

Interpretation of the observed $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ states as $1D$ bottom baryons

Bing Chen^{1,2,*}, Si-Qiang Luo^{3,2,†}, Xiang Liu^{3,2,‡} and Takayuki Matsuki^{4,5,§}

¹*School of Electrical and Electronic Engineering, Anhui Science and Technology University, Fengyang 233100, China*

²*Research Center for Hadron and CSR Physics, Lanzhou University & Institute of Modern Physics of CAS, Lanzhou 730000, China*

³*School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China*

⁴*Tokyo Kasei University, 1-18-1 Kaga, Itabashi, Tokyo 173-8602, Japan*

⁵*Theoretical Research Division, Nishina Center, RIKEN, Wako, Saitama 351-0198, Japan*



(Received 9 October 2019; published 26 November 2019)

The measured masses and strong decays of $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ required us to decode them as the $1D$ excited states with $J^P = 3/2^+$ and $J^P = 5/2^+$, respectively. Under the suggested assignment, the masses and total decay widths of $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ can be explained. As a $J^P = 3/2^+$ state, the $\Lambda_b(6146)^0$ should mainly decay into $\Sigma_b(5815)\pi$. However, this theoretical result is in contradiction to the measurement since no significant $\Lambda_b(6146)^0 \rightarrow \Sigma_b(5815)^\pm\pi^\mp$ signals were observed by the LHCb. The possible explanations for this difficulty are mentioned. For completeness, we present the prediction of masses and decay widths of other unseen $1D$ bottom baryons in this work. Our results could provide some important clues for the upcoming experiments.

DOI: [10.1103/PhysRevD.100.094032](https://doi.org/10.1103/PhysRevD.100.094032)

I. INTRODUCTION

Single heavy baryons occupy an important position in the hadron's jigsaw puzzle since chiral symmetry and heavy quark symmetry (HQS) can provide some qualitative insight into their dynamics. So the investigation of single heavy baryons, including the charm and bottom baryons, could be more helpful for improving our understanding of the confinement mechanism. With the experimental and theoretical efforts, the spectrum of both charm and bottom baryon families is being established step by step [1]. Here, we briefly review some important measurements of the excited charm and bottom baryons by different experiments in the past years.

In the Λ_c^+ sector, a new resonance, denoted as $\Lambda_c(2860)^+$, was discovered by the LHCb Collaboration [2]. The spin-parity of $\Lambda_c(2860)^+$ was identified as $3/2^+$. In the same work, the LHCb also confirmed the spin of $\Lambda_c(2880)^+$,

which was first measured by the Belle Collaboration [3]. Specifically, the preferred spin of the $\Lambda_c(2880)^+$ state was found to be $J = 5/2$. In addition, the first constraint on the spin-parity of the $\Lambda_c(2940)^+$ state was performed in the same analysis [2], where the $\Lambda_c(2940)^+$ was most likely a $J^P = 3/2^-$ state. Belle recently determined the isospin of $\Lambda_c(2760)^+/\Sigma_c(2760)^+$ and disentangled the identity of this puzzling state [4]. Since no evidence of $\Sigma_c(2760)^{++0}$ was observed in the $\Sigma_c(2455)^{++0}\pi^0$ channels, there is now no doubt that the puzzling state around 2.76 GeV is a Λ_c^+ state.

Experiments have also made progress in the study of excited Ξ_c and Ω_c states. The $\Xi_c(3055)^+$ and $\Xi_c(3080)^+$ were first observed in the decay channel of ΛD^+ , and their relevant ratios of branching fractions were also measured [5]. Five narrow Ω_c states, denoted as $\Omega_c(3000)$, $\Omega_c(3050)$, $\Omega_c(3065)$, $\Omega_c(3090)$, and $\Omega_c(3120)$, were found by the LHCb Collaboration [6] and were subsequently confirmed by the Belle Collaboration [7].

For the excited bottom baryons, the $1P$ Λ_b^0 states, i.e., $\Lambda_b^0(5912)^0$ and $\Lambda_b^0(5920)^0$, were established by LHCb [8] and CDF [9]. Last year, two bottom baryons, denoted as $\Xi_b(6227)^-$ [10] and $\Sigma_b(6097)^\pm$ [11], were first observed by the LHCb Collaboration. Very recently, the LHCb Collaboration again announced their new discovery of two bottom baryon states, the $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$, by analyzing the $\Lambda_b^0\pi^+\pi^-$ invariant mass spectrum from pp collisions [12]. The resonance parameters of these two states are listed below,

*chenbing@ahstu.edu.cn

†luosq15@lzu.edu.cn

‡Corresponding author;

xiangliu@lzu.edu.cn

§matsuki@tokyo-kasei.ac.jp

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](https://creativecommons.org/licenses/by/4.0/). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

$$\begin{aligned}
m_{\Lambda_b(6146)^0} &= 6146.17 \pm 0.33 \pm 0.22 \pm 0.16 \text{ MeV}, \\
m_{\Lambda_b(6152)^0} &= 6152.51 \pm 0.26 \pm 0.22 \pm 0.16 \text{ MeV}, \\
\Gamma_{\Lambda_b(6146)^0} &= 2.9 \pm 1.3 \pm 0.3 \text{ MeV}, \\
\Gamma_{\Lambda_b(6152)^0} &= 2.1 \pm 0.8 \pm 0.3 \text{ MeV}.
\end{aligned} \tag{1}$$

These newly observed single heavy baryons have motivated a wide discussion and study of their inner structures [13]. In recent years, our group has also carried a systematic study of the low-lying charm and bottom baryons, mainly including the $2S$ and $1P$ states in the single heavy baryon family [14–19]. In this work, we will further investigate the probable assignment of the new $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ and try to establish the $1D$ bottom baryons.

Heavy quark symmetry can provide us with some qualitative insight of the $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ when we compare them with the established Λ_c^+ baryon states. At present, the $1S$, $1P$, and $1D$ Λ_c^+ baryons have been established [1]. As shown later, the excited energies of $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ indicate that the $1D$ assignment is suitable for these two new bottom baryons. The predicted masses and strong decays can further test this assignment.

With the coming LHCb Upgrade I in 2020 [20], more and more excited bottom baryons are expected to be discovered in the near future. So it is necessary for us to systematically investigate the properties of the whole D -wave bottom baryons by including their mass spectrum and strong decay behaviors, which can provide some valuable information for further experiments to explore the excited bottom baryons.

The paper is organized as follows. In Sec. II, a brief review of our theoretical methods will be presented, where the parameters will also be given for the reader's convenience. We decode the newly reported $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ as the D -wave states in Sec. III, where the analysis of the mass spectrum and the investigation of strong decays will be presented. For completeness, we will also predict the mass spectra and strong decays of other D -wave bottom baryons in Sec. IV. Finally, the paper ends with a discussion and conclusions in Sec. V.

II. SCHEME OF THE QUARK POTENTIAL MODEL AND THE QUARK PAIR CREATION (QPC) MODEL

Heavy quark symmetry plays an important role in the dynamics of a single heavy baryon system. In the heavy quark limit, one heavy quark within the heavy baryon system is decoupled from two light quarks. Although heavy quark symmetry is broken due to the finite charm or bottom quark mass, much evidence indicates that heavy quark symmetry is a good approximation for the single heavy baryons.

Under the scenario of heavy quark symmetry, the dynamics of a heavy baryon state could be well separated

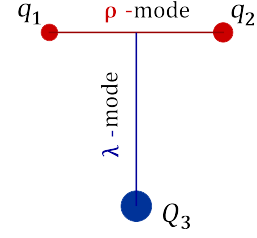


FIG. 1. The so-called “ ρ -mode” and “ λ -mode” excitations in a single heavy baryon system. The quarks q_1 and q_2 represent the light u , d , and s quarks, while the Q_3 quark refers to a heavy charm or bottom quark.

into two parts [21]. As illustrated in Fig. 1, the degree of freedom between two light quarks (q_1 and q_2) is usually called the “ ρ -mode,” while the degree between the center of mass of two light quarks and the heavy quark is called the “ λ -mode.” If adopting the ordinary confining potential like the linear or harmonic form to depict the interaction of quarks, the excited energy of the ρ -mode is higher than that of the λ -mode excitations [21,22]. So it seems to be easier for the experiments to detect the λ -mode excited heavy baryons. The available theoretical investigations confirm the point that almost all observed single heavy baryons could be explained as the λ -mode excited states.

In Refs. [18,19], we assigned the $\Xi_b(6227)^-$ and $\Sigma_b(6097)^\pm$ as the P -wave λ -mode excited bottom baryons. In this work, we also examine the possibility of $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ as the λ -mode excitations. Since two light quarks in a bottom baryon are always in the ground state with the antitriplet color structure, the λ -mode bottom heavy baryon can be simplified as a quasi-two-body system, which means that the diquark could be an effective degree of freedom in these λ -mode excited bottom baryons. To depict the interaction between the light diquark and the bottom quark, the following Schrödinger equation can be constructed,

$$\left(-\frac{\nabla^2}{2m_\mu} - \frac{4\alpha}{3r} + br + C + \frac{32\alpha\sigma^3}{9\sqrt{\pi}m_{di}m_b} \vec{s}_{di} \cdot \vec{s}_b \right) \psi_{nL} = E\psi_{nL}. \tag{2}$$

Here, \vec{s}_{di} and \vec{s}_b denote the spin of the light diquark and the b quark, respectively. The reduced mass is defined as $m_\mu \equiv m_{di}m_b/(m_{di} + m_b)$. The parameters α , b , and C stand for the strength of the color Coulomb potential, the strength of linear confinement, and a mass-renormalized constant, respectively. The parameter σ comes from a Gaussian-smearred contact hyperfine interaction between the light diquark and the bottom quark. Since the spin of the light diquark in the Λ_b and Ξ_b baryons is zero, the spin-spin contact hyperfine interaction in Eq. (2) is only important to calculate the radially excited Σ_b , Ξ'_b , and Ω_b states. The values of the parameters which have been given in Ref. [18] are also listed in Table I for the reader's convenience.

TABLE I. Values of the parameters for the bottom baryons in the nonrelativistic quark potential model where the mass of the b quark is taken as 4.96 GeV. Here, m_{di} refers to the mass of different diquarks.

Parameters	m_{di} (GeV)	α	b (GeV ²)	σ (GeV)	C (GeV)
Λ_b	0.45	0.20	0.112	–	0.265
Ξ_b	0.63	0.26	0.118	–	0.176
Σ_b	0.66	0.22	0.116	1.20	0.185
Ξ'_b	0.78	0.22	0.116	1.20	0.152
Ω_b	0.91	0.26	0.120	1.07	0.120

By solving the Schrödinger equation, the spin-averaged masses of these excited bottom baryons can be obtained. When the spin-orbit and tensor interactions are incorporated, all masses of D -wave λ -mode excited bottom baryons can be calculated. Furthermore, the simple harmonic oscillator (SHO) wave function $\psi_{nL}^m = \mathcal{R}_{nL}(\beta, \mathbf{K})\mathcal{Y}_{nL}^m(\mathbf{K})$ is usually adopted to construct the spatial wave function of the bottom baryon states and other hadron states when one uses the QPC model to calculate the strong decays of excited bottom baryons. The SHO wave-function scale β is also fixed by solving the above Schrödinger equation. In other word, the parameters in the potential model and in the QPC decay model have the same values except for the unique parameter γ of the QPC model.

As mentioned above, we will use the QPC model to study the strong decays of D -wave excited bottom baryons. For an OZI-allowed decay process of a hadron state, the QPC model assumes that a $q\bar{q}$ pair is created from the vacuum and then regroups into two outgoing hadrons by a quark rearrangement process [23–25]. For a process “ $A \rightarrow B + C$,” the transition matrix element in the QPC model is written as $\langle BC|\hat{T}|A\rangle = \delta^3(\mathbf{K}_B + \mathbf{K}_C)\mathcal{M}^{j_A j_B j_C}(p)$ where the transition operator \hat{T} reads as

$$\hat{T} = -3\gamma \sum_m \langle 1, m; 1, -m | 0, 0 \rangle \iint d^3\mathbf{k}_\mu d^3\mathbf{k}_\nu \delta^3(\mathbf{k}_\mu + \mathbf{k}_\nu) \times \mathcal{Y}_1^m\left(\frac{\mathbf{k}_\mu - \mathbf{k}_\nu}{2}\right) \omega_0^{(\mu,\nu)} \varphi_0^{(\mu,\nu)} \chi_{1,-m}^{(\mu,\nu)} d_\mu^\dagger(\mathbf{k}_\mu) d_\nu^\dagger(\mathbf{k}_\nu), \quad (3)$$

in a nonrelativistic limit. Here, the dimensionless parameter γ describes the strength of the quark-antiquark pair created from the vacuum. The $\omega_0^{(\mu,\nu)}$ and $\varphi_0^{(\mu,\nu)}$ are the color and flavor wave functions of the $q_\mu \bar{q}_\nu$ pair created from the vacuum. Therefore, $\omega_0^{(\mu,\nu)} = (R\bar{R} + G\bar{G} + B\bar{B})/\sqrt{3}$ and $\varphi_0^{(\mu,\nu)} = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$ are color and flavor singlets. The $\chi_{1,-m}^{(\mu,\nu)}$ represents the pair production in a spin-triplet state. When the mock state [26] is adopted to describe the wave function of a hadron state, the partial wave amplitudes $\mathcal{M}_{LS}(p)$ can be obtained by the following formula,

$$\mathcal{M}_{LS}^{A \rightarrow B+C}(p) = \frac{\sqrt{2L+1}}{2J_A+1} \sum_{j_B j_C} \langle L0Sj_A | J_A j_A \rangle \times \langle J_B j_B, J_C j_C | S j_A \rangle \mathcal{M}^{j_A j_B j_C}(p). \quad (4)$$

Here, p represents the momentum of an outgoing meson in the rest frame of a meson A . The J_i and j_i ($i = A, B$, and C) denote the total angular momentum and the projection of initial and final hadron states, respectively, and L denotes the orbital angular momenta between the final state B and C . Finally, the partial width of $A \rightarrow BC$ can be obtained in terms of the partial wave amplitudes

$$\Gamma(A \rightarrow BC) = 2\pi \frac{E_B E_C}{M_A} p \sum_{L,S} |\mathcal{M}_{LS}^{A \rightarrow B+C}(p)|^2, \quad (5)$$

in the A rest frame. Interested readers can consult Refs. [15,16] for more details of our method.

III. DECODING $\Lambda_b(6146)^0$ AND $\Lambda_b(6152)^0$ AS THE D -WAVE EXCITED STATES

The masses of $1D$ Λ_b^0 baryons have been calculated in our previous works [14,19] and in Refs. [27–29]. The results in Refs. [14,19] are in good agreement with the measured masses of $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$, while the predicted masses from Refs. [27–29] are about 30–40 MeV larger than the corresponding experimental results.¹ Here, we should point out that the authors in Refs. [27–29] adopted the same parameters to predict the masses of whole heavy baryon states, including the bottom and charm baryons. In fact, the input parameters of the quark potential model could be slightly different for the charm and bottom baryons. And, when considering the uncertainties of the quark potential model, the results presented in Refs. [27–29] do not contradict the $1D$ assignment to $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$.

In this work, we also give a semiquantitative analysis to illustrate the possibility of $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ as the $1D$ Λ_c^+ states. With the observations of $\Lambda_c(2286)^+$, $\Lambda_c(2595)^+$, $\Lambda_c(2625)^+$, $\Lambda_c(2860)^+$, and $\Lambda_c(2880)^+$ states, the $1S$, $1P$, and $1D$ Λ_c^+ baryons have been established by experiments [1]. In the Λ_b^0 sector, the $1S$ and $1P$ states have also been established [1]. By comparing with the excited energies of $\Lambda_c(2860)^+$ and $\Lambda_c(2880)^+$, it is possible to further decode the $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ as the $1D$ excited states.

The mass gaps between the $1P/1S$ and $1D/1P$ states are illustrated in Fig. 2 for the Λ_c^+ and Λ_b^0 baryons. The mass gap between the $1P$ and $1S$ Λ_c^+ states, which we denote as

¹The mass of the ground Λ_b^0 state was given as 5585 MeV in Ref. [27], which is about 35 MeV below the $\Lambda_b(5620)^0$ state. If one shifts up the predicted mass to match the measured mass of $\Lambda_b(5620)^0$, then the predicted masses of $1P$ and $1D$ Λ_b^0 states in Ref. [27] are also about 30–45 MeV larger than the measurements [8,9,12].

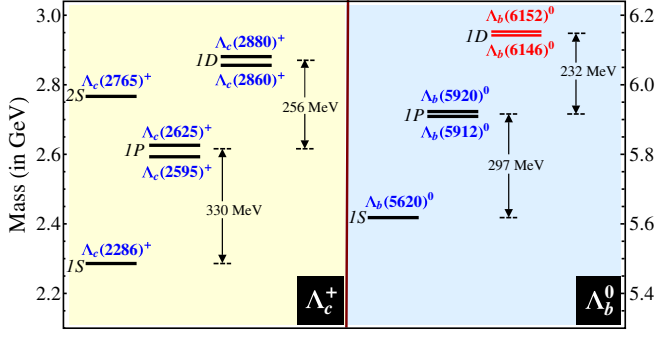


FIG. 2. The established Λ_c^+ and Λ_b^0 states and the newly observed $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$.

$\Delta_{1P-1S}(\Lambda_c^+)$, is about 330 MeV. The gap between the 1D and 1P Λ_c^+ states is about 256 MeV, i.e., $\Delta_{1D-1P}(\Lambda_c^+) \approx 256$ MeV. Then, the ratio

$$\varepsilon(\Lambda_c^+) = \frac{\Delta_{1D-1P}(\Lambda_c^+)}{\Delta_{1P-1S}(\Lambda_c^+)} = \frac{256 \text{ MeV}}{330 \text{ MeV}} = 0.776 \quad (6)$$

is obtained. Similarly, the ratio of Λ_b^0 states is given as

$$\varepsilon(\Lambda_b^0) = \frac{\Delta_{1D-1P}(\Lambda_b^0)}{\Delta_{1P-1S}(\Lambda_b^0)} = \frac{232 \text{ MeV}}{297 \text{ MeV}} = 0.781. \quad (7)$$

Obviously, the ratios of $\Delta_{1D-1P}/\Delta_{1P-1S}$, which are nearly equal for the Λ_c^+ and Λ_b^0 baryons, imply the similarity of dynamics between the charm and bottom baryons. The result of $\varepsilon(\Lambda_c^+) \approx \varepsilon(\Lambda_b^0)$ can be partly explained by the potential model. For simplicity, we replace the Cornell potential in Eq. (2) by the following confining potential,

$$V(r) = \kappa r^\nu, \quad (8)$$

to depict the interaction between the light diquark and heavy quark (c or b quark) in the single heavy baryon system. In this way, the orbitally excited energy can be obtained analytically as

$$E_L = \kappa^{\frac{2}{\nu+2}} \left(\frac{1}{2m_\mu} \right)^{\frac{\nu}{\nu+2}} \left[\sqrt{\pi\nu} \left(L + \frac{3}{2} \right) \frac{\Gamma(\frac{1}{\nu} + \frac{3}{2})}{\Gamma(\frac{1}{\nu})} \right]^{\frac{2\nu}{\nu+2}}, \quad (9)$$

by the Wentzel-Kramers-Brillouin (WKB) approach. Then the ratio defined by Eqs. (6) and (7) can be given by

$$\varepsilon = \frac{5^{\frac{2\nu}{\nu+2}} - 7^{\frac{2\nu}{\nu+2}}}{3^{\frac{2\nu}{\nu+2}} - 5^{\frac{2\nu}{\nu+2}}}, \quad (10)$$

which is independent of the parameter κ and the reduced mass m_μ . When the parameter ν is taken as 0.8 ± 0.2 , the ε is predicted as 0.835 ± 0.035 .

According to the above analysis, the new $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ states could be treated as the partners of the

$\Lambda_c(2860)^+$ and $\Lambda_c(2880)^+$ in the bottom baryon family. Furthermore, the mass splitting of $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$, which was measured as $\Delta m = 6.34 \pm 0.32 \pm 0.02$ MeV, is also consistent with the expectations of 1D Λ_b^0 states [14,29–31]. So according to the measured masses, the $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ states could be good candidates for D -wave bottom baryons.

Thus, it is desirable to further test the D -wave assignment of $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ by exploring their strong decays. A method based on the QPC model (see Sec. II) has successfully explained the strong decays of known bottom baryons [18,19] including the new $\Xi_b(6227)^-$ and $\Sigma_b(6097)^\pm$ states. With the same method and parameters, we calculate the strong decays of the $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ and list the results in Table II.

The total widths of $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ are obtained as 4.18 MeV and 4.39 MeV when they are assigned as the D -wave states with $J^P = 3/2^+$ and $J^P = 5/2^+$, respectively. The predicted total widths of $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ are comparable with the experimental measurements [12]. However, the LHCb announced that they did not observe significant $\Lambda_b(6146)^0 \rightarrow \Sigma_b(5815)^\pm \pi^\mp$ signals in their measurements [12]. This experimental result seems to contradict the theoretical results in Table II. The plausible explanations of this contradiction are given as follows.

As almost degenerate states, the $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ have the same decay channels (see Table II). Then, the possible interference effects, which were ignored in the LHCb analysis [12], might be important to pin down the decay behaviors of $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$. Another possible explanation is that the mass inversion may occur for the 1D Λ_b^0 states [32,33]. That is to say, the $\Lambda_b(6146)^0$ with smaller mass should be the $J^P = 5/2^+$ state, while the $\Lambda_b(6152)^0$ should be a $J^P = 3/2^+$ state. However, this assignment of $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ disagrees with the expectations given by the quark potential models [27–29]. So more experimental and theoretical efforts are required for the $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ states in the future.

TABLE II. Partial widths of strong decays of $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ as the D -wave excited states (in MeV). The superscript letters p and f mean that the corresponding decays occur via the p -wave and f -wave, respectively.

Decay mode	$\Lambda_b(6146)^0$ [$3/2^+(1D)$]	$\Lambda_b(6152)^0$ [$5/2^+(1D)$]
$\Sigma_b(5815)\pi$	3.25^p	0.22^f
$\Sigma_b^*(5835)\pi$	$0.65^p, 0.28^f$	$4.03^p, 0.14^f$
Total width	4.18	4.39
Expt. [12]	$2.9 \pm 1.3 \pm 0.3$	$2.1 \pm 0.8 \pm 0.3$

IV. OTHER PREDICTED $1D$ BOTTOM BARYONS

The basis $|s_{di}, L_\lambda, j_{di}, s_b, J\rangle$ including heavy quark symmetry will be used in our scheme. Here, s_{di} and s_b denote the spins of the light diquark and bottom quark, respectively; L_λ denotes the orbital quantum number between the light diquark and the bottom quark. Then, we define $\vec{j}_{di} = \vec{s}_{di} + \vec{L}_\lambda$ and $\vec{J} = \vec{s}_b + \vec{j}_{di}$. For the D -wave Λ_b^0 and $\Xi_b^{0,-}$ baryons, $j_{di} = L_\lambda = 2$ since $s_{di} = 0$. For the D -wave $\Sigma_b^{0,\pm}$, $\Xi_b^{0,-}$, and Ω_b^- baryons, $j_{di} = 1 \otimes 2 = 1, 2$, and 3 with $s_{di} = 1$. For simplicity, we denote these bottom baryons as $|J^P, 1D\rangle_{j_{di}}$. Finally, we list the abbreviated basis in Table III.

In practice, the mass of the b quark is not infinite. Then, two physical baryons with the same spin-parity should be the mixing states of $|J^P, 1D\rangle_{J-1/2}$ and $|J^P, 1D\rangle_{J+1/2}$. We take the D -wave Σ_b baryons with $J^P = 3/2^+$ as an example. In principle, two physical $3/2^+$ Σ_b baryons should be the mixing states of $|3/2^+, 1D\rangle_1$ and $|3/2^+, 1D\rangle_2$,

$$\begin{pmatrix} |3/2^+, 1D\rangle_H \\ |3/2^+, 1D\rangle_L \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |3/2^+, 1D\rangle_1 \\ |3/2^+, 1D\rangle_2 \end{pmatrix}. \quad (11)$$

Here, the physical $J^P = 3/2^+$ states are distinguished by their different masses. Specifically, the states with the lower and higher masses are denoted by the subscripts “ L ” and “ H ,” respectively. Our practical calculations indicate that the mixing effect is quite small for these D -wave bottom states with the same J^P . For simplicity, we could ignore these small mixing effects and denote the physical states by the $|J^P, 1D\rangle_{j_{di}}$ basis directly.

A. $1D$ Ξ_b baryons

Due to the approximate $SU(3)$ flavor symmetry, the Λ_b and Ξ_b states may have similar properties. With the observation of $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$, the $1S$, $1P$, and $1D$ Λ_b states may have been established by experiments. Differently from Λ_b , only the ground states, $\Xi_b(5792)^0/\Xi_b(5797)^-$, have been observed in the Ξ_b sector [1]. With the running of LHCb, we may expect the excited Ξ_b baryons to be observed in the near future. The masses and decay properties of the $1P$ and $2S$ Ξ_b states have been investigated in our previous work [18]. In the following, we give the predicted properties, especially the strong decay behaviors of the D -wave Ξ_b baryons.

TABLE III. The abbreviated basis of D -wave bottom baryons.

Bottom baryons	Basis		
$\Lambda_b^0, \Xi_b^{0,-}$	$ 3/2^+, 1D\rangle_2$	$ 5/2^+, 1D\rangle_2$	
$\Sigma_b^{0,\pm}, \Xi_b^{0,-}, \Omega_b^-$	$ 1/2^+, 1D\rangle_1$	$ 3/2^+, 1D\rangle_1$	$ 3/2^+, 1D\rangle_2$
	$ 5/2^+, 1D\rangle_2$	$ 5/2^+, 1D\rangle_3$	$ 7/2^+, 1D\rangle_3$

The masses of two D -wave Ξ_b baryons with $J^P = 3/2^+$ and $J^P = 5/2^+$ were predicted as 6327 MeV and 6330 MeV, respectively (see Fig. 2 in Ref. [18]). With the predicted masses, the partial and total decay widths obtained by the QPC model are listed in Table IV.

According to the results, the D -wave Ξ_b baryons seem to be the narrow resonances. It is possible to find the $3/2^+(1D)\Xi_b$ baryon in the $\Xi_b'(5935)\pi$ and $\Sigma_b(5815)K$ channels, while the $5/2^+(1D)\Xi_b$ baryon might be found in the decay channel of $\Xi_b^*(5955)\pi$. A similar conclusion was also obtained by the chiral quark model [34].

B. $1D$ Σ_b baryons

The masses of $1D$ Σ_b baryons have been predicted in our previous works [18,19] (see Fig. 1 in Ref. [19]). For clarity, we list the predicted masses of $1D$ Σ_b and all other bottom baryons together in Table V. According to the predictions, six D -wave Σ_b baryons may exist in the region 6.29–6.40 GeV. Then many decay channels are open for these high excited Σ_b baryons.

The predicted partial and total widths of the D -wave Σ_b baryons are presented in Table VI. In our calculation of strong decays, if the final states have been found, we take the measured masses as inputs. Otherwise, we take the predicted masses of the unknown final states as inputs. As shown in Table VI, the $|1/2^+\rangle_1$, $|3/2^+\rangle_1$, $|3/2^+\rangle_2$, and $|5/2^+\rangle_2$ Σ_b states might be broad since their decay widths are predicted around 100 MeV. So it is a challenge to search for them in experiments. Differently from these states, the $|5/2^+\rangle_3$ and $|7/2^+\rangle_3$ Σ_b states are much narrower.

TABLE IV. Partial widths of strong decays of the D -wave Ξ_b baryon states (in MeV). The superscript letters p and f represent the corresponding decays occurring via the p -wave and the f -wave, respectively.

Decay mode	$\Xi_b(6327) [3/2^+(1D)]$	$\Xi_b(6330)^0 [5/2^+(1D)]$
$\Xi_b'(5935)\pi$	0.39^p	0.09^f
$\Sigma_b(5815)K$	1.73^p	0.00^f
$\Xi_b^*(5955)\pi$	$0.09^p, 0.15^f$	$0.51^p, 0.07^f$
$\Sigma_b^*(5835)K$	$0.02^p, 0.00^f$	$0.09^p, 0.00^f$
Total width	2.38	0.76

TABLE V. Predicted masses of $1D$ Λ_b , Ξ_b , Σ_b , Ξ_b' , and Ω_b baryons (in MeV).

States	$ 1/2^+\rangle_1$	$ 3/2^+\rangle_1$	$ 3/2^+\rangle_2$	$ 5/2^+\rangle_2$	$ 5/2^+\rangle_3$	$ 7/2^+\rangle_3$
Λ_b			6149	6153		
Ξ_b			6327	6330		
Σ_b	6400	6402	6358	6359	6291	6292
Ξ_b'	6486	6488	6456	6457	6407	6408
Ω_b	6599	6602	6578	6580	6542	6543

TABLE VI. The predicted partial and total widths of D -wave Σ_b baryons (in MeV). The superscript letters s , p , \dots , and h mean that the corresponding decays occur via s -, p -, \dots , and h -waves, respectively. If the partial width is predicted to be smaller than 0.01 MeV, it is denoted as “0.00.” If a decay channel is forbidden, it is denoted by “ \times .” If the mass of an initial state is below the threshold of a decay channel, the partial width is denoted by “-.” The masses of $1P$ and $2S$ Σ_b baryons are borrowed from our previous work [19].

Final states	$\Sigma_b(6400)$	$\Sigma_b(6402)$	$\Sigma_b(6358)$	$\Sigma_b(6359)$	$\Sigma_b(6291)$	$\Sigma_b(6292)$
$\Lambda_b(5620)\pi$	1.89^p	1.88^p	\times	\times	7.14^f	7.13^f
$\Xi_b(5795)K$	2.16^p	2.16^p	\times	\times	0.00^f	0.00^f
$\Sigma_b(5815)\pi$	0.27^p	0.06^p	1.47^p	2.05^f	1.64^f	0.93^f
$\Sigma_b^*(5835)\pi$	0.17^p	0.41^p	$0.37^p, 3.96^f$	$2.17^p, 1.77^f$	2.08^f	2.83^f
$\Lambda_b(5912)\pi$	12.35^s	1.49^d	10.63^d	4.73^d	8.72^d	3.17^g
$\Lambda_b(5920)\pi$	2.18^d	$12.58^s, 1.17^d$	10.53^d	16.41^d	$2.51^d, 4.62^g$	$11.27^d, 1.95^g$
$\Sigma_b(6150)\pi$	\times	\times	0.39^d	0.40^d	\times	\times
$\Sigma_b(6134)\pi$	13.91^s	1.62^d	1.74^d	0.79^d	0.01^d	0.00^g
$\Sigma_b(6139)\pi$	2.74^d	$14.93^s, 1.43^d$	1.50^d	2.39^d	$0.00^{d,g}$	$0.01^d, 0.00^g$
$\Sigma_b(6094)\pi$	8.45^d	4.30^d	$26.41^s, 1.16^d$	$0.34^d, 0.15^g$	$1.18^d, 0.00^g$	$0.15^d, 0.00^g$
$\Sigma_b(6098)\pi$	5.36^d	9.67^d	$0.47^d, 0.18^g$	$27.80^s, 1.26^d, 0.06^g$	$0.24^d, 0.00^g$	$1.15^d, 0.00^g$
$\Lambda_b(6086)\pi$	13.05^p	13.30^p	\times	\times	0.06^f	0.07^f
$\Lambda_b(6146)\pi$	15.88^p	$1.57^p, 0.04^f$	$11.00^p, 0.14^f$	$0.83^p, 0.14^f$	$0.00^{p,f}$	$0.00^{p,f}$
$\Lambda_b(6152)\pi$	0.06^f	$14.49^p, 0.03^f$	$1.08^p, 0.14^f$	$1.10^p, 0.15^f$	-	$0.00^{p,f,h}$
BN	2.13^p	$0.12^p,$	8.09^p	11.63^f	0.20^f	7.50^f
B^*N	7.76^p	20.81^p	$13.74^p, 14.54^f$	$36.11^p, 8.66^f$	1.12^f	0.73^f
Total width	88.36	102.06	107.54	118.94	29.52	36.89

Experimentalists may search these two states in the $\Lambda_b(5620)\pi$, $\Lambda_b(5912)\pi$, and $\Lambda_b(5920)\pi$ channels.

Here we would like to stress the particularity of $\Lambda_b(5912)\pi$ and $\Lambda_b(5920)\pi$ channels to search the D -wave $|5/2^+\rangle_3$ and $|7/2^+\rangle_3$ Σ_b states. Since the $\Lambda_b(5912)$ and $\Lambda_b(5920)$ are below the threshold of $\Sigma_b(5815)\pi$, they can only decay through the three-body decay channel, i.e., $\Lambda_b\pi\pi$. As illustrated in Fig. 3, the $\Lambda_b(5912)$ and $\Lambda_b(5920)$ states with long lifetimes can fly far away from the decay vertex of the D -wave $|5/2^+\rangle_3$ and $|7/2^+\rangle_3$ Σ_b states. This kind of process is very helpful for the LHCb detector to distinguish two pions which are produced from the P -wave Λ_b^0 state and the third pion from the initial D -wave Σ_b state.

C. $1D$ Ξ'_b baryons

Up to now, only the $1S$ Ξ'_b baryons, $\Xi'_b(5935)$ and $\Xi_b^*(5955)$, have been established by experiments [1]. The

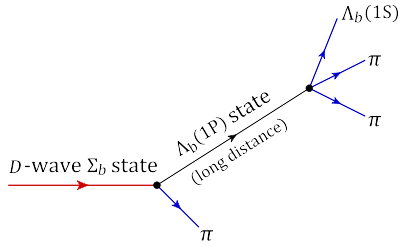


FIG. 3. The diagram to illustrate the D -wave Σ_b baryon decaying into a P -wave Λ_b baryon and a π meson. Subsequently, the P -wave Λ_b baryon decays into the ground Λ_b baryon and two π mesons.

masses and decay widths of $1P$ and $2S$ Ξ'_b baryons were given in Ref. [18].

As shown in Table V, the six $1D$ Ξ'_b baryons are expected to appear in the region 6.40–6.50 GeV. Many decay channels are allowed for these excited Ξ'_b baryons (see Table VII). The $|5/2^+\rangle_3$ and $|7/2^+\rangle_3$ states among the six D -wave Ξ'_b baryons may have narrower decay widths, while the other four states seem to be broad. This prediction is similar to the D -wave Σ_b baryons.

According to the results in Table VII, it is promising for experimentalists to find the $|5/2^+\rangle_3$ and $|7/2^+\rangle_3$ Ξ'_b states in the $\Lambda_b(5620)K$ channel. We also notice that the decay modes of $B\Lambda$ and $B^*\Lambda$ may make a significant contribution to the decays of $|1/2^+\rangle_1$, $|3/2^+\rangle_1$, $|3/2^+\rangle_2$, and $|5/2^+\rangle_2$ Ξ'_b states. Experimentalists could try to search for these four broad Ξ'_b states in the $B\Lambda$ and $B^*\Lambda$ channels.

D. $1D$ Ω_b baryons

So far, only the $\Omega_b(6046)^-$ state in the bss baryon sector has been established [1]. However, there are reasons to believe that the situation could be changed in the near future. The widths of the $2S$ and $1P$ Ω_b states which were predicted in Ref. [19] imply that these excited Ω_b states are usually narrow. This feature can help experimentalists distinguish the signals of these low-lying bss baryons from the background.

With the same method and parameters as before, we also calculate the masses and decays of six D -wave Ω_b states. The results are presented in Tables V and VIII, respectively. It is interesting to note that the D -wave excited Ω_b baryons

TABLE VII. The predicted partial and total widths of D -wave Ξ'_b baryons (in MeV). The superscript letters s , p , ..., and h mean that the corresponding decays occur via s -, p -, ..., and h -waves, respectively. If the partial width is predicted to be smaller than 0.01 MeV, it is denoted as "0.00." If a decay channel is forbidden, it is denoted by "×." If the mass of an initial state is below the threshold of a decay channel, the partial width is denoted by "–." The masses of $1P$, $2S$, and $1D$ $\Xi_b^{(l)}$ baryons are borrowed from our previous work [18].

Final states	$\Xi'_b(6486)$	$\Xi'_b(6488)$	$\Xi'_b(6456)$	$\Xi'_b(6457)$	$\Xi'_b(6407)$	$\Xi'_b(6408)$
$\Lambda_b(5620)K$	1.21 ^p	1.23 ^p	×	×	8.47 ^f	8.47 ^f
$\Sigma_b(5815)K$	4.87 ^p	1.18 ^p	15.01 ^p	2.38 ^f	0.93 ^f	0.54 ^f
$\Sigma_b^*(5835)K$	3.06 ^p	7.48 ^p	3.54 ^p , 3.39 ^f	21.17 ^p , 1.52 ^f	0.70 ^f	0.96 ^f
$\Xi_b(5795)\pi$	0.32 ^p	0.32 ^p	×	×	1.67 ^f	1.67 ^f
$\Xi'_b(5935)\pi$	0.03 ^p	0.01 ^p	0.14 ^p	0.26 ^f	0.23 ^f	0.13 ^f
$\Xi_b^*(5955)\pi$	0.02 ^p	0.05 ^p	0.04 ^p , 0.52 ^f	0.21 ^p , 0.23 ^f	0.31 ^f	0.42 ^f
$\Lambda_b(5912)K$	0.79 ^s	1.11 ^d	1.94 ^d	0.87 ^d	–	0.00 ^g
$\Lambda_b(5920)K$	2.24 ^d	1.75 ^s , 1.13 ^d	1.38 ^d	1.39 ^d	–	–
$\Xi_b(6096)\pi$	5.23 ^s	0.10 ^d	3.59 ^d	1.61 ^d	5.53 ^d	0.22 ^g
$\Xi_b(6102)\pi$	0.27 ^d	4.88 ^s , 0.13 ^d	3.39 ^d	5.32 ^d	1.49 ^d , 0.31 ^g	6.76 ^d , 0.13 ^g
$\Xi'_b(6150)\pi$	×	×	0.19 ^d	0.20 ^d	×	×
$\Xi'_b(6134)\pi$	3.10 ^s	0.32 ^d	0.36 ^d	0.17 ^d	0.02 ^d	0.00 ^g
$\Xi'_b(6139)\pi$	0.53 ^d	3.31 ^s , 0.28 ^d	0.31 ^d	0.50 ^d	0.00 ^{d,g}	0.02 ^d , 0.00 ^g
$\Xi'_b(6094)\pi$	1.09 ^d	0.56 ^d	6.04 ^s , 0.10 ^d	0.03 ^d , 0.01 ^g	0.33 ^d , 0.00 ^g	0.04 ^d , 0.00 ^g
$\Xi'_b(6098)\pi$	0.69 ^d	1.24 ^d	0.04 ^d , 0.01 ^g	6.41 ^s , 0.10 ^d , 0.00 ^g	0.07 ^d , 0.00 ^g	0.32 ^d , 0.00 ^g
$\Xi_b(6225)\pi$	0.88 ^p	0.94 ^p	×	×	0.00 ^f	0.00 ^f
$\Xi_b(6327)\pi$	2.24 ^p	0.26 ^p , 0.00 ^f	–	–	–	–
$\Xi_b(6330)\pi$	0.00 ^f	1.88 ^p , 0.00 ^f	–	–	–	–
$B\Lambda$	20.46 ^p	1.24 ^p ,	17.16 ^p	1.49 ^f	0.00 ^f	0.03 ^f
$B^*\Lambda$	17.13 ^p	49.43 ^p	5.51 ^p , 0.07 ^f	15.60 ^p , 0.05 ^f	–	–
Total width	64.16	78.83	62.73	59.52	20.06	19.71

TABLE VIII. The predicted widths of $1D$ Ω_b baryons (in MeV).

$1D$ Ω_b states	$\Xi_b(5795)K$	$\Xi'_b(5935)K$	$\Xi_b^*(5955)K$	Total
$\Omega_b(6599)$	0.10 ^p	0.65 ^p	0.41 ^p	1.16
$\Omega_b(6602)$	0.11 ^p	0.15 ^p	0.99 ^p	1.25
$\Omega_b(6578)$	×	1.90 ^p	0.45 ^p , 0.53 ^f	2.88
$\Omega_b(6580)$	×	0.41 ^f	2.68 ^p , 0.24 ^f	3.33
$\Omega_b(6542)$	3.32 ^f	0.24 ^f	0.19 ^f	3.75
$\Omega_b(6543)$	3.33 ^f	0.14 ^f	0.26 ^f	3.73

are also expected to be narrow. Experimentalists may try to search for the $|1/2^+\rangle_1$, $|3/2^+\rangle_1$, $|3/2^+\rangle_2$, and $|5/2^+\rangle_2$ Ω_b baryons in the $\Xi'_b(5935)K$ and $\Xi_b^*(5955)K$ channels. For the $|5/2^+\rangle_3$ and $|7/2^+\rangle_3$ Ω_b states, experiments may detect them around 6.54 GeV in the $\Xi_b(5795)K$ channel.

V. DISCUSSIONS AND CONCLUSIONS

There is not doubt that the LHCb Collaboration has played an important role in the research of the excited bottom baryons over the past years. Last year, two candidates for the P -wave bottom baryons, i.e., $\Xi_b(6227)^-$ [10] and $\Sigma_b(6097)^\pm$ [11], were discovered by the LHCb. In particular, the $\Xi_b(6227)^-$ was the first excited

bottom baryon which was identified by the OZI-allowed decay modes. So the observation of $\Xi_b(6227)^-$ could be treated as a starting point for constructing the highly excited bottom baryon spectroscopy [18].

Just as expected, the LHCb Collaboration recently made a breakthrough in the research of the higher excited Λ_b^0 states [12]. Two almost degenerate narrow states, the $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$, were observed in the $\Lambda_b^0\pi^+\pi^-$ spectrum. By comparing the excited energies of $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ to the corresponding Λ_c^+ states, we have found that they could be assigned as the $1D$ Λ_b^0 states with $J^P = 3/2^+$ and $J^P = 5/2^+$, respectively.

The masses of the $1D$ Λ_b^0 states predicted in Refs. [14,19,27] and the further investigation of the total decay widths also support the $1D$ assignment of $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$. However, the $\Sigma_b(5815)\pi$ is a main decay mode for the $\Lambda_b(6146)^0$ state if it is a $J^P = 3/2^+$ state. This theoretical result is in disagreement with the measurement since no significant $\Lambda_b(6146)^0 \rightarrow \Sigma_b^\pm\pi^\mp$ signals were observed at LHCb. Obviously, more experimental and theoretical efforts are desirable for the $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ in the future.

With the upcoming LHCb Upgrade I [20], more excited bottom baryons will be discovered. In this work, we have

also predicted the mass spectra and strong decay widths of other unseen $1D$ excited bottom baryons. The following clues are suggested for searching for these unknown bottom baryons.

- (1) We suggest to search for the $|5/2^+\rangle_3$ and $|7/2^+\rangle_3 \Sigma_b$ baryons in the $\Lambda_b(5620)\pi$, $\Lambda_b(5912)\pi$, and $\Lambda_b(5920)\pi$ channels.
- (2) We suggest to search for the $|5/2^+\rangle_3$ and $|7/2^+\rangle_3 \Xi'_b$ baryons in the $\Lambda_b(5620)K$ channel.
- (3) Due to the very narrow decay widths and the simple decay modes, the D -wave Ξ_b and Ω_b baryons are the potential states which could be found by experiments in the future.

Obviously, the heavy baryon family including the charm and bottom baryons is being established step by step. This interesting area of research deserves more attention from theorists and experimentalists.

ACKNOWLEDGMENTS

X. L. is supported by the China National Funds for Distinguished Young Scientists under Grant No. 11825503 and the National Program for Support of Top-notch Young Professionals. B. C. is partly supported by the National Natural Science Foundation of China under Grants No. 11305003 and No. 11647301.

-
- [1] M. Tanabashi *et al.* (Particle Data Group), Review of particle physics, *Phys. Rev. D* **98**, 030001 (2018).
 - [2] R. Aaij *et al.* (LHCb Collaboration), Study of the $D^0 p$ amplitude in $\Lambda_b^0 \rightarrow D^0 p \pi^-$ decays, *J. High Energy Phys.* **05** (2017) 030.
 - [3] K. Abe *et al.* (Belle Collaboration), Experimental Constraints on the Possible J^P Quantum Numbers of the $\Lambda_c(2880)^+$, *Phys. Rev. Lett.* **98**, 262001 (2007).
 - [4] A. Abdesselam *et al.* (Belle Collaboration), Experimental determination of the isospin of $\Lambda_c(2765)^+/\Sigma_c(2765)^+$, [arXiv:1908.06235](https://arxiv.org/abs/1908.06235).
 - [5] Y. Kato *et al.* (Belle Collaboration), Studies of charmed strange baryons in the ΛD final state at Belle, *Phys. Rev. D* **94**, 032002 (2016).
 - [6] R. Aaij *et al.* (LHCb Collaboration), Observation of Five New Narrow Ω_c^0 States Decaying to $\Xi_c^\pm K^-$, *Phys. Rev. Lett.* **118**, 182001 (2017).
 - [7] J. Yelton *et al.* (Belle Collaboration), Observation of excited Ω_c charmed baryons in e^+e^- collisions, *Phys. Rev. D* **97**, 051102 (2018).
 - [8] R. Aaij *et al.* (LHCb Collaboration), Observation of Excited Λ_b^0 Baryons, *Phys. Rev. Lett.* **109**, 172003 (2012).
 - [9] T. A. Aaltonen *et al.* (CDF Collaboration), Evidence for a bottom baryon resonance Λ_b^{*0} in CDF data, *Phys. Rev. D* **88**, 071101 (2013).
 - [10] R. Aaij *et al.* (LHCb Collaboration), Observation of a New Ξ_b^- Resonance, *Phys. Rev. Lett.* **121**, 072002 (2018).
 - [11] R. Aaij *et al.* (LHCb Collaboration), Observation of Two Resonances in the $\Lambda_b^0 \pi^\pm$ Systems and Precise Measurement of Σ_b^\pm and $\Sigma_b^{*\pm}$ Properties, *Phys. Rev. Lett.* **122**, 012001 (2019).
 - [12] R. Aaij *et al.* (LHCb Collaboration), Observation of New Resonances in the $\Lambda_b^0 \pi^+ \pi^-$ system, *Phys. Rev. Lett.* **123**, 152001 (2019).
 - [13] H. X. Chen, W. Chen, X. Liu, Y. R. Liu, and S. L. Zhu, A review of the open charm and open bottom systems, *Rep. Prog. Phys.* **80**, 076201 (2017).
 - [14] B. Chen, K. W. Wei, and A. Zhang, Assignments of Λ_Q and Ξ_Q baryons in the heavy quark-light diquark picture, *Eur. Phys. J. A* **51**, 82 (2015).
 - [15] B. Chen, K. W. Wei, X. Liu, and T. Matsuki, Low-lying charmed and charmed-strange baryon states, *Eur. Phys. J. C* **77**, 154 (2017).
 - [16] B. Chen and X. Liu, New Ω_c^0 baryons discovered by LHCb as the members of $1P$ and $2S$ states, *Phys. Rev. D* **96**, 094015 (2017).
 - [17] B. Chen, X. Liu, and A. Zhang, Newly observed $\Lambda_c(2860)^+$ at LHCb and its D -wave partners $\Lambda_c(2880)^+$, $\Xi_c(3055)^+$ and $\Xi_c(3080)^+$, *Phys. Rev. D* **95**, 074022 (2017).
 - [18] B. Chen, K. W. Wei, X. Liu, and A. Zhang, Role of newly discovered $\Xi_b(6227)^-$ for constructing excited bottom baryon family, *Phys. Rev. D* **98**, 031502 (2018).
 - [19] B. Chen and X. Liu, Assigning the newly reported $\Sigma_b(6097)$ as a P -wave excited state and predicting its partners, *Phys. Rev. D* **98**, 074032 (2018).
 - [20] R. Aaij *et al.* (LHCb Collaboration), Physics case for an LHCb Upgrade II—Opportunities in flavour physics, and beyond, in the HL-LHC era, [arXiv:1808.08865](https://arxiv.org/abs/1808.08865).
 - [21] T. Yoshida, E. Hiyama, A. Hosaka, M. Oka, and K. Sadato, Spectrum of heavy baryons in the quark model, *Phys. Rev. D* **92**, 114029 (2015).
 - [22] L. A. Copley, N. Isgur, and G. Karl, Charmed baryons in a quark model with hyperfine interactions, *Phys. Rev. D* **20**, 768 (1979); , Erratum, *Phys. Rev. D* **23**, 817(E) (1981).
 - [23] L. Micu, Decay rates of meson resonances in a quark model, *Nucl. Phys.* **B10**, 521 (1969).
 - [24] A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, Naive quark pair creation model of strong interaction vertices, *Phys. Rev. D* **8**, 2223 (1973).
 - [25] A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, *Hadron Transitions in the Quark Model* (Gordon and Breach, New York, 1988), p. 311.
 - [26] C. Hayne and N. Isgur, Beyond the wave function at the origin: Some momentum dependent effects in the non-relativistic quark model, *Phys. Rev. D* **25**, 1944 (1982).
 - [27] S. Capstick and N. Isgur, Baryons in a relativized quark model with chromodynamics, *Phys. Rev. D* **34**, 2809 (1986).
 - [28] W. Roberts and M. Pervin, Heavy baryons in a quark model, *Int. J. Mod. Phys. A* **23**, 2817 (2008).

- [29] D. Ebert, R. N. Faustov, and V. O. Galkin, Spectroscopy and Regge trajectories of heavy baryons in the relativistic quark-diquark picture, *Phys. Rev. D* **84**, 014025 (2011).
- [30] H. X. Chen, Q. Mao, A. Hosaka, X. Liu, and S. L. Zhu, D -wave charmed and bottomed baryons from QCD sum rules, *Phys. Rev. D* **94**, 114016 (2016).
- [31] K. Chen, Y. Dong, X. Liu, Q. F. Lü, and T. Matsuki, Regge-like relation and a universal description of heavy-light systems, *Eur. Phys. J. C* **78**, 20 (2018).
- [32] W. Liang, Q. F. Lü, and X. H. Zhong, Canonical interpretation of the newly observed $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ via strong decay behaviors, *Phys. Rev. D* **100**, 054013 (2019).
- [33] K. L. Wang, Q. F. Lü, and X. H. Zhong, Interpretation of the newly observed $\Lambda_b(6146)^0$ and $\Lambda_b(6152)^0$ states in a chiral quark model, [arXiv:1908.04622](https://arxiv.org/abs/1908.04622).
- [34] Y. X. Yao, K. L. Wang, and X. H. Zhong, Strong and radiative decays of the low-lying D -wave singly heavy baryons, *Phys. Rev. D* **98**, 076015 (2018).