Entropy production in inflation from spectator loops

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Perturbations in cosmic microwave background (CMB) photons and large scale structure of the Universe are sourced primarily by the curvature perturbation which is widely believed to be produced during inflation. In this paper we present a two-field inflationary model in which the inflaton couples biquadratically to a spectator field. We show that the spectator induces a rapid growth of the momentum of the curvature perturbation and the associated Gaussian van Neumann entropy during inflation such that the initial conditions at the end of inflation are substantially different from the standard ones. Consequently, one ought to reconsider the kinetic equations describing the evolution of the photon, dark matter, and baryonic fluids in the radiation and matter eras and take into account the fact that the curvature perturbation and its canonical momentum are two *a priori* independent stochastic fields. We also briefly analyze possible imprints on the CMB temperature fluctuations from the more general inflationary scenario which contains light spectator fields coupled to the inflaton.

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I. OVERVIEW AND MOTIVATION

It is a remarkable fact that all of the modern cosmic microwave background (CMB) data, together with various large scale structure (LSS) probes, can be described by a class of simple cosmological models containing just six parameters [1,2]. Two of these parameters—the amplitude (A_s) and spectral slope ($n_s - 1$) of the curvature spectrum—are primordial in origin, while four—the Hubble parameter today (H_0) [or equivalently the angular scale of the first acoustic peak ($\theta = r_*/D_A$)], the reionization optical depth (τ_r), (relative) baryonic density (Ω_b), and cold dark matter density (Ω_c)—are late-time observables. Since the simplest "vanilla" cosmological model assumes a spatially flat Universe ($\Omega_\kappa = 0$), the dark energy density $\Omega_{\rm de} = \Omega_\Lambda$ is not an independent parameter, i.e., $\Omega_{\rm de} = 1 - \Omega_b - \Omega_c$. For more details we refer to [2,3].

Cosmological models have been tested for various other features, such as various probes of isotropy and homogeneity, statistical Gaussianity (the amplitude of primordial bispectrum and trispectrum), and the amplitude and slope of tensor perturbations, but for all of these only upper bounds exist, albeit there is statistically weak evidence supporting some of the probes that indicate deviation from statistical isotropy or Gaussianity [4].

Another interesting class of features is encoded in isocurvature modes (see, e.g., Ref. [5]). Even though there are many potential physical degrees of freedom (d.o.f.) which can play the role of isocurvature modes, there is no strong evidence in the data that would suggest that any of these contribute dominantly to the CMB photon temperature fluctuations. Indeed, the authors of [3] looked for traces of cold dark matter density isocurvature (CDI), neutrino density isocurvature (NDI), and neutrino velocity isocurvature (NVI) modes in the data, and they placed upper limits on the relative amount of CDI, NDI, and NVI of 2.5%, 7.4%, and 6.8%, respectively, at the scale of $k = 0.002\ \mathrm{Mpc}^{-1}$. Signatures that are analogous to isocurvature modes are produced by topological defects and therefore similar upper bounds can be placed on the contribution of various classes of topological defects (which include cosmic strings, monopoles, and textures) to the observed spectrum [6].

In this paper we study an idea with similar effects, namely, how spectator fields during inflation decohere the Gaussian density matrix of the curvature perturbation on super-Hubble scales by means of quantum loop interactions.[1] This decoherence is manifested as an increase of entropy during inflation and can produce similar signals as isocurvature modes and topological defects in the effective CMB temperature fluctuations. This is so because isocurvature modes tend to produce peaks which are out of phase with the adiabatic mode, and therefore tend to wash out the coherent CMB oscillations. Let us be a bit more precise about the last statement and recap the form of the

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[1]A similar problem has recently been addressed in [7].

effective photon temperature fluctuation $\Delta\hat{T}$ in momentum space before recombination in a simple approximation which we review in Appendix B,

$$\Delta\hat{T}(\vec{k},\eta) \approx \frac{1}{2}\hat{\Psi}(\eta_{\rm cmb},\vec{k})\cos[kr_s(\eta)] + 2\frac{\Psi'(\eta_{\rm cmb},\vec{k})}{kc_s(\eta_{\rm cmb})}\sin[kr_s(\eta)]. \tag{1}$$

Here, $c_s(\eta)$ denotes the speed of sound and the sound horizon $r_s(\eta)$ is its integral over conformal time. The stochastic variable $\hat{\Psi}(\eta_{\rm cmb},\vec{k})$ is the gauge-invariant perturbation of the trace of the spatial metric at conformal time $\eta = \eta_{\rm cmb}$ within the radiation era some time before recombination such that it is observable in the CMB. Its derivative in conformal time, $\hat{\Psi}'(\eta_{\rm cmb},\vec{k})$, is an *a priori* stochastically independent variable. We can conclude that coherent CMB oscillations are possible if the stochastic operators $\hat{\Psi}(\eta_{\rm cmb},\vec{k})$ and $\hat{\Psi}'(\eta_{\rm cmb},\vec{k})$ are linearly related (which induces a phase shift) or if either of them is much smaller than the other. As we pointed out above, Planck data are mostly consistent with coherent CMB oscillation such that the standard case is to discard the initial time derivative of the gravitational potential and consider only the adiabatic mode whose associated operator is conserved on super-Hubble scales. Still, the constraints to wash out the CMB oscillation reside in the range of percent so it is worth studying mechanisms that can contribute to it. This allows us to either target those effects by precision cosmology or to rule them out. We remind ourselves in Appendix C that the linear dynamics of single-field inflation on super-Hubble scales effectively decreases the number of independent stochastic operators to the aforementioned adiabatic mode. Thus, one way of obtaining a nonvanishing and stochastic independent time derivative of the initial gravitational potential in (1) is to work with nontrivial background trajectories in a multifield inflationary model, leading to the aforementioned isocurvature modes whose stochastic independence can be traced back to independent quantum fluctuations whose presence is guaranteed by vacuum expectation values of the additional fields.

We obtain a significant amount of decoherence at the end of inflation by going beyond the tree-level analysis and relying purely on *interactions* of the inflaton perturbation φ with a *spectator field* χ that has a zero expectation value. We chose such a simple model because the inflaton coupling to the spectator field is controlled by a separate coupling constant, which is independent on the loop counting parameter of quantum gravity, $\kappa^2H^2 \sim H^2/M_{\rm P}^2 \sim 10^{-12}$ (here H is the inflationary Hubble parameter and $M_{\rm P} \simeq 2.4\times 10^{18}$ GeV is the reduced Planck mass), which governs the strength of interactions in the inflation sector. Moreover, since the spectator field does not acquire an expectation value, it is invariant under coordinate transformations to first order in perturbations. Thus, if we express corrections to the inflaton propagator in terms of the gauge-invariant curvature perturbation $\mathcal{R}$ and take corrections to the inflaton expectation value $\bar{\phi}$ into account, our results are to first order in perturbations gauge invariant and we may compare them to the tree-level analysis at the end of inflation.

The effect of quantum corrections to the power spectrum of the curvature perturbation has been studied in [8,9] with the conclusion that loop corrections on super-Hubble scales can at most be enhanced as powers of logarithms of the scale factor. However, the power spectrum of the $\mathcal{R}\mathcal{R}$-correlator remains approximately frozen due to the coupling constant suppression and the limit on how long inflation lasts. In this paper, we reconsider these observations with a concrete calculation in the above mentioned model involving spectator fields. The model consists of two canonical scalar fields on locally de Sitter background that interact via a cubic interaction which is derived by expanding a biquadratic action around the vacuum expectation value (VEV) of the inflaton. While the interactions with the spectator indeed produce logarithmic corrections to the comoving curvature perturbations, the corrections to the canonical momentum of the comoving curvature perturbations grow exponentially in time (inverse power in conformal time) and may induce considerable fluctuations. The question whether these field excitations are stochastically independent can be answered by calculating the Gaussian part of the von Neumann entropy $S_{\rm vN}$ associated to $\hat{\mathcal{R}}$ and $\hat{\pi}_{\mathcal{R}}$, which is conveniently represented in momentum space by

$$S_{\rm vN}[\mathcal{R},\pi_{\mathcal{R}}] = \frac{1}{2}\sum_{\vec{k}} s_{\rm vN}(\eta,k),$$
$$s_{\rm vN} = \frac{\Delta_{\mathcal{R}}+1}{2}\log\frac{\Delta_{\mathcal{R}}+1}{2} - \frac{\Delta_{\mathcal{R}}-1}{2}\log\frac{\Delta_{\mathcal{R}}-1}{2}, \tag{2}$$

which depends on the Gaussian invariant $\Delta^2_{\mathcal{R}}$ (see, e.g., [10]),

$$\Delta^2_{\mathcal{R}}(\eta,k) = 4[\Delta_{\mathcal{R}\mathcal{R}}(\eta,k)\Delta_{\pi_{\mathcal{R}}\pi_{\mathcal{R}}}(\eta,k) - \Delta^2_{\mathcal{R}\pi_{\mathcal{R}}}(\eta,k)], \tag{3}$$

where $\Delta_{\mathcal{R}\mathcal{R}}$, $\Delta_{\pi_{\mathcal{R}}\pi_{\mathcal{R}}}$, and $\Delta_{\mathcal{R}\pi_{\mathcal{R}}}$ are the equal-time momentum space two-point functions. The Gaussian invariant $\Delta^2_{\mathcal{R}}$ is identical to 1 for linearly evolved fields prepared in a pure Gaussian initial state (an important example of which is the Bunch-Davies vacuum) and thus yields *zero* Gaussian von Neumann entropy. A large Gaussian invariant on the other hand would indicate a big uncertainty in the phase-space which is spanned by the operators $\hat{\mathcal{R}}$ and $\hat{\pi}_{\mathcal{R}}$.

In order to see how quantum interactions with spectators during inflation influence the CMB, we relate the gauge-invariant gravitational potential $\hat{\Psi}$ shortly before the end of inflation to the gauge-invariant curvature perturbation $\hat{\mathcal{R}}$ and evolve it to the radiation era where we assume a simple scenario in which we switch off the interactions after inflation. In Appendix B we review that Eq. (1) then takes the following form:

$$\Delta\hat{T}(\eta,\vec{k}) \approx \frac{1}{2}\left[\frac{2}{3}\hat{\mathcal{R}}(\eta_e,\vec{k}) - \frac{a^3(\eta_e)}{a^3(\eta_{\rm cmb})}\frac{H}{2M_p^2k^2a(\eta_e)}\hat{\pi}_{\mathcal{R}}(\eta_e,\vec{k})\right] \times \cos[kr_s(\eta)] + \frac{6H}{kc_s(\eta_{\rm cmb})}\frac{a^4(\eta_e)}{a^4(\eta_{\rm cmb})} \times \left[\frac{H}{2M_p^2k^2}\hat{\pi}_{\mathcal{R}}(\eta_e,\vec{k}) + a(\eta_e)\hat{\mathcal{R}}(\eta_e,\vec{k})\right] \times \sin[kr_s(\eta)], \tag{4}$$

where the parameter H is the Hubble scale at the beginning of inflation and the argument η_e is some time shortly before the end of inflation such that the slow-roll parameter $\epsilon = 1 - \mathcal{H}^{-2}\mathcal{H}'$, with $\mathcal{H}a = a'$, is still small, $\epsilon(\eta_e) \ll 1$. We see that the main contribution $\propto \hat{\pi}_{\mathcal{R}}$ in (4) could wash out the Sakharov oscillations if it were able to balance the heavy suppression by the prefactor $\propto a^{-4}$, which is for initially small amplitudes only possible if $\hat{\pi}_{\mathcal{R}}$ were growing during inflation. As we review in Appendix C, linear single-field inflation yields the following relation on super-Hubble scales in the slow-roll regime:

$$\hat{\pi}_{\mathcal{R}}^{(\rm lin)}(\eta_e,\vec{k}) = -\frac{2M_p^2a(\eta_e)\epsilon(\eta_e)}{H}[\hat{\mathcal{R}}(\eta_e,\vec{k}) + \mathcal{O}(k\eta_e)], \tag{5}$$

such that the stochastic independent off-peak contribution in (4) can safely be neglected. However, in models in which the inflaton couples to other matter fields with unsuppressed couplings (a notable example being Higgs inflation), there is no reason to *a priori* expect that the standard tree-level results apply and thus spectator fields without VEVs might still contribute to stochastic independent modes.

While this work is inspired by the large literature on decoherence and classicalization of cosmological perturbations [11–18], it also differs from it in important aspects. In contrast with the effective approaches based on studying the approximate evolution of the reduced density matrix [19], we use standard perturbative methods of the quantum field theory [10,20–23]. Furthermore, we identify the late-time (CMB) observables that can be used to quantify the amount of decoherence in the curvature perturbation (expressed through the Gaussian part of the von Neumann entropy) that occurs during inflation and subsequent epochs, while most of the existing works base their analysis on standard criteria for classicalization often used in condensed matter systems, such as the diagonalization rate of the reduced density matrix in a suitably chosen pointer basis. While early works [11–18] used the late-time observer's inability to get complete access to the state of cosmological perturbations as the principal source of decoherence and classicalization (the so-called "decoherence without decoherence"), later works used more realistic settings, in which (dissipative) interactions among quantum fields during (or after) inflation are the principal cause for decoherence. The interactions considered range from self-interactions of the inflaton field [24–27], interactions with gravitational waves [28,29], interactions with other scalar fields [30–33], and interactions with massive fermionic fields [34].

Encouraged by the result of [18] we decided to investigate the effect of one-loop interactions between the spectator and the inflaton where the fields interact biquadratically. When this work was nearing completion, we became aware that a similar problem was addressed in [7] based on the density matrix formalism developed in [35,36]. While the authors of Refs. [7,35,36] start from a cubic interaction and make use of the density matrix formalism, we start from a biquadratic interaction which provides a stable theory for a positive coupling. By expanding around the inflaton condensate, we also obtain an effective cubic vertex which turns out to yield the dominant contributions to decoherence. However, we approach the problem differently by providing a one-loop evaluation of the inflaton propagator $\Delta_{\varphi\varphi}(\eta,\eta',k)$ from which we can fully reconstruct the Gaussian part of the density matrix.

The paper is organized as follows. In Sec. II we explain the model setup and how to relate the various two-point functions. In the follow-up Sec. III, we present the main steps in the calculation, including renormalization, the solution of the equation of motion for the statistical propagator, symmetry properties, and the super-Hubble limit. In Sec. IV we come back to the implications of our results and discuss extensions of the presented analysis. Moreover, we make a comparison with the findings of Refs. [7,35,36]. Some important technical details of the calculations are presented in several appendixes.

We work in natural units in which $c = \hbar = 1$ and with the metric tensor with a mostly plus signature, $(-,+,+,+)$.

II. GROWING CURVATURE MOMENTUM FROM QUANTUM INTERACTIONS

Coupling of the comoving curvature perturbation to other fields can be mediated not only via tree-level processes, but can also be studied at the quantum (loop) level. Take a simple two-scalar-field inflationary model with a biquadratic interaction term,

$$S[\phi,\chi] = S_{\rm EH} + \int d^Dx\sqrt{-g}\left(-\frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - \frac{1}{2}g^{\mu\nu}(\partial_\mu\chi)(\partial_\nu\chi) - V(\phi,\chi)\right), \tag{6}$$

where $S_{\rm EH}$ is the Einstein-Hilbert action,

$$S_{\rm EH} = \frac{1}{16\pi G}\int d^D x\sqrt{-g}R, \tag{7}$$

D is the number of space-time dimensions, $R = R[g_{\mu\nu}]$ is the Ricci curvature scalar, the field ϕ is the inflaton with the perturbation,

$$\hat{\varphi} = \hat{\phi} - \bar{\phi}, \qquad \bar{\phi}(t) = \langle\hat{\phi}(x)\rangle, \tag{8}$$

the field χ is a spectator with a vanishing expectation value, $\langle\hat{\chi}\rangle = 0$, and the potential $V(\phi,\chi)$ reads

$$V(\phi,\chi) = \frac{m_\phi^2}{2}\phi^2 + \frac{m_\chi^2}{2}\chi^2 + \frac{g}{4}\chi^2\phi^2, \tag{9}$$

where both fields are assumed to be light,

$$H \gg m_\chi, m_\phi. \tag{10}$$

We are interested in studying the dynamics of the metric and field perturbations around a cosmological background, with the metric tensor (in the plasma rest frame) given by

$$\bar{g}_{\mu\nu} = \mathrm{diag}\Big(-\bar{N}^2(t), \underbrace{a^2(t), \ldots, a^2(t)}_{D-1\ \text{times}}\Big),$$
$$\bar{g} = \det[\bar{g}_{\mu\nu}] = \bar{N}^2 a^{2(D-1)}, \tag{11}$$

where $\bar{N}(t)$ is the lapse function and $a(t)$ is the scale factor. While it would be of interest to study both the dynamics of the quantum gravitational and quantum scalar perturbations, for simplicity in this work we limit ourselves to studying the dynamics of the scalar curvature perturbation induced by its biquadratic interaction term given in Eq. (9). This process is controlled by the coupling constant g which is generally different from the gravitational coupling constant $\kappa = 1/\sqrt{16\pi G}$, where G denotes the Newton constant, and therefore can be separately studied. To show that, in what follows we recall some of the basics of the quantum perturbative gravity in inflationary space-times.

The theory (6) has two dynamical scalar d.o.f., which in the comoving gauge, in which $\varphi = 0$, are the scalar metric perturbation $\psi = -\mathrm{Tr}[\delta g_{ij}]/(6a^2)$ and the isocurvature field, χ, and one transverse, traceless tensor perturbation, $h_{ij} = \delta g_{ij}/a^2$, with $\delta_{ij}h_{ij} = 0 = \partial_i h_{ij}$. In addition, there are constraint d.o.f.: one scalar and one transverse vector d.o.f., namely, the lapse function $N(x)$ and the shift vector $N_i(x)$ (with $\partial_i N_i = 0$). Since one can choose a gauge in which the lapse and shift decouple from the dynamical d.o.f., one can ignore them [37,38].

The dynamics of the linear scalar cosmological perturbations is governed by the well-known Mukhanov-Sasaki action [39]. When written for the curvature perturbation $\mathcal{R}$, the action reads [39–41]

$$S_s^{(2)}[\mathcal{R}] = \int d^D x \bar{N} a^{D-1} 2\epsilon M_{\rm P}^2\left[\frac{1}{2}\dot{\mathcal{R}}^2 - \frac{1}{2a^2}(\partial_i\mathcal{R})^2\right],$$
$$\epsilon = -\frac{\dot{H}}{H^2}, \qquad M_{\rm P}^2 \equiv \frac{1}{8\pi G}, \tag{12}$$

and the quadratic action for the tensor perturbations, $h_{ij} = \delta g_{ij}/a^2$, in the traceless and transverse gauge ($\delta_{ij}h_{ij} = 0 = \partial_i h_{ij}$) reduces to

$$S_t^{(2)} = \frac{M_{\rm P}^2}{8}\int d^D x \bar{N} a^{D-1}\left[\dot{h}_{ij}^2 - \frac{1}{a^2}(\partial_l h_{ij})^2\right], \tag{13}$$

where a *dot* signifies a reparametrization-invariant derivative with respect to time, $\dot{X} \equiv \bar{N}^{-1}\partial_t X$. Note that both actions (12) and (13) are manifestly gauge invariant, as they are written for the gauge-invariant curvature perturbation $\mathcal{R}$ and gauge-invariant tensor perturbation h_{ij}. If one fixes a gauge completely, one can easily get the corresponding gauge fixed action from (12). For example, in the comoving gauge ($\varphi = 0$), in which $\mathcal{R} \to \psi$, the action for ψ is identical in form as the action (12) for $\mathcal{R}$; in the zero-curvature gauge ($\psi = 0$), the action for φ is obtained by exacting the replacement, $\mathcal{R} \to \varphi/(\sqrt{2\epsilon}M_{\rm P})$ in (12),

$$S_s^{(2)}[\varphi] = \int d^D x \bar{N} a^{D-1}\left[\frac{1}{2}\dot{\varphi}^2 - \frac{1}{2}\left(\frac{\partial_i\varphi}{a}\right)^2 + \frac{1}{4}\left(\frac{(a^{D-1}\dot{\epsilon})^{\cdot}}{a^{D-1}\epsilon} - \frac{1}{2}\frac{\dot{\epsilon}^2}{\epsilon^2}\right)\varphi^2\right], \tag{14}$$

such that the linear dynamics of the inflaton perturbation corresponds to that of a harmonic oscillator with a time-dependent frequency. Since χ remains invariant to first order under gauge transformations, the quadratic action for χ is by itself gauge invariant,

$$S_s^{(2)}[\chi] = \int d^D x \bar{N} a^{D-1}\left[\frac{1}{2}\dot{\chi}^2 - \frac{1}{2}\left(\frac{\partial_i\chi}{a}\right)^2 - \frac{1}{2}\left(m_\chi^2 + \frac{g}{2}\bar{\phi}^2\right)\chi^2\right]. \tag{15}$$

In addition, there are two physical constraint fields, the lapse and (transverse) shift function, but they decouple from the dynamical d.o.f. $\mathcal{R}$ and h_{ij}. While this decoupling is clearly evident (from the Helmholz decomposition) at the linear order in the perturbations, one has to work harder to show that it also works at higher order in perturbations [37,38]. In fact, there are gauges in which the constraint fields can play an important role [42]. The leading order actions (13)–(15) are supplemented by the higher order actions describing cubic, quartic, and higher order interactions [37,38,43]. Generically, while all gravitational interactions are suppressed by powers of the gravitational coupling constant $\kappa = 1/\sqrt{16\pi G}$, the interactions involving the scalar curvature perturbation are in addition suppressed by powers of the slow-roll parameters, $\epsilon = -\dot{H}/H^2$, and/or its derivatives (no such suppression occurs in the tensor interactions). However, that does not mean that scalar loops are suppressed when compared with the tensor loops, since the scalar curvature propagator is enhanced by a factor $\sim 1/\epsilon$ when compared with the tensor propagator, thus nullifying the slow-roll vertex suppression. The result is that, quite generically, each gravitational loop contributes as $\sim\kappa^2 H^2 \sim H^2/M_{\rm P}^2$. In addition, Weinberg's theorem [8,9] allows for a secular enhancement in the form of powers of the number of e-foldings, $\mathcal{N} = \ln(a)$. Since not much is known about such secular enhancements of the gravitational loops (most notably because the problem of gauge dependence of gravitational loops is *not* well understood [44,45]), for the sake of simplicity we neglect them in what follows.

From Eq. (15) we see that the inflaton condensate $\bar{\phi} \sim HM_{\rm P}/m_\phi$ generates a mass for the spectator field χ of the order,

$$\delta m_\chi^2 = \frac{g}{2}\bar{\phi}^2 \sim gH^2\frac{M_{\rm P}^2}{m_\phi^2}. \tag{16}$$

Since light scalar field fluctuations grow during inflation, their effect on the inflaton fluctuation will be larger than from a heavy scalar field. Demanding that χ remain light during inflation, $\delta m_\chi^2 \ll H^2$, leads to the following condition on the coupling constant:

$$0 < g \lesssim \frac{m_\phi^2}{M_{\rm P}^2} \sim 10^{-12}. \tag{17}$$

Let us first consider the tadpole contribution to the expectation value of the inflaton field $\bar{\phi}$, which contributes to the inflaton equation of motion as

$$(\Box - m_\phi^2)\bar{\phi} = \frac{g}{2}\bar{\phi}\, i\Delta_\chi(x;x). \tag{18}$$

This ought to be renormalized by the nonminimal coupling counterterm, $\int d^D x(-\frac{1}{2}\delta\xi R\bar{\phi}^2)$. According to (10), we assume that the coincident scalar propagator is that of the massless scalar in de Sitter space. The finite part of the coincident propagator is given by

$$i\Delta_\chi(x;x)_{\rm fin} \simeq [H^2/(4\pi^2)]\ln(a), \tag{19}$$

which exhibits a secular growth and modifies the inflaton mass by $\delta m_\phi^2 = [gH^2/(8\pi^2)]\ln(a) \ll m_\phi^2$ by a negligibly small amount. Moreover, this contribution changes the expansion rate and slow-roll parameters, but by a small amount. These corrections are important for maintaining gauge invariance of the corrected comoving curvature perturbation at linear order. The reason is that the inflaton VEV enters the definition of the curvature perturbation and

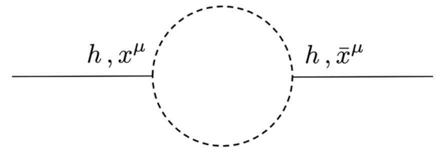

FIG. 1. The one-loop Feynman diagram for the inflaton two-point function (solid lines) generated by the cubic interaction in (20). The spectator field χ (dashed lines) runs in the loop. The vertex coupling strength is $h = g\bar{\phi}$.

its corrections are of a similar order as the nonlocal self-mass corrections. However, local terms will not induce dissipative effects that could affect the entropy of cosmological perturbations [46] and they are negligible for the canonical momentum of the comoving curvature perturbation and correlators thereof, as we will see explicitly later on.

The interaction between inflaton perturbation and spectator fields is governed by cubic and quartic interactions, whose actions are

$$S_s^{(3)}[\varphi,\chi] = \int d^D x \bar{N} a^{D-1}\left(-\frac{h}{2}\varphi\chi^2\right), \qquad h = g\bar{\phi}, \tag{20}$$

$$S_s^{(4)}[\varphi,\chi] = \int d^D x \bar{N} a^{D-1}\left(-\frac{g}{4}\varphi^2\chi^2\right). \tag{21}$$

Let us first make a rough comparison of the effects induced by these two interactions on the dynamics of the inflaton perturbation.

The one-loop $\mathcal{O}(g)$ contribution generated by the quartic interaction (21) will (upon renormalization) generate a time-dependent mass term for the inflaton fluctuations, $\delta m_\phi^2 = (g/2)i\Delta_\varphi(x;x)$, where $i\Delta_\varphi(x;x) = \langle\hat{\varphi}(x)^2\rangle$ denotes the coincident two-point function for the inflaton perturbation, and thus will not generate any entropy or any other dissipative effects in the scalar sector of the theory.

Next, at order g^2 there are two contributions: the one-loop contribution in Fig. 1 which is generated by the cubic action (20) and the two-loop contribution in Fig. 2 generated by the quartic interaction (21). Since we are primarily interested in super-Hubble fluctuations, we shall compare the size of these two diagrams for super-Hubble distances, $\|\vec{x}-\vec{x}'\| \gg 1/H$, and at equal time, $t = t'$.

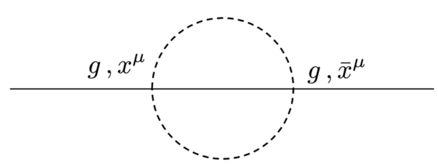

FIG. 2. The two-loop diagram generated by the quartic interaction in (21) with the spectator χ (dashed lines) and inflaton (solid lines) running in the loops. The vertex coupling strength is g.

It is not hard to see that the ratio of the two-loop to the one-loop contribution scales roughly as

$$\frac{i\Delta_\varphi(t,\vec{x};t,\vec{x}')}{\bar{\phi}(t)^2} \sim \frac{m_\phi^2 \ln(a)}{M_{\rm P}^2} \ll 1, \tag{22}$$

where we made use of $i\Delta_\varphi(t,\vec{x};t,\vec{x}') \sim H^2\ln(a)$, $\bar{\phi} \sim HM_{\rm P}/m_\phi$, and $m_\phi \ll H$ (in the above estimate, factors of order 1 such as powers of π have been neglected). This means that the principal diagram that contributes (in a dissipative manner) to the dynamics of the inflaton perturbation, and therefore also to the curvature perturbation, is the one-loop diagram in Fig. 1.

In what follows we shall compare the size of the one-loop spectator diagram with that of the quantum gravitational loops. From Eq. (30) we see that the ratio of the one-loop to the tree-level Hadamard function is of the order $\delta F_\varphi/F_{\varphi,dS} \sim (h^2/H^2)\ln^3(a)$, which ought to be compared with the corresponding quantum gravitational contribution, $\kappa^2 H^2 \ln^{n_g}(a) \sim (H^2/M_{\rm P}^2)\ln^{n_g}(a)$, where n_g is an unspecified positive integer which parametrizes our ignorance of the quantum gravitational loops. Upon dividing the two contributions we get

$$\frac{(g^2\bar{\phi}^2/H^2)\ln^3(a)}{\kappa^2 H^2 \ln^{n_g}(a)} \lesssim \frac{m_\phi^2}{H^2}[\ln(a)]^{3-n_g}. \tag{23}$$

Knowing the secular terms can be crucial, since each power of $\ln(a)$ produces an enhancement by a factor $\sim 10^2$, and that can be detrimental for determining whether the quantum gravitational or spectator contributions in (23) dominate. From the estimate in (23) we see that the condition that χ remain light in inflation implies that the contribution from the spectator loop can be comparable to the quantum gravitational loops. This means that, before one makes any definite conclusion concerning the strength of decoherence during inflation, one also ought to investigate the effect of the quantum gravitational loops. In fact, there have been several attempts to do precisely that [26,27,30,31,47]. In addition, a lot of work has been invested into a much easier set of problems, namely, into studying how the inflaton coupling with the other quantum fields (scalar, fermionic, or vector) induces decoherence in the inflaton sector [16,18,31,34]. While the earlier works considered simple models with bilinear couplings [16,18] (since these couplings are nondissipative, they are not true interactions), more recent works studied true interactions [31,34]. These types of studies are much easier, since the hardest problem—the problem of gauge dependence—is absent in these studies.

While these attempts represent important first steps, it is fair to say that no definite answer to that question has been given as yet. The principal reason is that none of the existing works has seriously addressed the issue of gauge (in)dependence, nor have the authors performed a complete quantum calculation which must include (a) a complete set of Feynman rules, with all relevant vertices and propagators included (currently there exists no propagator that encompasses the dynamics of both scalar and tensor perturbations in inflation); (b) a complete calculation of the one-loop diagrams that includes (preferably dimensional) regularization and renormalization, with the notable exception of Refs. [7,35], where normal ordering was used to renormalize the self-mass; (c) a study of how the inflaton two-point function gets modified by the one-loop quantum fluctuations, which also includes a detailed analysis of how it depends on the choice of gauge. Before we have good understanding of all of these steps and problems, we cannot say anything definite regarding the importance of the quantum gravitational loops for the evolution of cosmological perturbations.

As a final remark, we point out that, because the spectator loop is controlled by a different coupling constant (g) from that governing the quantum gravitational loops (κ), one can unambiguously separate the two. In other words, the quantum gravitational loops cannot cancel or compensate the effects of the spectator loop studied in this work.

In principle we could include slow-roll corrections in our study. However, including them would significantly complicate the spectator propagator, and thus also the whole calculation. Therefore, for simplicity, we shall consider a nearly de Sitter inflation, in which the effects due to slow-roll corrections are negligibly small. We point out that the spectator field is very different from the inflaton in that taking the limit $\epsilon \to 0$ in the scalar sector of the graviton is a delicate one, because the curvature propagator is in that limit enhanced as $\propto 1/\epsilon$; cf. the action for the curvature perturbation (14). No such enhancement is present in the spectator sector of the theory, implying that there is no subtlety involved in taking the limit $\epsilon \to 0$. Moreover, the tensor-to-scalar ratio $r \simeq 16\epsilon \leq 0.065$ is known to be small, implying that $\epsilon < 1/200$, such that taking the limit $\epsilon \to 0$ should give reasonably accurate answers. Next, the spectral slope of the curvature perturbation is also quite small, $n_s - 1 \simeq -0.035 = -2\epsilon - \epsilon_2 \approx -\epsilon_2$, and it is controlled by the second slow-roll parameter $\epsilon_2 = \dot{\epsilon}/(\epsilon H) \simeq 0.035$. This near scale invariance of the scalar perturbation also tells us that approximating the tree-level equation for the inflaton perturbation by that of a massless scalar, $\Box\varphi = 0$, constitutes a reasonably accurate approximation, where $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$ is the d'Alembertian operator.

With these remarks in mind, we can now proceed to the calculation of the Hadamard function induced by the one-loop diagram shown in Fig. 1. The calculation will be done entirely on spatially flat sections of de Sitter space (Poincaré patch), in which the scale factor in conformal time $d\eta = dt/a$ reads

$$a(\eta) = -\frac{1}{H\eta}, \quad (\eta < 0). \tag{24}$$

The relevant action is simply

$$S[\varphi,\chi] \approx \int d^4x\sqrt{-\bar{g}_{\rm dS}}\bigg(-\frac{1}{2}\bar{g}^{\mu\nu}_{\rm dS}(\partial_\mu\varphi)(\partial_\nu\varphi) - \frac{1}{2}\bar{g}^{\mu\nu}_{\rm dS}(\partial_\mu\chi)(\partial_\nu\chi) - \frac{h}{2}\varphi\chi^2\bigg), \tag{25}$$

with a de Sitter background metric $\bar{g}^{\rm dS}_{\mu\nu}$. The free theory is solved in momentum space, with $k = \|\vec{k}\|$, for each field by the Bunch-Davies vacuum whose positive (+) and negative (−) frequency mode functions are given by

$$u^{\pm}_{\rm dS}(\eta,k) = \frac{H}{\sqrt{2k^3}}(1 \pm ik\eta)e^{\mp ik\eta}. \tag{26}$$

In Appendix A, we give the definition of the Wightman functions $\Delta^{\mp\pm}_{\varphi}$ as well as the spectral (causal) two-point function Δ^c_φ and the Hadamard (statistical) two-point function F_φ in momentum space. For the Bunch-Davies vacuum they read

$$i\Delta^{\mp\pm}_{\varphi,\rm dS}(\eta,\eta',k) = \frac{H^2}{2k^3}(1\pm ik\eta)(1\mp ik\eta')e^{\mp ik(\eta-\eta')}, \tag{27}$$

$$\Delta^c_{\varphi,\rm dS}(\eta,\eta',k) = \frac{H^2}{k^3}[k(\eta-\eta')\cos[k(\eta-\eta')] - (1+k^2\eta\eta')\sin[k(\eta-\eta')]], \tag{28}$$

$$F_{\varphi,\rm dS}(\eta,\eta',k) = \frac{H^2}{2k^3}[(1+k^2\eta\eta')\cos[k(\eta-\eta')] + k(\eta-\eta')\sin[k(\eta-\eta')]]. \tag{29}$$

In the following Sec. III, we compute the one-loop correction to the statistical propagator in the super-Hubble limit as

$$\begin{aligned}\delta F_\varphi(\eta,\eta',k) &= [F_\varphi - F_{\varphi,\rm dS}](\eta,\eta',k)\\ &= \frac{h^2}{2^6 3^3 k^3\pi^2}\bigg\{6\bigg[4\log\bigg(\frac{H}{2k}\bigg) - 4\gamma_E + 5\bigg]\log(-2k\eta)\log(-2k\eta')\\ &\quad - \bigg[(106-48\gamma_E)\log\bigg(\frac{H}{2k}\bigg) - 18\log\bigg(\frac{\mu}{k}\bigg) + 36\gamma_E(\gamma_E-3) + \pi^2 + \frac{208}{3}\bigg]\log(4k^2\eta\eta')\\ &\quad + \bigg[12\log\bigg(\frac{H}{2k}\bigg) - 5\bigg][\log^2(-2k\eta) + \log^2(-2k\eta')]\\ &\quad + 4[\log^3(-2k\eta) + \log^3(-2k\eta')] + \mathcal{O}(k\eta,k\eta')\bigg\},\end{aligned} \tag{30}$$

where the parameter μ is the renormalization scale and the quantity $\gamma_E = -\psi(1) \approx 0.577216$ is Euler's constant, where $\psi(z) = (d/dz)\ln(\Gamma(z))$ is the digamma function (not to be confused with the spatial scalar metric perturbation ψ). We now have to express these results in terms of the comoving curvature perturbation which we achieve in a first approximation by using linear relations. The comoving curvature perturbation $\mathcal{R}$ and its canonical momentum $\pi_{\mathcal{R}}$ read to linear order in zero curvature gauge $\psi = 0$,

$$\mathcal{R} \equiv \psi + \frac{H}{\dot\phi}\varphi \to \frac{H}{\dot\phi}\varphi = \frac{1}{\sqrt{2\epsilon}}\frac{\varphi}{M_p}, \tag{31}$$

$$\pi_{\mathcal{R}} \equiv 2a^2M_p^2\epsilon\partial_\eta\mathcal{R} \to \sqrt{2\epsilon}M_pa^2\bigg[\partial_\eta\varphi - (\partial_\eta\epsilon)\frac{\varphi}{2\epsilon}\bigg]. \tag{32}$$

This procedure gives results that are gauge invariant to first order in coordinate gauge transformations if the one-loop corrections discussed above are consistently taken into account. However, our primary goal is to calculate the entropy increase from the dissipative part of the spectator loop in Fig. 1, which is controlled by the coupling constant g and which is different—and thus independent—from the gravitational coupling $\kappa = \sqrt{16\pi G}$. This observation provides evidence that our final result for the entropy is gauge independent.

Using the linear relations (32) we can express the statistical two-point functions of the comoving curvature perturbation and its canonical momentum in terms of the inflaton correlator to linear order as

$$\Delta_{\mathcal{R}\mathcal{R}}(\eta,k) \equiv F_{\mathcal{R}}(\eta,\eta,k) = \frac{1}{2\epsilon M_p^2}F_\varphi(\eta,\eta,k), \tag{33}$$

$$\begin{aligned}\Delta_{\mathcal{R}\pi_{\mathcal{R}}}(\eta,k) &\equiv 2a^2M_p^2\epsilon\partial_{\eta'}F_{\mathcal{R}}(\eta,\eta',k)|_{\eta=\eta'}\\ &= a^2\bigg[\frac{1}{2}\partial_\eta - \frac{(\partial_\eta\epsilon)}{2\epsilon}\bigg]F_\varphi(\eta,\eta,k),\end{aligned} \tag{34}$$

$$\begin{aligned}\Delta_{\pi_{\mathcal{R}}\pi_{\mathcal{R}}}(\eta,k) &\equiv (2a^2M_p^2\epsilon)^2\partial_\eta\partial_{\eta'}F_{\mathcal{R}}(\eta,\eta',k)|_{\eta=\eta'}\\ &= 2\epsilon M_p^2a^4\bigg[\partial_\eta\partial_{\eta'}F_\varphi(\eta,\eta',k)|_{\eta=\eta'}\\ &\quad - \frac{(\partial_\eta\epsilon)}{2\epsilon}\partial_\eta F_\varphi(\eta,\eta,k) + \frac{(\partial_\eta\epsilon)^2}{4\epsilon^2}F_\varphi(\eta,\eta,k)\bigg].\end{aligned} \tag{35}$$

Thus, shortly before the end of inflation at $\eta = \eta_e$ such that the slow-roll parameter $\epsilon(\eta_e)$ is still small and to leading order a constant, we have the following leading order corrections to the comoving curvature correlators on super-Hubble scales $|k\eta_e| \ll 1$:

$$\Delta_{\mathcal{R}\mathcal{R}}(\eta_e, k) \approx \frac{H^2}{4M_p^2 k^3 \epsilon(\eta_e)} \left[1 + \frac{h^2}{108\pi^2 H^2}[\log^3(-2k\eta_e) + \mathcal{O}(\log^2(-2k\eta_e))] + \mathcal{O}(\epsilon(\eta_e), k\eta_e)\right], \tag{36}$$

$$\Delta_{\mathcal{R}\pi_\mathcal{R}}(\eta_e, k) \approx -\frac{Ha(\eta_e)}{2k} \left[1 + \frac{a^2(\eta_e)h^2}{72\pi^2 k^2}[\log^2(-2k\eta_e) + \mathcal{O}(\log(-2k\eta_e))] + \mathcal{O}(\epsilon(\eta_e), k\eta_e)\right], \tag{37}$$

$$\Delta_{\pi_\mathcal{R}\pi_\mathcal{R}}(\eta_e, k) \approx kM_p^2 a^2(\eta_e)\epsilon(\eta_e)\left[1 + \frac{h^2 a^4(\eta_e)H^4}{36\pi^2 H^2 k^4}\left[\log\left(\frac{H}{2k}\right) + \frac{5}{4} - \gamma_E\right] + \mathcal{O}(\epsilon(\eta_e), k\eta_e)\right]. \tag{38}$$

We note that our result satisfies Weinberg's theorem, since the $\Delta_{\mathcal{R}\mathcal{R}}$ correlator in (36) receives only logarithmic corrections in time multiplying a constant:

$$\propto h^2 H^{-2} = g^2 \bar{\phi}^2 H^{-2} \sim g^2 M_P^2 m^{-2} \lesssim 10^{-12}. \tag{39}$$

The one-loop corrections to ϵ are also small as argued below (18). Although corrections to $\Delta_{\mathcal{R}\mathcal{R}}$ are negligible, the corrections to $\Delta_{\mathcal{R}\pi_\mathcal{R}}$ and $\Delta_{\pi_\mathcal{R}\pi_\mathcal{R}}$, which are induced by dissipative effects, can become very large since they multiply powers of the scale factor.

In order to study the physical implications at the end of inflation on supper-Hubble scales, we will rescale $\pi_\mathcal{R}$ by its linear relation to the gauge-invariant gravitational potential (B14),

$$\Psi = -\frac{\mathcal{H}}{2M_p^2 k^2 a^2}\pi_\mathcal{R}. \tag{40}$$

We quantify possibly large corrections of the $\pi_\mathcal{R}\pi_\mathcal{R}$-correlator to the tree-level result $\bar{\Delta}_{\pi_\mathcal{R}\pi_\mathcal{R}}$ by the ratio

$$\begin{aligned}\Delta_{\text{infl}} &\equiv \frac{H}{2M_p^2 k^2 a(\eta_e)}\left|\frac{\Delta_{\pi_\mathcal{R}\pi_\mathcal{R}} - \bar{\Delta}_{\pi_\mathcal{R}\pi_\mathcal{R}}}{\Delta_{\mathcal{R}\mathcal{R}}}\right|^{1/2} \\ &\approx \frac{\epsilon(\eta_e)h}{6\pi H}\frac{a^2(\eta_e)H^2}{k^2}\left|\log\left(\frac{H}{k}\right)\right|^{1/2} \\ &\lesssim 10^{-12}\frac{\epsilon(\eta_e)}{6\pi}\frac{a^2(\eta_e)H^2}{k^2}\left|\log\left(\frac{H}{k}\right)\right|^{1/2},\end{aligned} \tag{41}$$

where we kept only the dominant logarithmic contribution and substituted the estimate for the coupling constant $h = g\bar{\phi}$ from (17). We note that the quantity Δ_{inf} in (41) is order 1 after the mode k spends

$$N_{\text{dec}} \approx \frac{1}{2}\log\left[\frac{6\pi H}{\epsilon(\eta_e)h|\log(H/k)|^{1/2}}\right] \gtrsim 20 \tag{42}$$

e-folds on super-Hubble scales. This marks the time at which quantum corrections dominate the tree-level result for the $\pi_\mathcal{R}\pi_\mathcal{R}$-correlator and the decoherence sets in. In fact, the timescale (42) is a couple of e-folds longer than the decoherence time associated with the growth of entropy, which is controlled by the time at which the momentum-momentum correlator (38) becomes loop dominated, $N_{\text{entropy}} \simeq \frac{1}{2}\log[\frac{6\pi H}{h|\log(H/k)|^{1/2}}] = N_{\text{dec}} - \frac{1}{2}\log(1/\epsilon(\eta_e))$. Furthermore, the decoherence timescale N_{dec} in (42) differs essentially from the breakdown time of standard perturbation theory which is governed by the perturbativity time associated with the $\mathcal{R}\mathcal{R}$-correlator (36),[2]

$$N_{\text{pert}} \approx \left[\frac{108\pi^2 H^2}{h^2}\right]^{\frac{1}{3}} \gtrsim 10^9, \tag{43}$$

which is a much larger timescale because the correlators entering loop calculations grow only logarithmically with the scale factor. We can also quantify possibly large corrections of the $\mathcal{R}\pi_\mathcal{R}$-correlator to the tree-level result by the ratio

$$\begin{aligned}\theta_{\text{infl}} &\equiv \frac{H}{2M_p^2 k^2 a(\eta_e)}\left|\frac{\Delta_{\mathcal{R}\pi_\mathcal{R}} - \bar{\Delta}_{\mathcal{R}\pi_\mathcal{R}}}{\Delta_{\mathcal{R}\mathcal{R}}}\right| \\ &\approx \epsilon(\eta_e)\frac{a^2(\eta_e)h^2}{72\pi^2 k^2}\log^2(-2k\eta_e) \\ &\lesssim 10^{-24}\epsilon(\eta_e)\frac{a^2(\eta_e)H^2}{72\pi^2 k^2}\left|\log^2\left(\frac{Ha(\eta_e)}{k}\right)\right|.\end{aligned} \tag{44}$$

From (41) and (44), we see an enhancement of the $\hat{\pi}_\mathcal{R}$ operator by the factor $a^2(\eta_e)H^2/k^2$ at the end of inflation. The source of this amplification, however, lies in the vacuum quantum uncertainty of the spectator field χ which

[2]The standard estimate for the perturbativity time is larger, $N_{\text{pert}} \sim 10^{13}$ e-folds, and it is based on the assumption that there are only two powers of the logarithms in the $\mathcal{R}\mathcal{R}$-correlator (36). However, the detailed calculation performed in this work shows that there are in fact three powers of the logarithm, thus shortening significantly N_{pert}.

is coupled to the inflaton via the interaction term $\bar{\phi}\varphi\chi^2$. Since the quantum fluctuations of the spectator are independent of the inflaton quantum fluctuations they will lead to an independent, amplified late-time stochastic source. We can make the latter statement quantitative by invoking the Gaussian entropy of the corrected two-point functions. Since we used linear relations as a first approximation, the Gaussian invariant associated with the comoving curvature perturbation is identical to the Gaussian invariant associated with the inflaton perturbation,

$$\begin{aligned}\frac{\Delta^2_{\mathcal{R}}(\eta,k)}{4} &= \Delta_{\mathcal{R}\mathcal{R}}(\eta,k)\Delta_{\pi_{\mathcal{R}}\pi_{\mathcal{R}}}(\eta,k) - \Delta^2_{\mathcal{R}\pi_{\mathcal{R}}}(\eta,k)\\ &= a^4\bigg[F_\varphi(\eta,\eta,k)\partial_\eta\partial_{\eta'}F_\varphi(\eta,\eta',k)|_{\eta=\eta'}\\ &\quad -\frac{1}{4}(\partial_\eta F_\varphi(\eta,\eta,k))^2\bigg] = \frac{\Delta^2_\varphi(\eta,k)}{4}. \end{aligned}\tag{45}$$

The Gaussian invariant Δ^2_φ of the inflaton perturbation φ and hence of the comoving curbature perturbation $\Delta^2_{\mathcal{R}}$ is given by

$$\begin{aligned}\frac{\Delta^2_\varphi(\eta,k)}{4a^4} = \frac{\Delta^2_{\mathcal{R}}(\eta,k)}{4a^4} &= F_\varphi(\eta,\eta',k)\partial_\eta\partial_{\eta'}F_\varphi(\eta,\eta',k)\\ &\quad -[\partial_{\eta'}F_\varphi(\eta,\eta',k)]^2|_{\eta'=\eta},\end{aligned}\tag{46}$$

which can be used to calculate the Gaussian part of the von Neumann entropy,

$$\begin{aligned}S_{\mathrm{vN}}[\mathcal{R}] &= \frac{\Delta_{\mathcal{R}}+1}{2}\log\frac{\Delta_{\mathcal{R}}+1}{2} - \frac{\Delta_{\mathcal{R}}-1}{2}\log\frac{\Delta_{\mathcal{R}}-1}{2}\\ &= S_{\mathrm{vN}}[\varphi].\end{aligned}\tag{47}$$

The last equality follows from the fact that $\mathcal{R}$ and φ are related by a (time-dependent) rescaling, and since the von Neumann entropy is expressed in terms of the Gaussian invariant of the state Δ^2_φ, it cannot depend on a linear field redefinition. This is one way to understand why local mass corrections changing the VEV of the inflaton via (18) do not contribute to the entropy.

The mode functions of the noninteracting theory in the Bunch-Davies vacuum yield a Gaussian invariant that is identical to 1 and hence result in zero von Neumann entropy. The same reasoning holds for the spectator field χ which we also prepare in the Bunch-Davies vacuum. Thus, the Bunch-Davies vacuum for the fields φ and χ represents a state with minimal uncertainty which is solely due to the quantum nature of the theory. However, once interactions are taken into account, the Gaussian invariant and hence the entropy get perturbatively corrected:

$$\begin{aligned}\delta\left[\frac{\Delta^2_\varphi}{4a^4}\right] &= \delta[F_\varphi(\eta,\eta)\partial_\eta\partial_{\eta'}F_\varphi(\eta,\eta') - [\partial_{\eta'}F_\varphi(\eta,\eta')]^2]|_{\eta'=\eta}\\ &= [F_{\varphi,\mathrm{dS}}(\eta,\eta)\partial_\eta\partial_{\eta'}\delta F_\varphi(\eta,\eta')\\ &\quad + \delta F_\varphi(\eta,\eta)\partial_\eta\partial_{\eta'}F_{\varphi,\mathrm{dS}}(\eta,\eta')\\ &\quad - 2[\partial_{\eta'}F_{\varphi,\mathrm{dS}}(\eta,\eta')]\partial_{\eta'}\delta F_\varphi(\eta,\eta')]|_{\eta'=\eta}\\ &= \frac{H^2}{2k}[(1+k^2\eta^2)\partial_{k\eta}\partial_{k\eta'}\delta F_\varphi(\eta,\eta')\\ &\quad + \delta F_\varphi(\eta,\eta)k^2\eta^2 - 2\eta\partial_{\eta'}\delta F_\varphi(\eta,\eta')]|_{\eta'=\eta}.\end{aligned}\tag{48}$$

The correction to the Gaussian invariant of the inflaton perturbation is to leading order in the super-Hubble limit given by

$$\delta\left[\frac{\Delta^2_\varphi}{4}\right] = \frac{1}{9\pi^2}\frac{h^2}{H^2}\left(\frac{Ha}{2k}\right)^6\left[4\log\left(\frac{H}{2k}\right)+5-4\gamma_E+\mathcal{O}(k\eta)\right].\tag{49}$$

This expression is greater than zero for $H > 2k$, which is amply satisfied for the scales we will be interested in. We conclude that cubic interactions in inflation of the type $g\bar{\phi}\varphi\chi^2$ lead to a growth of the Gaussian invariant Δ^2_φ by a factor of a^6 on super-Hubble scales and correspondingly to a growth of the Gaussian entropy. This growth is to leading order due to the quantum loop corrected $\pi_{\mathcal{R}}\pi_{\mathcal{R}}$-correlator which grows much faster than the correction to the $\mathcal{R}\mathcal{R}$-correlator. This leads to two conclusions. First, the $\pi_{\mathcal{R}}\pi_{\mathcal{R}}$-correlator in (38) which was calculated with dissipative corrections is linearly gauge invariant. This follows from the fact that entropy production results only from dissipative effects [46] and the statement that the entropy [or the associated Gaussian invariant (46)] is to linear order gauge invariant. The second conclusion is that we can view the quantum loop corrected operators $\hat{\mathcal{R}}$ and $\hat{\pi}_{\mathcal{R}}$ as stochastically independent at the end of inflation, in contrast to the tree-level result (C16).

Let us visualize this statement by three snapshots of a phase-space diagram associated to $\mathcal{R}(\vec{k})$ and $\pi_{\mathcal{R}}(\vec{k})$ for a given mode $\vec{k}$. The first snapshot in Fig. 3 is taken while the

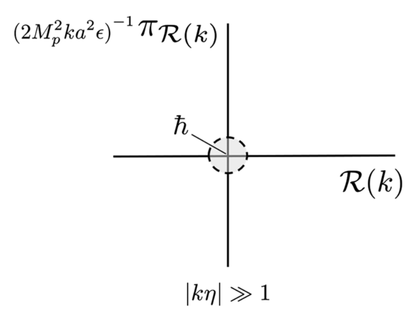

FIG. 3. Phase diagram for mode k early in the sub-Hubble regime. The rescaling for the momentum $\pi_{\mathcal{R}}$ follows from initial conditions of the linear evolution (C14) at early times.

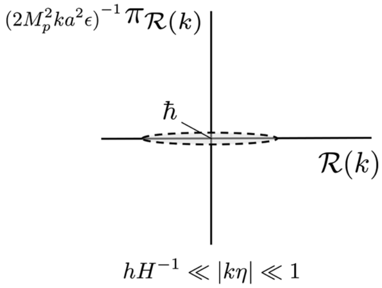

FIG. 4. Phase diagram for mode k at intermediate times such that k is super-Hubble but quantum loop corrections are still negligible. The Semiminor axis is enlarged to be visible and is substantially smaller than the one in Fig. 5.

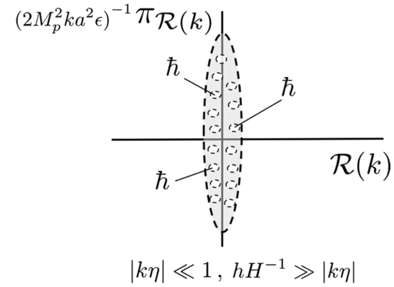

FIG. 5. Phase diagram for mode k which is super-Hubble at late times where quantum loop corrections balance the suppression from the small coupling constant. Note that the axes in this figure are compressed, which was necessary as the surface area of this state is very large when measured in units of $\hbar$.

mode is deep in the sub-Hubble regime where it is governed by tree-level dynamics due to the smallness of the coupling constant. The state is then approximately in its adiabatic, Gaussian vacuum, indicated by the circle on the phase space diagram, representing the set of points of equal probability amplitude. In an intermediate step in snapshot in Fig. 4, the mode becomes super-Hubble but the enhancement due to the factor of $k^{-2}a^2H^2$ in (41) is still too small to compensate the small coupling hH^{-1}. This phase is thus still dominated by the linear analysis and results in the usual squeezed state [48]. The final snapshot in Fig. 5 represents the end of inflation, more precisely, it is representative for all modes that have evolved for $\gtrsim 20$ e-folds on super-Hubble scales, cf. the estimate (42). For these modes, the enhancement of the $\pi_{\mathcal{R}}\pi_{\mathcal{R}}$-correlator due to the factor $k^{-2}a^2H^2$ in (41) is now big enough to overcome the suppression of the small coupling hH^{-1}. The state is still squeezed, but now mostly in the momentum direction.

A tempting question to ask is how the enhanced $\pi_{\mathcal{R}}$-operator at the end of inflation affects the effective temperature perturbation. In order to answer this question we still have to map these correlators to a time deep in the radiation era $\eta_{\rm cmb} \approx 10^{-1}\eta_{\rm rec}$, some time before recombination at $\eta = \eta_{\rm rec}$. As a first attempt, we pick the simplest possible scenario and assume that the comoving curvature perturbation $\mathcal{R}$ and the gauge-invariant gravitational potential Ψ will not be further affected on super-Hubble scales during the transition to radiation such that we can make use of standard linear relations. We review this process in Appendix B. The effective photon temperature perturbation relevant for the CMB at $\eta_{\rm cmb}$ (which is a conformal time early enough from the decoupling time such that the linear collisionless evolution still applies) may then be expressed according to (B18) in terms of the comoving curvature perturbation just before the end of inflation at η_e as follows:

$$\Delta\hat{T}(\eta,\vec{k}) \approx \frac{1}{2}\left[\frac{2}{3}\hat{\mathcal{R}}(\eta_e,\vec{k}) - \frac{a^3(\eta_e)}{a^3(\eta_{\rm cmb})}\frac{H}{2M_p^2k^2a(\eta_e)}\hat{\pi}_{\mathcal{R}}(\eta_e,\vec{k})\right]\cos[kr_s(\eta)] + \frac{6H}{kc_s(\eta_{\rm cmb})}\frac{a^4(\eta_e)}{a^4(\eta_{\rm cmb})}\left[\frac{H}{2M_p^2k^2}\hat{\pi}_{\mathcal{R}}(\eta_e,\vec{k}) + a(\eta_e)\hat{\mathcal{R}}(\eta_e,\vec{k})\right]\sin[kr_s(\eta)]. \tag{50}$$

We already know that the tree-level contribution to the sine term in Eq. (50) is insignificant in this scenario. Let us thus define here another quantity that allows us to measure the relative amplitude of orthogonal oscillations in (50) if we assume the quantum contributions to the $\pi_{\mathcal{R}}$ operator to be dominant,

$$\frac{\Delta_{\rm sin}}{\Delta_{\rm cos}} \equiv \frac{a^4(\eta_e)}{a^4(\eta_{\rm cmb})}\frac{18Ha(\eta_e)}{kc_s(\eta_{\rm cmb})}\Delta_{\rm infl} \sim \frac{h}{H}\frac{a^4(\eta_e)}{a^4(\eta_{\rm cmb})}\frac{3\epsilon(\eta_e)}{\pi}\frac{a^3(\eta_e)H^3}{k^3c_s(\eta_{\rm cmb})}\left|\log\left(\frac{H}{k}\right)\right|. \tag{51}$$

Putting in the estimate for our coupling constant h from (17) we get

$$\frac{\Delta_{\rm sin}}{\Delta_{\rm cos}} \lesssim 10^{-12}\frac{a(\eta_e)}{a(\eta_{\rm cmb})}\frac{3\epsilon(\eta_e)}{\pi}\frac{\mathcal{H}^3(\eta_{\rm cmb})}{k^3c_s(\eta_{\rm cmb})}\left|\log\left(\frac{H}{k}\right)\right| \ll 1. \tag{52}$$

It is thus not sufficient to have quantum loop enhancements of the $\pi_{\mathcal{R}}\pi_{\mathcal{R}}$-correlator only during inflation since the linear evolution throughout radiation suppresses it such that at the times of CMB it again becomes small. It is a natural question to ask whether quantum corrections during radiation will hinder this decay in a way that is similar to the quantum corrected processes that take place during inflation and we leave this for future studies.

III. KADANOFF-BAYM EQUATION FOR THE STATISTICAL PROPAGATOR

A. Effective action

In this section, we lay out in some detail how we calculate the quantum loop correction to the statistical propagator of the inflaton perturbation that we present in (30). We will perform this calculation in the Schwinger-Keldysh formalism for which the first step is to write down the two-particle-irreducible (2PI) effective action [49]. We will work with an accuracy of a two-loop effective action, where dissipative effects can occur. The 2PI effective action corresponding to the tree-level action (25) can be written in the two-loop approximation as

$$\Gamma[i\Delta_\varphi^{cd}, i\Delta_\chi^{cd}] = \Gamma_0[i\Delta_\varphi^{cd}, i\Delta_\chi^{cd}] + \Gamma_1[i\Delta_\varphi^{cd}, i\Delta_\chi^{cd}] + \Gamma_2[i\Delta_\varphi^{cd}, i\Delta_\chi^{cd}], \quad c,d=\pm, \tag{53}$$

where the three constituent functionals are given by

$$\Gamma_0[i\Delta_\varphi^{cd}, i\Delta_\chi^{cd}] = \frac{1}{2}\int d^D x d^D x' \sqrt{-\bar{g}_{\rm dS}(x)} \times \left(\sum_{c,d=\pm} \bar{\Box}_x^{\rm dS}\delta^D(x-x')c\delta^{cd} i\Delta_\varphi^{dc}(x',x) + \sum_{c,d=\pm}\bar{\Box}_x^{\rm dS}\delta^D(x-x')c\delta^{cd} i\Delta_\chi^{dc}(x',x)\right), \tag{54}$$

$$\Gamma_1[i\Delta_\varphi^{cd}, i\Delta_\chi^{cd}] = -\frac{i}{2}{\rm Tr}[\log(i\Delta_\varphi^{cd}(x;x'))] - \frac{i}{2}{\rm Tr}[\log(i\Delta_\chi^{cd}(x;x'))], \tag{55}$$

$$\Gamma_2[i\Delta_\varphi^{cd}, i\Delta_\chi^{cd}] = \int d^D x d^D x' \sqrt{-\bar{g}_{\rm dS}(x)}\sqrt{-\bar{g}_{\rm dS}(x')} \times \sum_{c,d=\pm} cd\frac{ih^2}{4}(i\Delta_\chi^{cd}(x,x'))^2 i\Delta_\varphi^{cd}(x,x'), \tag{56}$$

and the elements of the Keldysh propagators $i\Delta_{\varphi,\chi}^{cd}$ may be identified in terms of the statistical and spectral two-point functions,

$$i\Delta_{\varphi,\chi}^{\mp\pm}(x,x') = F_{\varphi,\chi}(x,x') \pm \frac{1}{2} i\Delta_{\varphi,\chi}^c(x,x'), \tag{57}$$

$$i\Delta_{\varphi,\chi}^{\pm\pm}(x,x') = F_{\varphi,\chi}(x,x') \pm \frac{1}{2}{\rm sign}[x^0-(x^0)'] i\Delta_{\varphi,\chi}^c(x,x'). \tag{58}$$

Applying the variational principle yields the following equations of motion:

$$\bar{\Box}_x^{\rm dS} i\Delta_\varphi^{ab}(x;x'') = \frac{a\delta^{ab} i\delta^D(x-x'')}{\sqrt{-\bar{g}_{\rm dS}(x)}} + \int d^D x' \sqrt{-\bar{g}_{\rm dS}(x')} \times \sum_{c=\pm} c\, iM_\varphi^{ac}(x,x') i\Delta_\varphi^{cb}(x',x''), \tag{59}$$

$$\bar{\Box}_{x''}^{\rm dS} i\Delta_\varphi^{ab}(x;x'') = \frac{a\delta^{ab} i\delta^D(x-x'')}{\sqrt{-\bar{g}_{\rm dS}(x)}} + \int d^D x' \sqrt{-\bar{g}_{\rm dS}(x')} \times \sum_{c=\pm} c\, i\Delta_\varphi^{ac}(x,x') iM_\varphi^{cb}(x',x''), \tag{60}$$

where the corresponding self-masses $iM_\varphi^{ab}(x,x')$ read

$$iM_\varphi^{ab}(x,x') = -\frac{ih^2}{2}(i\Delta_\chi^{cd}(x,x'))^2. \tag{61}$$

B. Renormalizing the self-mass

We attempt to solve Eq. (59) by using the expression for the free propagators in the Bunch-Davies vacuum,

$$iM_\varphi^{ab}(x,x') = -\frac{ih^2}{2}(i\Delta_\chi^{ab}(x,x'))^2 \approx -\frac{ih^2}{2}(i\Delta_{\rm dS}^{ab}(x,x'))^2. \tag{62}$$

The self-masses (62) are products of distributions that have local contributions $\propto \delta^D(x,x')$ which would yield indefinite answers when integrated against a test function. The singularities can be isolated by differential, dimensional regularization in position space where they take the form $\propto (D-4)^{-1}\delta^D(x,x')$ (and/or derivatives thereof). We renormalize the self-mass (62) by adding suitable local counterterms to the effective action which can be used to subtract these divergent contributions, yielding eventually finite answers in the limit $D\to 4$.

Let us first write down the de Sitter Feynman propagator in position space in D space-time dimensions which has been computed in terms of the quantity

$$y \equiv y_{++}, \tag{63}$$

where in de Sitter–invariant length functions

$$y_{ab} = aa'H^2\Delta x_{ab}^2 = a(\eta)a(\eta')H^2\Delta x_{ab}^2(\eta-\eta',\vec{x}-\vec{x}') = \frac{\Delta x_{ab}^2(\eta-\eta',\vec{x}-\vec{x}')}{\eta\eta'} \tag{64}$$

can be expressed with the Lorentz-invariant length functions

$$\Delta x_{\pm\pm}^2 = -(|\eta-\eta'| \mp i\varepsilon)^2 + \|\vec{x}-\vec{x}'\|^2, \tag{65}$$

$$\Delta x_{\pm\mp}^2 = -(\eta-\eta' \pm i\varepsilon)^2 + \|\vec{x}-\vec{x}'\|^2. \tag{66}$$

The de Sitter propagator in position space has been given by [50]

$$i\Delta_{\rm dS}^{++} = \frac{H^{D-2}}{(4\pi)^{D/2}}\left[-\sum_{n=0}^{\infty}\frac{1}{n-\frac{D}{2}+1}\frac{\Gamma[n+\frac{D}{2}]}{\Gamma[n+1]}\left(\frac{y}{4}\right)^{n-\frac{D}{2}+1} - \frac{\Gamma[D-1]}{\Gamma[\frac{D}{2}]}\pi\cot\left[\pi\frac{D}{2}\right] + \sum_{n=1}^{\infty}\frac{1}{n}\frac{\Gamma[n+D-1]}{\Gamma[n+\frac{D}{2}]}\left(\frac{y}{4}\right)^{n} + \frac{\Gamma[D-1]}{\Gamma[\frac{D}{2}]}\log[aa']\right], \tag{67}$$

where we use in this section the notation $a' = a(\eta')$ and it should be clear from the context whether a prime denotes a time derivative or refers to a coordinate. We can expand expression (67) around $D = 4$ and get

$$i\Delta_{\rm dS}^{++} = \frac{H^{D-2}}{(4\pi)^{D/2}}\left[\Gamma\left[\frac{D-2}{2}\right]\left(\frac{y}{4}\right)^{1-\frac{D}{2}} - 2\log\left[\frac{\sqrt{e}y}{4aa'}\right]\right] + \mathcal{O}(D-4). \tag{68}$$

Taking the square leads to

$$(i\Delta_{\rm dS}^{++})^2 = \frac{H^{2D-4}}{(4\pi)^{D}}\left[\Gamma^2\left[\frac{D-2}{2}\right]\left(\frac{y}{4}\right)^{2-D} - \frac{16}{y}\log\left[\frac{\sqrt{e}y}{4aa'}\right] + 4\log^2\left[\frac{\sqrt{e}y}{4aa'}\right]\right] + \mathcal{O}(D-4), \tag{69}$$

and we note that the nonintegrable piece of the self-mass is contained in the first term $\propto y^{2-D}$. Let us simplify the notation and denote the de Sitter d'Alembert operator as[3]

$$\frac{\square}{H^2} \equiv \frac{\bar{\square}^{\rm dS}}{H^2} = \eta^2\left[-\frac{\partial^2}{\partial\eta^2} + \frac{D-2}{\eta}\frac{\partial}{\partial\eta} + \delta^{ij}\frac{\partial^2}{\partial x^i\partial x^j}\right]. \tag{71}$$

We will make use of two relations that were established in [51],

$$\left(\frac{y_{\pm\pm}}{4}\right)^{2-D} = \left[\frac{2}{(D-3)(D-4)}\frac{\square}{H^2} - \frac{D(D-2)}{2(D-3)(D-4)} + \frac{D-6}{2(D-3)}\right]\left(\frac{y_{\pm\pm}}{4}\right)^{3-D} - \left[\frac{2}{(D-3)(D-4)}\frac{\square}{H^2} - \frac{D(D-2)}{2(D-3)(D-4)}\right]\left(\frac{y_{\pm\pm}}{4}\right)^{1-(D/2)} \pm \frac{2(4\pi)^{D/2}}{(D-3)(D-4)\Gamma[\frac{D}{2}-1]}\frac{i\delta^D(x-x')}{(Ha)^D}, \tag{72}$$

as well as

$$\frac{\square}{H^2}\left(\frac{y_{\pm\pm}}{4}\right)^{1-(D/2)} = \pm\frac{(4\pi)^{D/2}}{\Gamma[\frac{D}{2}-1]}\frac{i\delta^D(x-x')}{(Ha)^D} + \frac{D(D-2)}{4}\left(\frac{y_{\pm\pm}}{4}\right)^{1-(D/2)}. \tag{73}$$

Let us introduce the renormalization parameter μ with energy dimension 1. We can rewrite (72) by adding a μ-dependent term that vanishes on $D = 4$ in such a way that the divergence in the self-mass may be removed with a mass counterterm in the action $\propto (D-4)^{-1}\mu^{D-4}a^{-D}\delta^D(x-x')$. Moreover, we use

$$\left(\frac{y_{\pm\pm}}{4}\right)^{3-D} = \left(\frac{y_{\pm\pm}}{4}\right)^{1-(D/2)}\left[1 - \frac{D-4}{2}\log[y_{\pm\pm}] + \mathcal{O}[(D-4)^2]\right] \tag{74}$$

and expand the nonsingular terms in (72),

$$\left(\frac{y_{\pm\pm}}{4}\right)^{2-D} = \pm\frac{2(4\pi)^{D/2}}{(D-3)(D-4)\Gamma[\frac{D}{2}-1]}\left(\frac{\mu}{H}\right)^{D-4}\frac{i\delta^D(x-x')}{(Ha)^D} - \frac{\square}{H^2}\left(\frac{4}{y_{\pm\pm}}\log\left[\frac{\mu^2 y_{\pm\pm}}{H^2}\right]\right) - \frac{4}{y_{\pm\pm}}\left(2\log\left[\frac{\mu^2 y_{\pm\pm}}{H^2}\right] - 1\right) + \mathcal{O}(D-4), \tag{75}$$

which leads to

[3]We would like to remark that due to symmetry reasons we may use in the following derivations also derivatives acting on primed coordinates:

$$\frac{\square'}{H^2} = (\eta')^2\left[-\frac{\partial^2}{\partial(\eta')^2} + \frac{D-2}{\eta'}\frac{\partial}{\partial\eta'} + \delta^{ij}\frac{\partial^2}{\partial(x^i)'\partial(x^j)'}\right]. \tag{70}$$

$$
\begin{aligned}
(i\Delta_{\rm dS}^{\pm\pm})^2 = &\pm \frac{2\Gamma[\frac{D}{2}-1]\mu^{D-4}}{(4\pi)^{D/2}(D-3)(D-4)}\frac{i\delta^D(x-x')}{a^D} \\
&-\frac{H^{2D-4}}{(4\pi)^D}\left[\frac{\Box}{H^2}\left(\frac{4}{y_{\pm\pm}}\log\left[\frac{\mu^2 y_{\pm\pm}}{H^2}\right]\right)-\frac{4}{y_{\pm\pm}}\left(2\log\left[\frac{\mu^2 y_{\pm\pm}}{H^2}\right]-1\right)\right. \\
&\left.+\frac{16}{y_{\pm\pm}}\log\left[\frac{\sqrt{e}y}{4aa'}\right]-4\log^2\left[\frac{\sqrt{e}y_{\pm\pm}}{4aa'}\right]\right]+\mathcal{O}(D-4).
\end{aligned} \tag{76}
$$

The divergent local contribution in the first line of (76) yields a divergent contribution to the self-mass (62),

$$
(iM_\varphi^{cd}(x,x'))_{\rm div} = h^2\frac{\Gamma[\frac{D}{2}-1]\mu^{D-4}}{(4\pi)^{D/2}(D-3)(D-4)}\frac{\delta^D(x-x')}{a^D}c\delta^{cd}, \tag{77}
$$

which can be removed by adding the following counterterm action[4]:

$$
S_{\rm ct} = \int d^Dx a^D\left(-\frac{1}{2}\delta m^2\sum_{c,d=\pm}c\delta^{cd}i\Delta_\varphi^{cd}(x,x)\right), \tag{79}
$$

where δm^2 is proportional to the inflaton condensate squared,

$$
\delta m^2 = -g^2\bar\phi^2\frac{\Gamma(\frac{D}{2}-1)\mu^{D-4}}{(4\pi)^{D/2}(D-3)(D-4)}, \tag{80}
$$

and diverges as $\propto 1/(D-4)$. Clearly, the counterterm (79) is the divergent mass counterterm of the 2PI formalism. It is easy to check that varying the action (79) and adding it to the equations of motion (59)–(60) removes the divergent parts of the self-masses. The resulting renormalized self-mass $iM_{\phi,\rm ren}^{++}$ is

$$
iM_{\varphi,\rm ren}^{++}(x,x') = \frac{ih^2}{2}\frac{H^4}{(4\pi)^4}\left[\frac{\Box}{H^2}\left(\frac{4}{y}\log\left[\frac{\mu^2 y}{H^2}\right]\right)-\frac{4}{y}\left(2\log\left[\frac{\mu^2 y}{H^2}\right]-1\right)+\frac{16}{y}\log\left[\frac{\sqrt{e}y}{4aa'}\right]-4\log^2\left[\frac{\sqrt{e}y}{4aa'}\right]\right]. \tag{81}
$$

The other renormalized self-masses, $iM_{\varphi,\rm ren}^{ab}(x,x')$ $(a,b=\pm)$, are obtained simply by replacing $y(x,x')=y_{++}(x,x')$ in (81) by $y_{ab}(x,x')$.

C. Self-mass in momentum space

Ultimately, we will be interested in the Wigner transform of the spatially dependent piece of the self-mass. This may be conveniently achieved by extracting d'Alembert's operators and dropping homogeneous (momentum-independent) contributions. If the d'Alembertian in de Sitter space-time is acting on nonsingular functions (not containing y^{-1}), we have

$$
\frac{\Box}{H^2}f(y) = (4-y)yf''(y)+4(2-y)f'(y), \tag{82}
$$

which gives the identities

$$
\frac{1}{y} = \frac{1}{4}\frac{\Box}{H^2}\log(y)+\frac{3}{4}, \tag{83}
$$

$$
\frac{\log(y)}{y} = \frac{1}{8}\frac{\Box}{H^2}[\log^2(y)-2\log(y)]+\frac{3}{4}\log(y)-\frac{1}{2}. \tag{84}
$$

These identities allow us to rewrite the self-mass (81) as

$$
\begin{aligned}
iM_{\varphi,\rm ren}^{++}(x,x') = \frac{ih^2}{2}\frac{H^4}{(4\pi)^4}&\left\{\frac{\Box^2}{H^4}\left[\frac{1}{2}\log^2\left(\frac{y}{4}\right)+\log\left[\frac{4\mu^2}{eH^2}\right]\log\left(\frac{y}{4}\right)\right]\right. \\
&+2\frac{\Box}{H^2}\left[\frac{1}{2}\log^2\left(\frac{y}{4}\right)+\log\left[\frac{eH^2}{4\mu^2}\right]\log\left(\frac{y}{4}\right)\right]+2[1-2\log(aa')]\frac{\Box}{H^2}\log\left(\frac{y}{4}\right) \\
&\left.+2[1+4\log(aa')]\log\left(\frac{y}{4}\right)-4\log^2\left(\frac{y}{4}\right)\right\}+\text{hom}.,
\end{aligned} \tag{85}
$$

where hom. encode spatially homogeneous (y-independent) contributions, which are of no importance for this study. At this stage, we would like to emphasize that the expression for the self-mass (85) could have also been written with the de Sitter d'Alembertian operators acting on the primed space-time coordinates. We now perform the spatial Wigner transform of the self-mass (85) according to

$$iM^{++}_{\varphi,\mathrm{ren}}(\eta,\eta',k) = \int d^3(x-x')\, iM^{++}_{\phi,\mathrm{ren}}(x,x')e^{-i\vec{k}\cdot(\vec{x}-\vec{x}')}. \tag{86}$$

Furthermore, spatially homogeneous contributions are proportional to delta functions in k-space or derivatives thereof:

$$\int_0^\infty dr r\sin(kr) = -\pi\partial_k\delta(k). \tag{87}$$

We will drop again such contributions. In Appendix D we establish the following Wigner transformation:

$$\begin{aligned}
&\int d^3(x-x')e^{-i\vec{k}\cdot(\vec{x}-\vec{x}')}\left[\frac{1}{2}\log^2\left(\frac{y}{4}\right) + f(\eta,\eta')\log\left(\frac{y}{4}\right)\right] \\
&= -\frac{4\pi^2}{k^3}\left[2 + [1+ik|\Delta\eta|]\left(\log\left[\frac{aa'H^2|\Delta\eta|}{2k}\right] + i\frac{\pi}{2} - \gamma_E + f(\eta,\eta')\right)\right]e^{-ik|\Delta\eta|} \\
&\quad + \frac{4\pi^2}{k^3}(1-ik|\Delta\eta|)[\mathrm{ci}[2k|\Delta\eta|] - i\mathrm{si}[2k|\Delta\eta|]]e^{+ik|\Delta\eta|},
\end{aligned} \tag{88}$$

where $\Delta\eta = \eta - \eta'$ and $f(\eta,\eta')$ is some k-independent function. We make use of the Wigner transform (88); rewrite the scale factor as $a = -(H\eta)^{-1}$; and obtain, after some simplifications, the self-mass in momentum space as follows:

$$\begin{aligned}
iM^{++}_{\varphi,\mathrm{ren}}(\eta,\eta',k) = &-\frac{4\pi^2}{k^3}\frac{ih^2}{2}\frac{H^4}{(4\pi)^4}\left\{\frac{\Box_k^2}{H^4}\left(\left[2 + [1+ik|\Delta\eta|]\left(\log\left[\frac{2|\Delta\eta|\mu^2}{k\eta\eta'H^2}\right] + i\frac{\pi}{2} - \gamma_E - 1\right)\right]e^{-ik|\Delta\eta|}\right.\right. \\
&\left. - (1-ik|\Delta\eta|)[\mathrm{ci}[2k|\Delta\eta|] - i\mathrm{si}[2k|\Delta\eta|]]e^{+ik|\Delta\eta|}\right) \\
&+ 2\frac{\Box_k}{H^2}\left(\left[2 + [1+ik|\Delta\eta|]\left(\log\left[\frac{|\Delta\eta|H^2}{8k\eta\eta'\mu^2}\right] + i\frac{\pi}{2} - \gamma_E + 1\right)\right]e^{-ik|\Delta\eta|}\right. \\
&\left. - (1-ik|\Delta\eta|)[\mathrm{ci}[2k|\Delta\eta|] - i\mathrm{si}[2k|\Delta\eta|]]e^{+ik|\Delta\eta|}\right) \\
&+ 2[1 + 2\log(H^2\eta\eta')]\frac{\Box_k}{H^2}[[1+ik|\Delta\eta|]e^{-ik|\Delta\eta|}] \\
&- 8\left(\left[2 + [1+ik|\Delta\eta|]\left(\log\left[\frac{H^2|\Delta\eta|}{2k}\right] + i\frac{\pi}{2} - \gamma_E - \frac{1}{4}\right)\right]e^{-ik|\Delta\eta|}\right. \\
&\left.\left. - (1-ik|\Delta\eta|)[\mathrm{ci}[2k|\Delta\eta|] - i\mathrm{si}[2k|\Delta\eta|]]e^{+ik|\Delta\eta|}\right)\right\} + \mathrm{hom.},
\end{aligned} \tag{89}$$

where

$$\frac{\Box_k}{H^2} = -\eta^2\left(\partial_\eta^2 - \frac{2}{\eta}\partial_\eta + k^2\right) \tag{90}$$

is the d'Alembertian in momentum space. For a computational convenience we shall split the self-mass (89) in the following way:

$$M^{++}_{\phi,\mathrm{ren}}(\eta,\eta',k) = -2[1 + 2\log(H^2\eta\eta')]\frac{\Box_k}{H^2}\hat{M}^{++}(|\Delta\eta|,k) + \sum_{n=0}^{2}\left(\frac{\Box_k}{H^2}\right)^n M^{++}_{(n)}(\eta,\eta',k), \tag{91}$$

which is based on the definitions,

$$M^{++}_{(n)}(\eta,\eta',k) \equiv \alpha_{(n)}[\tilde{M}^{++}_{I}(|\Delta\eta|,k) + \tilde{M}^{++}_{II}(|\Delta\eta|,k)] + \beta_{(n)}\hat{M}^{++}(|\Delta\eta|,k) + \gamma_{(n)}\hat{M}^{++}(|\Delta\eta|,k)\log\left[\frac{\eta\eta' H^4}{4\mu^2}\right], \tag{92}$$

where

$$\alpha_{(n)} = \{-8,2,1\}, \qquad \beta_{(n)} = \left\{-2,-2+8\log\left[\frac{2\mu}{H}\right],1\right\}, \qquad \gamma_{(n)} = \{0,2,1\}, \tag{93}$$

and

$$\hat{M}^{++}(|\Delta\eta|,k) \equiv \frac{4\pi^2}{k^3}\frac{h^2}{2}\frac{H^4}{(4\pi)^4}[1+ik|\Delta\eta|]e^{-ik|\Delta\eta|}, \tag{94}$$

$$\tilde{M}^{++}_{I}(|\Delta\eta|,k) \equiv -\frac{4\pi^2}{k^3}\frac{h^2}{2}\frac{H^4}{(4\pi)^4}\left[2+[1+ik|\Delta\eta|]\left(\log\left[\frac{H^2|\Delta\eta|}{2k}\right]+i\frac{\pi}{2}-\gamma_E\right)\right]e^{-ik|\Delta\eta|}, \tag{95}$$

$$\tilde{M}^{++}_{II}(|\Delta\eta|,k) \equiv -\frac{4\pi^2}{k^3}\frac{h^2}{2}\frac{H^4}{(4\pi)^4}(1-ik|\Delta\eta|)\mathrm{E}_1[2ik|\Delta\eta|]e^{+ik|\Delta\eta|}. \tag{96}$$

Here, we made use of the identity for the exponential integral function,

$$\mathrm{E}_1[2ik|\Delta\eta|] = i\,\mathrm{si}[2k|\Delta\eta|] - \mathrm{ci}[2k|\Delta\eta|], \tag{97}$$

which holds when $k>0$. The sine (si) and cosine (ci) integrals are defined in Eqs. (A7) and (A8), respectively. The calculation of the other self-masses $iM^{\pm\mp}_{\phi,\mathrm{ren}}$ and $iM^{--}_{\phi,\mathrm{ren}}$ proceeds similarly. By writing

$$\log(\Delta x^2_{--}) = \log(|\Delta\eta^2 - \|\vec{x}-\vec{x}'\|^2|) - i\pi\theta(\Delta\eta^2 - \|\vec{x}-\vec{x}'\|^2), \tag{98}$$

we see that

$$M^{--}_{\phi,\mathrm{ren}} = [M^{++}_{\phi,\mathrm{ren}}]^*. \tag{99}$$

Moreover, due to

$$\log(\Delta x^2_{\mp\pm}) = \log(|\Delta\eta^2 - \|\vec{x}-\vec{x}'\|^2|) \pm i\,\mathrm{sign}(\eta,\eta')\pi\theta(\Delta\eta^2 - \|\vec{x}-\vec{x}'\|^2), \tag{100}$$

we see that

$$M^{ab}_{\phi,\mathrm{ren}}(\eta,\eta',k) = -4[1+\log(\eta\eta' H^2)]\frac{\Box_k}{H^2}\hat{M}^{ab}(|\Delta\eta|,k) + \sum_{n=0}^{2}\left(\frac{\Box_k}{H^2}\right)^n M^{ab}_{(n)}(\eta,\eta',k), \tag{101}$$

where

$$M^{\mp\pm}_{(n)} = M^{\pm\pm}_{(n)}\theta(\Delta\eta) + M^{\mp\mp}_{(n)}\theta(-\Delta\eta), \qquad \hat{M}^{\mp\pm}_{(n)} = \hat{M}^{\pm\pm}_{(n)}\theta(\Delta\eta) + \hat{M}^{\mp\mp}_{(n)}\theta(-\Delta\eta), \tag{102}$$

where $\mathrm{sign}(\eta,\eta') = \theta(\eta-\eta') - \theta(\eta'-\eta)$ and θ is the Heaviside step function. It will be convenient to define

$$M^F_{(n)} \equiv \frac{1}{2}[M^{++}_{(n)} + M^{--}_{(n)}] = \mathrm{Re}M^{++}_{(n)}, \tag{103}$$

$$M^c_{(n)}(\eta,\eta') \equiv \frac{\mathrm{sign}(\Delta\eta)}{i}[M^{++}_{(n)} - M^{--}_{(n)}](\eta,\eta') = 2\mathrm{sign}(\Delta\eta)\mathrm{Im}M^{++}_{(n)}, \tag{104}$$

$$\hat{M}^F \equiv \frac{1}{2}[\hat{M}^{++} + \hat{M}^{--}] = \mathrm{Re}\hat{M}^{++}, \tag{105}$$

$$\hat{M}^c(\eta,\eta') \equiv \frac{\mathrm{sign}(\Delta\eta)}{i}[\hat{M}^{++} - \hat{M}^{--}](\eta,\eta') = 2\mathrm{sign}(\Delta\eta)\mathrm{Im}\hat{M}^{++}, \tag{106}$$

and note the relations

$$[M^{++}_{(n)} - M^{--}_{(n)} \pm (M^{-+}_{(n)} - M^{+-}_{(n)})](\eta,\eta') = \pm 2\theta(\pm\Delta\eta) i M^{c}_{(n)}(\eta,\eta'), \tag{107}$$

$$[M^{++}_{(n)} + M^{--}_{(n)}](\eta,\eta') + \mathrm{sign}(\tau-\tau')[M^{-+}_{(n)} + M^{+-}_{(n)}](\eta,\eta') = 4\theta(\tau-\tau') M^{F}_{(n)}(\eta,\eta'), \tag{108}$$

which also hold for $\hat{M}^{ab}$.

D. Perturbative solution for the statistical propagator

Let us look at the renormalized version of equations of motion (59) for the Keldysh propagators $i\Delta^{ab}_{\varphi}$. By rewriting the two-point functions in terms of real and imaginary parts, we obtain

$$\begin{aligned}\Box_x F_\varphi(x,x'') = \frac{i}{2}\int d\eta' d^3x' (\eta' H)^{-4}[M^{++}_{\varphi,\mathrm{ren}} - M^{--}_{\varphi,\mathrm{ren}} + M^{-+}_{\varphi,\mathrm{ren}} - M^{+-}_{\varphi,\mathrm{ren}}](x,x') F_\varphi(x',x'') \\ - \frac{1}{4}\int d\eta' d^3x' (\eta' H)^{-4}[\mathrm{sign}(\eta'-\eta'')(M^{++}_{\varphi,\mathrm{ren}} + M^{--}_{\varphi,\mathrm{ren}}) - M^{-+}_{\varphi,\mathrm{ren}} - M^{+-}_{\varphi,\mathrm{ren}}](x,x') \Delta^c_\varphi(x',x'').\end{aligned} \tag{109}$$

We will solve for the statistical propagator perturbatively by approximating F_φ and Δ^c_φ on the right-hand side of (109) by the expressions for the Bunch-Davies vacuum (28) and (29), respectively. Inserting the concrete expressions (91) for our model in momentum space, we find

$$\begin{aligned}\Box_k F_\varphi(\eta,\eta'',k) \approx & -\sum_{n=0}^{2}\left(\frac{\Box_k}{H^2}\right)^n \int_{-\infty}^{\eta} d\eta' (\eta' H)^{-4} M^c_{(n)}(\eta,\eta',k) F_{\varphi,dS}(\eta',\eta'',k) \\ & + \sum_{n=0}^{2}\left(\frac{\Box_k}{H^2}\right)^n \int_{-\infty}^{\eta''} d\eta' (\eta' H)^{-4} M^F_{(n)}(\eta,\eta',k) \Delta^c_{\varphi,dS}(\eta',\eta'',k) \\ & + 2\int_{-\infty}^{\infty} d\eta' \frac{1+2\log(\eta\eta' H^2)}{(\eta' H)^4}\frac{\Box_k}{H^2}[\theta(\eta-\eta')\hat{M}^c(\eta,\eta',k)] F_{\varphi,dS}(\eta',\eta'',k) \\ & - 2\int_{-\infty}^{\eta''} d\eta' \frac{1+2\log(\eta\eta' H^2)}{(\eta' H)^4}\frac{\Box_k}{H^2}[\hat{M}^F(\eta,\eta',k)] \Delta^c_{\varphi,dS}(\eta',\eta'',k).\end{aligned} \tag{110}$$

Expanding the last two terms and rearranging the integration boundaries gives

$$\begin{aligned}\Box_k F_\phi(\eta,\eta'',k) \approx & -2\mathrm{Im}\sum_{n=0}^{2}\left(\frac{\Box_k}{H^2}\right)^n \int_{-\infty}^{\eta} d\eta' (\eta' H)^{-4} M^{++}_{(n)}(\eta,\eta',k) i\Delta^{+-}_{\varphi,dS}(\eta',\eta'',k) \\ & - \sum_{n=0}^{2}\left(\frac{\Box_k}{H^2}\right)^n \int_{\eta''}^{\eta} d\eta' (\eta' H)^{-4} \mathrm{Re} M^{++}_{(n)}(\eta,\eta',k) \Delta^c_{\varphi,dS}(\eta',\eta'',k) \\ & + 4\mathrm{Im}\int_{-\infty}^{\eta} d\eta' \frac{1+2\log(\eta\eta' H^2)}{(\eta' H)^4}\left[\frac{\Box_k}{H^2}\hat{M}^{++}(\eta,\eta',k)\right] i\Delta^{+-}_{\varphi,dS}(\eta',\eta'',k) \\ & + 2\int_{\eta''}^{\eta} d\eta' \frac{1+2\log(\eta\eta' H^2)}{(\eta' H)^4}\left[\frac{\Box_k}{H^2}\mathrm{Re}\hat{M}^{++}(\eta,\eta',k)\right] \Delta^c_{\varphi,dS}(\eta',\eta'',k).\end{aligned} \tag{111}$$

We will solve Eq. (111) by using a retarded Green's function $G_{\mathrm{ret}}(\eta,\eta',k)$ (which yields no contributions of the particular solution to the initial values) for the d'Alembertian operator in momentum space:

$$\begin{aligned}\Box_k G_{\mathrm{ret}}(\eta,\eta',k) = H^2\eta^2\left[-\partial_\eta^2 + \frac{D-2}{\eta}\partial_\eta - k^2\right] G_{\mathrm{ret}}(\eta,\eta',k) = a^{-4}(\eta')\delta(\eta-\eta'), \\ G_{\mathrm{ret}}(\eta,\eta',k) = \theta(\eta-\eta')\frac{H^2}{k^3}[k(\eta-\eta')\cos[k(\eta-\eta')] - (1+k^2\eta\eta')\sin[k(\eta-\eta')]].\end{aligned} \tag{112}$$

We have

$$F_\varphi(\eta,\eta'',k) = \frac{\Box_k}{H^2}\mathcal{B}_{(2)}(\eta,\eta'',k) + \mathcal{B}_{(1)}(\eta,\eta'',k) + H^2\int_{-\infty}^{\infty} d\tau \frac{G_{\rm ret}(\eta,\tau,k)}{(\tau H)^4}[\mathcal{B}_{(0)}(\tau,\eta'',k) + \mathcal{B}^{\log}_{(0)}(\tau,\eta'',k)] + F_{\rm hom.}(\eta,\eta'',k), \tag{113}$$

where

$$\mathcal{B}_{(n)}(\eta,\eta'',k) \equiv -\frac{2}{H^2}{\rm Im}\int_{-\infty}^{\eta}\frac{d\eta'}{(\eta' H)^4}M^{++}_{(n)}(\eta,\eta',k)i\Delta^{+-}_{\varphi,dS}(\eta',\eta'',k) - \frac{1}{H^2}\int_{\eta''}^{\eta}\frac{d\eta'}{(\eta' H)^4}{\rm Re}M^{++}_{(n)}(\eta,\eta',k)\Delta^{c}_{\varphi,dS}(\eta',\eta'',k), \tag{114}$$

$$\mathcal{B}^{\log}_{(0)}(\eta,\eta'',k) \equiv 4{\rm Im}\int_{-\infty}^{\eta}d\eta'\frac{1+2\log(\eta\eta' H^2)}{(\eta' H)^4}\left[\frac{\Box_\eta(k)}{H^2}\hat{M}^{++}(\eta,\eta',k)\right]i\Delta^{+-}_{\varphi,dS}(\eta',\eta'',k) + 2\int_{\eta''}^{\eta}d\eta'\frac{1+2\log(\eta\eta' H^2)}{(\eta' H)^4}\left[\frac{\Box_\eta(k)}{H^2}{\rm Re}\hat{M}^{++}(\eta,\eta',k)\right]\Delta^{c}_{\varphi,dS}(\eta',\eta'',k), \tag{115}$$

and

$$\Box_k F_{\rm hom.}(\eta,\eta'',k) = 0. \tag{116}$$

Let us define

$$\hat{F}(\eta,\eta'',k) \equiv F_\varphi(\eta,\eta'',k) - F_{\rm hom.}(\eta,\eta'',k). \tag{117}$$

The homogeneous solution has to be chosen in such a way that the symmetry properties of the statistical two-point function are satisfied,

$$F_{\rm hom.}(\eta,\eta'',k) - F_{\rm hom.}(\eta'',\eta,k) = \hat{F}(\eta'',\eta,k) - \hat{F}(\eta,\eta'',k), \tag{118}$$

and the full solution reads

$$F_\varphi(\eta,\eta'',k) = \frac{1}{2}[\hat{F}(\eta,\eta'',k) + \hat{F}(\eta'',\eta,k)] + \frac{1}{2}[F_{\rm hom.}(\eta,\eta'',k) + F_{\rm hom.}(\eta'',\eta,k)]. \tag{119}$$

We immediately get the consistency requirement

$$\Box_k\Box''_k[\hat{F}(\eta,\eta'',k) - \hat{F}(\eta'',\eta,k)] = 0, \tag{120}$$

which can be used as a nontrivial check of the result of the calculation. Let us also fix a common prefactor for the subsequent integrals

$$\lambda \equiv \frac{h^2}{256\pi^2 k^3}, \tag{121}$$

which gives the statistical two-point function F_φ correct dimensions in momentum space if all other factors and ratios are dimensionless.

Let us proceed with the calculation of (113). The integrals with logarithms $\mathcal{B}^{\log}_{(0)}$ in (115) combine to give the following expression:

$$\begin{aligned}\mathcal{B}^{\log}_{(0)}(\eta,\eta'',k) &= -8\lambda\{\cos[k(\eta-\eta'')](2+\log[H^2\eta^2]) \\ &\quad + \sin[k(\eta-\eta'')](2k(\eta-\eta'') - k\eta''\log[H^2\eta^2] + k\eta\log[H^2\eta\eta'']) \\ &\quad + k\eta({\rm ci}[-2k\eta] + {\rm ci}[-2k\eta''])(k\eta''\cos[k(\eta+\eta'')] - \sin[k(\eta+\eta'')]) \\ &\quad + k\eta(\pi + {\rm si}[-2k\eta] + {\rm si}[-2k\eta''])(\cos[k(\eta+\eta'')] + k\eta''\sin[k(\eta+\eta'')])\} \\ &\to -8\lambda(2+\log[H^2\eta^2]),\end{aligned} \tag{122}$$

where the arrow denotes the super-Hubble limit. The next step is to tackle the $\mathcal{B}_{2,1,0}$ terms in (114) for which we note that the integrals containing negative infinity as a boundary may be rewritten as

$$\int_{-\infty}^{\eta}\frac{d\tau}{(\tau H)^4}M^{++}_{(n)}(\eta,\tau,k)i\Delta^{+-}_{\varphi,dS}(\tau,\eta'',k)=\frac{1}{2}(1+ik\eta'')\frac{e^{-ik(\eta''-\eta)}}{H^2}\int_0^\infty dx\left[\alpha_{(n)}\left[\tilde{M}^{++}_{I}\left(\frac{x}{k},k\right)+\tilde{M}^{++}_{II}\left(\frac{x}{k},k\right)\right]\right.$$
$$\left.+\beta_{(n)}\hat{M}^{++}\left(\frac{x}{k},k\right)+\gamma_{(n)}\hat{M}^{++}\left(\frac{x}{k},k\right)\log\left[\frac{\eta(k\eta-x)H^4}{4k\mu^2}\right]\right]\frac{1-i(k\eta-x)}{(k\eta-x)^4}e^{-ix}. \tag{123}$$

We then have to solve the following integrals ($\eta,\eta''<0,k>0$):

$$I_{\tilde{R}}(\eta,\eta'',k)\equiv-\frac{1}{\lambda H^2}\int_{\eta''}^{\eta}d\tau(\tau H)^{-4}\mathrm{Re}[\tilde{M}^{++}_{I}+\tilde{M}^{++}_{II}](\eta,\tau,k)\Delta^{c}_{\varphi,dS}(\tau,\eta'',k), \tag{124}$$

$$I_{\hat{R}}(\eta,\eta'',k)\equiv-\frac{1}{\lambda H^2}\int_{\eta''}^{\eta}d\tau(\tau H)^{-4}\mathrm{Re}\hat{M}(\eta,\tau,k)\Delta^{c}_{\varphi,dS}(\tau,\eta'',k), \tag{125}$$

$$I_{R_{\log}}(\eta,\eta'',k)\equiv-\frac{1}{\lambda H^2}\int_{\eta''}^{\eta}d\tau(\tau H)^{-4}\mathrm{Re}\hat{M}(\eta,\tau,k)\log\left[\frac{\eta\tau H^4}{4\mu^2}\right]\Delta^{c}_{\varphi,dS}(\tau,\eta'',k), \tag{126}$$

$$I_{\tilde{M}}(\eta,k)\equiv-\frac{e^{ik\eta}}{2\lambda H^4}\int_0^\infty dx\left[\tilde{M}^{++}_{I}\left(\frac{x}{k},k\right)+\tilde{M}^{++}_{II}\left(\frac{x}{k},k\right)\right]\frac{1-i(k\eta-x)}{(k\eta-x)^4}e^{-ix}, \tag{127}$$

$$I_{\hat{M}}(\eta,k)\equiv-\frac{e^{ik\eta}}{2\lambda H^4}\int_0^\infty dx\hat{M}^{++}\left(\frac{x}{k},k\right)\frac{1-i(k\eta-x)}{(k\eta-x)^4}e^{-ix}, \tag{128}$$

$$I_{M_{\log}}(\eta,k)\equiv-\frac{e^{ik\eta}}{2\lambda H^4}\int_0^\infty dx\log\left[\frac{\eta(k\eta-x)H^4}{4k\mu^2}\right]\hat{M}^{++}\left(\frac{x}{k},k\right)\frac{1-i(k\eta-x)}{(k\eta-x)^4}e^{-ix}. \tag{129}$$

We are able to solve all integrals except for the first one in terms of the finite sums of exponentials, exponential integrals, and generalized hypergeometric functions. However, for the integral $I_{\tilde{R}}$ we have to define the function

$$\mathcal{J}(\eta,\eta'',k)\equiv\int_0^1 dxE_1[-2ik(x(\eta-\eta'')+\eta'')]\frac{1-e^{-2ik(\eta-\eta'')(x-1)}}{x-1}. \tag{130}$$

We note that (130) approaches a constant in the super-Hubble limit. We solve the I_R integrals in Appendix E and the I_M integrals in Appendix F. We then have

$$\mathcal{B}_{(n)}(\eta,\eta'',k)=\lambda\alpha_{(n)}[2\mathrm{Im}((1+ik\eta'')e^{-ik\eta''}I_{\tilde{M}}(\eta,k))+I_{\tilde{R}}(\eta,\eta'',k)]$$
$$+\lambda\beta_{(n)}[2\mathrm{Im}((1+ik\eta'')e^{-ik\eta''}I_{\hat{M}}(\eta,k))+I_{\hat{R}}(\eta,\eta'',k)]$$
$$+\lambda\gamma_{(n)}[2\mathrm{Im}((1+ik\eta'')e^{-ik\eta''}I_{M_{\log}}(\eta,k))+I_{R_{\log}}(\eta,\eta'',k)], \tag{131}$$

where the coefficients are given in (93). If we now act with the de Sitter d'Alembertian on $\mathcal{B}_{(2)}$ we have

$$\frac{\Box_k}{H^2}\mathcal{B}_{(2)}(\eta,\eta'',k)=2\lambda\Big\{\cos[k(\eta-\eta'')]-k(\eta-\eta'')\sin[k(\eta-\eta'')]$$
$$+(\cos[k(\eta-\eta'')]+k(\eta-\eta'')\sin[k(\eta-\eta'')])\left(\mathrm{ci}(2k|\eta-\eta''|)+\gamma_E-\log\left[\frac{2\mu^2|\eta-\eta''|}{H^2k\eta\eta''}\right]\right)$$
$$+\mathrm{sign}(\eta-\eta'')[\pi k(\eta-\eta'')\cos[k(\eta-\eta'')]-\frac{1}{2}\sin[k(\eta-\eta'')]$$
$$-(k(\eta-\eta'')\cos[k(\eta-\eta'')]-\sin[k(\eta-\eta'')])\mathrm{si}(2k|\eta-\eta''|)]\Big\}$$
$$\to 2\lambda\left(2\gamma_E-1+\log\left[\frac{H^2k^2\eta\eta''}{\mu^2}\right]\right), \tag{132}$$

where we made use of

$$E_1[2ik(\eta-\eta'')] = -\mathrm{ci}(2k|\eta-\eta''|) + i\,\mathrm{sign}(\eta-\eta')\mathrm{si}(2k|\eta-\eta''|) - i\frac{\pi}{2}\mathrm{sign}(\eta-\eta'). \tag{133}$$

The expression for $\mathcal{B}_{(1)}$ is unfortunately much lengthier which is why we give here only the super-Hubble limit:

$$\begin{aligned}\mathcal{B}_{(1)}(\eta,\eta'',k) \to \frac{2}{3}\lambda\Bigg[&\log^2(-2k\eta) + \log^2(-2k\eta'') + 2\log(-2k\eta)\log(-2k\eta'')\\&+\frac{4}{3}\log[4k^2\eta\eta'']\left(3\log\left[\frac{2\mu}{H}\right] + 3\gamma_E - 4\right)\\&+\frac{17}{4} - \frac{32}{3}\gamma_E + 4\gamma_E^2 + \frac{\pi^2}{3} + 2(4\gamma_E - 5)\log\left[\frac{2\mu}{H}\right]\Bigg].\end{aligned} \tag{134}$$

We see that the above expressions are already symmetric and we will not need a homogeneous solution for symmetrizing them. Finally, we turn to the integral that involves the Green's function,

$$\mathcal{G}(\eta,\eta'',k) \equiv H^2\int_{-\infty}^{\infty} d\tau \frac{G_{\mathrm{ret}}(\eta,\tau,k)}{(\tau H)^4}[\mathcal{B}_{(0)}(\tau,\eta'',k) + \mathcal{B}_{(0)}^{\log}(\tau,\eta'',k)] + F_{\mathrm{hom.}}(\eta,\eta'',k). \tag{135}$$

We realize that the integral boundary at negative infinity will lead to logarithmic divergences, which is why we add a homogeneous solution to cancel them:

$$\mathcal{G}(\eta,\eta'',k) = H^2\int_{\eta''}^{\infty} d\tau \frac{G_{\mathrm{ret}}(\eta,\tau,k)}{(\tau H)^4}[\mathcal{B}_{(0)}(\tau,\eta'',k) + \mathcal{B}_{(0)}^{\log}(\tau,\eta'',k)] + \tilde{F}_{\mathrm{hom.}}(\eta,\eta'',k). \tag{136}$$

We computed (136) in terms of finite sums of exponentials, exponential integrals, and generalized hypergeometric functions, as well as an additional integral which contains similar functions as (130) but is more complicated. We also find that the consistency condition (120) applies which is a highly nontrivial statement with regard to how the various terms contribute. However, since the result fills pages and includes a lot of partial integration, we decided to give only the super-Hubble limit in this paper. We note that the Green's function has the super-Hubble limit,

$$G_{\mathrm{ret}}(\eta,\eta',k) \to \theta(\eta-\eta')\frac{H^2}{k^3}\left[-\frac{1}{3}k^3(\eta-\eta')^3\right], \tag{137}$$

such that the full integral in the super-Hubble limit reduces to a rather simple expression:

$$\begin{aligned}\mathcal{G}(\eta,\eta'',k) \to -\frac{1}{3}\int_{\eta''}^{\eta} d\tau\frac{(\eta-\tau)^3}{\tau^4}\Bigg[&\frac{16}{3}\log^2(-2k\tau) + \frac{4}{3}\left(8\log\left[\frac{H}{2k}\right] - 9\right)\log(-2k\tau)\\&+\frac{4}{3}\left(8\log\left[\frac{H}{2k}\right] + 7 - 8\gamma_E\right)\log(-2k\eta'')\\&-\frac{70}{3} + 40\gamma_E - 16\gamma_E^2 - \frac{4}{9}\pi^2 + \frac{64}{3}(\gamma_E - 2)\log\left[\frac{H}{2k}\right]\Bigg] + \tilde{F}_{\mathrm{hom.}}(\eta,\eta'',k).\end{aligned} \tag{138}$$

The last step is to symmetrize the result by means of a homogeneous solution that should also include the tree-level solution for the Bunch-Davies vacuum,

$$\tilde{F}_{\mathrm{hom.}}(\eta,\eta'',k) = F_{\varphi,dS}(\eta,\eta'',k) + \lambda[h_1(\eta'') + i(k\eta'')^{-3}h_2(\eta'')](1+ik\eta)e^{-ik\eta} + \lambda[h_1(\eta'') - i(k\eta'')^{-3}h_2(\eta'')](1-ik\eta)e^{ik\eta}, \tag{139}$$

where $h_{1,2}$ are real functions that we determine perturbatively as

$$
\begin{aligned}
h_1(\eta'') \to & -\frac{733}{486} + \frac{1}{81}\left(18\gamma_E(3+2\gamma_E) + 7\pi^2 + 16(6\gamma_E - 11)\log\left[\frac{H}{2k}\right] + 216(\gamma_E - 2)\log\left[\frac{2\mu}{H}\right]\right) \\
& + \frac{2}{81}\log(-2k\eta'')\left(355 - 12\gamma_E(47+18\gamma_E) + 6\pi^2 - 288(\gamma_E - 2)\log\left[\frac{H}{2k}\right]\right) \\
& + \frac{4}{27}\log^2(-2k\eta'')\left(1 + 12\gamma_E - 24\log\left[\frac{H}{2k}\right]\right) - \frac{16}{27}\log^3(-2k\eta''),
\end{aligned} \tag{140}
$$

$$
\begin{aligned}
h_2(\eta'') \to & -\frac{353}{81} + \frac{2}{27}\left(18\gamma_E(2\gamma_E - 5) + \pi^2 - 8(11 - 6\gamma_E)\log\left[\frac{H}{2k}\right]\right) \\
& - \frac{4}{27}\log(-2k\eta'')\left(1 - 12\gamma_E + 24\log\left[\frac{H}{2k}\right]\right) - \frac{8}{9}\log^2(-2k\eta'').
\end{aligned} \tag{141}
$$

Adding up all contributions for the statistical two-point function,

$$
\begin{aligned}
F_\varphi(\eta,\eta'',k) = & F_{\varphi,dS}(\eta,\eta'',k) + \lambda\left(\frac{\square_k}{H^2}\mathcal{B}_{(2)}(\eta,\eta'',k) + \mathcal{B}_{(1)}(\eta,\eta'',k) + \mathcal{G}(\eta,\eta'',k)\right. \\
& \left. + [h_1(\eta'') + i(k\eta'')^{-3}h_2(\eta'')](1+ik\eta)e^{-ik\eta} + [h_1(\eta'') - i(k\eta'')^{-3}h_2(\eta'')](1-ik\eta)e^{ik\eta}\right),
\end{aligned} \tag{142}
$$

yields expression (30) in the super-Hubble limit.

IV. CONCLUSION AND OUTLOOK

In the literature on cosmological perturbations, their properties are often specified solely in terms of an equal time two-point function of the comoving curvature perturbation $\mathcal{R}$. This picture is correct if the fields are Gaussian distributed and if the decaying mode on super-Hubble scales makes the (canonical) momentum perturbation $\pi_{\mathcal{R}}$ small and/or stochastically dependent on $\mathcal{R}$, such that no useful or additional information is contained in it. This rationale can be extended at the linear level to include isocurvature modes stemming from additional field perturbation in a multifield inflation scenario. On the other hand, one can discuss the self-interactions of the inflaton perturbation. There is another possibility that we discuss in this paper, namely, that the momentum of the comoving curvature perturbation can become significant at the end of inflation via quantum interactions with a spectator field.

We study this scenario for a simple two-field model of inflation (6) in which the inflaton field couples biquadratically to a light spectator scalar field. Expanding around the inflaton condensate yields a dominant cubic coupling at the level of perturbations in which the inflaton perturbation couples linearly to the spectator (cf. Fig. 1). We investigate how the spectator field affects the curvature perturbation by performing an explicit one-loop calculation with renormalized self-masses in the 2PI formalism. Quantum gravitational interactions during inflation have been addressed in [8,9] with the conclusion that, in the single field inflationary models, corrections to the curvature perturbation grow on super-Hubble scales at most with powers of logarithms of the scale factor. We confirm this observation for our model in (30).

However, the momentum correlators (37)–(38) grow as powers of the scale factor, such that they are not necessarily suppressed at the end of inflation. We calculate the Gaussian, von Neumann entropy of the curvature perturbation (47) and show that during inflation and on super-Hubble scales it grows as $\sim 6\ln(a)$. This rapid growth of the entropy indicates a rapid classicalization of the curvature perturbation on super-Hubble scales during inflation, and it is a consequence of the rapid growth [$\propto a^6$; see Eq. (49)] of the Gaussian invariant of the state (46), which in turn can be attributed to the rapid growth of momentum correlators (37)–(38). This then implies that the momentum operator of the curvature perturbation (32) should be regarded as stochastically independent from the curvature perturbation.

When this work was nearing completion, we became aware that the idea of obtaining decoherence from spectators has been addressed in [7], based on the work in [35,36]. Strictly speaking, the theory with a cubic interaction studied in [7,35,36] is unstable and not the same as the biquadratic theory we start from in Eqs. (6)–(9), which is a stable theory for a positive coupling g. However, since the two-loop diagram in Fig. 2 is suppressed, the principle source of decoherence in our theory is incidentally a diagram that is topologically the same as the diagram used in [7,35,36], provided one identifies our coupling $h = g\bar{\phi}$ with their coupling λ. Since $\bar{\phi} = \bar{\phi}(t)$, this identity is never exact and at some level the theories do differ.

We also emphasize that our approach (based on the one-loop evaluation of the inflaton two-point function) differs significantly from the reduced density matrix approach used in [7,35,36]. Furthermore, our results qualitatively differ in that we find the leading order growth of the inflaton two-point function correlator to be $\log^3(-k\eta)$, which differs by one power from the result obtained in [7,35,36]. Moreover, our result differs by a sign. Namely, we get that the two-point function increases at late times while the above mentioned references find a suppression. Since both calculational frameworks differ significantly and bear a lot of complexity, we leave it as an important task for the future to explain how this difference comes about.

Our study shows that the effects of interactions are typically *large* at the end of inflation, which can be clearly seen from Eq. (41) and is illustrated in Figs. 3–5. On the other hand, if interactions switch off rapidly after inflation, quite generically by the end of the radiation era the momentum fluctuations will decay such that their effects will be too small to leave any observable imprint in the CMB or LSS, which is corroborated by the estimate given in Eqs. (51)–(52). This conclusion holds, however, only if the inflaton-spectator interactions are switched off rapidly enough after inflation, such that the postinflationary evolution of cosmological perturbations on super-Hubble scales can be well approximated by the corresponding free, linear evolution, according to which the large curvature momentum perturbation from the end of inflation decays swiftly during radiation. One way to hinder the decay of the momentum correlators is to keep the inflaton-spectator interactions active during the early parts of the radiation era. This can be achieved, for example, by delaying the postinflationary decays of the inflaton and spectator fluctuations, and by demanding that both fields are light enough such that, for some time during radiation, they remain approximately massless, i.e., $m_\phi, m_\chi \ll H(t)$, where $H(t) \simeq 1/(2t)$ is the Hubble rate in the radiation era. We leave a detailed study of decoherence on super-Hubble scales during radiation for future work.

Broadly speaking, investigations of quantum loop corrections to cosmological perturbations in an inflationary setting, a simple example of which is performed in this work, can be used to test the consistency of various inflationary models and can be considered as complementary to effective field theory methods, which can be very useful for studying the internal consistency of inflationary models such as Higgs inflation [52,53]. Furthermore, since quantum loop corrections from light matter fields may leave observable imprints in the CMB and large scale structure, one can use the signatures imprinted in the CMB and large scale structure by the momentum correlators of cosmological perturbations as a means to study inflationary interactions, thus opening a novel observational window to inflationary physics.

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APPENDIX A: DEFINITIONS AND CONVENTIONS

We make use of the d'Alembert operator in de Sitter space-time where

$$\frac{\Box}{H^2} \equiv \frac{\bar{\Box}_{\rm dS}}{H^2} = \eta^2\left[-\partial_\eta^2 + \frac{D-2}{\eta}\partial_\eta + \delta^{ij}\partial_i\partial_j\right], \qquad (A1)$$

where the constant parameter H is the Hubble rate at the beginning of inflation and η denotes conformal time. We use the following general notation for the Wightman functions and causal and statistical propagators in the cosmological context:

$$i\Delta_\varphi^{\mp\pm}(\eta,\eta',k) = F_\varphi(\eta,\eta',k) \pm \frac{i}{2}\Delta_\varphi^c(\eta,\eta',k), \qquad (A2)$$

$$F_\varphi(\eta,\eta',k) = \int d^3(x-x')\mathrm{e}^{-i\vec{k}\cdot(\vec{x}-\vec{x}')}F_\phi(x;x'), \qquad (A3)$$

$$i\Delta_\varphi^c(\eta,\eta',k) = \int d^3(x-x')\mathrm{e}^{-i\vec{k}\cdot(\vec{x}-\vec{x}')}i\Delta_\phi^c(x;x'), \qquad (A4)$$

with

$$F_\varphi(x;x') = \frac{1}{2}\mathrm{Tr}[\hat{\rho}(\eta_0)\{\hat{\varphi}(x),\hat{\varphi}(x')\}],$$
$$i\Delta_\varphi^c(x;x') = \mathrm{Tr}[\hat{\rho}(\eta_0)[\hat{\varphi}(x'),\hat{\varphi}(x)]], \qquad (A5)$$

where $\hat{\rho}_0 \equiv \hat{\rho}(\eta_0)$ is the initial density matrix (defined at $\eta = \eta_0$). Moreover, we define the correlators

$$\Delta_{XY}(x;x') \equiv \frac{1}{2}\mathrm{Tr}[\hat{\rho}(\eta_0)\{\hat{X}(x),\hat{Y}(x')\}]. \qquad (A6)$$

We make frequent use of the following functions:

$$\mathrm{si}(z) = -\int_z^\infty \frac{\sin(t)dt}{t} = \int_0^z \frac{\sin(t)dt}{t} - \frac{\pi}{2} = \mathrm{Si}(z) - \frac{\pi}{2}, \qquad (A7)$$

$$\mathrm{ci}(z) = -\int_z^\infty \frac{\cos(t)dt}{t}, \qquad (A8)$$

$$\mathrm{E}_1(ix) = -\gamma_E - \log(ix) - \sum_{k=1}^{\infty}\frac{(-ix)^k}{kk!} = i\mathrm{si}(x) - \mathrm{ci}(x), \quad x > 0, \tag{A9}$$

$$\mathrm{Ein}(z) = \int_0^z \frac{1-e^{-t}}{t}dt = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}z^n}{n!n} = E_1(z) + \log(z) + \gamma_E, \tag{A10}$$

$${}_3F_3\left[\begin{matrix}1 & 1 & 1\\ 2 & 2 & 2\end{matrix}; z\right] = \frac{1}{z}\int_0^z \frac{\mathrm{Ein(t)}}{t}dt. \tag{A11}$$

APPENDIX B: PHOTON KINETIC EQUATION

The starting point for our recapitulation is the Boltzmann equation for the temperature perturbation of the photon fluids which takes the following form in Fourier space (we follow closely the standard literature; see, e.g., [54] or [55]):

$$\partial_\eta\Theta(k,\mu) + ik\mu[\Theta(k,\mu) + \Phi(k)] - \partial_\eta\Psi(k) = -(\partial_\eta\tau)\left[\Theta_0(k) - \Theta(k,\mu) + \mu v_b(k) - \frac{1}{2}\mathcal{P}_2(\mu)\Sigma(k)\right], \quad \mu = \frac{\vec{k}\cdot\vec{p}}{kp}. \tag{B1}$$

Here, $\Theta(k,\mu)$ is the time-dependent gauge-invariant (integrated) photon temperature perturbation that is obtained from

$$\Theta(k,\mu) = \sum_{l=0}^{\infty}(-i)^l\theta_l(\eta,k)\mathcal{P}_l(\mu) \propto \int dp p^3\delta f(\eta,\vec{p},\vec{k}), \tag{B2}$$

where δf is the perturbed, gauge-invariant photon distribution function as defined in [54], and P_l are Legendre polynomials. The Bardeen potentials Φ and Ψ (as defined in [40]) are related to temporal and spatial metric perturbations. The time-dependent variable $\tau(\eta)$ is the optical depth related to Thomson scattering with v_b the (longitudinal) baryon velocity perturbation and Σ the anisotropic stress which depends on the polarization and quadrupole moment Θ_2, both of which are usually neglected in a first approximation [54,55]. We note that the gravitational slip is given by

$$\Psi - \Phi = \frac{a^2}{M_p^2}\Sigma, \tag{B3}$$

and we can identify the two potentials once anisotropic stress is absent or neglected. Moreover, we have to consider the speed of sound c_s of the photon-baryon fluid, which is defined via

$$\delta P = c_s^2\delta\rho + \delta P_{\mathrm{nad}}, \tag{B4}$$

where δP, $\delta\rho$, and δP_{nad} are the pressure, density, and nonadiabatic pressure perturbations, respectively. The speed of sound in radiation domination is related to the background density of photons $\rho_\gamma^{(0)}$ and baryons $\rho_b^{(0)}$ via

$$c_s^2(\eta) = \frac{1}{3(1+R(\eta))}, \qquad R(\eta) \equiv \frac{3}{4}\frac{\rho_b^{(0)}(\eta)}{\rho_\gamma^{(0)}(\eta)}. \tag{B5}$$

Since the baryon density is much smaller than the photon density in the radiation dominated phase, we can take as another approximation $c_s^2 \approx 1/3$ during this time, which also determines the baryon velocity to first order through the photon dipole moment as

$$v_b = -3i\Theta_1 + \mathcal{O}(R). \tag{B6}$$

Putting it all together, one can derive a second-order differential equation for the effective temperature fluctuation $\Delta T = \Theta_0 + \Phi$ and the gravitational potentials [54,55],

$$\left[\frac{d^2}{d\eta^2} + \frac{R}{1+R}\mathcal{H}\frac{d}{d\eta} + k^2c_s^2\right]\Delta T = k^2\left[c_s^2 - \frac{1}{3}\right]\Phi + \left[\frac{d^2}{d\eta^2} + \frac{R}{1+R}\mathcal{H}\frac{d}{d\eta}\right][\Psi + \Phi]. \tag{B7}$$

We see that if we neglect the damping term by $c_s^2 \approx 1/3$, we have a forced harmonic oscillator, whose homogeneous solutions are determined by the monopole density $\Theta_0(\eta_{\mathrm{cmb}})$ and its time derivative $\Theta_0'(\eta_{\mathrm{cmb}})$ as well as the gravitational potential $\Psi(\eta_{\mathrm{cmb}})$ and its time derivative $\Psi'(\eta_{\mathrm{cmb}})$ at some time within the radiation dominated phase η_{cmb} that is close to recombination $\eta_{\mathrm{cmb}} \approx 10^{-1}\eta_{\mathrm{rec}}$,

$$\Delta T(\eta) \approx [\Theta_0 + \Phi](\eta_{\mathrm{cmb}})\cos[kr_s(\eta)] + \left[\frac{\Theta_0' + \Phi'}{kc_s}\right](\eta_{\mathrm{cmb}})\sin[kr_s(\eta)] + \frac{\sqrt{3}}{k}\int_{\eta_{\mathrm{cmb}}}^{\eta}[\Phi''(\bar{\eta}) + \Psi''(\bar{\eta})]\sin[kr_s(\eta) - kr_s(\bar{\eta})]d\bar{\eta}, \tag{B8}$$

where we defined the sound horizon,

$$r_s(\eta) = \int_{\eta_{\mathrm{cmb}}}^{\eta} c_s(\bar{\eta})d\bar{\eta}, \tag{B9}$$

and we keep the time dependence of the speed of sound only in the phases. By making use of (B1) in the super-Hubble limit, in which also the gravitational slip vanishes, we obtain $\Theta_0'(\eta_{\mathrm{cmb}}) = \Psi'(\eta_{\mathrm{cmb}}) = \Phi'(\eta_{\mathrm{cmb}})$ and the temporal integration turns out to yield $2\Theta_0(\eta_{\mathrm{cmb}}) = -\Psi(\eta_{\mathrm{cmb}}) = -\Phi(\eta_{\mathrm{cmb}})$ [54,55]. We also recall that the gravitational potential Ψ obeys in the absence of gravitational slip the following differential equation [40]:

$$\partial_\eta^2\Psi + 3(1+c_s^2)\mathcal{H}\partial_\eta\Psi + [2\partial_\eta\mathcal{H} + (1+3c_s^2)\mathcal{H}^2 + c_s^2k^2]\Psi = \frac{1}{2M_p^2}\delta P_{\rm nad}. \tag{B10}$$

As a first approximation to the inhomogeneous solution in (B8), we can solve (B10) for vanishing nonadiabatic pressure, $\delta P_{\rm nad}\to 0$, with $c_s^2\approx 1/3$ during radiation unless it will appear as a phase in conjunction with the momentum k. Thus, we write

$$\partial_\eta^2\Psi + 4\mathcal{H}\partial_\eta\Psi + c_s^2k^2\Psi \approx 0. \tag{B11}$$

We see that the solution will stay constant on super-Hubble scales or decay otherwise during radiation and we thus neglect the integral in (B8). We then have

$$\Delta T(k,\eta) \approx \frac{1}{2}\Psi_k(\eta_{\rm cmb})\cos[kr_s(\eta)] + 2\frac{\Psi_k'(\eta_{\rm cmb})}{kc_s(\eta_{\rm cmb})}\sin[kr_s(\eta)]. \tag{B12}$$

Finally, in order to make contact with the era of inflation, we would like to relate Eq. (B12) to the gauge-invariant curvature perturbation $\mathcal{R}$ in the case of vanishing (linear) nonadiabatic pressure ($\delta P_{\rm nad}=0$). First we note that the gauge-invariant curvature perturbation $\mathcal{R}$ may be expressed in terms of the gauge-invariant gravitational potential Ψ via [40]

$$\mathcal{R} \equiv \Psi + \frac{\Phi}{\epsilon} + \frac{\partial_\eta\Psi}{\mathcal{H}\epsilon} = \Psi + \frac{\Psi}{\epsilon} + \frac{\partial_\eta\Psi}{\mathcal{H}\epsilon}, \qquad \epsilon = 1 - \frac{\partial_\eta\mathcal{H}}{\mathcal{H}^2}, \tag{B13}$$

where we neglected the gravitational slip in the second equality, which is justified on super-Hubble scales.

The squared adiabatic sound speed may be expressed as

$$c_s^2 \equiv \frac{\partial_\eta\bar{P}}{\partial_\eta\bar{\rho}} = -1 + \frac{2}{3}\epsilon - \frac{\partial_\eta\epsilon}{3\mathcal{H}\epsilon},$$

where P and ρ are the background pressure and energy density, respectively. Taking a derivative of (B13), using (B11), we find

$$\partial_\eta\mathcal{R} = -\frac{c_s^2k^2}{\epsilon\mathcal{H}}\Psi.$$

Note that the latter relation holds also in an inflationary context with c_s^2 set equal to 1. We can solve for Ψ and $\partial_\eta\Psi$ in terms of $\mathcal{R}$ and $\partial_\eta\mathcal{R}$, which yields

$$\Psi = -\frac{\epsilon\mathcal{H}}{c_s^2k^2}\partial_\eta\mathcal{R} \equiv -\frac{\mathcal{H}}{2M_p^2k^2a^2}\pi_{\mathcal{R}},$$

$$\partial_\eta\Psi = \epsilon\mathcal{H}\mathcal{R} + (1+\epsilon)\frac{\mathcal{H}^2}{2M_p^2k^2a^2}\pi_{\mathcal{R}}, \tag{B14}$$

where we defined the canonical momentum $\pi_{\mathcal{R}}$ associated to $\mathcal{R}$ as in Appendix C. We now would like to evolve the gravitational potential on super-Hubble scales from the end of inflation deep into the radiation era by using linear relations. Therefore, we make use of Weinberg's theorem [56], according to which there are always two solutions for the gravitational potential on super-Hubble scales which take the following form:

$$\Psi_{\rm ad}(\eta) = -\left[\frac{1}{2M_p^2k^2}\pi_{\mathcal{R}}(\eta_e) + \frac{a^2(\eta_e)}{\mathcal{H}(\eta_e)}\mathcal{R}(\eta_e)\right]\frac{\mathcal{H}(\eta)}{a^2(\eta)} + \mathcal{R}(\eta_e)\left[1 - \frac{\mathcal{H}(\eta)}{a^2(\eta)}\int_{\eta_e}^{\eta} a^2(\bar\eta)d\bar\eta\right], \tag{B15}$$

where the time η_e signals some time shortly before the end of inflation such that we still have that $\epsilon(\eta_e)\ll 1$. In order to set initial conditions for the CMB spectrum at time $\eta_{\rm cmb}\approx 10^{-1}\eta_{\rm rec}$ close to recombination, we stick to a simplified scenario in which we neglect small contributions due to the transition from inflation to radiation and keep only leading order terms in each variable,

$$\Psi_{\rm ad}(\eta_{\rm cmb}) \approx -\frac{1}{2M_p^2k^2}\frac{Ha^2(\eta_e)}{a^3(\eta_{\rm cmb})}\pi_{\mathcal{R}}(\eta_e) + \frac{2}{3}\mathcal{R}(\eta_e), \tag{B16}$$

$$\Psi'_{\rm ad}(\eta_{\rm cmb}) \approx 3\frac{H^2a^3(\eta_e)}{a^4(\eta_{\rm cmb})}\left[\frac{1}{2M_p^2k^2}\pi_{\mathcal{R}}(\eta_e) + \frac{a^2(\eta_e)}{H}\mathcal{R}(\eta_e)\right]. \tag{B17}$$

Inserting the above super-Hubble initial conditions into the approximate solution for the effective CMB temperature perturbation (B12) and making the stochastic character of the involved operators manifest, we have

$$\Delta\hat{T}(\eta,\vec{k}) \approx \frac{1}{2}\left[\frac{2}{3}\hat{\mathcal{R}}(\eta_e,\vec{k}) - \frac{a^3(\eta_e)}{a^3(\eta_{\rm cmb})}\frac{H}{2M_p^2k^2a(\eta_e)}\hat\pi_{\mathcal{R}}(\eta_e,\vec{k})\right]\cos[kr_s(\eta)] + \frac{6H}{kc_s(\eta_{\rm cmb})}\frac{a^4(\eta_e)}{a^4(\eta_{\rm cmb})}\left[\frac{H}{2M_p^2k^2}\hat\pi_{\mathcal{R}}(\eta_e,\vec{k}) + a(\eta_e)\hat{\mathcal{R}}(\eta_e,\vec{k})\right]\sin[kr_s(\eta)]. \tag{B18}$$

This relation is used in Eqs. (50)–(52) of Sec. II to estimate the size of the photon temperature fluctuations induced by the enhanced inflationary momentum perturbation.

APPENDIX C: LINEAR EVOLUTION OF CURVATURE PERTURBATION

The gauge-invariant curvature perturbation can be defined in terms of the metric perturbation ψ and the perturbation of the velocity potential φ_v [57] (in single-field inflationary models, φ_v reduces to the inflaton field perturbation) as

$$\mathcal{R} = \psi + \frac{H}{\sqrt{\rho + P}}\varphi_v, \tag{C1}$$

where $\psi = -\mathrm{Tr}[\delta g_{ij}]/(6a^2)$, and ρ and P are the background fluid density and pressure [in inflation $\rho + P \to (\dot{\phi})^2$, where $\phi(t) \equiv \langle\hat{\phi}(x)\rangle$ is the inflaton expectation value]. Let us solve for the curvature perturbation $\mathcal{R}$ in postinflationary epochs. The quadratic (reparametrization-invariant) action for $\mathcal{R}$ reads (see, e.g., [38,57])

$$S[\mathcal{R}] = (2M_p^2)\int d^3x dt \bar{N}(t) a^3\epsilon\left(\frac{1}{2c_s^2}\dot{\mathcal{R}}^2 - \frac{1}{2a^2}(\partial_i\mathcal{R})^2\right), \tag{C2}$$

where $\bar{N} = \bar{N}(t)$ is the lapse function of the ADM decomposition (defined on a global equal time hypersurface Σ_t),

$$\epsilon(t) = -\frac{\dot{H}}{H^2} \tag{C3}$$

is the principal slow-roll parameter, and $\dot{X}(t) \equiv \bar{N}^{-1}\partial/\partial t$ is the time derivative invariant under time reparametrizations. In inflation $\epsilon \ll 1$, in radiation $\epsilon = 2$, and in the matter era $\epsilon = 3/2$. From (C2) one easily finds the canonical momentum of $\mathcal{R}$,

$$\pi_{\mathcal{R}}(t,\vec{x}) \equiv \frac{\delta S}{\delta\partial_t\mathcal{R}(t,\vec{x})} = \frac{2M_p^2 a^3\epsilon}{\bar{N}c_s^2}\partial_t\mathcal{R}, \tag{C4}$$

and the Hamiltonian,

$$H(t) = \int d^3x\left(\frac{\bar{N}c_s^2}{4M_p^2 a^3\epsilon}\pi_{\mathcal{R}}^2 + M_p^2\bar{N}a\epsilon(\partial_i\mathcal{R})^2\right). \tag{C5}$$

From (C5) one easily arrives at the Heisenberg equations,

$$\partial_t\hat{\mathcal{R}} = \frac{\bar{N}c_s^2}{2M_p^2 a^3\epsilon}\hat{\pi}_{\mathcal{R}}, \qquad \partial_t\hat{\pi}_{\mathcal{R}} = 2M_p^2\bar{N}a\epsilon\partial_i^2\hat{\mathcal{R}}, \tag{C6}$$

where $\hat{\mathcal{R}}$ and $\hat{\pi}_{\mathcal{R}}$ are the canonical pair obeying

$$[\hat{\mathcal{R}}(t,\vec{x}), \hat{\pi}_{\mathcal{R}}(t,\vec{x}')] = i\hbar\delta^3(\vec{x} - \vec{x}'). \tag{C7}$$

One can solve (C6) in space-times of constant ϵ as follows. Let us introduce a time, $ad\eta = \bar{N}dt$ (notice that time η reduces to the usual conformal time in the gauge, $\bar{N} = a$), and (C6) reduces to

$$\partial_\eta[a^2\partial_\eta\hat{\mathcal{R}}] - a^2c_s^2\nabla^2\hat{\mathcal{R}} = 0, \tag{C8}$$

where we made use of $\dot{\epsilon} = 0$ and $\dot{c}_s = 0$. Since we are primarily interested in the spectra, it is convenient to perform the following mode decomposition:

$$\hat{\mathcal{R}}(\eta,\vec{x}) = \int\frac{d^3k}{(2\pi)^3}(e^{i\vec{k}\cdot\vec{x}}\mathcal{R}(\eta,k)\hat{a}(\vec{k}) + e^{-i\vec{k}\cdot\vec{x}}\mathcal{R}^*(\eta,k)\hat{a}^+(\vec{k})) \equiv \int\frac{d^3k}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{x}}\hat{\mathcal{R}}(\eta,\vec{k})$$
$$\hat{\pi}_{\mathcal{R}}(\eta,\vec{x}) = \int\frac{d^3k}{(2\pi)^3}(e^{i\vec{k}\cdot\vec{x}}\pi_{\mathcal{R}}(\eta,k)\hat{a}(\vec{k}) + e^{-i\vec{k}\cdot\vec{x}}\pi_{\mathcal{R}}^*(\eta,k)\hat{a}^+(\vec{k})) \equiv \int\frac{d^3k}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{x}}\hat{\pi}_{\mathcal{R}}(\eta,\vec{k}), \tag{C9}$$

where

$$[\hat{a}(\vec{k}), \hat{a}^+(\vec{k}')] = (2\pi)^3\delta^3(\vec{k} - \vec{k}'),$$
$$\mathcal{R}(\eta,k)\pi_{\mathcal{R}}^*(\eta,k) - \mathcal{R}^*(\eta,k)\pi_{\mathcal{R}}(\eta,k) = i. \tag{C10}$$

The equation of motion for the modes $\mathcal{R}(\eta,k)$ then becomes

$$\left[\frac{d^2}{d\eta^2} + c_s^2k^2 - (\mathcal{H}^2 + \partial_\eta\mathcal{H})\right](a\mathcal{R}) = 0, \tag{C11}$$

where $\mathcal{H} = \partial_\eta\ln(a) = aH$ is the conformal Hubble rate. For inflation we have $c_s^2 = 1$ and set to leading order in the slow-roll parameters $a(\eta) = -H\eta^{-1}$ ($H \approx$ const). Thus, the two fundamental solutions in inflation are to leading order given by

$$\frac{1}{\sqrt{2\epsilon}M_P}\frac{H}{\sqrt{2k^3}}(1 \mp ik\eta)e^{\pm ik\eta}, \tag{C12}$$

such that

$$\hat{\mathcal{R}}(\eta,\vec{k}) = \frac{1}{\sqrt{2\epsilon}M_P}\frac{H}{\sqrt{2k^3}} \times[(1 + ik\eta)e^{-ik\eta}\hat{a}(-\vec{k}) + (1 - ik\eta)e^{ik\eta}\hat{a}^+(\vec{k})], \tag{C13}$$

$$\hat{\pi}_{\mathcal{R}}(\eta,\vec{k}) = \frac{1}{\sqrt{2\epsilon}M_P}\frac{H}{\sqrt{2k^3}}2M_p^2a^2\epsilon k^2\eta \times [e^{-ik\eta}\hat{a}(-\vec{k}) + e^{ik\eta}\hat{a}^+(\vec{k})]. \quad \text{(C14)}$$

We now restrict the d.o.f. of (let us say) a Gaussian state associated to $\mathcal{R}$ and $\pi_{\mathcal{R}}$ to the Bunch-Davies vacuum with $\hat{a}(\vec{k})|0\rangle = 0$ by picking up only the commutator in any two-point function. However, the dynamics of single-field inflation on super-Hubble scales within the standard linear treatment reduces the effective d.o.f. of the Gaussian state in any case to only one stochastic variable. In other words, if we look on super-Hubble scales $|k\eta| \ll 1$, we find that $\hat{\mathcal{R}}$ and $\hat{\pi}_{\mathcal{R}}$ are effectively no longer independent operators,

$$\hat{\mathcal{R}}(\eta,\vec{k}) \to \frac{1}{\sqrt{2\epsilon}M_P}\frac{H}{\sqrt{2k^3}}[\hat{a}(-\vec{k}) + \hat{a}^+(\vec{k}) + \mathcal{O}(k\eta)], \quad \text{(C15)}$$

$$\hat{\pi}_{\mathcal{R}}(\eta,\vec{k}) \to \frac{1}{\sqrt{2\epsilon}M_P}\frac{H}{\sqrt{2k^3}}2M_p^2a^2\epsilon k^2\eta \times [\hat{a}(-\vec{k}) + \hat{a}^+(\vec{k}) + \mathcal{O}(k\eta)] = -\frac{2\epsilon M_p^2a^2}{\mathcal{H}}k^2[\hat{\mathcal{R}}(\eta,\vec{k}) + \mathcal{O}(k\eta)]. \quad \text{(C16)}$$

In conclusion, we would like to emphasize that $\hat{\mathcal{R}}$ and $\hat{\pi}_{\mathcal{R}}$ are for every $\vec{k}$ *a priori* independent and it is either the choice of state or the dynamics that could effectively cease this independence.

APPENDIX D: WIGNER TRANSFORM OF LOGARITHMS

In this appendix we show that

$$\begin{aligned}
&\int d^3(x-x')e^{-i\vec{k}\cdot(\vec{x}-\vec{x}')}\left[\frac{1}{2}\log^2\left(\frac{y}{4}\right) + f(\eta,\eta')\log\left(\frac{y}{4}\right)\right] \\
&= -\frac{4\pi^2}{k^3}\left[2 + [1+ik|\Delta\eta|]\left(\log\left[\frac{aa'H^2|\Delta\eta|}{2k}\right] + i\frac{\pi}{2} - \gamma_E + f(\eta,\eta')\right)\right]e^{-ik|\Delta\eta|} \\
&\quad + \frac{4\pi^2}{k^3}(1-ik|\Delta\eta|)[\mathrm{ci}[2k|\Delta\eta|] - i\mathrm{si}[2k|\Delta\eta|]]e^{+ik|\Delta\eta|},
\end{aligned} \quad \text{(D1)}$$

where $\Delta\eta = \eta - \eta'$ and $f(\eta,\eta')$ is some k-independent function. We need integrals of the following type:

$$\mathcal{I}_n(x) \equiv x^2\int_0^\infty dzz\sin[xz]\log^n(|1-z^2|) \quad \text{(D2)}$$

$$= x^2\left[\frac{d^n}{db^n}\int_0^\infty dzz\sin[xz]|1-z^2|^b\right]_{b=0}. \quad \text{(D3)}$$

By using

$$\int_0^\infty dzz\sin[xz]|1-z^2|^b = \frac{\sqrt{\pi}}{2}\left(\frac{2}{x}\right)^{b+\frac{1}{2}}\Gamma[b+1] \times [J_{b+\frac{3}{2}}(x) + Y_{-b-\frac{3}{2}}(x)], \quad x>0, -1<b<0, \quad \text{(D4)}$$

with J_n, Y_m being the Bessel functions of the first and second kind, by analytically extending we find

$$\mathcal{I}_1(x) = -\pi[\cos(x) + x\sin(x)] \quad \text{(D5)}$$

and

$$\mathcal{I}_2(x) = 2\pi\left[-2\cos(x) + [\cos(x) + x\sin(x)] \times \left[\mathrm{ci}(2x) + \gamma_E - \log\left(\frac{2}{x}\right)\right] + [\sin(x) - x\cos(x)]\mathrm{si}(2x)\right], \quad \text{(D6)}$$

where we used

$$\mathrm{ci}(x) = -\int_x^\infty \frac{\cos(y)}{y}dy, \qquad \mathrm{si}(x) = -\int_x^\infty \frac{\sin(y)}{y}dy. \quad \text{(D7)}$$

Remembering that

$$\log(\Delta x_{++}^2) = \log(|\Delta\eta^2 - |\vec{x}-\vec{x}'|^2|) + i\pi\theta(\Delta\eta^2 - |\vec{x}-\vec{x}'|^2), \quad \text{(D8)}$$

we get

$$
\begin{aligned}
\int d^3(x-x')e^{-i\vec{k}\cdot(\vec{x}-\vec{x}')}\log\left(\frac{y}{4}\right) &= \frac{4\pi}{k}\int_0^\infty dr r\sin(kr)\log\left(\frac{y}{4}\right) \\
&= \frac{4\pi}{k}\int_0^\infty dr r\sin(kr)\log(|1-r^2\Delta\eta^{-2}|) + i\frac{4\pi^2}{k}\int_0^\infty dr r\sin(kr)\theta(\Delta\eta^2-r^2) + \text{hom}. \\
&= \frac{4\pi}{k^3}[k\Delta\eta]^2\int_0^\infty dz z\sin[k|\Delta\eta|z]\log(|z^2-1|) + i\frac{4\pi^2}{k}\int_0^{|\Delta\eta|} dr r\sin(kr) + \text{hom}. \\
&= -\frac{4\pi^2}{k^3}[1+ik|\Delta\eta|]e^{-ik|\Delta\eta|} + \text{hom}.
\end{aligned} \tag{D9}
$$

Moreover, we calculate

$$
\begin{aligned}
\int d^3(x-x')e^{-i\vec{k}\cdot(\vec{x}-\vec{x}')}\log^2\left(\frac{y}{4}\right) &= \frac{4\pi}{k}\int_0^\infty dr r\sin(kr)\log^2(|1-r^2\Delta\eta^{-2}|) \\
&\quad + \frac{8\pi}{k}\log\left(\frac{aa'H^2\Delta\eta^2}{4}\right)\int_0^\infty dr r\sin(kr)\log(|1-r^2\Delta\eta^{-2}|) \\
&\quad + i\frac{8\pi^2}{k}\int_0^\infty dr r\sin(kr)\log(|1-r^2\Delta\eta^{-2}|)\theta(\Delta\eta^2-r^2) - \frac{4\pi^3}{k}\int_0^\infty dr r\sin(kr)\theta(\Delta\eta^2-r^2) \\
&\quad + i\frac{8\pi^2}{k}\log\left(\frac{aa'H^2\Delta\eta^2}{4}\right)\int_0^\infty dr r\sin(kr)\theta(\Delta\eta^2-r^2) + \text{hom}. \\
&= \frac{4\pi}{k^3}[k\Delta\eta]^2\int_0^\infty dz z\sin[k|\Delta\eta|]\log^2(|1-z^2|) \\
&\quad + \frac{8\pi}{k^3}\log\left(\frac{aa'H^2\Delta\eta^2}{4}\right)[k\Delta\eta]^2\int_0^\infty dz z\sin[k|\Delta\eta|z]\log(|1-z^2|) \\
&\quad + i\frac{8\pi^2}{k^3}[k\Delta\eta]^2\int_0^1 dz z\sin[k|\Delta\eta|z]\log(|1-z^2|) \\
&\quad - \frac{4\pi^2}{k}\left[\pi - 2i\log\left(\frac{aa'H^2\Delta\eta^2}{4}\right)\right]\int_0^{|\Delta\eta|} dr r\sin(kr) + \text{hom}. \\
&= \frac{8\pi^2}{k^3}[-2\cos(k|\Delta\eta|) + [\cos(k|\Delta\eta|) + k|\Delta\eta|\sin(k|\Delta\eta|)]\left[\text{ci}(2k|\Delta\eta|) + \gamma_E - \log\left(\frac{2}{k|\Delta\eta|}\right)\right] \\
&\quad + [\sin(k|\Delta\eta|) - k|\Delta\eta|\cos(k|\Delta\eta|)]\text{si}(2k|\Delta\eta|)] \\
&\quad - \frac{8\pi^2}{k^3}\log\left(\frac{aa'H^2\Delta\eta^2}{4}\right)[\cos(k|\Delta\eta|) + k|\Delta\eta|\sin(k|\Delta\eta|)] \\
&\quad + i\frac{8\pi^2}{k^3}\left[2\sin(k|\Delta\eta|) + [\sin(k|\Delta\eta|) - k|\Delta\eta|\cos(k|\Delta\eta|)]\left[\text{ci}(2k|\Delta\eta|) - \gamma_E + \log\left(\frac{2}{k|\Delta\eta|}\right)\right]\right. \\
&\quad \left. - [\cos(k|\Delta\eta|) + k|\Delta\eta|\sin(k|\Delta\eta|)]\left[\text{si}(2k|\Delta\eta|) + \frac{\pi}{2}\right]\right] \\
&\quad - \frac{4\pi^2}{k^3}\left[\pi - 2i\log\left(\frac{aa'H^2\Delta\eta^2}{4}\right)\right][\sin[k|\Delta\eta|] - k|\Delta\eta|\cos[k|\Delta\eta|]] + \text{hom}. \\
&= -\frac{8\pi^2}{k^3}\left[2 + [1+ik|\Delta\eta|]\left(\log\left[\frac{aa'H^2|\Delta\eta|}{2k}\right] + i\frac{\pi}{2} - \gamma_E\right)\right]e^{-ik|\Delta\eta|} \\
&\quad + \frac{8\pi^2}{k^3}(1-ik|\Delta\eta|)[\text{ci}[2k|\Delta\eta|] - i\,\text{si}[2k|\Delta\eta|]]e^{+ik|\Delta\eta|},
\end{aligned} \tag{D10}
$$

where $\gamma_E \approx 0.57721$ is Euler's constant. We combine the results (D9) and (D10) in order to get (D1).

APPENDIX E: I_R INTEGRALS

In this appendix we calculate the integrals (124), (125), and (126). Let us start with

$$\begin{aligned} I_{\tilde{R}}(\eta,\eta'',k) &= -\lambda^{-1}H^{-2}\mathrm{sign}(\eta,\eta'')\int_{\mathcal{C}(\eta,\eta'')} d\tau(\tau H)^{-4}\mathrm{Re}[\tilde{M}_I^{++}+\tilde{M}_{II}^{++}](\eta,\tau,k)\Delta^c_{\phi,BD}(\tau,\eta'',k) \\ &= \mathrm{Im}\{[(1+ik\eta'')e^{-ik\eta''}][e^{ik\eta}\tilde{\mathcal{R}}_1(\eta,\eta'',k)+e^{-ik\eta}\tilde{\mathcal{R}}_2(\eta,\eta'',k)]\}, \end{aligned} \tag{E1}$$

where

$$\begin{aligned} \tilde{I}_{R_1}(\eta,\eta'',k) \equiv \frac{1}{k^3}\int_\eta^{\eta''} d\tau\Bigg[2+[1-ik(\eta-\tau)]\Bigg(\log\left[\frac{iH^2(\eta-\tau)}{2k}\right]-i\pi\mathrm{sign}(\eta-\eta'') \\ -\gamma_E+E_1[2ik(\eta-\tau)]\Bigg)\Bigg]\frac{1-ik\tau}{\tau^4}, \end{aligned} \tag{E2}$$

$$\begin{aligned} \tilde{I}_{R_2}(\eta,\eta'',k) \equiv \frac{1}{k^3}\int_\eta^{\eta''} d\tau\Bigg[2+[1+ik(\eta-\tau)]\Bigg(\log\left[-i\frac{H^2(\eta-\tau)}{2k}\right]+i\pi\mathrm{sign}(\eta-\eta'') \\ -\gamma_E+E_1[-2ik(\eta-\tau)]\Bigg)\Bigg]e^{2ik\tau}\frac{1-ik\tau}{\tau^4}. \end{aligned} \tag{E3}$$

We were able to drop the absolute value signs in the above expressions since the function that effectively appears has no branch cut as one might expect naively due to the logarithm and the exponential integral. The branch cut is exactly canceled and we are dealing with the entire function $\mathrm{Ein}(z)$, the complementary exponential integral,

$$\mathrm{Ein}(z) = \int_0^z \frac{1-e^{-t}}{t}dt = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}z^n}{n!n} = E_1(z)+\log(z)+\gamma_E, \tag{E4}$$

converging for all finite values of $|z|$. We define

$$\mathcal{J}(\eta,\eta'',k) \equiv \int_0^1 dx E_1[-2ik(x(\eta-\eta'')+\eta'')]\frac{1-e^{-2ik(\eta-\eta'')(x-1)}}{x-1}, \tag{E5}$$

and have the following result:

$$\begin{aligned} I_{\tilde{R}}(\eta,\eta'',k) = \mathrm{Im}\Bigg[(1+ik\eta'')e^{-ik\eta''}\Bigg\{e^{ik\eta}\Bigg[&-\frac{4}{3k^3\eta^3}+\frac{4i}{3k^2\eta^2}-\frac{4}{3k\eta}+\frac{2}{3k^3(\eta'')^3}-\frac{2i}{3k^2(\eta'')^2}+\frac{2}{3k\eta''} \\ &+\left(\frac{i}{3}+\frac{1}{3k^3(\eta'')^3}-\frac{ik\eta}{3k^3(\eta'')^3}-\frac{k\eta}{2k^2(\eta'')^2}+\frac{1}{k\eta''}\right)\left(E_1[2ik(\eta-\eta'')]+\log\left[\frac{iH^2(\eta-\eta'')}{2k}\right]-\gamma_E-i\pi\mathrm{sign}(\eta-\eta'')\right) \\ &-\left(\frac{2}{3k^3\eta^3}-\frac{2i}{3k^2\eta^2}+\frac{1}{k\eta}+\frac{2i}{3}\right)\left(\log\left[\frac{H^2}{4k^2}\right]-2\gamma_E\right)+\frac{i}{3}\log\left[\frac{\eta}{\eta''}\right]\Bigg] \\ &+e^{-ik\eta}\Bigg[E_1[-2ik\eta'']\left(i+\frac{2}{3}(i-k\eta)\left[E_1[-2ik(\eta-\eta'')]+\log\left[\frac{H^2(\eta-\eta'')}{2ik}\right]+i\pi\mathrm{sign}(\eta-\eta'')-\gamma_E\right]\right) \\ &-E_1(-2ik\eta)\left(i+\frac{2}{3}[i-k\eta]\left[\log\left[\frac{H^2}{4k^2}\right]+i\pi\mathrm{sign}(\eta-\eta'')-2\gamma_E\right]\right)\Bigg] \\ &-e^{-ik\eta+2ik\eta''}\Bigg[\frac{2}{3k^3(\eta'')^3}-\frac{2i}{3k^2(\eta'')^2}+\frac{2}{3k\eta''} \\ &+\left(\frac{1}{3k^3(\eta'')^3}+\frac{ik\eta}{3k^3(\eta'')^3}-\frac{2i}{3k^2(\eta'')^2}+\frac{k\eta}{6k^2(\eta'')^2}+\frac{1}{3k\eta''}+\frac{ik\eta}{3k\eta''}\right) \\ &\times\left(E_1[-2ik(\eta-\eta'')]+\log\left[\frac{H^2(\eta-\eta'')}{2ik}\right]+i\pi\mathrm{sign}(\eta-\eta'')-\gamma_E\right)\Bigg] \\ &+\frac{2}{3}i(1+ik\eta)e^{-ik\eta}\mathcal{J}(\eta,\eta'',k)\Bigg\}\Bigg]. \end{aligned} \tag{E6}$$

On super-Hubble scales this simplifies to

$$I_{\tilde{R}}(\eta,\eta'',k)\to -\frac{4}{3}\left(\log\left[\frac{\eta}{\eta''}\right]-\frac{1}{3}+\frac{1}{3}\frac{(\eta'')^3}{\eta^3}\right)\left(\gamma_E-1-\log\left[\frac{H}{2k}\right]\right). \tag{E7}$$

The next integral we calculate is

$$\begin{aligned}
I_{\hat{R}}(\eta,\eta'',k) &= -\lambda^{-1}H^{-2}\int_{\eta''}^{\eta}d\tau(\tau H)^{-4}[\mathrm{Re}\hat{M}(\eta,\tau,k)]\Delta^c_{\phi,BD}(\tau,\eta'',k)\\
&= -\lambda^{-1}H^{-2}\frac{(2\pi)^3H^2h^2}{2k^6(4\pi)^5}\mathrm{Im}\bigg\{(1+ik\eta'')e^{-ik\eta''}\int_{\eta}^{\eta''}d\tau\tau^{-4}[[1+ik(\eta-\tau)]e^{ik(\tau-\eta)}\\
&\quad+[1-ik(\eta-\tau)]e^{-ik(\tau-\eta)}][(1-ik\tau)e^{ik\tau})]\bigg\}\\
&= -\mathrm{Im}\bigg[(1+ik\eta'')e^{-ik\eta''}\bigg\{-\frac{2}{3}(k\eta-i)e^{-ik\eta}E_1[-2ik\eta]\\
&\quad\times e^{ik\eta}\left(\frac{i}{3}+\frac{2}{3k^3\eta^3}-\frac{2i}{3k^2\eta^2}+\frac{1}{k\eta}-\frac{1}{3k^3(\eta'')^3}+\frac{ik\eta}{3k^3(\eta'')^3}+\frac{k\eta}{2k^2(\eta'')^2}-\frac{1}{k(\eta'')}\right)\\
&\quad+e^{2ik\eta''-ik\eta}\left(-\frac{1}{3k^3(\eta'')^3}-\frac{ik\eta}{3k^3(\eta'')^3}+\frac{2i}{3k^2(\eta'')^2}-\frac{k\eta}{6k^2(\eta'')^2}-\frac{1}{3k\eta''}-\frac{ik\eta}{3k\eta''}\right)\\
&\quad+\frac{2}{3}(k\eta-i)e^{-ik\eta}E_1[-2ik\eta'']\bigg\}\bigg],
\end{aligned} \tag{E8}$$

where we again made use of the fact that the absolute value sign does not matter for the real part of $\hat{M}$. On super-Hubble scales we have here

$$I_{\hat{R}}(\eta,\eta'',k)\to -\frac{2}{3}\left(\log\left[\frac{\eta}{\eta''}\right]-\frac{1}{3}+\frac{1}{3}\frac{(\eta'')^3}{\eta^3}\right). \tag{E9}$$

We also have to calculate

$$\begin{aligned}
\hat{\mathcal{R}}_{\log}(\eta,\eta'',k) &= -\lambda^{-1}H^{-2}\int_{\eta''}^{\eta}d\tau(\tau H)^{-4}\left[\mathrm{Re}\hat{M}(\eta,\tau,k)\log\left[\frac{\eta\tau H^4}{4\mu^2}\right]\right]\Delta^c_{\phi,BD}(\tau,\eta'',k)\\
&= \lambda^{-1}H^{-2}\log\left[\frac{-\eta H^4}{4\mu^2}\right]\hat{\mathcal{R}}(\eta,\eta'',k)-\lambda^{-1}H^{-2}\frac{(2\pi)^3H^2h^2}{2k^6(4\pi)^5}\mathrm{Im}\frac{\partial}{\partial\nu}\\
&\quad\times\left\{(1+ik\eta'')e^{-ik\eta''}\int_{\eta}^{\eta''}d\tau(-\tau)^{-4+\nu}[[1+ik(\eta-\tau)]e^{ik(\tau-\eta)}+[1+ik(\tau-\eta)]e^{ik(\eta-\tau)}][(1-ik\tau)e^{ik\tau})]\right\}_{\nu=0}\\
&= -\mathrm{Im}\bigg\{(1+ik\eta'')e^{-ik\eta''}\bigg[\left(-\frac{14i}{9}+\frac{5k\eta}{9}-\frac{2}{3}[i-k\eta]\log\left[\frac{\eta\tau H^4}{4\mu^2}\right]\right)e^{-ik\eta}E_1[-2ik\tau]\\
&\quad+\left(-\frac{1}{9k^3\tau^3}+\frac{ik\eta}{9k^3\tau^3}+\frac{k\eta}{4k^2\tau^2}-\frac{1}{k\tau}+\left(-\frac{1}{3k^3\tau^3}+\frac{ik\eta}{3k^3\tau^3}+\frac{k\eta}{2k^2\tau^2}-\frac{1}{k\tau}\right)\log\left[\frac{\eta\tau H^4}{4\mu^2}\right]\right)e^{ik\eta}\\
&\quad+\left(-\frac{1}{9k^3\tau^3}-\frac{ik\eta}{9k^3\tau^3}+\frac{2i}{9k^2\tau^2}+\frac{k\eta}{36k^2\tau^2}-\frac{7}{9k\tau}-\frac{5ik\eta}{18k\tau}\right.\\
&\quad+\left.\left(-\frac{1}{3k^3\tau^3}-\frac{ik\eta}{3k^3\tau^3}+\frac{2i}{3k^2\tau^2}-\frac{k\eta}{6k^2\tau^2}-\frac{1}{3k\tau}-\frac{ik\eta}{3k\tau}\right)\log\left[\frac{\eta\tau H^4}{4\mu^2}\right]\right)e^{ik(2\tau-\eta)}\\
&\quad+\left(\frac{\pi}{3}(-14-5ik\eta)-\frac{\pi^2}{6}(i-k\eta)-2\gamma_E(i-k\eta)\log[-2ik\tau]-\gamma_E^2(i-k\eta)\right.\\
&\quad+\left.4k\tau(1+ik\eta)_3F_3\left[\begin{matrix}1, & 1, & 1\\ 2, & 2, & 2\end{matrix};2ik\tau\right]-(i-k\eta)\log[-2ik\tau]^2\right)\frac{e^{-ik\eta}}{3}\bigg]_{\tau=\eta}^{\tau=\eta''}\bigg\},
\end{aligned} \tag{E10}$$

where ${}_3F_3[{}^{1}_{2}\ {}^{1}_{2}\ {}^{1}_{2}; 2ik\tau]$ is a generalized hypergeometric function. On super-Hubble scales we get

$$
\begin{aligned}
I_{R_{\log}}(\eta,\eta'',k) \to & \frac{2}{3}\log(-2k\eta)\log(-2k\eta'') - \log^2(-2k\eta) + \frac{1}{3}\log^2(-2k\eta'') \\
& - \frac{4}{9}\log(-2k\eta)\frac{(\eta'')^3}{\eta^3} + \frac{2}{9}\log(4k^2\eta\eta'') - \frac{4}{3}\log\left[\frac{\eta}{\eta''}\right]\log\left[\frac{H^2}{4k\mu}\right] \\
& + \frac{4}{9}\left(1 - \frac{(\eta'')^3}{\eta^3}\right)\log\left[\frac{H^2}{4k\mu}\right] + \frac{2}{27}\left(1 - \frac{(\eta'')^3}{\eta^3}\right). \qquad \text{(E11)}
\end{aligned}
$$

APPENDIX F: I_M INTEGRALS

In this appendix we will calculate the integrals (127), (128), and (129). Let us start by calculating the following integral:

$$
\begin{aligned}
I_{\tilde{M}_I}(\eta,k) = & \int_0^\infty dx \frac{1 - i(k\eta - x)}{(k\eta - x)^4}\left[2 + (1+ix)\left(\log\left[\frac{H^2 x}{2k^2}\right] + i\frac{\pi}{2} - \gamma_E\right)\right]e^{-2ix} \\
= & \frac{1}{6k^3}\left[2 + (1+ik\eta)\left(\log\left[\frac{H^2}{2k^2}\right] + i\frac{\pi}{2} - \gamma_E\right)\right]\partial_\eta^3\int_0^\infty dx \frac{e^{-2ix}}{x - k\eta} \\
& + \frac{i}{2k^2}\left[2 + (2+ik\eta)\left(\log\left[\frac{H^2}{2k^2}\right] + i\frac{\pi}{2} - \gamma_E\right)\right]\partial_\eta^2\int_0^\infty dx \frac{e^{-2ix}}{x - k\eta} \\
& - \frac{1}{k}\left[\log\left[\frac{H^2}{2k^2}\right] + i\frac{\pi}{2} - \gamma_E\right]\partial_\eta\int_0^\infty dx \frac{e^{-2ix}}{x - k\eta} + \frac{1}{6k^3}[1 + ik\eta]\partial_\eta^3\int_0^\infty dx \frac{\log[x]e^{-2ix}}{x - k\eta} \\
& - \frac{1}{2k^2}[k\eta - 2i]\partial_\eta^2\int_0^\infty dx \frac{\log[x]e^{-2ix}}{x - k\eta} - \frac{\partial_\eta}{k}\int_0^\infty dx \frac{\log[x]e^{-2ix}}{x - k\eta}. \qquad \text{(F1)}
\end{aligned}
$$

In order to proceed, we will make use of the following identities:

$$
\int_0^\infty dx \frac{e^{-2ix}}{x - k\eta} = e^{-2ik\eta}E_1[-2ik\eta], \qquad \text{(F2)}
$$

$$
\frac{\partial_\eta}{k}[e^{-2ik\eta}E_1[-2ik\eta]] = -2ie^{-2ik\eta}E_1[-2ik\eta] - \frac{1}{k\eta}, \qquad \text{(F3)}
$$

$$
\frac{\partial_\eta^2}{k^2}[e^{-2ik\eta}E_1[-2ik\eta]] = -4ie^{-2ik\eta}E_1[-2ik\eta] + \frac{1 + 2ik\eta}{k^2\eta^2}, \qquad \text{(F4)}
$$

$$
\frac{\partial_\eta^3}{k^3}[e^{-2ik\eta}E_1[-2ik\eta]] = 8ie^{-2ik\eta}E_1[-2ik\eta] - \frac{2 + 2ik\eta - 4k^2\eta^2}{k^3\eta^3}, \qquad \text{(F5)}
$$

$$
\begin{aligned}
\int_0^\infty dx \frac{\log[x]e^{-2ix}}{x - k\eta} = & -\partial_\nu \int_0^\infty dx \frac{e^{-2ix}}{x^\nu(x - k\eta)}\bigg|_{\nu=0} = -\partial_\nu\left[\frac{\Gamma(1-\nu)}{(-k\eta)^\nu}e^{-2ik\eta}\Gamma[\nu, -2ik\eta]\right]_{\nu=0} \\
= & -e^{-2ik\eta}\left(\gamma_E + i\frac{\pi}{2} + \log(2)\right)E_1[-2ik\eta] \\
& - e^{-2ik\eta}\frac{1}{2}\left[\gamma_E^2 + \frac{\pi^2}{6} + 4ik\eta\, {}_3F_3\left[\begin{matrix}1, & 1, & 1\\ 2, & 2, & 2\end{matrix}; 2ik\eta\right] + 2\gamma_E\log[-2ik\eta] + \log^2[-2ik\eta]\right], \qquad \text{(F6)}
\end{aligned}
$$

and

$$
\frac{d}{dx}\left[x\, {}_3F_3\left[\begin{matrix}1 & 1 & 1\\ 2 & 2 & 2\end{matrix}; x\right]\right] = -\frac{\gamma_E + \log(-x) + E_1(-x)}{x}. \qquad \text{(F7)}
$$

Plugging these expressions into (F1), we find

$$I_{\tilde{M}_I}(\eta,k) = \left[\frac{1}{3k^3\eta^3}+\frac{i}{3k^2\eta^2}+\frac{1}{2k\eta}+\frac{2i}{3}\left(2\gamma_E-2-\log\left[\frac{H^2}{4k^2}\right]\right)+\frac{2}{3}k\eta\left(\log\left[\frac{H^2}{4k^2}\right]-2\gamma_E\right)\right]e^{-2ik\eta}E_1[-2ik\eta] + \frac{4\gamma_E-1-2\log[\frac{H^2}{4k^2}]}{6k^3\eta^3}+i\frac{3-4\gamma_E+2\log[\frac{H^2}{4k^2}]}{6k^2\eta^2}+\frac{6\gamma_E-5-3\log[\frac{H^2}{4k^2}]}{6k\eta}+\frac{i}{3}\left(2\gamma_E-\log\left[\frac{H^2}{4k^2}\right]\right) + \frac{1}{3}(i-k\eta)e^{-2ik\eta}\left(\gamma_E^2+\frac{\pi^2}{6}+4ik\eta\,{}_3F_3\left[\begin{matrix}1, & 1, & 1\\ 2, & 2, & 2\end{matrix};2ik\eta\right]+2\gamma_E\log[-2ik\eta]+\log^2[-2ik\eta]\right). \tag{F8}$$

The next integral we calculate is

$$I_{\tilde{M}_{II}}(\eta,k) = \int_0^\infty \frac{dx}{(k\eta-x)^4}(1-ix)E_1[i2x][1-i(k\eta-x)] = -\frac{1}{2k^3\eta^3}+\frac{i}{6k^2\eta^2}+\frac{1}{6k\eta}-\frac{1}{3}\left[\frac{1}{k^3\eta^3}+\frac{i}{k^2\eta^2}+\frac{3}{2k\eta}-i\right]e^{-2ik\eta}E_1[-2ik\eta], \tag{F9}$$

where we used the indefinite integrals

$$\int\frac{dx}{x^2}E_1[ax+b] = \frac{1}{b}\left[ae^{-b}E_1[ax]-\frac{1}{x}(ax+b)E_1[ax+b]\right], \tag{F10}$$

$$\int\frac{dx}{x^3}E_1[ax+b] = \frac{ae^{-ax-b}}{2bx}-\frac{a^2(1+b)e^{-b}E_1[ax]}{2b^2}+\left(\frac{a^2}{2b^2}-\frac{1}{2x^2}\right)E_1[ax+b], \tag{F11}$$

$$\int\frac{dx}{x^4}E_1[ax+b] = \frac{1}{3}\frac{a^3}{b^3}e^{-ax-b}\left[-\frac{b}{ax}+\frac{1}{2}(1-ax)\frac{b^2}{a^2x^2}\right] + \frac{1}{3}\frac{a^3}{b^3}\left[1+b+\frac{b^2}{2}\right]e^{-b}E_1[ax]-\frac{1}{3}\left[\frac{1}{x^3}+\frac{a^3}{b^3}\right]E_1[ax+b]. \tag{F12}$$

Adding up the last two major integrals we find the integral (127),

$$I_{\tilde{M}}(\eta,k) = -\frac{e^{ik\eta}}{2\lambda H^4}\int_0^\infty dx\left[\tilde{M}_I^{++}\left(\frac{x}{k},k\right)+\tilde{M}_{II}^{++}\left(\frac{x}{k},k\right)\right]\left[\frac{1}{(k\eta-x)^4}-\frac{i}{(k\eta-x)^3}\right]e^{-ix} = e^{ik\eta}[I_{\tilde{M}_I}+I_{\tilde{M}_{II}}](\eta,k) = \frac{2}{3}\left[i\left(2\gamma_E-\frac{3}{2}-\log\left[\frac{H^2}{4k^2}\right]\right)+k\eta\left(\log\left[\frac{H^2}{4k^2}\right]-2\gamma_E\right)\right]e^{-ik\eta}E_1[-2ik\eta] + \frac{1}{3}e^{-ik\eta}\left[\frac{2\gamma_E-2-\log[\frac{H^2}{4k^2}]}{k^3\eta^3}+i\frac{2-2\gamma_E+\log[\frac{H^2}{4k^2}]}{k^2\eta^2}+\frac{6\gamma_E-4-3\log[\frac{H^2}{4k^2}]}{2k\eta}+i\left(2\gamma_E-\log\left[\frac{H^2}{4k^2}\right]\right)\right] + \frac{1}{3}(i-k\eta)e^{-ik\eta}\left(\gamma_E^2+\frac{\pi^2}{6}+4ik\eta\,{}_3F_3\left[\begin{matrix}1, & 1, & 1\\ 2, & 2, & 2\end{matrix};2ik\eta\right]+2\gamma_E\log[-2ik\eta]+\log^2[-2ik\eta]\right). \tag{F13}$$

The next integral we calculate is (128) and we have

$$
\begin{aligned}
I_{\hat{M}}(\eta,k) &= -\frac{e^{ik\eta}}{2\lambda H^4}\int_0^\infty dx \hat{M}^{++}\left(\frac{x}{k},k\right)\frac{1-i(k\eta-x)}{(k\eta-x)^4}e^{-ix} = -e^{ik\eta}\int_0^\infty dx\frac{1-i(k\eta-x)}{(k\eta-x)^4}(1+ix)e^{-2ix} \\
&= -\frac{e^{ik\eta}}{6k^3}[1+ik\eta]\partial_\eta^3\int_0^\infty dx\frac{e^{-2ix}}{x-k\eta} - e^{ik\eta}\frac{i}{2k^2}[2+ik\eta]\partial_\eta^2\int_0^\infty dx\frac{e^{-2ix}}{x-k\eta} + e^{ik\eta}\frac{1}{k}\partial_\eta\int_0^\infty dx\frac{e^{-2ix}}{x-k\eta} \\
&= -\frac{e^{ik\eta}}{6k^3}[1+ik\eta]\partial_\eta^3(e^{-2ik\eta}E_1[-2ik\eta]) - e^{ik\eta}\frac{i}{2k^2}[2+ik\eta]\partial_\eta^2(e^{-2ik\eta}E_1[-2ik\eta]) + e^{ik\eta}\frac{1}{k}\partial_\eta(e^{-2ik\eta}E_1[-2ik\eta]) \\
&= e^{ik\eta}\left(\frac{1}{3k^3\eta^3}-\frac{i}{3k^2\eta^2}+\frac{1}{2k\eta}-\frac{i}{3}\right)+\frac{2}{3}(i-k\eta)e^{-ik\eta}E_1[-2ik\eta]. \qquad \text{(F14)}
\end{aligned}
$$

The last integral we calculate in this appendix is (129), where we use similar techniques as above:

$$
\begin{aligned}
I_{M_{\log}}(\eta,k) &= -\frac{e^{ik\eta}}{2\lambda H^4}\int_0^\infty dx \log\left[\frac{\eta(k\eta-x)H^4}{4k\mu^2}\right]\hat{M}^{++}\left(\frac{x}{k},k\right)\frac{1-i(k\eta-x)}{(k\eta-x)^4}e^{-ix} \\
&= e^{ik\eta}\Bigg\{\left[\frac{1}{3k^3\eta^3}-\frac{i}{3k^2\eta^2}+\frac{1}{2k\eta}+\frac{i}{3}+\frac{2}{3}(i-k\eta)e^{-2ik\eta}E_1[-2ik\eta]\right]\log\left[\frac{H^4\eta^2}{4\mu^2}\right] \\
&\quad + \frac{1}{9k^3\eta^3} - i\frac{1}{9k^2\eta^2}+\frac{3}{4k\eta}+i\frac{5}{18}+\frac{e^{-2ik\eta}}{9}(14i-5k\eta)E_1[-2ik\eta] \\
&\quad + \frac{e^{-2ik\eta}}{3}(i-k\eta)\left(\gamma_E^2+\frac{\pi^2}{6}+4ik\eta\, {}_3F_3\left[\begin{matrix}1, & 1, & 1\\ 2, & 2, & 2\end{matrix};2ik\eta\right]+2\gamma_E\log[-2ik\eta]+\log^2[-2ik\eta]\right)\Bigg\}. \qquad \text{(F15)}
\end{aligned}
$$

We are also interested in the super-Hubble limit of the integrals that we calculated in this appendix. However, let us multiply them with $(1+ik\eta'')e^{-ik\eta''}$ before, since these are the expressions that enter the calculation via (123). Thus, on super-Hubble scales we have

$$
\begin{aligned}
\mathrm{Im}[(1+ik\eta'')e^{-ik\eta''}I_{\tilde{M}}(\eta,k)] &\to -\frac{1}{3}\log^2(-2k\eta)+\left(\frac{2}{3}\gamma_E-1-\frac{2}{3}\log\left[\frac{H^2}{4k^2}\right]\right)\log(-2k\eta)+\frac{2}{9}\frac{(\eta'')^3}{\eta^3}\left(\gamma_E-1-\log\left[\frac{H}{2k}\right]\right) \\
&\quad +\frac{8}{9}-\frac{26}{9}\gamma_E+\gamma_E^2+\frac{\pi^2}{36}+\frac{1}{9}(17-12\gamma_E)\log\left[\frac{H}{2k}\right], \qquad \text{(F16)}
\end{aligned}
$$

$$
\mathrm{Im}[(1+ik\eta'')e^{-ik\eta''}I_{\hat{M}}(\eta,k)] \to \frac{2}{3}\log(-2k\eta)+\frac{1}{18}\left(12\gamma_E-17+2\frac{(\eta'')^3}{\eta^3}\right), \qquad \text{(F17)}
$$

$$
\begin{aligned}
\mathrm{Im}[(1+ik\eta'')e^{-ik\eta''}I_{M_{\log}}(\eta,k)] &\to \log^2(-2k\eta)+\frac{1}{3}\log(-2k\eta)\left(4\log\left[\frac{H^2}{4k\mu}\right]+2\gamma_E-1+\frac{2}{3}\frac{(\eta'')^3}{\eta^3}\right) \\
&\quad +\frac{1}{27}\frac{(\eta'')^3}{\eta^3}\left(1+6\log\left[\frac{H^2}{4k\mu}\right]\right) \\
&\quad -\frac{115}{108}-\frac{14}{9}\gamma_E-\frac{1}{3}\gamma_E^2+\frac{\pi^2}{36}-\frac{1}{9}(17-12\gamma_E)\log\left[\frac{H^2}{4k\mu}\right]. \qquad \text{(F18)}
\end{aligned}
$$

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