Direct anthropic bound on the weak scale from supernovæ explosions

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Core-collapse supernovæ presumably explode because trapped neutrinos push the material out of the stellar envelope. This process is directly controlled by the weak scale v: we argue that supernova explosions happen only if fundamental constants are tuned within a factor of few as $v \sim \Lambda_{QCD}^{3/4} M_{Pl}^{1/4}$, such that neutrinos are trapped in supernovæ for a time comparable to the gravitational timescale. We provide analytic arguments and simulations in spherical approximation, that need to be validated by more comprehensive simulations. The above result can be important for fundamental physics, because core-collapse supernova explosions seem anthropically needed, as they spread intermediate-mass nuclei presumably necessary for "life." We also study stellar burning, finding that it does not provide anthropic boundaries on v.

I. INTRODUCTION

Nature contains two relative mass scales: the vacuum energy density $V \sim (10^{-30} M_{Pl})^4$ and the weak scale $v^2 \sim (10^{-17} M_{Pl})^2$ where v is the Higgs vacuum expectation value. Their smallness with respect to the Planck scale $M_{Pl} = 1.210^{19}$ GeV is not understood and is considered as "unnatural" in relativistic quantum field theory, because it seems to require precise cancellations among much larger contributions. If these cancellations happen for no fundamental reason, they are "unlikely," in the sense that summing random order one numbers gives 10^{-120} with a "probability" of about 10^{-120}. Worrying about naturalness of the Higgs mass and of the vacuum energy has been a major theme in fundamental physics in the last decades [1–3]. Many theories alternative to small tuned values have been proposed. Most theorists expected that the Higgs should have been accompanied by new physics that keeps its mass naturally light, but experiments discovered just the Higgs [3]. Collider data and cosmological observations are so far consistent with small tuned values of the weak scale and of the cosmological constant.

A controversial but lapalissian anthropic tautology seems relevant to understand what goes on in fundamental physics: observers can only observe physics compatible with their existence.

On the cosmological constant side, its smallness has been interpreted through an anthropic argument: a cosmological constant $\sim 10^3$ times larger than its physical value would have prevented structure formation [4,5]. No natural theoretical alternatives are known (for example, supergravity does not select $V = 0$ as a special point [1]), and anthropic selection of the cosmological constant seems possible in theories with some tens of scalars such that their potential has more than 10^{120} different vacua, which get "populated" forming a "multiverse" through eternal inflation. String theory could realize this scenario [6–8].

On the Higgs side, it has been noticed that light quark and lepton masses m_e, m_u, m_d are anthropically restricted in a significant way: a nontrivial nuclear physics with more nuclei than just H and/or He (and thereby chemistry, and life) exists because m_e/Λ_{QCD}, m_u/Λ_{QCD}, m_d/Λ_{QCD} have appropriate values which allow for the existence of a hundred of nuclear species [9–12]. Such anthropic boundaries in m_e, m_u, m_d give extra indicative support to the possibility that physics is described by a theory where fundamental constants have different values in different local minima.

We point out that, however, anthropic selection of fermion masses m_f does not lead to clean implications for theories of the weak scale. Rather, it leads to a confusing

and paradoxical situation. Indeed, in the Standard Model, quark and lepton masses are given by $m_f = y_f v$ where y_f are Yukawa couplings, which can be naturally small. If Yukawa couplings have unique values, the weak scale v must be anthropically small: changing v by a factor of few from its physical value $v_{SM} = 174$ GeV changes fermion masses removing complex chemistry [9].

But the multiple anthropic boundaries on m_e, m_u, m_d indicate that Yukawa couplings do not have unique values. Then one loses the anthropic interpretation of $v \ll M_{Pl}$, and the Standard Model appears uselessly fine-tuned: one can easily find anthropically acceptable alternative theories less tuned that the Standard Model. An example is a SM-like theory with the same m_f, obtained from a bigger v times smaller Yukawa couplings y_f. The paradox is that, in a multiverse landscape, more tuned vacua are relatively rarer. We should have expected to live in a less tuned vacuum. There are two main classes of less tuned vacua, leading to two aspects of the paradox:

(a) Natural theories: extensions of the SM where v is naturally small compared to M_{Pl}.
(b) Less unnatural theories: SM-like theories with smaller y_f and bigger v/M_{Pl}.

A. Natural theories

Collider bounds suggest that nature did not use a natural theory to achieve the observed small weak scale. Unnaturalness of the weak scale, hinted by previous colliders [13,14], has been confirmed by the Large Hadron Collider, and might be established by future colliders. We here assume that the weak scale is not natural.

This is not necessarily a paradox: in a generic multiverse context, nature might have avoided a natural extension of the SM if all such extensions have a "multiverse proba-bility" so much lower than unnatural models such as the SM, that the gain in probability due to naturalness ($\sim 10^{34}$ for the weak scale, possibly times $\sim 10^{60}$ for a partial suppression of the cosmological constant down to the weak scale) is not enough to statistically favor natural models over unnatural models. This possibility seems reasonable, given that very few natural extensions of the SM have been proposed, and they employ special ingredients, such as supersymmetry, which might be rare or absent in a landscape.

In the string context, model building focused on effective 4-dimensional theories below the string scale where the weak scale is naturally small thanks to $N = 1$ weak-scale supersymmetry. If the weak scale is instead unnatural and anthropically selected, the paradox is avoided provided that the landscape distribution of SUSY breaking scales is dominated by the largest energies (maybe because breaking supersymmetry dynamically at low energy needs somehow contrived model building in string models with a dilaton), suggesting that we live in a vacuum with no supersymmetry below the string scale.

Natural theories based on ingredients different from supersymmetry have more evident theoretical problems that can explain why nature did not use them. Concerning large extra dimensions, the problem is dynamically stabi-lizing their size in a natural way. Concerning composite Higgs models, the only dynamics proposed so far that gives phenomenologically acceptable models (with Yukawa cou-plings and thereby fermion masses) has similar naturalness issues as the SM itself, as it employs a fundamental scalar [15] with new strong interactions. Other scenarios where the weak scale is only partially natural involve baroque model-building.

B. Less unnatural theories

The SM appears uselessly more tuned than similar theories where the anthropic bounds on fermion masses $m_f = y_f v$ is satisfied using smaller Yukawas y_f and larger v/M_{Pl} (possibly up to the weak-less limit $v \sim M_{Pl}$). This is a real paradox, because (unlike in the case of natural SM extensions), it looks not plausible that SM-like theories have a drastically lower "multiverse probability" than the SM itself. The SM indicates that some mechanism can generate small Yukawas such as $y_e \sim 10^{-6}$, so that it is difficult to argue that smaller Yukawas are highly unlikely.

One way out from this paradox is that the SM itself is natural: this requires special theories of quantum gravity that do not employ particles much heavier than the Higgs and significantly coupled to it [16–18]. A different way out is the possible existence of an extra anthropic boundary that restricts *directly* the scale v of weak interactions.

C. Searching for a direct anthropic boundary on the weak scale

The Higgs vacuum expectation value v determines the Fermi coupling $G_F = 1/(2\sqrt{2}v^2)$ that controls neutrino interactions at low energies, $\sqrt{s} \lesssim M_{W,Z}$. The weak scale v might then be anthropically relevant in two situations [19–21] where nontrivial physics arises because of a numerical coincidence. In both cases the numerical coincidence is

$$u \equiv \frac{M_{Pl}^{1/4} \Lambda_{QCD}^{3/4}}{v} \sim 1, \qquad (1)$$

where $\Lambda_{QCD} \approx 300$ MeV is the QCD scale that naturally generates nucleon masses $m_{n,p} \sim \Lambda_{QCD}$ through dimen-sional transmutation. In the definition of u we ignored $\mathcal{O}(1)$ factors, which actually happen to be $\mathcal{O}(100)$. The two situations are:

(1) Big bang nucleosynthesis (BBN) predicts an order-one ratio between the number of neutrons N_n and of protons N_p because the neutrino decoupling temper-ature $T_{\nu dec}$ is comparable to the proton-neutron mass difference, and because the neutron lifetime τ_n is

comparable to the age of the Universe at BBN time, t_{BBN} [22]:

$$\frac{N_n}{N_p} \approx \exp\left(-\frac{m_n - m_p}{T_{\nu\text{dec}}} - \frac{t_{\text{BBN}}}{\tau_n}\right) \approx \frac{1}{7}. \qquad (2)$$

Varying v and M_{Pl} with quark masses fixed, the first term in the exponent scales as $(M_{\text{Pl}}/v^4)^{1/3}$ and the second as M_{Pl}/v^4: they depend on the same combination of M_{Pl} and v, as in Eq. (1). A larger (smaller) v increases (decrease) the He/H ratio, for not too large variations of v, at fixed M_{Pl} and fixed baryon asymmetry. Some authors discuss the possibility that a large $v \gtrsim 100 v_{\text{SM}}$ might be anthropically excluded because of a too low Hydrogen abundance, as H is used in life and molecular H plays a role in gas cooling that leads to star formation [22,23]. However p are not strongly suppressed at BBN; furthermore extra p can be later produced by stars (as they make heavy nuclei that contain more n than p in view of electric repulsion) as well as by cosmic rays.

(2) As we will discuss, core-collapse supernova explosions crucially depend on v, mainly because neutrinos push the material that surrounds the core, spreading intermediate-mass elements (O, F, Ne, Na, Mg, Al and possibly N, Cl, K, Ca) which seem produced almost exclusively in core collapse SN [24,25] and seem needed for life [21].

Indeed nuclei form as follows: H and He are dominantly synthesised during BBN; core-collapse supernovæ (known as type II, with rarer type Ib and Ic) lead to the ejection of the shells of burnt star materials, which include relatively light elements. Cosmic rays (possibly dominantly produced through supernovæ explosions) and dying light stars (formed thanks to the elements produced by core-collapse supernovæ) contribute to the production of relatively light elements (Li^3, Be^4, B^5, C) as well as to heavy elements thanks to neutron capture. Merging neutron stars, explosions of white dwarfs (accretion supernovæ known as type Ia SN) and dying light stars make elements heavier than Si, in particular Fe [26]. Accretion supernovæ are binary objects just below the threshold for carbon fusion, such that capture of extra mass triggers runaway nuclear reactions, heating and giving an explosive meltdown that proceeds up to the most stable nucleus, Fe, leaving negligible amounts of intermediate-mass nuclei.[1] As a result, nuclei produced almost exclusively by core collapse supernovæ include O (the primary element of terrestrial life, together with C, N, H, P, S), Na, Mg and possibly K, Ca, Cl (the secondary elements of terrestrial life). It has been argued that the chemistry of O is generically needed

for life [21]. If true, explosions of core collapse supernovæ are anthropically relevant, and their existence is related to weak interactions.

Needless to say, neither astrobiology nor the physics of supernovæ are fully understood and established. In particular, some authors claim that, unlike what believed earlier, neutrinos and weak interactions might be not needed for core-collapse supernovæ explosions, which can also explode through collapse-induced thermonuclear explosions [27–29]. If this alternative mechanism is confirmed and if it efficiently spreads nuclei such as O, our direct anthropic bound on the weak scale would not be present.

This paper is structured as follows. In Sec. II we discuss how core-collapse SN explosions would behave for a different value of v, keeping fermion masses fixed.[2] In Sec. III we briefly comment on how star evolution behaves as function of v, finding no anthropic boundaries. Conclusions are given in Sec. IV.

II. SUPERNOVA EXPLOSIONS

A. Standard supernova explosions

We start by summarizing the standard theoretical picture of core-collapse supernova explosions. While not established, it is consistent with the observation of neutrinos from SN1987A (see [30–33] for reviews). We keep all fundamental parameters to their physical values and we provide estimates which exhibit the dependence on fundamental parameters m_n, M_{Pl}, v ignoring order one factors, in order to later study modified weak scale or Planck mass. Gravity and weak interactions give competing comparable effects: due to this coincidence supernova explosions are a complex phenomenon and computing order one factors through numerical simulations is needed to understand what happens [34]. We summarize the results of numerical simulations, and clarify which features follow from dynamical adjustments or from numerical coincidences (as needed to later consider different values of fundamental parameters).

1. Collapse

The life of stars proceeds through subsequent stages controlled by a balance between gravity and the energy released in nuclear fusion reactions, that form heavier elements from lighter ones. During the first stage, the hydrogen in the core of the star is converted into helium. As the hydrogen in the core is exhausted, gravity—no longer balanced by hydrogen burning—causes the core to contract. Hydrogen burning is still active in a shell surrounding the core, made now primarily of helium nuclei.

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[1]We do not explore the possibility that O, rather than Fe, might be the most stable nucleus in vacua with different values of $m_{u,d}$ or α_{em}.

[2]Light fermion masses are independently anthropically constrained [9–12]. For $v < v_{\text{SM}}$ a top Yukawa coupling larger than one would be needed to keep the top quark mass fixed; anyhow, the top quark plays no anthropic role.
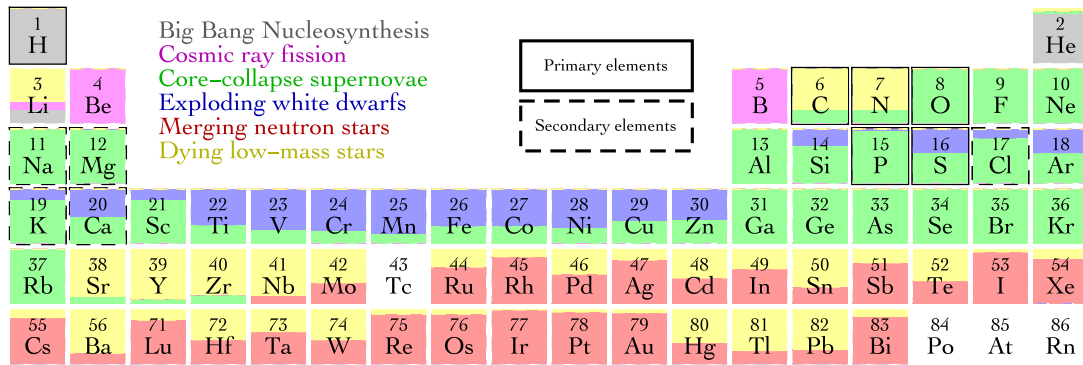
FIG. 1. Each stable element in the periodic table is colored according to its present relative contributions of nucleosynthesis sources [24]. The primary (secondary) elements of terrestrial life are highlighted by a continuous (dashed) contour.

The temperature of the core rises because of the contraction up to the point where helium fusion begins. For stars with low mass $M \lesssim 8\ M_\odot$ the fusion processes end with the creation of an electron degenerate carbon core (they never becomes hot enough to ignite carbon fusion), that eventually forms a white dwarf while the outer material drifts off into space forming a planetary nebula, giving rise to the elements in Fig. 1. For massive stars with $M \gtrsim 8\ M_\odot$, once helium fuel runs out in the core, a further contraction raises the inner temperature sufficiently so that carbon burning begins. Neon, oxygen and silicon burning stages follow similarly. Each stage is faster than the previous stage because the temperature is higher: neutrinos carry away more energy, while nuclear reactions releases less energy.

Eventually, the silicon burning stage causes the formation of an iron core in the innermost part of the star. The formation of the iron core stops the fusion chain because iron is the most stable element. The endpoint of this chain is an onion structure with shells of successively lighter elements burning around an iron core. We denote with $R_{\rm Fe}$ the radius of the iron core.

The iron core of a star is protected against the crushing force of gravity only by the electron degeneracy pressure, and not because of the energy released by nuclear fusion. This means that the iron core has a maximum mass determined by the Chandrasekhar limit, $M_{\rm Ch} \approx 1.4\ M_\odot$. As the silicon shell surrounding the iron core continues to burn, the iron core mass slowly increases approaching the Chandrasekhar limit above which the iron core starts collapsing under its weight. In terms of fundamental parameters the Chandrasekhar mass is given by

$$M_{\rm Ch} \sim Y_e^2 M_{\rm Pl}^3 / m_n^2 \tag{3}$$

where $Y_e \sim 1/2$ is the electron fraction per nucleon.

2. Deleptonization

Weak interactions contribute to star burning, and start playing a crucial role when the inert iron core with radius $R_{\rm Fe}$ of a few thousand of km and mass $M \sim M_{\rm Ch}$ reaches a high enough sub-nuclear density, $\rho \sim 10^{-6} m_n^4$, that electrons and protons are converted into neutrons and neutrinos [35]. Their main inverse β decay reaction is $e + {}^{56}{\rm Fe} \rightarrow {}^{56}{\rm Mn} + \nu_e$, kinematically allowed at energies larger than $M_{\rm Mn} - M_{\rm Fe} = 3.7$ MeV. While electrons support the core with their degeneracy pressure, neutrinos freely escape. The core is deleptonized in a timescale $\tau_{\rm weak} \sim 1/\sigma_{\rm weak} n_n \propto v^4$ faster than the free-falling timescale of the gravitational collapse

$$\tau_{\rm grav} \sim \frac{M_{\rm Pl}}{\sqrt{\rho}} \sim \frac{M_{\rm Pl}}{m_n^2} \quad \text{for } \rho \sim m_n^4. \tag{4}$$

Detailed computations find $\tau_{\rm weak} \approx 10^{-3}$ sec $\ll \tau_{\rm grav} \approx 0.1$ sec. As a consequence Y_e and thereby $M_{\rm Ch}$ suddenly decrease: after an order one decrease, a order one inner fraction of the iron core (called "inner core") starts collapsing. More precisely, the iron core breaks into two distinct regions: a subsonically and homologously collapsing inner core, with mass $M_{\rm ic}$ and radius $R_{\rm ic}$, and a supersonic outer core. In the inner core the inward collapse velocity is proportional to the radial distance (giving a "homologous" collapse), and its boundary $R_{\rm ic}$ is defined as the point where the inward radial velocity equals the sound speed of the fluid (which, on the contrary, decreases with density and, therefore, radial distance). The time evolution of the inner core radius $R_{\rm ic}$ is sketched as a red curve in Fig. 2.

3. Rebounce

When the collapsing inner core (with mass $M_{\rm ic} \approx Y_e^2 M_{\rm Pl}^3 / m_n^3$, more precisely equal to about 0.6 $M_\odot$ according to simulations) reaches nuclear density the collapse is halted by the nuclear force, which is repulsive at nucleon distances below 1 fm. At this point the radius of the inner core collapsed down to about $R_{\rm ic} \approx 20$ km. This can be estimated as $R_{\rm ic} \sim {\rm fm}\, N^{1/3} \sim M_{\rm Pl}/m_n^2$ taking into account that there are $N \sim M_{\rm Ch}/m_n \sim (M_{\rm Pl}/m_n)^3 \sim 10^{57}$ nucleons at distance fm $\sim 1/m_n$. The inner core radius is

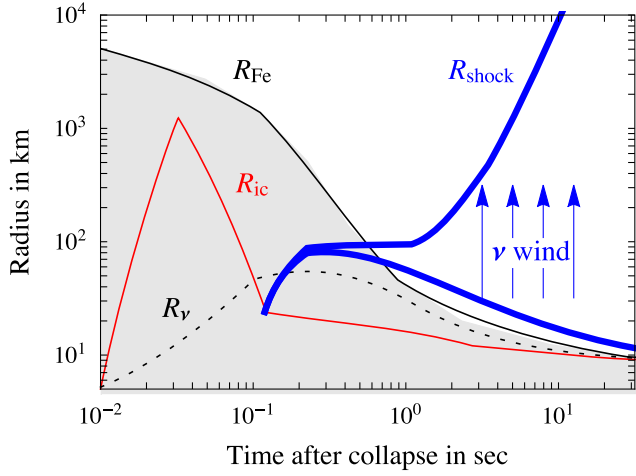FIG. 2. Sketchy plot of the time evolution of the main characteristic radii: (1) R_{Fe} is the radius of the iron core; (2) R_{ic} is the radius of the inner iron core which starts collapsing; (3) R_ν is the radius of the "neutrino-sphere"; (4) R_{shock} is the radius of the bounce shock, that stalls and gets possibly revitalized through neutrinos emitted from the neutrino-sphere.

parametrically the same as the Schwarzschild radius $R_{\text{Sch}} = 2M_{\text{Ch}}/M_{\text{Pl}}^2$. Simulations find $R_{\text{Sch}} > R_{\text{ic}}$ for $M \lesssim 40\,M_\odot$, such that only heavier neutron stars collapse directly into black holes.

If the collapse does not proceed with the formation of a black hole, the inner core, being in sonic communication throughout the process, coherently bounces [35]. The rebound of the inner core generates an outward-going shock wave (blue curve in Fig. 2) propagating through the still infalling outer core. To generate a supernova explosion, the shock wave must cross the outer iron core. In doing so, the shock wave loses energy due to iron dissociation (giving 8.8 MeV binding energy per nucleon) and neutrino production via electron capture on the way through the outer core: the shock slows down and stalls at $R_{\text{shock}} \approx 100$ km. Order one factors, such as the distinction between the inner and outer core, are important for causing the stall. After crossing the iron core with radius R_{Fe} the temperature is not high enough to dissociate nuclei, and the shock stalls.

Weak interactions give a second crucial effect: the stalled shock wave is rejuvenated by the outflowing neutrinos.

4. Neutrino trapping

As matter density increases, neutrinos get momentarily trapped in the collapsing star, up to a neutrino-sphere radius R_ν which is slightly larger than the inner core radius R_{ic} (dashed and red curves, respectively, in fig. 2), because of a numerical coincidence which involves the Fermi and Newton constants. Thereby the gravitational energy produced by the collapse, $E_{\text{tot}} \sim G_N M_{\text{ic}}^2/R_{\text{ic}} \sim M_{\text{Pl}}^3/m_n^2 \sim 3 \, 10^{53}$ erg, remains trapped

behind the shock wave, and is released as neutrinos with a timescale (comparable to the timescale of the shock, about a second), given by

$$\tau_\nu \sim \max(\tau_{\text{volume}}, \tau_{\text{surface}}) \qquad (5)$$

where the two factors are the timescales for energy transmission inside the trapping volume, and from its neutrino-sphere surface.

The first factor is controlled by diffusion of trapped neutrinos: the neutrino cross section is $\sigma_{\text{weak}} \sim T^2/v^4$ such that the neutrino mean free path at temperature $T_{\text{ic}} \sim m_n$ is

$$\ell_\nu \sim \frac{1}{n_n \sigma_{\text{weak}}} \sim \frac{v^4}{m_n^3 T_{\text{ic}}^2}. \qquad (6)$$

The diffusion time of neutrinos inside the inner core can be computed in a simple way in terms of random walk, given that the inner core has a constant matter density. A neutrino covers a distance R_{ic} in $N_\nu \sim R_{\text{ic}}^2/\ell_\nu^2$ diffusion steps. Neutrinos diffuse on a time

$$\tau_{\text{volume}} \sim \max(N_\nu, 1)\ell_\nu \sim \max\left(\frac{m_n M_{\text{Pl}}^2}{v^4}, \frac{M_{\text{Pl}}}{m_n^2}\right). \qquad (7)$$

The second factor in Eq. (5), τ_{surface}, depends on the radius R_ν of the neutrino-sphere: as it is bigger than the inner core radius R_{ic}, we need to know the profile density of the supernova outside the inner core. Simulations find that the nucleon number density $n(r)$ varies outside the inner core (after that it is stabilized) as

$$n \sim n_{\text{ic}}(R_{\text{ic}}/r)^3, \qquad n_{\text{ic}} \sim m_n^3. \qquad (8)$$

The temperature of the material is given by the Fermi momentum, $T \sim n^{1/3}$ and thereby scales as $T \sim T_{\text{ic}} R_{\text{ic}}/r$, having assumed that conduction of energy inside the inner core controlled by τ_{volume} is fast enough to compensate energy losses from the surface (otherwise the surface temperature gets lower). Imposing $n_n(R_\nu)\sigma_{\text{weak}} R_\nu \sim 1$ determines the radius of the neutrino-sphere

$$R_\nu \sim \frac{n_{\text{ic}}^{1/4} R_{\text{ic}}^{5/4} T_{\text{ic}}^{1/2}}{v} \sim \frac{1}{v}\left(\frac{M_{\text{Pl}}}{m_n}\right)^{5/4}, \qquad T_\nu \sim v\left(\frac{m_n}{M_{\text{Pl}}}\right)^{1/4}. \qquad (9)$$

The power emitted in neutrinos thereby is

$$L_\nu \sim R_\nu^2 T_\nu^4 \sim v^2(M_{\text{Pl}}/m_n)^{3/2}, \qquad (10)$$

and the cooling time of the surface is

$$\tau_{\text{surface}} = \frac{E_{\text{tot}}}{L_\nu} \sim \frac{M_{\text{Pl}}^{3/2}}{v^2 m_n^{1/2}}. \qquad (11)$$

Detailed numerical computations including order one factors (such as $n_{\text{ic}} \approx (0.2 m_n)^3$, $T_{\text{ic}} \approx 0.1 m_n$, …) find

$\ell_\nu \sim$ few cm $< R_{\rm ic}$ so that neutrinos are trapped. The various timescales happen to be comparable with the fine structure $\tau_{\rm volume} \sim$ sec $\gtrsim \tau_{\rm grav}, \tau_{\rm surface}$ for the timescale over which neutrinos emitted from the neutrino-sphere drain the gravitational energy of the collapse.

5. Heating due to out-going neutrinos

Nucleons immediately outside the neutrino-sphere are heated with rate [36]

$$Q_\nu \sim \sigma_{\rm weak} \frac{L_\nu}{4\pi R_\nu^2} \sim v^2 \left(\frac{M_{\rm Pl}}{m_n}\right)^{3/2} \quad \text{where } \sigma_{\rm weak} \sim G_{\rm F}^2 T_\nu^2. \quad (12)$$

More precisely the interactions of neutrinos with matter outside the neutrino-sphere result in two antagonistic processes: cooling and heating. On the one hand, outgoing neutrinos heat free nucleons (present at $r < R_{\rm shock}$) more efficiently than nuclei (present at $r > R_{\rm shock}$) through weak reactions $n + \nu_e \rightarrow e^- + p$ and $p + \bar\nu_e \rightarrow e^+ + n$. The heating rate per nucleon Q_ν^+ at generic radial distance $R_\nu \leq r \leq R_{\rm shock}$ is estimated as

$$Q_\nu^+(r) \sim \frac{G_{\rm F}^2 L_\nu \langle E_\nu^2 \rangle}{4\pi r^2} \sim \frac{1}{r^2} \frac{M_{\rm Pl}}{m_n}, \quad (13)$$

where $\langle E_\nu^2 \rangle \sim T_\nu^2$ is the mean squared energy of neutrinos. On the other hand, nucleons cool down by radiating neutrinos as a consequence of electron and positron capture with a typical cooling rate

$$Q_\nu^-(r) \sim G_{\rm F}^2 T^6, \quad (14)$$

where $T \sim T_{\rm ic} R_{\rm ic}/r$ is the temperature of the material, as already discussed above. The net heating rate per nucleon due to neutrinos is given by $Q_\nu \equiv Q_\nu^+ - Q_\nu^-$. Cooling typically dominates at small radial distances where the material is hotter but, since Q_ν^+ decreases less steeply with r than Q_ν^-, neutrino heating dominates over energy losses above some gain radius $r > R_{\rm gain}$ given by

$$R_{\rm gain} \sim \frac{1}{v}\left(\frac{M_{\rm Pl}}{m_n}\right)^{5/4}. \quad (15)$$

Having omitted order one factors we find $R_{\rm gain} \sim R_\nu$: in this approximation the expression for the gain radius is more simply found imposing $T_\nu \sim T$ irrespectively of the specific processes that dominate energy exchanges. Heating and cooling would be in equilibrium if matter at radius r had temperature $T_{\rm eq} = T_\nu \sqrt{R_{\rm gain}/r}$. This is hotter than $T = T_\nu R_{\rm gain}/r$ at $r > R_{\rm gain}$, confirming that neutrinos heat matter at $r > R_{\rm gain}$ and cool matter at $r < R_{\rm gain}$.

The presence of the region with positive net heating rate is considered crucial for a successful revival of the stagnant shock. Numerical computations find $R_{\rm gain} \approx 3R_\nu$ such that

$R_\nu < R_{\rm gain} < R_{\rm shock}, R_{\rm Fe}$: outgoing neutrinos can push the stalled shock wave from below in a way considered crucial for finally getting a "delayed explosion." If at least a few % of the gravitational energy $E_{\rm tot}$ emitted in neutrinos is transferred to the shock wave, it explodes the whole star, spreading its nuclei. This fails by a small margin (10–20%) according to simulations done in spherical approximation [37]. Successful explosions seem to require taking into account asphericity and possibly rotation, magnetic fields, etc [38–45]. The conclusion seems that stars with mass $8\,M_\odot \lesssim M \lesssim 40\,M_\odot$ can make SN explosions thanks to neutrinos.[3] It is believed that most core-collapse SN explode, that 99% of their energy is emitted in neutrinos and 1% in other particles, spreading elements needed for life.

B. Supernova explosions for different v and $M_{\rm Pl}$: Analytic discussion

In the previous discussion we provided simple analytic expressions in terms of fundamental parameters, v, $M_{\rm Pl}$, m_n. These analytic expressions help understanding what happens if such fundamental parameters had values different from their physical values.

Neutrino-induced delayed supernova explosions can only arise if neutrinos are trapped, such that the gravitational energy of the inner core is released gradually pushing the shock wave. This condition can be written either as $R_\nu \gtrsim R_{\rm ic}$ or as $\tau_\nu \gtrsim \tau_{\rm grav}$ or as $\tau_{\rm volume} \gtrsim \tau_{\rm grav}$ or as $N_\nu \gtrsim 1$. These conditions give the critical value of Eq. (1): neutrinos are trapped if

$$v < v_{\rm trap} \equiv \mathcal{O}(1) \times m_n (M_{\rm Pl}/m_n)^{1/4}. \quad (16)$$

The $\mathcal{O}(1)$ coefficient happens to be ~ 0.01 such that $v_{\rm trap}$ is a factor of few above $v_{\rm SM}$.

1. Supernova explosions for smaller/larger Fermi constant

Changing v affects the initial deleptonization phase, and, more importantly, the final phase that leads to the explosion.

Concerning deleptonization, it takes place at the physical value of v because weak interactions are faster than gravity, $\tau_{\rm weak} \sim 0.01\tau_{\rm grav}$. This remains true until $v \lesssim 3v_{\rm SM}$: we expect that for such values of v the decrease of Y_e keeps happening so fast that only the inner core collapses, such that the shock wave in all cases needs to cross roughly the same amount of outer material. We expect that deleptonization also happens for larger $v \gtrsim 3v_{\rm SM}$ because the

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[3]First stars with low metallicity and large mass $130 < M/M_\odot \lesssim 250$ are believed to undergo explosions when photons get energetic enough to produce e^+e^- pairs, removing pressure support, such that the consequent compression triggers runaway nuclear fusion.

gravitational collapse raises the temperature and density increasing the weak interaction rate until deleptonization of the core finally happens. Increasing v even more, massive enough stars reach temperatures and densities where the relevant physics is QCD rather than nuclear physics: we then expect that there is no rebounce. Finally, no delepto-nization can occur if v is so large that the extreme weakless limit ($G_F = 0$) is relevant. This was studied in [26,46], that suggested that core-collapse supernova explosions do not occur.[4] This would need to be established through numeri-cal simulations.[5]

Coming now to the final neutrino-driven core-collapse supernova explosion we expect that it is more critically affected by v and that explosions arise in a narrow range of v

$$v_{\min} < v < v_{\max} < v_{\text{trap}}. \tag{17}$$

The reason is the following.

(1) If $v > v_{\text{trap}}$ neutrinos are not trapped so they escape immediately having negligible interactions with matter outside: neutrinos cannot rejuvenate the shock that would lead to a supernova explosion.

(2) If $v < v_{\text{trap}}$ neutrinos are trapped, and one needs to study if neutrinos can trigger a supernova explosion. In first approximation, the total energy and momentum transmitted by outward-going neu-trinos to matter outside does not depend on v and is of order of (a few percent of) the total energy E_{tot}. The reason is that, for any v and for any matter density profile, neutrinos undergo about a scattering after exiting the neutrino-sphere. On the contrary, the spatial and temporal structures of neutrino heating depend significantly on v: both $R_{\text{gain}} \sim$ few $\times R_\nu \propto 1/v$ [see Eq. (9)] and τ_ν [see Eq. (5)] grow with G_F.

(3) Supernovae explosions take place in the usual way for $v_{\min} \lesssim v \lesssim v_{\max}$: the collapsing inner core gets halted by nuclear repulsion, giving a rebounce shock wave that stalls and is rejuvenated by neutrinos. This process is maximally efficient when the timescale of neutrino cooling is comparable to the timescale of the shock. In such a case, the shock can reach the gain region, $R_{\text{shock}} \gtrsim R_{\text{gain}}$. This happens around the physical value of v. For larger $v_{\max} < v < v_{\text{trap}}$ neutrinos escape too fast and deposit energy to more interior regions subject to larger gravitational po-tentials: on the timescale relevant for the shock the net effect of neutrinos is cooling the shock.

(4) We expect no supernova explosion in the opposite limit where v becomes too small, $v \lesssim v_{\min}$, because neutrinos interact so much that neutrino energy is released on a timescale much longer than the time-scale of the shock-wave. Furthermore, at even smaller v the gain radius and/or the neutrino-sphere radius R_ν become bigger than the radius reached by the stalled shock wave, so that shock is cooled and/ or not pushed. If supernova explosions get prevented by $R_{\text{shock}} < R_{\text{gain}}$, we expect that this roughly hap-pens at $v_{\min} \sim 0.2 v_{\text{SM}}$.

Numerical simulations indicate that explosions happen at the physical value of v only if asphericity is taken into account. Simulations in spherical approximation give explosions only if the neutrino luminosity is artificially enhanced by 10%–20% [47,48]. Possibly $v_{\max}$ could be $\mathcal{O}(20\%)$ bigger than the physical Higgs vev v_{SM}, but computing its value better than an order-of-magnitude estimate would need dedicated simulations.

2. Supernova explosions for smaller/larger Planck mass

Given that the critical parameter is the dimensionless combination of Eq. (1), $M_{\text{Pl}} m_n^3 / v^4$, we expect that the range in v argued in the previous section becomes a strip in the (v, M_{Pl}) plane, if both v and M_{Pl} are varied.

Furthermore, and less importantly, the energies involved in the SN explosion scale proportionally to M_{Pl}^3. These are the gravitational energy $E_{\text{tot}} \sim M_{\text{Pl}}^3 / m_n^2$ and the comparable energy needed to dissociate $N \sim (M_{\text{Pl}}/m_n)^3$ iron nuclei such that the shock wave can lead to a SN explosion.

However, we could not validate the above expectations trough reliable numerical simulations at different values of the Planck mass.[6]

C. Supernova explosions for different v: Simulations

Numerical simulations are needed to validate the above analytical discussion because it involves not only simple rescaling of cross sections but also disentangling dynamical adaptive features (which result in nontrivial scaling laws) from numerical coincidences (which make order one factors crucial).

We thereby run the public numerical SN code of [49] in its version 2, that involves an improved treatment of neutrino energy transport [50]. We rescale all weak interactions changing the value of v or equivalently of the Fermi constant $G_F = 1/(2\sqrt{2}v^2)$. The code employs the spherical approximation. Despite this simplification, simulations involve difficult numerical aspects and

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[4]The authors of [46] suggested that core-collapse supernovæ can be replaced by accretion supernovæ, which however do not spread some of the light elements which seem needed for life.

[5]An alternative possibility is that the inner core still rebounces and its mass $M_{\text{ic}} \sim Y_e^2 M_{\text{Pl}}^3 / m_n^2$ is bigger than in the physical case (because deleptonizaton does not reduce Y_e): this might allow the shock wave to avoid stalling even without the help of outflowing neutrinos.

[6]Varying the Planck mass, we only managed to run the numerical SN code of [49] in its version 1 (faster simplified treatment of neutrino energy transport) and adapting progenitors computed at the physical value of the Planck mass. Such simulations are possibly inadequate, so we do not report their results.
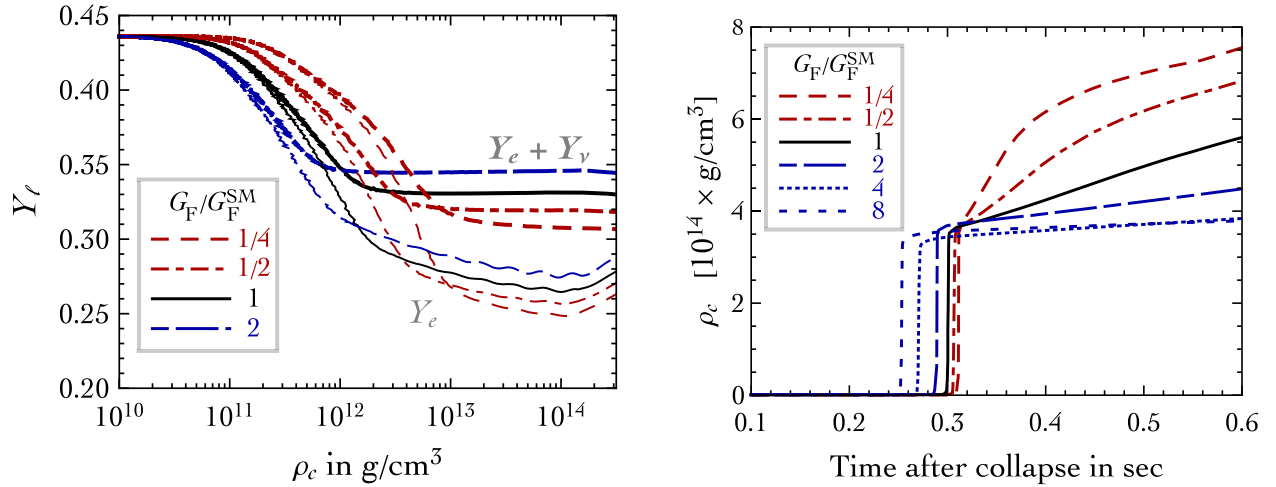
FIG. 3. Left: electron fraction Y_e (thin curves) and total lepton fraction $Y_\ell = Y_e + Y_\nu$ (thick curves) as function of matter density in the SN center, in the prebounce phase after collapse. Right: Time evolution of the density in the SN center for different values of the Fermi constant $G_F = \{1/4, 1/2, 1, 2, 4, 8\}G_F^{SM}$. We simulated a SN with $M = 15 \, M_\odot$ running the code in [49].

different timescales: the code has been optimized for the physical value $v = v_{SM}$. By changing v optimization gets lost, until numerical issues get out of control (especially for larger $G_F \gtrsim$ few $\times G_F^{SM}$). We thereby limit ourselves to run the code for relatively small deviations of v from v_{SM}, and emphasize that the expertise of the authors of numerical SN codes seems needed for fully reliable simulations. With this caveat in mind, we describe our results.

We start numerical simulations from a fixed star configuration with mass $M = 15 \, M_\odot$ and simulate the initial collapse—a phase nearly universal independently on the progenitor. Weak interactions start playing a crucial role providing deleptonization: simulations indicate that Y_e decreases in time triggering the collapse of the *inner* part of the core in roughly the usual way, see Fig. 3: for smaller G_F deleptonization proceeds slower; furthermore deleptonization stops later because a larger density is needed to trap neutrinos: as a result the final Y_e becomes only slightly lower. Thereby, for all simulated v, the shock wave must cross the outer part of the core and can stall.

Figure 4 shows the time evolution of the key radii discussed above, for bigger (left) and smaller (right) values of v. We find that, as expected, increasing G_F increases $R_\nu \propto G_F^{1/2}$ [Eq. (9)]. Figure 5 also shows that the heating rate is smaller for large G_F as expected in view of the larger gain radius $R_{gain} \sim$ few $\times R_\nu$. Furthermore, Figs. 4 and 5 show that the timescale of neutrino cooling grows for larger G_F, as expected. Figure 6 additionally shows that the luminosity of emitted neutrinos scales as expected in Eq. (10), and that their average energy scales as expected in Eq. (9). Numerical simulations thereby confirm the expected scalings.

While our results indicate that supernova explosions need values of the weak scale around its physical value, we have not presented analytic estimates about the behavior of

the shock wave around the physical value of G_F, as this is a complicated issue and because nonsphericity seems anyhow needed for explosions. However, we can discuss results of numerical simulations that also compute how the above changes in the dynamics of supernovæ affect the evolution of the shock radius. The most evident result that emerges from Fig. 4 is that the shock radius R_{shock} is smaller (bigger) for decreasing (increasing) values of G_F. To understand this behavior, it is instructive to consider the deleptonization of the material behind the shock. This is indeed an important physical quantity that keeps track of the evolution of the shock. The reason is the following. As the shock propagates, it loses energy by dissociating heavy nuclei into nucleons; in turn, this change of composition favors electron-capture processes because the electron capture rate on free protons is significantly larger (because of its much smaller Q-value) than on neutron-rich elements. Consequently, the outward-moving shock leaves behind, in its passing, a smaller value of the electron fraction Y_e. We summarize our results in Fig. 7 below. In the top panel we show the value of Y_e as a function of the enclosed mass M at different instants of time t for the Standard Model value of G_F. The deleptonization effect is evident going from the prebounce phase (dashed lines) to the postbounce phase (solid lines). In addition, the plot shows that it is possible, in the postbounce phase, to identify the sharp rise in Y_e with the position of the shock front in terms of M. We checked that this alternative definition precisely matches the values of R_{shock} in our Fig. 4. We now turn to discuss the G_F-dependence of R_{shock}. To gain some insight in this direction, we computed the matter pressure as a function of the radial distance in the postbounce phase (with the corresponding time indicated with the subscript t_{pb}). We show our results in the bottom panel of Fig. 7. The sharp discontinuity in the radial profile of matter pressure indicates the position of the
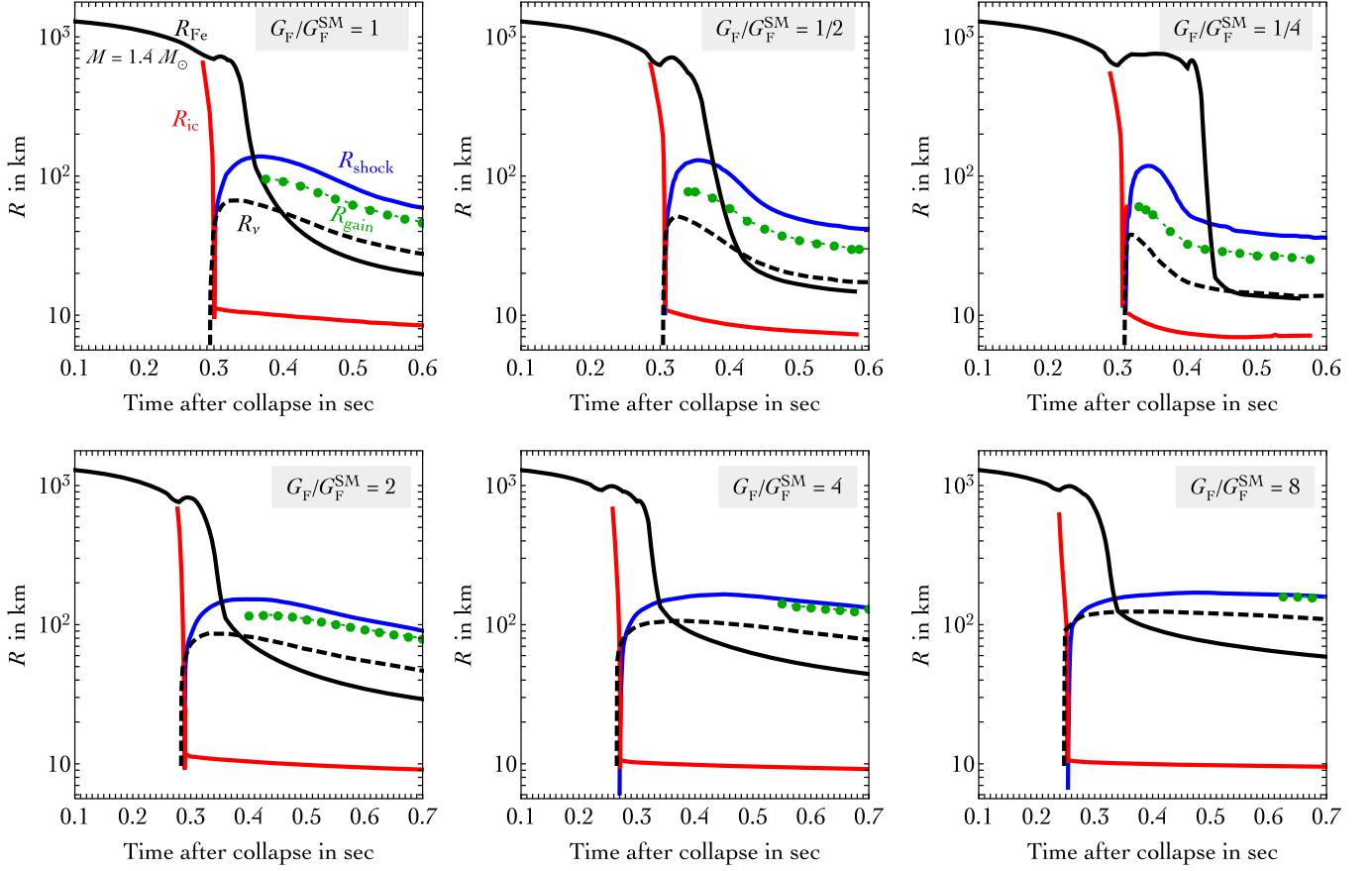
FIG. 4. Time evolution of the relevant radii for $G_F = \{1/4, 1/2, 1, 2, 4, 8\} G_F^{SM}$. We plot R_{Fe} (radius of the iron core, in black), R_{ic} (radius of the inner iron core, in red), R_{shock} (radius of the shock wave, in blue), R_ν (radius of the neutrino-sphere, black-dashed), and R_{gain} (above which neutrino heating wins over cooling, green-dotted). We simulated the collapse of a star with total mass 15 $M_\odot$ running the code in [49]. As a proxy for R_{Fe} we plot the radius that encloses the mass $M = 1.4\, M_\odot$. Before the bounce, the inner core radius R_{ic} delimits the region of subsonic collapse from the supersonic outer core; after the bounce, it represents the compact inner region of the nascent neutron star (here defined as the radial distance at which the entropy per baryon equals 3 [50]).

shock front. To facilitate the comparison among different values of G_F, we plot the matter pressure as a function of the radial distance in units of R_{shock}. Right after the bounce at $t_{pb} = 0.01s$ (left panel), the radial profile of matter pressure does not significantly depend on G_F. The situation changes if we consider, for instance, $t_{pb} = 0.25s$ (right panel). As relativistic electrons dominate the matter pressure in the region behind the shock, larger values of G_F correspond to a more efficient electron-capture rate thus decreasing the matter pressure. The pressure in the region beyond the shock is also modified (being related to the value behind the shock as a consequence of conservation of mass, momentum and energy across the discontinuity of the shock front, as dictated by Rankine-Hugoniot conditions). The increased value of pressure makes the pressure gradient more negative, and, consequently, it pushes the shock inwards. To justify this point, remember that in general relativity for a perfect fluid with mass-energy density $\varepsilon(r)$ and pressure $P(r)$, the Tolman-Oppenheimer-Volkoff equation reads

$$\frac{dP(r)}{dr} = -\frac{G_N[P(r) + \varepsilon(r)]}{r[r - 2G_N m(r)]}[m(r) + 4\pi r^3 P(r)], \quad (18)$$

with G_N the Newton's constant and $m(r)$ representing the mass-energy inside the shell of radial coordinate r. Although written in a stationary approximation that is not suitable for our purposes, the right-hand side of Eq. (18) shows that pressure gravitates exactly like the mass-energy distribution, thus contributing to the inward-directed pull of gravity.

Finally, we see from Figs. 4 and 5 that for $v \sim v_{SM}$ R_{shock} manages to reach the gain region, $R_{shock} \gtrsim R_{gain} \propto G_F^{1/2}$. Furthermore, the numerical code of [49] stops working at values of G_F so large that the neutrino-sphere becomes comparable to the stalled shock wave: simulating point 4 above would need a dedicated code. The numerical simulations of Fig. 4 confirm that the critical v_{trap} is a factor of few above the physical value $v_{SM} = 174$ GeV, but cannot determine it precisely.
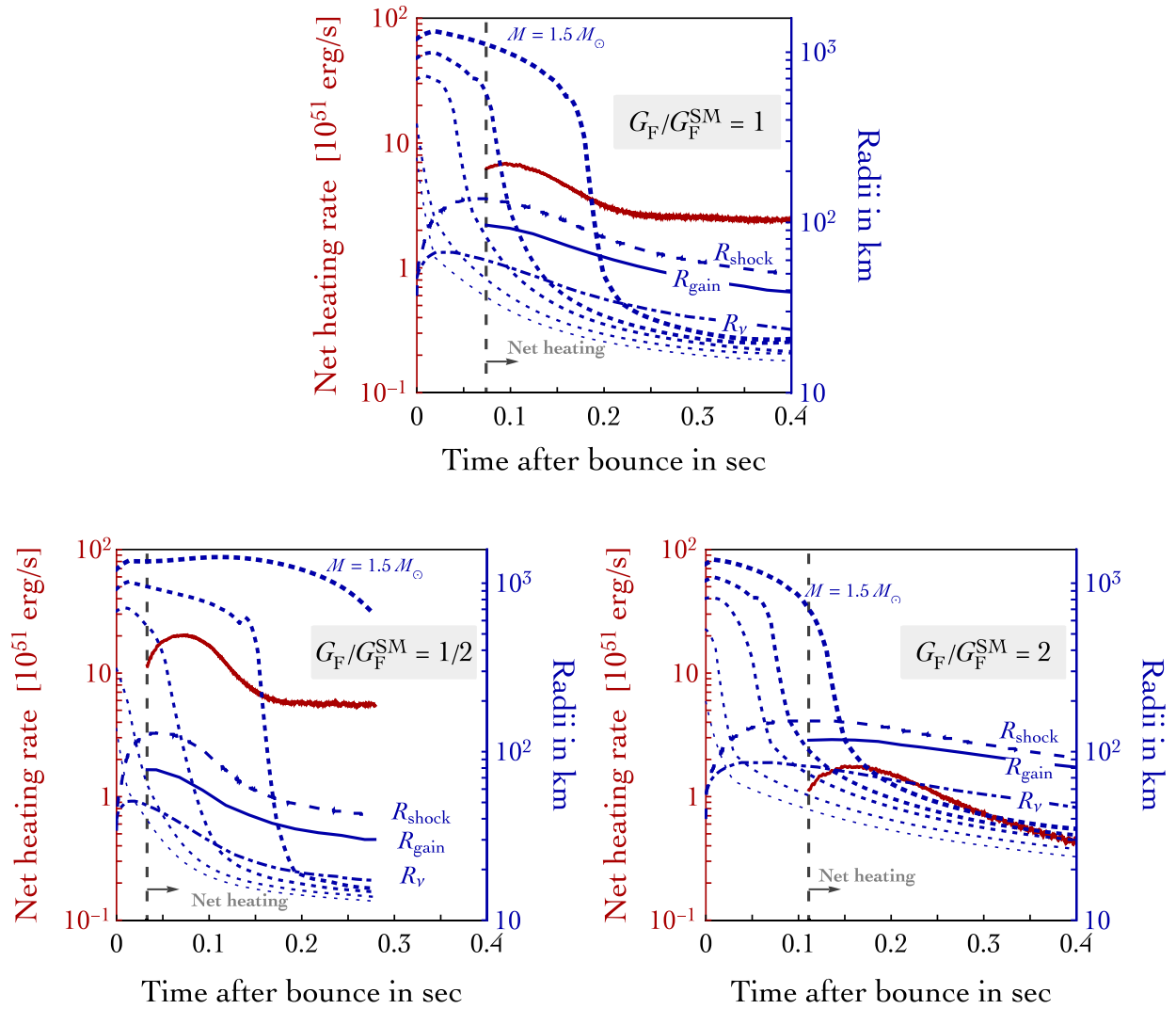
FIG. 5. Time evolution of the total net heating rate for $G_F = \{1/2, 1, 2\}G_F^{SM}$ (left-side of the y-axes, in red). We also show (right-side of the y-axes, in blue) the gain radius and, again, the shock and neutrino-sphere radii. Notice the qualitative scaling $Q_\nu \propto 1/r^2$ as in Eq. (13) neglecting cooling in Q_ν. The dotted blue curves show the contours of constant baryonic mass (in progression, from thicker to thinner, $M = \{1.5, 1.45, 1.4, 1.3, 1.2\}\, M_\odot$: the shock wave prevents the fall of the outer $\sim 15\, M_\odot$ during the simulated 0.4 sec).
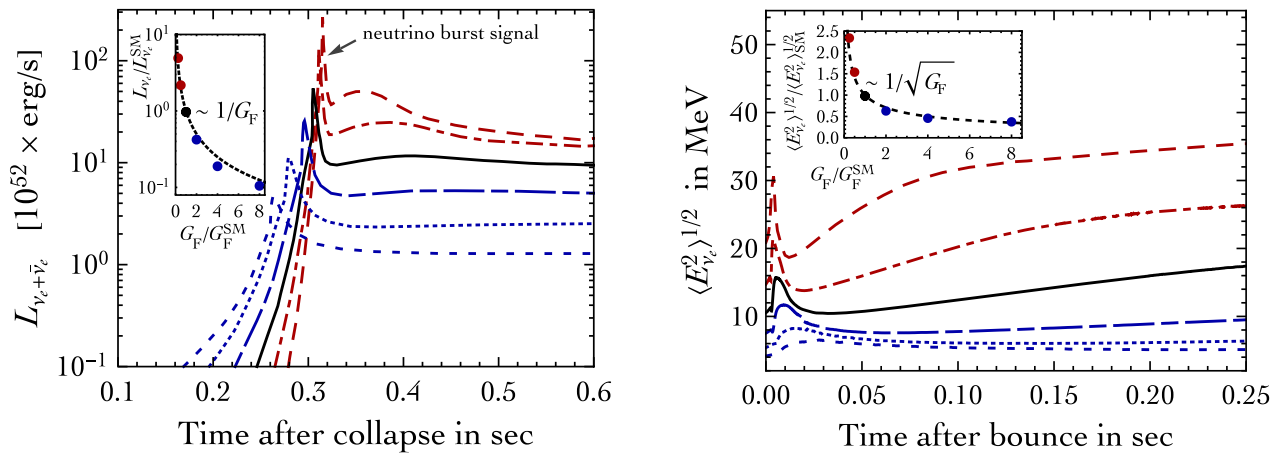

FIG. 6. Time evolution of the power emitted in electron neutrinos (left) and of their average quadratic energy (right) for different values of the Fermi constant $G_F = \{1/4, 1/2, 1, 2, 4, 8\}G_F^{SM}$. We simulated a SN with $M = 15\, M_\odot$ running the code in [49].
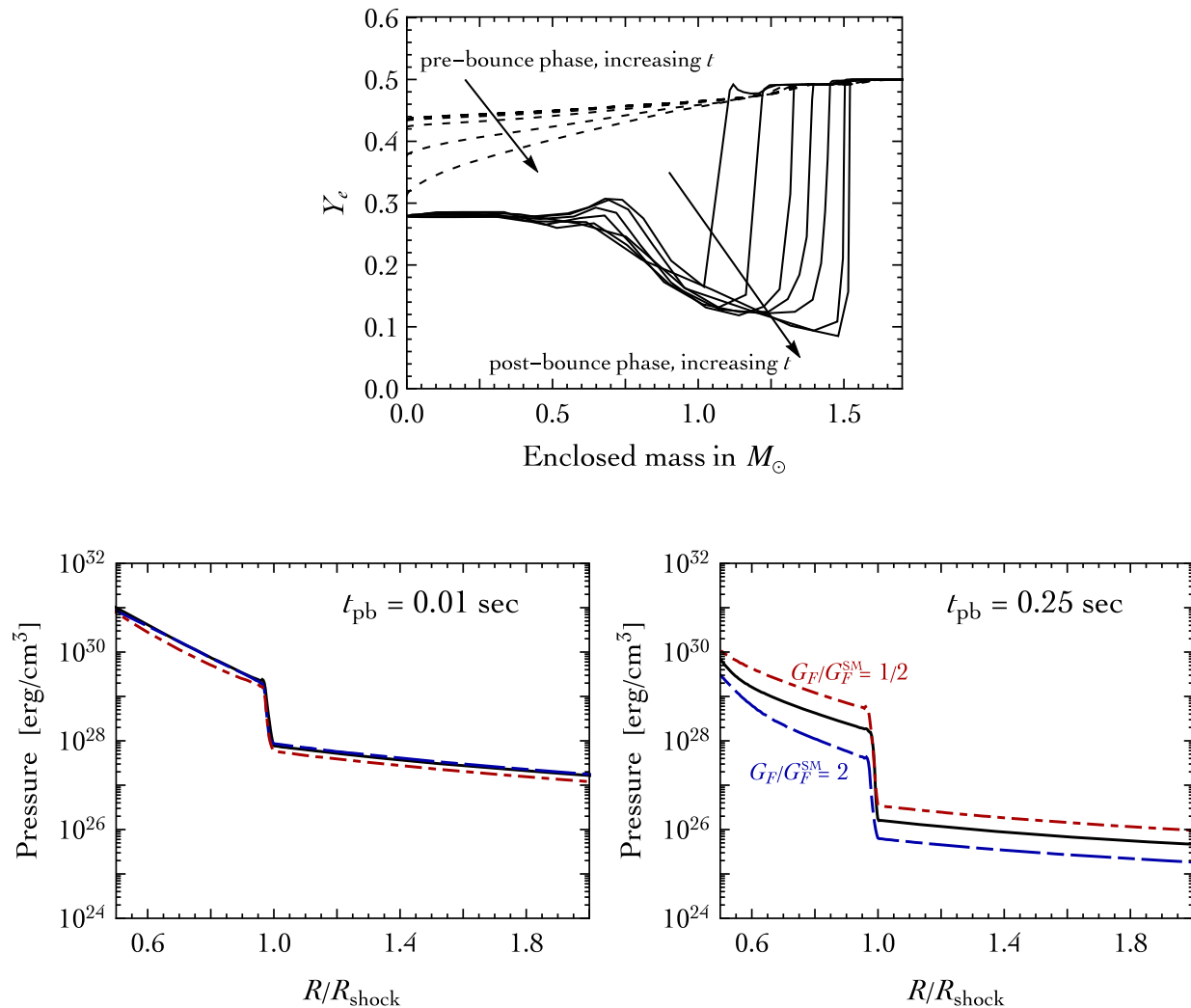
FIG. 7. Top: Electron fraction as a function of the enclosed mass for $G_F = G_F^{SM}$ and for different instants of time with dashed (solid) lines referring to the pre(post)bounce phase. Bottom: Matter pressure as a function of the radial distance R (in units of R_{shock}) for different values of G_F ($G_F = G_F^{SM}/2$ and $G_F = 2G_F^{SM}$, see labels) evaluated at postbounce time $t_{pb} = 0.01s$ (left panel) and $t_{pb} = 0.25s$ (right panel).

Indeed, we never get any explosion in spherical approximation [37]. As well known from simulations at the physical value of the weak scale, explosions seems to need 2d [38–41] or 3d [42–45] simulations, that are computationally much more intensive than simulations in spherical approximation. We hope that experts in supernova physics can test our main findings: that neutrino-driven SN explosions happen in a restricted range of the weak scale v that contains its physical value v_{SM}. The upper bound of v is especially important for fundamental theory, given that it seems to have anthropic relevance. Furthermore, once explosions are simulated, it would be interesting to compute the fraction of the total energy that explodes into nuclei as function of v. At the physical v_{SM} this fraction is about 1%: SN explosions spread the elements needed for life, and also damage life in the nearby ~ 30 ly (in the solar

neighborhood this corresponds to a rate comparable to the time span of life, about 0.5 Gyr).

III. STELLAR EVOLUTION

A too small v is anthropically excluded because ordinary matter at temperature T would cool too fast losing energy into neutrinos with mass $m_\nu \lesssim T$, with a timescale $\tau_{cool} \sim v^4/\alpha^3 (m_e T)^{3/2}$. Complex chemistry and life is possible at the "ambient" temperature comparable to the binding energy of atoms, $T \sim \alpha^2 m_e$. At this temperature $\tau_{cool} \sim v^4/\alpha^8 m_e^5$ is much larger than the age of the Universe. Furthermore, matter is heated by stars. Since stars have higher temperature, weak interactions play a role in stellar evolution. A nonstandard value of v would modify stellar evolution in two ways: by changing

(1) energy losses into neutrinos, and

(2) weak interactions that contribute to star burning.

The second factor is not crucial, because the cross sections that depend on the weak scale, also depend more strongly on the Coulomb barrier factor $e^{-3E_{\text{Gamow}}(T)/T}$ such that a steady state of stellar burning is restored by a small change in T. Even in the extreme weakless limit, stars can anyhow burn through purely nuclear interactions which do not involve weak interactions. In particular, BBN at large v leads to an equal number of neutrons and protons, i.e., to Helium; Helium nuclei can burn to $^{12}_{6}C$ through the triple α process mediated by the Hoyle resonance.

The first factor depends strongly on the stellar temperature and density. Considering, for example, the Sun, only a small (10^{-10}) fraction of its energy is emitted as thermal radiation of neutrinos with energy comparable to its central temperature $T = 1.3$ keV [51].[7] Since the thermal neutrino rate is proportional to $1/v^4$, a small $v \lesssim v_{\text{SM}}/300$ would modify the solar behavior. Different processes ($eZ \to eZ\nu\bar{\nu}$, $e\gamma \to e\nu\bar{\nu}$, pair production, plasmon decay $\gamma \to \nu\bar{\nu}$) dominate energy losses into neutrinos for different stellar temperatures and densities, and their rates grow with the temperature proportionally to T^{3-9}, depending on the process. For the physical value of v, neutrino radiation dominates energy losses of stars with central temperatures hotter than $T \gtrsim 50$ keV, making the later stages of stellar evolution very fast.

The stellar temperature is determined by the stellar mass (and chemical composition) as dictated by stellar evolution. All stars have masses around M_{Pl}^3/m_p^2 due to physics which does not involve weak nor strong interactions [20,52]. More precisely, star masses M must lie in a range $M_{\text{min}} < M < M_{\text{max}}$. The minimal stellar mass $M_{\text{min}} \sim (T_{\text{nuc}}/m_e)^{3/4} M_{\text{Pl}}^3/m_p^2$ arises because the star must reach the critical temperature $T_{\text{nuc}} \sim \alpha^2 m_n$ before that nuclear reactions proceed igniting the star. The maximal stellar mass $M_{\text{max}} \sim M_{\text{Pl}}^3/m_p^2$ arises because radiation pressure $p \sim T^4$ dominates if T is too large, making stars unstable. Precise computations find $M_{\text{min}} \approx 0.08\,M_\odot$ and $M_{\text{max}} \approx 100\,M_\odot$ at the physical values of the fundamental constants [52]. Varying M_{Pl} only, order one factors change and the range closes ($M_{\text{min}} = M_{\text{max}}$) and stars disappear if $M_{\text{Pl}} \lesssim M_{\text{Pl}}^{\text{SM}}/100$ [52]. A stronger limit $M_{\text{Pl}} \lesssim M_{\text{Pl}}^{\text{SM}}/12$ arises demanding that stars are long-lived enough [53].

We studied how stellar evolution changes for different values of v, finding that for wide ranges of v stars settle to different steady-state regimes, which can be slow enough to support life as well as fast enough to produce the first nuclei.[8] We do not document our findings, as the same conclusion has been recently reached in [54]. While stellar evolution depends on v (such that the weak constant could be measured from stellar data), no anthropic boundary on v is found from stellar evolution. Presumably the only anthropic boundary is $M_{\text{Pl}} \gtrsim M_{\text{Pl}}^{\text{SM}}/100$ if both v and M_{Pl} are varied.

IV. CONCLUSION AND DISCUSSION

Anthropic arguments, despite their controversial reputation, are important for indicating main directions in fundamental physics. We explored whether anthropic selection played a role in selecting the value of the weak scale v. As discussed in the introduction, fermion masses $m_f = y_f v$ are anthropically relevant, but they depend on Yukawa couplings y_f times v, so that they do not directly restrict v, leading to a paradoxical situation.

We focused on physical processes *directly* affected by weak interactions: the ones of possible anthropic relevance are BBN, stellar evolution and core-collapse supernovæ. We studied what changes if the Higgs vacuum expectation value v (and thereby the Fermi constant $G_F = 1/(2\sqrt{2}v^2)$ that controls weak interactions) differs from its physical value, with the quark and lepton masses kept fixed.

BBN was studied in [22] and stellar evolution in [54]: they do not seem to lead to anthropic boundaries on v. Indeed, stellar evolution changes qualitatively if v is changed by more than one order in magnitude in either direction, but stars still burn in a slow stable way. For large v weak nuclear processes (such as pp burning) no longer lead to stable stars, which anyhow continue existing thanks to strong nuclear processes (such as the triple α process). Stellar dynamics seem to lead to a weak anthropic bound on the Planck mass: it must be larger than 1% of its observed value [52].

The situation with supernovæ (SN) seems more interesting, as core-collapse supernova explosions seem anthropically relevant and driven by weak interactions. We assume

(i) the dominant (but not necessarily correct) paradigm according to which core-collapse SN explosions happen thanks to weak interactions: neutrinos push the stalled shock wave generated by the rebounce of the inner core when the supernova reaches nuclear density, such that the rejuvenated shock manages to spread the outer SN material.

(ii) that the observed core-collapse SN explosions are anthropically relevant, given that they are the largely

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[7]The sun emits another $\approx 3\%$ of its energy in MeV neutrinos, because the same nuclear/weak interactions that produce the solar energy (in particular $pp \to d\bar{e}\nu_e$) also emit neutrinos. Such neutrinos cannot be considered as energy loss.

[8]Stellar evolution has been studied in collaboration with Giada Valle, Matteo Dell'Omodarme, Scilla Degl'Innocenti and Pier Giorgio Prada Moroni.
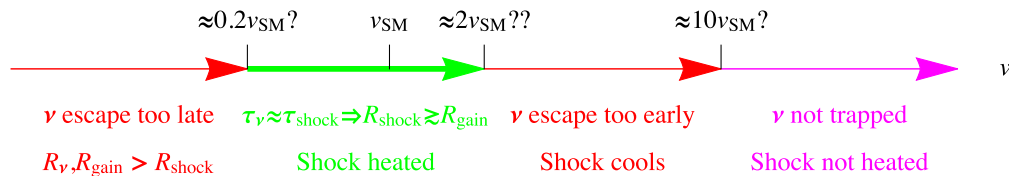
FIG. 8.   Behavior of core-collapse supernovæ as function of the weak scale v. Although numerical factors are only indicative, we conclude that neutrino-driven SN explosions arise in a narrow range of v.

dominant process that spreads elements possibly needed for life, in particular oxygen (see Fig. 1). The two points above are plausible, but establishing them is difficult. Just to be very clear, we repeat that our subsequent discussion relies on the assumption that light elements needed for life are significantly produced only by neutrino-driven explosions of core-collapse supernovæ.

We argued that core-collapse supernovæ no longer explode if v is increased or decreased by a factor of few, as illustrated in Fig. 8. For the physical value of v, the timescale of neutrino trapping matches the gravitational timescale of the supernova (with a related matching of the spatial scales).

(i) Increasing v reduces weak interactions, such that neutrinos escape too fast and no longer push the shock wave when it stalls.

   Ultimately, at large $v \gtrsim 10 v_{\rm SM}$ neutrinos become not trapped.

(ii) Decreasing v increases weak interactions, such that neutrinos exit too late for rejuvenating the shock wave before the collapse of the exterior material.

   Ultimately, at small $v \lesssim 0.2 v_{\rm SM}$ the gain-sphere (the region where neutrinos push matter) and the neutrino-sphere (which is the region where neutrinos are trapped) grow bigger than the shock wave, such that neutrinos no longer push the shock outwards.

We provided analytic estimates that capture supernova physics, disentangling adaptive dynamical features from accidental numerical coincidences. For example, the energy transmitted by interactions of trapped neutrinos is comparable to the total energy, independently of the value of v. Disentangling dynamics from tunings is needed to correctly identify anthropic features [55]. We validated aspects of our analytic understanding relying on numerical simulations in spherical approximation. However this approximation does not lead to supernova explosions. As well known, supernova explosions at $v = v_{\rm SM}$ seem so much critical that nonspherical simulations are needed to get enhancements by a few 10% which lead to explosions. We hope that dedicated work by experts can firmly establish (or revise) our results.

This finding has important implications for fundamental physics. As hinted by our subtitle, like most theorists we would have preferred an understanding of the weak scale based on natural supersymmetry rather than on anthropic supernovæ. Paraphrasing Bohr, anthropic arguments work even when physicists do not believe in them. As discussed in the Introduction, our direct anthropic boundary on the weak scale (if confirmed by future studies) avoids the paradox raised by previous anthropic bounds: on fermion masses $m_f = y_f v$: a SM-like theory with fixed m_f and bigger v would need a less unlikely tuning of $v^2 / M_{\rm Pl}^2$.

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