]. Later, the complete order-Q^3 Lagrangian was constructed in Refs. [24,25] for the baryon octet fields. To the best knowledge of the authors, such counterterms have not been constructed so far involving the baryon decuplet fields. We are aware of the recent Ref. [26], which, however, provides partial results only. Since we wish to derive sum rules for the Q^3 low-energy constants from large-N_c QCD [27,28], a reliable construction of the latter terms is the target of the first part of our work in Sec. II. The second part follows with Sec. III, in which we apply large-N_c QCD in order to derive sum rules for the set of Q^3 low-energy constants. Here we follow the framework previously established in Refs. [29–31]. In our case, we compute the contributions of the Q^3 counterterms to the correlation function with two axial-vector currents and one vector current in the baryon ground states. From a study of the latter, the desired sum rules will be derived.

II. CHIRAL LAGRANGIAN WITH BARYON OCTET AND DECUPLET FIELDS

We recall the conventions for the chiral Lagrangian as used in the current work [5,8,32,33]. The hadronic fields as decomposed into their isospin multiplets are

$$\begin{aligned}
\Phi &= \tau\cdot\pi(140)+\alpha^\dagger\cdot K(494)+K^\dagger(494)\cdot\alpha+\eta(547)\lambda_8,\\
\sqrt{2}B &= \alpha^\dagger\cdot N(939)+\lambda_8\Lambda(1115)+\vec{\tau}\cdot\vec{\Sigma}(1195)\\
&\quad +\Xi^T(1315)i\sigma_2\cdot\alpha,\\
\alpha^\dagger &= \frac{1}{\sqrt{2}}(\lambda_4+i\lambda_5,\lambda_6+i\lambda_7),\qquad \vec{\tau}=(\lambda_1,\lambda_2,\lambda_3),
\end{aligned} \tag{1}$$

where the matrices λ_i are the Gell-Mann generators of the $SU(3)$ algebra. The numbers in the brackets recall the approximate masses of the particles in units of MeV. Of central importance is the covariant derivative

$$(D_\mu B)^i_j = \partial_\mu B^i_j + (\Gamma_\mu)^i_h B^h_j - B^i_h(\Gamma_\mu)^h_j, \tag{2}$$

as introduced in terms of the chiral connection Γ_μ. The chiral connection with $\Gamma_\mu = -\Gamma_\mu^\dagger$ and other convenient chiral building blocks are constructed in terms of the chiral fields Φ in a nonlinear fashion such that all the chiral Ward identities of QCD are recovered in systematic applications of the chiral Lagrangian [32,34,35]. We write

$$\begin{aligned}
\Gamma_\mu &= \frac{1}{2}u^\dagger[\partial_\mu - i(v_\mu+a_\mu)]u+\frac{1}{2}u[\partial_\mu - i(v_\mu-a_\mu)]u^\dagger,\\
U_\mu &= \frac{1}{2}u^\dagger(\partial_\mu e^{i\frac{\Phi}{f}})u^\dagger - \frac{i}{2}u^\dagger(v_\mu+a_\mu)u\\
&\quad +\frac{i}{2}u(v_\mu-a_\mu)u^\dagger,\qquad u=e^{i\frac{\Phi}{2f}},\\
H_{\mu\nu} &= D_\mu iU_\nu + D_\nu iU_\mu,\qquad D_{\mu\nu}=D_\mu D_\nu + D_\nu D_\mu,
\end{aligned} \tag{3}$$

where we emphasize the presence of the classical vector and axial-vector source fields, v_μ and a_μ of QCD [34,35]. The important merit of all building blocks B, U_μ, $H_{\mu\nu}$, and $D_{\mu\nu}$ lies in their identical chiral transformation properties. Thus, the action of the covariant derivatives is implied by the example case of Eq. (2).

As derived first in Ref. [8], there are ten independent symmetry-conserving Q^3 terms that are needed in the baryon octet sector. Such terms were studied in momentum space properly projected onto the kinematics required in meson-baryon scattering process. Initially, there were 20 terms considered. It was shown in Ref. [8] that only ten terms are independent. This result was established by an evaluation of the s- and p-wave projections of their contributions to the scattering amplitudes. Explicit expressions for how such terms contribute to the meson-baryon interaction kernel were provided in Appendix B of that work.

This result was confirmed later in Refs. [24,25] based on a complementary strategy. In fact, initially the authors of Ref. [25] claimed the relevance of 11 terms in Ref. [36], a result inconsistent with the original finding in Ref. [8]. This error was corrected first in Ref. [24]. In the current work, we use the ten terms in the following representation:

$$\begin{aligned}\mathcal{L}^{(3)}_{[8][8]} =& -u_1 \mathrm{tr}\bar{B}\gamma^\mu B[U^\nu,H_{\mu\nu}]_- - u_2\mathrm{tr}\bar{B}[U^\nu,H_{\mu\nu}]_-\gamma^\mu B - \frac{1}{2}u_3(\mathrm{tr}\bar{B}U^\nu\gamma^\mu \mathrm{tr}H_{\mu\nu}B + \mathrm{H.c.})\\ &-\frac{1}{2}u_4(\mathrm{tr}\bar{B}\gamma^\lambda(D^{\mu\nu}B)[U_\lambda,H_{\mu\nu}]_- + \mathrm{tr}(D^{\mu\nu}\bar{B})\gamma^\lambda B[U_\lambda,H_{\mu\nu}]_-) - \frac{1}{2}u_5(\mathrm{tr}\bar{B}[U_\lambda,H_{\mu\nu}]_-\gamma^\lambda(D^{\mu\nu}B) + \mathrm{tr}(D^{\mu\nu}\bar{B})[U_\lambda,H_{\mu\nu}]_-\gamma^\lambda B)\\ &-\frac{1}{4}u_6(\mathrm{tr}\bar{B}U_\lambda\gamma^\lambda \mathrm{tr}H_{\mu\nu}(D^{\mu\nu}B) + \mathrm{tr}(D^{\mu\nu}\bar{B})U_\lambda\gamma^\lambda \mathrm{tr}H_{\mu\nu}B + \mathrm{H.c.}) - \frac{1}{2}u_7(\mathrm{tr}\bar{B}\sigma^{\lambda\mu}(D^\nu B)[U_\lambda,H_{\mu\nu}]_+ - \mathrm{tr}(D^\nu\bar{B})\sigma^{\lambda\mu}B[U_\lambda,H_{\mu\nu}]_+)\\ &-\frac{1}{2}u_8(\mathrm{tr}\bar{B}[U_\lambda,H_{\mu\nu}]_+\sigma^{\lambda\mu}(D^\nu B) - \mathrm{tr}(D^\nu\bar{B})[U_\lambda,H_{\mu\nu}]_+\sigma^{\lambda\mu}B) - \frac{1}{4}u_9(\mathrm{tr}\bar{B}U_\lambda\sigma^{\lambda\mu}(D^\nu B)H_{\mu\nu} - \mathrm{tr}(D^\nu\bar{B})U_\lambda\sigma^{\lambda\mu}BH_{\mu\nu} + \mathrm{H.c.})\\ &-\frac{1}{2}u_{10}(\mathrm{tr}\bar{B}\sigma^{\lambda\mu}(D^\nu B)\mathrm{tr}U_\lambda H_{\mu\nu} - \mathrm{tr}(D^\nu\bar{B})\sigma^{\lambda\mu}B\mathrm{tr}U_\lambda H_{\mu\nu}). \end{aligned} \tag{4}$$

We turn to the decuplet sector. The construction of the chiral Lagrangian is straightforward following the rules established by Krause in Ref. [32]. We use here the conventional Rarita-Schwinger fields to interpolate to the decuplet of the spin-3 half-states. The baryon decuplet field B^{ijk}_μ comes with three fully symmetric flavor indices, i, j, $k = 1, 2, 3$ as

$$\begin{aligned}&B^{111}_\mu = \Delta^{++}_\mu, \quad B^{112}_\mu = \Delta^{+}_\mu/\sqrt{3}, \quad B^{122}_\mu = \Delta^{0}_\mu/\sqrt{3}, \quad B^{222}_\mu = \Delta^{-}_\mu,\\ &B^{113}_\mu = \Sigma^{+}_\mu/\sqrt{3}, \quad B^{123}_\mu = \Sigma^{0}_\mu/\sqrt{6}, \quad B^{223}_\mu = \Sigma^{-}_\mu/\sqrt{3},\\ &B^{133}_\mu = \Xi^{0}_\mu/\sqrt{3}, \quad B^{233}_\mu = \Xi^{-}_\mu/\sqrt{3},\\ &B^{333}_\mu = \Omega^{-}_\mu, \end{aligned} \tag{5}$$

where the components are identified with the states in the particle basis for convenience. The covariant derivative takes the form

$$(D_\mu B_\nu)^{ijh} = \partial_\mu B^{ijh}_\nu + \Gamma^i_{\mu,l}B^{ljh}_\nu + \Gamma^j_{\mu,l}B^{ilh}_\nu + \Gamma^h_{\mu,l}B^{ijl}_\nu, \tag{6}$$

where again the chiral connection Γ_μ is needed. In order to keep track of the various flavor index contraction in the many terms of the chiral Lagrangian, we use here a powerful notation already introduced by one of the authors in Ref. [8]. The idea behind the notation is to introduce a few auxiliary objects in terms of which any interaction term can be written down in terms of simple 3×3 matrix products, as is the case in the baryon octet sector. Indeed, this is achieved by the consideration of suitable "dot" products of the decuplet fields. We need to discriminate among the following three cases only:

$$\begin{aligned}&(\bar{B}^\mu\cdot B_\nu)^i_j = \bar{B}^\mu_{jkl}B^{ikl}_\nu, \qquad (\bar{B}^\mu\cdot\Phi)^i_j = \epsilon^{kli}\bar{B}^\mu_{kmj}\Phi^m_l,\\ &(\Phi\cdot B_\nu)^i_j = \epsilon_{klj}\Phi^l_m B^{kmi}_\nu, \end{aligned} \tag{7}$$

where any of such products yields a two-index object that transforms as a flavor octet field again. Note that it takes a bit of group theory, but indeed all our terms in the chiral Lagrangian can be written down in such a notation. Given this fact, it is, however, rather convenient to apply such a notation, since the painful write-down of flavor-redundant terms can be avoided to a large extent. In Refs. [8,33], all terms at order Q^2 that are relevant for meson-baryon scattering were written down for the first time. Such terms were recently rediscovered in Refs. [26,37] using a less transparent notation. The first partial list of Q^2 terms involving the baryon decuplet field was published in Ref. [38].

We now turn to the symmetry-preserving Q^3 terms that involve a decuplet field. A complete list of $14 = 8 + 6$ terms is readily worked out with

$$\begin{aligned}\mathcal{L}^{(3)}_{[10][10]} =& \frac{1}{2}v_1(\mathrm{tr}(\bar{B}_\tau\cdot U^\nu)\gamma^\mu(H_{\mu\nu}\cdot B^\tau) + \mathrm{H.c.}) + \frac{1}{2}v_2(\mathrm{tr}(\bar{B}_\lambda\cdot U^\lambda)\gamma^\mu(H_{\mu\nu}\cdot B^\nu) + \mathrm{H.c.}) + \frac{1}{2}v_3(\mathrm{tr}(\bar{B}^\nu\cdot U^\lambda)\gamma^\mu(H_{\mu\nu}\cdot B_\lambda) + \mathrm{H.c.})\\ &+\frac{1}{4}v_4(\mathrm{tr}(\bar{B}_\tau\cdot U_\lambda)\gamma^\lambda(H_{\mu\nu}\cdot(D^{\mu\nu}B^\tau)) + \mathrm{tr}((D^{\mu\nu}\bar{B}_\tau)\cdot U_\lambda)\gamma^\lambda(H_{\mu\nu}\cdot B^\tau) + \mathrm{H.c.}) + \frac{1}{2}v_5(\mathrm{tr}(\bar{B}_\tau\cdot\sigma^{\lambda\mu}(D^\nu B^\tau))[U_\lambda,H_{\mu\nu}]_+\\ &-\mathrm{tr}((D^\nu\bar{B}_\tau)\cdot\sigma^{\lambda\mu}B^\tau)[U_\lambda,H_{\mu\nu}]_+) + \frac{1}{4}v_6(\mathrm{tr}(\bar{B}_\tau\cdot U_\lambda)\sigma^{\lambda\mu}(H_{\mu\nu}\cdot(D^\nu B^\tau)) - \mathrm{tr}((D^\nu\bar{B}_\tau)\cdot U_\lambda)\sigma^{\lambda\mu}(H_{\mu\nu}\cdot B^\tau) + \mathrm{H.c.})\\ &+\frac{1}{2}v_7(\mathrm{tr}(\bar{B}_\tau\cdot\sigma^{\lambda\mu}(D^\nu B^\tau))\mathrm{tr}U_\lambda H_{\mu\nu} - \mathrm{tr}((D^\nu\bar{B}_\tau)\cdot\sigma^{\lambda\mu}B^\tau)\mathrm{tr}U_\lambda H_{\mu\nu}) + \frac{1}{4}v_8(\mathrm{tr}(\bar{B}^\mu\cdot U_\lambda)(H_{\mu\nu}\cdot(D^\lambda B^\nu))\\ &-\mathrm{tr}((D^\lambda\bar{B}^\mu)\cdot U_\lambda)(H_{\mu\nu}\cdot B^\nu) + \mathrm{H.c.}) \end{aligned} \tag{8}$$

and

$$\mathcal{L}^{(3)}_{[8][10]} = \frac{1}{2} w_1 (\mathrm{tr}(\bar{B}^\nu \cdot [U_\lambda, H_{\mu\nu}]_+) i\sigma^{\lambda\mu}\gamma_5 B + \mathrm{H.c.}) + \frac{1}{4} w_2 (\mathrm{tr}(\bar{B}^\lambda \cdot [U_\lambda, H_{\mu\nu}]_+) i\gamma^\mu \gamma_5 (D^\nu B)$$
$$- \mathrm{tr}((D^\nu \bar{B}^\lambda) \cdot [U_\lambda, H_{\mu\nu}]_+) i\gamma^\mu\gamma_5 B + \mathrm{H.c.}) + \frac{1}{4} w_3 (\mathrm{tr}(\bar{B}^\lambda \cdot [U_\mu, H_{\lambda\nu}]_+) i\gamma^\mu\gamma_5 (D^\nu B)$$
$$- \mathrm{tr}((D^\nu \bar{B}^\lambda) \cdot [U_\mu, H_{\lambda\nu}]_+) i\gamma^\mu\gamma_5 B + \mathrm{H.c.}) + \frac{1}{2} w_4 (\mathrm{tr}(\bar{B}^\nu \cdot [U_\lambda, H_{\mu\nu}]_-) i\sigma^{\lambda\mu}\gamma_5 B + \mathrm{H.c.})$$
$$+ \frac{1}{4} w_5 (\mathrm{tr}(\bar{B}^\lambda \cdot [U_\lambda, H_{\mu\nu}]_-) i\gamma^\mu\gamma_5 (D^\nu B) - \mathrm{tr}((D^\nu \bar{B}^\lambda) \cdot [U_\lambda, H_{\mu\nu}]_-) i\gamma^\mu\gamma_5 B + \mathrm{H.c.})$$
$$+ \frac{1}{4} w_6 (\mathrm{tr}(\bar{B}^\lambda \cdot [U_\mu, H_{\lambda\nu}]_-) i\gamma^\mu\gamma_5 (D^\nu B) - \mathrm{tr}((D^\nu \bar{B}^\lambda) \cdot [U_\mu, H_{\lambda\nu}]_-) i\gamma^\mu\gamma_5 B + \mathrm{H.c.}). \tag{9}$$

We observe a significant mismatch with the number of seven terms claimed in Ref. [26].

III. CORRELATION FUNCTION FROM THE CHIRAL LAGRANGIAN

We consider QCD's axial-vector and vector currents,

$$A^{(a)}_\mu(x) = \bar{\Psi}(x)\gamma_\mu\gamma_5 \frac{\lambda_a}{2}\Psi(x), \qquad V^{(a)}_\mu(x) = \bar{\Psi}(x)\gamma_\mu \frac{\lambda_a}{2}\Psi(x), \tag{10}$$

where we recall their definitions in terms of the Heisenberg quark-field operators $\Psi(x)$. With λ_a we denote the Gell-Mann flavor matrices. Our target is an evaluation of the following matrix elements:

$$C^{(abe)}_{\mu\nu\lambda}(q,q') = \int d^4x\, d^4y\, e^{+iq\cdot(x-y)} e^{+iq'\cdot y} \times \langle \bar{p},\bar{\chi}|\mathcal{T} A^{(a)}_\mu(x) A^{(b)}_\nu(0) V^{(e)}_\lambda(y)|p,\chi\rangle \tag{11}$$

in the baryon ground states. Here the spin projections of the initial and final baryon states we denote by χ and $\bar{\chi}$. Similarly, the initial and final three-momenta of the states are p and $\bar{p}$. The flavor structure in Eq. (11) is incomplete, since the initial and final baryon states also come in different flavor copies. We return to this issue below in more detail. Given the chiral Lagrangian, it is well defined how to derive the contributions to such matrix elements in application of the classical matrices of source functions, a_μ and v_μ.

The particular correlation function is chosen so as to selectively probe our Q^3 terms. This is so because any such term in the chiral Lagrangian is linear not only in the U_λ field but also in the $H_{\mu\nu}$ field. Upon an expansion of those building blocks in powers of the meson fields, one finds

$$iU_\mu = a_\mu + \cdots, \qquad iH_{\mu\nu} = [v_\mu, a_\nu]_- + [v_\nu, a_\mu]_- + \cdots. \tag{12}$$

From here we conclude that the tree-level evaluation of the chiral Lagrangian is characterized by the symmetry-conserving Q^3 terms, as anticipated above.

The motivation for our study of this correlation function is twofold. First, it serves as a convenient tool to verify whether we use only independent sets of the symmetry-conserving Q^3 terms. We checked for the flavor octet case, that any additional term leads to a contribution that can be linear-combined in terms of the ten terms originally used in Ref. [8] and confirmed later in Refs, [24,25]. An analogous computation consolidates our claim about the smallest set of independent terms in the decuplet sector. Second, such a correlation function can be scrutinized also in large-N_c QCD. This will lead to sum rules among the set of low-energy constants introduced in this work. We will turn to this issue in the next section.

We close this section with explicit results for the correlation function. It suffices to evaluate the matrix elements in the strict flavor $SU(3)$ limit. In this case, a baryon octet or a decuplet state,

$$|p,\chi,c\rangle, \qquad |p,\chi,klm\rangle, \tag{13}$$

is specified by its three-momentum p and the flavor indices $c = 1,\ldots,8$ or $k,\, l,\, m = 1,\, 2,\, 3$. The spin polarization label is $\chi = 1,\, 2$ for the octet and $\chi = 1,\ldots,4$ for the decuplet states. In order to discriminate flavor structures from the currents versus those from the baryon states, we introduce the operator

$$\mathcal{O}^{(abe)}_{ijh}(q,q') = \int d^4x\, d^4y\, e^{+iq\cdot(x-y)} e^{+iq'\cdot y} \times \mathcal{T} A^{(a)}_i(x) A^{(b)}_j(0) V^{(e)}_h(y), \tag{14}$$

by which the matrix elements in the baryon states [Eq. (13)] are considered in the following. Note that in Eq. (14) we already focus on the space components of the three currents. From the study of such components, the anticipated large-N_c sum rules for the low-energy constants, the main target of our work, can be derived.

Since we will encounter many flavor indices in our work, which either run from 1 to 3 or from 1 to 8, we found it useful to split the alphabet into two parts. We use the roman small letters from a to g for flavor indices with $a = 1,\ldots,8$ and the letters from h to z for indices with $h = 1,\ldots,3$. With this convention it is easily confirmed over which range a given flavor index goes.

We are now prepared to present results for the matrix elements introduced with Eq. (11). A somewhat tedious but straightforward evaluation leads to the explicit results

$$
\begin{aligned}
\langle \bar{p},\bar{\chi},d|\mathcal{O}_{ijh}^{(abe)}(q,q')|p,\chi,c\rangle = {}& \bar{u}(\bar{p},\bar{\chi})\frac{2g^{ij}\gamma^h + g^{ih}\gamma^j + g^{jh}\gamma^i}{4}u(p,\chi)\Big\{-(u_1+u_2)\delta_{ab}d_{dce} - 3(u_1+u_2)d_{abg}d_{efg}d_{dcf} \\
& - (u_1-u_2)\delta_{ab}if_{dce} - 3(u_1-u_2)d_{abg}d_{efg}if_{dcf} + (u_1+u_2)(\delta_{ae}d_{bdc}+\delta_{be}d_{adc}) \\
& + (u_1-u_2)(\delta_{ae}if_{bdc}+\delta_{be}if_{adc}) - \frac{1}{2}u_3(\delta_{ad}if_{bec}+\delta_{bd}if_{aec}-\delta_{ac}if_{bed}-\delta_{bc}if_{aed})\Big\} \\
& + \bar{u}(\bar{p},\bar{\chi})\frac{g^{ih}\gamma^j - g^{jh}\gamma^i}{4}u(p,\chi)\Big\{(u_1+u_2)f_{abg}f_{efg}d_{dcf} + (u_1-u_2)f_{abg}f_{efg}if_{dcf} \\
& - \frac{1}{2}u_3(\delta_{ad}if_{bec}-\delta_{bd}if_{aec}-\delta_{ac}if_{bed}+\delta_{bc}if_{aed})\Big\} \\
& + \bar{u}(\bar{p},\bar{\chi})\frac{i\sigma^{ih}\delta_{jk}+i\sigma^{jh}\delta_{ik}}{8}u(p,\chi)(\bar{p}+p)^k\Big\{-(u_7+u_8-u_9)d_{abg}if_{efg}d_{dcf} \\
& - (u_7-u_8)d_{abg}if_{efg}if_{dcf} - \frac{1}{2}u_9(\delta_{ad}if_{bec}+\delta_{bd}if_{aec}+\delta_{ac}if_{bed}+\delta_{bc}if_{aed})\Big\} \\
& + \bar{u}(\bar{p},\bar{\chi})\frac{2i\sigma^{ij}\delta_{hk}+i\sigma^{ih}\delta_{jk}-i\sigma^{jh}\delta_{ik}}{8}u(p,\chi)\Big\{-\Big(\frac{4}{3}u_7+\frac{4}{3}u_8-\frac{1}{3}u_9+2u_{10}\Big)if_{abe}\delta_{dc} \\
& + (u_7+u_8-u_9)(if_{aeg}d_{bgf}-if_{beg}d_{agf})d_{dcf} + (u_7-u_8)(if_{aeg}d_{bgf}-if_{beg}d_{agf})if_{dcf} \\
& - \frac{1}{2}u_9(\delta_{ad}if_{bec}-\delta_{bd}if_{aec}+\delta_{ac}if_{bed}-\delta_{bc}if_{aed})\Big\}(\bar{p}+p)^k \\
& + \bar{u}(\bar{p},\bar{\chi})\frac{\bar{p}^i\gamma^j+\bar{p}^j\gamma^i}{2}u(p,\chi)(\bar{p}+p)^h\Big\{-(u_4+u_5)\delta_{ab}d_{dce} - 3(u_4+u_5)d_{abg}d_{efg}d_{dcf} \\
& - (u_4-u_5)\delta_{ab}if_{dce} - 3(u_4-u_5)d_{abg}d_{efg}if_{dcf} + (u_4+u_5)(\delta_{ae}d_{bdc}+\delta_{be}d_{adc}) \\
& + (u_4-u_5)(\delta_{ae}if_{bdc}+\delta_{be}if_{adc}) - \frac{1}{2}u_6(\delta_{ad}if_{bec}+\delta_{bd}if_{aec}-\delta_{ac}if_{bed}-\delta_{bc}if_{aed})\Big\} \\
& - \bar{u}(\bar{p},\bar{\chi})\frac{\bar{p}^i\gamma^j-\bar{p}^j\gamma^i}{2}u(p,\chi)\Big\{(u_4+u_5)f_{abg}f_{efg}d_{dcf} + (u_4-u_5)f_{abg}f_{efg}if_{dcf} \\
& - \frac{1}{2}u_6(\delta_{ad}if_{bec}-\delta_{bd}if_{aec}-\delta_{ac}if_{bed}+\delta_{bc}if_{aed})\Big\}(\bar{p}-p)^h. \qquad (15)
\end{aligned}
$$

Corresponding expressions for matrix elements in the baryon decuplet states are collected in Appendix A. We wish to emphasize that the computation of such matrix elements serves as a powerful consistency check of whether the terms of the chiral Lagrangian were constructed properly. Our results (15) show that all terms shown are independent—i.e., it is not possible to eliminate any term.

IV. CURRENT CORRELATION FUNCTION IN LARGE-N_c QCD

Consider $\mathcal{O}_{\rm QCD}$ to be the time-ordered product of any combination of local currents in large-N_c QCD, where Eq. (14) may serve as a specific example for $N_c = 3$. The generic form of the large-N_c operator expansion can be taken as

$$\langle \bar{p},\bar{\chi}|\mathcal{O}_{\rm QCD}|p,\chi\rangle = \sum_{n=0}^{\infty} c_n(\bar{p},p)(\bar{\chi}|\mathcal{O}_{\rm static}^{(n)}|\chi), \qquad (16)$$

where it is important to note that unlike the physical baryon states, $|p,\chi\rangle$, the effective baryon states, $|\chi)$, do not depend on the three-momentum p. All dynamical information in Eq. (16) is moved into appropriate coefficient functions $c_n(\bar{p},p)$. Moreover, in the decomposition of Eq. (16), the coefficients $c_n(\bar{p},p)$ depend on neither the flavor nor the spin quantum number of the initial or the final baryon state. The merit of Eq. (16) lies in the fact that the contributions on its right-hand side can be sorted according to their relevance at large values of N_c.

The effective baryon states $|c,\chi\rangle$ and $|klm,\chi\rangle$ have a mean-field structure that can be generated in terms of effective quark operators. They correspond to the baryon states already introduced with Eq. (13) for the particular choice $N_c = 3$. A complete set of color-neutral one-body operators may be constructed in terms of the very same static quark operators:

$$\mathbb{1} = q^\dagger(\mathbf{1}\otimes\mathbf{1}\otimes\mathbf{1})q, \quad J_i = q^\dagger\left(\frac{\sigma_i}{2}\otimes\mathbf{1}\otimes\mathbf{1}\right)q,$$
$$T^a = q^\dagger\left(\mathbf{1}\otimes\frac{\lambda_a}{2}\otimes\mathbf{1}\right)q, \quad G_i^a = q^\dagger\left(\frac{\sigma_i}{2}\otimes\frac{\lambda_a}{2}\otimes\mathbf{1}\right)q, \qquad (17)$$

with operators $q = (u, d, s)^T$ introduced for the up, down, and strange quarks. With λ_a we denote the Gell-Mann matrices. While the action of any of the spin-flavor operators introduced in Eq. (17) on the tower of large-N_c states is quite involved at large $N_c \neq 3$, matters turn quite simple and straightforward at the physical value $N_c = 3$. For this physical case where there is a flavor octet with spin 1/2 or a flavor decuplet with spin 3/2 only, we recall the well-established results of Refs. [8,33] with

$$\begin{aligned}
\mathbb{1}|c,\chi\rangle &= 3|c,\chi\rangle,\\
J_i|c,\chi\rangle &= \frac{1}{2}\sigma^{(i)}_{\bar\chi\chi}|c,\bar\chi\rangle, \quad T^a|c,\chi\rangle = if_{cda}|d,\chi\rangle,\\
G_i^a|c,\chi\rangle &= \sigma^{(i)}_{\bar\chi\chi}\left(\frac{1}{2}d_{cda} + \frac{i}{3}f_{cda}\right)|d,\bar\chi\rangle\\
&\quad + \frac{1}{2\sqrt{2}}S^{(i)}_{\bar\chi\chi}\Lambda^{klm}_{ac}|klm,\bar\chi\rangle,\\
\mathbb{1}|klm,\chi\rangle &= 3|klm,\chi\rangle,\\
J_i|klm,\chi\rangle &= \frac{3}{2}(\vec S\sigma_i\vec S^\dagger)_{\bar\chi\chi}|klm,\bar\chi\rangle,\\
T^a|klm,\chi\rangle &= \frac{3}{2}\Lambda^{a,nop}_{klm}|nop,\chi\rangle,\\
G_i^a|klm,\chi\rangle &= \frac{3}{4}(\vec S\sigma_i\vec S^\dagger)_{\bar\chi\chi}\Lambda^{a,nop}_{klm}|nop,\bar\chi\rangle\\
&\quad + \frac{1}{2\sqrt{2}}(S_i^\dagger)_{\bar\chi\chi}\Lambda^{ac}_{klm}|c,\bar\chi\rangle, \qquad (18)
\end{aligned}$$

with the Pauli matrices σ_i and the spin-transition matrices S_i characterized by

$$\begin{aligned}
&S_i^\dagger S_j = \delta_{ij} - \frac{1}{3}\sigma_i\sigma_j, \qquad S_i\sigma_j - S_j\sigma_i = -i\varepsilon_{ijk}S_k,\\
&\vec S\cdot\vec S^\dagger = \mathbf{1}_{(4\times4)}, \qquad \vec S^\dagger\cdot\vec S = 2\,\mathbf{1}_{(2\times2)},\\
&\vec S\cdot\vec\sigma = 0, \qquad \epsilon_{ijk}S_iS_j^\dagger = i\vec S\sigma_k\vec S^\dagger. \qquad (19)
\end{aligned}$$

We recall some instrumental flavor structures,

$$\begin{aligned}
&\Lambda^{klm}_{ab} = [\varepsilon_{ijk}\lambda^{(a)}_{li}\lambda^{(b)}_{mj}]_{\mathrm{sym}(klm)}, \qquad \delta^{klm}_{nop} = [\delta_{kn}\delta_{lo}\delta_{mp}]_{\mathrm{sym}(nop)},\\
&\Lambda^{ab}_{klm} = [\varepsilon_{ijk}\lambda^{(a)}_{il}\lambda^{(b)}_{jm}]_{\mathrm{sym}(klm)}, \qquad \Lambda^{a,klm}_{nop} = [\lambda^{(a)}_{kn}\delta_{lo}\delta_{mp}]_{\mathrm{sym}(nop)}, \qquad (20)
\end{aligned}$$

that occur frequently in our previous and current works [5,31,39].

In the sum of Eq. (16), there are infinitely many terms one may write down. The static operators $\mathcal{O}^{(n)}_{\mathrm{static}}$ are finite products of the one-body operators J_i, T^a, and G_i^a. In contrast, the counting of N_c factors is intricate, since there is a subtle balance of suppression and enhancement effects. An r-body operator consisting of the r products of any of the spin and flavor operators receives the suppression factor N_c^{-r}. This is counteracted by enhancement factors for the flavor and spin-flavor operators T^a and G_i^a that are produced by taking baryon matrix elements at $N_c \neq 3$. This leads to the enhancement factors [40]

$$J_i \sim N_c^0, \qquad T^a \sim N_c, \qquad G_i^a \sim N_c. \qquad (21)$$

Together with the suppression factor N_c^{-r} from the expansion coefficients in Eq. (16), this implies the effective scaling $J_i \sim 1/N_c$ and $T^a \sim G_i^a \sim N_c^0$ [31,40,41]. According to Eq. (21), there are an infinite number of terms contributing at a given order in the $1/N_c$ expansion. Taking higher products of flavor and spin-flavor operators does not reduce the N_c scaling power. A systematic $1/N_c$ expansion is made possible by a set of operator identities [31,40] that allows a systematic summation of the infinite number of relevant terms. As a consequence of the $SU(6)$ Lie algebra, any commutator of one-body operators can be expressed in terms of one-body operators again. Therefore, it suffices to consider anticommutators of the one-body operators [31,40]. For instance, consider the following two identities that hold in matrix elements of the baryon states:

$$\begin{aligned}
d_{gab}[T_a, T_b]_+ &= -2T_g + 2[J^i, G^i_g]_+,\\
d_{gab}[G^i_a, G^j_b]_+ &= \frac{1}{3}\delta^{ij}\left(\frac{9}{2}T_g - \frac{3}{2}[J^k, G^k_g]_+\right)\\
&\quad + \frac{1}{6}([J^i, G^j_g]_+ + [J^j, G^i_g]_+), \qquad (22)
\end{aligned}$$

where we consider $N_c = 3$ for simplicity. The relations in Eq. (22) have been verified in Ref. [31] by the application of Eq. (18).

Altogether, the expansion scheme is implied by two reduction rules:

(1) All operator products in which two flavor indices are contracted using δ_{ab}, f_{abc}, or d_{abc}, or in which two spin indices on G's are contracted using δ_{ij} or ε_{ijk}, can be eliminated.

(2) All operator products in which two flavor indices are contracted using symmetric or antisymmetric

combinations of two different d and/or f symbols can be eliminated. The only exception to this rule is the antisymmetric combination $f_{acg}d_{bch} - f_{bcg}d_{ach}$. As a consequence, the infinite tower of spin-flavor operators truncates at any given order in the $1/N_c$ expansion. We can now turn to the $1/N_c$ expansion of the baryon matrix elements of our specific product of QCD's axial-vector and vector currents. In the application of the operator reduction rules, the baryon matrix elements of time-ordered products of the current operators are expanded in powers of the effective one-body operators according to the counting rule [Eq. (21)] supplemented by the reduction rules.

V. SUM RULES FOR THE LOW-ENERGY CONSTANTS

As compared to previous works [5,29–31] that dealt with correlation functions of one or two currents only, it turned out that the systematic construction of the large-N_c operator hierarchy for the correlation function of three currents is considerably more involved. While it is straightforward to write down a set of operators to a given order, almost any single term cannot be matched to the matrix elements as implied by the chiral Lagrangian. This is so because the role of charge conjugation and parity invariances is not so transparent in the given framework. We derived our operators by considering all possible combinations and then performed the matching in application of a suitable computer algebra code by using matrix elements presented in Appendix B. This then generated the following leading-order decomposition:

TABLE I. Matching of the large-N_c operators to the LEC.

$u_1 = 0$	$v_1 = 0$	$w_1 = 0$
$u_2 = 0$	$v_2 = 0$	$w_2 = -4\hat{g}_2$
$u_3 = 0$	$v_3 = 0$	$w_3 = 4\hat{g}_2$
$u_4 = \frac{1}{2}\hat{g}_1 + \frac{1}{2}\hat{g}_4$	$v_4 = -3\hat{g}_1$	$w_4 = 0$
$u_5 = -\frac{1}{2}\hat{g}_1 - \frac{1}{2}\hat{g}_4$	$v_5 = -3\hat{g}_2 - 18\hat{g}_6$	$w_5 = 0$
$u_6 = 3\hat{g}_4$	$v_6 = 0$	$w_6 = 0$
$u_7 = \frac{1}{3}\hat{g}_2 - 2\hat{g}_6$	$v_7 = 2\hat{g}_2 + 3\hat{g}_6 + 12\hat{g}_6$	
$u_8 = \frac{5}{3}\hat{g}_2 + 2\hat{g}_6$	$v_8 = 0$	
$u_9 = 0$		
$u_{10} = -\frac{4}{3}\hat{g}_2 - \hat{g}_5$		

$$\mathcal{O}^{ijh}_{abe} = \delta_h^{(ij)_+}\Big\{\hat{g}_1(\delta_{ab}T_e - (\delta_{ae}T_b + \delta_{be}T_a) + 3d_{abg}d_{efg}T_f) - \frac{1}{2}\hat{g}_4\{d_{aeg}[J^l,([T_g,G^l_b]_+ - [T_b,G^l_g]_+)]_+ + d_{beg}[J^l,([T_g,G^l_a]_+ - [T_a,G^l_g]_+)]_+ - 2d_{abg}[J^l,([T_g,G^l_e]_+ - [T_e,G^l_g]_+)]_+\}\Big\}$$
$$+ (\bar{p}+p)^q\{(i\epsilon^{iju}\delta_{hq} + \delta^{(ij)_-}_{(vq)_-}i\epsilon^{huv})[\hat{g}_2(if_{aeg}d_{gbf} - if_{beg}d_{gaf})G^u_f + \hat{g}_5 if_{abe}J^u + \hat{g}_6(if_{aeg}d_{bfg} - if_{beg}d_{afg})[J^u,T_f]_+]$$
$$+ \delta^{(ij)_+}_{(vq)_+}i\epsilon^{huv}[\hat{g}_3 d_{abg}if_{feg}G^u_f + \hat{g}_7 d_{abg}if_{efg}[J^u,T_f]_+]\}, \tag{23}$$

where the parameters $\hat{g}_{1-3}$ and $\hat{g}_{4-7}$ are relevant at leading and subleading orders, respectively. In Eq. (23), we use the notation

$$\delta^{(ij)_\pm}_{(mn)_\pm} = \frac{1}{2}(\delta_{mi}\delta_{nj} \pm \delta_{mj}\delta_{ni}),$$
$$\delta^{(ij)_\pm}_h = \frac{1}{2(\bar{M}+M)}(\bar{p}^i\bar{p}^h + p^ip^h)(\bar{p}+p)^j \pm (i \leftrightarrow j). \tag{24}$$

Owing to the matching condition

$$\hat{g}_2 + \hat{g}_3 = 0, \tag{25}$$

there are two leading-order operators only. This is a nontrivial result in view of the fact that one may write down many more leading-order operators. For instance, consider the particular term

$$\delta^{(ij)_+}_h(2[[T_a,T_b]_+,T_e]_+ - ([[T_a,T_e]_+,T_b]_+ + [[T_b,T_e]_+,T_a]_+)), \tag{26}$$

for which the matrix elements can be shown to be proportional to the matrix elements of the operator associated with $\hat{g}_1$. At subleading order, we find three additional operators only. Here the matching condition

$$\hat{g}_6 + \hat{g}_7 = 0 \tag{27}$$

eliminates one term.

The number of independent coupling constants in the chiral Lagrangian is 24. At leading order in the $1/N_c$ expansion, all of them can be expressed in terms of $\hat{g}_1$ and $\hat{g}_2$ as detailed in Table I. At subleading order, the additional three parameters $\hat{g}_4$, $\hat{g}_5$, and $\hat{g}_6$ enter. The desired sum rules follow upon eliminating the parameters $\hat{g}_n$. There are 15 common sum rules applicable at LO and NLO:

$$u_{1,2,3} = 0 = u_9, \quad v_{1,2,3} = 0 = v_{6,8}, \quad w_{4,5,6} = 0 = w_1,$$
$$u_5 = -u_4, \quad w_3 = -w_2. \tag{28}$$

They are supplemented by four and seven additional sum rules at NLO and LO, respectively, as

$$v_5 = 6u_7 - 3u_8, \qquad v_4 = 6u_5 + u_6,$$
$$v_7 = -6u_7 - 3u_{10}, \qquad w_3 = 2(u_7 + u_8), \tag{29}$$

and

$$v_5 = -9u_7 = \frac{9}{4}u_{10}, \qquad v_4 = 6u_5, \qquad u_6 = 0,$$
$$v_7 = 6u_7, \qquad w_3 = 12u_7 = \frac{12}{5}u_8. \tag{30}$$

VI. SUMMARY

In this work, we further prepared the ground for realistic applications of the chiral Lagrangian with the baryon octet and the baryon decuplet fields. For the first time, all symmetry-preserving Q^3 counterterms were constructed as they are relevant for any two-body meson-baryon reaction process. Altogether, we find 24 terms. In order to pave the way toward applications of this set of low-energy parameters, we derived a set of sum rules. We considered matrix elements of a correlation function with two axial-vector currents and one vector current in the baryon ground states as they arise in QCD at a large number of colors (N_c). From a systematic operator expansion thereof, we deduced our set of 22 sum rules valid at leading order in the $1/N_c$ expansion. At subleading order, there remain 19 relations.

With our result we now deem it feasible to perform significant coupled-channel studies of meson-baryon scattering processes considering channels with the baryon octet and decuplet fields on an equal footing as it is requested by large-N_c QCD. Increasing the accuracy request by one order from Q^2 to Q^3 involves only a few further independent parameters once the constraints of large-N_c QCD are imposed to subleading order. Of particular interest are the sectors with two or three strangeness units, for which the empirical dataset is quite limited.

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APPENDIX A: MATRIX ELEMENTS FROM THE CHIRAL LAGRANGIAN

$$\begin{aligned}
&\langle \bar{p},\bar{\chi}, nop|\mathcal{O}^{(abe)}_{ijh}(q,q')|p,\chi,klm\rangle \\
&= -\frac{1}{4}\delta^{nop}_{xyz}\Lambda^{d,xyz}_{klm}(3d_{abg}d_{edg} + \delta_{ab}\delta_{ed} - \delta_{ae}\delta_{bd} - \delta_{ad}\delta_{be})\bigg\{\frac{1}{2}v_1\bar{u}_\tau(\bar{p},\bar{\chi})(2\gamma^h g^{ij} + \gamma^i g^{hj} + \gamma^j g^{hi})u^\tau(p,\chi) \\
&\quad + \frac{1}{2}v_2(\bar{u}^i(\bar{p},\bar{\chi})\gamma^j u^h(p,\chi) + \bar{u}^j(\bar{p},\bar{\chi})\gamma^i u^h(p,\chi) + \bar{u}^i(\bar{p},\bar{\chi})\gamma^h u^j(p,\chi) + \bar{u}^j(\bar{p},\bar{\chi})\gamma^h u^i(p,\chi)) \\
&\quad + \frac{1}{2}v_3(\bar{u}^i(\bar{p},\bar{\chi})\gamma^h u^j(p,\chi) + \bar{u}^j(\bar{p},\bar{\chi})\gamma^h u^i(p,\chi) + \bar{u}^h(\bar{p},\bar{\chi})\gamma^i u^j(p,\chi) + \bar{u}^h(\bar{p},\bar{\chi})\gamma^j u^i(p,\chi))\bigg\} \\
&\quad - \frac{1}{4}\delta^{nop}_{xyz}\Lambda^{d,xyz}_{klm}f_{abg}f_{edg}\bigg\{\frac{1}{2}v_1\bar{u}_\tau(\bar{p},\bar{\chi})(\gamma^i g^{hj} - \gamma^j g^{hi})u^\tau(p,\chi) - \frac{1}{2}v_2(\bar{u}^i(\bar{p},\bar{\chi})\gamma^j u^h(p,\chi) - \bar{u}^j(\bar{p},\bar{\chi})\gamma^i u^h(p,\chi) \\
&\quad + \bar{u}^i(\bar{p},\bar{\chi})\gamma^h u^j(p,\chi) - \bar{u}^j(\bar{p},\bar{\chi})\gamma^h u^i(p,\chi)) + \frac{1}{2}v_3(\bar{u}^i(\bar{p},\bar{\chi})\gamma^h u^j(p,\chi) - \bar{u}^j(\bar{p},\bar{\chi})\gamma^h u^i(p,\chi) \\
&\quad + \bar{u}^h(\bar{p},\bar{\chi})\gamma^i u^j(p,\chi) - \bar{u}^h(\bar{p},\bar{\chi})\gamma^j u^i(p,\chi))\bigg\} \\
&\quad - \frac{1}{8}\bar{u}_\tau(\bar{p},\bar{\chi})(i\sigma^{ih}(\bar{p}+p)^j + i\sigma^{jh}(\bar{p}+p)^i)u^\tau(p,\chi)\bigg\{\left(v_5 + \frac{3}{4}v_6\right)d_{abg}if_{efg}\delta^{nop}_{xyz}\Lambda^{f,xyz}_{klm} \\
&\quad - \frac{3}{8}v_6\delta^{nop}_{rst}((\Lambda^{a,rst}_{xyz}if_{beg} + \Lambda^{b,rst}_{xyz}if_{aeg})\Lambda^{g,xyz}_{klm} + \Lambda^{g,rst}_{xyz}(\Lambda^{a,xyz}_{klm}if_{beg} + \Lambda^{b,xyz}_{klm}if_{aeg}))\bigg\} \\
&\quad - \frac{1}{8}\bar{u}_\tau(\bar{p},\bar{\chi})(2i\sigma^{ij}(\bar{p}+p)^h + i\sigma^{ih}(\bar{p}+p)^j - i\sigma^{jh}(\bar{p}+p)^i)u^\tau(p,\chi)\bigg\{2\left(\frac{2}{3}v_5 + \frac{1}{2}v_6 + v_7\right)\delta^{nop}_{klm}if_{abe} \\
&\quad + \left(v_5 + \frac{3}{4}v_6\right)(if_{beg}d_{agf} - if_{aeg}d_{bgf})\delta^{nop}_{xyz}\Lambda^{f,xyz}_{klm} \\
&\quad - \frac{3}{8}v_6\delta^{nop}_{rst}((\Lambda^{a,rst}_{xyz}if_{beg} - \Lambda^{b,rst}_{xyz}if_{aeg})\Lambda^{g,xyz}_{klm} + \Lambda^{g,rst}_{xyz}(\Lambda^{a,xyz}_{klm}if_{beg} - \Lambda^{b,xyz}_{klm}if_{aeg}))\bigg\}
\end{aligned}$$

$$
\begin{aligned}
&-\frac{1}{8}\delta^{nop}_{xyz}\Lambda^{f,xyz}_{klm}(3d_{abg}d_{feg}+\delta_{ab}\delta_{fe}-\delta_{af}\delta_{be}-\delta_{ae}\delta_{bf})\bigg\{\frac{1}{2}v_8((\bar{u}^h(\bar{p},\bar{\chi})u^j(p,\chi)+\bar{u}^j(\bar{p},\bar{\chi})u^h(p,\chi))(\bar{p}+p)^i\\
&+(\bar{u}^h(\bar{p},\bar{\chi})u^i(p,\chi)+\bar{u}^i(\bar{p},\bar{\chi})u^h(p,\chi))(\bar{p}+p)^j)\bigg\}\\
&-\frac{1}{8}\delta^{nop}_{xyz}\Lambda^{f,xyz}_{klm}f_{abg}f_{feg}\bigg\{\frac{1}{2}v_8((\bar{u}^h(\bar{p},\bar{\chi})u^j(p,\chi)+\bar{u}^j(\bar{p},\bar{\chi})u^h(p,\chi))(\bar{p}+p)^i\\
&-(\bar{u}^h(\bar{p},\bar{\chi})u^i(p,\chi)+\bar{u}^i(\bar{p},\bar{\chi})u^h(p,\chi))(\bar{p}+p)^j)\bigg\}\\
&+\frac{1}{4}\delta^{nop}_{xyz}\Lambda^{d,xyz}_{klm}(3d_{abg}d_{edg}+\delta_{ab}\delta_{ed}-\delta_{ae}\delta_{bd}-\delta_{ad}\delta_{be})\bigg\{v_4\bar{u}_\tau(\bar{p},\bar{\chi})(\bar{p}^i\gamma^j+\bar{p}^j\gamma^i)u^\tau(p,\chi)(\bar{p}+p)^h\bigg\}\\
&-\frac{1}{4}\bar{u}_\tau(\bar{p},\bar{\chi})(\bar{p}^i\gamma^j-\bar{p}^j\gamma^i)u^\tau(p,\chi)(\bar{p}-p)^h\delta^{nop}_{xyz}\Lambda^{d,xyz}_{klm}v_4f_{abg}f_{edg}, \qquad \text{(A1)}
\end{aligned}
$$

and

$$
\begin{aligned}
&\langle\bar{p},\bar{\chi},nop|\mathcal{O}^{(abe)}_{ijh}(q,q')|p,\chi,c\rangle\\
&=\frac{1}{8\sqrt{2}}(\bar{u}^i(\bar{p},\bar{\chi})i\sigma^{jh}+\bar{u}^j(\bar{p},\bar{\chi})i\sigma^{ih})\gamma_5u(p,\chi)\{-w_1d_{abf}if_{egf}-w_4(3d_{abf}d_{gef}+\delta_{ab}\delta_{ge}-(\delta_{ag}\delta_{be}+\delta_{bg}\delta_{ae}))\}\Lambda^{nop}_{gc}\\
&+\frac{1}{16\sqrt{2}}((\bar{u}^i(\bar{p},\bar{\chi})\gamma^j+\bar{u}^j(\bar{p},\bar{\chi})\gamma^i)(\bar{p}+p)^h+(\bar{u}^i(\bar{p},\bar{\chi})(\bar{p}+p)^j+\bar{u}^j(\bar{p},\bar{\chi})(\bar{p}+p)^i)\gamma^h)\gamma_5u(p,\chi)\{-w_2d_{abf}if_{egf}\\
&-w_5(3d_{abf}d_{gef}+\delta_{ab}\delta_{ge}-(\delta_{ag}\delta_{be}+\delta_{bg}\delta_{ae}))\}\Lambda^{nop}_{gc}\\
&+\frac{1}{16\sqrt{2}}((\bar{u}^i(\bar{p},\bar{\chi})\gamma^j+\bar{u}^j(\bar{p},\bar{\chi})\gamma^i)(\bar{p}+p)^h+\bar{u}^h(\bar{p},\bar{\chi})(\gamma^i(\bar{p}+p)^j+\gamma^j(\bar{p}+p)^i))\gamma_5u(p,\chi)\{-w_3d_{abf}if_{egf}\\
&-w_6(3d_{abf}d_{gef}+\delta_{ab}\delta_{ge}-(\delta_{ag}\delta_{be}+\delta_{bg}\delta_{ae}))\}\Lambda^{nop}_{gc}\\
&+\frac{1}{8\sqrt{2}}(2\bar{u}^h(\bar{p},\bar{\chi})i\sigma^{ij}-(\bar{u}^i(\bar{p},\bar{\chi})i\sigma^{jh}-\bar{u}^j(\bar{p},\bar{\chi})i\sigma^{ih}))\gamma_5u(p,\chi)\{w_1(if_{aef}d_{bgf}-if_{bef}d_{agf})-w_4f_{abf}f_{gef}\}\Lambda^{nop}_{gc}\\
&+\frac{1}{16\sqrt{2}}((\bar{u}^i(\bar{p},\bar{\chi})\gamma^j-\bar{u}^j(\bar{p},\bar{\chi})\gamma^i)(\bar{p}+p)^h+(\bar{u}^i(\bar{p},\bar{\chi})(\bar{p}+p)^j\\
&-\bar{u}^j(\bar{p},\bar{\chi})(\bar{p}+p)^i)\gamma^h)\gamma_5u(p,\chi)\{w_2(if_{aef}d_{bgf}-if_{bef}d_{agf})-w_5f_{abf}f_{gef}\}\Lambda^{nop}_{gc}\\
&+\frac{1}{16\sqrt{2}}(-(\bar{u}^i(\bar{p},\bar{\chi})\gamma^j-\bar{u}^j(\bar{p},\bar{\chi})\gamma^i)(\bar{p}+p)^h\\
&+\bar{u}^h(\bar{p},\bar{\chi})(\gamma^i(\bar{p}+p)^j-\gamma^j(\bar{p}+p)^i))\gamma_5u(p,\chi)\{w_3(if_{aef}d_{bgf}-if_{bef}d_{agf})-w_6f_{abf}f_{gef}\}\Lambda^{nop}_{gc}. \qquad \text{(A2)}
\end{aligned}
$$

APPENDIX B: MATRIX ELEMENTS AT $N_c = 3$

The matrix elements for one- and two-body operators are presented in Ref. [33]. Those for three-body operators considered in this work are collected here.

$$
\begin{aligned}
(d,\bar\chi|[J^i,[T_a,J^j]_+]_+|c,\chi) &= \delta_{\bar\chi\chi}\delta^{ij} i f_{acd},\\
(d,\bar\chi|[J^i,[T_a,G^j_b]_+]_+|c,\chi) &= \frac{1}{3}\delta_{\bar\chi\chi}\delta^{ij}\Bigg(\delta_{ab}\delta_{dc}-\delta_{ad}\delta_{bc}-\delta_{bd}\delta_{ca}+3d_{abe}d_{cde}\\
&\quad+\frac{3}{2}i(\underbrace{f_{cde}d_{abe}-f_{bce}d_{ade}}_{=+f_{ace}d_{bde}}-f_{ade}d_{bce})\Bigg),\\
(nop,\bar\chi|[J^i,[T_a,J^j]_+]_+|klm,\chi) &= \frac{27}{2}\left(\frac{5}{9}\delta^{ij}\mathbf{1}_4-\frac{2}{3}\left(S^iS^{j\dagger}+S^jS^{i\dagger}-\frac{2}{3}\delta^{ij}\mathbf{1}_4\right)\right)_{\bar\chi\chi}\delta^{nop}_{xyz}\Lambda^{a,xyz}_{klm},\\
(nop,\bar\chi|[J^i,[T_a,G^j_b]_+]_+|klm,\chi) &= \frac{27}{8}\left(\frac{5}{9}\delta^{ij}\mathbf{1}_4-\frac{2}{3}\left(S^iS^{j\dagger}+S^jS^{i\dagger}-\frac{2}{3}\delta^{ij}\mathbf{1}_4\right)\right)_{\bar\chi\chi}\delta^{nop}_{rst}(\Lambda^{a,rst}_{xyz}\Lambda^{b,xyz}_{klm}+\Lambda^{b,rst}_{xyz}\Lambda^{a,xyz}_{klm}),\\
(nop,\bar\chi|[J^i,[T_a,J^j]_+]_+|c,\chi) &= 0,\\
(nop,\bar\chi|[J^i,[T_a,G^j_b]_+]_+|c,\chi) &= \frac{1}{4\sqrt{2}}(S^i\sigma^j+S^j\sigma^i+3i\epsilon^{ijk}S^k)_{\bar\chi\chi}(2if_{ace}\Lambda^{nop}_{be}-if_{abe}\Lambda^{nop}_{ce}),\\
(d,\bar\chi|[[T_a,T_b]_+,T_e]_+|c,\chi) &= \delta_{\bar\chi\chi}[2\delta_{ab}if_{ecd}+3d_{abg}(d_{cgf}if_{def}-d_{dgf}if_{cef})\\
&\quad-\delta_{bd}if_{eca}-\delta_{ad}if_{ecb}-\delta_{ac}if_{ebd}-\delta_{bc}if_{ead}],\\
(d,\bar\chi|[[G^i_a,G^j_b]_+,T_e]_+|c,\chi) &= \frac{1}{4}\delta_{\bar\chi\chi}\delta^{ij}\Bigg[\frac{10}{3}\delta_{ab}if_{ecd}-d_{abg}(d_{gdf}if_{ecf}-d_{gcf}if_{edf})+\frac{8}{3}d_{abg}(f_{gdf}f_{ecf}+f_{gcf}f_{edf})\\
&\quad+\frac{1}{3}(\delta_{ad}if_{ebc}+\delta_{bd}if_{eac}-\delta_{ac}if_{ebd}-\delta_{bc}if_{ead})\Bigg]\\
&\quad+\frac{1}{4}i\epsilon^{ijk}\sigma^k_{\bar\chi\chi}\Bigg[\delta_{ad}if_{ebc}-\delta_{bd}if_{eac}+\delta_{ac}if_{ebd}-\delta_{bc}if_{ead}+2if_{abg}(d_{gdf}if_{ecf}-d_{gcf}if_{edf})\\
&\quad+\frac{5}{3}if_{abg}(f_{gdf}f_{ecf}+f_{gcf}f_{edf})\Bigg],\\
(d,\bar\chi|[[T_a,T_b]_+,G^i_e]_+|c,\chi) &= \frac{1}{2}\sigma^i_{\bar\chi\chi}\Bigg[2\delta_{ab}d_{ecd}+\frac{4}{3}\delta_{ab}if_{ecd}+3d_{abg}d_{fgd}d_{ecf}+2d_{abg}d_{fgd}if_{ecf}+3d_{abg}d_{cgf}d_{efd}\\
&\quad+2d_{abg}d_{cgf}if_{efd}-\delta_{ac}d_{ebd}-\delta_{bc}d_{ead}-\delta_{bd}d_{eca}-\delta_{ad}d_{ecb}-\frac{2}{3}\delta_{ac}if_{ebd}\\
&\quad-\frac{2}{3}\delta_{bc}if_{ead}-\frac{2}{3}\delta_{bd}if_{eca}-\frac{2}{3}\delta_{ad}if_{ecb}\Bigg],\\
(d,\bar\chi|[[T_a,T_b]_+,J^i]_+|c,\chi) &= \sigma^i_{\bar\chi\chi}[\delta_{ab}\delta_{cd}-\delta_{ac}\delta_{bd}-\delta_{ad}\delta_{bc}+3d_{abg}d_{cgd}],\\
(d,\bar\chi|[[G^l_a,G^l_b]_+,J^l]_+|c,\chi) &= \frac{1}{4}\sigma^i_{\bar\chi\chi}\left[\frac{1}{3}(5\delta_{ab}\delta_{cd}-\delta_{ac}\delta_{bd}-\delta_{ad}\delta_{bc})-d_{dcg}d_{abg}+\frac{8}{3}if_{cdg}d_{abg}\right],\\
(nop,\bar\chi|[[T_a,T_b]_+,T_e]_+|klm,\chi) &= \frac{27}{8}\delta_{\bar\chi\chi}\delta^{nop}_{rst}\{\Lambda^{a,rst}_{xyz}\Lambda^{b,xyz}_{uvw}\Lambda^{e,uvw}_{klm}+\Lambda^{e,rst}_{uvw}\Lambda^{a,uvw}_{xyz}\Lambda^{b,xyz}_{klm}+(a\leftrightarrow b)\},\\
(nop,\bar\chi|[[G^i_a,G^j_b]_+,T_e]_+|klm,\chi) &= \frac{3}{2}\delta^{nop}_{rst}\Bigg\{-\frac{3}{8}(S^iS^{j\dagger}+S^jS^{i\dagger}-\frac{3}{2}\delta^{ij}\mathbf{1}_{(4\times4)})_{\bar\chi\chi}[\Lambda^{a,rst}_{xyz}\Lambda^{b,xyz}_{uvw}\Lambda^{e,uvw}_{klm}\\
&\quad+\Lambda^{e,rst}_{uvw}\Lambda^{a,uvw}_{xyz}\Lambda^{b,xyz}_{klm}+(a\leftrightarrow b)]\\
&\quad+\frac{3}{16}i\epsilon^{ijk'}(\vec S\sigma^{k\prime}\vec S^\dagger)_{\bar\chi\chi}[\Lambda^{a,rst}_{xyz}\Lambda^{b,xyz}_{uvw}\Lambda^{e,uvw}_{klm}+\Lambda^{e,rst}_{uvw}\Lambda^{a,uvw}_{xyz}\Lambda^{b,xyz}_{klm}-(a\leftrightarrow b)]\\
&\quad+\frac{1}{16}(S^iS^{j\dagger}+S^jS^{i\dagger})_{\bar\chi\chi}[\Lambda^{rst}_{ag}\Lambda^{bg}_{uvw}\Lambda^{e,uvw}_{klm}+\Lambda^{e,rst}_{uvw}\Lambda^{uvw}_{ag}\Lambda^{bg}_{klm}+(a\leftrightarrow b)]\\
&\quad+\frac{1}{16}i\epsilon^{ijk'}(\vec S\sigma^{k\prime}\vec S^\dagger)_{\bar\chi\chi}[\Lambda^{rst}_{ag}\Lambda^{bg}_{uvw}\Lambda^{e,uvw}_{klm}+\Lambda^{e,rst}_{uvw}\Lambda^{uvw}_{ag}\Lambda^{bg}_{klm}-(a\leftrightarrow b)]\Bigg\},
\end{aligned}
$$

$$(nop,\bar\chi|[[T_a,T_b]_+,G^i_e]_+|klm,\chi)=\frac{27}{16}(\vec S\sigma^i\vec S^\dagger)_{\bar\chi\chi}\delta^{nop}_{rst}\{\Lambda^{a,rst}_{xyz}\Lambda^{b,xyz}_{uvw}\Lambda^{e,uvw}_{klm}+\Lambda^{e,rst}_{xyz}\Lambda^{a,xyz}_{uvw}\Lambda^{b,uvw}_{klm}+(a\leftrightarrow b)\},$$

$$(nop,\bar\chi|[[T_a,T_b]_+,J^i]_+|klm,\chi)=\frac{27}{4}(\vec S\sigma^i\vec S^\dagger)_{\bar\chi\chi}\delta^{nop}_{rst}\{\Lambda^{a,rst}_{xyz}\Lambda^{b,xyz}_{klm}+(a\leftrightarrow b)\},$$

$$(nop,\bar\chi|[[G^i_a,G^l_b]_+,J^l]_+|klm,\chi)=\frac{3}{2}(\vec S\sigma^i\vec S^\dagger)_{\bar\chi\chi}\delta^{nop}_{rst}\left\{\frac{13}{8}\Lambda^{a,rst}_{xyz}\Lambda^{b,xyz}_{klm}-\frac{1}{12}\Lambda^{rst}_{ag}\Lambda^{bg}_{klm}+(a\leftrightarrow b)\right\},$$

$$(nop,\bar\chi|[[T_a,T_b]_+,T_e]_+|c,\chi)=0,$$

$$\begin{aligned}(nop,\bar\chi|[[G^i_a,G^j_b]_+,T_e]_+|c,\chi)=\frac{1}{8\sqrt2}\delta^{nop}_{rst}\Bigg\{&(S^i\sigma^j+S^j\sigma^i)_{\bar\chi\chi}\Bigg[if_{ecf}(d_{afg}+if_{afg})\Lambda^{rst}_{bg}\\&+\frac{3}{2}\Lambda^{e,rst}_{uvw}(d_{acg}+if_{acg})\Lambda^{uvw}_{bg}+(a\leftrightarrow b)\Bigg]\\&+i\epsilon^{ijk}S^k_{\bar\chi\chi}\Bigg[if_{ecf}\left(\left(d_{afg}+\frac{2}{3}if_{afg}\right)\Lambda^{rst}_{bg}+\frac{5}{3}(if_{afg}\Lambda^{rst}_{bg}-if_{abg}\Lambda^{rst}_{fg})\right)\\&+\frac{3}{2}\Lambda^{e,rst}_{uvw}\left(\left(d_{acg}+\frac{2}{3}if_{acg}\right)\Lambda^{uvw}_{bg}+\frac{5}{3}(if_{acg}\Lambda^{uvw}_{bg}-if_{abg}\Lambda^{uvw}_{cg})\right)-(a\leftrightarrow b)\Bigg]\Bigg\},\end{aligned}$$

$$\begin{aligned}(nop,\bar\chi|[[T_a,T_b]_+,G^i_e]_+|c,\chi)=\frac{1}{2\sqrt2}S^i_{\bar\chi\chi}\Bigg\{&\frac{9}{4}\delta^{nop}_{rst}(\Lambda^{a,rst}_{xyz}\Lambda^{b,xyz}_{uvw}+\Lambda^{b,rst}_{xyz}\Lambda^{a,xyz}_{uvw})\Lambda^{uvw}_{ec}+\Lambda^{nop}_{ef}(\delta_{ab}\delta_{cf}-\delta_{ac}\delta_{bf}\\&-\delta_{af}\delta_{bc}+3d_{abg}d_{cgf})\Bigg\},\end{aligned}$$

$$(nop,\bar\chi|[[T_a,T_b]_+,J^i]_+|c,\chi)=0,$$

$$\begin{aligned}(nop,\bar\chi|[[G^i_a,G^l_b]_+,J^l]_+|c,\chi)=\frac{1}{8\sqrt2}S^i_{\bar\chi\chi}\Bigg\{&5(d_{acg}+if_{acg})\Lambda^{nop}_{bg}+5(d_{bcg}+if_{bcg})\Lambda^{nop}_{ag}-3\left(d_{acg}+\frac{2}{3}if_{acg}\right)\Lambda^{nop}_{bg}\\&+3\left(d_{bcg}+\frac{2}{3}if_{bcg}\right)\Lambda^{nop}_{ag}-5(if_{acg}\Lambda^{nop}_{bg}-if_{bcg}\Lambda^{nop}_{ag}-2if_{abg}\Lambda^{nop}_{cg})\Bigg\},\end{aligned}\tag{B1}$$

where Eq. (A.2) in Ref. [31] was used. We correct a typo with

$$\begin{aligned}(nop,\bar\chi|[G^i_a,G^j_b]_+|c,\chi)=&\frac{1}{8\sqrt2}i\epsilon^{ijk}S^k_{\bar\chi\chi}\left[\left(d_{ace}+\frac{2}{3}if_{ace}\right)\Lambda^{nop}_{be}+\frac{5}{3}(if_{ace}\Lambda^{nop}_{be}-if_{abe}\Lambda^{nop}_{ce})-(a\leftrightarrow b)\right]\\&+\frac{1}{8\sqrt2}(S^i\sigma^j+S^j\sigma^i)_{\bar\chi\chi}[(d_{ace}+if_{ace})\Lambda^{nop}_{be}+(a\leftrightarrow b)].\end{aligned}\tag{B2}$$

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