Constraining a causal dissipative cosmological model

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In this paper, a cosmological solution of the polynomial type $H \approx (t + \mathrm{const})^{-1}$ for the causal thermodynamical approach of Isarel-Stewart [Phys. Rev. D **96**, 124020 (2017); Phys. Lett. B **767**, 103 (2017)] is constrained using the joint of the latest measurements of the Hubble parameter (OHD) and type Ia supernovae (SNIa). Since the expansion described by this solution does not present a transition from a decelerated phase to an accelerated one, both phases can be well modeled, connecting both phases by requiring the continuity of the Hubble parameter at $z = z_t$, the accelerated-decelerated transition redshift. Our best fit constrains the main free parameters of the model to be $A_1 = 1.58^{+0.08}_{-0.07}$ ($A_2 = 0.84^{+0.02}_{-0.02}$) for the accelerated (decelerated) phase. For both phases, we obtain $q = -0.37^{+0.03}_{-0.03}$ ($0.19^{+0.03}_{-0.03}$) and $\omega_{\mathrm{eff}} = -0.58^{+0.02}_{-0.02}$ ($-0.21^{+0.02}_{-0.02}$) for the deceleration parameter and the effective equation of state, respectively. Comparing our model and LCDM statistically through the Akaike information criterion and the Bayesian information criterion, we obtain that the LCDM model is preferred by the OHD + SNIa data. Finally, it is shown that the constrained parameters values satisfy the criterion for a consistent fluid description of a dissipative dark matter component, but with a high value of the speed of sound within the fluid, which is a drawback for a consistent description of the structure formation. We briefly discuss the possibilities to overcome this problem with a nonlinear generalization of the causal linear thermodynamics of bulk viscosity and also with the inclusion of some form of dark energy.

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I. INTRODUCTION

The Universe is currently in an accelerated expansion epoch that has been observed through the type Ia supernovae (SNIa) [1,2] and the large-scale structure (LSS) [3]. Typically, this phenomenon is associated to a component known as dark energy (DE), and together with the one named dark matter (DM), it constitutes the dark sector that corresponds to about 96% of the Universe [4]. The simplest cosmological model to explain this dark sector and also compatible with the observational data is the so-called Λ-cold dark matter (ΛCDM). This model proposes a cosmological constant responsible for the accelerated expansion of the Universe and a nonrelativistic entity without pressure as the dark matter. However, one of the open problems in the investigation of the dark sector is its division into DM and DE, which has been proven to be merely conventional since there exists a degeneracy between both components, resulting from the fact that gravity only measures the total energy tensor [5,6]. So, in the lack of a well confirmed detection (nongravitational) of the DM, only the overall properties of the dark sector can be inferred from the cosmological data at the background and perturbative levels. These results have driven the research to explore alternative models which consider a single fluid that behaves as DM but also presents the effects of an effective negative pressure at some stage of the cosmic evolution. They are called unified DM models (UDM), and examples of them are (generalized) Chaplygin fluids [7–10], a logotropic dark fluid [11], and more recently, generalized perfect fluid models [12,13]. Apart from them, there exists the possibility of explaining the accelerated expansion of the Universe at late times as an effect of the effective negative pressure due to bulk viscosity in the cosmic fluids that was first considered in [14,15]. Several models regarding this approach have been studied and constrained using cosmological data [16–21].

A consistent description of the relativistic thermodynamics of nonperfect fluids is the causal description framework given by the Israel-Stewart (IS) theory [22]. Due to the high degree of nonlinearity of the differential equations involved, only some exact solution has been

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found for a simple ansatz of the bulk viscosity coefficient ξ, as a function of the energy density ρ of the fluid with dissipation. For the election $\xi = \xi_0\rho^{1/2}$, a cosmological solution of the polynomial type $H \approx (t + \mathrm{const})^{-1}$ for the Hubble rate was found as an ansatz in [23,24], which can describe accelerated, decelerated, or even a phantom type cosmic expansion.

This solution can also be obtained in a systematic way by applying the factorization method to the dynamics equation for the Hubble rate. The factorization of second-order linear ordinary differential equations (ODEs) is a well established method to get solutions in an algebraic manner. It goes back to some works of Dirac to solve the spectral problem for the quantum oscillator [25], and was further developed due to Schrodinger's works on the factorization of the Sturm-Liouville equation [26,27]. However, in recent times, the factorization technique has been developed and applied to find exact solutions of nonlinear second-order ODEs [28–34]. The basic concept follows the same pattern already used in linear equations, and it works efficiently for ODEs with polynomial nonlinearities. The method is well adapted to the Hubble rate ODE, which raises, for instance, in viscous cosmological models [35,36].

The main aim of this work is to constrain this solution using the latest measurements of the Hubble parameter (OHD) and type Ia supernovae (SNIa), reported in [37] and [38], respectively. Despite the fact that the expansion described by this solution does not present a transition from a decelerated phase to an accelerated one, which is an ultimate feature supported by the observational data, both phases can be well modeled by separation using the analytical solution obtained, as we will show in our results.

In the case of the noncausal Eckart's approach, ξ_0 can be estimated, for example, directly from the observational data [39]. Nevertheless, in the case of our solution, the observational constraints lead to allowed regions for ξ_0 and the parameter ϵ, which is related to the nonadiabatic contribution to the speed of sound in the viscous fluid as will be discussed in Sec. II. Since the above mentioned parameters are involved in a constraint which is a necessary condition for maintaining the thermal equilibrium, we will discuss our results considering such a constraint.

This paper is organized as follows: in Sec. II, we describe briefly the causal Israel-Stewart theory, showing the general differential equation to be solved. In Sec. III, we solve this differential equation by using the factorization technique. In Sec. IV, we present the constraints for our model using the observational data coming from the direct measurements of the Hubble parameter and SNIa. Finally, in Sec. V, we discuss our results.

II. ISRAEL-STEWART-HISCOCK FORMALISM

In what follows, we shall present briefly the Israel-Stewart-Hiscock formalism to describe the thermodynamic properties and evolution of a Universe filled with only one fluid as the main component, which experiments a dissipative process during its cosmic evolution. We assume that this fluid obeys a barotropic EoS, $p = \omega\rho$, where p is the barotropic pressure and $0 \leq \omega < 1$. For a flat Friedman-Lemaitre-Robertson-Walker Universe, the equation of constraint is

$$3H^2 = \rho. \tag{1}$$

In the ISH framework, the transport equation for the viscous pressure Π is given by [22]

$$\tau\dot{\Pi} + \left(1 + \frac{1}{2}\tau\Delta\right)\Pi = -3\xi(\rho), \tag{2}$$

where the "dot" accounts for the derivative with respect to the cosmic time. τ is the relaxation time, $\xi(\rho)$ is the bulk viscosity coefficient, for which we assume the dependence upon the energy density ρ, H is the Hubble parameter, and Δ is defined by

$$\Delta = 3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{\xi}}{\xi} - \frac{\dot{T}}{T}, \tag{3}$$

where T is the barotropic temperature, which takes the form $T = \beta\rho^{\omega/(\omega+1)}$ that is the Gibbs integrability condition when $p = \omega\rho$ and β is a positive parameter. We also have that [40]

$$\frac{\xi}{(\rho + p)\tau} = c_b^2, \tag{4}$$

where c_b is the speed of bulk viscous perturbations (the nonadiabatic contribution to the speed of sound in a dissipative fluid without heat flux or shear viscosity), $c_b^2 = \epsilon(1 - \omega)$, and $0 < \epsilon \leq 1$, in order to ensure causality, with a dissipative speed of sound lower to or equal to the speed of light. We shall also assume a power law dependence for ξ in terms of the energy density of the main fluid, i.e., $\xi = \xi_0\rho^s$, where s is an arbitrary parameter and ξ_0 a positive constant, in order to satisfy the second law of thermodynamics [41]. This particular election of $\xi(\rho)$ is rather arbitrary but allows us to obtain a differential equation for the Hubble parameter that can be integrated for some particular values of s, obtaining well-known analytic solutions. As we will be discussed below, the case $s = 1/2$ leads to the most simple form of the differential equation involved.

Using the barotropic equation of state (EoS) in Eq. (4), we obtain the following expression for the relaxation time:

$$\tau = \frac{\xi_0}{\epsilon(1 - \omega^2)}\rho^{s-1}, \tag{5}$$

and according to Eq. (3),

$$\Delta = \frac{3H}{\delta(\omega)}\left(\delta(\omega) - \frac{\dot{H}}{H^2}\right), \tag{6}$$

where we have defined the $\delta(\omega)$ parameter by

$$\delta(\omega) \equiv \frac{3}{4}\left(\frac{1+\omega}{1/2+\omega}\right). \tag{7}$$

So, for $0 \leq \omega < 1$, $\delta(\omega) > 0$. Using Eqs. (1) and (5), we can write

$$\tau H = \frac{3^{s-1}\xi_0}{\epsilon(1-\omega^2)} H^{2(s-1/2)}. \tag{8}$$

For the particular case $s = 1/2$, we obtain that

$$\tau H = \frac{\xi_0}{\sqrt{3}\epsilon(1-\omega^2)}. \tag{9}$$

In this case, the necessary condition for keeping the fluid description of the dissipative dark matter component is given by $\tau H < 1$, which leads to the upper limit for ξ_0,

$$\xi_0 < \sqrt{3}\epsilon(1-\omega^2). \tag{10}$$

We will discuss later this condition when a cold dark matter fluid with dissipation, as the main component of a late time Universe, can be constrained by the observational data.

The differential equation for the Hubble parameter can be constructed by using the conservation equation,

$$\dot{\rho} + 3H[(1+\omega)\rho + \Pi] = 0, \tag{11}$$

the Eqs. (1) and (2), and the relation $\xi(\rho) = \xi_0\rho^s$. So, we can obtain the following differential equation:

$$\left[\frac{2}{3(1-\omega^2)}\left(\frac{3(1+\omega)\dot{H}}{H^2} + \frac{\ddot{H}}{H^3}\right) - 3\right] H^{2(s-1/2)} + \frac{1}{3^s\xi_0}\left[1 + \frac{3^{s-1}\xi_0\Delta H^{2(s-1)}}{2(1-\omega^2)}\right]\left[3(1+\omega) + \frac{2\dot{H}}{H^2}\right] = 0. \tag{12}$$

For $s = 1/2$, the following ansatz:

$$H(t) = A(t_s - t)^{-1} \tag{13}$$

is a solution of Eq. (12) with a big rip singularity [23], and the ansatz,

$$H(t) = A(t - t_s)^{-1}, \tag{14}$$

is also a solution which can describe cosmic evolutions with accelerated, linear, and decelerated expansion [24]. In the next section, we will show that by using the factorization method, this ansatz can be obtained as a particular solution of the differential equation (12), which gives a deeper understanding of its particularity and its dependence on the initial conditions.

III. SOLVING THE DIFFERENTIAL EQUATION FOR THE HUBBLE RATE

The nonlinear differential equation for the Hubble function (12) can be rewritten for $s = 1/2$ as follows:

$$\ddot{H} + \frac{\alpha_1}{H}\dot{H}^2 + \alpha_2 H\dot{H} + \alpha_3 H^3 = 0, \tag{15}$$

where

$$\alpha_1 = -\frac{3}{2\delta}, \tag{16}$$

$$\alpha_2 = \frac{3}{2} + 3(1+\omega) - \frac{9}{4\delta}(1+\omega) + \frac{\sqrt{3}\epsilon(1-\omega^2)}{\xi_0}, \tag{17}$$

$$\alpha_3 = \frac{9}{4}(1+\omega) + \frac{9}{2}\epsilon(1-\omega^2)\left[\frac{1+\omega}{\sqrt{3}\xi_0} - 1\right], \tag{18}$$

are constant coefficients.

Let us consider the following factorization scheme [29–31] to obtain an exact particular solution of the Eq. (15). The nonlinear second order differential equation,

$$\ddot{H} + f(H)\dot{H}^2 + g(H)\dot{H} + j(H) = 0, \tag{19}$$

where $\dot{H} = \frac{dH}{dt} = D_t H$, can be factorized in the form,

$$[D_t - \phi_1(H)\dot{H} - \phi_2(H)][D_t - \phi_3(H)]H = 0, \tag{20}$$

where $\phi_i(H)$ ($i = 1, 2, 3$) are factoring functions to be found. Expanding Eq. (20), one is able to group terms as follows [31]:

$$\ddot{H} - \phi_1\dot{H}^2 + \left(\phi_1\phi_3 H - \phi_2 - \phi_3 - \frac{d\phi_3}{dH}H\right)\dot{H} + \phi_2\phi_3 H = 0. \tag{21}$$

Then, by comparing Eq. (19) with Eq. (21), we get the following conditions:

$$f(H) = -\phi_1, \tag{22}$$

$$g(H) = \phi_1\phi_3 H - \phi_2 - \phi_3 - \frac{d\phi_3}{dH}H, \tag{23}$$

$$j(H) = \phi_2\phi_3 H. \tag{24}$$

Any factorization like (20) of an scalar ODE in the form given in (19) allows us to find a compatible first order ODE [28],

$$[D_t - \phi_3(H)]H = D_t H - \phi_3(H)H = 0, \tag{25}$$

whose solution provides a particular solution of Eq. (19).

We apply now the previous scheme to Eq. (15). The factoring function $\phi_1 = -\frac{\alpha_1}{H}$, since $f(H)$ is explicitly given in Eq. (15). Also, according to Eq. (24), the two unknown functions ϕ_2 and ϕ_3 are easily obtained by merely factoring the polynomial expression $j(H) = \alpha_3 H^3$ given as well in Eq. (15). Then, the functions,

$$\phi_2 = a_1^{-1} H, \quad \text{and} \quad \phi_3 = a_1 \alpha_3 H, \tag{26}$$

where $a_1 (\neq 0)$ is an arbitrary constant, are proposed.

The explicit value of a_1 is obtained by substituting $g(H) = \alpha_2 H$ and the ϕ_i functions into Eq. (23). Then, we get the constraint equation,

$$\alpha_2 H = -(a_1 \alpha_1 \alpha_3 + a_1^{-1} + 2 a_1 \alpha_3) H, \tag{27}$$

and equating both sides of the equation provides

$$a_1 = \frac{-\alpha_2 \pm \sqrt{\alpha_2^2 - 4\alpha_3(2+\alpha_1)}}{2\alpha_3(2+\alpha_1)}. \tag{28}$$

Therefore, the Eq. (15) admits the factorization,

$$\left[D_t + \frac{\alpha_1}{H}\dot{H} - a_1^{-1} H\right][D_t - a_1 \alpha_3 H] H = 0, \tag{29}$$

with the compatible first order ODE,

$$\dot{H} - a_1 \alpha_3 H^2 = 0, \tag{30}$$

whose solution is also a particular solution of the Eq. (15) factorized in the form (29).

The integration of this equation generates one arbitrary integration constant, which can be written in explicit terms of an initial condition. If we consider the initial condition $H(t_0) = H_0$, where H_0 is the Hubble constant, then we get the following particular solution of Eq. (12) with $s = 1/2$:

$$H(t) = \frac{A_\pm}{t - (t_0 - \frac{A_\pm}{H_0})}, \tag{31}$$

where $A_\pm = -\frac{1}{\alpha_3 a_1}$, or equivalently,

$$A_\pm = \frac{2\sqrt{3}\epsilon(\omega^2 - 1) - 6\xi_0 \pm 2\sqrt{3}\epsilon\sqrt{\epsilon(\omega^2-1)^2 + 6\xi_0^2(1-\omega)}}{3(\omega+1)(-3\xi_0 + 2\epsilon(\omega-1)[\sqrt{3}(1+\omega) - 3\xi_0])}, \tag{32}$$

with the restriction equation $\xi_0 \neq \frac{2\sqrt{3}\epsilon(\omega^2-1)}{3+6\epsilon(\omega-1)}$, which avoids $A_\pm$ to be an indeterminate function. The above particular solution (31) can also be written in the form,

$$H(t) = \frac{A_\pm}{t - t_s}, \tag{33}$$

where $t_s = t_0 - \frac{1}{H_0(1+q_0)}$, and q_0 is the initial value of the deceleration parameter, but since

$$1 + q = -\frac{\dot{H}}{H^2} = \frac{1}{A_\pm}, \tag{34}$$

this means that this solution represents an expansion with a constant deceleration parameter. Once a q_0 is given, a value is obtained for $A_\pm$ and a family of possible values for the parameters ϵ, ω, and ξ_0 can be evaluated from Eq. (32). Or, once the value of $A_\pm$ is given, or constrained from the data, as it will be done in the next section, q_0 and the other ranges of the parameters can be evaluated.

The solution (33) can also be written in terms of the redshift variable. For the scale factor, one obtains

$$\frac{a}{a_0} = \left(\frac{t - t_s}{t_0 - t_s}\right)^{A_\pm} = \frac{1}{1+z}. \tag{35}$$

Therefore,

$$H(z) = H_0 (1+z)^{1/A_\pm}, \tag{36}$$

where $H_0 = 100\ h\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, and h denotes the dimensionless Hubble constant. Notice that this form of the Hubble parameter is defined for both phases of the Universe, the accelerated and decelerated one. However, we can connect both phases by requiring the continuity of the Hubble parameter function at $z = z_t$, where z_t is the accelerated-decelerated transition redshift. Then, we obtain

$$H(z) = \begin{cases} H_0 (1+z)^{1/\hat{A}_1}, & z \leq z_t, \\ H_0 (1+z_t)^{1/\hat{A}_1 - 1/\hat{A}_2} (1+z)^{1/\hat{A}_2}, & z > z_t. \end{cases} \tag{37}$$

In the above expression, $\hat{A}_1$ and $\hat{A}_2$ are the free parameters corresponding to the accelerated and decelerated phases, respectively.

TABLE I. Priors considered for the model parameters.

Parameter	Prior
$\hat{A}_1$	Flat in [1, 5]
$\hat{A}_2$	Flat in [0, 1]
h	Gaus(0.7324, 0.0174)

TABLE II. Best fit values of the free parameters of the UDM model.

Data	χ^2	$\hat{A}_1$	$\hat{A}_2$	h	$\mathcal{M}$	BIC	AIC
OHD	34.10	$1.58^{+0.15}_{-0.12}$	$0.84^{+0.02}_{-0.02}$	$0.700^{+0.014}_{-0.014}$	…	57.69	40.10
SNIa	1029.48	$1.62^{+0.11}_{-0.10}$	$0.71^{+0.16}_{-0.14}$	$0.732^{+0.017}_{-0.017}$	$5.76^{+0.05}_{-0.05}$	1085.12	1035.48
OHD + SNIa	1064.91	$1.58^{+0.08}_{-0.07}$	$0.84^{+0.02}_{-0.02}$	$0.700^{+0.010}_{-0.010}$	$5.67^{+0.02}_{-0.02}$	1120.93	1070.91

IV. COSMOLOGICAL CONSTRAINTS

In this section, we describe the observational data used and build the χ^2-function to perform the confidence regions of the free model parameters. We employ a chain Markov Monte Carlo analysis based on emcee module [42] by setting 5000 chains with 500 steps. The nburn is stopped up to a value of 1.1 on each free parameter in the Gelman-Rubin criteria [43]. Table I presents the priors considered for each parameter. We also set the redshift of the accelerated-decelerated transition as $z_t = 0.64$ [44] in the Eq. (37). Then, in order to constrain the model parameters, we use the Hubble parameter measurements and supernovae data, and the combined data.

A. Hubble observational data

The direct way to observe the expansion rate of the Universe is through measurements of the Hubble parameter (OHD) as a function of the redshift, $H(z)$. The latest OHD obtained using the differential age (DA) method [45] are compiled in [37] and consist of 51 Hubble parameter points covering the redshift range [0, 1.97]. We constrain the free model parameters by minimizing the chi-square function,

$$\chi^2_{OHD} = \sum_i \left(\frac{H_{th}(z_i) - H_{\rm obs}}{\sigma^i_{\rm obs}} \right)^2, \tag{38}$$

where $H_{th}(z_i)$ and $H_{\rm obs}(z_i) \pm \sigma^i_{\rm obs}$ are the theoretical and observational Hubble parameter at the redshift z_i, respectively.

B. Type Ia supernovae

We use the Pantheon data set [38] consisting of 1048 type Ia supernovae (SNIa) located into the range $0.01 < z < 2.3$. The comparison between data and model is obtained with the expression,

$$\chi^2_{SNIa} = (m_{th} - m_{\rm obs}) \cdot {\rm Cov}^{-1} \cdot (m_{th} - m_{\rm obs})^T, \tag{39}$$

where $m_{\rm obs}$ is the observational bolometric apparent magnitude and ${\rm Cov}^{-1}$ is the inverse of the covariance matrix. m_{th} is the theoretical estimation and is computed by

$$m_{th}(z) = \mathcal{M} + 5 \log_{10} [d_L(z)/10pc]. \tag{40}$$

Here, $\mathcal{M}$ is a nuisance parameter and $d_L(z)$ is the dimensionless luminosity distance given by

$$d_L(z) = (1+z) c \int_0^z \frac{dz'}{H(z')}, \tag{41}$$

where c is the speed of light.

C. Joint analysis

We also perform a joint analysis by defining the merit-of-function as

$$\chi^2_{\rm joint} = \chi^2_{OHD} + \chi^2_{SNIa}, \tag{42}$$

where χ^2_{OHD} and χ^2_{SNIa} are given in Eqs. (38) and (39), respectively. The best fitting parameters are obtained by setting the acceleration-deceleration transition $z_t = 0.64$ [44]. Table II presents the summary of the best estimates of the parameters for the dissipative unified dark matter (DUDM) model [see Eq. (37)].

Figure 1 shows the best fit curves over OHD and SNIa samples at the top and bottom panel, respectively, using the

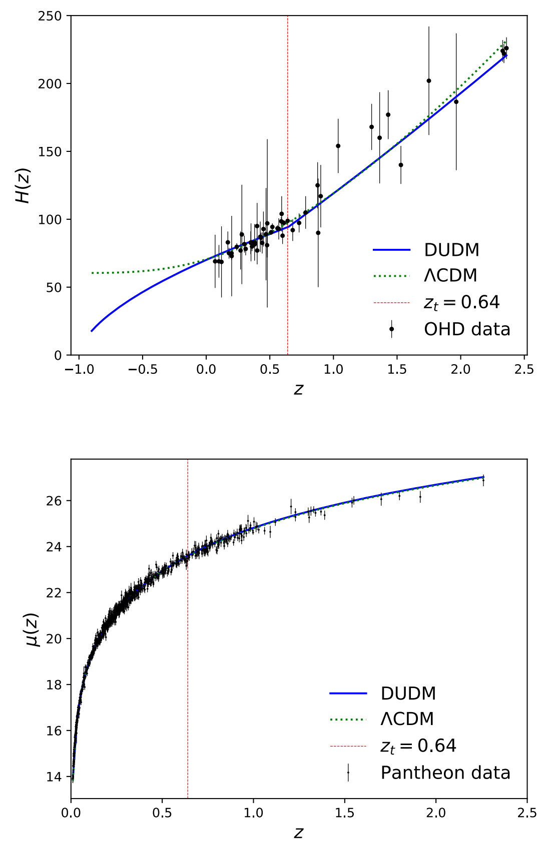

FIG. 1. Joint best fit of DUDM and ΛCDM using the best fitting values of joint analysis.

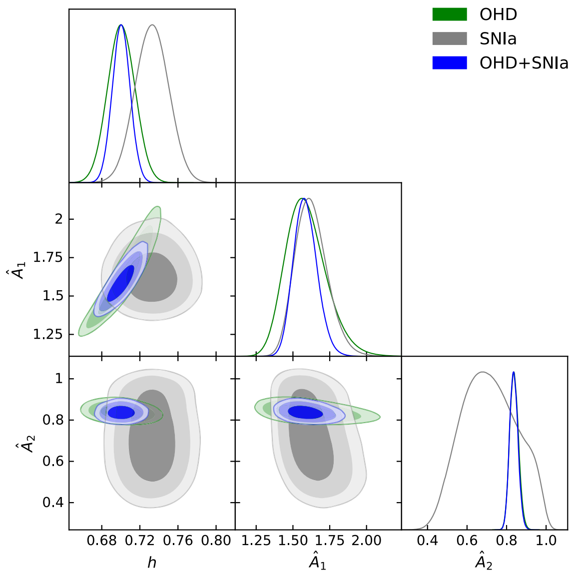

FIG. 2. 2D contours considering OHD (green), SNIa (gray), and joint analysis (blue) at the 68%, 95%, 99.7% confidence level.

joint analysis. We observe an evident behavior in the Hubble parameter between the DUDM and LCDM at $z < 0$ (the future). While LCDM gives a Universe expansion smoothly, the DUDM model has a Universe expansion as a big rip. From Eq. (34) and the joint analysis values, we estimate the decelerated parameter $q = -0.37$ and 0.19 for the accelerated and decelerated phases, respectively. Notice that q is constant during each phase.

Figure 2 shows the 2D contours at the 68%, 95%, and 99.7% (1σ, 2σ, and 3σ) confidence level (CL) and the 1D posterior distributions of the free model parameters. It shows a good agreement between the best fits within 1σ CL.

V. DISCUSSION

In the following, we will refer the mathematical expressions given in Eq. (32) as A_+ or A_-, and the numerical values of each expression could be $\hat{A}_1$ or $\hat{A}_2$. By using Eq. (32), which gives $A_\pm$ as a function of the model parameters, we explore the behavior of ξ_0 as a function of ϵ. These curves are shown in Fig. 3 for several values of $\omega = 0, 0.05, 0.1$ (from bottom curve to the top one) and are obtained when we consider the positive (top panel) and negative (bottom panel) sign in Eq. (32), i.e., A_+ and A_-, respectively. For A_+, we find values $\xi_0 > 0$ in the region $0.5 < \epsilon < 1$ and $\xi_0 < 0$ for $0 < \epsilon < 0.5$. Similarly, when we consider A_-, we find positive values of ξ_0 in the allowed region $\xi_0 < \sqrt{3}\epsilon$ in $0.5 < \epsilon < 1$ for both epochs. In contrast, we find values of $\xi_0 < 0$ within $0 < \epsilon < 0.5$ for both epochs when any sign is considered. Then, we discard the phase space of ϵ, ξ_0 where $\xi_0 < 0$ because the second law of thermodynamics would be infringed. It is

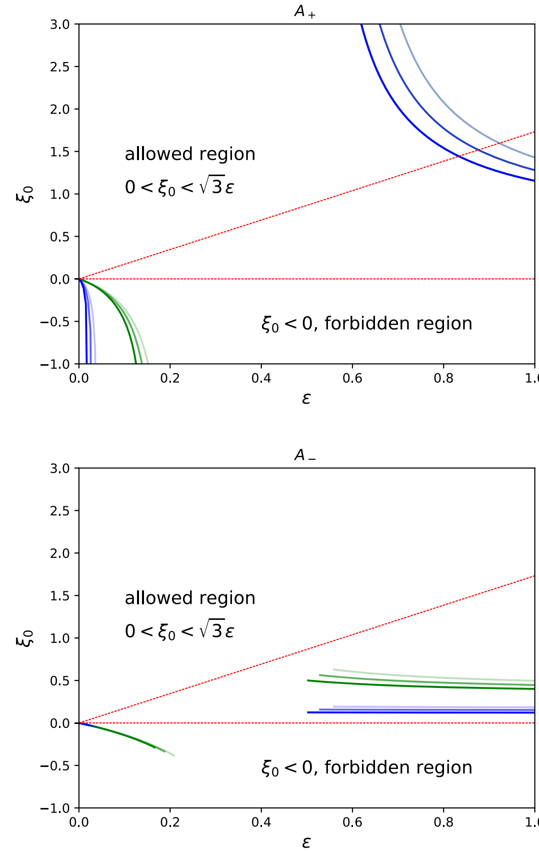

FIG. 3. Top (bottom) panel displays the behavior of ξ_0 as a function of ϵ considering the positive (negative) sign of the Eq. (32). The green (blue) color lines correspond to the $\hat{A}_1$ ($\hat{A}_2$) value. In the top panel, the green and blue lines in the region $\xi_0 > 0$ are superimposed. For both plots and each color, from bottom to top, the green (blue) lines refer to $\omega = 0, 0.05, 0.1$, respectively.

interesting to note that when we use A_+ and $\xi_0 > 0$, the curves for decelerated/accelerated epochs are not sensitive to the $\hat{A}_{1,2}$ values (see Table II).

A further insight of the previous results can be done considering the effective EoS, $\omega_{\rm eff}$, which is defined by

$$\omega_{\rm eff} = -1 - \frac{2}{3}\frac{\dot{H}}{H^2} = -1 + \frac{2}{3}\frac{1}{\hat{A}}, \tag{43}$$

where $\hat{A}$ takes the values $\hat{A}_1$ or $\hat{A}_2$ for the accelerated or decelerated phase, respectively, For the accelerated phase, we take, $\hat{A}_1 = 1.58^{+0.08}_{-0.07}$, corresponding to the obtained value using OHD + SNIa data. In this case, we obtain from Eq. (43) that $\omega_{\rm eff} = -0.58^{+0.02}_{-0.02}$, which means that the dissipative effects drive a quintessence like behavior. For the same set of data, $\hat{A}_2 = 0.84^{+0.02}_{-0.02}$ in the decelerated phase and $\omega_{\rm eff} = -0.21^{+0.02}_{-0.02}$; therefore, in this case, even with dissipative effects present in the dark matter fluid, they are not enough to drive acceleration. We find a deviation of 6.2σ over the region of quintessence ($\omega < -1/3$).

Despite the fact that the solution found does not display a smooth transition in the deceleration parameter, it allows us to describe both phases separately by using such a solution with the parameters $\hat{A}_1$ and $\hat{A}_2$, derived from cosmological data.

On the other hand, the condition to keep the fluid description of the dark matter component, which is an essential assumption of the thermodynamical formalism invoked, given by Eq. (10), provides the upper limit $\xi_0 < \sqrt{3}\epsilon$ for a pressureless dark matter fluid. By a simple inspection of the curves displayed in Fig. 3, it is easy to see that the constraint can be satisfied by both solutions A_+ and A_- and also in the case of the decelerated and accelerated expansions. Nevertheless, this constraint is fulfilled for approximately $\epsilon > 0.82$ for the A_+ solution and for $\epsilon > 0.5$ in the A_- solution. This fact indicates that it needs a great nonadiabatic contribution to the speed of sound within the fluid. It is well known that the structure formation observed implies a very low speed of sound, consistent with a cold dark matter component. Therefore, this issue represents a weakness of the model. Moreover, in the case of the solution with the accelerated expansion, the thermal equilibrium of the fluid can not be maintained. Besides, a positive entropy production and the convexity condition, $d^2S/dt^2 < 0$, are only satisfied by the decelerated solution, as it was shown in [24].

The solution analyzed in this work takes the simple form given by Eq. (33), which is too simple and clearly not a general solution of the IS formalism. In fact, only one initial condition is enough to determine the solution, and since it represents a cosmic expansion with a constant deceleration parameter, the other initial condition, q_0, necessary to determine the solution of a second order differential equation in the Hubble parameter, plays no role at all. It is hoped that more general solutions could overcome the above spotlighted difficulties.

Finally, we compare the DUDM and LCDM statistically through the Akaike information criterion (AIC) [46,47] and the Bayesian information criterion (BIC) [48]. The AIC and BIC are defined by $\mathrm{AIC} = \chi^2 + 2k$ and $\mathrm{BIC} = \chi^2 + 2k\ln(N)$, respectively, where χ^2 is the χ^2 function, k is the number of degree of freedom, and N is the data size. The preferred model by data is the one with the minimum value of these quantities. In order to compare the models, we use the full data sample, OHD + SNI, and obtain the χ^2 as the sum of the ones obtained in the decelerated and accelerated phases for the DUDM model. Then, we estimate a yield value of $\Delta AIC = AIC^{DUDM} - AIC^{LCDM} = 7.96$ and $\Delta BIC = BIC^{DUDM} - BIC^{LCDM} = 19.96$, which suggest that the LCDM is the model preferred by the OHD + SNIa data used. This result is expected since the DUDM model contains a degree of freedom greater than LCDM.

In summary, we analyze an exact solution of a DUDM model using the most recent cosmological data of the Hubble parameter and SNIa, that cover the redshift region $0.01 < z < 2.3$. Although the exact solution under study was proposed as an ansatz in [23,24], we are able to obtain it in a systematic way by following a factorization procedure [29–31]. Due to the inability of the model to drive accelerated and decelerated phases with the same value of the main free parameters A as is shown in Eq. (36), we build the Hubble parameter of the model by connecting both phases as is expressed in Eq. (37) and the free parameters $\hat{A}_+$ and $\hat{A}_-$. Then, we employ an analysis using the combined data, OHD + SNIa, and considering the transition redshift $z_t = 0.64$ [44] to constrain their values. According to Eq. (37) and (34), the model presents an acceleration (deceleration) phase when $\hat{A}_1 > 1$ ($\hat{A}_2 < 1$). In these epochs, we infer a constant value of $q = -0.37^{+0.03}_{-0.03}$ ($0.19^{+0.03}_{-0.03}$), and an effective EoS $\omega_{\mathrm{eff}} = -0.58^{+0.02}_{-0.02}$ ($-0.21^{+0.02}_{-0.02}$). It is interesting to see that ω_{eff} is in the quintessence region for the accelerated epoch, while the decelerated phase is characterized by a negative effective EoS, even though it is not enough to drive an accelerated expansion of the Universe. We have also found that our solution can fit well the cosmological data, and the evaluated values of ξ_0 from the constrained values of A_+ and A_- always satisfy the condition of a fluid description for both phases, required from the thermodynamics formalism. Nevertheless, the high value of the speed of sound within the fluid is an undesirable behavior of the model.

It is important to point out that our solution is obtained assuming a Universe filled with only one fluid with dissipation; therefore, it is clear that it can describe only the late time evolution. An extension of this model to early ages of the Universe requires us to introduce radiation and evaluate the behavior of the linear perturbations. In the framework of the Eckart theory, the discussion of the linear perturbations has been realized, for example, in [49]. The found results indicate that viscous dark matter leads to modifications of the large-scale cosmic microwave background spectrum, weak lensing and cosmic microwave background-galaxy cross-correlations, which implies difficulties in order to fit the astronomical data. In the case of a perturbative study in the framework of the causal thermodynamics, it was found in [50] that numerical solutions for the gravitational potential seem to disfavor causal theory, whereas the truncated theory leads to results similar to those of the ΛCDM model for a very small bulk viscous speed.

Let us discuss here what can be a possible way to overcome this difficulty, which is present in this type of models. As we mentioned above, the division into DM and DE is merely conventional due to the degeneracy between both components, resulting from the fact that gravity only measures the total energy tensor. In the case of DUDM models, the viscous stress provides the negative pressure which allows accelerated phases, but the near equilibrium condition demanded in the thermodynamics approaches of relativistic viscous fluids implies that the viscous stress must be lower than the equilibrium pressure of the fluid. In general, this condition is not fulfilled, and one possibility is to go further and to consider a nonlinear generalization of the causal linear thermodynamics of bulk viscosity, where deviations from equilibrium are allowed (see, for example, [51]). Another possibility is to consider a cosmological scenario with dissipative DM and some other DE component. In [21], an introduction of a cosmological constant is considered together with a dissipative DM component. This also allows in some regions the satisfaction of the near equilibrium condition. Of course, in this scenario, UDM models with dissipation are abandoned as consistent models to describe the evolution of the Universe, and, on the other hand, we are assuming the division into DM and DE.

As a conclusion of the above discussion, we can say that the solution found within the full causal Israel-Stewart-Hiscock formalism indicates that accelerated expansion compatible with OHD and SNIa data can be obtained with only one dissipative DM component, but having a great nonadiabatic contribution to the speed of sound within the fluid, which is not compatible with the structure formation. Further investigations are required to solve this drawback including some form of DE, along with the dissipative component.

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