Constraints on fifth forces through perihelion precession of planets

Bing Sun,[1] Zhoujian Cao,[2,*] and Lijing Shao[2]

[1]Department of Astronomy, Beijing Normal University, Beijing 100875, China

[2]Kavli Institute for Astronomy and Astrophysics, Peking University, Beijing 100871, China

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The equivalence principle is important in fundamental physics. The fifth force, as a formalism describing the equivalence principle, may indicate the property of an unknown theory. Dark matter is one of the most mysterious objects currently in the natural sciences. It is interesting to constrain the fifth force of dark matter. We propose a new method to use the perihelion precession of planets to constrain the long-range fifth force of dark matter. Due to the high accuracy of perihelion precession observation and the large difference of matter composition between the Sun and planets, we get one of the strongest constraints on the fifth force of dark matter. In the near future, the BepiColombo mission will be capable of improving the test by another factor of 10.

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I. INTRODUCTION

The equivalence principle (EP) is fundamental to both Newtonian theory and Einsteinian theory [1]. Consequently, the experimental examination of the EP is very important to fundamental physics [2–4]. We have the Eötvös parameter to describe the violation of the EP,

$$\eta^{(A,B)} \equiv \frac{2(a_A - a_B)}{a_A + a_B}, \qquad (1)$$

where a_i ($i = A, B$) denotes the acceleration of two test particles A and B relative to the central attractor. In general, the attractive force arising from the central attractor depends on the composition of the two test particles and the central object when the EP is violated, as does the above Eötvös parameter. We need to distinguish between different Eötvös parameters arising from different objects. For example, (i) the MICROSCOPE satellite [4] considered a Ti-Pt pair with respect to the Earth and obtained $\eta_{\oplus}^{(\mathrm{Ti,Pt})} \lesssim 10^{-14}$; (ii) the Eöt-Wash experiments [2] considered Be-Al and Be-Ti pairs with respect to the Sun, and obtained $\eta_{\odot}^{(\mathrm{Be,Al})}$, $\eta_{\odot}^{(\mathrm{Be,Ti})} \lesssim 10^{-13}$; and (iii) lunar laser ranging [3] considered the Earth-Moon pair with respect to the Sun and obtained $\eta_{\odot}^{(\oplus,\mathbb{C})} \lesssim 10^{-13}$. Using the idea proposed by Stubbs [5] and the above results [2,3], we have $\eta_{\mathrm{DM}}^{(\mathrm{Be,Al})}$, $\eta_{\mathrm{DM}}^{(\mathrm{Be,Ti})}$, $\eta_{\mathrm{DM}}^{(\oplus,\mathbb{C})} \lesssim 10^{-5}$ when considering the Galactic dark matter (DM) as the attractor (see Refs. [2,6] for the framework of the effective field theory).

Equivalently, we can use the concept of fifth force to describe the violation of the EP [7]. Compared to the Eötvös parameter, the concept of the fifth force is more straightforward and as such can be related to a fundamental theory. Based on the fifth force, we can take the advantage of the large difference in the matter composition between two test bodies and better constrain the fifth force of an object in question. This idea has been successfully applied in Ref. [8], where the binary pulsar PSR J1713 + 0747 constrained a neutron star (NS)-white dwarf (WD) pair with respect to the DM, $\eta_{\mathrm{DM}}^{(\mathrm{NS,WD})} \lesssim 0.004$ [9].

Dark matter is one of the most mysterious objects in current natural sciences. It is interesting to study the fifth force behavior of dark matter which will help people to advance their knowledge about dark matter. Due to the great efforts in searching for dark matter particles, we nowadays have stringent constraints on the interaction cross section between ordinary matter and dark matter. Those studies mostly focus on the possible short-range interactions between dark matter and the nucleons. We here investigate another possibility with the *long-range* fifth-force formalism originally proposed by the E. Fischbach *et al.* [10]. Notice that the long-range fifth force we are studying is extremely weak from the point of view of particle physics. We will see that it is even weaker than the gravity interaction; thus, it does not contradict any constraints from dark matter searches. This is a largely unexplored territory; thus, it is interesting to see whether dark matter could have a sizeable long-range interaction with ordinary matter. The strongest constraint for the long-range fifth force of dark matter comes from the lunar laser ranging (LLR) experiment. The NS-WD binary PSR J1713 + 0747 also gives an interesting constraint using the large difference of the composition of matter between NS and WD

*Corresponding author.
zjcao@amt.ac.cn

[8,9,11]. Here, we propose a new method to use the perihelion precession of planets to constrain the fifth force of dark matter. Due to both the large difference of the composition of matter between the Sun and the planets, and the high observational accuracy of perihelion precession, we can get a very good constraint of the fifth force of the dark matter in the Galaxy.

The most well-known system of celestial mechanics is the Sun-Mercury system. The famous "43 arcseconds" problem hastened the birth of general relativity. Later, the observation about the Mercury perihelion precession became more and more accurate [12]. The current most accurate detection was done by the MESSENGER mission [13]. In the near future, much more accurate detection will be achieved by the BepiColombo mission [12], which was launched last year. These missions directly detect the relative distance and velocity between Mercury and the Earth. Combining these data with other related information, people can construct accurate ephemerides for Mercury. Based on the accurate ephemerides, it is straightforward to get the perihelion advance of Mercury. Representative ephemerides for Mercury include the JPL DE series [14], the EPM series [15], and the INPOP series [16]. Take the EPM2004 ephemerides as an example; the estimated accuracy for the perihelion advance of Mercury is about 10^{-3} as/cy [17]. EPM2004 is rather old; more recent ephemerides give much more accurate perihelion advance of Mercury.

Although our knowledge is limited about the perihelion precession observation for planets other than Mercury, we find that Jupiter may result in a better constraint than Mercury. This is because when one converts the constraint of the extra perihelion precession to the Eötvös parameter, the planet orbit information plays an important role. We will calculate the detailed conversion relation and explain such a dependence in the next section. There an analysis of the Eötvös parameter based on the perihelion precession observation will also be presented. After that, we relate the Eötvös parameter constraint to the fifth force in Sec. III. At last, some related discussions and the summary are given in Sec. IV. To complement the main text, some necessary calculation details are included in the Appendix.

II. EFFECTS OF THE FIFTH FORCE ON PERIHELION ADVANCE OF THE MERCURY

The fifth force results in a relative acceleration of the Mercury with respect to the Sun [8],

$$\vec{a}_{\eta_{\rm DM}} = \eta_{\rm DM}\vec{a}_{\rm DM}, \tag{2}$$

where $\vec{a}_{\rm DM}$ is the gravitational acceleration acted on the Mercury-Sun binary system by DM in the Galaxy.

The acceleration in Eq. (2) generates an additional perihelion precession. This kind of a physical picture has been investigated before by other authors including Schäfer [18] and Freire *et al.* [19]. These authors were concerned about the orbital eccentricity variation. Differently, our concern is the precession of the perihelion. The variation of the longitude of the perihelion can be expressed as (see the detailed calculation in the Appendix),

$$\dot{\varpi}_5 = -\frac{3\eta_{\rm DM}a_{\rm DM}\pi a^2}{GMP}\left(\frac{\sqrt{1-e^2}}{e}F_1 + \frac{e}{\sqrt{1-e^2}}F_2\right), \tag{3}$$

$$F_1 = \cos(\Phi-\Omega)\cos\omega\sin\Theta + \cos\Theta\sin\iota\sin\omega + \sin\omega\cos\iota\sin\Theta\sin(\Phi-\Omega) \tag{4}$$

$$F_2 = \tan\frac{\iota}{2}\sin\omega[\cos\iota\cos\Theta - \sin\iota\sin\Theta\sin(\Phi-\Omega)]. \tag{5}$$

Here, G is the gravitational constant, M is the total mass of the Sun and the planet, and P is the orbital period. Above we have adopted the traditional notation for the planet orbit, where Ω is the angle between the x direction and the ascending node, ω is the angle between the perihelion and the ascending node of the ecliptic plane, a is the semimajor axis, e is the orbit eccentricity, and ι is the inclination of the orbit. In addition, Θ is the angle between the Galactic center and the spin axis of the Sun, Φ is the angle between the x direction and the projected direction of the Galactic center to the x-y plane. The orbit layout is illustrated in Fig. 1. Specifically, for the Galaxy center, we have $\Theta = 117.1°$ and $\Phi = 192.9°$.

The perihelion precession of the planets is detectable. Until now, the detected perihelion precession of planets can be explained without the fifth force contribution (3). So we can constrain $\eta_{\rm DM}$ through Eq. (3) based on the observational precision. Note that Eq. (3) can be viewed as

$$\dot{\varpi}_5 = \frac{\eta_{\rm DM}}{\Xi}, \tag{6}$$

where the factor Ξ is determined completely by the planet's orbit information and the dark matter distribution. Based on the observational precision $\epsilon_{\dot{\varpi}}$, we can roughly constrain the Eötvös parameter to

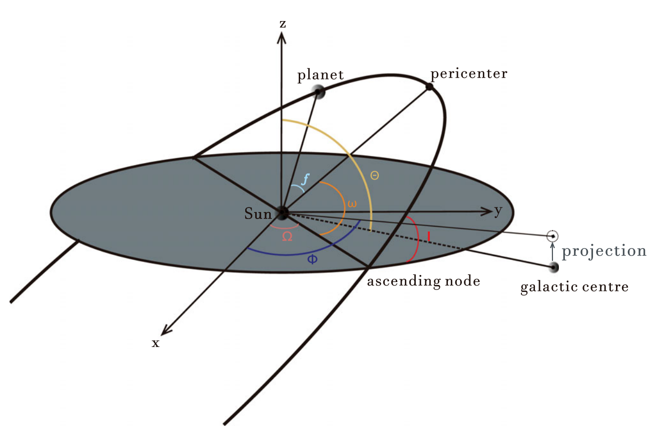

FIG. 1. The layout of the planet orbit with respect to the Galaxy center.

$$\eta_{\mathrm{DM}} < \Xi \cdot \epsilon_{\dot{\varpi}}. \tag{7}$$

In order to determine the factor Ξ, we need to investigate the dark matter distribution in our Galaxy. We assume for the dark matter halo of our Galaxy,

$$\rho = \begin{cases} \frac{\rho_{\mathrm{sp}}(r)\rho_{\mathrm{in}}(r)}{\rho_{\mathrm{sp}}(r)+\rho_{\mathrm{in}}(r)}, & r_0 \le r < R_{\mathrm{sp}} \\ \rho_{\mathrm{GNFW}}(r), & r \ge R_{\mathrm{sp}} \end{cases} \tag{8}$$

$$\rho_{\mathrm{GNFW}}(r) = \frac{\rho_0}{(r/R_s)^{\gamma}(1+r/R_s)^{3-\gamma}}. \tag{9}$$

The inner part of the halo is called the "spike". $\rho_{\mathrm{sp}} = \alpha r^{-\gamma_{\mathrm{sp}}}$ is the distribution of the spike and R_{sp} is its radius; $\gamma_{\mathrm{sp}} = \frac{9-2\gamma}{4-\gamma}$ [20]. $\rho_{\mathrm{in}} = \beta r^{-\gamma_{\mathrm{in}}}$ has taken into account the DM particles' annihilation cross section. γ_{in} depends on the annihilation mechanism: for s-wave annihilation $\gamma_{\mathrm{in}} \simeq 0.5$, and for p-wave annihilation $\gamma_{\mathrm{in}} \simeq 0.34$ [21]. The outer part is the generalized Navarro-Frenk-White (GNFW) profile. Specifically, for our Galaxy, γ ranging from 1.0 to 1.4 [8] and $R_s = 20$ kpc. Moreover, the parameter $\rho_0 = (2/7)^{\gamma} \times 3.2 \times 10^{64}\ \mathrm{GeV/kpc^3}$, which is determined through the condition that $\rho_{\mathrm{GNFW}}(r \simeq 8\ \mathrm{kpc}) = 1.2 \times 10^{64}\ \mathrm{GeV/kpc^3}$.

Integrating the density from $r_0 = 10^{-5}$ kpc [21] to $R = 8$ kpc, where our Solar System is located, we obtain the total DM mass,

$$m_{\gamma} = \int_{r_0}^{R} 4\pi r^2 \rho(r) \mathrm{d}r \tag{10}$$

$$\in [8.4 \times 10^{40}\ \mathrm{kg}, 1.0 \times 10^{41}\ \mathrm{kg}], \tag{11}$$

for $\gamma \in [1.0, 1.4]$. We find that the spike part contributes little to the total DM mass. So the parameter R_{sp} is negligible in the current discussion [8]. Consequently, the gravitational acceleration of DM can be estimated as

$$a_{\mathrm{DM}} = G\frac{m_{\gamma}}{R^2} \in [9.2 \times 10^{-11}\ \mathrm{m/s^2}, 1.1 \times 10^{-10}\ \mathrm{m/s^2}]. \tag{12}$$

In the following analysis, we take $a_{\mathrm{DM}} \approx 10^{-10}\ \mathrm{m/s^2}$.

Based on the above dark matter distribution information and planet orbit information, we can determine the precession factor Ξ. Besides Mercury, we have also investigated several other planets, including Mars, Jupiter, and Saturn. We list the corresponding Ξ's in Table I.

Currently, the observational accuracy of the perihelion precession of Mercury is 10^{-3} as/century [13,17,22]. In the near future, the European-Japanese BepiColombo mission will improve it for Mercury to about 10^{-4} as/century [12,23]. Currently, the observational accuracy of the perihelion precession of Saturn is 10^{-3} as/century [24]. If a more dedicated analysis is paid to construct an ephemerides, a more accurate perihelion precession will be obtained [25]. In the current paper, we are only concerned with the precession directly deduced from experiments. Based on the current observational accuracy, the Eötvös parameter can be constrained to $\eta_{\mathrm{DM}}^{(\mathrm{Sun,Mercury})} < 2.71 \times 10^{-4}$. In the near future, the BepiColombo mission will improve such a constraint to $\eta_{\mathrm{DM}}^{(\mathrm{Sun,Mercury})} \lesssim 3 \times 10^{-5}$. A comparable accuracy to Mercury for Mars and Saturn will make the constraint 1 order of magnitude better. For Jupiter, it will make the constraint 2 orders of magnitude better. But we are not sure about the observational accuracy of the perihelion precession for Mars, Jupiter, and Saturn. In the following analysis for the fifth force constraint, we only discuss the result from the Mercury observation.

TABLE I. Perihelion precession factor Ξ of Mercury, Mars, Jupiter, and Saturn. Here the unit for Ξ is $(\mathrm{as/centrury})^{-1}$.

Planet	Mercury	Mars	Jupiter	Saturn
Ξ	0.271	0.038	0.0081	0.031

III. THE FIFTH FORCE OF THE GALACTIC DM

We assume that the fifth force between DM and ordinary matter can be described by a Yukawa potential [2,10],

$$V(r) = \mp \frac{g^2}{4\pi}\frac{q q_{\mathrm{DM}}}{r} e^{-r/\xi}, \tag{13}$$

where g is the coupling constant, q_{DM} and q are the dimensionless charges of DM and ordinary matter, respectively, and ξ is the range of effective interaction, which is related to the mass of the intermediate particle through $\xi = \hbar/(mc)$. The $\mp$ sign corresponds to the scalar $(-)$ and vector $(+)$ interactions, respectively. For an electrically neutral body consisting of atoms, the charge q is parametrized as [2]

$$q = Z\cos\psi + N\sin\psi, \tag{14}$$

where Z is the proton number, N is the neutron number, $\tan\psi \equiv q_n/(q_p + q_e)$, and q_p, q_n, q_e are the fifth force charges carried by a proton, a neutron, and an electron, respectively. For a specific mixing, the value of ψ can be derived; for example, in the $B - L$ scenario, $\psi = 90°$.

The Yukawa potential (13) gives rise to a relative acceleration of two test bodies,

$$\Delta a = \mp \frac{g^2}{4\pi} q_{\mathrm{DM}}\left(\frac{q_A}{m_A} - \frac{q_B}{m_B}\right)\left(\frac{1}{r^2} + \frac{1}{r\xi}\right) e^{-r/\xi}, \tag{15}$$

where $q_{A,B}$ are the fifth force charges of these two bodies. Notice that in the above equation if we had replaced the fifth-force charge $q_{\rm DM}$ with, say, $q_\odot$, the abnormal acceleration would be tightly constrained by equivalence-principle experiments (e.g., by the Eöt-Wash group and lunar laser ranging). Therefore, the fifth-force charges for ordinary matter are already constrained to be extremely small. In contrast, the fifth-force charges for dark matter are not so well bounded. In principle, dark matter can possess large fifth-force charges yet to be unnoticed. This is exactly why it is interesting, and our study in this work is motivating. Due to this relative acceleration, the EP will be violated with an Eötvös parameter [2],

$$\eta_{\rm DM}^{(A,B)} \approx \frac{\Delta a}{a_{\rm DM}} = \mp \frac{g^2}{4\pi G u^2}\left(\frac{q}{\mu}\right)_{\rm DM}\left[\left(\frac{q}{\mu}\right)_A - \left(\frac{q}{\mu}\right)_B\right]\left(1+\frac{r}{\xi}\right)e^{-r/\xi}, \tag{16}$$

where $a_{\rm DM}$ is the acceleration resulting from the gravitational force of the DM,

$$a_{\rm DM} = \frac{G m_{\rm DM}}{r^2}, \tag{17}$$

and we have introduced an atomic mass unit u to express the mass $m = u\mu$. Based on the long-range interaction approximation to the fifth force $\xi \to \infty$ [8], the Eötvös parameter becomes

$$\eta_{\rm DM}^{(A,B)} \simeq \mp \frac{g^2}{4\pi G u^2}\left(\frac{q}{\mu}\right)_{\rm DM}\left[\left(\frac{q}{\mu}\right)_A - \left(\frac{q}{\mu}\right)_B\right]. \tag{18}$$

Using this expression and replacing the source of the DM by some ordinary objects such as the Earth, the Sun, or various man-made objects, the EP violation test can be formulated [2].

If we separate the acceleration of the ordinary material results from the DM into the gravitational part $a_{\rm DM}$ and the fifth force part $a_{\eta_{\rm DM}}$, as $a_{\rm tol} = a_{\rm DM} + a_{\eta_{\rm DM}}$, the Eötvös parameter is related to the acceleration ratio through [26],

$$\frac{a_{\eta_{\rm DM}}}{a_{\rm tol}} = \frac{\eta_{\rm DM}^{(A,B)}\cos\psi}{\sin\psi[\Delta(N/\mu)] + \cos\psi[\Delta(Z/\mu)]}, \tag{19}$$

where $\Delta(\cdot)$ means the difference between bodies A and B. The acceleration ratio is determined up to an unknown ψ once the magnitude of $\eta_{\rm DM}$ and the compositions of matter in the experiment are given.

We list the involved matter composition for kinds of objects in Table II. Based on the current constraint $\eta_{\rm DM}^{(\rm Sun,Mercury)} < 2.71 \times 10^{-4}$ and a future expected constraint $\eta_{\rm DM}^{(\rm Sun,Mercury)} \lesssim 3 \times 10^{-5}$, we can determine the constraint of the fifth force as shown in Fig. 2 for neutral hydrogen. In the figure, the regions above the curves represent the excluded parameter space. For a comparison, we reproduce the result for the NS-WD binary PSR J1713 + 0747 and that for LLR [8]. Although the observational constraint of PSR J1713 + 0747 on the Eötvös parameter is not as tight $\eta_{\rm DM}^{(\rm NS,WD)} \lesssim 0.004$, the large difference of the matter composition makes the resulting fifth force constraint comparable to other experiments. Regarding the LLR, although the observation accuracy makes the constraint to the Eötvös parameter very tight $\eta_{\rm DM}^{(\oplus,☾)} \lesssim 10^{-5}$, the similar matter composition for the Moon and the Earth makes the constraint to the fifth force not quite outstanding among the experiments. Interestingly, our new perihelion precession method takes both the advantage of a high observation accuracy and a mediate matter composition difference. As shown in Fig. 2, the constraint resulting from the current observation is already similar to the result of LLR. In the near future, one more order of magnitude improvement will be achieved.

TABLE II. The ratios of the proton number and neutron number to the (dimensionless) mass for related objects in the current paper [6,8]. Here, the solid planets mean planets like the Earth, Mercury, Mars, and others.

	Z/μ	N/μ
NS	0	1.19
WD	0.5	0.5
Solid planets	0.49	0.51
Sun	0.86	0.14
Moon	0.502	0.498

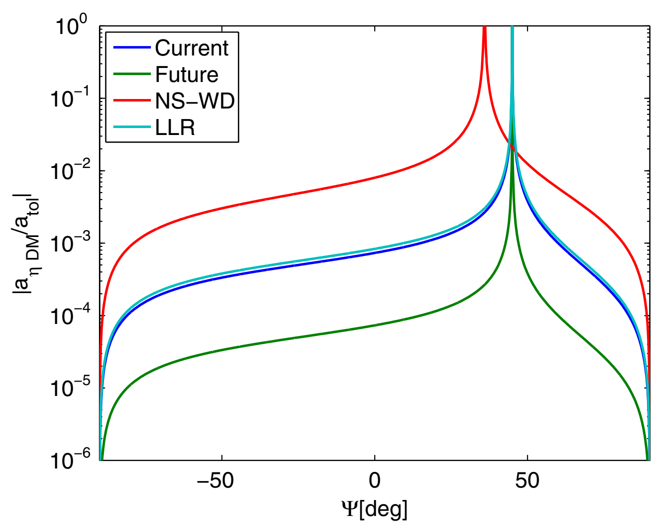

FIG. 2. The fifth force constraint for neutral hydrogen from the current perihelion precession measurement and an expected near-future measurement based on the BepiColombo mission. For comparison, the constraints from the variation of the orbital eccentricity of the NS-WD binary PSR J1713 + 0747 [8] and that from the LLR measurement [3,27] are also included.

IV. DISCUSSION AND SUMMARY

The equivalence principle (EP) is important to both gravitational and high energy physics theories. Alternatively, the equivalence principle can be expressed as a fifth force. Since the equivalence principle, or the fifth force, depends on the difference in the matter composition, it is useful to investigate different kinds of matter. Due to the mysteries of dark matter and the possibility that dark matter might possess unnoticed large fifth-force charges, it is quite interesting to investigate the equivalence principle related to the dark matter from experiments and observations.

Regarding the dark matter in our Galaxy, the most stringent constraint comes from the LLR detection. The authors of Ref. [8] took the advantage of the large matter composition difference between a NS and a WD, and obtained a compelling constraint.

In this paper, we propose a new method to use the perihelion precession observation of the planets to constrain the fifth force acted by the dark matter in our Galaxy. We interestingly find that the Eötvös parameter is proportional to an extra perihelion precession rate. The proportional coefficient depends on, and only on, the gravitational acceleration in the Solar System acted by dark matter and the orbit information of the specific planet. Such a dependence is shown in Eq. (3). Based on this relation, the Eötvös parameter can be constrained by the observational accuracy of the perihelion precession [see Eq. (7)]. For a given observational accuracy of the perihelion precession, different planets result in different constraints on the Eötvös parameter. Due to the much smaller factor Ξ, Jupiter would give 2 orders of magnitude stronger constraint than Mercury if comparable accurate precession can be detected for Jupiter.

Thanks to the high observation accuracy of the perihelion precession, the current constraint on the Eötvös parameter has achieved $\eta_{\rm DM}^{(\rm Sun,Mercury)} < 2.71 \times 10^{-4}$. Thanks to the big difference of the matter composition between planets and the Sun, the constraint on the Eötvös parameter will result in a good constraint on the fifth force. Besides our results, currently the strongest constraint on the fifth force of the dark matter comes from the LLR. Current observational accuracy of the perihelion precession for the Mercury results in a similar constraint. After the BepiColombo mission, 1 more order of magnitude improvement in the constraint is expected in the near future. It is worth mentioning that it is amazing to see that an extremely weak fifth force from the viewpoint of particle physics, even weaker than gravity, can be constrained with celestial dynamics. This kind of study complements the searches for dark matter particles from, say, the Large Hadron Collider and underground laboratories.

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APPENDIX: THE EFFECT OF THE FIFTH FORCE ON THE PLANET PERIHELION PRECESSION

We consider the fifth force as a perturbation to the binary motion. Then the perturbative equations of the binary orbit elements are (see p. 158 in Ref. [28])

$$\frac{da}{dt} = 2\sqrt{\frac{p^3}{\mu}}(1-e^2)^{-2}[-\sin f F_e + (e+\cos f)F_m] \quad \text{(A1)}$$

$$\frac{d\iota}{dt} = \sqrt{\frac{p}{\mu}}\frac{\cos(\omega+f)}{1+e\cos f}F_l \quad \text{(A2)}$$

$$\sin\iota\frac{d\Omega}{dt} = \sqrt{\frac{p}{\mu}}\frac{\sin(\omega+f)}{1+e\cos f}F_l \quad \text{(A3)}$$

$$\frac{de}{dt} = \sqrt{\frac{p}{\mu}}\frac{1}{1+e\cos f}[(1+\cos^2 f + 2e\cos f)F_m - \sin f(e+3\cos f+2\cos^2 f)F_e] \quad \text{(A4)}$$

$$\frac{d\omega}{dt} = \sqrt{\frac{p}{\mu}}\frac{1}{e(1+e\cos f)}\left[\frac{1}{2}(\cos 2f - 2e\cos f - 3)F_e + \sin f\cos f F_m - e\cot\iota\sin(\omega+f)F_l\right] \quad \text{(A5)}$$

$$\frac{dM}{dt} = n - \frac{1-e^2}{nae(1+e\cos f)}[(2e\sin f + e\cos^2 f - e\sin f\cos^2 f + 2\cos f - \sin f\cos f)F_e + (-e\cos^3 f + 2e\cos f - e\sin f\cos f - 2\sin f - \cos^2 f)F_m] \quad \text{(A6)}$$

$$\frac{df}{dt} = \sqrt{\frac{\mu}{p^3}}(1+e\cos f)^2 + \sqrt{\frac{p}{\mu}}\frac{1}{e(1+e\cos f)}[(2\sin f + \cos^2 f + e\cos^3 f + e\sin f\cos f)F_e + (\cos f\sin f + e\cos^2 f\sin f - 2\cos f - e\cos^2 f)F_m], \quad \text{(A7)}$$

where $\mu = GM_{\rm tot}$, $p = a(1-e^2)$, $n = \sqrt{\mu/a^3}$, $F_e = \vec{F}\cdot\hat{e}$, $F_m = \vec{F}\cdot\hat{m}$, $F_l = \vec{F}\cdot\hat{l}$ with $M_{\rm tot}$ the total mass of the binary, $\hat{e}$ the unit vector from the center of mass towards the perihelion, $\hat{l}$ the unit vector pointing along the orbital angular momentum, and $\hat{m} = \hat{l}\times\hat{e}$. $\vec{F}$ is the perturbed force. $(a, \iota, \Omega, e, \omega, M, f)$ are the binary orbit elements, respectively, semimajor axis, inclination angle, ascending angle, periastron angle, mean anomaly, and true anomaly.

In order to investigate the secular change of the orbital elements, we transform the derivatives with respect to t to the ones with respect to the true anomaly f,

$$\frac{da}{df}=\frac{2p^3}{\mu}(1-e^2)^{-2}\left[-\frac{\sin f}{(1+e\cos f)^2}F_e+\frac{(e+\cos f)}{(1+e\cos f)^2}F_m\right] \tag{A8}$$

$$\frac{d\iota}{df}=\frac{p^2}{\mu}\frac{\cos(\omega+f)}{(1+e\cos f)^3}F_l \tag{A9}$$

$$\sin\iota\frac{d\Omega}{df}=\frac{p^2}{\mu}\frac{\sin(\omega+f)}{(1+e\cos f)^3}F_l \tag{A10}$$

$$\frac{de}{df}=\frac{p^2}{\mu}\frac{1}{(1+e\cos f)^3}[(1+\cos^2 f+2e\cos f)F_m-\sin f(e+3\cos f+2\cos^2 f)F_e] \tag{A11}$$

$$\frac{d\omega}{df}=\frac{p^2}{\mu}\frac{1}{e(1+e\cos f)^3}\left[\frac{1}{2}(\cos 2f-2e\cos f-3)F_e+\sin f\cos fY-e\cot\iota\sin(\omega+f)F_l\right] \tag{A12}$$

$$\frac{dM}{df}=-\frac{p^2}{\mu}\frac{\sqrt{1-e^2}}{e(1+e\cos f)^3}[(2e\sin f+e\cos^2 f-e\sin f\cos^2 f+2\cos f-\sin f\cos f)F_e+(-e\cos^3 f+2e\cos f-e\sin f\cos f-2\sin f-\cos^2 f)F_m]. \tag{A13}$$

For the Sun-planet binary systems with respect to the dark matter in the Galaxy, the fifth force can be approximated as a constant force. Using this fact, and integrating the above equations with respect to f over the range $(0,2\pi)$, we obtain the change for a period of the binary orbit. Combining this perturbation from the fifth force and the general relativistic effect up to the first post-Newtonian order, the secular change of the binary elements can be expressed as

$$\left(\frac{da}{dt}\right)_{\rm sec}=0 \tag{A14}$$

$$\left(\frac{d\iota}{dt}\right)_{\rm sec}=-\frac{3e}{2an\sqrt{1-e^2}}\cos\omega F_l \tag{A15}$$

$$\left(\frac{d\Omega}{dt}\right)_{\rm sec}=-\frac{3e}{2an\sqrt{1-e^2}}\frac{\sin\omega}{\sin\iota}F_l \tag{A16}$$

$$\left(\frac{de}{dt}\right)_{\rm sec}=\frac{3}{2an}\sqrt{1-e^2}F_m \tag{A17}$$

$$\left(\frac{d\omega}{dt}\right)_{\rm sec}=\frac{3\mu n}{c^2a(1-e^2)}-\frac{3}{2an}\times\left(\frac{\sqrt{1-e^2}}{e}F_e-\frac{e}{\sqrt{1-e^2}}\cot\iota\sin\omega F_l\right) \tag{A18}$$

$$\left(\frac{dM}{dt}\right)_{\rm sec}=-\frac{n^3a^2}{c^2e^2\sqrt{1-e^2}}\times\left[5e^2+(6-7\eta)\left(1-\sqrt{1-e^2}\right)\right]+\frac{1}{2na}\times\left\{\frac{2}{e^3}[-1+3e^2+e^4+(1-e^2)^{5/2}]F_e+(5-2e^2)F_m\right\} \tag{A19}$$

$$\left(\frac{d\varpi}{dt}\right)_{\rm sec}=\frac{3\mu n}{c^2a(1-e^2)}-\frac{3}{2an}\times\left(\frac{\sqrt{1-e^2}}{e}F_e+\frac{e}{\sqrt{1-e^2}}\tan\frac{\iota}{2}\sin\omega F_l\right), \tag{A20}$$

where $\varpi=\omega+\Omega$. Equivalently, we can use vector notation $\vec{l}\equiv\sqrt{1-e^2}\hat{l}$, $\vec{e}\equiv e\hat{e}$ to denote the above relations,

$$(d\vec{e}/dt)_{\rm sec}=\frac{3}{2na}\vec{F}\times\vec{l}+\frac{3\mu n}{c^2a(1-e^2)}\vec{e}_z\times\vec{e}, \tag{A21}$$

$$(d\vec{l}/dt)_{\rm sec}=\frac{3}{2na}\vec{F}\times\vec{e}. \tag{A22}$$

These two equations correspond to the second and the third expressions of Eq. (3) in [18].

Converting to the variables involved in the main text, Eq. (A20) reduces to Eq. (3).

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