Nonsingular bouncing cosmology from general relativity

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We investigate a particular type of classical nonsingular bouncing cosmology, which results from general relativity if we allow for degenerate metrics. The simplest model has a matter content with a constant equation-of-state parameter and we get the modified Hubble diagrams for both the luminosity distance and the angular diameter distance. Based on these results, we present a *Gedankenexperiment* to determine the length scale of the spacetime defect which has replaced the big bang singularity. A possibly more realistic model has an equation-of-state parameter which is different before and after the bounce. This last model also provides an upper bound on the defect length scale.

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I. INTRODUCTION

There have been discussions from various physics perspectives of the possible existence of a pre-big-bang phase, with or without a bounce-type behavior of the cosmic scale factor. See, e.g., Refs. [1–3] and references therein.

Recently, we have obtained a surprising hint for the actual existence of a pre-big-bang phase [4], where we worked with the established theory of general relativity in four spacetime dimensions but allowed for degenerate metrics. (See the last two paragraphs of Sec. I in Ref. [4] for a brief comparison of this extended version of general relativity and the standard version, which considers only nondegenerate metrics.) With an appropriate differential structure and trivial spacetime topology, a nonsingular spatially flat Friedmann-type solution of the Einstein gravitational field equation has been obtained, where the curvature and the matter energy density remain finite (these quantities diverge for the standard Friedmann solution). Most interestingly, this nonsingular Friedmann-type solution suggests the existence of a "pre-big-bang" phase (in standard terminology) with a bounce-type behavior of the cosmic scale factor.

The aim of the present article is to review this nonsingular bounce, which remains within the realm of general relativity, and to obtain a better understanding of the nonsingular bouncing cosmology by performing exploratory calculations of certain cosmological observables. Even though, at this moment, these cosmological observables are only accessible through *Gedankenexperiments*, it is instructive to discuss them. In the Appendix, we also give an explicit realization of a particular classical nonsingular bouncing cosmology that was discussed in Ref. [3] (this cosmology has a different matter content before and after the bounce). The model of the Appendix allows us to obtain a qualitative upper bound on the length scale of the spacetime defect which has replaced the big bang singularity.

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II. NONSINGULAR BOUNCE WITH A CONSTANT EQUATION OF STATE

We start from the classical spacetime manifold of Ref. [4], but use a simplified version of the cosmic time coordinate T and consider only the T-even solution for the cosmic scale factor $a(T)$. In this way, we obtain a modified spatially flat Friedmann–Lemaître–Robertson–Walker (FLRW) universe with a bounce-type behavior of $a(T)$. We can be relatively brief in this section, as further details can be found in Refs. [4–7]. Throughout, we use natural units with $c = 1$ and $\hbar = 1$.

With a cosmic time coordinate T and comoving spatial Cartesian coordinates $\{x^1, x^2, x^3\}$, an appropriate *Ansatz* for the metric is given by [4]

$$ds^2|_{\text{mod. FLRW}} \equiv g_{\mu\nu}(x)dx^\mu dx^\nu|_{\text{mod. FLRW}} = -\frac{T^2}{b^2+T^2}dT^2 + a^2(T)\delta_{kl}dx^k dx^l, \tag{2.1a}$$

$$b > 0, \tag{2.1b}$$

$$T \in (-\infty, \infty), \tag{2.1c}$$

$$x^k \in (-\infty, \infty), \tag{2.1d}$$

where the function $a(T)$ in (2.1a) is, strictly speaking, not the same as the function $a(\tau)$ in (3.6) of Ref [4]. The parameter b in the metric (2.1a) corresponds to the characteristic length scale of the spacetime defect localized at $T = 0$ (see Refs. [4–7] and references therein). For the moment, b is simply a model parameter and we remain agnostic as to its physical origin. It may be that b is related to the Planck length, but it is also possible that b is related to a new fundamental length scale of quantum spacetime [8].

We observe that the metric (2.1a) is degenerate, with $\det\, g_{\mu\nu} = 0$ at $T = 0$. The corresponding $T = 0$ spacetime slice may be interpreted as a 3-dimensional "defect" of spacetime with topology $\mathbb{R}^3$. The standard elementary-flatness condition does not hold at the location of this spacetime defect; see Appendix D in Ref. [5] and Sec. 2 D in Ref. [6] for further discussion. As will be seen shortly, the metric (2.1a) removes the big bang curvature singularity, but does so at the price of introducing a spacetime defect. We also remark that a degenerate metric evades certain singularity theorems; cf. Sec. 3. 1. 5 in Ref. [7].

Later on, we will simplify the calculations away from the spacetime defect by use of the auxiliary coordinate τ instead of T. These two coordinates are related as follows (Fig. 1):

$$T(\tau) = \begin{cases} +\sqrt{\tau^2 - b^2}, & \text{for } \tau \geq b, \\ -\sqrt{\tau^2 - b^2}, & \text{for } \tau \leq -b, \end{cases} \tag{2.2a}$$

$$\tau \in (-\infty, -b] \cup [b, \infty). \tag{2.2b}$$

The inverted relation reads

$$\tau(T) = \begin{cases} +\sqrt{b^2 + T^2}, & \text{for } T \geq 0, \\ -\sqrt{b^2 + T^2}, & \text{for } T \leq 0, \end{cases} \tag{2.3}$$

which is multivalued at $T = 0$. The advantage of using the auxiliary coordinate τ is that the metric (2.1a) takes the standard spatially flat FLRW form, $ds^2 = -d\tau^2 + a^2(\tau)\delta_{kl}dx^k dx^l$, and that the reduced field equations are nonsingular. But it is important to realize that the coordinate transformation from T to τ is not a diffeomorphism (an invertible C^∞ function by definition): the function (2.3) is discontinuous between $T = 0^-$ and $T = 0^+$, as is the (suitably defined) second derivative. In short, the differential structure of the metric (2.1a) in terms of T is different from the one of the standard spatially flat FLRW metric in terms of τ; see Ref. [6] for a related discussion.

Taking the metric (2.1a) with spacetime coordinates $\{T, x^1, x^2, x^3\}$ and the energy-momentum tensor of a homogeneous perfect fluid, the Einstein equation gives the following modified spatially flat Friedmann equation and energy-conservation equation, together with an assumed equation-of-state parameter $W(T)$:

$$\left(1 + \frac{b^2}{T^2}\right)\left(\frac{1}{a(T)}\frac{da(T)}{dT}\right)^2 = \frac{8\pi G_N}{3}\rho(T), \tag{2.4a}$$

$$\frac{d}{da}[a^3\rho(a)] + 3a^2 P(a) = 0, \tag{2.4b}$$

$$W(T) \equiv \frac{P(T)}{\rho(T)} = w = 1, \tag{2.4c}$$

where the last equation corresponds to a particular choice for the constant equation-of-state parameter w. The actual value $w = 1$ in (2.4c) matches the one used in Ref. [3] and the reason for this choice will be discussed further in Sec. IV. As mentioned in Ref. [4], the only new ingredient in (2.4) is the singular factor $(1 + b^2/T^2)$ on the left-hand side of the modified Friedmann equation (2.4a).

The T-even bounce-type solution $a(T)$ from (2.4) with normalization $a(T_0) = 1$ at $T_0 > 0$ is given by

$$a(T)\Big|^{(\text{w=1},\, T\text{-even sol.})}_{\text{mod. FLRW}} = \sqrt[6]{(b^2 + T^2)/(b^2 + T_0^2)}, \tag{2.5}$$

which is perfectly smooth at $T = 0$ as long as $b \neq 0$ (see Fig. 2 for a comparison with the singular solution). The corresponding Kretschmann curvature scalar $K \equiv R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$ and matter energy density ρ are then finite at $T = 0$, provided $b \neq 0$,

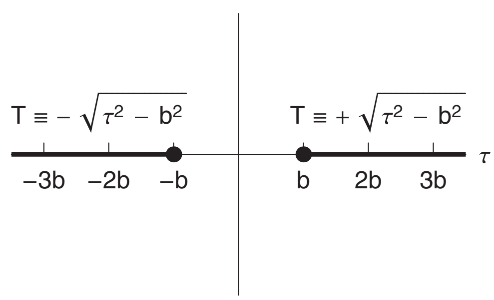

FIG. 1. Surgery on the real line with coordinate $\tau \in \mathbb{R}$ gives the cosmic time axis $\tau \in (-\infty, -b] \cup [b, \infty)$, where the points $\tau = -b$ and $\tau = b$ are identified (as indicated by the dots). A suitable cosmic time coordinate is given by $T \in \mathbb{R}$ from (2.2a). Each point of the cosmic time axis corresponds to a unique value of the coordinate T.

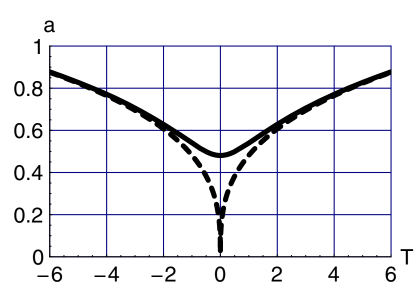

FIG. 2. Cosmic scale factor (full curve) of the modified spatially flat FLRW universe with $w = 1$ matter, as given by (2.5) with $b = 1$ and $T_0 = 4\sqrt{5}$. Also shown is the cosmic scale factor (dashed curve) of the standard FLRW universe with an extended cosmic time coordinate T, as given by (2.5) with $b = 0$ and $T_0 = 4\sqrt{5}$.

$$K(T) \propto (b^2 + T^2)^{-2}, \tag{2.6a}$$

$$\rho(T) \propto (b^2 + T^2)^{-1}. \tag{2.6b}$$

In terms of the auxiliary coordinate τ from (2.3), the bounce solution reads

$$a(\tau)\Big|^{(\mathrm{w}=1,\,\tau\text{-even sol.})}_{\text{mod. FLRW}} = \sqrt[6]{\tau^2/\tau_0^2}, \tag{2.7}$$

with $\tau_0^2 \equiv b^2 + T_0^2$.

We emphasize that the new input for this particular nonsingular bouncing cosmology is the metric *Ansatz* (2.1a). The other two inputs are standard [9], the Einstein equation and the energy-momentum tensor of the matter (here, matter with $w = 1$). The resulting modified Friedmann equation (2.4a), together with the standard equations (2.4b) and (2.4c), then gives the bounce-type scale factor (2.5). In the next section, we calculate some cosmological observables for this bounce-type FLRW universe.

III. COSMOLOGICAL OBSERVABLES

A. Null geodesics

The background metric is given by (2.1a). Particles travel on straight lines in the coordinate system $\{T, x^1, x^2, x^3\}$. So, we can consider geodesics of light that start at $T = T_1 < 0$ and end at $T = T_0 > 0$, while moving in the $x^1 \equiv X$ direction. Then, the reduced metric is

$$0 = ds^2\Big|^{(\text{light})}_{\text{mod. FLRW}} = -\frac{T^2}{b^2+T^2}dT^2 + a^2(T)dX^2, \tag{3.1}$$

where c has been set to unity. For matter with equation-of-state parameter $w = 1$, the cosmic scale factor $a(T)$ is given by (2.5).

With boundary condition $X(0) = 0$, we now have the following geodesic solution $X = X(T)$ from the reduced metric (3.1) and the cosmic scale factor (2.5):

$$X(T) = \begin{cases} +\frac{3}{2}\sqrt[6]{b^2+T_0^2}\,[\sqrt[3]{T^2+b^2} - \sqrt[3]{b^2}], & \text{for } T > 0, \\ -\frac{3}{2}\sqrt[6]{b^2+T_0^2}\,[\sqrt[3]{T^2+b^2} - \sqrt[3]{b^2}], & \text{for } T \le 0. \end{cases} \tag{3.2}$$

A plot of this null geodesic is given in Fig. 3.

The conclusion is that particles, in particular photons and gravitons, can travel from the prebounce phase to the postbounce phase without hindrance whatsoever.

B. Past particle horizon

At cosmic time $T_0 > 0$, the past particle horizon is infinite, as the universe extends back in time indefinitely. Explicitly, the particle horizon at $T_0 > 0$ reads

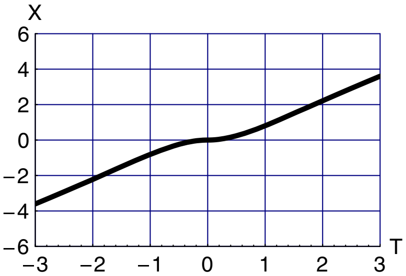

FIG. 3. Null geodesic (3.2) with $b = 1$ and $T_0 = 4\sqrt{5}$.

$$d_{\text{hor}}(T_0) = a(T_0)\lim_{\tau_1\to-\infty}\left[\int_{\tau_1}^{-b}\frac{d\tau''}{a(\tau'')} + \int_b^{\tau(T_0)}\frac{d\tau'}{a(\tau')}\right], \tag{3.3}$$

where $\tau(T_0) \equiv \tau_0$ is given by (2.3) and $a(\tau)$ by (2.7). For positive and finite values of b and τ_0, we get

$$\begin{aligned} d_{\text{hor}}(T_0) &= \frac{3}{2}a(T_0)\lim_{\tau_1\to-\infty}\left(\sqrt[3]{\tau_1^2\tau_0} - 2\sqrt[3]{b^2\tau_0} + \tau_0\right) \\ &= \frac{3}{2}a(T_0)\lim_{\tau_1\to-\infty}\sqrt[3]{\tau_1^2\tau_0}, \end{aligned} \tag{3.4}$$

which goes to $+\infty$. In other words, the past particle horizon at a finite positive cosmic time T_0 diverges for this particular bounce-type universe.

With an infinite particle horizon, there may be no horizon and flatness problems to worry about, and no need for inflation [10] (further references on inflationary models can be found in Ref. [11]). A succinct comparison between nonsingular bouncing cosmology models and big bang inflationary models appears in Ref. [3].

C. Modified Hubble diagrams

It is a straightforward exercise to calculate the luminosity distance d_L as a function of the redshift z, provided we distinguish two cases:

(1) the light is emitted by a comoving source in the expanding phase of the universe ($T_1 > 0$);
(2) the light is emitted by a comoving source in the contracting phase of the universe ($T_1 \le 0$).

In both cases, the light is detected by a comoving observer in the expanding phase at cosmic time $T_0 > 0$ with $T_0 > T_1$.

Using the auxiliary time coordinate τ from (2.3) with scale factor (2.7) and adapting the relevant formulae in Secs. 14. 4 and 14. 6 of Ref. [9], we obtain

$$d_L(\tau_0, \tau_1)\big|^{(\text{case 1})} = \frac{a^2(\tau_0)}{a(\tau_1)}\int_{\tau_1}^{\tau_0}\frac{d\tau'}{a(\tau')}, \tag{3.5a}$$

$$d_L(\tau_0,\tau_1)|^{(\text{case 2})} \equiv d_L^{(\text{pre})}(\tau_1) + d_L^{(\text{post})}(\tau_0) = \frac{a^2(-b)}{a(\tau_1)}\int_{\tau_1}^{-b}\frac{d\tau''}{a(\tau'')} + \frac{a^2(\tau_0)}{a(b)}\int_b^{\tau_0}\frac{d\tau'}{a(\tau')}, \quad (3.5b)$$

where light is emitted at cosmic time $\tau = \tau_1$ (with $\tau_1 > b$ for case 1 and $\tau_1 \le -b$ for case 2) and observed at $\tau = \tau_0 > b > 0$ with $\tau_0 > \tau_1$. Taking the positive function $a(\tau)$ from (2.7) and introducing the redshift,

$$z \equiv \sqrt{a^2(\tau_0)/a^2(\tau_1)} - 1 = a(\tau_0)/a(\tau_1) - 1, \quad (3.6)$$

the integrals in (3.5) give

$$d_L(z)\Big|_{z\in[0,z_{\max})}^{(\text{case 1})} = 3\tau_0\frac{1}{2}\left[1+z-\frac{1}{1+z}\right], \quad (3.7a)$$

$$d_L(z)\Big|_{z\in(-1,z_{\max}]}^{(\text{case 2})} = 3\tau_0\frac{1}{2}\left[1+z_{\max}-\frac{1}{1+z_{\max}} + \frac{1}{(1+z_{\max})^2}\left(\frac{1}{1+z}-\frac{1+z}{(1+z_{\max})^2}\right)\right], \quad (3.7b)$$

with definition

$$z_{\max} \equiv a(\tau_0)/a(b) - 1 = \sqrt[3]{\tau_0/b} - 1. \quad (3.7c)$$

The length scale $3\tau_0$ entering (3.7a) and (3.7b) is determined by (2.7),

$$3\tau_0 = \left[\frac{1}{a(\tau)}\frac{da(\tau)}{d\tau}\right]_{\tau=\tau_0}^{-1} \equiv [H_0^{(\tau-\text{def.})}]^{-1}, \quad (3.8)$$

where the Hubble constant $H_0^{(\tau-\text{def.})}$ differs from $H_0^{(T-\text{def.})} \equiv [da(T)/dT]/a(T)|_{T=T_0}$ by a factor close to unity, as long as $\tau_0 \gg b$.

The corresponding expressions for the angular diameter distance d_A read

$$d_A(\tau_0,\tau_1)|^{(\text{case 1})} = \frac{a^2(\tau_1)}{a^2(\tau_0)}d_L(z)|^{(\text{case 1})}, \quad (3.9a)$$

$$d_A(\tau_0,\tau_1)|^{(\text{case 2})} = \frac{a^2(\tau_1)}{a^2(-b)}d_L^{(\text{pre})}(\tau_1) + \frac{a^2(b)}{a^2(\tau_0)}d_L^{(\text{post})}(\tau_0). \quad (3.9b)$$

With the definitions in (3.5) and $a(\tau)$ from (2.7), the integrals give

$$d_A(z)\Big|_{z\in[0,z_{\max})}^{(\text{case 1})} = 3\tau_0\frac{1}{2}\frac{1}{(1+z)^2}\left[1+z-\frac{1}{1+z}\right], \quad (3.10a)$$

$$d_A(z)\Big|_{z\in(-1,z_{\max}]}^{(\text{case 2})} = 3\tau_0\frac{1}{2}\left[\frac{1}{(1+z)^3}-\frac{1}{(1+z_{\max})^3} + \frac{z_{\max}+z(1+z_{\max})}{(1+z_{\max})^2(1+z)}\right]. \quad (3.10b)$$

The modified Hubble diagram with the luminosity distance $d_L(z)$ is plotted in the left-panel of Fig. 4 and the one with the angular diameter distance $d_A(z)$ in the right-panel. The nonsmooth behavior at $z = z_{\max}$ in Fig. 4 is a direct manifestation of the spacetime defect and will be discussed further in Sec. III D.

The results in Fig. 4 are shown for a relatively small value of $z_{\max}$, in order to display the main characteristics of the modified Hubble diagrams. For very large values of $z_{\max}$ (as will appear in Sec. IV), it makes more sense to use a compactified redshift coordinate and to compress (or compactify) the distance axis. Specifically, we can use the following compactified variables:

$$\zeta \equiv \frac{z}{z+2} \in (-1,1), \quad (3.11a)$$

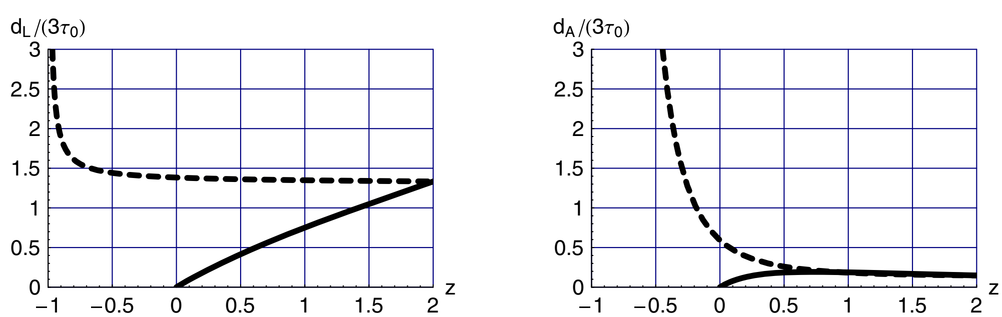

FIG. 4. Modified Hubble diagrams for, on the left, the luminosity distance $d_L(z)$ from (3.7) and, on the right, the angular diameter distance $d_A(z)$ from (3.10). The model parameters are $b/\tau_0 = 1/27$ and $z_{\max} = 2$. With a comoving observer in the expanding phase, the full curves correspond to case 1 (light emitted by a comoving source in the expanding phase of the universe) and the dashed curves to case 2 (light emitted by a comoving source in the contracting phase).

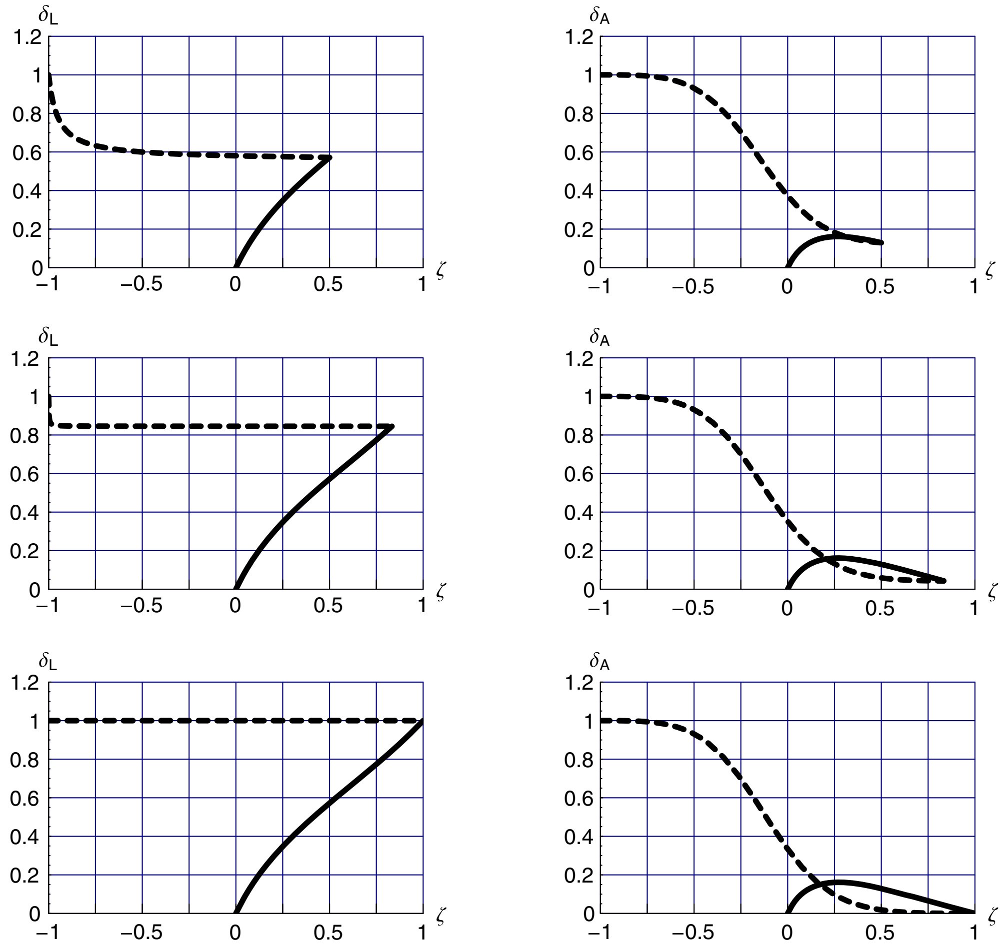

FIG. 5. Modified Hubble diagrams from (3.7) and (3.10), using the compactified redshift variable ζ from (3.11a) and the compactified distance variables δ_L from (3.11b) and δ_A from (3.11c). The top row has $z_{\max} = 2$, the middle row $z_{\max} = 10$, and the bottom row $z_{\max} = 10^{15}$. The top-row curves correspond to those of Fig. 4.

$$\delta_L \equiv \frac{d_L}{d_L + 3\tau_0} \in [0, 1), \tag{3.11b}$$

$$\delta_A \equiv \frac{d_A}{d_A + 3\tau_0} \in [0, 1). \tag{3.11c}$$

The resulting modified Hubble diagrams are shown in Fig. 5 for three values of $z_{\max}$.

After we completed our calculation of the luminosity distance, we became aware of Ref. [12], which discusses certain phenomenological aspects of a nonsingular bouncing cosmology but not the dynamics of the bounce. The behavior of the $n = 1/2$ curve in Fig. 1 of Ref. [12] agrees with the more or less constant behavior of the dashed curve in Fig. 4 of a previous version of this article [13], which considered the $w = 1/3$ case.

In the next two subsections, we will discuss these modified Hubble diagrams in more detail.

D. Cusps in the modified Hubble diagrams

The cusp behavior seen in Fig. 4 is of interest in that it shows that the spacetime defect at $T = 0$ (or $\tau = \pm b$) is not just a coordinate artifact, as it leads to observable effects. The discontinuity of the derivative $d_L'(z)$ at $z_{\max}$ from (3.7a) and (3.7b) traces back to the nontrivial τ_1 behavior in (3.5), due to the sharp change in slope of $a(\tau_1)$ between $\tau_1 \le -b$ and $\tau_1 \ge b$. This change of slope is explained by two facts (the overdot stands for differentiation with respect to τ). First, the modified first-order Friedmann equation (2.4a) in terms of the auxiliary coordinate τ implies that the value of $(\dot{a}/a)^2$ at $\tau = \pm b$ is nonvanishing if the value of ρ is. Second, the nonzero value of $\dot{a}/a$ can change sign between $\tau = -b$ and $\tau = +b$, because the interval between these two points ($\Delta\tau = 2b$) is nonvanishing, as long as b is nonzero. Incidentally, we have also calculated $d_L(z)$ from (3.5) with an *ad hoc* function $a(\tau) = 1 + (\tau^2/b^2 - 1)^2$ and find that the cusp in the resulting modified Hubble diagram is absent.

The possible cusp behavior of the luminosity distance $d_L(z)$ has, to the best of our knowledge, not been obtained before in other bouncing models. In Ref. [12], the authors did calculate the luminosity distances for different contracting phases but did not give a complete description, from contraction to expansion. Needless to say, a complete description of the luminosity distance is far from trivial for most of the bouncing models in the literature, as it depends on the details of the bouncing models (especially the dynamics at the bounce moment). As shown in Sec. III C, our bouncing model not only gives a complete description of the luminosity distance (or the angular diameter distance) but also displays a nontrivial effect such as the cusp behavior, which may be regarded as a characteristic of our bouncing model.

E. Gedankenexperiment

Assume that modified Hubble diagrams for $d_L(z)$ and $d_A(z)$ have been established. Then, we may consider a *Gedankenexperiment* to determine the numerical value of b (presupposing the relevance of the nonsingular bounce model of Sec. II to the real Universe). The simplest possible *Gedankenexperiment* uses only the modified $d_L(z)$ Hubble diagram and proceeds in three steps.

First, determine the numerical value of $z_{\max}$ from the modified $d_L(z)$ Hubble diagram, where the $z_{\max}$ value corresponds to the z-coordinate of the intersection point of the case-1 curve (3.7a) and the case-2 curve (3.7b); compare with, respectively, the full and dashed curves in the left-panel of Fig 4. With the obtained $z_{\max}$ value, calculate

$$\Xi_L \equiv 1 + z_{\max}. \tag{3.12}$$

In this subsection, we use upper-case Greek letters to denote experimental quantities.

Second, determine, in the modified $d_L(z)$ Hubble diagram, the numerical value of the slope of the upper (case-2) curve just below $z = z_{\max}$,

$$\Sigma_L \equiv \frac{d}{dz}\left[d_L(z)\right|^{(\text{case-2})}]_{z=z_{\max}} = \frac{-3}{1+z_{\max}}\, b, \tag{3.13}$$

where the last identification results from (3.7b). In principle, it is also possible to obtain other experimental quantities [for example, from the behavior of the curvature of the case-2 $d_L(z)$ curve just above $z = -1$], but the choice (3.13) suffices for the moment.

Third, the numerical value of b then follows from the previously determined values Ξ_L and Σ_L by calculating

$$b_{\text{num}} = -\frac{1}{3}\Sigma_L \Xi_L. \tag{3.14}$$

With $z_{\max}$ significantly larger than 1, the numerical value of b in (3.14) results from multiplying a reduced value (Σ_L) by a large number (Ξ_L). Note also that the $d_L(z)$ function obtained for the $w = 1/3$ case [13] gives the same result as in (3.14) but with the fraction $1/3$ on the right-hand side replaced by $1/2$.

Needless to say, we neglect all practical difficulties in this *Gedankenexperiment*, if at all feasible. A discussion of the cosmological context is given in Sec. IV.

IV. DISCUSSION

The construction of the spacetime manifold in Ref. [4] is entirely classical. But it could very well be that the classical length parameter b appearing in the metric (2.1a) has its origin in the (unknown) theory of "quantum spacetime," with a fundamental length scale related to the Planck length or not [8]. It is, then, possible to imagine that this quantum theory removes the classical times $\tau \in (-b, b)$ in Fig. 1 and ties together $\tau = -b$ and $\tau = b$, so that the resulting interval of the emerging classical time coordinate $T = T(\tau)$ has no boundary. In that case, there must be a classical "pre-big-bang" phase $T \leq 0$ and, in this article, we have studied some cosmological consequences (one example being the cusps in the modified Hubble diagrams of Fig. 4, as explained in Sec. III D).

Assuming the relevance of the nonsingular bounce model of Sec. II, we have discussed, in Sec. III E, a *Gedankenexperiment* to determine the numerical value of the length scale b entering the metric (2.1a). The required observations for this *Gedankenexperiment* would concern images originating *before* the known epoch of the "hot big bang" (postbounce in our model universe), which contains a hot plasma that would strongly scatter the light of any assumed "standard candles" (or "standard-size objects") in the prebounce phase. But it may very well be that the required standard candles emit gravitational waves instead of electromagnetic waves (light). For example, it is possible to consider a gravitational standard candle from a binary-black-hole merger [14] with definite masses (giving a recognizable "chirp"); see Ref. [12] for further discussion.

Hence, the *Gedankenexperiment* of Sec. III E relies on gravitational standard candles. The cosmological scenario for which this *Gedankenexperiment* may be relevant is as follows. In the prebounce phase and just after the bounce, the matter content of the universe can be described by a homogeneous perfect fluid with a constant equation-of-state parameter $w = 1$. The particular value $w \geq 1$ agrees with the absence of instabilities in the prebounce phase, as discussed in the third and fourth paragraphs of Sec. IV in Ref. [3], which also contains further references. After the bounce, the matter content of the universe changes to that of a homogeneous perfect fluid with $w \sim 1/3$, attributed to the presence of ultrarelativistic particles. In the Appendix, we present a model with a postbounce change of the equation-of-state parameter.

Even if the cosmological scenario as discussed holds true, there are formidable hurdles to overcome before the *Gedankenexperiment* can be converted into a realistic experiment. We only mention two. The first major hurdle (perhaps the most important one) concerns the actual presence and identification of the required gravitational standard candles (or gravitational standard-size objects), present before and after the bounce. The second major hurdle concerns the measurement of b by use of (3.14). Using the known postbounce age of the universe ($c\tau_0 \approx 10^{10}$ lyr $\approx 10^{26}$ m) and taking the maximum allowed value for b from (A6b), we get the following estimate from (3.7c) adapted to the post-bounce expansion $a(\tau) \propto \tau^{1/2}$ for the model of the Appendix:

$$1 + z_{\mathrm{max}} \sim 10^{15} \left(\frac{c\tau_0}{10^{26}\ \mathrm{m}}\right)^{1/2} \left(\frac{10^{-3}\ \mathrm{m}}{b}\right)^{1/2}. \tag{4.1}$$

This large number 10^{15} (or an even larger number if, for example, b is very much below the millimeter scale) makes the determination of b difficult, as mentioned in the sentence below (3.14). The slope entering (3.13), in particular, is suppressed by, at least, a factor 10^{-15}, making it hard to measure.

The experiment as outlined above will stay, for a long time to come, a *Gedankenexperiment* and the discussion will remain entirely academic. Still, the general idea appears to be valid and may perhaps be adapted to other circumstances.

Up till now, we have been talking primarily about direct images of prebounce structures (e.g., hypothetical binary-black-hole mergers emitting gravitational waves). But, as mentioned in Fig. 4 of Ref. [3], the currently observed "superhorizon" patterns in the cosmic microwave background may also be due to a prebounce phase, assuming that there has been such a phase. Hence, the crucial question is whether or not a cosmic bounce has occurred and, if so, what physics is responsible. The intriguing result from general relativity, extended to allow for degenerate metrics, is that a particular "regularization" of the standard big bang singularity indeed suggests the occurrence of a cosmic bounce.

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Note added.—The present article is a follow-up paper of Ref. [4]. There are now two more follow-up papers. The first of these papers [15] provides a scalar-field model for the type of time-asymmetric nonsingular bounce constructed in the Appendix. The second of these papers [16] gives a detailed analysis of the dynamics near the bounce.

APPENDIX: NONSINGULAR BOUNCE WITH A VARIABLE EQUATION OF STATE

In this Appendix, we give some results for a modified spatially flat FLRW universe with a time-dependent equation-of-state parameter. In fact, we take our cue from the general discussion of a particular classical nonsingular bouncing cosmology in Ref. [3]. With the notation $\epsilon \equiv (3/2)(1 + P/\rho) \equiv (3/2)(1 + W)$, Sec. IV of that paper states: "According to the bouncing scenario, at some point during or shortly after the bounce, the kinetic energy stored in scalar fields is converted to the matter and radiation we observe, with $\epsilon \leq 2$. The irreversible reheating process accounts for the asymmetry in ϵ about the bounce point." The main characteristics of that nonsingular bouncing cosmology are summarized in Fig. 3 of Ref. [3] and the goal of the present Appendix is to present a "fully-computable bounce model," as mentioned in Sec. 6 of Ref. [3].

With reduced-Planckian units ($8\pi G_N = c = \hbar = 1$), the modified spatially-flat Friedmann equation, the energy-conservation equation, and the assumed equation of state are given by

$$\left(1 + \frac{b^2}{T^2}\right)\left(\frac{1}{a(T)}\frac{da(T)}{dT}\right)^2 = \frac{1}{3}\rho(T), \tag{A1a}$$

$$\frac{d}{da}[a^3\rho(a)] + 3a^2 P(a) = 0, \tag{A1b}$$

$$P(T) = W(T)\rho(T), \tag{A1c}$$

$$W(T) = \begin{cases} \frac{1}{3} + \frac{2}{3}\exp\left[-\left(\sqrt{1 + T^2/b^2} - 1\right)^2\right], & \text{for } T > 0, \\ 1, & \text{for } T \leq 0, \end{cases} \tag{A1d}$$

where (A1c) and (A1d) provide an explicit realization of the required equation-of-state behavior of the nonsingular bouncing cosmology as displayed in Fig. 3 of Ref. [3]. The particular function $W(T)$ from (A1d) is shown, for model parameter $b = 1$, in the top-left panel of Fig. 6.

By reverting temporarily to the auxiliary coordinate τ from (2.3) and by focusing on the Hubble parameter $h(\tau) \equiv a^{-1}(\tau)[da(\tau)/d\tau]$ it is possible to get an analytic result:

$$H(T) \equiv \left[\frac{1}{a(T)}\frac{da(T)}{dT}\right] = \sqrt{\frac{T^2}{b^2 + T^2}}\,\bar{h}(T), \tag{A2a}$$

$$\rho(T) = 3\bar{h}^2(T), \tag{A2b}$$

$$\bar{h}(T) = \begin{cases} \left(b + 2\sqrt{b^2 + T^2} + \frac{1}{2}b\sqrt{\pi}\,\mathrm{erf}\left[\sqrt{1 + T^2/b^2} - 1\right]\right)^{-1}, & \text{for } T > 0, \\ (-3\sqrt{b^2 + T^2})^{-1}, & \text{for } T \leq 0, \end{cases} \tag{A2c}$$

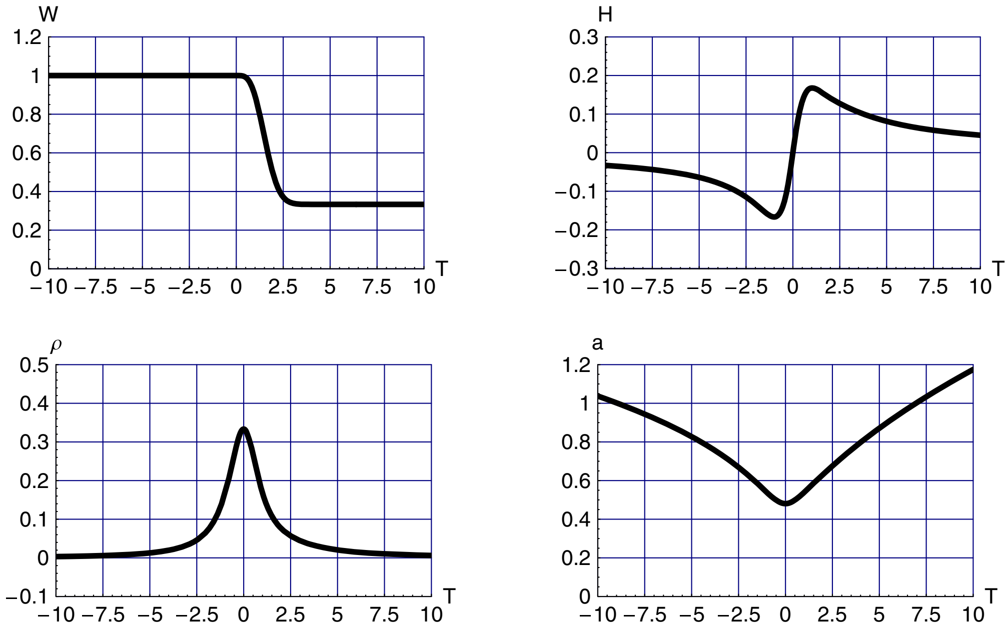

FIG. 6. Bounce-type universe from the modified Friedmann equation (A1a) with a postbounce change of the equation of state (A1c). Top-left panel: equation-of-state parameter $W(T)$ from (A1d). Top-right panel: Hubble parameter $H(T)$ from (A2a) and (A2c). Bottom-left panel: energy density $\rho(T)$ from (A2b) and (A2c). Bottom-right panel: numerical solution for the cosmic scale factor $a(T)$ from the ordinary differential equation (A2a) with boundary condition $a(-T_0) = 1$. The time-asymmetric behavior of $a(T)$ in the bottom-right panel is manifest [having, for example, $a(10) \neq a(-10)$] and differs from the symmetric behavior in Fig. 2. The model parameters are $\{b, \tau_0, T_0\} = \{1, 9, 4\sqrt{5}\}$ in reduced-Planckian units with $8\pi G_N = c = \hbar = 1$.

in terms of the error function

$$\mathrm{erf}(z) \equiv \frac{2}{\sqrt{\pi}} \int_0^z dt \exp(-t^2). \tag{A3}$$

From (A2a) and (A2c), we have $H(T) \sim (1/3)T^{-1} < 0$ for $T \ll -b$ and $H(T) \sim (1/2)T^{-1} > 0$ for $T \gg b$.

It does not appear possible to get $a(T)$ in an explicit analytic form, but the ordinary differential equation from (A2a) can be solved numerically for $a(T)$. Figure 6 shows the cosmological functions for a particular choice of model parameters, where the bottom-right panel displays the time asymmetry of the cosmic scale factor, $a(T) \neq a(-T)$ for $T \neq 0$. The corresponding luminosity distance d_L and angular diameter distance d_A (Fig. 7) are found to be qualitatively the same as those from Sec. III (Fig. 4).

From (A2b) and (A2c), we find that the maximum value of the energy density (which occurs at $T = 0$) remains finite, provided the defect length scale b is nonzero

$$\rho(0) = \frac{1}{3} E_{\mathrm{planck}}^2 b^{-2}, \tag{A4}$$

in terms of the reduced Planck energy,

$$E_{\mathrm{planck}} \equiv \sqrt{\hbar c^5/(8\pi G_N)} \approx 2.44 \times 10^{18}\ \mathrm{GeV}. \tag{A5}$$

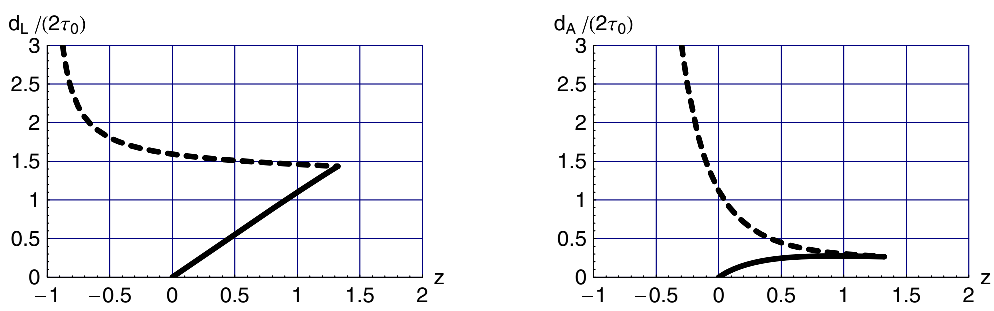

FIG. 7. Modified Hubble diagrams based on numerical results for the luminosity distance d_L from (3.5) and the angular diameter distance d_A from (3.9) in the bounce-type universe of Fig. 6. For the model parameters chosen, the maximum redshift is given by $z_{\mathrm{max}} \equiv a(T_0)/a(0) - 1 \approx 1.32425$.

Demanding

$$\rho(0) \gtrsim (\mathrm{TeV})^4, \tag{A6a}$$

in order to reproduce, in the postbounce phase, the hot-big-bang model with temperatures $\mathcal{T} \lesssim$ TeV, we have the following qualitative upper bound on the defect length scale b from (A4):

$$b \lesssim \left(\frac{E_{\mathrm{planck}}}{\mathrm{TeV}}\right) \hbar c/\mathrm{TeV} \approx 10^{15}\, \hbar c/\mathrm{TeV} \approx 10^{-3}\ \mathrm{m}. \tag{A6b}$$

The millimeter scale has, of course, appeared before in higher-dimensional TeV gravity [17], essentially tracing back to the Einstein equation [which, here, gives rise to (A4)].

For the record, we can mention that we also have a qualitative lower bound on the defect length scale b. Demanding

$$\rho(0) \lesssim (E_{\mathrm{planck}})^4, \tag{A7a}$$

in order that the classical Einstein theory be applicable, we have the following qualitative lower bound on b from (A4):

$$b \gtrsim \hbar c/E_{\mathrm{planck}} \equiv l_{\mathrm{planck}} \approx 8.10 \times 10^{-35}\ \mathrm{m}. \tag{A7b}$$

Such a minimal length scale is also expected, on general grounds, for the emerging classical spacetime [2].

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