Dynamical supersymmetry breaking in large N_c supersymmetric QCD

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We study dynamical supersymmetry breaking in supersymmetric QCD theories for $N_f < N_c$. We consider a model with a singlet chiral superfield coupled to the infrared meson chiral superfield through a classical superpotential. We examine the vacuum structure of this model and show that in a particular limit of the parameter space with the large N_c limit, it has a vacuum that dynamically breaks supersymmetry. The supersymmetric vacuum, in this limit, is being pushed to infinity.

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I. INTRODUCTION

Supersymmetric quantum field theories have provided us with some useful toy models in which one can track the strong coupling dynamics of phenomena like quark confinement, chiral symmetry breaking, and the mass gap. Moreover, they are promising candidates to replace the standard model of particle physics. The minimal supersymmetric standard model (MSSM) is a theory which minimally embeds supersymmetry in the standard model.

Although the MSSM can address the hierarchy problem, improve the flow of the running coupling constants (so that they all meet at one high energy scale point), and provide particle candidates for dark matter, it is not yet quite clear how to consistently handle the problem of supersymmetry breaking in this model. A hierarchy of energy scales can be naturally achieved through dynamical supersymmetry breaking. In SQCD theories, for instance, this is triggered by the generation of nonperturbative superpotentials [1–3]. In fact, depending on the number of colors, N_c, and the number of the flavors, N_f, one gets different nonperturbative superpotentials. Therefore, in principle, it is natural to think of models which classically have a supersymmetric vacuum, but at the quantum level a nonperturbative superpotential is generated resulting in a nonsupersymmetric vacuum. Constructing a realistic model of this kind, however, turns out to be difficult and, in fact, it is nongeneric. For $N_c = N_f$, there are some interesting models for which F-flatness conditions are in conflict with the dynamical constraint on the quantum vacuum moduli space, and hence supersymmetry is dynamically broken [4,5]. This example, however, is noncalculable, in the sense that the vacuum is not in the weakly coupled region.

The idea of having a long-lived metastable vacuum instead of a nonsupersymmetric absolute minimum opened a new avenue in constructing phenomenologically viable models of dynamical supersymmetry breaking [6,7]. Metastable vacua are easier to construct and appear in more generic models. In [6] Seiberg *et al.* proposed a SQCD model with $N_c < N_f < 3/2N_c$, which, instead of having a stable nonsupersymmetric vacuum, has a metastable vacuum. They showed that, by a suitable adjustment of the parameters of the model, this vacuum can be long lived.

In this paper, we study SQCD with $N_f < N_c$ deformed by a singlet chiral superfield. The singlets are coupled to the low energy meson chiral superfields through a classical superpotential. We examine the vacuum moduli space and observe that, apart from a supersymmetric solution, there are also nonsupersymmetric solutions. We choose a maximally symmetric solution and look at the small fluctuations around it to determine its stability. The analysis of the stability simplifies in a corner of the parameter space of the model; however, to keep the potential finite, we need to take a simultaneous large N_c limit. We discuss the spectrum of small fluctuations and show that there are no tachyonic modes in this particular limit.[1] Furthermore, we show that it is also possible to keep the nonsupersymmetric vacuum in the weakly coupled region of the field space and very far from the supersymmetric one. In fact, the supersymmetric vacuum will be pushed to infinity in that limit.

The organization of this paper is as follows. In the next section, we introduce the model and discuss its vacuum structure. We observe that the model, in addition to

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[1]Large N supersymmetry breaking in lower dimensions has been studied in [8].

supersymmetric vacua, admits nonsupersymmetric extrema. To discuss the classical stability, in Sec. III, we work out the mass matrix around a maximally symmetric solution. In Sec. IV, we look at a particular limit in the parameter space of the model which simplifies the stability analysis. We derive the corresponding eigenvalues and show how this vacuum can be placed far away from the supersymmetric one. Conclusions and an outlook are given in Sec. V. In the Appendix, we derive the details of the computations of the successive derivatives of the Kähler potential.

II. THE MODEL

Supersymmetric QCD has a dynamical mass scale Λ below which the coupling constant becomes strong so that N_f quark chiral superfields ϕ_i^a and N_f antiquark $\tilde{\phi}^{ai}$ condense into the singlet meson chiral superfields $M_i^j = \phi_i^a \tilde{\phi}_a^j$.[2] These mesons constitute the low energy degrees of freedom (d.o.f.) of the theory. Nonrenormalization theorems forbid any perturbative quantum corrections to the superpotential. However, a superpotential is generated nonperturbatively [1]. For $N_f < N_c$, it reads

$$(N_c - N_f)\left(\frac{\Lambda^{3N_c - N_f}}{\det M}\right)^{\frac{1}{N_c - N_f}}. \tag{1}$$

With no classical superpotential, the model will only have supersymmetric runaway vacua at infinity. To modify the vacuum structure and get some possible metastable vacua, we introduce singlet [under the gauge group $SU(N_c)$] chiral superfields S_{ij} which couple to the low energy meson fields M_{ij} through the following superpotential:

$$W_{cl} = \eta M_{ij} S^{ji} + \gamma S_{ij} S^{ji} + m_{ij} M^{ji}, \tag{2}$$

where η, γ, and m_{ij} are complex parameters, with $i, j, \ldots = 1, 2, \ldots, N_f$. This superpotential should be regarded as an effective superpotential, which, in principle, could be derived from a more fundamental theory. It can arise, for example, from higher dimensional operators induced from supergravity or by integrating out some massive fields at higher energies. Given the above superpotential, one can discuss the low energy effective description and especially the vacuum structure of the model. If the mass parameters of the fields M and S are large compared to Λ, then they can be integrated out before hitting the scale Λ. One is then left with a pure supersymmetric gauge theory which confines at Λ, and because of the mass gap, a nonperturbative superpotential is generated. Upon integrating in the M fields, superpotential (1) is then obtained. On the other hand, if the mass parameters are small compared to Λ, those terms survive and need to be included in the low energy effective theory. However, because of the nonrenormalization theorems, the classical superpotential is perturbatively exact, and one only needs to add the nonperturbative superpotential (1) to the tree-level superpotential to describe the low energy effective theory [3,9]. Hence, taking the nonperturbative superpotential into account, the superpotential for $N_f < N_c$ becomes

$$W = (N_c - N_f)\left(\frac{\Lambda^{3N_c - N_f}}{\det M}\right)^{\frac{1}{N_c - N_f}} + \eta M_{ij} S^{ji} + \gamma S_{ij} S^{ji} + m_{ij} M^{ji}. \tag{3}$$

Let us now discuss the vacuum structure of this model. First we need to compute the F-terms,

$$\frac{\partial W}{\partial M_{ij}} = -\left(\frac{\Lambda^{3N_c - N_f}}{\det M}\right)^{\frac{1}{N_c - N_f}} M_{ji}^{-1} + \eta S_{ji} + m_{ji}, \tag{4}$$

and for S_{ij} we get

$$\frac{\partial W}{\partial S_{ij}} = \eta M_{ji} + 2\gamma S_{ji}. \tag{5}$$

If we set the F-terms to zero, we get the supersymmetric vacuum:

$$-\left(\frac{\Lambda^{3N_c - N_f}}{\det M}\right)^{\frac{1}{N_c - N_f}} M_{ij}^{-1} - \frac{\eta^2}{2\gamma} M_{ij} + m_{ij} = 0, \tag{6}$$

and

$$S_{ij} = -\frac{\eta}{2\gamma} M_{ij}. \tag{7}$$

For a maximally symmetric vacuum solution, we have

$$M_{ij} = \lambda \delta_{ij}, \qquad S_{ij} = s\delta_{ij}. \tag{8}$$

Moreover, if we set $m_{ij} = m\delta_{ij}$, the F-term equation (6) for a supersymmetric solution reduces to

$$\Lambda^{\frac{3N_c - N_f}{N_c - N_f}} \lambda^{\frac{N_c}{N_f - N_c}} + \frac{\eta^2}{2\gamma}\lambda - m = 0, \tag{9}$$

whereas Eq. (7) becomes

$$s = -\frac{\eta}{2\gamma}\lambda. \tag{10}$$

Next, we want to examine whether there are nonsupersymmetric vacua. For the potential, with a canonical Kähler potential for S_{ij}, we have

$$V = K_{ij,kl}^{-1} \frac{\partial W}{\partial M_{ij}} \frac{\partial \bar{W}}{\partial \bar{M}_{kl}} + \left|\frac{\partial W}{\partial S_{ij}}\right|^2, \tag{11}$$

[2] Note that ϕ_i^a and $\tilde{\phi}^{ai}$ transform in the fundamental and antifundamental representations of the $SU(N_c)$ gauge group, respectively.

where $K^{-1}_{ij,kl}$ is the inverse of the metric

$$K_{ij,kl} = \frac{\partial^2 K}{\partial M_{ij} \partial \bar{M}_{kl}}, \tag{12}$$

with K the Kähler potential

$$K = \mathrm{tr}\sqrt{\bar{M}M} = \sqrt{\bar{M}M}_{ii}. \tag{13}$$

This Kähler potential is actually induced from the microscopic theory simply by imposing the D-flatness condition and expressing the canonical Kähler potential in terms of the low energy d.o.f. M_{ij} [3]. Therefore, this form of K is valid only in that region of the moduli space where M_{ij} is large so that the microscopic theory is weakly coupled and the use of its canonical potential is justified. In the present model, we will see that the nonsupersymmetric solution can be located in the weakly coupled region of field space, and hence it makes sense to use (13) as the Kähler potential.

For the minima we derive

$$\begin{aligned}\frac{\partial V}{\partial M_{rs}} = {}& \frac{\partial K^{-1}_{ij,kl}}{\partial M_{rs}} \frac{\partial W}{\partial M_{ij}} \frac{\partial \bar{W}}{\partial \bar{M}_{kl}} \\ & + K^{-1}_{ij,kl}\left[\hat{\Lambda}\left(\frac{1}{N_c - N_f} M^{-1}_{rs} M^{-1}_{ji} + M^{-1}_{jr} M^{-1}_{si}\right)\right] \\ & \times (-\bar{\hat{\Lambda}} \bar{M}^{-1}_{lk} + \bar{\eta}\bar{S}_{lk} + \bar{m}_{lk}) + \eta(\bar{\eta}\bar{M}_{sr} + 2\bar{\gamma}\bar{S}_{sr}) = 0,\end{aligned} \tag{14}$$

and

$$\begin{aligned}\frac{\partial V}{\partial S_{rs}} = {}& \eta K^{-1}_{sr,kl}(-\hat{\Lambda}\bar{M}^{-1}_{kl} + \bar{\eta}\bar{S}_{kl} + \bar{m}_{kl}) \\ & + 2\gamma(\bar{\eta}\bar{M}_{sr} + 2\bar{\gamma}\bar{S}_{sr}) = 0,\end{aligned} \tag{15}$$

where we have defined

$$\hat{\Lambda} = \left(\frac{\Lambda^{3N_c - N_f}}{\det M}\right)^{\frac{1}{N_c - N_f}}. \tag{16}$$

Now, to examine nonsupersymmetric vacua we assume that the F-terms are nonvanishing, and hence we plug (15) into (14) to obtain

$$\begin{aligned}& \left[\frac{\partial K^{-1}_{ij,kl}}{\partial M_{rs}}(-\hat{\Lambda}M^{-1}_{ji} + \eta S_{ji} + m_{ji})\right. \\ & \left. + K^{-1}_{ij,kl}\left(\hat{\Lambda}\frac{M^{-1}_{rs} M^{-1}_{ji}}{N_c - N_f} + \hat{\Lambda} M^{-1}_{jr} M^{-1}_{si} - \frac{\eta^2}{2\gamma}\delta_{jr}\delta_{is}\right)\right] \\ & \times \frac{\partial \bar{W}}{\partial \bar{M}_{kl}} = 0.\end{aligned} \tag{17}$$

As in the case of the supersymmetric vacuum, here we can also look for a maximally symmetric extremum, where M_{ij} and S_{ij} are proportional to the identity matrix as in (8). In the Appendix, we show that with this assumption we have

$$K^{-1}_{ij,kl} = 4|\lambda|\delta_{jk}\delta_{il}, \qquad \frac{\partial K^{-1}_{ij,kl}}{\partial M_{rs}} = \frac{\bar{\lambda}}{|\lambda|}(\delta_{jk}\delta_{ir}\delta_{ls} + \delta_{il}\delta_{kr}\delta_{sj}). \tag{18}$$

Therefore, assuming supersymmetry is broken, Eq. (17) requires that we set the first bracket in that equation to zero. With (18), this implies

$$\left(-\frac{\hat{\Lambda}}{\lambda} + \eta s + m\right) + \frac{2\hat{\Lambda}}{\xi\lambda} - \frac{\eta^2}{\gamma}\lambda = 0, \tag{19}$$

from which we get

$$s = \frac{1}{\eta}\left(\left(\frac{\xi - 2}{\xi}\right)\frac{\hat{\Lambda}}{\lambda} + \frac{\eta^2\lambda}{\gamma} - m\right). \tag{20}$$

Plugging s into (15) we get an equation which determines λ,

$$\frac{\hat{\Lambda}}{\lambda}\left[\frac{2}{\xi} + \left(\frac{2}{\xi} - 1\right)\frac{|\gamma|^2}{|\eta|^2|\lambda|}\right] - \frac{\eta^2}{\gamma}\left(1 + \frac{3|\gamma|^2}{2|\eta|^2|\lambda|}\right)\lambda + \frac{m|\gamma|^2}{|\eta|^2|\lambda|} = 0, \tag{21}$$

where we have defined

$$\xi = \frac{N_c - N_f}{N_c}, \tag{22}$$

so that $0 < \xi < 1$.

III. CLASSICAL STABILITY

In this section we discuss the stability of nonsupersymmetric solutions (20) and (21) by looking at the mass matrix of small fluctuations around them. With the canonical Kähler potential, the bosonic mass matrix reads

$$\begin{pmatrix} W_{ij}\bar{W}^{im} & W^i\bar{W}_{ijm} \\ \bar{W}^i W_{ijm} & \bar{W}_{ij}W^{im} \end{pmatrix}, \tag{23}$$

where the indices on the superpotential W indicate derivatives with respect to the chiral superfields, e.g., $W_i = \frac{\partial W}{\partial \phi^i}$, and so on. Since here we have N_f^2 chiral superfields S_{ij} and N_f^2 chiral superfields M_{ij} of mesons in the low energy limit of SQCD, Eq. (23) is a $2(2N_f^2) \times 2(2N_f^2)$ mass matrix.

To obtain the mass matrix for a generic Kähler potential, however, one has to directly expand the potential to quadratic order. As we have chosen a maximally symmetric vacuum, the potential of small fluctuations splits into two parts. The first part consists of quadratic fluctuations of symmetric (antisymmetric) matrices $\delta M_{ij}\delta M^{ij}$, and the second part contains the quadratic trace terms $\delta M^i_i \delta M^j_j$,

$$\frac{\partial^2 V}{\partial M_{kl}\partial \bar{M}_{rs}}\delta M_{kl}\delta \bar{M}_{rs} = (A\delta_{kr}\delta_{ls} + A'\delta_{kl}\delta_{rs})\delta M_{kl}\delta \bar{M}_{rs} = A\delta M_{kl}\delta \bar{M}^{kl} + N_f^2 A' \frac{\delta M_k^k}{N_f}\frac{\delta \bar{M}_l^l}{N_f}. \quad (24)$$

Let us further decompose the symmetric part of δM_{ij} as follows:

$$\delta M_{ij} = \frac{1}{N_f}\delta M_k{}^k \delta_{ij} + \delta \hat{M}_{ij}, \quad (25)$$

where $\delta M_k{}^k$ is the trace part, and $\delta \hat{M}_{ij}$ is the traceless part. In doing so, Eq. (26) can be decomposed into two independent d.o.f.,

$$A\delta M_{kl}\delta \bar{M}^{kl} + A'\delta M_k{}^k \delta \bar{M}_l^l = A\delta \hat{M}_{kl}\delta \bar{\hat{M}}^{kl} + N_f^2(A' + A/N_f)\frac{\delta M_k{}^k}{N_f}\frac{\delta \bar{M}_l^l}{N_f}. \quad (26)$$

Using the formula for the derivatives of K^{-1}_{ijkl} in the Appendix to compute the second derivative of potentials (11) and (19), we obtain

$$A = \frac{1}{|\lambda|}\left|2(\xi - 1)\frac{\hat{\Lambda}}{\xi\lambda} + \frac{\eta^2}{\gamma}\lambda\right|^2 + |\eta|^2, \quad (27)$$

$$A_t \equiv N_f^2\left(A' + \frac{A}{N_f}\right) = N_f|\lambda|\left|\frac{\eta^2}{\gamma}\right|^2 + N_f|\eta|^2. \quad (28)$$

Similarly, for $\delta S_{kl}\delta \bar{S}_{rs}$ we get

$$\frac{\partial^2 V}{\partial S_{kl}\partial \bar{S}_{rs}}\delta S_{kl}\delta \bar{S}_{rs} = (B\delta_{kr}\delta_{ls})\delta S_{kl}\delta \bar{S}_{rs} = B\delta \hat{S}_{kl}\delta \bar{\hat{S}}^{kl} + B_t\frac{\delta S_k{}^k}{N_f}\frac{\delta \bar{S}_l^l}{N_f}, \quad (29)$$

where

$$B = 4|\lambda||\eta|^2 + 4|\gamma|^2, \quad (30)$$

and

$$B_t = N_f B. \quad (31)$$

Next, consider quadratic fluctuations of $\delta S_{kl}\delta \bar{M}_{rs}$,

$$\frac{\partial^2 V}{\partial S_{kl}\partial \bar{M}_{rs}}\delta S_{kl}\delta \bar{M}_{rs} = (\alpha\delta_{kr}\delta_{ls} + \alpha'\delta_{kl}\delta_{rs})\delta S_{kl}\delta \bar{M}_{rs} = \alpha\delta \hat{S}_{kl}\delta \bar{\hat{M}}^{kl} + N_f^2(\alpha' + \alpha/N_f)\frac{\delta S_k^k}{N_f}\frac{\delta \bar{M}_l^l}{N_f}, \quad (32)$$

where

$$\alpha = 2|\lambda|\bar{\eta}\left(2\left(1 - \frac{1}{\xi}\right)\frac{\hat{\Lambda}}{\lambda^2} + \frac{\eta^2}{\gamma}\right) + 2\eta\bar{\gamma}, \quad (33)$$

$$\alpha_t = N_f^2(\alpha' + \alpha/N_f) = \frac{2\eta N_f}{\gamma}(|\lambda||\eta|^2 + |\gamma|^2) = \frac{\eta}{2\gamma}B_t. \quad (34)$$

Finally, we compute the off-diagonal elements in (23). For $\delta S_{kl}\delta M_{rs}$ we get

$$\frac{\partial^2 V}{\partial S_{kl}\partial M_{rs}}\delta S_{kl}\delta M_{rs} = (e\delta_{kl}\delta_{rs})\delta S_{kl}\delta M_{rs}, \quad (35)$$

where

$$e = 2\left(-\frac{2\hat{\Lambda}}{\xi\lambda^2} + \frac{\eta^2}{\gamma}\right)|\lambda|\eta, \quad (36)$$

with

$$e_t = N_f e. \quad (37)$$

Moreover, the coefficient of $\delta M_{kl}\delta M_{rs}$ reads

$$\frac{\partial^2 V}{\partial M_{kl}\partial M_{rs}}\delta M_{kl}\delta M_{rs} = (C\delta_{kr}\delta_{ls} + C'\delta_{kl}\delta_{rs})\delta M_{kl}\delta M_{rs} = C\delta \hat{M}_{kl}\delta \hat{M}^{kl} + N_f^2(C' + C/N_f)\frac{\delta M_k^k}{N_f}\frac{\delta M_l^l}{N_f},$$

where

$$C = 4|\lambda|\left(\frac{2\bar{\hat{\Lambda}}}{\xi\bar{\lambda}^2} - \frac{\bar{\eta}^2}{\bar{\gamma}}\right)\left(\frac{\hat{\Lambda}}{2\xi\lambda^2} + \frac{\eta^2}{4\gamma}\right)\left(\frac{\bar{\lambda}}{\lambda}\right), \quad (38)$$

and

$$C_t = N_f^2(C' + C/N_f) = 4N_f|\lambda|\left(\frac{2\bar{\hat{\Lambda}}}{\xi\bar{\lambda}^2} - \frac{\bar{\eta}^2}{\bar{\gamma}}\right)\left((2 - \xi)\frac{\hat{\Lambda}}{2\xi^2\lambda^2} + \frac{\eta^2}{4\gamma}\right)\left(\frac{\bar{\lambda}}{\lambda}\right). \quad (39)$$

We are now ready to write down the mass matrix (23). For the traceless part it reads

$$\begin{pmatrix} A & \alpha & C & e \\ \bar{\alpha} & B & e & 0 \\ \bar{C} & \bar{e} & A & \bar{\alpha} \\ \bar{e} & 0 & \alpha & B \end{pmatrix}, \quad (40)$$

where A, B, α, and C are derived in (27), (30), (33), and (38), respectively. There is a similar matrix for the trace part with components A_t, B_t, α_t, and C_t as defined in (28), (31), (34), and (39), respectively.

Let us discuss the trace part, where we can identify the Goldstino. In diagonalizing (40), we obtain the following characteristic equation of the eigenvalues:

$$(|e_t|^2 + |\alpha_t|^2 - A_t B_t + k_t(A_t + B_t) - k_t^2)^2 = |(B_t - k_t)C_t - 2e_t\alpha_t|^2. \tag{41}$$

For the mass matrix of fermions, on the other hand, the off-diagonal 2×2 matrices of (40) are zero. Moreover, from (28), (31), and (34) we deduce

$$A_t B_t = |\alpha_t|^2, \tag{42}$$

and so, using this in (41), we see that for fermions

$$k_t^2 - k_t(A_t + B_t) = 0. \tag{43}$$

Hence, we get a massless mode which we identify as the Goldstino coming from the supersymmetry breaking on solution (21).

IV. LARGE N_c LIMIT

There are certain limits in which Eq. (41) reduces to a quadratic equation and the analysis of the stability simplifies. For instance, we can take either C_t or $B_t C_t - 2e_t\alpha_t$ in (41) to vanish. Together with (21), this constrains the space of parameters of the model. However, in both of these cases we will end up with some negative eigenvalues of (41) resulting in the instability of the solution.

Alternatively, we can go to a limit where both η and γ vanish so that Eq. (21) and the stability analysis greatly simplify. However, a look at the potential shows that to have a finite value for V, we need to take a large N_f limit simultaneously. Thus, we take the following limit:

$$\eta = C_1/\sqrt{N_f}, \qquad \gamma = C_2/N_f, \tag{44}$$

with C_1 and C_2 two finite complex constants (with η^2/γ fixed) and large N_f. Explicitly, first note that from Eqs. (15) and (19) we see that the extremum of the potential satisfies the following equations:

$$-\frac{\hat{\Lambda}}{\lambda} + \eta s + m = -\frac{\bar{\gamma}}{2|\lambda|\bar{\eta}}(\eta\lambda + 2\gamma s) = -\frac{2\hat{\Lambda}}{\xi\lambda} + \frac{\eta^2}{\gamma}\lambda. \tag{45}$$

So assuming that λ and s at the extremum are finite, the last two equations of (45) imply that in the limit of (44) we have

$$-\frac{2\hat{\Lambda}}{\xi\lambda} + \frac{\eta^2}{\gamma}\lambda = -\frac{C_2\lambda}{2N_f|\lambda|}\left(\frac{\eta}{\bar{\eta}}\right) + \mathcal{O}(1/N_f^{3/2}), \tag{46}$$

where we have used (44) to replace for γ on the right-hand side. Now we can use (44) and (45) to see how potential (11) scales at large N_f,

$$\begin{aligned} V &= 4N_f|\lambda|\left|-\frac{\hat{\Lambda}}{\lambda} + \eta s + m\right|^2 + N_f|\eta\lambda + 2\gamma s|^2 \\ &= \left(N_f\left|\frac{\gamma}{\eta}\right|^2\frac{1}{|\lambda|} + N_f\right)|\eta\lambda + 2\gamma s|^2 \\ &= 4|\lambda|^2\left(\left|\frac{C_2}{C_1}\right|^2\frac{1}{|\lambda|} + N_f\right)\left|\frac{\eta}{\gamma}\right|^2\left|-\frac{2\hat{\Lambda}}{\xi\lambda} + \frac{\eta^2}{\gamma}\lambda\right|^2 \\ &= |C_1\lambda|^2 + \mathcal{O}(1/N_f), \end{aligned} \tag{47}$$

where in the last equality we have used (46). Hence, we observe that in the limit (44) the potential, evaluated on the extremum, approaches a finite value.

To discuss the mass matrix in the limit (44), first note that, assuming λ is finite, from (45) we infer

$$\left(-\frac{2\hat{\Lambda}}{\xi\lambda^2} + \frac{\eta^2}{\gamma}\right) \sim 1/N_f. \tag{48}$$

Now, looking at the elements of the trace part of the mass matrix, A_t, B_t, α_t, e_t, and C_t—as derived in Eqs. (28), (31), (34), (37), and (39)— (48) implies that, in the large N_f limit, they scale as

$$N_f, \quad 1, \quad N_f^{1/2}, \quad N_f^{-1/2}, \quad 1, \tag{49}$$

respectively. We can now analyze the quartic characteristic eigenvalue equation (41). Using the explicit expressions for the roots of this equation and keeping only the dominant terms in the large N_f limit, we find that the eigenvalues scale as follows[3]:

$$\mathrm{diag}(N_f, N_f, 1/N_f, -1/N_f). \tag{50}$$

Hence, in the limit $N_f \to \infty$, the first two modes get very heavy and decouple from the spectrum, and we are left with two zero-mode eigenvalues. For the nontrace part, the matrix elements, A, B, α, e, and C, in the large N_f limit scale as

$$1, \quad {N_f}^{-1}, \quad N_f^{-1/2}, \quad {N_f}^{-3/2}, \quad N_f^{-1}, \tag{51}$$

respectively. Analyzing the quartic eigenvalue equation, we find that the leading terms of the eigenvalues scale as follows:

$$\mathrm{diag}(1, 1, 1/N_f, -1/N_f), \tag{52}$$

so in the limit $N_f \to \infty$ we get two extra zero modes. Note that there is a similar expression for the modes coming from the antisymmetric part of the quadratic fluctuations, so, in

[3]We have also checked the leading order terms using *Mathematica*.

sum, we get $2N_f^2$ real massless scalars. Here we get a factor of 2 as matrix (40) is 4×4, and hence the eigenvalues are doubly degenerate. We conclude that in the large N_f limit, there are no negative eigenvalues of the mass matrix of small fluctuations around the nonsupersymmetric solution, and hence no instability.

Now let us discuss what happens to solution (21) in the limit $\gamma/\eta \to 0$ and large N_c. Setting $N_c = aN_f$, with $a > 1$, and since η^2/γ is held fixed and finite, solution (21) in this limit reduces to the following equation for a nonsupersymmetric vacuum:

$$\left(\frac{\Lambda^{3N_c-N_f}}{\lambda^{2N_c-N_f}}\right)^{\frac{1}{N_c-N_f}} = \frac{\eta^2}{2\gamma}\left(\frac{N_c-N_f}{N_c}\right). \tag{53}$$

Note that this equation is the same as (46) in the large N_f limit. Further, let us define

$$\frac{\eta^2}{2\gamma}\left(\frac{N_c-N_f}{N_c}\right) \equiv \frac{1}{\Lambda L^{\frac{2N_c-N_f}{N_c-N_f}}}, \tag{54}$$

where $L > 1$ is a finite number, and on the right-hand side, we have included a factor of Λ for dimensions to match. Equation (53) now implies

$$\lambda = L\Lambda^2. \tag{55}$$

Therefore, we observe that, by taking $L \gg 1$, the nonsupersymmetric solution (55) can be located in the weakly coupled (in terms of the microscopic d.o.f.) region of the field space. As mentioned in Sec. II, this also justifies the use of the Kähler potential (13) induced from the microscopic theory. Further, note that Eq. (45) shows that, in the limit of vanishing η and γ, for any finite and constant s we have a solution. Therefore, s is a modulus, and this coincides exactly with $2N_f^2$ zero modes in the large N_f limit that we obtained in (50) and (52).

Having obtained λ, we can now derive the explicit minimum value of the potential in the large N_f limit. From (47) and (55), we have

$$V = |C_1|^2|\lambda|^2 = L^2|C_1|^2|\Lambda|^4, \tag{56}$$

which gives an energy scale for the nonsupersymmetric small excitations in this vacuum, and hence a scale of supersymmetry breaking:

$$M_s^4 = L^2|C_1|^2|\Lambda|^4. \tag{57}$$

So, if we take

$$L|C_1| \ll 1, \tag{58}$$

then $M_s \ll \Lambda$, and supersymmetry breaking happens at a scale well below Λ which, in turn, justifies the use of meson low energy d.o.f. to describe this region of field space.

In the end, let us examine the supersymmetric vacuum and see how it compares with the nonsupersymmetric one in the limit. The supersymmetric solutions (4) and (5) are

$$-\frac{\hat{\Lambda}}{\lambda} + \eta s + m = 0,$$
$$\eta\lambda + 2\gamma s = 0. \tag{59}$$

The second equation implies $\eta s = -\frac{\eta^2}{2\gamma}\lambda$ and since we have kept η^2/γ fixed, ηs is finite. Plugging ηs into the first equation, we can derive λ. However, since $s = -\frac{\eta}{2\gamma}\lambda$, it diverges in the limit (44). Therefore, for the supersymmetric solution, s is located at infinity. On the other hand, recall that the model also admits nonsupersymmetric solutions with finite s. The moduli space of nonsupersymmetric solutions is thus the whole complex plane with the potential at its constant value (56) approaching zero at the boundary ($s \sim \sqrt{N_f}$; the supersymmetric solution). Therefore, any nonsupersymmetric solution with finite s is very far from the supersymmetric solution, and since the potential along the modulus is flat, it is also stable. This can also be seen from the Coleman bounce action [10]

$$S_B \sim \frac{(\Delta M)^2}{V_{\rm meta}}, \tag{60}$$

where ΔM is the distance between the metastable and supersymmetric vacua and $V_{\rm meta}$ is the metastable potential. In the large N_c limit, $V_{\rm meta}$ is finite, whereas ΔM goes to infinity; hence the probability of quantum tunneling approaches zero.

V. CONCLUSIONS

Dynamical supersymmetry breaking in models with metastable vacua have been extensively studied. These models, in particular, serve as the hidden sector of MSSM-like theories where supersymmetry first breaks and then is mediated to the visible sector. In SQCD, for instance, interesting generic models for $N_c = N_f$ and $N_c < N_f < 3/2N_c$ have been constructed that admit metastable vacua [4–6]. In this paper, we addressed the existence of such vacua for $N_f < N_c$.

We showed that supersymmetric QCD with $N_f < N_c$ coupled to a singlet chiral superfield can have stable nonsupersymmetric vacua. We observed that, apart from a supersymmetric solution, the model also admits nonsupersymmetric solutions. We discussed the mass matrix of small fluctuations around a maximally symmetric solution. We noticed that there are certain limits on the space of parameters where the quartic equation of eigenvalues reduces to a quadratic equation, and thus the question of stability is tractable. However, the appearance of tachyonic modes in such cases led us to examine the potential and consider instead a limit of large N_f with $N_f\gamma^2/\eta^2$ kept fixed. We analyzed the mass matrix of small fluctuations and showed that the spectrum, in this limit, contains no tachyons.

In the large N_f limit, the spectrum showed some extra zero modes. We argued that these zero modes correspond to the pseudomoduli space of flat directions along s. The expectation value of the meson field on the field space turned out to be proportional to a finite number L, which we took to be large for the solution to be in the weakly coupled region of field space. Further, as the supersymmetric solution gets pushed to infinity in the limit, the two solutions become infinitely far apart. The nonsupersymmetric vacuum thus remains stable under small fluctuations.

It is very interesting to further examine the characteristic equation and see if there are any other limits where the spectrum of the mass matrix is free from tachyons. In this model, we encountered a pseudomoduli space of flat directions along s which was the characteristic of the limit taken in the parameter space of the model. It is important to see whether this modulus is lifted when quantum corrections are included.

APPENDIX: COMPUTATION OF SUCCESSIVE DERIVATIVES OF KÄHLER POTENTIAL

Although computing the derivative of the square root of a matrix in terms of the derivative of the matrix itself is not straightforward, in this appendix, we show how to derive such derivatives calculated on the solutions of type $M_{ij} = \lambda\delta_{ij}$. This is done by taking the successive derivatives. In each step, we compute the value of the derivative on the solution and then use it to compute the derivative in the next step. Let us start from the definition of the square root of a matrix,

$$\bar{M}_{il}M_{lj} = \sqrt{\bar{M}M}_{il}\sqrt{\bar{M}M}_{lj}. \tag{A1}$$

Taking the first derivative we deduce

$$M_{sj}\delta_{ir} = \frac{\partial\sqrt{\bar{M}M}_{il}}{\partial\bar{M}_{rs}}\sqrt{\bar{M}M}_{lj} + \sqrt{\bar{M}M}_{il}\frac{\partial\sqrt{\bar{M}M}_{lj}}{\partial\bar{M}_{rs}}. \tag{A2}$$

Multiplying the right by $\sqrt{\bar{M}M}^{-1}$ results in

$$M_{sj}\delta_{ir}\sqrt{\bar{M}M}^{-1}_{jk} = \frac{\partial\sqrt{\bar{M}M}_{ik}}{\partial\bar{M}_{rs}} + \sqrt{\bar{M}M}_{il}\frac{\partial\sqrt{\bar{M}M}_{lj}}{\partial\bar{M}_{rs}}\sqrt{\bar{M}M}^{-1}_{jk}. \tag{A3}$$

Now, summing over i and k,

$$2\frac{\partial\sqrt{\bar{M}M}_{ii}}{\partial\bar{M}_{rs}} = M_{sj}\sqrt{\bar{M}M}^{-1}_{jr} = \bar{M}^{-1}_{sj}\sqrt{\bar{M}M}_{jr}, \tag{A4}$$

and then taking the second derivative gives

$$2\frac{\partial^2\sqrt{\bar{M}M}_{ii}}{\partial M_{pq}\partial\bar{M}_{rs}} = \bar{M}^{-1}_{sj}\frac{\partial\sqrt{\bar{M}M}_{jr}}{\partial M_{pq}}. \tag{A5}$$

Since we need the value of the derivatives on the solution, let us set $M_{ij} = \lambda\delta_{ij}$ and $\sqrt{\bar{M}M}_{ij} = |\lambda|\delta_{ij}$ in (A2) to obtain

$$\frac{\partial\sqrt{\bar{M}M}_{ij}}{\partial\bar{M}_{rs}} = \frac{\lambda}{2|\lambda|}\delta_{ir}\delta_{js}. \tag{A6}$$

Using this in (A5) we get

$$K_{pq,rs} \equiv \frac{\partial^2 K}{\partial M_{pq}\partial\bar{M}_{rs}} = \frac{\partial^2\sqrt{\bar{M}M}_{ii}}{\partial M_{pq}\partial\bar{M}_{rs}} = \frac{\lambda^2}{4|\lambda|^3}\delta_{sp}\delta_{rq} \tag{A7}$$

on the solution.

Taking the second derivative of (A2) we find

$$\begin{aligned}\delta_{sp}\delta_{jq}\delta_{ir} = {}& \frac{\partial^2\sqrt{\bar{M}M}_{il}}{\partial M_{pq}\partial\bar{M}_{rs}}\sqrt{\bar{M}M}_{lj} + \frac{\partial\sqrt{\bar{M}M}_{il}}{\partial M_{pq}}\frac{\partial\sqrt{\bar{M}M}_{lj}}{\partial\bar{M}_{rs}} \\ & + \frac{\partial\sqrt{\bar{M}M}_{il}}{\partial\bar{M}_{rs}}\frac{\partial\sqrt{\bar{M}M}_{lj}}{\partial M_{pq}} + \sqrt{\bar{M}M}_{il}\frac{\partial^2\sqrt{\bar{M}M}_{lj}}{\partial M_{pq}\partial\bar{M}_{rs}},\end{aligned} \tag{A8}$$

which, upon using (A6), reads

$$\frac{\partial^2\sqrt{\bar{M}M}_{ij}}{\partial M_{pq}\partial\bar{M}_{rs}} = \frac{1}{8|\lambda|}(3\delta_{sp}\delta_{jq}\delta_{ir} - \delta_{ip}\delta_{rq}\delta_{js}). \tag{A9}$$

Similarly

$$\frac{\partial^2\sqrt{\bar{M}M}_{ij}}{\partial M_{pq}\partial M_{rs}} = -\frac{\bar{\lambda}^2}{8|\lambda|^3}(\delta_{sp}\delta_{jq}\delta_{ir} + \delta_{ip}\delta_{rq}\delta_{js}). \tag{A10}$$

Next, we take the derivative of (A8) and then use the value of the first and the second derivatives on the solutions (A6), (A9), and (A10) to get

$$\begin{aligned}\frac{\partial K_{pq,rs}}{\partial\bar{M}_{mn}} &= \frac{\partial^3\sqrt{\bar{M}M}_{ii}}{\partial\bar{M}_{mn}\partial M_{pq}\partial\bar{M}_{rs}} \\ &= \frac{-1}{16\bar{\lambda}|\lambda|}(\delta_{sm}\delta_{pn}\delta_{rq} + \delta_{sp}\delta_{mq}\delta_{rn})\end{aligned} \tag{A11}$$

and

$$\begin{aligned}\frac{\partial K_{pq,rs}}{\partial M_{mn}} &= \frac{\partial^3\sqrt{\bar{M}M}_{ii}}{\partial M_{mn}\partial M_{pq}\partial\bar{M}_{rs}} \\ &= \frac{-\bar{\lambda}}{16|\lambda|^3}(\delta_{ps}\delta_{mq}\delta_{rn} + \delta_{qr}\delta_{ms}\delta_{np}).\end{aligned} \tag{A12}$$

By iteration of the above procedure, we obtain the fourth derivatives,

$$\frac{\partial^2 K_{pq,rs}}{\partial M_{ef}\partial \bar{M}_{mn}} = \frac{\partial^4 \sqrt{\bar{M}M}_{ii}}{\partial M_{ef}\partial \bar{M}_{mn}\partial M_{pq}\partial \bar{M}_{rs}}$$
$$= \frac{-1}{32|\lambda|^3}(-\delta_{sm}\delta_{pn}\delta_{eq}\delta_{rf} + \delta_{qm}\delta_{en}\delta_{sp}\delta_{rf} - \delta_{sm}\delta_{en}\delta_{pf}\delta_{rq} + \delta_{fm}\delta_{pn}\delta_{rq}\delta_{se} - \delta_{fp}\delta_{mq}\delta_{rn}\delta_{se} - \delta_{sp}\delta_{qe}\delta_{mf}\delta_{rn}), \quad \text{(A13)}$$

with

$$\frac{\partial^2 K_{pq,rs}}{\partial \bar{M}_{ef}\partial \bar{M}_{mn}} = \frac{\partial^4 \sqrt{\bar{M}M}_{ii}}{\partial \bar{M}_{ef}\partial \bar{M}_{mn}\partial M_{pq}\partial \bar{M}_{rs}}$$
$$= \frac{-1}{32\bar{\lambda}^2|\lambda|}(3\delta_{se}\delta_{fm}\delta_{pn}\delta_{rq} + \delta_{se}\delta_{fp}\delta_{mq}\delta_{rn} + 3\delta_{sm}\delta_{en}\delta_{pf}\delta_{rq} + \delta_{sm}\delta_{pn}\delta_{eq}\delta_{rf} - \delta_{sp}\delta_{eq}\delta_{mf}\delta_{rn} - \delta_{sp}\delta_{qm}\delta_{en}\delta_{rf}). \quad \text{(A14)}$$

Finally, we obtain

$$\frac{\partial K^{-1}_{ij,kl}}{\partial M_{mn}} = -K^{-1}_{ij,pq}\frac{\partial K_{pq,rs}}{\partial M_{mn}}K^{-1}_{rs,kl}, \quad \text{(A15)}$$

where the inverse of the metric on the solution reads

$$K^{-1}_{pq,rs} = 4|\lambda|\delta_{qr}\delta_{ps}. \quad \text{(A16)}$$

Equation (A15), together with (A12), yields

$$\frac{\partial K^{-1}_{ij,kl}}{\partial M_{mn}} = \frac{\bar{\lambda}}{|\lambda|}(\delta_{jk}\delta_{im}\delta_{ln} + \delta_{il}\delta_{km}\delta_{nj}), \quad \text{(A17)}$$

as claimed in the text. Taking the next derivative of (A15), and using (A13), (A14), and (A17), we derive the second derivative of the inverse metric on the solution, which we needed to calculate the mass spectrum.

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