Generalized disformal coupling leads to spontaneous tensorization

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We show that gravity theories involving disformally transformed metrics in their matter coupling lead to spontaneous growth of various fields in a similar fashion to the spontaneous scalarization scenario in scalar-tensor theories. Scalar-dependent disformal transformations have been investigated in this context, and our focus is understanding the transformations that depend on more general fields. We show that vector-dependent disformal couplings can be obtained in various different ways, each leading to spontaneous vectorization as indicated by the instabilities in linearized equations of motion. However, we also show that spontaneous growth is not evident beyond vectors. For example, we could not identify a spontaneous growth mechanism for a spinor field through disformal transformations, even though there is a known example for conformal transformations. This invites further work on the fundamental differences between the two types of metric transformations. We argue that our results are relevant for observations in strong gravity such as gravitational wave detections due to their promise of large deviations from general relativity in this regime.

I. INTRODUCTION

Possible modifications to general relativity (GR) have been a topic of interest for many decades, but until recently, ideas in this line could only be tested in the weak field regime, where GR has been confirmed in all attempts [1]. Gravitational wave (GW) observations are changing this picture, and dynamical strong field gravity can now be directly investigated [2–4]. Despite these advances, the precision of GW detections is limited, which has led to an increased interest in modifications of GR that provide large deviations in strongly gravitating systems [5]. Spontaneous scalarization in scalar-tensor theories where scalar fields grow near neutron stars to provide nontrivial solutions provides exactly this type of modification [6]. This growth occurs due to a specific conformal transformation of the metric in the matter coupling. In this study, we investigate theories with disformal couplings that depend on fields beyond scalars (such as vectors) and show that in many cases we can observe the spontaneous growth of the field.

Spontaneous scalarization contains a fundamental scalar degree of freedom (d.o.f.) that governs gravity in addition to the metric tensor—that is, it is a scalar-tensor theory. Any solution in GR is also a solution in the spontaneous scalarization scenario, and it corresponds to a vanishing scalar field. However, such solutions are unstable in the presence of neutron stars [6]. Arbitrarily small scalar field perturbations go through exponential growth, and the eventual stable solution is a neutron star surrounded by a scalar cloud. The amplitude of the scalar dies off away from the star, hence known weak field tests of gravity are satisfied. More strikingly, the value of the scalar field is large in the vicinity of the neutron star, which leads to order-of-unity deviations from GR, making spontaneous scalarization a prime target for strong gravity observations.

We will explain the basic mechanism of spontaneous scalarization and its generalizations in the following section, but the central idea is a tachyonic instability. In the original theory of Damour and Esposito-Farèse (DEF) [6], the matter fields couple to a metric that is conformally scaled by a function of the scalar (in the so-called Einstein frame). At the level of the scalar equation of motion (EOM), this leads to an imaginary effective mass in the presence of matter. This is the famous tachyon, and it grows exponentially in time instead of oscillating. The growth is quenched by nonlinear terms, and the end point is a stable scalarized neutron star.

The essence of spontaneous scalarization is in an instability that is eventually suppressed at large field values. It has been shown that a similar mechanism exists in many other theories as well [7,8]. One idea to generalize spontaneous scalarization utilizes the fact that the scalar nature of the spontaneously growing field is not crucial. One can have a spontaneously growing vector field as well, as long as there is a conformal scaling of the metric that is a function of the vector field, and the conformal function has a similar form to that of the DEF theory [7]. The spontaneous growth idea applied to any field in this manner is named "spontaneous tensorization" [8].

Another place where spontaneous growth appears is a theory where matter fields couple to a disformally transformed metric rather than a conformally scaled one, whose

technical details we will explain in the next section. Minamitsuji and Silva demonstrated that such a theory contains an instability that causes spontaneous growth, and they also numerically constructed explicit scalarized star solutions, but they did not consider spontaneous growth for other types of fields [9].

Our main task is combining the two aforementioned approaches that generalize spontaneous scalarization. Namely, we will study theories of gravity where disformal transformations play a role, but these transformations are based on fields other than scalars. Such gravity theories have been in the literature, as we will discuss in more detail, but the fact that they give rise to spontaneous tensorization has been overlooked to the best of our knowledge. Spontaneous growth generically leads to order-of-unity deviations from GR, hence identification of its existence in any theory of gravity is especially important, since it dramatically increases the chances of studying the theory using GWs or other means of strong field observations that are becoming more commonly available. Our work demonstrates that spontaneous growth is not merely a scenario specific to a single theory, but it is a ubiquitous mechanism that exists in a wide variety of gravity theories. On the other hand, we will show that not every theory of spontaneous growth with a conformal transformation can be automatically turned to one with a disformal transformation, and there are limits to known mechanisms that provide spontaneous growth.

In Sec. II, we give a basic explanation of spontaneous scalarization and its generalizations to both disformally transformed metrics and nonscalar fields, basically summarizing the literature. In Sec. III, we present three different forms of disformal transformations that can lead to spontaneous growth of vector fields. In Sec. IV, we investigate the spontaneous growth of spinor and rank-2 tensor fields through disformal couplings, and we see that for various reasons, results from vectors cannot be extended to all fields. In Sec. V, we summarize our results and their limitations, discuss other related theories such as disformal transformations beyond matter as in extended Gauss-Bonnet gravity, and comment on connections to observations. We employ geometric units $G = c = 1$ throughout the paper.

II. SPONTANEOUS SCALARIZATION THROUGH CONFORMAL AND DISFORMAL COUPLINGS

The first example of spontaneous growth in the gravity literature was devised by DEF in scalar-tensor theories as in the action

$$\frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - \overbrace{2g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi}^{T_\phi} - \overbrace{2m_\phi^2\phi^2}^{V_\phi} \right]$$
$$+ S_m[f_m, \tilde{g}_{\mu\nu}], \tag{1}$$

where

$$\tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu}, \tag{2}$$

and f_m represents any matter d.o.f. [6]. If the conformal coupling is of the form $A(\phi) = 1 + \beta\phi^2/2 + \cdots$, such as the original choice $A(\phi) = e^{\beta\phi^2/2}$, $\phi = 0$ is a solution that corresponds to GR, but it is an unstable one in the presence of matter. When β is negative and of the order of unity, neutron stars spontaneously grow scalar clouds around them that typically lead to large deviations from GR. Scalarization weakens away from the star, guaranteeing conformity with known tests of gravitation. Thus, investigation of such modified theories is a realistic target for gravitational wave science. We should add that the $V(\phi)$ term in Eq. (1) actually inhibits spontaneous growth, and it was not present in the original DEF theory, but it is strongly favored to satisfy recent binary star observations. This, and the details of other aspects of spontaneous scalarization through this Lagrangian, can be found in Ref. [10].

The origin of the instability can be seen in the linearized EOM for the scalar

$$\Box_g\phi = \left(-8\pi A^4 \frac{d(\ln A(\phi))}{d(\phi^2)} \tilde{T} + m_\phi^2 \right)\phi$$
$$\approx (-4\pi\beta\tilde{T} + m_\phi^2) \, \phi \equiv m_{\text{eff}}^2\phi. \tag{3}$$

The trace of the stress-energy tensor in the frame of $\tilde{g}_{\mu\nu}$ is negative as long as matter is not ultrarelativistic, since $\tilde{T} = -\tilde{\rho} + 3\tilde{p} \approx -\tilde{\rho}$. So, for appropriate densities and β values, the effective mass m_{eff} is imaginary, which causes the lowest-frequency Fourier modes of the scalar to have a tachyonic instability, since $\omega^2 \sim \vec{k}^2 + m_{\text{eff}}^2 < 0$. An equally important fact is that this instability is eventually quenched by nonlinear effects as the scalar grows to the nonperturbative regime, and we end up with a stable scalar cloud.

To understand the first path to generalize the idea of DEF, note that there is nothing specific to the scalar nature of the field in the mechanism that incites the growth, or the nonlinear terms that later suppress it. That is, one can replace the scalar, for example, with a vector X_μ, as in the action [7]

$$\frac{1}{16\pi} \int d^4x \sqrt{-g} [R - F^{\mu\nu}F_{\mu\nu} - 2m_X^2 X^\mu X_\mu]$$
$$+ S_m[f_m, A_X^2(\eta)g_{\mu\nu}], \qquad \eta = g^{\mu\nu}X_\mu X_\nu, \tag{4}$$

and still have spontaneous growth. Here, $F_{\mu\nu} = \nabla_\mu X_\nu - \nabla_\nu X_\mu$, and A_X is an appropriate function of the vector field such as $e^{\beta_X\eta/2}$. The vector EOM

$$\nabla_\rho F^{\rho\mu} = (-4\pi A_X^4\beta_X\tilde{T} + m_X^2)X^\mu \tag{5}$$

has a tachyonlike nature as in the DEF theory, and observational signatures of the theories are very similar.

This line of thought can also be extended to spinors [11] and gauge bosons [12].

We should also add that the conformal coupling terms in these theories can depend on the derivatives of the fields as well as the fields themselves. For example, in the simplest case of the scalar field, the action

$$\frac{1}{16\pi} \int d^4x \sqrt{-g}[R - 2\nabla_\mu\phi\nabla^\mu\phi - 2m_\phi^2\phi^2]$$
$$+ S_{\rm m}[f_{\rm m}, A_\partial^2(K)g_{\mu\nu}], \qquad K = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \quad (6)$$

leads to the equation of motion [8]

$$\nabla_\mu[(-8\pi\tilde{T}A_\partial^3 A_\partial' + 1)\nabla^\mu\phi] = m_\phi^2\phi. \quad (7)$$

This is radically different from the case in Eq. (3), since there is no modification to the mass term. However, for $A_\partial = e^{\beta_\partial K/2}$ with $\beta_\partial < 0$, the principal part of the equation (the part with the highest order of derivatives in the partial differential equation) reads

$$(-4\pi\tilde{T}\beta_\partial + 1)\Box\phi = \cdots. \quad (8)$$

For large enough $\tilde{T}\beta_\partial$, it is the kinetic term rather than the mass-squared term that changes sign, which means that we have a ghostlike instability rather than a tachyonic one. This instability also grows exponentially from arbitrary perturbations despite its different nature, which is the essence of spontaneous growth. This is called "ghost-based spontaneous scalarization." We can also obtain "ghost-based spontaneous vectorization" by changing the dependence of the conformal factor in Eq. (4) $A_X \to A_F(F_{\mu\nu}F^{\mu\nu})$.

Another path to generalize spontaneous scalarization, and the one we are going to examine in more detail in this study, uses the fact that a conformal coupling is not the only way to obtain an instability that causes spontaneous growth. The most general scalar-dependent disformal transformation is [13]

$$\tilde{g}_{\mu\nu} = A^2(\phi)[g_{\mu\nu} + \Lambda B^2(\phi)\partial_\mu\phi\partial_\nu\phi]. \quad (9)$$

If we use this in the action Eq. (1), the resulting EOM is [9,14]

$$\Box\phi = m_\phi^2\phi + \frac{4\pi}{1 + \Lambda B^2 \partial_\mu\phi\partial^\mu\phi}$$
$$\times \{\Lambda B^2[(\delta-\alpha)T^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi + T^{\rho\sigma}\partial_\rho\partial_\sigma\phi] - \alpha T\}, \quad (10)$$

where $\alpha(\phi) \equiv A^{-1}(dA/d\phi)$, $\delta(\phi) \equiv B^{-1}(dB/d\phi)$, and the stress-energy tensor and its trace T are in the frame of $g_{\mu\nu}$. The linearized EOM arising from Eq. (10) is more complicated than the case of conformal coupling in Eq. (3), but they were analyzed similarly to the conformal case, which shows the existence of instabilities. Scalarized

neutron star solutions for disformal couplings have been explicitly constructed using numerical methods [9].

To understand the disformal transformation case better, first see that the last αT term in Eq. (10) arises from the overall conformal scaling A^2 in Eq. (9), and behaves as an effective mass term as in tachyonic spontaneous scalarization. The novel contribution of the disformal transformation can best be seen when we set $A(\phi) = B(\phi) = 1$, in which case the principal part of the linearized EOM becomes

$$(-4\pi\Lambda T^{\rho\sigma} + g^{\rho\sigma})\partial_\rho\partial_\sigma\phi = \cdots. \quad (11)$$

Hence, one can see that the character of the highest-derivative term can change for large enough Λ and/or stress-energy density. This is very similar to ghost-based spontaneous growth, but it arises from a completely different form of coupling.

In the following sections, we are going to unite the two paths we discussed which generalize spontaneous scalarization—that is, using fields other than scalars, and using disformal rather than conformal transformations. This way, we will investigate spontaneous tensorization of vectors and other fields through disformal couplings that depend on these fields.

III. SPONTANEOUS GROWTH FROM VECTOR DISFORMAL COUPLINGS

A. Field disformal coupling

The simplest disformal transformation of a metric by a vector field X_μ is given by [15,16]

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + B(x)X_\mu X_\nu, \quad (12)$$

where $x = X_\mu X^\mu$. Here, we ignore the overall conformal scaling that is present in Eq. (9) to concentrate our efforts on the purely disformal part of the transformation. We can devise a related modified gravity theory in analogy to DEF given by the action

$$\frac{1}{16\pi} \int d^4x \sqrt{-g}[R - F_{\mu\nu}F^{\mu\nu} - 2m_X^2 X_\mu X^\mu]$$
$$+ \int d^4x \sqrt{-\tilde{g}}\mathcal{L}_{\rm m}[f_{\rm m}, \tilde{g}_{\mu\nu}], \quad (13)$$

where $F_{\mu\nu} = \nabla_\mu X_\nu - \nabla_\nu X_\mu$, and we express the matter term explicitly in terms of the matter Lagrangian $\mathcal{L}_{\rm m}$. Replacing all occurrences of $\tilde{g}_{\mu\nu}$ with $g_{\mu\nu}$ corresponds to minimal matter coupling, hence GR.

Varying the action provides the EOM

$$\nabla_\mu F^{\mu\nu} = (m_X^2 - 4\pi\sqrt{1+xB}B'\tilde{T}^{\rho\sigma}X_\rho X_\sigma)X^\nu$$
$$- 4\pi\sqrt{1+xB}B\tilde{T}^{\mu\nu}X_\mu, \quad (14)$$

where $B' = dB/dx$. Here, the two stress-energy tensors defined with respect to the bare and tilde metrics are related through

$$T^{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-\bar{g}}\mathcal{L}_{\mathrm{m}})}{\delta g_{\mu\nu}} = \frac{2}{\sqrt{-g}} \frac{\delta \tilde{g}_{\rho\sigma}}{\delta g_{\mu\nu}} \frac{\delta(\sqrt{-\bar{g}}\mathcal{L}_{\mathrm{m}})}{\delta \tilde{g}_{\rho\sigma}}$$
$$= \sqrt{1+xB}(\tilde{T}^{\mu\nu} - B'\tilde{T}^{\rho\sigma} X_\rho X_\sigma X^\mu X^\nu). \quad (15)$$

Note that the lowering of the indices of the stress-energy tensors should be performed with their respective metrics.

Our main interest is the spontaneous growth of X_μ in compact stars; hence, we will have a closer look at the vector EOM in this setting. To the leading order in X_μ, the EOM becomes

$$\nabla_\mu F^{\mu\nu} \approx (m_X^2 \delta^\nu_{\ \mu} - 4\pi B(0)\tilde{T}^{\nu\rho} g_{\rho\mu})X^\mu = \mathcal{M}^\nu_{\ \mu}X^\mu, \quad (16)$$

where $\mathcal{M}$ can be interpreted as an effective mass-squared tensor, which is the analog of m_{eff} in Eq. (3). Then, all it takes to have an instability is to have one negative eigenvalue of $\mathcal{M}$. For matter that is not ultrarelativistic, the largest component of $\mathcal{M}$ is of the magnitude of the rest mass density $\tilde{\rho}$ of the matter; hence, a negative mass mode exists if $4\pi B(0)\tilde{\rho} \gtrsim m_X^2$, and suitable choices of B lead to spontaneous vectorization.

Seeing the instability is easier for sufficiently symmetric spacetimes where the metric and the stress-energy tensor are diagonal. One common example is a spherically symmetric star with perfect fluid matter, where the equation simplifies to

$$\nabla_\mu F^{\mu\nu} \approx (m_X^2 - 4\pi B(0)\tilde{T}^{\nu\nu} g_{\nu\nu})X^\nu = m_{\mathrm{eff}}^2 X^\nu. \quad (17)$$

Here, the repeated indices are not summed on the right-hand side. m_{eff}^2 is clearly negative for appropriate choices of B, and this indicates a tachyonic instability.

Let us remember that an instability around the GR solution $X_\mu = 0$ is desirable, but it is also essential that the instability shut off as it grows so that the final solution is stable. Inspired by the DEF theory, a natural choice is $B = \lambda_X e^{\beta_X X_\mu X^\mu}$ for some constants β_X and λ_X. For example, in an astrophysical system where X_0 is the dominant growing mode, $\beta_X > 0$ ensures that the negative contribution to $\mathcal{M}$ disappears as X_0 grows, killing the instability, while a $\lambda_X \sim 1$ would likely provide a powerful enough instability in analogy to the DEF theory.

It should be clear that there is nothing magical about the exponential form of B, and any function that behaves similarly when $x = 0$ and $x \to \infty$ provides spontaneous growth. However, ensuring that this recipe provides stable neutron star solutions—that is, that the instability indeed shuts off eventually—requires more thorough numerical studies, such as time evolution, which we will not attempt here.

It is curious to observe that the instability we have modifies the effective mass, and is of tachyonic nature, unlike the scalar-dependent disformal coupling in Eq. (9) which modifies the wave operator, leading to a ghostlike instability. This difference is not due to the nature of the field, but is related to the fact that the former directly uses the field in the disformal transformation, whereas the latter necessarily uses the derivatives, since scalars have no intrinsic indices. We will now see that ghostlike instabilities can arise for vector-dependent disformal couplings as well, if the transformation includes the derivatives of the field.

B. Derivative disformal coupling

It is also possible to have a derivative vector disformal coupling such as

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \lambda B_F(x) F_{\mu\alpha} F_\nu^{\ \alpha}, \quad (18)$$

where λ is a constant with dimensions of area that renders B_F dimensionless. This form of coupling has been discussed in the literature [17], but its consequences for any concrete theory, let alone in terms of spontaneous growth, have not been investigated. We will assume $B_F(0) = 1$ without loss of generality.[1]

If we insert Eq. (18) into Eq. (13), the vector EOM in the resulting theory becomes

$$\nabla_\mu F^{\mu\nu} = m_X^2 X^\nu + 4\pi\nabla_\mu[\sqrt{\chi}\lambda B_F(\tilde{T}^{\mu\beta}F_\beta^{\ \nu} - \tilde{T}^{\nu\beta}F_\beta^{\ \mu})]$$
$$- 4\pi\sqrt{\chi}\lambda B_F'\tilde{T}^{\mu\beta}F_{\mu\alpha}F_\beta^{\ \alpha}X^\nu. \quad (19)$$

In this and all the following cases with disformal coupling, we define the ratio of the determinants of the metrics in the two frames as

$$\sqrt{\chi} \equiv \sqrt{-\tilde{g}}/\sqrt{-g}. \quad (20)$$

The nature of this equation is less transparent compared to Eq. (14), but we can have a better idea by first linearizing, and then concentrating on the principal part—i.e., considering only the highest-derivative terms:

$$[-4\pi\lambda(\tilde{T}^{\rho\sigma}g^{\mu\nu} - \tilde{T}^{\nu\sigma}g^{\rho\mu}) + g^{\rho\sigma}g^{\mu\nu}]\nabla_\rho F_{\sigma\mu} = \cdots. \quad (21)$$

Note that the terms arising from the disformal transformation, $\tilde{T}^{\rho\sigma}g^{\mu\nu} - \tilde{T}^{\nu\sigma}g^{\rho\mu}$, generically would not vanish, and hence would change the overall sign of the kinetic term $\nabla_\rho F_{\sigma\mu}$ for appropriate (large enough) choices of λ. The "wrong-sign" kinetic term simply means that there is a ghostlike instability in such regions of spacetime similar to the case of scalar-dependent disformal transformation in Eq. (9). This is not a surprise, since we know that ghostlike

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[1]This choice rules out $B(0) = 0$, but we will soon see that this case is not relevant to the discussion of spontaneous growth.

instabilities arise from derivative couplings, which is the case in both theories.

The ghostlike instability can be more easily seen in specific cases, such as when the metric and the stress-energy tensor are diagonal, as for a nonrotating neutron star with perfect fluid matter. Let us also assume that the rest mass density, and hence $\tilde{T}^{00}$, is dominant for ease of analysis. Consider the ∂_t^2 terms in the EOM for the spatial components $\nu = i$:

$$[-4\pi\lambda B_F(0)\tilde{T}^{00} + g^{00}]g^{ii}\partial_t^2 X_i \approx \cdots \quad \text{(no sum)}, \quad (22)$$

where the right-hand side contains at most first time derivatives. The coefficient of $\partial_t^2 X_i$ can reverse its sign in the presence of matter. For this to happen, $\lambda \gtrsim \tilde{\rho}^{-1}$ should be satisfied. Our assumptions might look too restrictive, since the pressure terms can be comparable to density terms in $\tilde{T}^{\mu\nu}$, especially for more massive neutron stars. This would not qualitatively change our conclusions, since $\tilde{T}^{\rho\sigma}g^{\mu\nu} - \tilde{T}^{\nu\sigma}g^{\rho\mu}$ does not vanish in general, and it has a value of $\sim\tilde{\rho}$, hence $\lambda \gtrsim \tilde{\rho}^{-1}$ would still be sufficient with order of unity changes of λ.

Just as in Eq. (12), we still need the instability to shut off as the field grows. This can again be satisfied by nonlinear terms in X_μ—namely, a decaying function $B_F = e^{\beta_F \cdot X}$ with a choice of sign for β_F that ensures that B_F vanishes for growing values of X_μ, or $B_F = e^{\bar{\beta}_F x^2}$ for $\bar{\beta}_F < 0$, would ensure that the initial instability around $X_\mu = 0$ would vanish for larger fields.

The form of the disformal term in Eq. (18) is inspired by the standard kinetic term $F_{\mu\nu}F^{\mu\nu}$, in a similar fashion to the relationship between the standard Proca potential term $X_\mu X^\mu$ and Eq. (12). Other choices, such as replacing $F_{\mu\nu}$ with nonsymmetric $\nabla_\mu X_\nu$ might seem possible, but they may lead to unregularized ghosts in the flat-space limit, hence we choose to avoid them [18]. Such terms are in general possible with carefully chosen couplings in the most general vector-tensor theories which contain at most two derivatives to avoid Ostrogradsky's theorem [15,19]. Our choice in Eq. (18) is to demonstrate the relevance of derivative vector disformal couplings, especially in the context of spontaneous tensorization, and we will not attempt to construct the most general disformal vector coupling in this study.

Lastly, we remind the reader that m_X does not play an essential role in our discussion, which means that the case $m_X = 0$ still possesses the instability. Such a theory preserves the gauge symmetry $X_\mu \to X_\mu + \partial_\mu\rho$ for any scalar ρ, which might be desirable depending on the physical interpretation of X_μ.

C. Disformal coupling through the Abelian Higgs mechanism

So far, we have considered the intrinsically massive vector field, the Proca field, in Eq. (13). A second, and physically better motivated, way of introducing mass to a vector field is the Abelian Higgs mechanism, which preserves the gauge symmetry of the massless vector. This mechanism is given by the following action:

$$\frac{1}{16\pi}\int d^4x\sqrt{-g}[R - F_{\mu\nu}F^{\mu\nu}]$$
$$-\frac{1}{16\pi}\int d^4x\sqrt{-g}(2\overline{D_\mu\Phi}D^\mu\Phi + 2V(\bar{\Phi}\Phi))$$
$$+\int d^4x\sqrt{-\tilde{g}}\mathcal{L}_m[f_m, \tilde{g}_{\mu\nu}], \quad (23)$$

where Φ is a complex scalar, $D_\mu\Phi = (\nabla_\mu - ieX_\mu)\Phi$ is the gauge covariant derivative with a coupling constant e, and an overbar means complex conjugation. The gauge transformation is $X_\mu \to X_\mu - \nabla_\mu\rho$ and $\Phi \to e^{ie\rho}\Phi$. The Higgs mechanism introduces the vector field mass through the hidden $e^2\bar{\Phi}\Phi X_\mu X^\nu$ in the scalar kinetic term. This is thanks to the choice

$$V(\bar{\Phi}\Phi) = m_0^2(u^2 - \bar{\Phi}\Phi)^2/(2u^2), \quad (24)$$

which causes the ground state of Φ to attain a non-zero value.

The trivial matter coupling choice that preserves the gauge symmetry is

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \lambda_D B_D(\bar{\Phi}\Phi)\overline{D_{(\mu}\Phi}D_{\nu)}\Phi, \quad (25)$$

where $_{()}$ represents symmetrization, and we choose the normalization $B_D(0) = 1$. This disformal transformation contains both scalar and vector dependences through the gauge covariant derivative D. Similar disformal transformations have been investigated in the cosmology literature [20], but its effects in terms of spontaneous tensorization of compact objects is a novel concept to the best of our knowledge. The action in Eq. (23) together with a conformal, rather than disformal, transformation for $\tilde{g}_{\mu\nu}$ is known to cause spontaneous growth of vector and gauge boson fields [12].

The EOMs for the scalar and vector fields arising from Eq. (25) are

$$\nabla^\nu F_{\nu\mu} = \Delta^\nu_{\ \mu}(e^2\bar{\Phi}\Phi X_\nu + J^\Phi_\nu),$$
$$\Theta^{\mu\nu}[\nabla_\mu\nabla_\nu - e^2 X_\mu X_\nu - 2ieX_\mu\nabla_\nu - ie\nabla_\mu X_\nu]\Phi$$
$$- 4\pi\nabla_\mu(\lambda_D B_D T^{\mu\nu})D_\nu\Phi$$
$$= [m_0^2(\bar{\Phi}\Phi/u^2 - 1) + \mu_\Phi^2]\Phi, \quad (26)$$

where $J^\Phi_\mu = ie(\bar{\Phi}\nabla_\mu\Phi - \Phi\nabla_\mu\bar{\Phi})/2$, and

$$\Delta^\nu{}_\mu = -4\pi\lambda_D B_D T^\nu{}_\mu + \delta^\nu{}_\mu,$$
$$\Theta^{\mu\nu} = -4\pi\lambda_D B_D T^{\mu\nu} + g^{\mu\nu},$$
$$\mu_\Phi^2 = -4\pi\lambda_D B'_D T^{\mu\nu}\overline{D_\mu\Phi}D_\nu\Phi. \qquad (27)$$

Equation (26) behaves qualitatively similarly to the spontaneous growth cases we have seen so far. $\Delta^\nu{}_\mu$ and $\Theta^{\mu\nu}$ cause the principal parts of the equations for X_μ and Φ to change sign when λ_D is large enough, leading to ghostlike instabilities in both. Note that Φ also gets a contribution to its effective mass through μ_Φ^2 that can potentially drive a tachyonic instability for an appropriate form of B_D, but this term only appears beyond the linear order in perturbations of Φ around its equilibrium value $\bar\Phi\Phi = u^2$. This means it does not initiate spontaneous growth, but it can play a role once the fields grow to a level where nonlinear effects are dominant.

Remember that we require the shutoff of the instability as the fields grow, which suggests that we need B_D to decay as Φ grows. Inspired by our experience, $B_D = e^{\beta_D\bar\Phi\Phi}$ with $\beta_D < 0$ is a possible choice. Even though we considered a B_D that is only a function of Φ, it can be generalized to include vector dependence, $B_D(\Phi, X_\mu X^\mu)$, which would bring new effective mass terms to the vector as well. However, these would not change the qualitative picture of the spontaneous tensorization process we described.

The case for the spontaneous growth of a non-Abelian gauge field W_μ^a is very similar to its Abelian version. The action is given by

$$\frac{1}{16\pi}\int d^4x\sqrt{-g}[R - F^{a\mu\nu}F_{\mu\nu}^a]$$
$$-\frac{1}{16\pi}\int d^4x\sqrt{-g}[2(D_\mu\Phi)^\dagger D^\mu\Phi + 2V(\Phi^\dagger\Phi)]$$
$$+\int d^4x\sqrt{-\tilde g}\mathcal{L}_\mathrm{m}[f_\mathrm{m}, \tilde g_{\mu\nu}], \qquad (28)$$

where † indicates the Hermitian conjugate, and

$$F_{\mu\nu}^a = \nabla_\mu W_\nu^a - \nabla_\nu W_\mu^a + ef^{abc}W_\mu^b W_\nu^c,$$
$$V(\Phi^\dagger\Phi) = \frac{1}{2}\frac{m_0^2}{u^2}(u^2 - \Phi^\dagger\Phi)^2. \qquad (29)$$

The Higgs field Φ is now a multidimensional object that can be acted upon by T^a, generators of the Lie algebra of the gauge group. The letters a, b, and c label T^a, and the structure constants f^{abc} are defined as $[T^a, T^b] = if^{abc}T^c$. Then, the disformal transformation

$$\tilde g_{\mu\nu} = g_{\mu\nu} + \lambda_W B_W(\Phi^\dagger\Phi)D_{(\mu}\Phi^\dagger D_{\nu)}\Phi \qquad (30)$$

results in a theory where the non-Abelian fields grow spontaneously.

IV. DISFORMAL COUPLING BEYOND VECTORS

We have seen that spontaneous growth arising from disformal couplings can be easily adapted to vectors. However, the idea is even more general, and we can consider disformal coupling of any field. We will investigate the cases of spin-half and spin-2 particles in this section, and see that our approach to scalars and vectors does not proceed as smoothly in all cases in terms of obtaining spontaneous growth phenomena.

A. Spinor disformal coupling

Our extension of spontaneous growth through disformal coupling from scalars to vectors can be generalized to other fields. As in the vector case, we can get inspiration from spontaneous growth through conformal transformation, where the next targets after vectors were spinors [11]. Spontaneous growth of spinor fields in gravity is less known; hence, we will try to summarize all the basic aspects of a spinor-dependent conformal metric scaling first, which will be crucial in understanding the spinor-dependent disformal coupling and its role in spontaneous growth.

Consider the following action:

$$\frac{1}{16\pi}\int d^4x\sqrt{-g}R$$
$$+\frac{1}{16\pi}\int d^4x\sqrt{-g}[(\bar\psi\gamma^\mu(\nabla_\mu\psi) - (\nabla_\mu\bar\psi)\gamma^\mu\psi) - 2m\bar\psi\psi]$$
$$+\int d^4x\sqrt{-\tilde g}\mathcal{L}_\mathrm{m}[f_\mathrm{m}, \tilde g_{\mu\nu}], \qquad (31)$$

where ψ is a Dirac bispinor and $\bar\psi \equiv -i\psi^\dagger\hat\gamma^0$ is constructed with the flat-space gamma matrix $\hat\gamma^0$. Definitions for the flat-space gamma matrices $\hat\gamma^{(\mu}\hat\gamma^{\nu)} = \eta^{\mu\nu}$, curved-space gamma matrices $\gamma^{(\mu}\gamma^{\nu)} = g^{\mu\nu}$, covariant derivatives ∇_μ for spinors, and other relevant mathematical details can be found in Ref. [11]. The second line is simply the action for a minimally coupled spinor field in gravity. The spinor field spontaneously grows in the presence of matter if we have a conformal coupling of the form $\tilde g_{\mu\nu} = A_\psi^2 g_{\mu\nu}$ in the matter action with

$$A_\psi = e^{\beta_\psi(\bar\psi\hat\gamma^5\hat\gamma^\mu(\nabla_\mu\psi) - (\nabla_\mu\bar\psi)\hat\gamma^5\gamma^\mu\psi)/4} \equiv e^{\beta_\psi\mathcal{L}_\psi^{5,K}/2} \qquad (32)$$

for a constant β_ψ and

$$\gamma^5 \equiv \frac{i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma = \frac{i}{4!}\tilde\epsilon_{abcd}\hat\gamma^a\hat\gamma^b\hat\gamma^c\hat\gamma^d = \hat\gamma^5, \qquad (33)$$

which gives the EOM

$$\gamma^\mu \nabla_\mu \psi - \frac{\mathbb{I} - \zeta_\psi \hat{\gamma}^5}{1 - \zeta_\psi^2}[m - (\nabla_\mu \zeta_\psi)\hat{\gamma}^5 \gamma^\mu/2]\psi = 0, \quad (34)$$

with $\zeta_\psi \equiv 4\pi \tilde{T}\beta_\psi A_\psi^4$.

To understand why the above EOM leads to spontaneous growth, let us examine the purely tachyonic spinor EOM in flat space [21,22]:

$$(\hat{\gamma}^\mu \partial_\mu - \hat{\gamma}^5 m)\psi = 0. \quad (35)$$

Let us investigate a plane wave solution $\psi = u(\vec{k})e^{ik_\mu x^\mu} = e^{-i\omega t + i\vec{k}\cdot\vec{x}}$ of this equation:

$$(\hat{\gamma}^\nu \partial_\nu - \hat{\gamma}^5 m)(\hat{\gamma}^\mu \partial_\mu - \hat{\gamma}^5 m)\psi = 0$$
$$\Rightarrow [-\eta^{\mu\nu}k_\mu k_\nu - i(\hat{\gamma}^\mu \hat{\gamma}^5 + \hat{\gamma}^5 \hat{\gamma}^\mu)mk_\mu + \hat{\gamma}^5 \hat{\gamma}^5 m^2]\psi = 0$$
$$\Rightarrow \omega^2 = \vec{k}\cdot\vec{k} - m^2, \quad (36)$$

where we used $\hat{\gamma}^\mu \hat{\gamma}^5 + \hat{\gamma}^5 \hat{\gamma}^\mu = 0$. This clearly chows that ω is imaginary for large-wavelength (small-$|k|$) modes, and leads to exponential growth rather than oscillation in time. This is the instability mechanism of the tachyon, hence the name "tachyonic Dirac equation."

We have a mix of tachyonic and nontachyonic terms in Eq. (34), but this equation also has the exponential growth modes for large enough λ_ψ, details of which can be found in Ref. [11].

One important aspect of Eq. (34) is that it contains the term $\nabla_\mu \zeta_\psi$. ζ_ψ itself contains derivatives of Ψ [since it is a function of A_ψ in Eq. (32)], which means $\nabla_\mu \zeta_\psi$ has second derivatives of ψ, seemingly becoming the principal part of the EOM. This would be a radical change, since the principal part has a dominant effect on the behavior of the equation, as we have utilized so far. However, using the EOM to express the derivative terms in Eq. (32) leads to the expression

$$A_\psi^4 = \frac{\zeta_\psi}{4\pi\beta_\psi \tilde{T}} = \exp\left(-2m\beta_\psi \bar{\psi}\psi \frac{\zeta_\psi}{1 - \zeta_\psi^2}\right). \quad (37)$$

That is, ζ_ψ can be written as a function of ψ, albeit implicitly, rather than its derivatives. This means the equation of motion is still a first-order partial differential equation.[2] We will look at this important fact once more for the disformal coupling case.

The task at hand is finding a disformally transformed $\tilde{g}_{\mu\nu}$ that can possibly lead to spontaneous spinorization. Examination of the relationship between conformal and disformal transformations that lead to spontaneous growth for scalars and vectors immediately suggests that we lower

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[2]The $\nabla_\mu \zeta_\psi$ term was missing in the original publication, and is added in a recent erratum.

one of the contracted indices in Eq. (32) and add such a term to the metric

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \lambda_\psi B_\psi[\bar{\psi}\hat{\gamma}^5 \gamma_{(\mu}\nabla_{\nu)}\psi - (\nabla_{(\mu}\bar{\psi})\hat{\gamma}^5\gamma_{\nu)}\psi], \quad (38)$$

where $B_\psi(\bar{\psi}\psi)$ can be normalized as $B_\psi(0) = 1$. Varying the action gives

$$(\zeta^{\mu\nu}\gamma^5 + g^{\mu\nu}\mathbb{I})\gamma_\mu \nabla_\nu \psi - (\mu_\psi - (\nabla_\mu \zeta^{\mu\nu})\gamma_\nu)\psi = 0, \quad (39)$$

where

$$\zeta^{\mu\nu} = 8\pi\sqrt{\chi}\lambda_\psi B_\psi \tilde{T}^{\mu\nu},$$
$$\mu_\psi = m - 4\pi\sqrt{\chi}\lambda_\psi B_\psi' \tilde{T}^{\mu\nu}[\bar{\psi}\hat{\gamma}^5\gamma_\mu \nabla_\nu \psi - \nabla_\mu \bar{\psi}\hat{\gamma}^5\gamma_\nu \psi]. \quad (40)$$

At first sight, for large enough λ_ψ, the $\hat{\gamma}^5$ term may seem to dominate over $\mathbb{I}$ in Eq. (39). An instability occurs around $\psi = 0$ in a similar manner to our tachyonic dispersion relation in Eq. (36).

The above explanation of the tachyonic nature of Eq. (39) overlooks an important fact: the equation of motion contains the $\nabla_\mu \zeta^{\mu\nu}$ term. Note that $\zeta^{\mu\nu}$ already contains derivatives of ψ through χ. For conformal transformations, we were able to use the equation of motion to express ζ_ψ without any derivatives, as in Eq. (37), hence $\nabla_\mu \zeta_\psi$ stayed a first-order term in the differential equation.

We were not able to perform a similar procedure for $\zeta^{\mu\nu}$—that is, to the best of our knowledge, $\zeta^{\mu\nu}$ depends on the derivatives of ψ, which means that $\nabla_\mu \zeta^{\mu\nu}$ contains second derivatives of ψ. In other words, $\nabla_\mu \zeta^{\mu\nu}$ is the principal part of the EOM [Eq. (39)]. This means that even if its coefficient is small, it has a leading role in the time evolution of ψ, and as a consequence the above analysis for the existence of a tachyonic d.o.f. cannot be repeated verbatim. A change in the order of the EOM as a partial differential equation (from first to second in this case) is radical, and has never been the case in any of the spontaneous growth theories so far.

Moving beyond the above major modification to the spinor EOM, we were also not able to find a clear tachyonic mode in this second-order differential equation. This is an important difference from the case of vectors, where the instability directly appears both in conformal and in disformal transformations.

We should add that any spontaneously growing spinor should be considered as a classical object as opposed to a quantum field [11]. This is mainly because spinor fields obey the Pauli exclusion principle when quantized, which means that their occupation numbers cannot have arbitrary values. Spontaneous spinorization would result in a continuously adjustable spinor field value which is determined by the spinorizing object (e.g., the neutron star), hence the exclusion principle cannot be accommodated. A purely classical spinor is not a commonly encountered object, and

its spontaneous growth also calls for care in understanding the basics of half-integer spins in nonquantum contexts [23].

B. Spin-2 disformal coupling

Can we have a disformal transformation based on a spin-2 field, since it seems to be the natural choice after scalars (spin-0) and vectors (spin-1)? The answer is negative, based on our current knowledge of interacting spin-2 field theories.

To start with, we should note that there is no known theory of spontaneous growth of a spin-2 field, even when we have a conformal transformation [8]. A good starting point to understand this is looking at the known interacting spin-2 theories, which have only recently been developed [18]. Since the metric is also a spin-2 field, such theories can be considered as theories of two metrics $g_{\mu\nu}$ and $f_{\mu\nu}$ (bimetric theories), whose generic action is given by

$$S = S_{\text{EH}}(g) + S_{\text{EH}}(f) + S_{\text{int}}(f, g) + S_m[f_m, g_{\mu\nu}], \quad (41)$$

where S_{EH} is simply the Einstein-Hilbert action for a given metric, and the interaction term between the metrics S_{int} has to be of a specific form in order to avoid undesirable ghosts [24,25]. We also included a minimal matter coupling S_m with one of the metrics, which is known to preserve the ghost-free nature of the theory.

Looking at our previous examples of spontaneous growth through nonminimal matter coupling, the most straightforward attempt to induce an instability in this case would be replacing the metric in the matter coupling with a transformed one:

$$S_m[\psi_m, g_{\mu\nu}] \rightarrow S_m[\psi_m, \tilde{g}_{\mu\nu}], \quad (42)$$

where $\tilde{g}_{\mu\nu}$ is a function of both $g_{\mu\nu}$ and $f_{\mu\nu}$. For example, a possible disformal transformation is

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \lambda_f f_{\mu\nu}. \quad (43)$$

That is, matter field couples to both metrics at the same time. However, coupling to a single metric is crucial for having a ghost-free theory, and the ghost is known to reappear with composite metric couplings as in Eq. (43) [26]. Thus, stable spontaneous growth that comes with such matter couplings seems to be impossible. One potential way out of this can be a scenario where the ghost only comes in at extremely high-energy scales and the theory can be reconciled with observation [26]. Whether this happens for our proposed disformal transformation is not clear. We note that conformal transformations also contain couplings to both metrics in a similar fashion, and seem to fail due to the same reason [8]. Overall, all known coupling forms that lead to spontaneous growth seem to fail for spin-2 fields.

In summary, it is not straightforward to generalize disformal transformation-based spontaneous growth beyond vectors. This was not a surprise for spin-2 fields, since spontaneous growth is not known for any form of coupling. However, the spinor case is puzzling, since spontaneous spinorization does occur for conformal transformations. Understanding the deeper reasons for such differences between conformal and disformal transformations is an important part of future studies on spontaneous growth in gravity.

V. CONCLUSION

The original spontaneous scalarization theory of DEF is the quintessential example of spontaneous growth in gravity where large deviations from GR in strong fields provide an ideal target for GWs. This theory has a scalar-dependent conformal transformation in the metric that couples to matter, which provides an imaginary effective mass, hence the growth. This idea was recently generalized in various ways, and here we investigated the interaction of two such paths. First, one can replace the scalar in the conformal scaling of DEF with other fields, e.g., a vector, and obtain spontaneous tensorization for general fields. Second, one can replace the scalar-dependent conformal scaling of the metric with a still scalar-dependent disformal transformation, and obtain a novel form of spontaneous scalarization.

In this study, we combined the two approaches above, and showed that spontaneous growth also occurs when the scalar dependence of the disformal transformation is generalized to other fields. We have devised three vector-based disformal transformation theories, and showed that they generically possess the instabilities that incite spontaneous growth. The first of these theories can be said to be somehow simpler than the scalar-dependent case, since the transformation only contains the field, and not its derivatives. This is also reflected in the fact that the instability in this theory is tachyonlike, while all other cases contain a ghostlike instability which can lead to astrophysically unusual structures [8].

We have also showed that ideas to generalize spontaneous growth have their limitations. It was already known that the usual conformal transformations cannot be used to obtain a theory of spontaneously growing spin-2 field, and this continues to be the case for disformal transformations due to similar reasons. The case is more curious for spinor fields. Even though one can obtain spontaneous spinorization based on conformal transformations, it is not clear whether this happens for their disformal counterparts. Disformal coupling changes the nature of the equation of motion for the spinor from a first-order partial differential equation to a second-order one. This radical change also makes it hard to establish that there is an instability in the linearized equations, though we have not ruled out this possibility either.

Before we conclude, let us discuss one of the paths to generalize the spontaneous scalarization of DEF that we ignored so far: using couplings beyond the matter term. It has been recently shown that a scalar-dependent coupling to any term in the Lagrangian—for example, the Gauss-Bonnet terms as in the action

$$\frac{1}{16\pi} \int d^4x \sqrt{-g}[R - 2\nabla_\mu \phi \nabla^\mu \phi + \lambda^2 f(\phi)\mathcal{R}^2], \quad (44)$$

leads to the spontaneous growth of the scalar [27–29]. Here $\mathcal{R}^2 = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is the Gauss-Bonnet invariant of $g_{\mu\nu}$, and since it is nonzero purely due to curvature, one can obtain spontaneous growth near black holes as well as neutron stars. $\mathcal{R}^2$ can be replaced with other curvature- or field-dependent terms [30], or the scalar can be replaced with another field such as a vector [31], and one can still obtain more general spontaneous tensorization phenomena. Following the theme of the current study, we can hope to find spontaneous growth in the analogues of these theories with disformal transformations, e.g.,

$$\frac{1}{16\pi} \int d^4x \sqrt{-g}[R - 2\nabla_\mu \phi \nabla^\mu \phi + \lambda^2 \tilde{\mathcal{R}}^2], \quad (45)$$

where $\tilde{\mathcal{R}}^2$ is the Gauss-Bonnet invariant of a disformally transformed metric $\tilde{g}_{\mu\nu}$. Unlike the above case, the coupling of the scalar (or vector) field to the Gauss-Bonnet term is not straightforward anymore. Even though such theories have been considered in the past [32], their equations of motion are quite complicated, and we were not able to find clear signs of spontaneous growth with our linearized analysis. More thorough studies may shed more light on this issue.

We have taken the path of considering a single form of coupling and investigating its dependence on different fields, but there are alternative directions to explore the landscape for theories that feature spontaneous growth phenomena. For example, one can consider the most general gravity theory that contains a given field in addition to the metric, and then consider all possible coupling terms for the field. Such theories for scalar fields were pioneered by Horndeski [33], and all possible mechanisms of spontaneous scalarization in this case have been recently investigated [34]. This procedure can be repeated for other fields, which would form a systematic approach that would complement ours.

This study can be considered as a demonstration of the fact that spontaneous growth is widespread in gravity theories. Theories that have very different action formulations can have very similar behavior if they contain similar dynamical mechanisms. In our case, the mechanism is spontaneous growth based on an instability, which is eventually regularized due to nonlinear interactions. The similarity is not merely a theoretical one—observational signatures of these theories are also quite alike, and particularly prominent in the context of GWs. Thus, we think it will be fruitful to consider spontaneous tensorization theories as members of the same family as far as modifications to GR are considered, despite the diversity in their Lagrangian formulations. On the other hand, there are limitations to which fields can grow spontaneously by a given coupling in the action, and discovering the underlying reasons for these will be an important aspect of understanding spontaneous growth in gravity.

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