Primordial black hole remnants as dark matter produced in thermal, matter, and runaway-quintessence postinflationary scenarios

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We investigate the cosmology of mini primordial black holes (PBHs) produced by large density perturbations. The mini PBHs evaporate promptly in the early Universe, and we assume that a stable remnant is left behind. The PBH remnants can constitute all of the dark matter in the Universe for a wide range of remnant masses. We build inflationary models, in the framework of α-attractors utilizing exponential functions, in which the PBHs are produced during matter, radiation, and kination domination eras. The advantage of these inflationary models is that the spectral index takes values favorable to the Planck 2018 data. The PBH production from runaway inflaton models has the unique and very attractive feature to automatically reheat the Universe. In these models the PBHs are produced during the kination stage, and their prompt evaporation efficiently produces the required entropy. Such runaway models are remarkably economic, having interesting implications for the early Universe cosmology, possibly giving rise to a quintessence late time cosmology as well.

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I. INTRODUCTION

The primordial origin of black holes (PBH) is a rather attractive scenario because PBHs can constitute all of the cosmological dark matter (DM), or some significant fraction of it [1–3]. Contrary to stellar black holes, the mass range of the PBHs can be very wide, spanning over 30 dec of mass, from $10^{-18}\ M_\odot$ to $10^{15}\ M_\odot$.

A synergy of observations, including the cosmic microwave background (CMB) [4], the stochastic gravitational wave background [5], the Lyman α forest [6], lensing events [7–9], and dynamical studies of bound astrophysical systems [10–14] derive a combination of constraints on the PBH abundance for nearly the entire PBH mass range. Neglecting possible enhanced merging rates or an extended PBH mass distribution, the current allowed mass windows for the dominant PBH dark matter scenario are quite narrow and have central values $M \sim 10^{-15}\ M_\odot$ and $M \sim 10^{-12}\ M_\odot$. Remarkably, although triggered by LIGO events [15–17], PBH research has been extended into scenarios with vastly different mass scales. PBHs with very light masses are anticipated to Hawking radiate energetically, and this places strong constraints on their abundance. The lightest PBHs that can constitute a non-negligible part of the cosmic dark matter have mass $M \sim 10^{-17}\ M_\odot$. PBHs with smaller mass are prone to evaporation and hence are much constrained from the extragalactic gamma-ray background [18], CMB, and big bang nucleosynthesis (BBN) [19]. Mini PBHs with masses $M \ll 10^{-24}\ M_\odot \simeq 10^9$ g will have promptly evaporated in the very early Universe, potentially leaving no observational traces.

However, the scenario of mini PBHs is of major interest due to the theoretical expectation that black holes cannot evaporate into nothing; see, e.g., [20–22]. If the black hole evaporation halts at some point, then a stable state, called the black hole remnant, will survive. Remnants from the PBH's prompt evaporation have important cosmological consequences, with the most notable one being that PBH remnants can constitute all of the dark matter in the Universe [23,24]. The cosmological scenario of the mini PBH's evaporation and the PBH remnants has been studied in several contexts in [25–32].

The generation of mini PBHs implies that $\mathcal{P}_\mathcal{R}(k)$ has to feature a peak at very large wave numbers k. The most attractive mechanism to generate PBHs is inflation. Because of the natural generation of large scale perturbations from quantum fluctuations, inflation is the dominant paradigm that cosmologists follow to explain the origin of the large scale structure and has been, so far, successfully tested by the CMB precision measurements [33]. Nevertheless, inflation does not seed large scale perturbations only; it seeds perturbations in all scales. Hence, PBHs can form if perturbations strong enough to collapse are produced at scales $k^{-1} \ll k_{\rm cmb}^{-1}$ characteristic of the PBH mass.

There have been numerous inflationary models constructed to predict a significant abundance of PBHs; for a recent proposal see, e.g., [34–49]. Their common feature is that the spectrum of the curvature perturbations, $\mathcal{P}_\mathcal{R}(k)$,

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turns from red into blue at small scales. Since inflation models primarily generate the CMB anisotropies, these new inflationary proposals, though successful at generating PBHs, might fail at the $k_{\rm cmb}^{-1}$ scale. In general the predicted spectral index n_s and running α_s values are in best accordance with the CMB measured values when the PBHs have masses of minimal size; see, e.g., [50], where the growth of the power spectrum is discussed. Light PBHs imply that the $\mathcal{P}_\mathcal{R}(k)$ shape has to be modified at k values, far beyond the scales probed by the CMB. From a model building perspective, achieving an enhancement of $\mathcal{P}_\mathcal{R}(k)$ at the very end of the spectrum might seem to be an attractive feature. This is extra motivation to examine the mini PBH scenario.

In this work we build inflationary models that generate mini PBHs and examine the early and late Universe cosmology of the PBH remnants. Our inflationary models belong in the family of the α-attractors [51], and the building blocks we use are exponential potentials. The PBHs are generated by the presence of an inflection point at small field values where the inflaton velocity decreases significantly producing a spike in $\mathcal{P}_\mathcal{R}(k)$. Since PBHs form during the very early postinflationary cosmic stage and reheating might have not been completed, it is natural to examine the evolution of the PBHs and their remnants for different backgrounds and expansion rates.

We derive expressions for the relic abundance of the PBH remnants for an arbitrary barotropic parameter w and remnant mass $M_{\rm rem}$. These expressions are general and applicable to the stable PBH scenario as well. Afterward, we examine explicitly the radiation domination, matter domination, and kination domination cases. We explicitly construct and analyze three different inflationary potentials, and we compute the power spectrum of the comoving curvature perturbation, solving numerically the Mukhanov-Sasaki equation, and we estimate the fractional PBH remnant abundance. A great advantage of our inflationary models is that the predicted values for n_s and α_s are placed inside the 1σ C.L. region of the Planck 2018 data.

Moreover, we introduce the scenario of PBH production during the kination domination regime (called also the stiff phase), which has interesting cosmological implications. Kination is driven by the kinetic energy of the inflaton field itself. The duration of the kination regime is solely specified by the mass and the abundance of the PBHs produced. The fact that the PBHs promptly evaporate means that the Universe is automatically and successfully reheated without the need of special couplings or tailor-made resonance mechanisms. In addition, the inflaton field might play the role of the quintessence at late times giving rise to a testable wCDM cosmology, where cold dark matter, comprising PBHs remnants, is produced by the primordial fluctuations of the very same field, and w is the equation of state of the dark energy; see Fig. 1. Apparently, in terms of ingredients, this is a maximally economic cosmological scenario.

The analysis in this paper is structured as follows. In Sec. II we discuss the bounds on the masses of the PBHs and of their remnants, reviewing briefly theoretical considerations and deriving the associated cosmological bounds. In Sec. III the cosmology of the PBH remnants is presented for a general expansion rate, and the main formulas are derived. In Sec. IV we turn to the inflationary model building and formulate the constraints that $\mathcal{P}_\mathcal{R}(k)$ has to satisfy. In Sec. V we examine inflationary models based on the α-attractors that generate mini PBHs, we compute numerically $\mathcal{P}_\mathcal{R}(k)$, and we construct explicit examples in which the PBH remnants constitute all of the dark matter in the Universe. We present and illustrate our results with several plots and tables. In Sec. VI we conclude.

II. PBH EVAPORATION REMNANTS

Hawking predicted that black holes radiate thermally with a temperature [52,53]

$$T_{\rm BH} = \frac{\hbar c^3}{8\pi G M k_B} \sim 10^8 \left(\frac{M}{10^5\ {\rm g}}\right)^{-1} {\rm GeV} \tag{2.1}$$

and are expected to evaporate on a timescale $t_{\rm evap} \sim G^2 M^3/(\hbar c^4)$, which is found to be [19]

$$t_{\rm evap} = 407 \tilde{f}(M) \left(\frac{M}{10^{10}\ {\rm g}}\right)^3 {\rm s}, \tag{2.2}$$

where M is the mass of the PBH formed. We see that PBHs in the mass range $M \sim 10^9$–10^{12} g evaporate during or after the BBN cosmic epoch, and that β is significantly constrained by the abundance of the BBN relics. For PBH in the mass range $M \sim 10^{13}$–10^{14} g, the evaporation takes place during the cosmic epoch of recombination and the CMB observations put the stringent constraints on $\beta(M)$. Larger PBH masses contribute to the extragalactic gamma-ray background; for a review see [19]. Hence, the scenario of the PBH remnants as dark matter is motivated for $M < 10^9$ g. In particular, the remnants from the evaporation of the PBHs can constitute all or a significant portion of the cosmic dark matter if the PBH mass is smaller than

$$M < \left(\frac{\kappa m_{\rm Pl}^2 M_{\rm eq}^{1/2}}{1+w}\right)^{2/5}, \tag{2.3}$$

as we will show in Sec. III. The Planck mass is $m_{\rm Pl} = 2.2 \times 10^{-5}$ g, and $M_{\rm eq}$ is the horizon mass at the moment of radiation-matter equality. The w stands for the equation of state of the background cosmic fluid. For reasons that we will explain later, we call the upper mass bound $M_{\rm inter}$ and it has a size, roughly, of $2\kappa^{2/5} 10^6$ g. Remnants from PBHs with mass $M > M_{\rm inter}$ contribute only a small fraction to the total dark matter.

A. Theoretical considerations about the PBH remnant mass

There are several theoretical reasons for anticipating that black holes do not evaporate completely but leave behind a stable mass state. Hawking radiation is derived by treating matter fields quantum mechanically, while treating the space-time metric classically. When the mass of an evaporating black hole becomes comparable to the Planck scale, such a treatment would break down, and quantum gravitational effects would become relevant. Energy conservation [54], extra spatial dimensions [55,56], higher order corrections to the action of general relativity [23], and the information loss paradox [22] could be sufficient to prevent complete evaporation. Higher order correction to the Hawking radiation emerging from some quantum gravity theory are also expected to modify the evaporation rate. The mass of the final state of the evaporation, i.e., the PBH remnants, $M_{\rm rem}$ can be written in terms of the Planck mass

$$M_{\rm rem} = \kappa m_{\rm Pl}. \tag{2.4}$$

κ is a factor that parametrizes our ignorance. Different theories predict stable black hole relics of different mass. κ may be of order 1, with relic black hole masses characterized by the fundamental scale of gravity, $m_{\rm Pl} = G^{-2}$, but other values for κ are also admitted. If black holes have quantum hair, e.g., if they possess discrete electric and magnetic charges, the remnant mass depends on the value of the charge, $M_{\rm rem} \sim m_{\rm Pl}/g$, where g is the corresponding coupling constant [21]. Thus, $M_{\rm rem}$ can be orders of magnitudes larger than 1, $\kappa \gg 1$, for weakly coupled theories. In other theories, such as those where a generalized uncertainty principle is applied [57], the mass of the black hole remnants can be much smaller than $m_{\rm Pl}$ (see, e.g., [58]); hence, it is $\kappa \ll 1$. In our analysis and expressions κ is a free parameter. This is a justified approach since we know next to nothing about the physics at that energy scale. In explicit examples we will pick up the benchmark $\kappa \sim 1$ value, and in others $\kappa \ll 1$. We will also comment on the cosmology of different κ values; see Figs. 2 and 3.

This work aims at the cosmology of the PBH remnants, and we will remain agnostic about the fundamental physics that prevents black holes (or holes more properly) from complete evaporation. We will not enter into the details regarding the modification of the Hawking temperature with respect to the black hole mass either. Nevertheless, we remark that the formation of mini PBHs with mass $M < M_{\rm inter} \sim \kappa^{2/5} 10^6$ g takes place in the very early Universe, at the cosmic time $t_{\rm form}$, and we expect these PBHs to evaporate promptly with their temperature reaching a maximum value contrary to what the standard expression (2.1) dictates. If the temperature of the PBHs is initially smaller that the background cosmic temperature, $T_{\rm BH}(t_{\rm form}) < T(t_{\rm form})$, the accretion effects should be taken into account. Although the accretion decreases the temperature of the PBH, the decrease of the cosmic temperature due to the expansion is much faster, and the amount of matter that a PBH can accrete is small. Hence, the PBH lifetime will not be modified, and once the cosmic temperature falls below the value $T_{\rm BH} \sim 10^8 (M/10^5\ {\rm g})^{-1}$ GeV, the PBHs heat up and evaporate. Concerning the black hole temperature, $T_{\rm BH}$ is expected to reach a maximal value and afterward decrease as $M \to M_{\rm rem}$. In this last stage of PBH evaporation, the rate $dM/dT_{\rm BH}$ turns positive. The PBHs are expected to exist in stable equilibrium with the background only when their mass is already close to the remnant mass [23]; thus, possible related corrections can be considered negligible for the scope of this work.

B. Cosmological constraints on the mass of the PBH remnants

The examination of the PBH remnant cosmology provides us with observational constraints on the κ value. The corresponding analysis is presented in detail in Sec. III, and here, in advance, we will use part of the results to report the cosmological allowed values for the PBH remnant masses.

Let us first examine the minimal possible value for $M_{\rm rem}$. In the inflationary framework the formation of PBHs with mass M can be realized only if the horizon mass right after inflation, $M_{\rm end} = 4\pi M_{\rm Pl}^2/H_{\rm end}$ is smaller than M/γ. The γ parameter is the fraction of the Hubble mass that finds itself inside the black hole. In terms of the Hubble scale at the end of inflation, $H_{\rm end}$, the bound reads

$$M > \gamma 10^5\ {\rm g}\, \frac{10^9\ {\rm GeV}}{H_{\rm end}}. \tag{2.5}$$

The above inequality yields a lower bound for the PBH mass. The upper bound on the tensor-to-scalar ratio $r_* < 0.064$ [33] and the measured value of the scalar power spectrum amplitude constrain the Hubble scale, $H_* \simeq (\pi^2 A_s r_*/2)^{1/2} M_{\rm Pl}$. It is $H_{\rm end} < H_* < 2.6 \times 10^{-5} M_{\rm Pl} \simeq 6.5 \times 10^{13}$ GeV; hence, the minimal PBH mass that can be generated is $M/\gamma > \mathcal{O}(1)(r_*/0.06)^{-1/2}$ g. Assuming that a radiation domination phase follows inflation, the fractional abundance of the PBH remnants, given by Eq. (3.10), is maximal $f_{\rm rem} = 1$ for $\kappa \gtrsim 10^{-18.5}$. Hence, the PBH remnants are able to have a significant relic abundance only if they have mass

$$M_{\rm rem} > 1\ {\rm GeV} \simeq 1.8 \times 10^{-24}\ {\rm g}. \tag{2.6}$$

This lower bound has been derived assuming the minimum possible PBH mass, $M \sim 1$ g, and the maximum possible formation rate, $\beta \sim 1$; see Eq. (3.10). It is also valid for nonthermal postinflationary cosmic evolution. PBHs remnants with smaller mass can constitute only a negligible amount of the total dark matter energy density.

On the other side, the κ value has a maximum value, $\kappa_{\max} = M/m_{\rm Pl}$. Apparently, for $\kappa = \kappa_{\max}$ there is no Hawking radiation, and one does not talk about PBH remnants. For $\kappa \ll M/m_{\rm Pl}$ the Hawking radiation is important, and it might affect the BBN and CMB observables for light enough PBHs. Assuming again a radiation domination phase after inflation, Eq. (3.10) implies that $\kappa \propto 1/\beta$ for $f_{\rm rem} = 1$. According to Eq. (2.2), for $M \gtrsim 10^9$ g the PBH evaporates after a timescale of 1 s, and the BBN constrains $\beta < 10^{-22}$. However, the BBN β upper bounds cannot be satisfied for $\kappa \ll \kappa_{\max}$. Hence, for $M_{\rm rem} \ll M$ we conclude that only the remnants with mass in the window

$$10^{-24}\ {\rm g} < M_{\rm rem} \ll 10^8\ {\rm g} \tag{2.7}$$

can have a sufficient abundance to explain the observed dark matter in the Universe. The upper bound is determined by the BBN constraints on the parent PBH mass. It might be satisfied for $M_{\rm rem}$ about 1 order of magnitude less than 10^8 grams; its exact value depends on how Eqs. (2.1) and (2.2) are modified and on the equation of state w after inflation. In the following we derive the expressions for the relic abundance of the PBH remnants for a general expansion rate and $M_{\rm rem}$ parameter, and we examine separately the cases of radiation, matter, and kination domination eras.

III. THE EARLY UNIVERSE COSMOLOGY OF THE PBH REMNANTS

The cosmology of the PBH remnants originating from large primordial inhomogeneities was studied in detail in Refs. [23,24], where the basic expressions were derived. PBH remnants might also originate form micro black holes produced from high energy collisions in the early Universe [59]. In the following we will consider the formation of PBHs due to large inhomogeneities and will generalize the key expressions for arbitrary barotropic parameter w, introducing also the PBH–stiff fluid (kination) scenario.

Let us suppose that at the early moment $t_{\rm form}$ a fraction β of the energy density of the Universe collapses and forms primordial black holes. The mass density of the PBHs is $\rho_{\rm PBH} \simeq \gamma\beta\rho_{\rm tot}$ at the moment of formation, where $\rho_{\rm tot} = 3H^2 M_{\rm Pl}^2$ with $M_{\rm Pl} = m_{\rm Pl}/\sqrt{8\pi}$, the reduced Planck mass. The formation probability is usually rather small, $\beta \ll 1$, and the background energy density $\rho_{\rm bck} = (1-\gamma\beta)\rho_{\rm tot}$ is approximately equal to $\rho_{\rm tot}$.

The PBHs are pressureless nonrelativistic matter, and their number density $n_{\rm PBH}$ scales like a^{-3}. The background energy density scales as $\rho \propto a^{-3(1+w)}$, where $a(t) \propto t^{\frac{2}{3(1+w)}}$ and w is the equation of state of the background fluid. The perturbations evolve inside the curvature scale $1/H$, which has mass $M_H = (3/4)(1+w)m_{\rm Pl}^2 t$, called the Hubble scale mass. In the approximation of instantaneous evaporation, the moment right before evaporation that we label $t_{\rm evap}^<$, the energy density of the PBHs over the background energy density is

$$\frac{\rho_{\rm PBH}(t_{\rm evap}^<)}{\rho_{\rm bck}(t_{\rm evap}^<)} = \gamma\beta\gamma^{\frac{2w}{1+w}}\left(\frac{M_H(t_{\rm evap}^<)}{M}\right)^{\frac{2w}{1+w}}\tilde{g}(g_*, t_{\rm evap}), \tag{3.1}$$

where $\tilde{g}(g_*, t_{\rm evap})$ is equal to 1 unless the Universe is radiation dominated; in that case it is $\tilde{g}(g_*, t_{\rm evap}) \equiv (g_*(t_{\rm form})/g_*(t_{\rm evap}))^{-1/4}$, where g_* represents the thermalized degrees of freedom (d.o.f.), which we took to be equal to the entropic d.o.f., g_s. Substituting the Hubble mass at the evaporation moment of a PBH with mass M, $M_H(t_{\rm evap}) = 3M^3(1+w)/4m_{\rm Pl}^2$, a threshold $\beta(M)$ value is found. For

$$\beta < \tilde{h}^{-1}(\gamma, w, t_{\rm evap})\left(\frac{m_{\rm Pl}}{M}\right)^{\frac{4w}{1+w}}, \tag{3.2}$$

where $\tilde{h}(\gamma, w, t_{\rm evap}) \equiv \gamma^{\frac{1+3w}{1+w}}\left(\frac{3}{4}(1+w)\right)^{\frac{2w}{1+w}}\tilde{g}(g_*, t_{\rm evap})$, the Universe has never been PBH dominated.

At the moment right after the evaporation, which we label $t_{\rm evap}^>$, the energy density of the PBHs has decreased $(\kappa m_{\rm Pl}/M)^{-1}$ times. This factor is much larger than 1, and thus nearly the entire energy density of the initial PBHs turns into radiation apart from a tiny amount, reserved by the PBH remnants. The present density of the PBH remnants depends on the equation of state of the Universe after the PBH evaporation. If we assume that a radiation domination phase follows the PBH evaporation, the fractional abundance of the PBH remnants over the total DM abundance today is

$$f_{\rm rem}(M) = \tilde{c}\beta\left(\frac{M_{\rm eq}}{M_H(t_{\rm evap})}\right)^{1/2}\frac{\kappa m_{\rm Pl}}{M}\left(\frac{M}{m_{\rm Pl}}\right)^{\frac{4w}{1+w}}, \tag{3.3}$$

where $\tilde{c}(\gamma, w, t_{\rm eq}) = 2^{1/4}\tilde{h}(\gamma, w, t_{\rm eq})\Omega_{\rm m}/\Omega_{\rm DM}$. However, the assumption that there is a radiation domination phase after the PBH evaporation holds either when the Universe has become PBH dominated at the moment $t_{\rm evap}^<$ or when the equation of state of the background fluid is $w = 1/3$. Otherwise, one has to replace the $M_H(t_{\rm evap})$ with the $M_{\rm rh}$—that is, the Hubble radius mass at the completion of reheating—and include a w-dependent factor to account for the different expansion rate. Next, particular cases will be examined.

If the Universe has become PBH dominated at the moment $t_{\rm evap}^<$ and we ask for $f_{\rm rem}(M) = 1$, we get the mass

$$M_{\rm inter} = \tilde{\alpha}^{2/5}(w)(\kappa m_{\rm Pl}^2 M_{\rm eq}^{1/2})^{2/5}, \tag{3.4}$$

where $\tilde{\alpha}(w) = 2^{1/4}(\Omega_{\rm m}/\Omega_{\rm DM})(\sqrt{3}(1+w))^{-1}$. This is the intersection mass of Eq. (3.2) and the $f_{\rm rem}(M) = 1$ line, given by Eq. (3.3). We see that $M_{\rm inter}$ slightly depends on w.

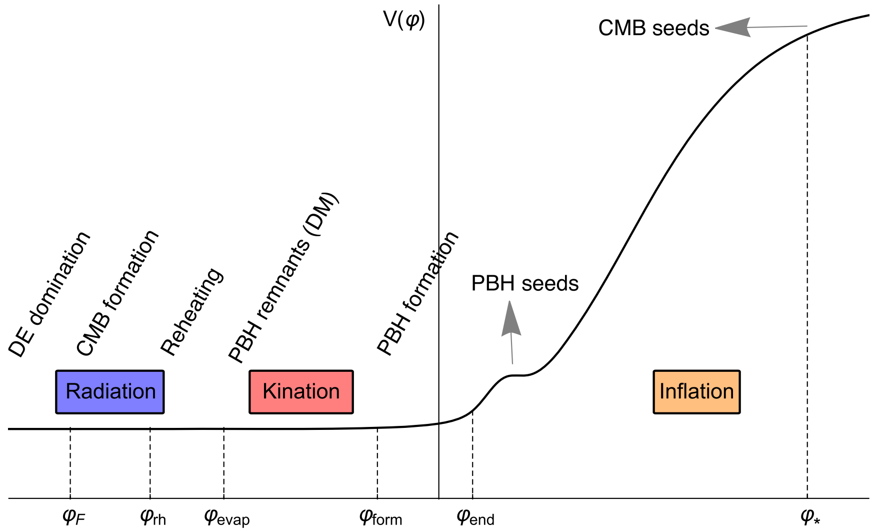

FIG. 1. A schematic illustration of the runaway inflationary model introduced in this work (the kination scenario) that produces PBHs, explains the dark matter with the PBH remnants, reheats the Universe via the PBH evaporation, and implements a wCDM late time cosmology, where the dark energy (DE) is identified as the energy density of the inflaton field itself.

This means that there is a single PBH mass scale such that the early Universe becomes PBH dominated and the evaporation remnants account for the total DM, for any positive value of the equation of state. Plugging in values, $M_{\mathrm{eq}} \simeq 6\times 10^{50}$ g, $g_*(T_{\mathrm{eq}}) = 3.36$, we obtain $M_{\mathrm{inter}} \simeq 2\kappa^{2/5} 10^6$ g. The intersection mass is the maximum M value in Figs. 2 and 3. For masses $M \geq M_{\mathrm{inter}}$ the upper bound on β is practically removed. Turning to β, the β_{inter} value that yields $f_{\mathrm{rem}} = 1$ and the momentary PBH domination phase is w dependent,

$$\beta_{\mathrm{inter}}(w) = \tilde{h}^{-1}(\gamma, w)\tilde{\alpha}^{\frac{-2w}{5+5w}}\left(\frac{m_{\mathrm{Pl}}}{M_{\mathrm{eq}}}\right)^{\frac{4w}{5+5w}}. \tag{3.5}$$

Larger values for w require smaller β, and hence minimal β values are achieved for $w = 1$; see Figs. 2 and 3.

For masses $M > M_{\mathrm{inter}}$ the relic abundance of the PBH remnants is always smaller than the total dark matter abundance even if PBHs dominate the early Universe. Hence, for $M \geq M_{\mathrm{inter}}$ there is no constraint on β from $f_{\mathrm{rem}}(M)$. This can be understood as follows. Let us assume that $M = M_{\mathrm{inter}}$ and $\beta = \beta_{\mathrm{inter}}$ such that $\Omega_{\mathrm{rem}} = \Omega_{\mathrm{DM}}$. This means that right before evaporation the PBH number density is $n_{\mathrm{PBH}}(t^{<}_{\mathrm{evap}}) \simeq \rho_{\mathrm{tot}}/M_{\mathrm{inter}}$. If it had been $\beta > \beta_{\mathrm{inter}}$, the PBH domination phase would have started at times $t < t_{\mathrm{evap}}$, but the number density of the PBH relics at the moment t_{evap} would have been the same. Hence, the value of $f_{\mathrm{rem}}(M_{\mathrm{inter}})$ does not increase for $\beta > \beta_{\mathrm{inter}}$. Also, for $M > M_{\mathrm{inter}}$ the number density of the PBHs is always smaller than $\rho_{\mathrm{tot}}/M_{\mathrm{inter}}$ at the moment $t^{<}_{\mathrm{evap}} \sim G^2 M^3_{\mathrm{max}}$, even

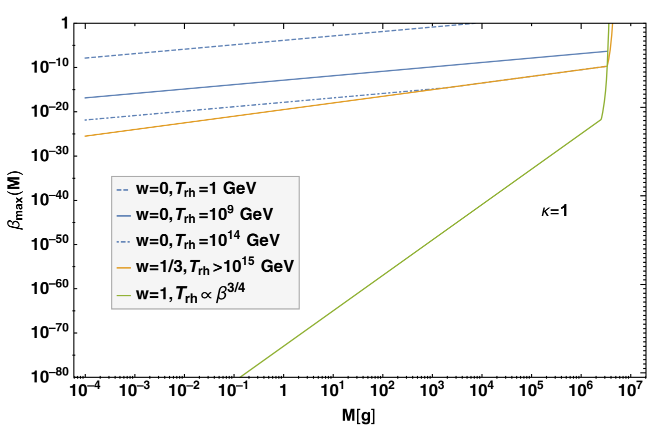

FIG. 2. β_{max}, given by the condition $f_{\mathrm{rem}} = 1$ for PBH formation during three different cosmic phases with equation of states: $w = 0$ (for three different reheating temperatures), $w = 1/3$, and $w = 1$. At the mass $M = M_{\mathrm{inter}}$ the requirement $f_{\mathrm{rem}} \leq 1$ does not imply any bound on β. We considered the mass of the PBH remnants to be equal to the Planck mass, $\kappa = 1$.

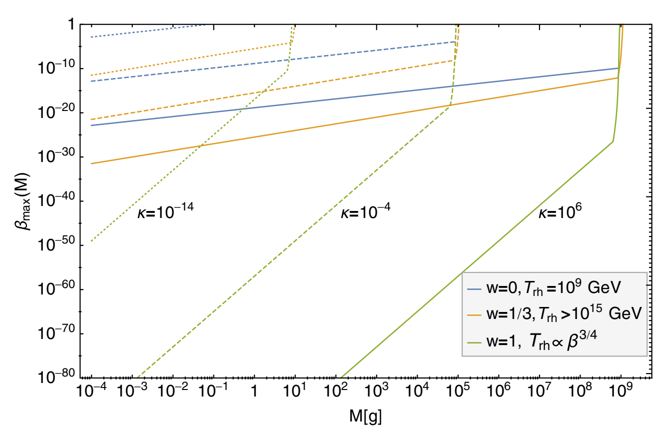

FIG. 3. As in Fig. 2, for PBH remnants with arbitrary chosen masses $10^6 m_{\mathrm{Pl}}$ (solid lines), $10^{-4} m_{\mathrm{Pl}}$ (dashed lines), and 10^2 TeV (dotted lines).

if $\beta(M) \sim 1$. Since the PBHs will evaporate into PBH remnants with the universal mass $\kappa m_{\rm Pl}$, the conclusion to be drawn is that the relic energy density parameter of PBH remnants with mass $M > M_{\rm inter}$ is always less than $\Omega_{\rm DM}$. Summing up, any constraint on $\beta(M)$ for $M_{\rm inter} < M < 10^{17}$ g comes only from the Hawking radiation of the PBHs, *not from the abundance* of the PBH remnants.

Let us now turn to the $M = M(k)$ relation, assuming a one-to-one correspondence between the scale k^{-1} and the mass M. Following the Press-Schechter formalism [60], there is a probability β an overdensity with wave number k to collapse when it enters the Hubble radius (or some time later if the Universe is matter dominated). The mass M of the PBH is related to the wave number $k = aH$ as

$$\frac{M}{M_{\rm rh}} = \gamma \frac{H^{-1}}{H^{-1}_{\rm rh}} = \gamma \left(\frac{k}{k_{\rm rh}}\right)^{\frac{-3(1+w)}{3w+1}}, \tag{3.6}$$

where we utilized the relation between the wave number and the scale factor,

$$\frac{k}{k_{\rm rh}} = \left(\frac{a}{a_{\rm rh}}\right)^{-\frac{1}{2}(3w+1)}. \tag{3.7}$$

The horizon mass at the completion of reheating, $M_{\rm rh} = 4\pi(\pi^2 g_*/90)^{-1/2} M^3_{\rm Pl}/T^2_{\rm rh}$, reads

$$M_{\rm rh} \simeq 10^{12}\ {\rm g} \left(\frac{T_{\rm rh}}{10^{10}\ {\rm GeV}}\right)^{-2} \left(\frac{g_*}{106.75}\right)^{-1/2}. \tag{3.8}$$

If PBHs form during the radiation domination era, it is $M/\gamma > M_{\rm rh}$, whereas if they form before the completion of the thermalization of the Universe, it is $M/\gamma < M_{\rm rh}$. We will return to the relation between the PBH mass and the wave number k in Sec. IV, where we will explicitly write the $M = M(k, T_{\rm rh}, w)$ formula in order to connect the PBH mass with the $\mathcal{P}_{\mathcal{R}}(k)$ peak.

Let us note that the formation of PBHs with mass M is possible only if the horizon mass right after inflation is smaller than M/γ; see Eq. (2.5). Equivalently, a PBH with mass M will form due to superhorizon perturbations only if the corresponding wavelength k^{-1} is larger than the Hubble scale at the end of inflation. Thus, a different way to express condition (2.5) is $k_{\rm end} > k$.

Next, we examine separately the interesting cosmological scenarios with barotropic parameter $w = 0$, $w = 1/3$, and $w = 1$.

A. PBH production during radiation domination

Let us assume that the bulk energy density is in the form of radiation. Thus, it is $\rho_{\rm PBH} \simeq \gamma\beta\rho_{\rm rad}$ after the approximation $\rho_{\rm rad} = (1-\beta)\rho_{\rm tot} \simeq \rho_{\rm tot}$, which is legitimate for $\beta \ll 1$. The PBH mass is $M = \gamma M_H$, where $M_H = m^2_{\rm Pl}/(2H) \simeq m^2_{\rm Pl} t$ is the Hubble radius mass during radiation domination (RD).

Assuming a RD phase until the moment of the evaporation and making the approximation of instantaneous evaporation, the energy density of the PBHs at the moment right before evaporation is $\rho_{\rm PBH}(t^<_{\rm evap}) = \gamma^{3/2}\beta M m^{-1}_{\rm Pl} \rho_{\rm rad}(t^<_{\rm evap})$. Thus, the assumption of a radiation dominated phase is valid for $\gamma^{3/2}\beta M m^{-1}_{\rm Pl} < 1$. In the opposite case the Universe becomes PBH dominated before the moment of evaporation. At the moment right after the PBH evaporation, the energy density of the PBH relics is $\kappa\gamma^{3/2}\beta$ times the energy density of the radiation background. For a RD phase until the epoch of matter-radiation equality, $t_{\rm eq}$, it is $\rho_{\rm rem}(t_{\rm eq}) \simeq (\kappa m_{\rm Pl}/M)\gamma\beta\rho_{\rm rad} T_{\rm evap}/T_{\rm eq}$, and the fractional abundance of the PBH remnants is found,

$$f_{\rm rem}(M) = \tilde{c}_{\rm R}\gamma^{3/2} \frac{\kappa m_{\rm Pl}}{M} \beta \left(\frac{M_{\rm eq}}{M}\right)^{1/2}, \tag{3.9}$$

where $\tilde{c}_{\rm R} = 2^{1/4}(g(T_{\rm form})/g(T_{\rm eq}))^{-1/4}\Omega_{\rm m}/\Omega_{\rm DM}$. $T_{\rm evap}$ and $T_{\rm eq}$ are the cosmic temperatures at the moment of evaporation and the epoch of matter-radiation equality. The effectively massless d.o.f. for the energy and entropy densities were taken to be equal. The maximum value $f_{\rm rem} = 1$ gives the maximum value for $\beta_{\rm max}(M)$; see Figs. 2 and 3. Equation (3.9) is rewritten after inserting the benchmark values,

$$f_{\rm rem}(M) \simeq \kappa \left(\frac{\beta}{10^{-12}}\right) \left(\frac{\gamma}{0.2}\right)^{\frac{3}{2}} \left(\frac{M}{10^5\ {\rm g}}\right)^{-3/2}, \tag{3.10}$$

where we omitted the factor $0.95(g(T_k)/106.75)^{-\frac{1}{4}}$ from the rhs and took $\Omega_{\rm DM}h^2 = 0.12$.

Assuming Gaussian statistics, the black hole formation probability for a spherically symmetric region is

$$\beta(M) = \int_{\delta_c} d\delta \frac{1}{\sqrt{2\pi\sigma^2(M)}} e^{-\frac{\delta^2}{2\sigma^2(M)}}, \tag{3.11}$$

which is approximately equal to $\beta\sqrt{2\pi} \simeq \sigma(M)/\delta_c e^{-\frac{\delta_c^2}{2\sigma^2(M)}}$. The PBH abundance has an exponential sensitivity to the variance of the perturbations $\sigma(k)$ and to the threshold value δ_c. In the comoving gauge Ref. [61] finds that δ_c has the following dependence on w,

$$\delta_c = \frac{3(1+w)}{5+3w} \sin^2 \frac{\pi\sqrt{w}}{1+3w}. \tag{3.12}$$

For $w = 1/3$ it is $\delta_c = 0.41$. The variance of the density perturbations in a window of k is given by the relation $\sigma^2 \sim \mathcal{P}_\delta$, where $\mathcal{P}_\delta$ is related to the power spectrum of the comoving curvature perturbation as

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{5+3w}{2(1+w)}\right)^2 \mathcal{P}_\delta; \tag{3.13}$$

hence, it is $\sigma^2 \sim (4/9)^2 \mathcal{P}_{\mathcal{R}}$. From the approximation of Eq. (3.11), we get $\mathcal{P}_{\mathcal{R}} \sim (9/4)^2 (\delta_c^2/2) \ln(1/(\sqrt{2\pi}\beta))^{-1}$. Benchmark values $\beta = 10^{-12}$, $\delta_c = 0.41$, $\kappa = 1$ yield the required value for the power spectrum $\mathcal{P}_{\mathcal{R}} \sim 1.6 \times 10^{-2}$. Increasing the value of κ by 1 order of magnitude or more gives only a slight decrease in the required value of $\mathcal{P}_{\mathcal{R}}$.

Finally, let us note that PBHs are expected to form with mass $M = \gamma M_H$ when the cosmic temperature is

$$T(M) \simeq 10^{11}\ \mathrm{GeV}\gamma^{1/2} \left(\frac{M}{10^{10}\ \mathrm{g}}\right)^{-1/2} \left(\frac{g_*}{106.75}\right)^{-1/4}. \tag{3.14}$$

For example, formation of PBHs with mass $M \sim 10^5$ g requires a reheating temperature $T_{\mathrm{rh}} > 10^{13}$ GeV. If the reheating temperature is lower than $T(M)$, then PBHs with mass M form during the nonthermal phase.

A particular example that yields $f_{\mathrm{rem}} = 1$ is described in Sec. V.

B. PBH production during matter domination

For PBH formation during the matter domination (MD) era, expression (3.9) has to be multiplied with $(t_{\mathrm{form}}/t_{\mathrm{rh}})^{1/2}$ to account for the absence of a relative redshift of ρ_{PBH} with respect to the background energy density. That is,

$$f_{\mathrm{rem}}(M, M_{\mathrm{rh}}) = \tilde{c}_{\mathrm{M}} \frac{\kappa m_{\mathrm{Pl}}}{M} \gamma \beta \left(\frac{M_{\mathrm{eq}}}{M_{\mathrm{rh}}}\right)^{1/2}, \tag{3.15}$$

where $\tilde{c}_{\mathrm{M}} = 2^{1/4} (g(T_{\mathrm{rh}})/g(T_{\mathrm{eq}}))^{-1/4} \Omega_{\mathrm{m}}/\Omega_{\mathrm{DM}}$ and for $M < M_{\mathrm{rh}}$. The Hubble scale mass at the completion of reheating reads $M_{\mathrm{rh}} = M_H(T_{\mathrm{rh}}, g_*) = 4\pi(\pi^2 g_*/90)^{-1/2} \times M_{\mathrm{Pl}}^3/T_{\mathrm{rh}}^2$. Equation (3.15) is rewritten after normalizing M, M_{rh} with benchmark values,

$$f_{\mathrm{rem}}(M, M_{\mathrm{rh}}) \simeq 3\kappa\gamma \left(\frac{\beta}{10^{-9}}\right) \left(\frac{M_{\mathrm{rh}}}{10^{10}\ \mathrm{g}}\right)^{-1/2} \times \left(\frac{M}{10^5\ \mathrm{g}}\right)^{-1} \left(\frac{g_*}{106.75}\right)^{-1/4}, \tag{3.16}$$

and the mass M is related to the scale k^{-1} of the inhomogeneity as

$$k = k_{\mathrm{end}} \left(\frac{4\pi M_{\mathrm{Pl}}^2}{H_{\mathrm{end}}}\right)^{1/3} \left(\frac{M}{\gamma}\right)^{-1/3} \quad \text{for } k > k_{\mathrm{rh}}. \tag{3.17}$$

In a pressureless background overdensities can grow and collapse more easily if the MD era is sufficiently long. A slow reheating stage after inflation was examined in Ref. [62]. Contrary to the RD case, nonsphericity and spin effects suppress the formation probability. Reference [63] examined the PBH production in the MD era and found that for not very small σ the PBH production rate tends to be proportional to σ^5,

$$\beta(M) = 0.056\sigma^5. \tag{3.18}$$

If the collapsing region has angular momentum, the formation rate is further suppressed and reads [64]

$$\beta(M) = 1.92 \times 10^{-7} f_q(q_c) \mathcal{I}^6 \sigma^2 e^{-0.147\frac{\mathcal{I}^{4/3}}{\sigma^{2/3}}}. \tag{3.19}$$

Benchmark values are $q_c = \sqrt{2}$, $\mathcal{I} = 1$, $f_q \sim 1$. According to [64], expression (3.19) applies for $\sigma \lesssim 0.005$, whereas Eq. (3.18) applies for $0.005 \lesssim \sigma \lesssim 0.2$.

During the MD era, an additional critical parameter is the duration of the gravitational collapse. Reference [64] concluded that the finite duration of the PBH formation can be neglected if the reheating time t_{rh} satisfies $t_{\mathrm{rh}} > (\frac{2}{5}\mathcal{I}\sigma)^{-1} t_k$, where t_k is the time of the horizon entry of the scale k^{-1} (it does not coincide with the formation time t_{form}). In terms of temperature this condition is rewritten as $T_{\mathrm{rh}} < (\frac{2}{5}\mathcal{I}\sigma)^{1/2} T_k$, where T_k is the temperature at which the scale k^{-1} would enter the horizon during RD. Let us define the temperature

$$T_{\mathrm{form}}^{\mathrm{MD}} = \left(\frac{2}{5}\mathcal{I}\sigma\right)^{1/2} T_k. \tag{3.20}$$

If the reheating temperature is smaller than $T_{\mathrm{form}}^{\mathrm{MD}}$, PBHs form during the MD era. Unless this condition is fulfilled, the time duration for the overdensity to grow and enter the nonlinear regime is not adequate. Hence, the formation rates (3.18) and (3.19) apply only for the scales k that experience a variance of the comoving density contrast at horizon entry that is larger than

$$\sigma > \sigma_{\mathrm{cr}} \equiv \frac{5}{2} \mathcal{I}^{-1} \left(\frac{k_{\mathrm{rh}}}{k}\right)^3. \tag{3.21}$$

In terms of temperature this translates into $\sigma > 5/2\mathcal{I}^{-1}(T_{\mathrm{rh}}/T)^2$. If $\sigma < \sigma_{\mathrm{cr}}$, one should consider the radiation era formation rate.

A particular example that yields $f_{\mathrm{rem}} = 1$ is described in Sec. V. σ in that example is less than 0.005 and larger than σ_{cr}; thus, the overdensity collapses during the matter era, with the spin effects being crucial.

C. PBH production during stiff fluid domination

Let us assume that the bulk energy density is in the form of stiff fluid (SD era), that is, a fluid with barotropic parameter $w = 1$, also called the kination phase. A nonoscillatory inflaton can give rise to a kination phase. It is $\rho_{\mathrm{PBH}} \simeq \gamma\beta\rho_{\mathrm{S}}$, where we have approximated $\rho_{\mathrm{S}} = (1-\beta)\rho_{\mathrm{tot}} \simeq \rho_{\mathrm{tot}}$ for $\beta \ll 1$. The PBH mass is $M = \gamma M_H(t_{\mathrm{form}})$, and the

Hubble scale mass for stiff fluid domination at the evaporation moment is $M_H(t_{\rm evap}) = (3/2)M^3/m_{\rm Pl}^2$.

Assuming that the SD era lasts at least until the moment of evaporation and making the approximation of instantaneous evaporation, the energy density of the PBHs at the moment right before evaporation is

$$\frac{\rho_{\rm PBH}(t_{\rm evap}^<)}{\rho_{\rm S}(t_{\rm evap}^<)} = \frac{3}{2}\gamma^2\beta\frac{M^2}{m_{\rm Pl}^2}. \tag{3.22}$$

The assumption of a kination phase is valid roughly for $\gamma^2\beta M^2 m_{\rm Pl}^{-2} < 1$; otherwise, the Universe becomes PBH dominated before the moment of evaporation.

Let us assume that $\gamma^2\beta M^2 m_{\rm Pl}^{-2} < 1$. At the moment right after PBH evaporation, the energy density of the PBH remnants is $\rho_{\rm rem}(t_{\rm evap}^>) = (3/2)\kappa\gamma^2\beta(M/m_{\rm Pl})\rho_{\rm tot}$. The background energy density is now partitioned between the stiff fluid, $\rho_{\rm S}$, and the entropy produced by the PBH evaporation, $\rho_{\rm rad}$. The latter is about $M/(\kappa m_{\rm Pl})$ times larger than $\rho_{\rm rem}(t_{\rm evap}^>)$. Assuming that the evaporation products thermalize fast, the radiation redshifts like $\rho_{\rm rad} \propto g_* g_s^{-4/3} a^{-4}$, whereas the stiff fluid background redshifts like $\rho_{\rm S} \propto a^{-6}$. At some moment the radiation dominates the background energy density, and we define it as the *reheating* moment $t_{\rm rh}$. The scale factor is

$$\frac{a(t_{\rm rh})}{a(t_{\rm evap})} = \left(\frac{2}{3}\frac{m_{\rm Pl}^2}{M^2}\frac{1}{\gamma^2\beta}\frac{g_*^{1/3}(t_{\rm rh})}{g_*^{1/3}(t_{\rm evap})}\right)^{1/2}. \tag{3.23}$$

At that moment we also define the reheating temperature of the Universe, which reads

$$T_{\rm rh} \equiv 6.3\ {\rm MeV}\left(\frac{\beta}{10^{-28}}\right)^{3/4}\gamma^{3/2}g_*^{-1/2}. \tag{3.24}$$

Until the moment $t_{\rm rh}$, the energy density of the PBH remnants increases relatively to the stiff fluid dominated background as $\rho_{\rm rem}/\rho_{\rm S} \propto a^3$, and afterward, that radiation dominates: it increases as $\rho_{\rm rem}/\rho_{\rm rad} \propto T^{-1}$. It is

$$f_{\rm rem}(M) = \tilde{c}_{\rm S}\frac{3}{2}\gamma^2\beta\frac{\kappa M}{m_{\rm Pl}}\left(\frac{a(t_{\rm rh})}{a(t_{\rm evap})}\right)^3\left(\frac{M_{\rm eq}}{M_{\rm rh}}\right)^{1/2}, \tag{3.25}$$

where $\tilde{c}_{\rm S} = 2^{1/4}(g(T_{\rm rh})/g(T_{\rm eq}))^{-1/4}\Omega_{\rm m}/\Omega_{\rm DM}$ and $M_{\rm rh} = M_H(t_{\rm rh})$. For times $t < t_{\rm rh}$ the Hubble radius mass increases like $M_H \propto a^3$, and, given that $M_H(t_{\rm evap}) = (3/2)M^3/m_{\rm Pl}^2$, we find the $M_{\rm rh}$ mass:

$$M_{\rm rh} = \sqrt{\frac{2}{3}}\gamma^{-3}\beta^{-3/2}g_*^{1/2}m_{\rm Pl}. \tag{3.26}$$

Therefore, the Eq. (3.25) is rewritten as

$$f_{\rm rem}(M) = \sqrt{\frac{2}{3}}\tilde{c}_{\rm S}\left(\frac{3}{2}\gamma^2\beta\right)^{1/4}\kappa\left(\frac{m_{\rm Pl}}{M}\right)^{3/2}\left(\frac{M_{\rm eq}}{M}\right)^{1/2}, \tag{3.27}$$

and normalizing with benchmark values, we attain

$$f_{\rm rem}(M) \simeq 4\kappa\sqrt{\gamma}\left(\frac{\beta}{10^{-32}}\right)^{1/4}\left(\frac{M}{10^5\ {\rm g}}\right)^{-2}. \tag{3.28}$$

For $\kappa \sim 1$ and $M \sim 10^5$ g, β values as small as 10^{-32} can explain the observed dark matter in the Universe. Pressure is maximal, and we expect the overdense regions to be spherically symmetric. Utilizing relation (3.12), PBH formation occurs when the density perturbation becomes larger than $\delta_c = 0.375$, and for the formation probability $\beta(M)$ given by Eq. (3.11), we find that power spectrum values $\mathcal{P}_{\mathcal{R}} \lesssim 3.5\times 10^{-3}$, for $\kappa \gtrsim 1$ and $M \sim 10^5$ g, can yield $f_{\rm rem} = 1$.

A particular example that yields $f_{\rm rem} = 1$ is described in Sec. V.

1. BBN constraints

A kination regime has to comply with the BBN constraints. Let us assume that a runaway inflaton φ is responsible for the kination regime. The energy density during BBN is partitioned between the kinetic energy of the φ field and the background radiation. Any modification to the simple radiation domination regime is parametrized by an equivalent number of additional neutrinos, and the Hubble parameter has to satisfy the constraint [65]

$$\left(\frac{H}{H_{\rm rad}}\right)^2\Bigg|_{T=T_{\rm BBN}} \leq 1 + \frac{7}{43}\Delta N_{\nu_{\rm eff}} \simeq 1.038, \tag{3.29}$$

where H is the actual Hubble parameter and $H_{\rm rad}$ is the Hubble parameter if the total energy density is equal to the radiation. The $\Delta N_{\nu_{\rm eff}} = 3.28\text{–}3.046$ is the difference between the cosmologically measured value and the SM prediction for the effective number of neutrinos. In order to prevent the Universe from expanding too fast during BBN due to the extra energy density $\dot{\varphi}^2/2$, the reheating temperature has to be larger than [66]

$$T_{\rm rh} > (\alpha-1)^{-1/2}\left(\frac{g_*(T_{\rm BBN})}{T_{\rm rh}}\right)^{1/4}T_{\rm BBN} = \mathcal{O}(10)\ {\rm MeV}, \tag{3.30}$$

where $\alpha \equiv 1 + 7/43\Delta N_{\nu_{\rm eff}} \simeq 1.038$.

An additional issue is that the gravitational wave energy in the gigahertz region gets enhanced during the kination regime [67–70]. The energy density of the gravitational waves does not alter BBN predictions if

$$I \equiv h^2 \int_{k_{\rm BBN}}^{k_{\rm end}} \Omega_{\rm GW}(k) d\ln k \leq 10^{-5}, \tag{3.31}$$

which is written as [71]

$$I = \frac{2\epsilon h^2 \Omega_{\rm rad}(t_0)}{\pi^{2/3}} \left(\frac{30}{g(T_{\rm reh})}\right)^{1/3} \frac{h_{\rm GW}^2 V_{\rm end}^{1/3}}{T_{\rm rh}^{4/3}}, \tag{3.32}$$

where $\epsilon \sim 81/(16\pi^3)$, $h^2\Omega_{\rm rad}(t_0) = 2.6\times 10^{-5}$, and $h_{\rm GW}^2 = H_{\rm end}^2/(8\pi M_{\rm Pl}^2)$. Substituting numbers, the observational constraint $I \lesssim 10^{-5}$ gives a lower bound on the reheating temperature,

$$T_{\rm rh} \gtrsim 10^6 \text{ GeV} \left(\frac{106.75}{g_*}\right)^{1/4} \left(\frac{H_{\rm end}^2}{10^{-6} M_{\rm Pl}}\right)^2. \tag{3.33}$$

Substituting the reheating temperature predicted by kination-PBH models, Eq. (3.24), into the bound (3.33), we obtain a lower bound on the formation rate

$$\beta \gtrsim 2\times 10^{-17} \left(\frac{106.75}{g_*}\right)^{1/3} \left(\frac{H_{\rm end}}{10^{-6} M_{\rm Pl}}\right)^{8/3}. \tag{3.34}$$

Smaller values for β mean that the radiation produced from the PBH evaporation dominates later during the early cosmic evolution, and the kination regime is dangerously extended. Asking for $f_{\rm rem} = 1$, the lower bound on β yields a lower bound on the ratio

$$\frac{M}{\sqrt{\kappa}} \gtrsim 1.6\times 10^7 \text{ g}\gamma^{1/4} \left(\frac{H_{\rm end}}{10^{-6} M_{\rm Pl}}\right)^{1/3} \left(\frac{g_*}{106.75}\right)^{1/12}. \tag{3.35}$$

We recall that the mass M has to satisfy the upper bound given by Eq. (3.4), $M < M_{\rm inter} \simeq 2\kappa^{2/5} 10^6$ g; otherwise, it is always $f_{\rm rem} < 1$. This bound gives a maximum value for κ,

$$\frac{\kappa}{10^{-10}} \lesssim 8.5\gamma^{-5/2} \left(\frac{H_{\rm end}}{10^{-6} M_{\rm Pl}}\right)^{-10/3} \left(\frac{g_*}{106.75}\right)^{-5/6}. \tag{3.36}$$

Unless $H_{\rm end} \ll 10^{-6} M_{\rm Pl}$, it must be $\kappa < 1$; hence, for high scale inflation the PBH remnants must have sub-Planckian masses. For $\kappa = \kappa_{\rm max}$ a maximum value for the mass of the PBHs, $M = M_{\rm inter}$, is obtained for the kination regime in order for the remnants to saturate $\Omega_{\rm DM}$.

PBH remnants with $\kappa \geq 1$ require $H_{\rm end} \lesssim 2\times 10^{-9}\gamma^{-3/4} M_{\rm Pl}$, which can be achieved either in a small field inflation model or by models where the CMB and PBH potential energy scales have a large difference, so the high frequency gravitational waves (GWs) have a smaller amplitude. We underline that the above results are valid only if the postinflationary equation of state of the inflaton field satisfies $w \simeq 1$, at least until the BBN epoch. If it is $w < 1$, the derived bounds get relaxed.

IV. BUILDING A $\mathcal{P}_{\mathcal{R}}(k)$ PEAK IN ACCORDANCE WITH OBSERVATIONS

A. The position of the $\mathcal{P}_{\mathcal{R}}(k)$ peak

The wave number that inflation ends is

$$k_{\rm end} = k_* \frac{H_{\rm end}}{H_*} e^{N_*}, \tag{4.1}$$

where N_* are the e-folds of the observable inflation and given by the expression

$$N_* \simeq 57.6 + \frac{1}{4}\ln\epsilon_* + \frac{1}{4}\ln\frac{V_*}{\rho_{\rm end}} - \frac{1-3w}{4}\tilde{N}_{\rm rh}. \tag{4.2}$$

ϵ_*, H_*, and V_* are, respectively, the first slow-roll parameter, the Hubble scale, and the potential energy when the CMB pivot scale exits the Hubble radius, while $H_{\rm end}$ and $\rho_{\rm end}$ are the Hubble scale and the energy density at the end of inflation.

The N_* value is related to the postinflationary reheating e-folds $\tilde{N}_{\rm rh}$ and the corresponding (averaged) equation of state w. We have implicitly assumed that w refers to the postinflationary equation of state until the moment reheating completes. The number of e-folds until the completion of the reheating $\tilde{N}_{\rm rh}$ are

$$\tilde{N}_{\rm rh}(T_{\rm rh}, H_{\rm end}, g_*, w) = -\frac{4}{3(1+w)} \ln\left[\left(\frac{\pi^2 g_*}{90}\right)^{1/4} \frac{T_{\rm rh}}{(H_{\rm end} M_{\rm Pl})^{1/2}}\right]. \tag{4.3}$$

An inhomogeneity of size k^{-1} crosses inside the horizon during radiation domination if $k < k_{\rm rh}$, where

$$k_{\rm rh} = k_{\rm end} e^{-\frac{3w+1}{2}\tilde{N}_{\rm rh}}. \tag{4.4}$$

For a general expansion rate determined by the effective equation of state value w, the $k(M)$ relation reads

$$k(M, w) = k_{\rm end} \left(\frac{M/\gamma}{M_{\rm end}}\right)^{-\frac{3w+1}{3(1+w)}}. \tag{4.5}$$

$M_{\rm end}$ and $k_{\rm end}$ depend on the details of the inflationary model, with the latter becoming larger for larger values of w. $k(M, w)$ can be written using the reheating completion moment as the reference period replacing, respectively, $k_{\rm end}$ and $M_{\rm end}$ with $k_{\rm rh}$ and $M_{\rm rh}$ in Eq. (4.5). Then we attain the more general $k(M, T_{\rm rh}, w)$ relation

$$k(M, T_{\rm rh}, w) \simeq 2\times 10^{17}\ {\rm Mpc}^{-1}\left(\frac{T_{\rm rh}}{10^{10}\ {\rm GeV}}\right)^{\frac{1-3w}{3(1+w)}} \times\left(\frac{M/\gamma}{10^{12}\ {\rm g}}\right)^{-\frac{3w+1}{3(1+w)}}\left(\frac{g_*}{106.75}\right)^{\frac{1}{4}\frac{1-3w}{3(1+w)}}. \quad (4.6)$$

For the case of kination domination the minor correction, $T_{\rm rh}\to 2^{1/4}T_{\rm rh}$, should be added due to the equipartition of the energy density between the radiation and the scalar field.

Assuming a one-to-one correspondence between the scale of perturbation and the mass of PBHs, an inflationary model builder who aims at generating PBHs with mass M has to produce a $\mathcal{P}_\mathcal{R}(k)$ peak at the wave number $k(M, T_{\rm rh}, w)$. Next, we briefly discuss the additional observational constraints, regarding the width of the peak, which one has to take into account in order for the inflationary model to be viable.

B. Observational constraints on $\mathcal{P}_\mathcal{R}(k)$ at small scales

A power spectrum peak at large wave number, $k\gg k_*$, is welcome for not spoiling the n_s and α_s values measured at k_*. Also, such a peak can generate PBHs abundant enough to compose all of the dark matter in the Universe, either as long-lived PBHs or as PBH remnants. However, shifting the peak at large wave number does not render $\mathcal{P}_\mathcal{R}(k)$ free from constraints. The impact of Hawking radiation on the BBN and CMB observables and the extragalactic γ-ray background put strong upper bounds on the $\mathcal{P}_\mathcal{R}(k)$ at large k-bands. These bounds rule out a great part of the PBH mass spectrum with range 10^9 g $< M < 10^{17}$ g by explaining $\Omega_{\rm DM}$ with PBH remnants (PBH remnants from holes with mass $M\sim 10^{10}$ g or larger could explain $\Omega_{\rm DM}$ if $\kappa\gg 1$). Moreover, even if the power spectrum peak produces PBHs at a mass scale where the PBH abundance can be maximal (e.g., $M\sim 10^{18}$ g), the width of the peak has to be particularly narrow. The stringent constraint comes from the CMB, at the mass scale $M\sim 10^{13}$ g, where the electrons and positron produced by PBH evaporation after the time of recombination scatter off the CMB photons and heat the surrounding matter damping small-scale CMB anisotropies, contrary to observations. The next stringent constraint is applied at the mass range $M = 10^{10}$–10^{13} g, where evaporation affects the BBN relics via hadrodissociation and photodissociation processes [19,72–74].

In Ref. [75] the observational constraints have been explicitly translated into $\mathcal{P}_\mathcal{R}(k)$ bounds. In a radiation dominated early Universe, utilizing Eq. (4.6), the CMB constraint for $M_{\rm cmb}\equiv 2.5\times 10^{13}$ g and $w_{\rm rh}=1/3$ yields the bound

$$\sigma(3\times 10^{17}k_*)\lesssim 0.035\left(\frac{\delta_c}{0.41}\right). \quad (4.7)$$

σ is the variance of the comoving density contrast, $\sigma^2\sim(4/9)^2\mathcal{P}_\mathcal{R}$, and the bound $\mathcal{P}_\mathcal{R}(4\times 10^{17}k_*)\lesssim\mathcal{O}(10^{-3})$ is derived.

Turning to a MD early Universe reheated at temperatures $T_{\rm rh}\lesssim 10^7$ GeV, the variance of the density perturbations has to satisfy the CMB bound,

$$\sigma(k(M_{\rm cmb}, T_{\rm rh}))\lesssim {\rm Exp}\left[-6.9-0.09\ln\frac{T_{\rm rh}}{\rm GeV} + 2\times 10^{-3}\left(\ln\frac{T_{\rm rh}}{\rm GeV}\right)^2 - 3\times 10^{-5}\left(\ln\frac{T_{\rm rh}}{\rm GeV}\right)^3\right], \quad (4.8)$$

where $k(M_{\rm cmb}, T_{\rm rh})\simeq 3\times 10^{17}k_*\gamma^{1/3}(T_{\rm rh}/10^7\ {\rm GeV})^{1/3}(g_*/106.75)^{1/12}$, according to Eq. (4.6) for $w_{\rm rh}=0$. During the matter domination era, it is $\sigma\sim(2/5)\mathcal{P}_\mathcal{R}{}^{1/2}$ and for $\gamma=0.1$ and $T_{\rm rh}=10^7$ GeV the constraint on the power spectrum reads $\mathcal{P}_\mathcal{R}(1.4\times 10^{17}k_*)\lesssim\mathcal{O}(9\times 10^{-7})$. If the reheating temperature is $10^7\ {\rm GeV}\lesssim T_{\rm rh}\lesssim 4\times 10^8$ GeV, then the BBN constraint on the power spectrum applies. For $M_{\rm BBN}\equiv 5\times 10^{10}$ and $T_{\rm rh}=10^8$ GeV, the constraint reads $\mathcal{P}_\mathcal{R}(10^{18}k_*)\lesssim\mathcal{O}(4\times 10^{-6})$, which is a bit weaker than the CMB. For larger reheating temperatures, $T_{\rm rh}\gtrsim 10^9$ GeV, the constraints get significantly relaxed and are given by Eq. (4.7).

For the case of kination domination the CMB constraint applies on the scale with wave number $k(M_{\rm cmb}, T_{\rm rh})$ and reads

$$\sigma(k(M_{\rm cmb}, T_{\rm rh}))\lesssim 0.032\left(\frac{\delta_c}{0.375}\right). \quad (4.9)$$

It is $k(M_{\rm cmb}, T_{\rm rh})\simeq 5\times 10^{18}k_*\gamma^{2/3}(T_{\rm rh}/10^7\ {\rm GeV})^{-1/3}(g_*/106.75)^{-1/12}$, according to Eq. (4.6) for $w_{\rm rh}=1$ and $M_{\rm cmb}=2.5\times 10^{13}$ g. Note here that the CMB bound (4.9) applies for reheating temperatures $T_{\rm rh}\lesssim 2\times 10^9$ GeV since the gravitational collapse can be considered instantaneous, contrary to the case of the matter era [75].

For smaller PBH masses that evaporate in less than a second, there are limits on the amount of thermal radiation from the PBH evaporation due to the production of entropy that may be in conflict with the cosmological photon-to-baryon ratio [76]. There are also constraints from the abundance of dark matter produced by the evaporation, e.g., the lightest supersymmetric particle (LSP). In our models the abundance of the PBH remnants saturates the dark matter density parameter $\Omega_{\rm DM}h^2=0.12$; thus, the LSP constraint does not apply. Finally, for ultrasmall PBH masses, the constraint comes only from the relic abundance of the Planck-mass remnants [23,24,77]. These constraints are labeled *entropy, LSP* (with a dotted-dashed line due to fact that in our models this constraint is raised), and *Planck*, respectively, in our figures.

V. PBHs FROM THE α-ATTRACTOR INFLATION MODELS

A. The inflaton potential and the computation of $\mathcal{P}_{\mathcal{R}}(k)$

The above constraints imply that, even in large wave numbers, the power spectrum peak has to be positioned in a particular range of k and, additionally, to be sufficiently narrow. In this section our goal is to generate PBHs that will evaporate fast enough in the early Universe without affecting the BBN and CMB observables and, at the same time, leave behind mass remnants that will saturate the dark matter abundance. In order to implement this scenario, we employ the machinery of α-attractors and build inflationary models with inflection point at large k.

If the inflaton potential features an inflection point, a large amplification in the power spectrum $\mathcal{P}_{\mathcal{R}}(k)$ can be achieved due to the acceleration and deceleration of the inflaton field in the region around the inflection point, as was pointed out in [38,78]. The presence of an inflection point requires $V' \approx 0$ and $V'' = 0$. In the context of supergravity such a model may arise from α-attractors, by choosing appropriate values for the parameters in the superpotential as described in Ref. [41].

We focus on the effective Lagrangian for the inflaton field φ in the α-attractor scenario that turns out to be

$$e^{-1}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}(\partial_\mu \varphi)^2 - f^2\left(\tanh\frac{\varphi}{\sqrt{6\alpha}}\right), \tag{5.1}$$

where $\mathrm{Re}\Phi = \phi = \sqrt{3}\tanh(\varphi/\sqrt{6\alpha})$ is a chiral superfield.

Polynomial and trigonometric forms for the function $f(\phi)$ can feature an inflection point plateau sufficient to generate a significant dark matter abundance in accordance with the observational constraints [41]. Nevertheless, other forms for the function $f(\phi)$ are plausible. Exponential potentials enjoy a theoretical motivation in several BSM frameworks, and their cosmology has been extensively studied; see, e.g., [79–86]. In the following we will examine the PBH formation scenario from α-attractor inflationary potentials built by exponential functions.

The form of the potential fully determines the subsequent adiabatic evolution of the Universe; see Fig. 4. Firstly, the number of e-folds N_*, which follow the moment the k_*^{-1} scale exits the quasi–de Sitter horizon, determine the duration of the nonthermal stage after inflation. Secondly, the position and the features of the inflection point plateau determine the mass and the abundance of the PBHs that form and, in particular, the moment the overdensities reenter the horizon. If PBHs of a given mass M form during the radiation era a specific inflationary potential has to be designed. On the contrary, PBH production during the matter era requires a different potential. Furthermore, an inflationary potential might be a runaway without a minimum at all. Such a potential is acceptable if it can realize an inflationary exit and a sufficient reheating of the Universe. Remarkably, both of these conditions can be satisfied in our modes with the generated mini PBHs to guarantee a successful reheating via their evaporation. Last but not least, at large scales $k \sim k_*$ we demand that the CMB observables, as specified by the Planck 2018 data, remain intact.

The PBH abundance is found only after the computation of the value of the comoving curvature perturbation $\mathcal{R}_k$. In the comoving gauge we have $\delta\varphi = 0$ and $g_{ij} = a^2[(1-2\mathcal{R})\delta_{ij} + h_{ij}]$. Expanding the inflaton-gravity action to second order in $\mathcal{R}$, one obtains

$$S_{(2)} = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}a^3\frac{\dot{\varphi}^2}{H^2}\left[\dot{\mathcal{R}}^2 - \frac{(\partial_i\mathcal{R})^2}{a^2}\right]. \tag{5.2}$$

After the variable redefinition $v = z\mathcal{R}$, where $z^2 = a^2\dot{\phi}^2/H^2 = 2a^2\epsilon_1$, and switching to conformal time τ (defined by $d\tau = dt/a$), the action is recast into

$$S_{(2)} = \frac{1}{2}\int \mathrm{d}\tau\mathrm{d}^3x\left[(v')^2 - (\partial_i v)^2 + \frac{z''}{z}v^2\right]. \tag{5.3}$$

The evolution of the Fourier modes v_k of $v(x)$ are described by the Mukhanov-Sasaki equation

$$v_k'' + \left(k^2 - \frac{z''}{z}\right)v_k = 0, \tag{5.4}$$

where z''/z is expressed in terms of the functions

$$\epsilon_1 \equiv -\frac{\dot{H}}{H^2}, \qquad \epsilon_2 \equiv \frac{\dot{\epsilon}_1}{H\epsilon_1}, \qquad \epsilon_3 \equiv \frac{\dot{\epsilon}_2}{H\epsilon_2} \tag{5.5}$$

as

$$\frac{z''}{z} = (aH)^2\left[2 - \epsilon_1 + \frac{3}{2}\epsilon_2 - \frac{1}{2}\epsilon_1\epsilon_2 + \frac{1}{4}\epsilon_2^2 + \frac{1}{2}\epsilon_2\epsilon_3\right]. \tag{5.6}$$

We are interested in the super-Hubble evolution of the curvature perturbation, that is, for $k^2 \ll z''/z$. The power of $\mathcal{R}_k$ on a given scale is obtained once the solution v_k of the Mukhanov-Sasaki equation is known and estimated at a time well after it exits the horizon and its value freezes out,

$$\mathcal{P}_{\mathcal{R}} = \frac{k^3}{2\pi^2}\frac{|v_k|^2}{z^2}\bigg|_{k \ll aH}. \tag{5.7}$$

After the numerical computation of the Mukhanov-Sasaki equation, $\mathcal{P}_{\mathcal{R}}$ at all of the scales is obtained; see Figs. 5, 6, and 7. As required, the $\mathcal{P}_{\mathcal{R}}(k)$ of our models satisfy the constraints given by Eq. (4.7), (4.8), and (4.9) for the

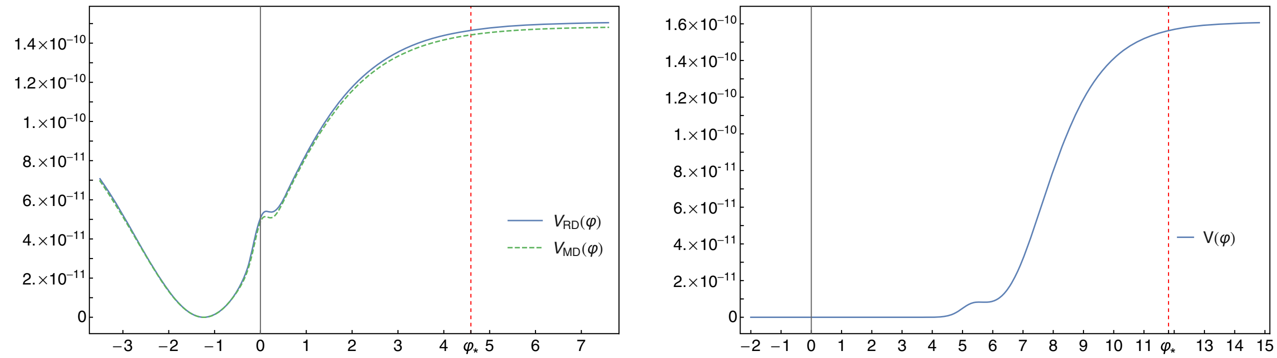

FIG. 4. (Left panel) The potentials from the superconformal attractors, Eq. (5.9), which trigger PBH formation during the radiation era (solid line) and during the matter era (dashed line). Although the potentials differ slightly, they yield very different power spectra (see Figs. 5 and 6) and PBH masses. (Right panel) The potential for the runaway model, Eq. (5.11), with the characteristic asymptotic flatness for large negative values of φ. In both panels the position of the inflection point and the total number of e-folds N_* determine the mass of the PBHs. The red dashed vertical line indicates the φ_* position of the field that corresponds to the Planck pivot scale k_*; it is $\varphi_*/M_{\rm Pl} = 4.59,\ 4.60,\ 11.81$ for the radiation, matter, and kination cases, respectively. The parameters of the potentials are listed in Table III.

radiation, matter, and kination eras, respectively. From the $\mathcal{P}_\mathcal{R}(k)$ we compute the $f_{\rm rem}$ as described in Sec. III; see Tables I and II. We note that we have neglected possible impacts on the power spectrum from non-Gaussianities [87–90] and quantum diffusion effects [91–94].

B. Inflaton potential for PBH production during the radiation and matter eras

1. Radiation era

A function $f(\phi)$ built by exponentials can feature a proper inflection point plateau. The form of the potential is chosen to produce PBHs of the right abundance. We ask for large reheating temperatures so that the large inhomogeneities can reenter the horizon after the thermalization of the Universe. This is achieved by sufficiently strong couplings of the inflaton field to the visible sector. We also demand values for n_s and α_s that are favored by the Planck 2018 data [33]. An example of a combination of exponentials that can fulfill the above requirements is of the form

$$f(\phi/\sqrt{3}) = f_0(c_0 + c_1 e^{\lambda_1\phi/\sqrt{3}} + c_2 e^{\lambda_2\phi^2/3}), \qquad (5.8)$$

which generates the potential

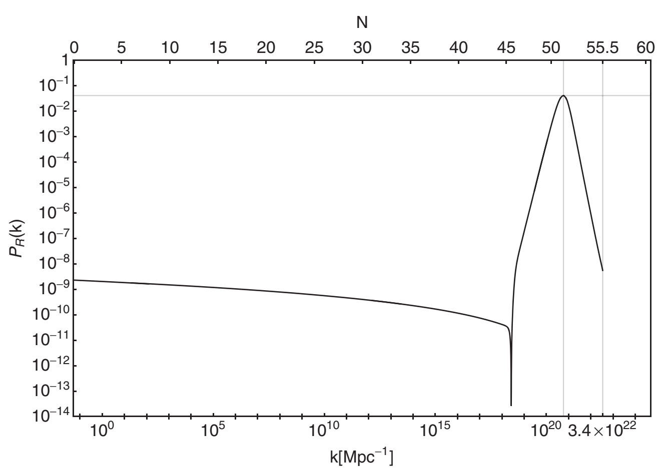

FIG. 5. The power spectrum of the comoving curvature perturbations for model (5.9) with parameters listed in Table III for the RD case and potential depicted in Fig. 4. A significant amplification of the power spectrum $\mathcal{P}_\mathcal{R} \simeq 4\times10^{-2}$ takes place at small scales $k = 5.9\times10^{20}$ Mpc^{-1} about $N \simeq 4$ e-folds before the end of inflation, triggering the production of mini PBHs. The duration of the reheating era is almost instantaneous.

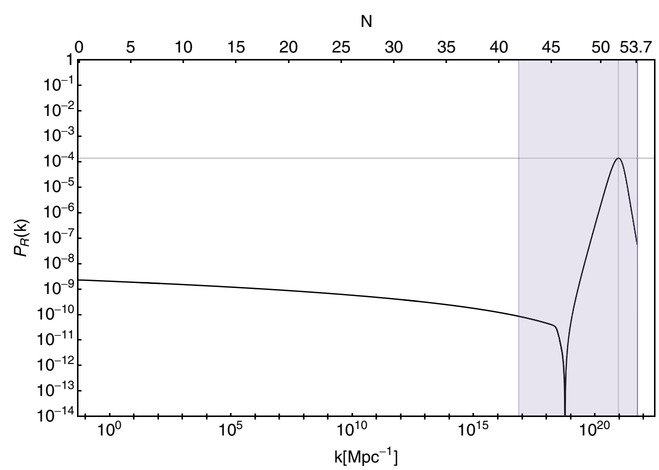

FIG. 6. The power spectrum of the comoving curvature perturbations for model (5.9), with parameters listed in Table III for the MD case and the potential depicted in Fig. 4. The power spectrum has a peak with amplitude $\mathcal{P}_\mathcal{R} \simeq 7\times10^{-5}$ at the scale $k = 9.6\times10^{20}$ Mpc^{-1} about $N \simeq 2$ e-folds before the end of inflation and before mini PBHs are produced. The shaded part of $\mathcal{P}_\mathcal{R}(k)$ corresponds to the scales that enter the horizon during the matter era.

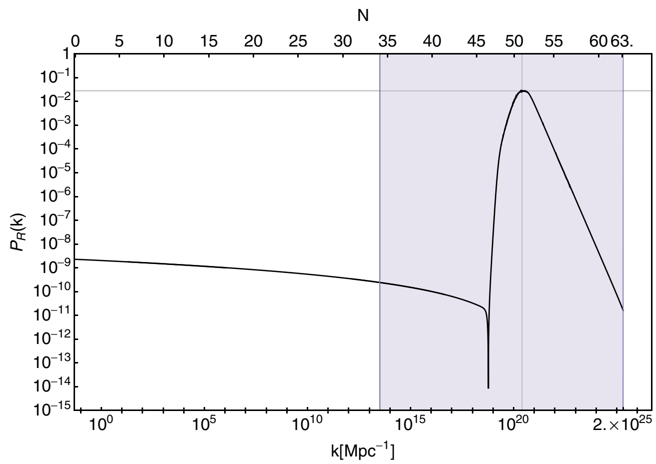

FIG. 7. The power spectrum of the comoving curvature perturbations for model (5.11) with parameters listed in Table III for the SD case and the potential depicted in Fig. 4. The power spectrum has a peak with amplitude $\mathcal{P}_{\mathcal{R}} \simeq 2.7 \times 10^{-2}$ at the scales $k = 2 \times 10^{25}\ \mathrm{Mpc}^{-1}$ about $N \simeq 12$ e-folds before the end of inflation, triggering the production of mini PBHs during the kination regime. The shaded part of $\mathcal{P}_{\mathcal{R}}(k)$ corresponds to the scales that enter the horizon during the kination era.

$$V(\varphi) = f_0^2 (c_0 + c_1 e^{\lambda_1 \tanh\varphi/\sqrt{6}} + c_2 e^{\lambda_2 (\tanh\varphi/\sqrt{6})^2})^2, \qquad (5.9)$$

having taken $\alpha = 1$. The determination of the parameter values requires a subtle numerical process that we outline. Firstly, a central PBH mass M has to be chosen, and from Eq. (3.10) the β value that saturates f_{PBH} is specified. M is the parameter that spots the k-position of the $\mathcal{P}_{\mathcal{R}}(k)$ peak. β is exponentially sensitive to the amplitude of the peak, and its exact value is found after a delicate selection of the potential parameters. $\mathcal{P}_{\mathcal{R}}(k)$ is produced by solving numerically the Mukhanov-Sasaki equation, following the method described in Ref. [41]. At the same time consistency with the CMB normalization and the measured

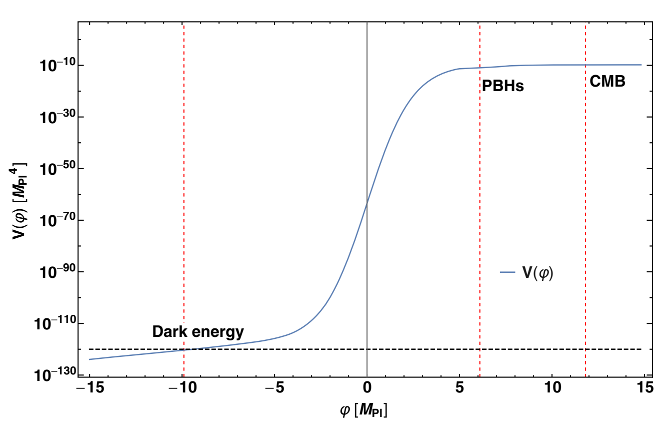

FIG. 8. Potential (5.11), depicted in Fig. 4, for the parameters listed in Table III (Kin.). The plot is in logarithmic scale in order to make the $V(\varphi)$ value visible both during inflation and today. See also Fig. 1.

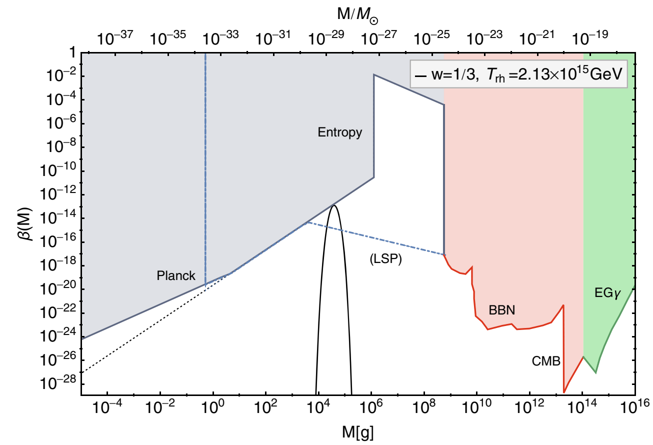

FIG. 9. The PBH formation rate $\beta(M)$ estimated by Eq. (3.11) for model (5.9) with the parameters listed in Table III, and $\mathcal{P}_{\mathcal{R}}(k)$ depicted in Fig. 5 for the RD case. The central mass of the PBHs is $M \simeq 4 \times 10^4$ g that evaporate, leaving behind Planck-mass remnants with $f_{\mathrm{rem}} = 1$. The dotted black line depicts the constraints if the reheating temperature is $T_{\mathrm{rh}} > 10^{15}$ GeV. The LSP upper bound is not applicable since the PBH remnants compose the total dark matter in our scenarios. EGγ stands for the extra galactic gamma ray background.

n_s and α_s values, as well as a large enough N_* value, is required.

For $\phi \to \sqrt{3}$ the potential energy drives the early Universe cosmic inflation, and CMB normalization gives the first constraint for the parameters. We also demand zero potential energy at the minimum of the potential that gives a second constraint. We also note that f_0 is a redundant parameter since it can be absorbed by c_0, c_1, and c_2. We keep it only for numeric convenience. An example of parameter values that realize the PBH production during

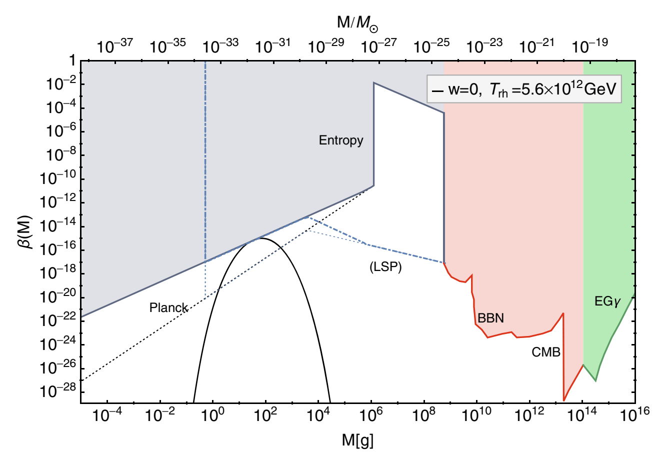

FIG. 10. The PBH formation rate $\beta(M)$ estimated by Eq. (3.19) for model (5.9) with the parameters listed in Table III, and $\mathcal{P}_{\mathcal{R}}(k)$ depicted in Fig. 6 for the MD case. The central mass of the PBHs is $M \simeq 61$ g that evaporate, leaving behind Planck-mass remnants with $f_{\mathrm{rem}} = 1$. The dotted black and blue lines depict the constraints at an arbitrary large reheating temperature.

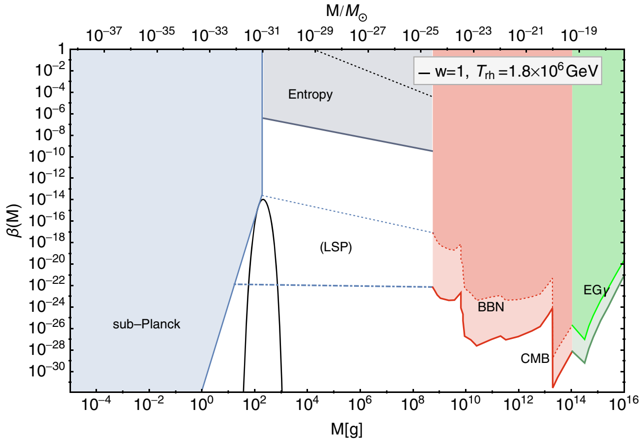

FIG. 11. The $\beta(M)$ predicted by the inflationary model (5.11), with parameters listed in Table III for the SD case. The PBHs with central mass $M = 2 \times 10^2$ g are produced and evaporate during the kination era (SD). The evaporation leaves behind sub-Planckian-mass remnants with mass $M_{\rm rem} = 4 \times 10^{-11} m_{\rm Pl} \simeq 6 \times 10^8$ GeV that compose all of the dark matter, $f_{\rm rem} = 1$. The reheating temperature is determined by Eq. (3.24) and is proportional to $\beta^{3/4}$. The $\beta_{\rm max}$ constraints for the entropy, BBN, CMB, and EGγ are $(M/M_{\rm rh})^{1/2}$ stringent compared to the RD case (dotted lines).

the RD era is listed in Table III, and the potential is depicted in the left panel of Fig. 4.

2. Matter era

An early Universe matter domination era can be realized if the shape of the inflationary potential around the minimum is approximated with a quadratic potential.

TABLE I. The predictions for the three inflationary models discussed in the text. The era indicates the production era of the PBHs, that is, the era in which the perturbations with the largest amplitude reenter the horizon. The PBH remnant abundance, $f_{\rm rem}$, is the maximal one. Depending on the parameters of the potentials, the initial PBH mass $M_{\rm PBH}^{\rm peak}$ varies in the range $M \sim 10$–10^4 g.

Era	β	$T_{\rm rh}$ (GeV)	$\tilde{N}_{\rm rh}$	$M_{\rm peak}$(g)	$f_{\rm rem}$
RD	1.25×10^{-13}	2.13×10^{15}	0	3.9×10^4	~1
MD	1×10^{-15}	5.6×10^{12}	7.5	69	~1
Kin.	1.07×10^{-14}	1.8×10^6	13.57	2×10^2	~1

TABLE II. Characteristic values for the curvature power spectrum are listed.

Era	$\mathcal{P}_{\mathcal{R}}{}^{\rm peak}$	$k_{\rm end}$	N_*
RD	4×10^{-2}	3.45×10^{22}	55.5
MD	1.4×10^{-4}	5.69×10^{21}	53.7
Kin.	2.7×10^{-2}	2×10^{25}	63.0

TABLE III. A set of values for the parameters of each potential in Eqs. (5.9) and (5.11) responsible for PBH production in the radiation, matter, and kination domination scenarios are listed. For the kination model we also took $\varphi_{\rm P} = 0.995 M_{\rm Pl}$. We add the parameter values $f_0^2 = 7.49267 \times 10^{-11}$, 7.37421×10^{-11}, 3.11497×10^{-62} for the radiation, matter, and kination cases, respectively.

Era	c_0	c_1	c_2	λ_1	λ_2
RD	−1.856	1.173	−0.14	−0.987	95.5904
MD	−1.856	1.173	−0.13	−0.987	124.57
Kin.	-8.70×10^{-27}	0.1045	-4×10^{25}	62.2	−4430.973

For moderately suppressed inflaton couplings the inflaton decays after a large number of oscillations. The inhomogeneities that reenter the horizon during the stage of the inflaton oscillations might collapse in a pressureless environment.

The calculation of PBH production during the matter era involves the same numerical steps as the case of radiation plus some extra conditions that have to be taken into account. Firstly, the PBH mass M value is not adequate to specify the k-position and the amplitude of the $\mathcal{P}_{\mathcal{R}}(k)$ peak since there is a crucial dependence on the reheating temperature. Hence, after choosing the PBH mass M, the required β is fixed for a particular reheating temperature. In turn, the $T_{\rm rh}$ fixes the number of e-folds that constrain the inflaton excursion in the field space. Moreover, the amplitude of the peak has an additional dependence on the reheating temperature—namely, the variance of the perturbations has to satisfy the bound (3.21), $\sigma > \sigma_{\rm cr}(T_{\rm rh})$—in order for the inhomogeneities to fully collapse during the matter domination era.

An inflationary example that predicts PBH formation during the matter era is given by Eq. (5.8) after proper parameter values are chosen. The set of the parameters, listed in Table III, yields an amplitude for $\mathcal{P}_{\mathcal{R}}(k)$ in which spin effects have to be considered in the estimation of the formation probability.

C. Inflaton potential for PBH production during the kination era

The period of kination domination has an interesting and distinct cosmology. It is able to be realized after inflation if the potential does not have a vacuum; see [71,95,96] for α-attractor kination models. A nonoscillatory inflaton field will run away without decaying, resulting in a period where the kinetic energy dominates over the potential energy. The attractive feature of such models is that the inflaton can survive until today and might play the role of quintessence. Moreover, such models are attractive because they lead to a different early Universe phenomenology since the effective equation of state is $w \sim 1$ and the expansion rate is reduced. This is the so-called stiff fluid or kination era, which gives rise to different predictions regarding some early Universe observables such as the spectrum of the tensor perturbations, a fact that renders such an era testable.

The kination scenarios usually suffer from radiation shortage since the inflaton field does not decay and special mechanisms have to be introduced. A source of radiation comes from the Hawking temperature of de Sitter space, called gravitational reheating, but this is very inefficient [97,98]. On the other hand, the Hawking radiation from mini PBHs formed by a runaway inflaton automatically reheats the Universe. So, in our models radiation is produced by the evaporation of the PBHs, which can be efficient enough. According to Eq. (3.24), common values for β imply large enough reheating temperatures.

The construction of kination inflation models that induce PBH production is very challenging. Firstly, the inflaton runs away until it freezes at some value φ_F, and this residual potential energy of the inflaton must not spoil the early and late time cosmology. The inflaton potential energy at φ_F has to be tuned to values $V(\varphi_F) \lesssim 10^{-120} M_{\rm Pl}^4$, and similarly to all of the quintessence models. Secondly, the kination inflaton model parameters are self-constrained. A particular PBH mass M specifies the k of $\mathcal{P}_\mathcal{R}(k)$ only if the reheating temperature is known. However, the reheating temperature is not a free parameter, as, e.g., in matter or radiation cases where the $T_{\rm rh}$ depends on the inflaton decay rate. In the kination scenario $T_{\rm rh}$ depends on β.β is found using the condition $f_{\rm rem} = 1$, and this fixes the reheating temperature.

Hence, the characteristics of the peak in the power spectrum determine

(i) the mass of the evaporating PBHs,
(ii) the dark matter abundance, and
(iii) the reheating temperature of the Universe.

In addition, the tail of the potential might lead to the observed late time acceleration of the Universe; see Fig. 8. Undoubtedly, this scenario is remarkably economic.

To be explicit, let us introduce the model

$$f(\phi/\sqrt{3}) = f_0(c_0 + c_1 e^{\lambda_1 \phi/\sqrt{3}} + c_2 e^{\lambda_2(\phi-\phi_{\rm P})^2/3}), \qquad (5.10)$$

which generates the potential

$$V(\varphi) = f_0^2[c_0 + c_1 e^{\lambda_1 \tanh\varphi/\sqrt{6}} + c_2 e^{\lambda_2(\tanh(\varphi/\sqrt{6}) - \tanh(\varphi_{\rm P}/\sqrt{6}))}]^2. \qquad (5.11)$$

$\varphi_{\rm P}$ is a fixed value in the field space that determines the position of the inflection point. Again here the f_0 can be absorbed in c_0, c_1, and c_2. For $\phi \to \sqrt{3}$ early Universe cosmic inflation takes place, and CMB normalization gives the first constraint for the parameters. For $\phi \to -\sqrt{3}$ we demand zero potential energy, and thus we get the second constraint,

$$c_0 = -c_1 e^{-\lambda_1} - c_2 e^{\lambda_2(\sqrt{3}+\phi_{\rm P}))^2/3}. \qquad (5.12)$$

The kination stage lasts until the moment that the radiation produced by the PBH evaporation dominates the energy density. Later the field freezes at some value ϕ_F and defreezes at the present Universe. The runaway potential is flat enough to lead to the currently observed accelerated expansion; hence, we implement a wCDM cosmology as a quintessence model.

Let us pursue some approximate analytic expressions that describe the postinflationary evolution of the field φ. After inflation φ rolls past the potential and a stage of kination commences, where $\dot{\varphi}^2/2 \gg V(\varphi)$. The Klein-Gordon equation for φ for negligible potential energy is $\ddot{\varphi} + 3H\dot{\varphi} \simeq 0$. During kination it is $a \propto t^{1/3}$, and for $t \gg t_{\rm end}$ the field value evolves as

$$\varphi - \varphi_{\rm end} \simeq -\sqrt{\frac{2}{3}} M_{\rm Pl} \ln\left(\frac{t}{t_{\rm end}}\right), \qquad (5.13)$$

where we considered negative initial velocity for φ. At the moment $t_{\rm form}$ the PBHs form and later, at $t_{\rm evap}$, they evaporate. Later, at the moment $t_{\rm rh}$ the Universe becomes radiation dominated and the kination regime ends. Until reheating it is $a \propto t^{1/3}$, and one finds that $t_{\rm rh} = (\Omega_{\rm rad}(t_{\rm evap}))^{-3/2} t_{\rm evap}$, where $\Omega_{\rm rad}(t_{\rm evap}) = (3/2)\gamma^2 \beta M^2/m_{\rm Pl}^2$, given by Eq. (3.22). At the moment of reheating the field value, $\varphi_{\rm rh}$, is

$$\varphi_{\rm rh} \simeq \varphi_{\rm end} - \sqrt{\frac{2}{3}}\left(-\frac{3}{2}\ln\Omega_{\rm rad}(t_{\rm evap}) + \ln\left(\frac{t_{\rm evap}}{t_{\rm end}}\right)\right) M_{\rm Pl}. \qquad (5.14)$$

After reheating it is $a \propto t^{1/2}$, and the field evolution slows down,

$$\varphi - \varphi_{\rm rh} \simeq -\frac{2}{\sqrt{3}} M_{\rm Pl}\left(1 - \sqrt{\frac{t_{\rm rh}}{t}}\right). \qquad (5.15)$$

For $t \gg t_{\rm rh}$ the field gets displaced $2M_{\rm Pl}/\sqrt{3}$ from $\varphi_{\rm rh}$, and thus, at some late moment t_F, the field freezes at the value φ_F,

$$\varphi_F \simeq \varphi_{\rm end} - \sqrt{\frac{2}{3}}\left(\sqrt{2} - \frac{3}{2}\ln\Omega_{\rm rad}(t_{\rm evap}) + \ln\left(\frac{t_{\rm evap}}{t_{\rm end}}\right)\right) M_{\rm Pl}. \qquad (5.16)$$

Asking for $f_{\rm rem} = 1$, we find from Eq. (3.28) that $\Omega_{\rm rad}(t_{\rm evap}) = 3 \times 10^{-13}(M/10^5\ {\rm g})^{10}(4\kappa)^{-4}$. Also, it is $t_{\rm evap} \sim 4 \times 10^2 (M/10^{10}\ {\rm g})^3$ s and $t_{\rm end} \simeq t_{\rm Pl}(m_{\rm Pl}/H_{\rm end})$. Therefore, we obtain an expression for φ_F that depends only on the initial mass of the PBH M and the mass of the PBH remnant $\kappa m_{\rm Pl}$,

$$\varphi_F \simeq \varphi_{\rm end} - \sqrt{\frac{2}{3}}[19 + 13\ln(M/10^5\ {\rm g}) + 4\ln(1/\kappa)] M_{\rm Pl}. \qquad (5.17)$$

This is a general approximate expression for any runaway potential that predicts PBH remnants as dark matter. It is general because we have omitted the potential $V(\varphi)$ from both the Friedman and Klein-Gordon equations for being negligible. The φ_F value depends only on the mass M and the parameter κ. For $\kappa = 1$ and $M = 10^5$ g it is $\varphi_F - \varphi_{\rm end} \sim -15 M_{\rm Pl}$. For $\kappa = 10^{-10}$ and $M = 10^2$ g it is $\varphi_F - \varphi_{\rm end} \sim -17 M_{\rm Pl}$. We note that the exact value of φ_F is found after the numerical solution of the Klein-Gordon and Friedman equations, and $|\varphi_F - \varphi_{\rm end}|$ is a bit less than the value of Eq. (5.17), as we neglected the potential $V(\varphi)$ and considered instant transitions between the kination and radiation regimes.

If we want to identify the dark energy as the energy density of the scalar field φ, then we have to tune the potential energy value at φ_F. For our model (5.11) we impose the condition

$$\frac{\rho_{\rm inf}}{\rho_0} \simeq \frac{V(\varphi \gg 1)}{V(\varphi_F)} \sim \frac{e^{2\lambda_1}}{e^{-2\lambda_1}} \sim 10^{108} \tag{5.18}$$

dictated by the hierarchy of energy scales between the α-attractor inflation and the dark energy. This condition gives a third constraint to the parameters of the potential, together with the CMB normalization and the requirement for zero vacuum energy as $\varphi \to -\infty$, Eq. (5.12). Equation (5.18) gives a rough relation for the size of the exponent parameter λ_1,

$$4\lambda_1 \sim 108 \ln(10). \tag{5.19}$$

In Table III we list a set of parameters in which the kination model (5.11) generates PBHs which after evaporation leave behind remnants with $f_{\rm rem} = 1$ and act as quintessence.

D. Observational constraints on $\mathcal{P}_{\mathcal{R}}(k)$

Let us now make the all-important checks of our PBH generation models: the spectral index at small wave number $k = 0.05\ \mathrm{Mpc}^{-1}$, and the BBN and CMB bounds from PBH evaporation at large wave number $k \gg 0.05\ \mathrm{Mpc}^{-1}$, as discussed in Sec. IV.

1. At large scales: The CMB spectral index

The n_s and r values in the standard α-attractors are expressed as the analytic relations $n_s \sim 1\text{–}2/N_*$ and $r \sim 12\alpha/N_*^2$. These expressions still apply in α-attractor models that feature an inflection point, with the essential difference that N_* is replaced by the number of e-folds ΔN that separate the moments of horizon exit of the CMB scale k_*^{-1} and the PBH scale k^{-1}. Thus, we have $n_s \sim 1\text{–}2/\Delta N$ and $r \sim 12\alpha/\Delta N^2$. In our models we get $\Delta N \gtrsim 50$; hence, the spectral index value is predicted to be

$$n_s \gtrsim 0.96 \tag{5.20}$$

and the tensor-to-scalar ratio

$$r < 0.048, \tag{5.21}$$

placing the prediction of our models in the 68% C.L. region of the Planck 2018 data [33] without assuming running for the n_s. Generally, the n_s value becomes larger than 0.96 if the PBHs have mass less than about 10^5 g.

2. At small scales: The width constraints on the $\mathcal{P}_{\mathcal{R}}(k)$ peak

In Sec. IV B we discussed the impact and the bounds of BBN and CMB to the variance of the comoving density contrast, which can be translated to the power spectrum $\mathcal{P}_{\mathcal{R}}(k)$ bounds. In the case of the radiation model the variance of the comoving density contrast is $\sigma_{\rm RD} \sim 4.5 \times 10^{-6}$ at small scales, satisfying easily the Eq. (4.7) bound. In the case of matter domination, the upper bound on the variance of the comoving density contrast is the stringent one. For our models we find that $\sigma_{\rm MD} \sim 2 \times 10^{-6}$ at $M_{\rm cmb}$ in accordance with the bound (4.8). Also, for the kination domination scenario the variance corresponding to the CMB mass scale is $\sigma_{\rm KN} \sim 2.6 \times 10^{-5}$ well below the upper bound (4.9).

VI. CONCLUSIONS

In this work we investigated the cosmology of mini PBHs. The very motivation for examining this scenario is the theoretical postulation that a stable or long-lived remnant is left behind after the evaporation of the "black" holes. The mass of the remnant is expected to depend on the unknown physics that operates at the Planck energy scale. Therefore, we examined the cosmology of PBH remnants with arbitrary mass $M_{\rm rem} = \kappa m_{\rm Pl}$ and with κ being a free parameter that might be orders of magnitude larger or smaller than 1. We computed the general relic abundance of the PBHs remnants and found the conditions that they constitute all of the cold dark matter in the Universe. We found that the PBH remnants have a significant cosmological abundance only if they have mass $M_{\rm rem} > 1$ GeV. Also, the mass of the remnants must have mass $M_{\rm rem} \ll 10^8$ g; otherwise, the parent PBH affects the BBN or the CMB observables.

Mini PBHs imply that the comoving curvature perturbation is enhanced at the extreme end of the $\mathcal{P}_{\mathcal{R}}(k)$. This is a rather attractive feature since the required large primordial inhomogeneities can be produced by the inflationary phase without spoiling the spectral index value n_s. The PBHs formed in the very early Universe after the inflationary phase; hence, the primordial inhomogeneities are expected to collapse during a nonthermal phase unless the inflaton field decays very fast.

In this work we built inflationary models in the framework of α-attractors. We produced a peak in power spectrum by constructing an inflection point and computed numerically $\mathcal{P}_{\mathcal{R}}(k)$ by solving the Mukhanov-Sasaki equation. The PBH remnants constitute all or a significant part of the dark matter in the Universe; see Figs. 9, 10, and 11. Our models yield a spectral index value $n_s > 0.96$, which places them in the 68% C.L. contour region of the Planck 2018 data. The building blocks of the inflationary potentials are exponential functions. We examined the PBH production for three different inflationary scenarios. In the first, the inflaton field decays nearly instantaneously after inflation, reheating the Universe at very large temperatures. In this scenario the mini PBHs are produced and evaporate during the radiation phase. In the second scenario the inflaton field decays a bit later, after oscillating several times about the minimum of its potential, resulting in a postinflationary stage of pressureless matter domination. During matter domination the primordial inhomogeneities collapse into PBHs. After the inflaton decay the Universe is reheated and the mini PBHs evaporate.

In the third scenario the PBH are produced during a kination regime. This is a novel scenario; hence, we examined it in more detail. A kination regime takes place if the inflaton potential has no minimum, and the inflaton runs away after the end of inflation. The radiation is produced by the PBH evaporation that gradually dominates the energy density and reheats the Universe. The resulting reheating temperature can be larger than 10^6 GeV, terminating the kination era fast enough to be in accordance with the BBN constraints. The PBH remnants can account for all of the dark matter in the Universe. Interestingly enough, the nondecaying inflaton can additionally act as a quintessence field, giving rise to the observed late time accelerated expansion implementing a wCDM cosmological model. Actually this model is remarkably economic in terms of ingredients.

Now that the existence of black holes and dark matter are unambiguous, the investigation of the PBH dark matter scenario is very motivated. Here we examined the less studied mini PBH scenario and derived general expressions complementing older results and put forward new and testable cosmological scenarios for the early and late Universe.

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