Symmetry and geometry in a generalized Higgs effective field theory: Finiteness of oblique corrections versus perturbative unitarity

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We formulate a generalization of Higgs effective field theory (HEFT) including an arbitrary number of extra neutral and charged Higgs bosons—a generalized HEFT (GHEFT)—to describe nonminimal electroweak symmetry breaking models. Using the geometrical form of the GHEFT Lagrangian, which can be regarded as a nonlinear sigma model on a scalar manifold, it is shown that the scalar boson scattering amplitudes are described in terms of the Riemann curvature tensor (geometry) of the scalar manifold and the covariant derivatives of the potential. The coefficients of the one-loop divergent terms in the oblique correction parameters S and U can also be written in terms of the Killing vectors (symmetry) and the Riemann curvature tensor (geometry). It is found that the perturbative unitarity of the scattering amplitudes involving the Higgs bosons and the longitudinal gauge bosons demands that the scalar manifold be flat. The relationship between the finiteness of the electroweak oblique corrections and the perturbative unitarity of the scattering amplitudes is also clarified in this language: we verify that once the tree-level unitarity is ensured, the one-loop finiteness of the oblique correction parameters S and U is automatically guaranteed.

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I. INTRODUCTION

What is the origin of electroweak symmetry breaking (EWSB)? In the standard model (SM) of particle physics, EWSB is caused by a vacuum expectation value of a complex scalar field (the SM Higgs field), which linearly transforms under the $SU(2)_W \times U(1)_Y$ electroweak gauge symmetry. The Higgs sector of the SM is constructed to be minimal, as it includes only a scalar boson (SM Higgs boson) and three would-be Nambu-Goldstone bosons eaten by massive gauge bosons after EWSB. There are no cousin particles of the Higgs in the SM. The scalar particle discovered by the ATLAS and CMS experiments in 2012 with a mass of 125 GeV [1,2] can now be successfully interpreted as the SM(-like) Higgs boson.

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The Higgs sector in the SM, however, does not ensure the stability of the EWSB scale against quantum corrections. In other words, the SM itself cannot explain why the EWSB scale is of the order of 100 GeV, which is much smaller than its cutoff scale, e.g., the Planck scale ($\sim 10^{18}$ GeV) or the grand unification scale ($\sim 10^{16}$ GeV). Thus, the SM Higgs sector is inherently incomplete and should be extended. Many extensions/generalizations of the SM Higgs sector—such as the two-Higgs-doublet model [3–24], composite Higgs models [25–34], and the Georgi-Machacek model [35–38]—have been proposed. The 125 GeV Higgs boson accompanies extra Higgs particles in these scenarios.

The effective field theory (EFT) approach is widely used to study these beyond-the-SM (BSM) physics in a model-independent manner. The physics below 1 TeV can be described by the standard model effective field theory (SMEFT) [39–66], which parametrizes the BSM contributions using the coefficients of SM field higher-dimensional operators. The SMEFT is successful if the BSM particles are much heavier than 1 TeV and they decouple from the low-energy physics. The SMEFT cannot be applied, however, if the heavy BSM particles do not decouple from the low-energy physics. The Higgs effective field theory (HEFT) [67–83] should be applied instead. These existing EFTs cannot be applied if there exist BSM particles lighter than 1 TeV. We should include these BSM particles explicitly in the EFT approach.

In this paper, we propose a generalization of the HEFT (GHEFT) for this purpose. As in the HEFT, the GHEFT is based on electroweak chiral perturbation theory (EWChPT) [84–89]. In the GHEFT, the BSM particles, as well as the 125 GeV Higgs boson, are introduced as matter particles in the Callan-Coleman-Wess-Zumino (CCWZ) construction [90–92] of EWChPT.

Note that the longitudinal gauge boson scattering amplitudes exceed perturbative unitarity limits at high energy in the EWChPT. The GHEFT couplings should satisfy special conditions—known as the unitarity sum rules [93–95]—to keep the amplitudes perturbative in the high-energy scatterings if the model is considered to be ultraviolet (UV) complete. We also note that the EWChPT is not renormalizable. The UV-completed GHEFT couplings should satisfy the finiteness conditions in order to cancel these UV divergences.

The GHEFT can also be described in a geometrical language using the scalar manifold metric, as discussed in Refs. [96,97] in the HEFT context. We point out that both the scalar scattering amplitudes and the one-loop UV divergences in the electroweak oblique correction parameters S and U [98] are described by using the Riemann curvature tensor (geometry) and the Killing vectors (symmetry) of the scalar manifold. Therefore, both the unitarity sum rules and the oblique correction finiteness conditions are described in terms of geometry and symmetry. We find that perturbative unitarity is ensured by the flatness of the scalar manifold (vanishing Riemann curvature). We also find that the divergences in the oblique correction parameters (S and U parameters) are canceled if a subset of the perturbative unitarity conditions and the $SU(2)_W \times U(1)_Y$ gauge symmetry are satisfied. These findings generalize our previous observation [99] which relates perturbative unitarity to the one-loop finiteness of the oblique correction parameters.[1]

This paper is organized as follows. In Sec. II we introduce the GHEFT Lagrangian at its lowest order [$\mathcal{O}(p^2)$]. We investigate the scalar boson scattering amplitudes in Sec. III. Sections IV and V present one-loop computations with and without the gauge boson contributions. The relationship between perturbative unitarity and the one-loop finiteness of the oblique correction parameters is clarified in Sec. VI. We conclude in Sec. VII.

II. GENERALIZED HEFT LAGRANGIAN OF $SU(2)_W \times U(1)_Y \to U(1)_{\rm em}$

EWChPT [84–89] provides a systematic framework to describe the low-energy phenomenologies of electroweak symmetry breaking physics. It utilizes the electroweak chiral Lagrangian method for parametrizing the nondecoupling corrections, which appear ubiquitously in models with a strongly interacting electroweak symmetry breaking sector. Although the original version of the EWChPT was constructed to be a Higgsless theory [84–87, 94,95,105–114], after the discovery of the 125 GeV Higgs particle the EWChPT was extended to the HEFT [67–83], incorporating the 125 GeV Higgs particle h as a neutral spin-0 matter particle in the electroweak chiral Lagrangian. Introducing functions the $\mathcal{F}(h)$ and $V(h)$ which parametrize the phenomenological properties of the 125 GeV Higgs, the HEFT provides a systematic description of a neutral spin-0 particle in the electroweak symmetry breaking sector, including the one-loop radiative corrections [72–83]. It can parametrize the low-energy properties of the 125 GeV Higgs particle in both the strongly and weakly interacting model context.

We need to generalize the HEFT further if we want to introduce extra Higgs particles other than the discovered 125 GeV Higgs particle. It is not trivial to introduce nonsinglet extra particles into the EWChPT, however, since the electroweak gauge symmetry $SU(2)_W \times U(1)_Y$ is realized nonlinearly in the EWChPT. The interaction Lagrangian needs to be arranged carefully to make the theory invariant under the electroweak gauge symmetry $SU(2)_W \times U(1)_Y$.

These extra nonsinglet Higgs particles can be regarded as matter particles in the EWChPT Lagrangian context. The CCWZ formulation [90–92] provides an ideal framework for the concrete construction of the matter particle interaction Lagrangian in a manner consistent with the symmetry structure of the nonlinear sigma model. See, e.g., Refs. [115,116] for earlier studies on these nonsinglet matter particles in QCD chiral perturbation theory and EWChPT, respectively.

In this section, we use the CCWZ formulation to construct the GHEFT Lagrangian.

A. Electroweak chiral Lagrangian

For simplicity, in this subsection we consider the EWChPT Lagrangian in the gaugeless limit, i.e., $g_W = g_Y = 0$. The couplings with the electroweak gauge fields will be introduced in Sec. II C. The electroweak symmetry $G = [SU(2)_W \times U(1)_Y]$ is broken spontaneously to the $H = U(1)_{\rm em}$ symmetry in the SM Higgs sector. The most general scalar sector Lagrangian consistent with the symmetry-breaking structure $G/H = [SU(2)_W \times U(1)_Y]/U(1)_{\rm em}$ can be constructed as the CCWZ nonlinear sigma model Lagrangian on the coset space G/H. The coset manifold $G/H = [SU(2)_W \times U(1)_Y]/U(1)_{\rm em}$ is parameterized by the Nambu-Goldstone (NG) boson fields π^a ($a = 1, 2, 3$) as

$$\xi_W(x) = \exp\left(i \sum_{a=1,2} \pi^a(x) \frac{\tau^a}{2}\right), \qquad (1)$$

[1] Possible relations between unitarity and renormalizability have also been investigated in gravity models. See Refs. [100–104].

$$\xi_Y(x) = \exp\left(i\pi^3(x)\frac{\tau^3}{2}\right), \tag{2}$$

where τ^a ($a = 1, 2, 3$) are the Pauli spin matrices. Under the $G = [SU(2)_W \times U(1)_Y]$ transformation,

$$\mathfrak{g}_W \in SU(2)_W, \qquad \mathfrak{g}_Y \in U(1)_Y, \tag{3}$$

these NG boson fields transform as

$$\xi_W(x) \to \xi'_W(x) = \mathfrak{g}_W \xi_W(x) \mathfrak{h}^\dagger(\pi, \mathfrak{g}_W, \mathfrak{g}_Y), \tag{4}$$

$$\xi_Y(x) \to \xi'_Y(x) = \mathfrak{h}(\pi, \mathfrak{g}_W, \mathfrak{g}_Y)\xi_Y(x)\mathfrak{g}_Y^\dagger. \tag{5}$$

Here $\mathfrak{h}(\pi, \mathfrak{g}_W, \mathfrak{g}_Y)$ is an element of the unbroken group H, which is determined to pull the coset space coordinates (ξ_W, ξ_Y) back to their original forms (1) and (2). Note that the H transformation $\mathfrak{h}(\pi, \mathfrak{g}_W, \mathfrak{g}_Y)$ depends not only on the $SU(2)_W$ and $U(1)_Y$ elements $\mathfrak{g}_W$ and $\mathfrak{g}_Y$, but also on the NG boson fields $\pi(x)$. The NG boson fields π^a ($a = 1, 2, 3$) therefore transform nonlinearly under the G symmetry.

It is useful to introduce objects called Maurer-Cartan (MC) one-forms, $\alpha^a_{\perp\mu}$ ($a = 1, 2, 3$), defined as

$$\alpha^a_{\perp\mu} = \mathrm{tr}\left[\frac{1}{i}\xi_W^\dagger(\partial_\mu \xi_W)\tau^a\right] \quad (a = 1, 2) \tag{6}$$

and

$$\alpha^3_{\perp\mu} = \mathrm{tr}\left[\frac{1}{i}\xi_W^\dagger(\partial_\mu \xi_W)\tau^3\right] + \mathrm{tr}\left[\frac{1}{i}(\partial_\mu \xi_Y)\xi_Y^\dagger \tau^3\right]. \tag{7}$$

Although the NG boson fields π transform nonlinearly, these MC one-forms transform homogeneously, i.e.,

$$\sum_{a=1,2}\alpha^a_{\perp\mu}\frac{\tau^a}{2} \to \mathfrak{h}(\pi, \mathfrak{g}_W, \mathfrak{g}_Y)\left(\sum_{a=1,2}\alpha^a_{\perp\mu}\frac{\tau^a}{2}\right)\mathfrak{h}^\dagger(\pi, \mathfrak{g}_W, \mathfrak{g}_Y), \tag{8}$$

$$\alpha^3_{\perp\mu}\frac{\tau^3}{2} \to \mathfrak{h}(\pi, \mathfrak{g}_W, \mathfrak{g}_Y)\left(\alpha^3_{\perp\mu}\frac{\tau^3}{2}\right)\mathfrak{h}^\dagger(\pi, \mathfrak{g}_W, \mathfrak{g}_Y), \tag{9}$$

under the G symmetry. We see that the MC one-forms transform as

$$\alpha^a_{\perp\mu} \to [\rho_\alpha(\mathfrak{h})]^a{}_b \alpha^b_{\perp\mu}, \tag{10}$$

with $\rho_\alpha(\mathfrak{h})$ being a 3×3 matrix,

$$\rho_\alpha(\mathfrak{h}) = \exp(i\theta_h(\pi, \mathfrak{g}_W, \mathfrak{g}_Y)Q_\alpha),$$
$$\mathfrak{h} = \exp\left(i\theta_h(\pi, \mathfrak{g}_W, \mathfrak{g}_Y)\frac{\tau^3}{2}\right). \tag{11}$$

In Eq. (10) and hereafter, the summation $\sum_{b=1,2,3}$ is implied whenever an index b is repeated in a product. Here the NG boson charge matrix Q_α is defined by

$$Q_\alpha = \begin{pmatrix} -\sigma_2 & \\ & 0 \end{pmatrix}, \tag{12}$$

with σ_2 being the Pauli spin matrix,

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ +i & 0 \end{pmatrix}. \tag{13}$$

It is now straightforward to construct the lowest-order $[\mathcal{O}(p^2)]$ G-invariant Lagrangian of the NG bosons:

$$\mathcal{L}_\pi = \frac{1}{2}G^{(0)}_{ab}\alpha^a_{\perp\mu}\alpha^{b\mu}_\perp, \tag{14}$$

with

$$G^{(0)}_{ab} = \frac{1}{4}\begin{pmatrix} v^2 & & \\ & v^2 & \\ & & v_Z^2 \end{pmatrix}. \tag{15}$$

The Lagrangian can be rewritten as

$$\mathcal{L}_\pi = \frac{v^2}{4}\mathrm{tr}[(\partial_\mu U^\dagger)(\partial^\mu U)] - \frac{v_Z^2 - v^2}{8}\mathrm{tr}[U^\dagger(\partial_\mu U)\tau^3]\mathrm{tr}[U^\dagger(\partial^\mu U)\tau^3], \tag{16}$$

with

$$U \coloneqq \xi_W \xi_Y. \tag{17}$$

It should be emphasized here that v and v_Z (the decay constants of $\pi^{1,2}$ and π^3) are independently adjustable parameters in the EWChPT on the $G/H = [SU(2)_W \times U(1)_Y]/U(1)_{\mathrm{em}}$ coset space. The phenomenologically preferred relation

$$\rho \coloneqq \frac{v^2}{v_Z^2} \simeq 1 \tag{18}$$

is realized only by a parameter tuning as $v \simeq v_Z$ in this setup.[2]

[2]It is possible to introduce custodial symmetry to justify the tuning. The standard model, in fact, possesses the custodial symmetry in its gaugeless and Yukawa-less limit. See, e.g., Ref. [117] for the HEFT power-counting rules and how custodial-symmetry-violating terms are organized therein. We do not introduce the custodial symmetry here, however, since it is not relevant to the main findings in the present paper. The restrictions on the GHEFT Lagrangian parameters coming from the custodial requirements and their power-counting rules will be studied in a separate publication.

B. Matter particles coupled with the electroweak chiral Lagrangian

Thanks to the homogeneous transformation properties of the MC one-forms (10), matter particles can be easily introduced into the CCWZ formulation of the EWChPT Lagrangian (16).

We consider a set of real scalar matter fields ϕ^I, which transforms homogeneously as

$$\phi^I \to [\rho_\phi(\mathfrak{h})]^I{}_J \phi^J \tag{19}$$

under the unbroken group H. Here $\rho_\phi(\mathfrak{h})$ stands for a representation matrix,

$$\rho_\phi(\mathfrak{h}) = \exp(i\theta_h Q_\phi), \qquad \mathfrak{h} = \exp\left(i\theta_h \frac{\tau^3}{2}\right), \tag{20}$$

with Q_ϕ being a Hermitian matrix. Note here that the $\mathfrak{h}$ transformation depends on the NG boson fields $\pi(x)$. It is therefore a local transformation depending on the spacetime point x. If the set of scalar matter particles consists of n_N species of neutral particles and n_C species of charged particles, the matrix Q_ϕ can be expressed as a $(2n_C + n_N) \times (2n_C + n_N)$ matrix,

$$Q_\phi = \begin{pmatrix} -q_1\sigma_2 & & & & & \\ & \ddots & & & & \\ & & -q_{n_C}\sigma_2 & & & \\ & & & 0 & & \\ & & & & \ddots & \\ & & & & & 0 \end{pmatrix}. \tag{21}$$

Here q_i $(i = 1, 2, \ldots n_C)$ are the charges of the scalar matter particles. Since $\mathfrak{h}$ is a local transformation, $\partial_\mu \phi^I$ transforms nonhomogeneously under $\mathfrak{h}$. In order to write a kinetic term for the matter field ϕ^I, we therefore introduce a covariant derivative of the matter field ϕ^I:

$$(\mathcal{D}_\mu\phi)^I = \partial_\mu\phi^I + i\mathcal{V}_\mu^3[Q_\phi]^I{}_J\phi^J, \quad (I, J = 1, 2, \ldots, 2n_C + n_N). \tag{22}$$

We take the connection $\mathcal{V}_\mu^3$ as

$$\mathcal{V}_\mu^3 = -\mathrm{tr}\left[\frac{1}{i}(\partial_\mu \xi_Y)\xi_Y^\dagger \tau^3\right] + c\alpha_{\perp\mu}^3, \tag{23}$$

with c being an arbitrary constant. Hereafter we take $c = 0$ for simplicity. The covariant derivative (22) transforms homogeneously,

$$(\mathcal{D}_\mu\phi)^I \to [\rho_\phi(\mathfrak{h})]^I{}_J (\mathcal{D}_\mu\phi)^J, \tag{24}$$

as we intended. It is now straightforward to write down an $\mathcal{O}(p^2)$ EWChPT Lagrangian including additional scalar bosons with arbitrary charges:

$$\mathcal{L} = \frac{1}{2}G_{ab}\alpha_{\perp\mu}^a \alpha_\perp^{b\mu} + G_{aI}\alpha_{\perp\mu}^a (\mathcal{D}^\mu\phi)^I + \frac{1}{2}G_{IJ}(\mathcal{D}_\mu\phi)^I(\mathcal{D}^\mu\phi)^J - V. \tag{25}$$

Here G_{ab}, G_{aI}, G_{IJ}, and V are functions of the scalar fields ϕ^I. Also, G_{ab}, G_{aI}, and G_{IJ} transform homogeneously as multiplets of corresponding representations. They satisfy[3]

$$G_{ab}\Big|_{\phi=0} = G_{ab}^{(0)}, \qquad G_{aI}\Big|_{\phi=0} = 0, \qquad G_{IJ}\Big|_{\phi=0} = \delta_{IJ} \tag{26}$$

and

$$\frac{\partial}{\partial\phi^I}V\Big|_{\phi=0} = 0, \qquad \frac{\partial}{\partial\phi^J}\frac{\partial}{\partial\phi^I}V\Big|_{\phi=0} = M_I^2\delta_{IJ}, \tag{27}$$

with M_I being the ϕ^I boson mass. The second and third conditions in Eq. (26) can be achieved by redefining the scalar field ϕ^I in the Lagrangian. The first condition in Eq. (26) ensures that the extended Lagrangian (25) reproduces the lowest-order EWChPT Lagrangian (14) in the absence of Higgs particles ϕ^I. The stability around the vacuum $\phi = 0$ is guaranteed by the conditions (27).

C. Electroweak gauge fields

It is easy to introduce the electroweak gauge fields W_μ^a $(a = 1, 2, 3)$ and B_μ into our EWChPT Lagrangian (25). When the gauge coupling is switched on, we just need to replace the derivatives $\partial_\mu \xi_W$ and $\partial_\mu \xi_Y$ by the covariant derivatives:

$$D_\mu \xi_W = \partial_\mu \xi_W - i g_W W_\mu^a \frac{\tau^a}{2}\xi_W, \tag{28}$$

$$D_\mu \xi_Y = \partial_\mu \xi_Y + i g_Y \xi_Y B_\mu \frac{\tau^3}{2}, \tag{29}$$

with g_W and g_Y being the $SU(2)_W$ and $U(1)_Y$ gauge coupling strengths, respectively.

The lowest-order $[\mathcal{O}(p^2)]$ GHEFT Lagrangian is therefore

[3]Equations (26) and (27) are understood to be the tree-level matching conditions between the GHEFT and EWChPT. They may be modified beyond the tree level. See, e.g., Refs. [118,119].

$$\mathcal{L} = \frac{1}{2}G_{ab}\hat{\alpha}^a_{\perp\mu}\hat{\alpha}^{b\mu}_{\perp} + G_{aI}\hat{\alpha}^a_{\perp\mu}(\mathcal{D}^\mu\phi)^I + \frac{1}{2}G_{IJ}(\mathcal{D}_\mu\phi)^I(\mathcal{D}^\mu\phi)^J - V - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \tag{30}$$

with

$$\hat{\alpha}^a_{\perp\mu} = \mathrm{tr}\left[\frac{1}{i}\xi^\dagger_W(\partial_\mu\xi_W)\tau^a\right] - g_W\mathrm{tr}\left[\xi^\dagger_W W^b_\mu\frac{\tau^b}{2}\xi_W\tau^a\right] \quad (a=1,2) \tag{31}$$

and

$$\hat{\alpha}^3_{\perp\mu} = \mathrm{tr}\left[\frac{1}{i}\xi^\dagger_W(\partial_\mu\xi_W)\tau^3\right] + \mathrm{tr}\left[\frac{1}{i}(\partial_\mu\xi_Y)\xi^\dagger_Y\tau^3\right] - g_W\mathrm{tr}\left[\xi^\dagger_W W^b_\mu\frac{\tau^b}{2}\xi_W\tau^3\right] + g_Y B_\mu. \tag{32}$$

We define the covariant derivative of the matter fields $(\mathcal{D}_\mu\phi)^I$ as

$$(\mathcal{D}_\mu\phi)^I = \partial_\mu\phi^I + i\hat{\mathcal{V}}^3_\mu[Q_\phi]^I{}_J\phi^J, \tag{33}$$

with

$$\hat{\mathcal{V}}^3_\mu = -\mathrm{tr}\left[\frac{1}{i}(\partial_\mu\xi_Y)\xi^\dagger_Y\tau^3\right] - g_Y B_\mu. \tag{34}$$

It should be noted that the GHEFT Lagrangian (30) reproduces the HEFT Lagrangian [67–83] for $n_N = 1$ and $n_C = 0$. Here $\phi^{I=h}$ stands for the 125 GeV Higgs boson field. In the HEFT, G_{aI} and G_{IJ} are taken as

$$G_{ah} = 0, \qquad G_{hh} = 1. \tag{35}$$

G_{ab} is tuned to be

$$G_{ab} = \frac{v^2}{4}\mathcal{F}(h)\delta_{ab}. \tag{36}$$

D. Geometrical form of the $\mathcal{O}(p^2)$ GHEFT Lagrangian

The lowest-order [$\mathcal{O}(p^2)$] GHEFT Lagrangian (30) can also be expressed in a geometrical form:

$$\mathcal{L} = \frac{1}{2}g_{ij}(\phi)D_\mu\phi^i D^\mu\phi^j - V(\phi) - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \tag{37}$$

where ϕ^i stands for a scalar field multiplet containing both Higgs bosons ϕ^I and the NG bosons π^a as its components, i.e.,

$$\{\phi^i\} = \{\pi^a, \phi^I\}. \tag{38}$$

The geometrical form of the GHEFT Lagrangian (37) can be understood as a gauged nonlinear sigma model on a scalar manifold. The scalar manifold (internal space) is parameterized by the scalar multiplet ϕ^i. Both the metric $g_{ij}(\phi)$ and the potential $V(\phi)$ are functions of ϕ^i. They should be invariant under the $SU(2)_W\times U(1)_Y$ transformation:

$$0 = w^k_a g_{ij,k} + (w^k_a)_{,i}g_{kj} + (w^k_a)_{,j}g_{ik}, \tag{39}$$

$$0 = y^k g_{ij,k} + (y^k)_{,i}g_{kj} + (y^k)_{,j}g_{ik}, \tag{40}$$

$$0 = w^k_a V_{,k}, \tag{41}$$

$$0 = y^k V_{,k}, \tag{42}$$

with

$$g_{ij,k} \coloneqq \frac{\partial}{\partial\phi^k}g_{ij}, \qquad V_{,k} \coloneqq \frac{\partial}{\partial\phi^k}V, \qquad (w^k_a)_{,i} \coloneqq \frac{\partial}{\partial\phi^i}w^k_a, \qquad (y^k)_{,i} \coloneqq \frac{\partial}{\partial\phi^i}y^k. \tag{43}$$

The $SU(2)_W$ and $U(1)_Y$ Killing vectors are denoted by w^k_a ($a = 1, 2, 3$) and y^k, respectively, in Eqs. (39)–(42). The GHEFT Lagrangian (30) provides the most general and systematic method to construct the geometrical form of the Lagrangian (37) with the symmetry properties (39)–(42). The translation dictionary from the GHEFT Lagrangian (30) to the geometrical form (37) is given in Appendix A.

The $SU(2)_W\times U(1)_Y$ gauge interactions are introduced into the scalar sector through the covariant derivative

$$D_\mu\phi^i = \partial_\mu\phi^i + g_W W^a_\mu w^i_a(\phi) + g_Y B_\mu y^i(\phi). \tag{44}$$

It should be noted that the gauge fields interact with the scalar sector through the $SU(2)_W\times U(1)_Y$ Killing vectors w^i_a and y^i.

The scalar potential $V(\phi)$ should be minimized at the point where

$$\langle\phi^i\rangle = \bar{\phi}^i. \tag{45}$$

We hereafter call the point Eq. (45) the vacuum. Note that, since the electroweak symmetry is spontaneously broken at the vacuum, the vacuum $\phi^i = \bar{\phi}^i$ cannot be a fixed point of the $SU(2)_W\times U(1)_Y$ transformation, i.e.,

$$w^i_a(\bar{\phi}) \neq 0, \qquad y^i(\bar{\phi}) \neq 0. \tag{46}$$

However, it should be a fixed point of the $U(1)_{\rm em}$ transformation,

$$w_3^i(\bar{\phi}) + y^i(\bar{\phi}) = 0. \tag{47}$$

The electroweak gauge bosons (W and Z) acquire masses,

$$M_W^2 \propto g_W^2 g_{ij}(\bar{\phi}) w_1^i(\bar{\phi}) w_1^j(\bar{\phi}),$$
$$M_Z^2 \propto (g_W^2 + g_Y^2) g_{ij}(\bar{\phi}) w_3^i(\bar{\phi}) w_3^j(\bar{\phi}). \tag{48}$$

Therefore, Killing vectors at the vacuum (46) play the role of the Higgs vacuum expectation value in the SM. It should be emphasized that the vanishing scalar vacuum expectation value $\bar{\phi}^i = 0$ does not imply the recovery of electroweak symmetry in the GHEFT Lagrangian. Actually, in the GHEFT coordinate (38), even though all of the vacuum expectation values of the scalar fields vanish ($\bar{\phi}^i = 0$), the electroweak symmetry is still spontaneously broken by the nonvanishing Killing vectors at the vacuum (46).

The dynamical excitation fields φ^i are obtained after expanding around the vacuum,

$$\phi^i = \bar{\phi}^i + \varphi^i. \tag{49}$$

The scalar manifold metric g_{ij} is expanded as

$$g_{ij} = \bar{g}_{ij} + \bar{g}_{ij,k}\varphi^k + \frac{1}{2}\bar{g}_{ij,kl}\varphi^k\varphi^l + \cdots, \tag{50}$$

with

$$\bar{g}_{ij} \coloneqq g_{ij}(\bar{\phi}), \qquad \bar{g}_{ij,k} \coloneqq \frac{\partial}{\partial\phi^k} g_{ij}(\phi)\bigg|_{\phi=\bar{\phi}},$$
$$\bar{g}_{ij,kl} \coloneqq \frac{\partial}{\partial\phi^l}\frac{\partial}{\partial\phi^k} g_{ij}(\phi)\bigg|_{\phi=\bar{\phi}}, \qquad \cdots. \tag{51}$$

In a similar manner, the potential term is expanded as

$$V(\phi) = \bar{V} + \bar{V}_{,i}\varphi^i + \frac{1}{2}\bar{V}_{,ij}\varphi^i\varphi^j + \frac{1}{3!}\bar{V}_{,ijk}\varphi^i\varphi^j\varphi^k + \frac{1}{4!}\bar{V}_{,ijkl}\varphi^i\varphi^j\varphi^k\varphi^l + \cdots, \tag{52}$$

with

$$\bar{V} \coloneqq V\bigg|_{\phi=\bar{\phi}}, \qquad \bar{V}_{,i} \coloneqq \frac{\partial}{\partial\phi^i} V\bigg|_{\phi=\bar{\phi}},$$
$$\bar{V}_{,ij} \coloneqq \frac{\partial}{\partial\phi^j}\frac{\partial}{\partial\phi^i} V\bigg|_{\phi=\bar{\phi}}, \qquad \cdots. \tag{53}$$

Since the potential V is minimized at the point where the potential should satisfy

$$\bar{V}_{,i} = 0. \tag{54}$$

The scalar manifold is parameterized by the scalar field multiplet ϕ^i. Hereafter, we normalize/diagonalize the coordinates ϕ^i as

$$\bar{g}_{ij} = \delta_{ij} \tag{55}$$

and

$$\bar{V}_{,ij} = \delta_{ij} m_i^2, \tag{56}$$

so that the excitation fields φ^i are canonically normalized and diagonalized.

III. SCALAR SCATTERING AMPLITUDES AND PERTURBATIVE UNITARITY

We next consider the implications of perturbative unitarity in the GHEFT framework. It is well known that, in the effective field theory framework, the longitudinally polarized EW gauge boson scattering amplitudes grow at high energy and tend to cause violations of perturbative unitarity [120,121]. The effective field theory coupling constants need to be arranged to maintain perturbative unitarity in the high-energy gauge boson scattering amplitudes.

For such a purpose, we use the equivalence theorem between the longitudinally polarized gauge boson scattering amplitudes and the corresponding would-be NG boson amplitudes [122–126]. The equivalence theorem allows us to estimate the longitudinally polarized gauge boson high-energy scattering amplitudes by using the NG boson amplitudes in the gaugeless limit i.e., $g_W = g_Y = 0$ with an uncertainty of $\mathcal{O}(M_W^2/E^2)$. The computation of the amplitudes is greatly simplified in the gaugeless limit.

Note that the individual Feynman-diagrams for longitudinally polarized gauge boson scattering amplitudes grow as positive powers of the energy E (energy growing behavior), which is, however, canceled exactly among the diagrams in the SM [122,127–129]. The energy growing behavior coming from the EW gauge boson exchange and contact interaction diagrams is exactly canceled by the Higgs exchange diagram in the SM. The Higgs boson plays an essential role in maintaining perturbative unitarity in the SM.

On the other hand, it is highly nontrivial whether the cancellation of the energy growing terms works in the GHEFT. In fact, in order to ensure the cancellation, the coupling strengths between the Higgs boson(s) and the EW gauge bosons should satisfy special conditions known as the "unitarity sum rules" [93–95]. The unitarity sum rules provide a guiding principle to investigate the extended Higgs scenarios in a model-independent manner. Model-independent studies on extended EWSB scenarios have been done based on the unitarity argument [70,93,99,130,131].

We estimate the longitudinally polarized EW gauge boson scattering amplitudes using the corresponding would-be NG boson amplitudes with the help of the equivalence theorem. In subsequent subsections, we explicitly calculate the on-shell amplitudes among the scalar fields φ^i in the gaugeless limit, and express the unitarity sum rules in terms of the scalar manifold's geometry.

A. Scalar scattering amplitudes

We consider here an N-point on-shell scalar scattering amplitude at the tree level,

$$i\mathcal{M}(123\cdots N) \coloneqq i\mathcal{M}(\varphi^{i_1}(p_1), \varphi^{i_2}(p_2), \varphi^{i_3}(p_3), \ldots, \varphi^{i_N}(p_N)), \tag{57}$$

with p_n and i_n ($n = 1, 2, \ldots, N$) being outgoing momenta and the particle species, respectively.

We define

$$s_{n_1} \coloneqq p_{n_1}^2, \qquad s_{n_1 n_2} \coloneqq (p_{n_1} + p_{n_2})^2, \qquad s_{n_1 n_2 n_3} \coloneqq (p_{n_1} + p_{n_2} + p_{n_3})^2, \qquad \cdots. \tag{58}$$

External momenta p_n are taken on shell,

$$s_n = m_{i_n}^2. \tag{59}$$

We note that

$$s_{n_1 n_2 n_3} = s_{n_1 n_2} + s_{n_2 n_3} + s_{n_1 n_3} - m_{i_{n_1}}^2 - m_{i_{n_2}}^2 - m_{i_{n_3}}^2, \qquad \cdots. \tag{60}$$

The N-point amplitude (57) can thus be written as a function of the scalar particle masses m_i^2 and the generalized Mandelstam variables $s_{n_1 n_2}$.

As we will show explicitly below, the three- and four-point on-shell scattering amplitudes are described in terms of the geometry of the scalar manifold,

$$i\mathcal{M}(123) = -i\bar{V}_{;(i_1 i_2 i_3)}, \tag{61}$$

$$\begin{aligned} i\mathcal{M}(1234) = iM(1234) &+ i\mathcal{M}(125)[D(s_{12})]_{i_5 i_6} i\mathcal{M}(346) \\ &+ i\mathcal{M}(135)[D(s_{13})]_{i_5 i_6} i\mathcal{M}(246) \\ &+ i\mathcal{M}(145)[D(s_{14})]_{i_5 i_6} i\mathcal{M}(236), \end{aligned} \tag{62}$$

with

$$\begin{aligned} iM(1234) = -i\bar{V}_{;(i_1 i_2 i_3 i_4)} &- \frac{i}{3}(\bar{R}_{i_1 i_3 i_4 i_2} + \bar{R}_{i_1 i_4 i_3 i_2}) s_{12} \\ &- \frac{i}{3}(\bar{R}_{i_1 i_2 i_4 i_3} + \bar{R}_{i_1 i_4 i_2 i_3}) s_{13} \\ &- \frac{i}{3}(\bar{R}_{i_1 i_2 i_3 i_4} + \bar{R}_{i_1 i_3 i_2 i_4}) s_{14}, \end{aligned} \tag{63}$$

and

$$[D(s)]_{ij} \coloneqq \frac{i}{s - m_i^2} \bar{g}_{ij}. \tag{64}$$

Here $\bar{V}_{;(i_1 i_2 i_3)}$, $\bar{V}_{;(i_1 i_2 i_3 i_4)}$, and $\bar{R}_{i_1 i_2 i_3 i_4}$ stand for the totally symmetrized covariant derivatives of the potential and the Riemann curvature tensor of the scalar manifold at the vacuum.

Let us start with the three-point scalar scattering amplitude $\mathcal{M}(123)$. The interaction vertices relevant for this amplitude are

$$\mathcal{L}_3 = \frac{1}{2}\bar{g}_{ij,k}\varphi^k(\partial_\mu \varphi^i)(\partial^\mu \varphi^j) - \frac{1}{3!}\bar{V}_{,ijk}\varphi^i \varphi^j \varphi^k \tag{65}$$

at the tree level. It is straightforward to evaluate the on-shell three-point amplitude

$$\begin{aligned} i\mathcal{M}(123) &= \frac{i}{2}(\bar{g}_{i_1 i_2, i_3} + \bar{g}_{i_2 i_1, i_3})(-p_1 \cdot p_2) + \frac{i}{2}(\bar{g}_{i_2 i_3, i_1} + \bar{g}_{i_3 i_2, i_1})(-p_2 \cdot p_3) + \frac{i}{2}(\bar{g}_{i_3 i_1, i_2} + \bar{g}_{i_1 i_3, i_2})(-p_3 \cdot p_1) - i\bar{V}_{,i_1 i_2 i_3} \\ &= \frac{i}{2}\bar{g}_{i_1 i_2, i_3}(m_{i_1}^2 + m_{i_2}^2 - s_{12}) + \frac{i}{2}\bar{g}_{i_2 i_3, i_1}(m_{i_2}^2 + m_{i_3}^2 - s_{23}) + \frac{i}{2}\bar{g}_{i_3 i_1, i_2}(m_{i_3}^2 + m_{i_1}^2 - s_{31}) - i\bar{V}_{,i_1 i_2 i_3}, \end{aligned} \tag{66}$$

from the vertices in Eq. (65). The conservation of the total momentum

$$p_1^\mu + p_2^\mu + p_3^\mu = 0$$

implies

$$s_{12} = (p_1 + p_2)^2 = p_3^2 = m_{i_3}^2,$$

and similarly

$$s_{23} = m_{i_1}^2, \qquad s_{31} = m_{i_2}^2.$$

The on-shell three-point amplitude (66) can therefore be expressed as

$$
\begin{aligned}
i\mathcal{M}(123) &= \frac{i}{2}\bar{g}_{i_1 i_2, i_3}(m_{i_1}^2 + m_{i_2}^2 - m_{i_3}^2) + \frac{i}{2}\bar{g}_{i_2 i_3, i_1}(m_{i_2}^2 + m_{i_3}^2 - m_{i_1}^2) + \frac{i}{2}\bar{g}_{i_3 i_1, i_2}(m_{i_3}^2 + m_{i_1}^2 - m_{i_2}^2) - i\bar{V}_{,i_1 i_2 i_3} \\
&= \frac{i}{2} m_{i_1}^2 (\bar{g}_{i_1 i_2, i_3} + \bar{g}_{i_1 i_3, i_2} - \bar{g}_{i_2 i_3, i_1}) + \frac{i}{2} m_{i_2}^2 (\bar{g}_{i_2 i_3, i_1} + \bar{g}_{i_2 i_1, i_3} - \bar{g}_{i_3 i_1, i_2}) + \frac{i}{2} m_{i_3}^2 (\bar{g}_{i_3 i_1, i_2} + \bar{g}_{i_3 i_2, i_1} - \bar{g}_{i_1 i_2, i_3}) - i\bar{V}_{,i_1 i_2 i_3}.
\end{aligned} \tag{67}
$$

Note that $m_{i_1}^2$, $m_{i_2}^2$, and $m_{i_3}^2$ are related to the second derivative of the potential $V_{,ij}$ by (56). The first derivative of the metric tensor in the interaction vertex (65) is related to the affine connection Γ^l_{jk},

$$
g_{il}\Gamma^l_{jk} \coloneqq \frac{1}{2}[g_{ij,k} + g_{ki,j} - g_{jk,i}]. \tag{68}
$$

The amplitude (67) can then be rewritten as

$$
i\mathcal{M}(123) = i\bar{V}_{,i_1 l}\bar{\Gamma}^l_{i_2 i_3} + i\bar{V}_{,i_2 l}\bar{\Gamma}^l_{i_3 i_1} + i\bar{V}_{,i_3 l}\bar{\Gamma}^l_{i_1 i_2} - i\bar{V}_{,i_1 i_2 i_3}, \tag{69}
$$

with $\bar{\Gamma}^l_{jk}$ being the affine connection at the vacuum

$$
\bar{\Gamma}^l_{jk} \coloneqq \Gamma^l_{jk}\bigg|_{\phi=\bar{\phi}}. \tag{70}
$$

Our final task is to rewrite the amplitude (69) in terms of the covariant derivatives of the potential V. It is straightforward to show that

$$
\begin{aligned}
V_{;ijk} &= V_{,ijk} - (\Gamma^l_{ij})_{,k}V_{,l} - \Gamma^l_{ij}V_{,lk} - \Gamma^l_{ik}V_{,lj} - \Gamma^l_{jk}V_{,li} \\
&\quad + \Gamma^l_{ik}\Gamma^m_{lj}V_{,m} + \Gamma^l_{jk}\Gamma^m_{li}V_{,m}.
\end{aligned} \tag{71}
$$

Since the first derivative of the potential vanishes at the vacuum, we obtain

$$
\bar{V}_{;ijk} = \bar{V}_{,ijk} - \bar{\Gamma}^l_{ij}\bar{V}_{,lk} - \bar{\Gamma}^l_{ik}\bar{V}_{,lj} - \bar{\Gamma}^l_{jk}\bar{V}_{,li}. \tag{72}
$$

Moreover, as we see in Eq. (72), $\bar{V}_{;ijk}$ is symmetric under the exchanges $i \leftrightarrow j$, $i \leftrightarrow k$, and $j \leftrightarrow k$. We therefore obtain

$$
\begin{aligned}
\bar{V}_{;(ijk)} &\coloneqq \frac{1}{3!}[\bar{V}_{;ijk} + \bar{V}_{;jki} + \bar{V}_{;kij} + \bar{V}_{;ikj} + \bar{V}_{;kji} + \bar{V}_{;jik}] \\
&= \bar{V}_{;ijk}.
\end{aligned} \tag{73}
$$

It is now easy to obtain a geometrical formula for the three-point amplitude:

$$
i\mathcal{M}(123) = -i\bar{V}_{;(i_1 i_2 i_3)}. \tag{74}
$$

We next consider the four-point amplitude,

$$
\begin{aligned}
i\mathcal{M}(1234) = iM_0(1234) &+ iM(12[5])[D(s_{12})]_{i_5 i_6} iM(34[6]) \\
&+ iM(13[5])[D(s_{13})]_{i_5 i_6} iM(24[6]) \\
&+ iM(14[5])[D(s_{14})]_{i_5 i_6} iM(23[6]),
\end{aligned} \tag{75}
$$

where the first line comes from the four-point contact interaction vertices, while the second, third, and fourth lines are from the i_5-particle exchange diagrams in the s_{12}, s_{13}, and s_{14} channels, respectively. The k-particle is allowed to be off-shell in the three-point amplitude $M(ij[k])$, while the particles i and j are assumed to be on-shell.

We first study $M(12[5])$,

$$
M(12[5]) = -\bar{V}_{;(125)} + \bar{g}_{i_5 i_5'}\bar{\Gamma}^{i_5'}_{i_1 i_2}(s_{12} - m_{i_5}^2), \tag{76}
$$

which can be related to the on-shell three-point amplitude $\mathcal{M}(125)$ as

$$
M(12[5]) = \mathcal{M}(125) + \bar{g}_{i_5 i_5'}\bar{\Gamma}^{i_5'}_{i_1 i_2}(s_{12} - m_{i_5}^2). \tag{77}
$$

It is easy to rewrite the amplitude (75) as

$$
\begin{aligned}
i\mathcal{M}(1234) = iM(1234) &+ i\mathcal{M}(125)[D(s_{12})]_{i_5 i_6} i\mathcal{M}(346) \\
&+ i\mathcal{M}(135)[D(s_{13})]_{i_5 i_6} i\mathcal{M}(246) \\
&+ i\mathcal{M}(145)[D(s_{14})]_{i_5 i_6} i\mathcal{M}(236),
\end{aligned} \tag{78}
$$

with $M(1234)$ being

$$
\begin{aligned}
M(1234) = M_0(1234) &- \bar{g}_{i_5 i_6}\bar{\Gamma}^{i_5}_{i_1 i_2}\bar{\Gamma}^{i_6}_{i_3 i_4}(s_{12} - m_{i_5}^2) \\
&- \bar{g}_{i_5 i_6}\bar{\Gamma}^{i_5}_{i_1 i_3}\bar{\Gamma}^{i_6}_{i_2 i_4}(s_{13} - m_{i_5}^2) \\
&- \bar{g}_{i_5 i_6}\bar{\Gamma}^{i_5}_{i_1 i_4}\bar{\Gamma}^{i_6}_{i_2 i_3}(s_{14} - m_{i_5}^2) + \bar{V}_{;(i_1 i_2 i_5)}\bar{\Gamma}^{i_5}_{i_3 i_4} \\
&+ \bar{V}_{;(i_1 i_3 i_5)}\bar{\Gamma}^{i_5}_{i_2 i_4} + \bar{V}_{;(i_1 i_4 i_5)}\bar{\Gamma}^{i_5}_{i_2 i_3} + \bar{V}_{;(i_2 i_3 i_5)}\bar{\Gamma}^{i_5}_{i_1 i_4} \\
&+ \bar{V}_{;(i_2 i_4 i_5)}\bar{\Gamma}^{i_5}_{i_1 i_3} + \bar{V}_{;(i_3 i_4 i_5)}\bar{\Gamma}^{i_5}_{i_1 i_2}.
\end{aligned} \tag{79}
$$

The evaluation of the four-point contact interaction contribution is a bit tedious but straightforward. We obtain

$$iM_0(1234) = -i\bar{V}_{,i_1i_2i_3i_4} + \frac{i}{2}[-(\bar{g}_{i_1i_2,i_3i_4} + \bar{g}_{i_3i_4,i_1i_2})s_{12} - (\bar{g}_{i_1i_3,i_2i_4} + \bar{g}_{i_2i_4,i_1i_3})s_{13} - (\bar{g}_{i_1i_4,i_2i_3} + \bar{g}_{i_2i_3,i_1i_4})s_{14} + (\bar{g}_{i_1i_2,i_3i_4} + \bar{g}_{i_1i_3,i_2i_4} + \bar{g}_{i_1i_4,i_2i_3})m_{i_1}^2 + (\bar{g}_{i_2i_1,i_3i_4} + \bar{g}_{i_2i_3,i_1i_4} + \bar{g}_{i_2i_4,i_1i_3})m_{i_2}^2 + (\bar{g}_{i_3i_1,i_2i_4} + \bar{g}_{i_3i_2,i_1i_4} + \bar{g}_{i_3i_4,i_1i_2})m_{i_3}^2 + (\bar{g}_{i_4i_1,i_2i_3} + \bar{g}_{i_4i_2,i_1i_3} + \bar{g}_{i_4i_3,i_1i_2})m_{i_4}^2]. \quad (80)$$

Combining these results, we obtain a geometrical formula for the on-shell four-point amplitude:

$$iM(1234) = -i\bar{V}_{;(i_1i_2i_3i_4)} - \frac{i}{3}(\bar{R}_{i_1i_3i_4i_2} + \bar{R}_{i_1i_4i_3i_2})s_{12} - \frac{i}{3}(\bar{R}_{i_1i_2i_4i_3} + \bar{R}_{i_1i_4i_2i_3})s_{13} - \frac{i}{3}(\bar{R}_{i_1i_2i_3i_4} + \bar{R}_{i_1i_3i_2i_4})s_{14}. \quad (81)$$

We used the on-shell condition

$$s_{12} + s_{13} + s_{14} = m_{i_1}^2 + m_{i_2}^2 + m_{i_3}^2 + m_{i_4}^2 \quad (82)$$

in the computation above. Here $\bar{R}_{ijkl}$ and $\bar{V}_{;(ijkl)}$ denote the Riemann curvature tensor and the totally symmetrized covariant derivatives of the potential at the vacuum:

$$\bar{R}_{ijkl} \coloneqq R_{ijkl}\Big|_{\phi=\bar{\phi}}, \qquad \bar{V}_{;(ijkl)} \coloneqq V_{;(ijkl)}\Big|_{\phi=\bar{\phi}}. \quad (83)$$

We here give formulas to compute $\bar{R}_{ijkl}$ and $\bar{V}_{;(ijkl)}$ from the metric tensor g_{ij} and the potential V:

$$\bar{R}_{ijkl} = \frac{1}{2}(\bar{g}_{il,jk} + \bar{g}_{jk,il} - \bar{g}_{ik,jl} - \bar{g}_{jl,ik}) + \bar{g}_{mn}(\bar{\Gamma}^m_{il}\bar{\Gamma}^n_{jk} - \bar{\Gamma}^m_{ik}\bar{\Gamma}^n_{jl}), \quad (84)$$

and

$$\bar{V}_{;(ijkl)} = \bar{V}_{,ijkl} - \bar{V}_{,ijm}\bar{\Gamma}^m_{kl} - \bar{V}_{,klm}\bar{\Gamma}^m_{ij} - \bar{V}_{,ikm}\bar{\Gamma}^m_{jl} - \bar{V}_{,jlm}\bar{\Gamma}^m_{ik} - \bar{V}_{,ilm}\bar{\Gamma}^m_{jk} - \bar{V}_{,jkm}\bar{\Gamma}^m_{il} + \bar{V}_{,mn}[\bar{\Gamma}^m_{ij}\bar{\Gamma}^n_{kl} + \bar{\Gamma}^m_{ik}\bar{\Gamma}^n_{jl} + \bar{\Gamma}^m_{il}\bar{\Gamma}^n_{jk}] + A_{ijkl} + A_{jikl} + A_{kijl} + A_{lijk}, \quad (85)$$

with

$$A_{ijkl} \coloneqq \frac{1}{6}\bar{V}_{,im}\bar{g}^{mn}[\bar{g}_{jk,nl} + \bar{g}_{kl,nj} + \bar{g}_{jl,nk} - 2(\bar{g}_{nj,kl} + \bar{g}_{nk,jl} + \bar{g}_{nl,jk})] + \bar{V}_{,im}[\bar{\Gamma}^m_{jn}\bar{\Gamma}^n_{kl} + \bar{\Gamma}^m_{kn}\bar{\Gamma}^n_{jl} + \bar{\Gamma}^m_{ln}\bar{\Gamma}^n_{jk}] + \frac{1}{3}\bar{V}_{,im}\bar{g}^{mp}[\bar{\Gamma}^q_{pj}\bar{\Gamma}^n_{kl} + \bar{\Gamma}^q_{pk}\bar{\Gamma}^n_{jl} + \bar{\Gamma}^q_{pl}\bar{\Gamma}^n_{jk}]\bar{g}_{qn}. \quad (86)$$

B. Perturbative unitarity

As we have shown in Eq. (81), the scalar four-point amplitude $M(1234)$ contains energy-squared terms proportional to s_{12}, s_{13}, and s_{14}. This implies that the perturbative unitarity of the scattering amplitude is generally violated at a certain high-energy scale in the GHEFT (37). In order to maintain perturbative unitarity in the high-energy limit, the GHEFT Lagrangian should satisfy special conditions known as the unitarity sum rules [93–95]. We here give a geometrical interpretation for the unitarity sum rules.

Applying the on-shell condition

$$s_{12} + s_{13} + s_{14} = m_{i_1}^2 + m_{i_2}^2 + m_{i_3}^2 + m_{i_4}^2 \quad (87)$$

to the four-point amplitude (81), we obtain

$$iM(1234) = -\frac{i}{3}(\bar{R}_{i_1i_3i_4i_2} + \bar{R}_{i_1i_4i_3i_2} - \bar{R}_{i_1i_2i_3i_4} - \bar{R}_{i_1i_3i_2i_4})s_{12} - \frac{i}{3}(\bar{R}_{i_1i_2i_4i_3} + \bar{R}_{i_1i_4i_2i_3} - \bar{R}_{i_1i_2i_3i_4} - \bar{R}_{i_1i_3i_2i_4})s_{13} + \mathcal{O}(E^0). \quad (88)$$

Therefore, the unitarity sum rules can be summarized in geometrical language as

$$\bar{R}_{i_1i_3i_4i_2} + \bar{R}_{i_1i_4i_3i_2} - \bar{R}_{i_1i_2i_3i_4} - \bar{R}_{i_1i_3i_2i_4} = 0, \quad (89)$$

$$\bar{R}_{i_1i_2i_4i_3} + \bar{R}_{i_1i_4i_2i_3} - \bar{R}_{i_1i_2i_3i_4} - \bar{R}_{i_1i_3i_2i_4} = 0. \quad (90)$$

Note that the Riemann curvature tensor R_{ijkl} is antisymmetric under the exchange $k \leftrightarrow l$:

$$R_{ijkl} \equiv -R_{ijlk}. \quad (91)$$

The unitarity sum rules (89) can thus be rewritten as

$$2\bar{R}_{i_1i_3i_4i_2} - \bar{R}_{i_1i_4i_2i_3} - \bar{R}_{i_1i_2i_3i_4} = 0. \quad (92)$$

The Bianchi identity

$$R_{ijkl} + R_{iklj} + R_{iljk} \equiv 0 \quad (93)$$

can be expressed as

$$-\bar{R}_{i_1i_4i_2i_3} - \bar{R}_{i_1i_2i_3i_4} \equiv \bar{R}_{i_1i_3i_4i_2}, \quad (94)$$

which enables us to further simplify the unitarity sum rules (92). The sum rules (89) can be expressed in a simple form:

$$3\bar{R}_{i_1 i_3 i_4 i_2} = 0. \tag{95}$$

The unitarity sum rules (89) and (90) can be expressed in a compact form:

$$\bar{R}_{ijkl} = 0. \tag{96}$$

Note that the unitarity sum rules (96) imply the flatness of the scalar manifold only at the vacuum. The unitarity conditions (96) are lifted to

$$R_{ijkl} = 0, \tag{97}$$

i.e., the complete flatness of the entire scalar manifold at least in the vicinity of the vacuum, by imposing perturbative unitarity in the arbitrary N-point amplitudes. See Appendix B for details.

Perturbative unitarity is violated at a certain high-energy scale in an extended Higgs scenario with a curved scalar manifold. For instance, if we consider the HEFT with $\mathcal{F}(h) = (1 + (\kappa_V h)/v)^2$ and take $\kappa_V < 1$, the corresponding scalar manifold has nonzero curvature proportional to $1 - \kappa_V^2$ [96,97]. The model violates perturbative unitarity at $\Lambda \sim 4\pi v/(1 - \kappa_V^2)^{1/2}$. In that case, we need to introduce new particles with mass $m \lesssim \Lambda$ and/or consider nonperturbative effects to ensure the unitarity of the model.

IV. ONE-LOOP DIVERGENCES IN THE GAUGELESS LIMIT

As we have shown in the previous section, the tree-level perturbative unitarity requires that the GHEFT scalar manifold be flat at the vacuum. What does this imply at the loop level? References [96,97] investigated the structure of the one-loop divergences in the nonlinear sigma model Lagrangian (37). They found that the logarithmic divergences in the scalar one-loop integral are described in the gaugeless limit by

$$\Delta\mathcal{L}_{\rm div}^{\varphi\text{–loop}} = \frac{1}{(4\pi)^2\epsilon}\left[\frac{1}{12}\mathrm{tr}(Y_{\mu\nu}Y^{\mu\nu}) + \frac{1}{2}\mathrm{tr}(X^2)\right]. \tag{98}$$

Here ϵ is defined as

$$\epsilon \coloneqq 4 - D, \tag{99}$$

with D being the spacetime dimension. $Y_{\mu\nu}$ and X are defined as

$$[Y_{\mu\nu}]^i{}_j = R^i{}_{jkl}(D_\mu\phi)^k(D_\nu\phi)^l + W^a_{\mu\nu}(w^i_a)_{;j} + B_{\mu\nu}(y^i)_{;j}, \tag{100}$$

$$[X]^i{}_k = R^i{}_{jkl}(D_\mu\phi)^j(D^\mu\phi)^l + g^{ij}V_{;jk}, \tag{101}$$

with

$$(w^i_a)_{;j} = \frac{\partial}{\partial\phi_j}w^i_a + \Gamma^i_{lj}w^l_a, \qquad (y^i)_{;j} = \frac{\partial}{\partial\phi_j}y^i + \Gamma^i_{lj}y^l, \tag{102}$$

and $R^i{}_{jkl} = g^{im}R_{mjkl}$.

Remember that perturbative unitarity implies the flatness at the vacuum,

$$\bar{R}_{ijkl} = 0. \tag{103}$$

It is easy to see that the unitarity condition (103) is enough to guarantee the absence of divergences in the $(\partial_\mu\phi)^4$-type operators, which affect the scalar boson high-energy four-point scattering amplitudes. The flatness of the scalar manifold at the vacuum (103) also automatically guarantees the absence of divergences in the anomalous triple gauge boson operators. These findings are in accord with the general expectations regarding the connections between perturbative unitarity and the absence of new counterterms in the one-loop divergences, and also with the explicit heat-kernel computations presented in Refs. [79,81,82,96,97].

However, the divergence structure in the operators proportional to

$$W^a_{\mu\nu}W^{b\mu\nu}, \qquad W^a_{\mu\nu}B^{\mu\nu}, \qquad B_{\mu\nu}B^{\mu\nu} \tag{104}$$

is not manifest. Note that the oblique correction parameters S and U [98] are related to the gauge-kinetic-type operators listed in Eq. (104). There is no obvious reason to ensure the absence of the one-loop divergences in the S and U parameters even in the perturbatively unitary models.

Moreover, the one-loop divergence formula (98) does not include quantum corrections arising from the gauge-boson loop diagrams, which should be evaluated to deduce the divergence structure of the oblique correction parameters.

In what follows, we explicitly perform the one-loop calculations for both the scalar and gauge loop diagrams. Our results are consistent with those of Refs. [81,82], in which the complete one-loop divergence formulas including gauge loops and fermionic loop corrections were obtained. Picking the UV-divergent parts from the one-loop functions, we investigate the relationship between the divergence structure and tree-level perturbative unitarity.

V. OBLIQUE CORRECTIONS AND FINITENESS CONDITIONS

A. Vacuum polarization functions at one loop

The electroweak oblique correction parameters S and U are defined as

$$S \coloneqq 16\pi(\Pi'_{33}(0) - \Pi'_{3Q}(0)), \tag{105}$$

$$U \coloneqq 16\pi(\Pi'_{11}(0) - \Pi'_{33}(0)), \tag{106}$$

with $\Pi'_A(0)$ being

$$\Pi'_A(0) \coloneqq \frac{d}{dp^2}\Pi_A(p^2)\bigg|_{p^2=0}. \tag{107}$$

Here $\Pi_A(p^2)$ stands for the non-SM contribution to the gauge boson vacuum polarization function in the A channel. $\Pi_{11}(p^2)$ and $\Pi_{33}(p^2)$ are charged and neutral weak $SU(2)_W$ current correlators at momentum p, respectively. $\Pi_{3Q}(p^2)$ is the correlator between the neutral weak $SU(2)_W$ current and the electromagnetic current. Note that, in the GHEFT, a number of scalar particles, in addition to the 125 GeV Higgs, contribute to $\Pi_A(p^2)$ at loop level.

The oblique correction parameter T is related to Veltman's ρ parameter [132],

$$\alpha T \coloneqq \rho - 1, \qquad \rho = \frac{\frac{v_0^2}{4} + \Pi_{11}(0)}{\frac{v_{Z0}^2}{4} + \Pi_{33}(0)}, \tag{108}$$

with v_0 and v_{Z0} being the "bare" parameters corresponding to the charged and neutral would-be NG boson decay constants v and v_Z. The GHEFT Lagrangian loses its ability to predict the T parameter if we introduce independent counterterms for v and v_Z.

On the other hand, if we assume that the counterterms for v and v_Z are related to each other,

$$v_0^2 = v^2(1+\delta_v), \qquad v_{Z0}^2 = v_Z^2(1+\delta_v), \tag{109}$$

then ρ is calculated as

$$\rho = \frac{v^2}{v_Z^2}\frac{1+\delta_v+\frac{4}{v^2}\Pi_{11}(0)}{1+\delta_v+\frac{4}{v_Z^2}\Pi_{33}(0)} \tag{110}$$

and we regain a counterterm-independent prediction for the parameter ρ,

$$\rho = \frac{v^2}{v_Z^2}(1+\alpha\tilde{T}), \tag{111}$$

with

$$\alpha\tilde{T} \coloneqq 4\left(\frac{1}{v^2}\Pi_{11}(0) - \frac{1}{v_Z^2}\Pi_{33}(0)\right). \tag{112}$$

In what follows, we calculate Π_{11}, Π_{33}, and Π_{3Q} at the one-loop level in the GHEFT and derive the required conditions to ensure the UV finiteness of Eqs. (105), (106), and (112). We apply a background-field method [133–138] to calculate the vacuum polarization functions to keep the gauge invariance. See Appendix C for the details of the calculation. Although UV divergences associated with tadpole diagrams exist in $\Pi_{11}(0)$ and $\Pi_{33}(0)$, as shown in Fig. 1, we assume that these UV divergences are canceled by the corresponding tadpole counterterms.

1. Scalar loop

Let us start with the scalar loop corrections to the vacuum polarization functions. The relevant Feynman diagrams are shown in Fig. 2, which are evaluated to be

$$\begin{aligned}\Pi_{3Q}^{\varphi\varphi}(p^2) = \frac{1}{(4\pi)^2}\Bigg[&-2\sum_{i,j}(\bar{w}_3^i)_{;j}((\bar{w}_3^j)_{;i} + (\bar{y}^j)_{;i}) \\ &\times B_{22}(p^2, m_i^2, m_j^2) \\ &+ \sum_{i,j}(\bar{w}_3^i)_{;j}((\bar{w}_3^j)_{;i} + (\bar{y}^j)_{;i})A(m_i^2)\Bigg],\end{aligned} \tag{113}$$

and

$$\begin{aligned}\Pi_{bc}^{\varphi\varphi}(p^2) = \frac{1}{(4\pi)^2}\Bigg[&-2\sum_{i,j}(\bar{w}_b^i)_{;j}(\bar{w}_c^j)_{;i}B_{22}(p^2, m_i^2, m_j^2) \\ &+ \sum_{i,j}[(\bar{w}_b^i)_{;j}(\bar{w}_c^j)_{;i} + \bar{g}^{ij}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}_{kilj}]A(m_i^2)\Bigg],\end{aligned} \tag{114}$$

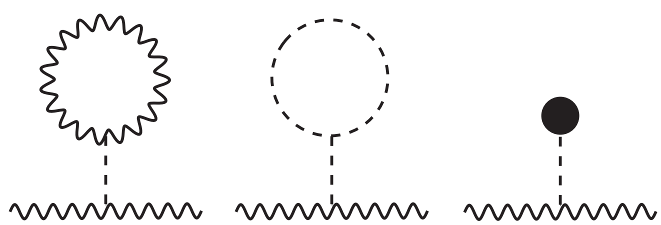

FIG. 1. Feynman diagrams for tadpole contributions to $\Pi_{11}(0)$ and $\Pi_{33}(0)$ and their counterterms.

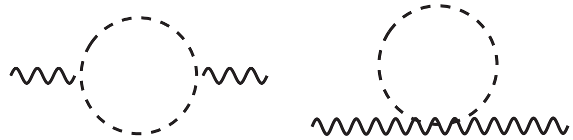

FIG. 2. Feynman diagrams for $\Pi_{11}^{\varphi\varphi}$, $\Pi_{33}^{\varphi\varphi}$, and $\Pi_{3Q}^{\varphi\varphi}$. The internal lines correspond to φ fields.

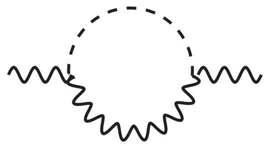

FIG. 3. Feynman diagrams for $\Pi_{11}^{\varphi V}$, $\Pi_{33}^{\varphi V}$, and $\Pi_{3Q}^{\varphi V}$. The internal lines correspond to φ and gauge fields.

for b, $c = 1, 2, 3$. Here $(\bar{w}_a^i)_{;j}$ and $(\bar{y}^i)_{;j}$ denote the covariant derivatives of the Killing vectors at the vacuum,

$$(\bar{w}_a^i)_{;j} \coloneqq (w_a^i)_{;j}\Big|_{\phi=\bar{\phi}}, \qquad (\bar{y}^i)_{;j} \coloneqq (y^i)_{;j}\Big|_{\phi=\bar{\phi}}. \tag{115}$$

A and B_{22} are loop functions defined as

$$\frac{i}{(4\pi)^2} A(m^2) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2}, \tag{116}$$

$$\frac{i}{(4\pi)^2} B_{22}(p^2; m_1^2, m_2^2) = \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k_\nu}{(k^2 - m_1^2)\{(k+p)^2 - m_2^2\}}\Bigg|_{g_{\mu\nu}}. \tag{117}$$

2. Scalar-gauge loop

We next calculate the Feynman diagrams shown in Fig. 3. In the 't Hooft–Feynman gauge, we obtain

$$\Pi_{3Q}^{\varphi V}(p^2) = 0 \tag{118}$$

and

$$\begin{aligned}\Pi_{bc}^{\varphi V}(p^2) &= -\frac{4}{(4\pi)^2}\Bigg[\sum_{a=1,2}\sum_{i,j}\bar{g}_{ij}(G_{Wa})_b^i(G_{Wa})_c^j B_0(p^2, M_W^2, m_i^2) \\ &\quad + \sum_{i,j}\bar{g}_{ij}(G_Z)_b^i(G_Z)_c^j B_0(p^2, M_Z^2, m_i^2) \\ &\quad + \sum_{i,j}\bar{g}_{ij}(G_A)_b^i(G_A)_c^j B_0(p^2, 0, m_i^2)\Bigg],\end{aligned} \tag{119}$$

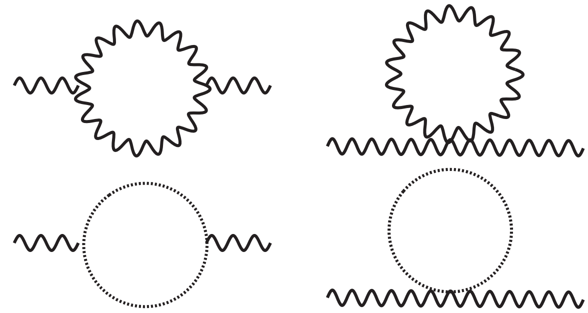

FIG. 4. Feynman diagrams for $\Pi_{11}^{\mathrm{Gauge},cc}$, $\Pi_{33}^{\mathrm{Gauge},cc}$, and $\Pi_{3Q}^{\mathrm{Gauge},cc}$. The wavy and dotted lines correspond to gauge fields and Faddeev-Popov ghost fields, respectively.

for b, $c = 1, 2, 3$. Here $(G_{Wa})_b^i$, $(G_Z)_b^i$, and $(G_A)_b^i$ are defined as

$$(G_{Wa})_b^i \coloneqq g_W(\bar{w}_a^i)_{;j}\bar{w}_b^j \quad (a = 1, 2), \tag{120}$$

$$(G_Z)_b^i \coloneqq \frac{1}{\sqrt{g_W^2 + g_Y^2}}[g_W^2(\bar{w}_3^i)_{;j} - g_Y^2(\bar{y}^i)_{;j}]\bar{w}_b^j, \tag{121}$$

$$(G_A)_b^i \coloneqq \frac{g_W g_Y}{\sqrt{g_W^2 + g_Y^2}}[(\bar{w}_3^i)_{;j} + (\bar{y}^i)_{;j}]\bar{w}_b^j, \tag{122}$$

and

$$M_W^2 = \frac{g_W^2}{4}v^2, \qquad M_Z^2 = \frac{g_W^2 + g_Y^2}{4}v_Z^2. \tag{123}$$

B_0 is defined as

$$\frac{i}{(4\pi)^2} B_0(p^2; m_1^2, m_2^2) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_1^2)\{(k+p)^2 - m_2^2\}}. \tag{124}$$

3. Gauge and Faddeev-Popov ghost loop

Finally, we calculate the contributions that are independent of the scalar interactions. The relevant Feynman diagrams are depicted in Fig. 4. In the 't Hooft–Feynman gauge, we find that the gauge bosons contributions are given by

$$\begin{aligned}\Pi_{11}^{\mathrm{Gauge}}(p^2) &= \Pi_{22}^{\mathrm{Gauge}}(p^2) \\ &= \frac{4}{(4\pi)^2}[-A(M_W^2) - c_W^2 A(M_Z^2) - s_W^2 A(0) + 2p^2(c_W^2 B_0(p^2; M_Z^2, M_W^2) + s_W^2 B_0(p^2; 0, M_W^2)) \\ &\quad + 4(c_W^2 B_{22}(p^2; M_Z^2, M_W^2) + s_W^2 B_{22}(p^2; 0, M_W^2))],\end{aligned} \tag{125}$$

$$\Pi_{33}^{\rm Gauge}(p^2) = \frac{8}{(4\pi)^2}[p^2 B_0(p^2; M_W^2, M_W^2) + 2B_{22}(p^2; M_W^2, M_W^2) - A(M_W^2)], \quad (126)$$

$$\Pi_{3Q}^{\rm Gauge}(p^2) = \frac{8}{(4\pi)^2}[p^2 B_0(p^2; M_W^2, M_W^2) + 2B_{22}(p^2; M_W^2, M_W^2) - A(M_W^2)], \quad (127)$$

and the Faddeev-Popov (FP) ghost contributions are calculated as

$$\Pi_{11}^{cc}(p^2) = \Pi_{22}^{cc}(p^2) = \frac{2}{(4\pi)^2}[A(M_W^2) + c_W^2 A(M_Z^2) + s_W^2 A(0) - 4(c_W^2 B_{22}(p^2; M_Z^2, M_W^2) + s_W^2 B_{22}(p^2; 0, M_W^2))], \quad (128)$$

$$\Pi_{33}^{cc}(p^2) = -\frac{4}{(4\pi)^2}[2B_{22}(p^2; M_W^2, M_W^2) - A(M_W^2)], \quad (129)$$

$$\Pi_{3Q}^{cc}(p^2) = -\frac{4}{(4\pi)^2}[2B_{22}(p^2; M_W^2, M_W^2) - A(M_W^2)]. \quad (130)$$

Here s_W and c_W are

$$s_W = \frac{g_Y}{g_Z}, \qquad c_W = \frac{g_W}{g_Z}, \qquad g_Z = \sqrt{g_W^2 + g_Y^2}. \quad (131)$$

B. Finiteness of the oblique corrections

We are now ready to derive the UV finiteness conditions for the oblique correction parameters at the one-loop level, i.e., the finiteness of Eqs. (105), (106), and (112).

To estimate the UV divergences, we regularize the loop functions A, B_0, and B_{22} by employing dimensional regularization. The loop functions are expanded as

$$A(m^2) = -\Lambda^2 + m^2 \ln\frac{\Lambda^2}{\mu^2} - (4\pi)^2 A_r(m), \quad (132)$$

$$B_0(p^2, m_1^2, m_2^2) = \ln\frac{\Lambda^2}{\mu^2} + (4\pi)^2 B_r(m_1, m_2, p^2), \quad (133)$$

$$B_{22}(p^2, m_1^2, m_2^2) = -\frac{1}{2}\Lambda^2 + \frac{1}{4}\left(m_1^2 + m_2^2 - \frac{p^2}{3}\right)\ln\frac{\Lambda^2}{\mu^2} + \frac{1}{4}(4\pi)^2 B_{0r}(m_1, m_2, p^2), \quad (134)$$

where the terms proportional to Λ^2 and $\ln\Lambda^2$ correspond to the terms proportional to $1/(2-D)$ and $1/(4-D)$, respectively. D and μ denote the spacetime dimension and the renormalization scale, respectively. A_r, B_r, and B_{0r} are Λ-independent (μ-dependent) functions. The explicit expressions for the Λ-independent functions are given in Ref. [99].

1. S and U parameters

Let us focus on the UV divergences in Eqs. (105) and (106). Combining the results derived in Sec. VA and Eqs. (132)–(134), we find that the UV-divergent parts of S and U are given as

$$S_{\rm div} = -\frac{1}{12\pi}(\bar{w}_3^i)_{;j}(\bar{y}^j)_{;i}\ln\frac{\Lambda^2}{\mu^2}, \quad (135)$$

$$U_{\rm div} = \frac{1}{12\pi}((\bar{w}_1^i)_{;j}(\bar{w}_1^j)_{;i} - (\bar{w}_3^i)_{;j}(\bar{w}_3^j)_{;i})\ln\frac{\Lambda^2}{\mu^2}. \quad (136)$$

The gauge-boson loops do not contribute to the one-loop divergences in the parameters S and U. These results are thus identical to the results computed in the gaugeless limit [96,97].

2. $\Pi_{11}(0)$ and $\Pi_{33}(0)$

The UV divergences in Eq. (112) other than the tadpole contributions can also be extracted using Eqs. (132)–(134). We obtain

$$\left(\frac{1}{v^2}\Pi_{11}(0) - \frac{1}{v_Z^2}\Pi_{33}(0)\right)_{\rm div} = \left(\frac{1}{v^2}\Pi_{11}(0) - \frac{1}{v_Z^2}\Pi_{33}(0)\right)_{\Lambda^2} + \left(\frac{1}{v^2}\Pi_{11}(0) - \frac{1}{v_Z^2}\Pi_{33}(0)\right)_{\ln\Lambda^2}, \quad (137)$$

where

$$\left(\frac{1}{v^2}\Pi_{11}(0) - \frac{1}{v_Z^2}\Pi_{33}(0)\right)_{\Lambda^2} = -\frac{1}{(4\pi)^2}\left[\frac{1}{v^2}(\bar{w}_1^i)(\bar{w}_1^j) - \frac{1}{v_Z^2}(\bar{w}_3^i)(\bar{w}_3^j)\right]\bar{R}_{ikjl}\bar{g}^{kl}\Lambda^2, \quad (138)$$

and

$$\left(\frac{1}{v^2}\Pi_{11}(0)-\frac{1}{v_Z^2}\Pi_{33}(0)\right)_{\ln\Lambda^2} = \frac{1}{(4\pi)^2}\left[\frac{1}{v^2}(\bar{w}_1^i)(\bar{w}_1^j)-\frac{1}{v_Z^2}(\bar{w}_3^i)(\bar{w}_3^j)\right] \times\{-4g_W^2(\bar{w}_a^k)_{;i}(\bar{w}_a^l)_{;j}\bar{g}_{kl}-4g_Y^2(\bar{y}^k)_{;i}(\bar{y}^l)_{;j}\bar{g}_{kl} + \bar{R}_{ikjl}(\tilde{M}^2)^{kl}\}\ln\frac{\Lambda^2}{\mu^2}, \tag{139}$$

with $(\tilde{M}^2)^{kl}$ being the scalar boson mass matrix in the 't Hooft–Feynman gauge:

$$(\tilde{M}^2)^{ij} \coloneqq \bar{g}^{ik}\bar{g}^{jl}\bar{V}_{;kl}+g_W^2(\bar{w}_a^i)(\bar{w}_a^j)+g_Y^2(\bar{y}^i)(\bar{y}^j), \quad \bar{V}_{;ij}\coloneqq V_{;ij}\Big|_{\phi=\bar{\phi}}. \tag{140}$$

VI. PERTURBATIVE UNITARITY VS FINITENESS CONDITIONS

We are now ready to discuss the implications of perturbative unitarity on the one-loop finiteness of the oblique correction parameters. We first concentrate on the parameter S, the UV divergence of which is given by Eq. (135). As we stressed in Sec. IV, since there are no obvious connections between the Riemann curvature tensor (geometry) R_{ijkl} and the $SU(2)_W\times U(1)_Y$ Killing vectors (symmetry) w_a^i and y^i, the relation between perturbative unitarity $\bar{R}_{ijkl}=0$ and the one-loop finiteness of S is not obvious in Eq. (135).

We note, however, that the scalar manifold should be invariant under the $SU(2)_W\times U(1)_Y$ transformations, and thus the Killing vectors should satisfy the Killing equations,

$$0=(w_a^k)g_{ij,k}+(w_a^k)_{,i}g_{kj}+(w_a^k)_{,j}g_{ik}, \quad 0=(y^k)g_{ij,k}+(y^k)_{,i}g_{kj}+(y^k)_{,j}g_{ik}. \tag{141}$$

There *do* exist connections between the geometry (R_{ijkl}) and the symmetry (w_a^i and y^i) embedded in the Killing equations (141). Moreover, the Killing vectors w_a^i and y^i should obey the $SU(2)_W\times U(1)_Y$ Lie algebra,

$$[w_a,w_b]=\varepsilon_{abc}w_c, \qquad [w_a,y]=0, \tag{142}$$

with

$$w_a\coloneqq w_a^i\frac{\partial}{\partial\phi^i}, \qquad y\coloneqq y^i\frac{\partial}{\partial\phi^i}. \tag{143}$$

The connections can be studied more easily if we use a Riemann normal coordinate (RNC) system around the vacuum $\bar{\phi}$, in which the metric tensor $g_{ij}(\phi)$ can be expressed in a Taylor-expanded form around $\bar{\phi}$ as

$$g_{ij}(\phi)=\delta_{ij}-\frac{1}{3}\bar{R}_{ikjl}\varphi^k\varphi^l+\cdots, \tag{144}$$

with

$$\delta_{ij}=\bar{g}_{ij}=g_{ij}(\phi)\Big|_{\phi=\bar{\phi}}, \qquad \bar{R}_{ijkl}=R_{ijkl}\Big|_{\phi=\bar{\phi}}. \tag{145}$$

Solving the Killing equations (141) in terms of the Taylor expansion around the vacuum,

$$w_a^i=\bar{w}_a^i+(\bar{w}_a^i)_{,j}\varphi^j+\frac{1}{2!}(\bar{w}_a^i)_{,jk}\varphi^j\varphi^k+\cdots, \tag{146}$$

$$y^i=\bar{y}^i+(\bar{y}^i)_{,j}\varphi^j+\frac{1}{2!}(\bar{y}^i)_{,jk}\varphi^j\varphi^k+\cdots, \tag{147}$$

we find that the Taylor expansion coefficients satisfy

$$0=\bar{g}_{ik}(\bar{w}_a^k)_{,j}+\bar{g}_{jk}(\bar{w}_a^k)_{,i}, \tag{148}$$

$$0=\bar{g}_{ik}(\bar{y}^k)_{,j}+\bar{g}_{jk}(\bar{y}^k)_{,i}, \tag{149}$$

$$(\bar{w}_a^i)_{,jk}=\frac{1}{3}(\bar{R}^i{}_{jkl}+\bar{R}^i{}_{kjl})\bar{w}_a^l, \tag{150}$$

$$(\bar{y}^i)_{,jk}=\frac{1}{3}(\bar{R}^i{}_{jkl}+\bar{R}^i{}_{kjl})\bar{y}^l, \tag{151}$$

$$\vdots$$

There are certainly connections between the geometry R_{ijkl} and the symmetry w_a^i and y^i in Eqs. (150) and (151). However, Eqs. (150) and (151) are not enough to clarify the relation between perturbative unitarity and the S-parameter coefficient in Eq. (135). Note that the S-parameter coefficient is written in terms of the first covariant derivative of the Killing vectors $(\bar{w}_a^i)_{;j}$ and $(\bar{y}^i)_{;j}$. We need physical principles to relate $(\bar{w}_a^i)_{;j}$ and $(\bar{y}^i)_{;j}$ to the second derivatives $(\bar{w}_a^i)_{,jk}$ and $(\bar{y}^i)_{,jk}$. Actually, the $SU(2)_W\times U(1)_Y$ Lie algebra (symmetry) (142) plays this role. Plugging Eqs. (150) and (151) into Eq. (142), we obtain

$$(T_a)_j{}^i=\frac{1}{2}\varepsilon_{abc}([T_b,T_c])_j{}^i+\frac{1}{2}\varepsilon_{abc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}, \tag{152}$$

$$0=([T_a,T_Y])_j{}^i+(\bar{w}_a^k)(\bar{y}^l)\bar{R}^i{}_{jkl}, \tag{153}$$

with T_a and T_Y being matrices denoting the first derivatives of the $SU(2)_W\times U(1)_Y$ Killing vectors at the vacuum,

$$(T_a)_j{}^i\coloneqq(\bar{w}_a^i)_{,j}, \qquad (T_Y)_j{}^i\coloneqq(\bar{y}^i)_{,j}. \tag{154}$$

It is now easy to show that

$$\begin{aligned} \mathrm{tr}(T_3T_Y) &= \frac{1}{2}\varepsilon_{3bc}\mathrm{tr}([T_b,T_c]T_Y) + \frac{1}{2}\varepsilon_{3bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}(T_Y)_i{}^j \\ &= \frac{1}{2}\varepsilon_{3bc}\mathrm{tr}([T_c,T_Y]T_b) + \frac{1}{2}\varepsilon_{3bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}(T_Y)_i{}^j \\ &= -\frac{1}{2}\varepsilon_{3bc}(\bar{w}_c^k)(\bar{y}^l)\bar{R}^i{}_{jkl}(T_b)_i{}^j + \frac{1}{2}\varepsilon_{3bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}(T_Y)_i{}^j \\ &= \frac{1}{2}\varepsilon_{3bc}(\bar{w}_c^k)(\bar{w}_3^l)\bar{R}^i{}_{jkl}(T_b)_i{}^j + \frac{1}{2}\varepsilon_{3bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}(T_Y)_i{}^j. \end{aligned} \tag{155}$$

In the last line of Eq. (155) we used the fact that $U(1)_{\mathrm{em}}$ is unbroken at the vacuum (47), i.e.,

$$0 = \bar{w}_3^i + \bar{y}^i. \tag{156}$$

Equation (155) can be rewritten in the covariant form

$$\begin{aligned} (\bar{w}_3^i)_{;j}(\bar{y}^j)_{;i} = \frac{1}{2}(&\varepsilon_{3bc}(\bar{w}_c^k)(\bar{w}_3^l)\bar{R}^i{}_{jkl}(\bar{w}_b^j)_{;i} \\ &+ \varepsilon_{3bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}(\bar{y}^j)_{;i}). \end{aligned} \tag{157}$$

In a similar manner, we obtain the divergent coefficient in U [Eq. (136)],

$$\begin{aligned} &(\bar{w}_1^i)_{;j}(\bar{w}_1^j)_{;i} - (\bar{w}_3^i)_{;j}(\bar{w}_3^j)_{;i} \\ &= \frac{1}{2}(\varepsilon_{1bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}(\bar{w}_1^j)_{;i} - \varepsilon_{3bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}(\bar{w}_3^j)_{;i}). \end{aligned} \tag{158}$$

Combining Eqs. (135), (136), (157), and (158), we find

$$\begin{aligned} S_{\mathrm{div}} = -\frac{1}{12\pi}(&\varepsilon_{3bc}(\bar{w}_c^k)(\bar{w}_3^l)\bar{R}^i{}_{jkl}(\bar{w}_b^j)_{;i} \\ &+ \varepsilon_{3bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}(\bar{y}^j)_{;i})\ln\frac{\Lambda^2}{\mu^2}, \end{aligned} \tag{159}$$

$$\begin{aligned} U_{\mathrm{div}} = \frac{1}{12\pi}(&\varepsilon_{1bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}(\bar{w}_1^j)_{;i} \\ &- \varepsilon_{3bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}(\bar{w}_3^j)_{;i})\ln\frac{\Lambda^2}{\mu^2}. \end{aligned} \tag{160}$$

The relation between the symmetry and the geometry hidden in Eqs. (135) and (136) is now unveiled in Eqs. (159) and (160). The one-loop divergences of both S and U are proportional to the Riemann curvature tensor $\bar{R}_{ijkl}$ at the vacuum. Once the four-point tree-level unitarity is ensured, i.e., $\bar{R}_{ijkl} = 0$, the one-loop finiteness of S and U is automatically guaranteed in Eqs. (159) and (160).[4]

[4]The relation between the one-loop S divergence and the flatness of the scalar manifold was also pointed out in Ref. [96] in the context of the HEFT framework, in which the connection can be seen more manifestly than in the GHEFT framework.

The physical implications of the S and U parameter formulas (159) and (160) can be studied more closely. Note that both of them vanish when

$$(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}_{ijkl} = 0, \tag{161}$$

which does not require $\bar{R}_{ijkl} = 0$. What does the condition (161) imply? Combining the equivalence theorem and the results presented in Sec. III, we see that the condition (161) ensures the tree-level unitarity of the high-energy p-wave scattering amplitude in the

$$\pi^b\pi^c \to \varphi^i\varphi^j \tag{162}$$

channel. In the high-energy limit, the amplitude (162) corresponds to the $V_L^b V_L^c \to \varphi^i\varphi^j$ scattering amplitudes because of the equivalence theorem. Here V_L^a stands for the longitudinally polarized massive gauge bosons, $V_L^{1,2} = W_L^{1,2}$ and $V_L^3 = Z_L$. The one-loop finiteness of S and U does not require a completely flat scalar manifold: the scattering amplitudes $\varphi^i\varphi^j \to \varphi^k\varphi^l$ other than the NG boson channels may still violate the tree-level unitarity. Once the p-wave tree-level unitarity in the channel (162) is somehow ensured, it is potentially possible to construct strongly interacting EWSB models without violating the one-loop finiteness of S and U.

Moreover, as we see in Appendix D, the covariant derivative of the Killing vector $(w_c^i)_{;j}$ is related to the light-fermion scattering amplitudes

$$f\bar{f} \to \varphi^i\varphi^j. \tag{163}$$

Here f $(\bar{f})$ stands for light quarks or leptons (light antiquarks or antileptons). The coefficients in front of the logarithmic divergences in Eqs. (159) and (160) can be expressed in the form

$$(w_b^k)(w_c^l)\bar{R}^i{}_{jkl}(\bar{w}_a^j)_{;i}. \tag{164}$$

This suggests that, assuming negligibly small tree-level $\mathcal{O}(p^4)$ contributions, precise measurements of S and U can be used to constrain the high-energy scattering amplitudes in the

$$V_L^b V_L^c \to \varphi^i \varphi^j \quad \text{and} \quad f\bar{f} \to \varphi^i \varphi^j \tag{165}$$

channels, which can be tested in future collider experiments.

Finally we make a comment on the UV finiteness condition of Eq. (137). We find that the UV finiteness of Eq. (137) is not ensured solely by the flatness of the scalar manifold. For example, even if we assume that the scalar manifold is completely flat and $v = v_Z$ at the tree level, the extra condition

$$[(\bar{w}_1^i)(\bar{w}_1^j) - (\bar{w}_3^i)(\bar{w}_3^j)][g_W^2(\bar{w}_a^k)_{;i}(\bar{w}_a^l)_{;j}\bar{g}_{kl} + g_Y^2(\bar{y}^k)_{;i}(\bar{y}^l)_{;j}\bar{g}_{kl}] = 0. \tag{166}$$

is required to ensure the finiteness of the one-loop T-parameter correction. The Georgi-Machacek model [35–38] is an example of models where the condition (166) is not satisfied. We need to introduce independent counterterms for v and v_Z in these models.

VII. SUMMARY

We have formulated a generalized Higgs effective field theory that includes extra Higgs particles other than the 125 GeV Higgs boson as a low-energy effective field theory describing electroweak symmetry breaking. The scalar scattering amplitudes are expressed by the geometry (Riemann curvature) and the symmetry (Killing vectors) of the scalar manifold in the GHEFT. The one-loop radiative corrections to electroweak oblique corrections are also expressed in terms of the geometry and symmetry of the scalar manifold. By using the results, we have clarified the relationship between perturbative unitarity and the UV finiteness of oblique corrections in the GHEFT.

In particular, we have shown that once the tree-level unitarity is ensured, the S and U parameters' one-loop finiteness is automatically guaranteed. The tree-level perturbative unitarity in the scalar amplitudes requires the scalar manifold to be completely flat at the vacuum. On the other hand, the one-loop finiteness of the electroweak oblique corrections does not require complete flatness. These findings enabled us to verify that the tree-level unitarity condition is stronger than the one-loop UV finiteness condition in extended Higgs scenarios.

We also found the coefficients of S and U parameter divergences [Eqs. (157) and (158)] have a connection with the particle scattering amplitudes which can be measured in future collider experiments.

We emphasize that future precision measurements of the discovered Higgs couplings, cross section, and oblique parameters are quite important for investigating the geometry and symmetry of the scalar manifold in the generalized Higgs sector. Combining collider and precision experimental data with our effective theoretical approach, we should be able to obtain new prospects for physics beyond the SM.

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APPENDIX A: A SYMMETRY-GEOMETRY DICTIONARY

The metric tensor $g_{ij}(\phi)$ in the geometrical form of the Lagrangian [Eq. (37)] can be computed from the symmetry form of the Lagrangian [Eq. (30)]. We obtain

$$g_{11} = G_{11} - G_{13}\pi^2 + \frac{1}{3}(-G_{11}\pi^2\pi^2 + G_{12}\pi^1\pi^2) + \frac{1}{4}G_{33}\pi^2\pi^2 + \mathcal{O}((\pi)^3), \tag{A1}$$

$$g_{12} = G_{12} + \frac{1}{2}(G_{13}\pi^1 - G_{23}\pi^2) + \frac{1}{6}(G_{11}\pi^1\pi^2 + G_{22}\pi^1\pi^2 - G_{12}\pi^1\pi^1 - G_{12}\pi^2\pi^2) - \frac{1}{4}G_{33}\pi^1\pi^2 + \mathcal{O}((\pi)^3), \tag{A2}$$

$$g_{13} = G_{13} - \frac{1}{2}G_{33}\pi^2 + \frac{1}{6}(-G_{13}\pi^2\pi^2 + G_{23}\pi^1\pi^2) - G_{1I}[iQ_\phi]^I{}_J\phi^J\left(1 - \frac{1}{6}\pi^2\pi^2\right) + \frac{1}{2}G_{3I}[iQ_\phi]^I{}_J\phi^J\pi^2 - \frac{1}{6}G_{2I}[iQ_\phi]^I{}_J\phi^J\pi^1\pi^2 + \mathcal{O}((\pi)^3), \tag{A3}$$

$$g_{22} = G_{22} + G_{23}\pi^1 + \frac{1}{3}(-G_{22}\pi^1\pi^1 + G_{12}\pi^1\pi^2) + \frac{1}{4}G_{33}\pi^1\pi^1 + \mathcal{O}((\pi)^3), \tag{A4}$$

$$g_{23} = G_{23} + \frac{1}{2}G_{33}\pi^1 + \frac{1}{6}(G_{13}\pi^1\pi^2 - G_{23}\pi^1\pi^1) - G_{2I}[iQ_\phi]^I{}_J\phi^J\left(1 - \frac{1}{6}\pi^1\pi^1\right) - \frac{1}{2}G_{3I}[iQ_\phi]^I{}_J\phi^J\pi^1 - \frac{1}{6}G_{1I}[iQ_\phi]^I{}_J\phi^J\pi^1\pi^2 + \mathcal{O}((\pi)^3), \tag{A5}$$

$$g_{33} = G_{33} - 2G_{3I}[iQ_\phi]^I{}_J\phi^J + G_{IJ}[iQ_\phi]^I{}_K[iQ_\phi]^J{}_L\phi^K\phi^L, \tag{A6}$$

$$g_{1I} = G_{1I} + \frac{1}{2}G_{3I}\pi^2 - \frac{1}{6}G_{1I}\pi^2\pi^2 + \frac{1}{6}G_{2I}\pi^1\pi^2 + \mathcal{O}((\pi)^3), \tag{A7}$$

$$g_{2I} = G_{2I} + \frac{1}{2}G_{3I}\pi^1 + \frac{1}{6}G_{1I}\pi^1\pi^2 - \frac{1}{6}G_{2I}\pi^1\pi^1 + \mathcal{O}((\pi)^3), \tag{A8}$$

$$g_{3I} = G_{3I} - G_{IJ}[iQ_\phi]^J{}_K\phi^K, \tag{A9}$$

$$g_{IJ} = G_{IJ}. \tag{A10}$$

Note that the scalar multiplet ϕ^i in the geometrical form Lagrangian (37) contains both the NGB bosons π^1, π^2, π^3 and the Higgs bosons ϕ^I as its components, i.e.,

$$\{\phi^i\} = \{\pi^1, \pi^2, \pi^3, \phi^I\}. \tag{A11}$$

The $SU(2)_W \times U(1)_Y$ Killing vectors w_a^i and y^i are introduced through the covariant derivative (44) in the geometrical form Lagrangian (37). These Killing vectors can be determined from the infinitesimal $SU(2)_W \times U(1)_Y$ transformation properties (4), (5), and (19). They are

$$(w_1)^1 = 1 - \frac{1}{3}\pi^2\pi^2 + \mathcal{O}((\pi)^4), \tag{A12}$$

$$(w_1)^2 = \frac{1}{3}\pi^1\pi^2 + \mathcal{O}((\pi)^4), \tag{A13}$$

$$(w_1)^3 = -\frac{1}{2}\pi^2 - \frac{1}{24}(\pi^1\pi^1 + \pi^2\pi^2)\pi^2 + \mathcal{O}((\pi)^4), \tag{A14}$$

$$(w_1)^I = -\frac{1}{2}\pi^2[iQ_\phi]^I{}_J\phi^J - \frac{1}{24}(\pi^1\pi^1 + \pi^2\pi^2)\pi^2[iQ_\phi]^I{}_J\phi^J + \mathcal{O}((\pi)^4), \tag{A15}$$

$$(w_2)^1 = \frac{1}{3}\pi^1\pi^2 + \mathcal{O}((\pi)^4), \tag{A16}$$

$$(w_2)^2 = 1 - \frac{1}{3}\pi^1\pi^1 + \mathcal{O}((\pi)^4), \tag{A17}$$

$$(w_2)^3 = \frac{1}{2}\pi^1 + \frac{1}{24}(\pi^1\pi^1 + \pi^2\pi^2)\pi^1 + \mathcal{O}((\pi)^4), \tag{A18}$$

$$(w_2)^I = \frac{1}{2}\pi^1[iQ_\phi]^I{}_J\phi^J + \frac{1}{24}(\pi^1\pi^1 + \pi^2\pi^2)\pi^1[iQ_\phi]^I{}_J\phi^J + \mathcal{O}((\pi)^4), \tag{A19}$$

$$(w_3)^1 = \pi^2, \tag{A20}$$

$$(w_3)^2 = -\pi^1, \tag{A21}$$

$$(w_3)^3 = 1, \tag{A22}$$

$$(w_3)^I = [iQ_\phi]^I{}_J\phi^J, \tag{A23}$$

$$(y)^1 = 0, \tag{A24}$$

$$(y)^2 = 0, \tag{A25}$$

$$(y)^3 = -1, \tag{A26}$$

$$(y)^I = 0. \tag{A27}$$

APPENDIX B: *N*-POINT AMPLITUDE

Let us consider the Taylor expansion of the scalar manifold metric tensor $g_{ij}(\phi)$ around the vacuum point $\bar{\phi}^i$,

$$g_{ij}(\phi) = \bar{g}_{ij} + \bar{G}_{ijk}\varphi^k + \frac{1}{2}\bar{G}_{ijkl}\varphi^k\varphi^l + \frac{1}{3!}\bar{G}_{ijklm}\varphi^k\varphi^l\varphi^m + \frac{1}{4!}\bar{G}_{ijklmn}\varphi^k\varphi^l\varphi^m\varphi^n + \cdots, \tag{B1}$$

with $\phi^i = \bar{\phi}^i + \varphi^i$. The Taylor coefficients can be expressed in terms of the covariant derivatives of the Riemann curvature tensor in RNC. They are [139,140]

$$\bar{g}_{ij} = \delta_{ij}, \tag{B2}$$

$$\bar{G}_{ijk} = 0, \tag{B3}$$

$$\bar{G}_{ijkl} = \frac{2}{3}\bar{R}_{iklj}, \tag{B4}$$

$$\bar{G}_{ijklm} = \bar{R}_{iklj;m}, \tag{B5}$$

$$\bar{G}_{ijklmn} = \frac{6}{5}\bar{R}_{iklj;mn} + \frac{16}{15}\bar{R}_{iklo}\bar{R}^o{}_{mnj} \\ \vdots \tag{B6}$$

with

$$\bar{R}_{ijkl} \coloneqq R_{ijkl}\Big|_{\phi=\bar{\phi}}, \qquad \bar{R}_{ijkl;m} \coloneqq R_{ijkl;m}\Big|_{\phi=\bar{\phi}}, \qquad \bar{R}_{ijkl;mn} \coloneqq R_{ijkl;mn}\Big|_{\phi=\bar{\phi}}, \qquad \cdots. \tag{B7}$$

The one-particle-irreducible on-shell N-point amplitude $M(12\cdots N)$ can thus be expressed[5] as

$$iM(12\cdots N) = -\frac{i}{2}\sum_{m<n} s_{mn}\bar{G}_{(i_m i_n)(i_1 i_2\cdots\check{i}_m\cdots\check{i}_n\cdots i_N)}, \tag{B8}$$

in the gaugeless flat-potential ($V = 0$) scalar model. Scalar particles are all massless in this model. The indices inside

[5]Equation (B8) can be regarded as a geometrical manifestation of Weinberg's soft-theorem in on-shell amplitudes. See, e.g., Ref. [141] for a recent review on the computational techniques of various on-shell amplitudes including nonlinear sigma models.

the parentheses are understood to be totally symmetrized. The inverted hats on top of $\check{i}_m$ and $\check{i}_n$ in the sequence $i_1 i_2 \cdots \check{i}_m \cdots \check{i}_n \cdots i_N$ denote the absence of the corresponding indices, i.e.,

$$i_1 i_2 \cdots \check{i}_m \cdots \check{i}_n \cdots i_N = i_1 i_2 \cdots i_{m-1} i_{m+1} \cdots i_{n-1} i_{n+1} \cdots i_N. \quad \text{(B9)}$$

In this Appendix we show that perturbative unitarity up to the N-point amplitudes requires

$$\bar{R}_{i_1 i_2 i_3 i_4} = 0, \qquad \bar{R}_{i_1 i_2 i_3 i_4; i_5} = 0, \\ \bar{R}_{i_1 i_2 i_3 i_4; i_5 i_6} = 0, \qquad \cdots \bar{R}_{i_1 i_2 i_3 i_4; i_5 \cdots i_N} = 0. \quad \text{(B10)}$$

The scalar manifold needs to be completely flat at least in the vicinity of the vacuum. It should be stressed here that, even though we already have a compact expression for the N-point amplitude (B8), it is nontrivial to obtain the unitarity condition (B10) since the generalized Mandelstam variables s_{mn} need to satisfy the momentum conservation conditions

$$\sum_{n=1}^{N} s_{mn} = 0 \quad \text{(B11)}$$

and the conditions coming from the four-dimensional spacetime (Gram determinant conditions) [142]. We need to make full use of the Riemann tensor's symmetry to deduce our conclusions (B10).

$N = 4$.—Let us start with the four-point scattering amplitude. We compute the amplitude in the limit

$$s \coloneqq s_{12} = s_{34} = -s_{13} = -s_{24} \neq 0, \qquad s_{14} = s_{23} = 0. \quad \text{(B12)}$$

Clearly, the momentum conservation conditions (B11) are satisfied in Eq. (B12). The Gram determinant conditions do not give extra conditions for $N = 4$.

In the above limit, the four-point on-shell amplitude behaves as

$$M(1234) \propto sA(1234), \quad \text{(B13)}$$

with

$$A(1234) \coloneqq \{(12)|(34)\} + \{(34)|(12)\} \\ - \{(13)|(24)\} - \{(24)|(13)\}. \quad \text{(B14)}$$

Here we introduce an abbreviation for the Riemann curvature tensor,

$$\{12|34\} \coloneqq \bar{R}_{i_1 i_3 i_4 i_2}. \quad \text{(B15)}$$

Again, the indices inside parentheses are understood to be totally symmetrized.

Considering the amplitude (B13) for large s, we see that perturbative unitarity requires

$$A(1234) = 0. \quad \text{(B16)}$$

Using the Riemann curvature tensor's symmetry

$$\{12|34\} = -\{32|14\} = -\{14|32\} = \{34|12\} = \{21|43\} \quad \text{(B17)}$$

and the first Bianchi identity

$$\{12|34\} + \{13|42\} + \{14|23\} = 0, \quad \text{(B18)}$$

the coefficient $A(1234)$ can be computed as

$$\begin{aligned} A(1234) &= 2\{(12)|(34)\} - 2\{(13)|(24)\} \\ &= \{12|34\} + \{12|43\} - \{13|24\} - \{13|42\} \\ &= \{12|34\} - \{13|42\} + \{14|23\} - \{13|42\} \\ &= -3\{13|42\}. \end{aligned} \quad \text{(B19)}$$

It is now easy to see that perturbative unitarity requires the Riemann curvature tensor to vanish at the vacuum,

$$\bar{R}_{i_1 i_4 i_2 i_3} = 0. \quad \text{(B20)}$$

Taking the external lines $i_1, \ldots, i_4$ as arbitrary, the result (B20) requires $\bar{R}_{ijkl} = 0$, which is enough to guarantee perturbative unitarity in the arbitrary four-point amplitudes given in the form of Eq. (B8). The considerations in the limit (B12) thus provide necessary and sufficient conditions for perturbative unitarity in the four-point amplitudes.

$N = 5$

We next consider the five-point scattering amplitude. Again, we consider the amplitude in the limit

$$s \coloneqq s_{12} = s_{34} = -s_{13} = -s_{24} \neq 0, \\ s_{14} = s_{23} = s_{15} = s_{25} = s_{35} = s_{45} = 0. \quad \text{(B21)}$$

Note that the fifth particle is considered to be very soft.

We introduce an abbreviation for the covariant derivative of the Riemann curvature tensor,

$$\{12|34; 5\} \coloneqq \bar{R}_{i_1 i_3 i_4 i_2; i_5}. \quad \text{(B22)}$$

The five-point amplitude in this limit behaves as

$$M(12345) \propto sA(12345), \quad \text{(B23)}$$

with

$$A(12345) \coloneqq \{(12)|(34;5)\} + \{(34)|(12;5)\} - \{(13)|(24;5)\} - \{(24)|(13;5)\}. \tag{B24}$$

Using

$$\{(12)|(34;5)\} = \frac{1}{3}\{(12)|(34);5\} + \frac{1}{3}\{(12)|(35);4\} + \frac{1}{3}\{(12)|(45);3\}, \tag{B25}$$

we obtain

$$\begin{aligned} A(12345) = &\frac{1}{3}\{\{(12)|(34);5\} + \{(34)|(12);5\} - \{(13)|(24);5\} - \{(24)|(13);5\}\} \\ &+ \frac{1}{3}\{\{(43)|(25);1\} - \{(42)|(35);1\}\} + \frac{1}{3}\{\{(34)|(15);2\} - \{(31)|(45);2\}\} \\ &+ \frac{1}{3}\{\{(21)|(45);3\} - \{(24)|(15);3\}\} + \frac{1}{3}\{\{(12)|(35);4\} - \{(13)|(25);4\}\}. \end{aligned} \tag{B26}$$

The first line in Eq. (B26) can be easily computed using the result for the four-point amplitude. The second and third lines can also be computed in a manner similar to Eq. (B19). We find

$$\begin{aligned} A(12345) = &-\{13|42;5\} - \frac{1}{2}\{42|53;1\} - \frac{1}{2}\{31|54;2\} \\ &- \frac{1}{2}\{24|51;3\} - \frac{1}{2}\{13|52;4\}. \end{aligned} \tag{B27}$$

Equation (B27) can be simplified further with the help of the second Bianchi identity,

$$\{12|34;5\} + \{14|35;2\} + \{15|32;4\} = 0. \tag{B28}$$

We obtain

$$\begin{aligned} A(12345) &= -\{13|42;5\} - \frac{1}{2}\{\{24|35;1\} + \{25|31;4\}\} \\ &\quad - \frac{1}{2}\{\{13|45;2\} + \{15|42;3\}\} \\ &= -\{13|42;5\} + \frac{1}{2}\{21|34;5\} + \frac{1}{2}\{12|43;5\} \\ &= -2\{13|42;5\}. \end{aligned} \tag{B29}$$

The perturbative unitarity in the five-point amplitude thus requires

$$\bar{R}_{i_1 i_3 i_4 i_2; i_5} = 0. \tag{B30}$$

It is easy to see that Eq. (B30) gives necessary and sufficient conditions for perturbative unitarity in the five-point amplitudes.

$N = 6$

It is now straightforward to derive perturbative unitarity conditions for the six-point amplitude $M(123456)$. It turns out that considering the limit

$$s \coloneqq s_{12} = s_{34} = -s_{13} = -s_{24} \neq 0 \tag{B31}$$

are enough. Generalized Mandelstam variables other than s_{12}, s_{34}, s_{13}. and s_{24} are taken to be zero. Note that the fifth and sixth particles are both considered to be very soft in this limit. Note also that this choice of Mandelstam variables is consistent with the momentum conservation constraints and the Gram determinant constraints.

We already know that the Riemann curvature tensor $\bar{R}_{ijkl}$ vanishes at the vacuum thanks to the perturbative unitarity of the four-point amplitude. We therefore concentrate on the $\bar{R}_{ijkl;mn}$ term in Eq. (B6). The six-point amplitude coming from the $\bar{R}_{ijkl;mn}$ term in Eq. (B6) behaves as

$$M(123456) \propto sA(123456), \tag{B32}$$

with

$$\begin{aligned} A(123456) \coloneqq &\{(12)|(34;56)\} + \{(34)|(12;56)\} \\ &- \{(13)|(24;56)\} - \{(24)|(13;56)\}. \end{aligned} \tag{B33}$$

Here we introduce the abbreviation

$$\{12|34;56\} \coloneqq \bar{R}_{i_1 i_3 i_4 i_2; i_5 i_6}. \tag{B34}$$

Using

$$\begin{aligned} &\{(12)|(34;56)\} \\ &= \frac{1}{6}\{(12)|(34);(56)\} + \frac{1}{6}\{(12)|(56);(34)\} \\ &\quad + \frac{1}{6}\{(12)|(35);(46)\} + \frac{1}{6}\{(12)|(46);(35)\} \\ &\quad + \frac{1}{6}\{(12)|(36);(45)\} + \frac{1}{6}\{(12)|(45);(36)\}, \end{aligned} \tag{B35}$$

we obtain

$$A(123456) \coloneqq A_1 + A_2 + A_3 + A_4, \tag{B36}$$

with

$$A_1 = \frac{1}{6}\{\{(12)|(34);56\} + \{(34)|(12);56\} - \{(13)|(24);56\} - \{(24)|(13);56\}\}, \tag{B37}$$

$$A_2 = \frac{1}{6}\{\{(12)|(35);46\} + \{(12)|(45);36\} + \{(34)|(15);26\} + \{(34)|(25);16\} \\ - \{(13)|(25);46\} - \{(13)|(45);26\} - \{(24)|(15);36\} - \{(24)|(35);16\}\}, \tag{B38}$$

$$A_3 = \frac{1}{6}\{\{(12)|(36);45\} + \{(12)|(46);35\} + \{(34)|(16);25\} + \{(34)|(26);15\} \\ - \{(13)|(26);45\} - \{(13)|(46);25\} - \{(24)|(16);35\} - \{(24)|(36);15\}\}, \tag{B39}$$

$$A_4 = \frac{1}{6}\{\{(12)|(56);34\} + \{(34)|(56);12\} - \{(13)|(56);24\} - \{(24)|(56);13\}\}. \tag{B40}$$

Here we used the fact that the covariant derivatives are commutable, justified by the vanishing curvature tensor $\bar{R}_{ijkl} = 0$ at the vacuum. The A_1 term can be easily computed by using the result for $A(1234)$. The A_2 and A_3 terms can be computed in a manner similar to the computation of $A(12345)$. We obtain

$$A_1 = A_2 = A_3 = -\frac{1}{2}\{13|42;56\}. \tag{B41}$$

The A_4 term can be computed as

$$\begin{aligned} A_4 &= \frac{1}{12}\{\{12|56;34\} + \{12|65;34\} + \{34|56;12\} + \{34|65;12\} \\ &\quad - \{13|56;24\} - \{13|65;24\} - \{24|56;13\} - \{24|65;13\}\} \\ &= \frac{1}{12}\{\{12|56;34\} + \{12|65;34\} + \{43|65;12\} + \{43|56;12\} \\ &\quad + \{16|53;24\} + \{15|63;24\} + \{45|62;13\} + \{46|52;13\}\} \\ &= \frac{1}{12}\{(\{12|56;34\} + \{16|53;24\}) + (\{12|65;34\} + \{15|63;24\}) \\ &\quad + (\{43|65;12\} + \{45|62;13\}) + (\{43|56;12\} + \{46|52;13\})\}. \end{aligned}$$

Applying the second Bianchi identity, it can be simplified further to obtain

$$\begin{aligned} A_4 &= -\frac{1}{12}\{\{13|52;64\} + \{13|62;54\} + \{42|63;15\} + \{42|53;16\}\} \\ &= -\frac{1}{12}\{\{31|25;46\} + \{26|31;45\} + \{24|36;15\} + \{35|24;16\}\} \\ &= -\frac{1}{12}\{(\{31|25;46\} + \{35|24;16\}) + (\{26|31;45\} + \{24|36;15\})\} \\ &= \frac{1}{12}\{\{34|21;56\} + \{21|34;65\}\} \\ &= -\frac{1}{6}\{13|42;56\}. \end{aligned} \tag{B42}$$

The second Bianchi identity is used in the first and fourth lines in the above calculation. Combining these results, we find that the six-point amplitude can be expressed in the simple form

$$A(123456) = -\frac{5}{3}\{13|42;56\}. \tag{B43}$$

The perturbative unitarity condition in the six-point amplitude $A(123456) = 0$ can now be written in terms of the covariant derivative of the Riemann curvature,

$$\bar{R}_{i_1 i_4 i_2 i_3; i_5 i_6} = 0. \tag{B44}$$

It is straightforward to generalize the calculation presented above to perturbative unitarity conditions in the N-point amplitude,

$$\bar{R}_{i_1 i_4 i_2 i_3; i_5 i_6 \cdots i_N} = 0. \tag{B45}$$

Since the Taylor expansion coefficients of $R_{ijkl}(\phi)$ are required to vanish at any order, the N-point perturbative unitarity requires the Riemann curvature to be

$$R_{ijkl}(\phi) = 0, \tag{B46}$$

at least in the vicinity of the vacuum. However, there may exist essential singularity type corrections to Eq. (B46) without affecting the Taylor expansion coefficients of $R_{ijkl}(\phi)$.

APPENDIX C: BACKGROUND-FIELD METHOD

In this Appendix we briefly summarize the interaction terms used in the calculation of the vacuum polarization functions in the background-field method at the one-loop level. For a review of the background-field method, see Refs. [133–138].

We start with the lowest-order [$\mathcal{O}(p^2)$] gauged nonlinear sigma model Lagrangian (37). Let us first decompose ϕ^i, W^a_μ, and B_μ into the background fields and the fluctuation fields as

$$\phi^i \coloneqq \tilde{\phi}^i + \xi^i - \frac{1}{2}\tilde{\Gamma}^i_{jk}\xi^j\xi^k + \cdots, \tag{C1}$$

$$W^a_\mu \coloneqq \tilde{W}^a_\mu + \mathcal{W}^a_\mu, \tag{C2}$$

$$B_\mu \coloneqq \tilde{B}_\mu + \mathcal{B}_\mu, \tag{C3}$$

where $\tilde{\phi}^i$, $\tilde{W}^a_\mu$, and $\tilde{B}_\mu$ are the background fields. The dynamical fluctuation fields are denoted by ξ^i, $\mathcal{W}^a_\mu$, and $\mathcal{B}_\mu$. $\tilde{\Gamma}^i_{jk}$ represents the Christoffel symbols for the metric g_{ij} at $\phi = \tilde{\phi}$. The metric tensor and the Killing vector fields are expanded as

$$g_{ij} = \tilde{g}_{ij} + \frac{1}{3}\tilde{R}_{iklj}\xi^k\xi^l + \cdots, \tag{C4}$$

$$w^i_a = \tilde{w}^i_a + (\tilde{w}^i_a)_{;j}\xi^j + \frac{1}{3}\tilde{R}^i_{klj}\tilde{w}^j_a\xi^k\xi^l + \cdots, \tag{C5}$$

$$y^i = \tilde{y}^i + (\tilde{y}^i)_{;j}\xi^j + \frac{1}{3}\tilde{R}^i_{klj}\tilde{y}^j\xi^k\xi^l + \cdots, \tag{C6}$$

where $\tilde{g}_{ij}$ and $\tilde{R}_{ikjl}$ denote the metric and the Riemann curvature tensor evaluated at $\phi = \tilde{\phi}$, respectively. $\tilde{w}^i_a$ and $\tilde{y}^i$ are the $SU(2)_W$ and $U(1)_Y$ Killing vectors, while $(\tilde{w}^i_a)_{;j}$ and $(\tilde{y}^i)_{;j}$ are the covariant derivatives of the Killing vectors evaluated at $\phi = \tilde{\phi}$.

The Lagrangian (37) is expanded as

$$\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots, \tag{C7}$$

where $\mathcal{L}^{(n)}$ is of order n in the fluctuation fields.

The quadratic terms $\mathcal{L}^{(2)}$ are given as

$$\begin{aligned}\mathcal{L}^{(2)} = &-\frac{1}{2}\mathcal{W}^a_\mu(-\tilde{D}^2\delta_{ab}\eta^{\mu\nu} + \tilde{D}^\nu\tilde{D}^\mu\delta_{ab} - g_W^2\tilde{g}_{ij}\tilde{w}^i_a\tilde{w}^j_b\eta^{\mu\nu} - g_W\tilde{W}^{c\mu\nu}\epsilon^{abc})\mathcal{W}^b_\nu - \frac{1}{2}\mathcal{B}_\mu(-\partial^2\eta^{\mu\nu} + \partial^\nu\partial^\mu - g_Y^2\tilde{g}_{ij}\tilde{y}^i\tilde{y}^j\eta^{\mu\nu})\mathcal{B}_\nu\\ &+ g_W g_Y\mathcal{W}^a_\mu(\tilde{g}_{ij}\tilde{w}^i_a\tilde{y}^j\eta^{\mu\nu})\mathcal{B}_\nu + \frac{1}{2}\xi^i(-\tilde{D}^2\tilde{g}_{ij} - \tilde{D}_\mu\tilde{\phi}^k\tilde{D}^\mu\tilde{\phi}^l\tilde{R}_{kilj} - \tilde{V}_{;ij})\xi^j + 2g_W\mathcal{W}^a_\mu(\tilde{g}_{jk}(\tilde{w}^k_a)_{;i}\tilde{D}^\mu\tilde{\phi}^j)\xi^i\\ &+ 2g_Y\mathcal{B}_\mu(\tilde{g}_{jk}(\tilde{y}^k)_{;i}\tilde{D}^\mu\tilde{\phi}^j)\xi^i - g_W\tilde{g}_{ji}\tilde{w}^j_a(\tilde{D}^\mu\mathcal{W}^a_\mu)\xi^i - g_Y\tilde{g}_{ji}\tilde{y}^j(\partial^\mu\mathcal{B}_\mu)\xi^i,\end{aligned} \tag{C8}$$

with $\eta^{\mu\nu}$ being the spacetime metric. Here we define

$$\tilde{D}_\mu\mathcal{W}^a_\mu \coloneqq \partial_\mu\mathcal{W}^a_\mu - g_W\epsilon^{abc}\tilde{W}^b_\mu\mathcal{W}^c_\mu, \tag{C9}$$

$$\tilde{D}_\mu\tilde{\phi}^i \coloneqq \partial_\mu\tilde{\phi}^i + g_W\tilde{W}^a_\mu\tilde{w}^i_a + g_Y\tilde{B}_\mu\tilde{y}^i, \tag{C10}$$

$$\tilde{D}_\mu\xi^i \coloneqq \partial_\mu\xi^i + \tilde{\Gamma}^i_{kj}(\partial_\mu\tilde{\phi})^j\xi^k + g_W\tilde{W}^a_\mu(\tilde{w}^i_a)_{;j}\xi^j + g_Y\tilde{B}_\mu(\tilde{y}^i)_{;j}\xi^j, \tag{C11}$$

$$\tilde{V}_{;ij} \coloneqq V_{;ij}\Big|_{\phi=\tilde{\phi}}. \tag{C12}$$

In order to compute radiative corrections, we introduce the gauge-fixing action

$$\mathcal{L}_{\mathrm{GF}} := -\frac{1}{2\alpha_W} G_W^a G_W^a - \frac{1}{2\alpha_Y} G_Y G_Y, \tag{C13}$$

where

$$G_W^a := \tilde{D}^\mu \mathcal{W}_\mu^a - g_W \alpha_W \tilde{g}_{ij} \tilde{w}_a^i \xi^j, \tag{C14}$$

$$G_Y := \partial^\mu \mathcal{B}_\mu - g_Y \alpha_Y \tilde{g}_{ij} \tilde{y}^i \xi^j, \tag{C15}$$

with α_W and α_Y being gauge-fixing parameters for $SU(2)_W$ and $U(1)_Y$ symmetry, respectively. In the one-loop calculation performed in Sec. V A, we take $\alpha_W = \alpha_Y = 1$ ('t Hooft–Feynman gauge).

We then obtain

$$\begin{aligned}\mathcal{L}^{(2)} + \mathcal{L}_{\mathrm{GF}} = &-\frac{1}{2}\mathcal{W}_\mu^a \left(-\tilde{D}^2 \delta_{ab}\eta^{\mu\nu} + \left(1 - \frac{1}{\alpha_W}\right)\tilde{D}^\mu \tilde{D}^\nu \delta_{ab} - g_W^2 \tilde{g}_{ij}\tilde{w}_a^i \tilde{w}_b^j \eta^{\mu\nu} - 2g_W \tilde{W}^{c\mu\nu}\varepsilon^{abc}\right)\mathcal{W}_\nu^b \\ &-\frac{1}{2}\mathcal{B}_\mu\left(-\partial^2 \eta^{\mu\nu} + \left(1 - \frac{1}{\alpha_Y}\right)\partial^\mu\partial^\nu - g_Y^2 \tilde{g}_{ij}\tilde{y}^i\tilde{y}^j \eta^{\mu\nu}\right)\mathcal{B}_\nu + g_W g_Y \mathcal{W}_\mu^a (\tilde{g}_{ij}\tilde{w}_a^i \tilde{y}^j \eta^{\mu\nu})\mathcal{B}_\nu \\ &+\frac{1}{2}\xi^i(-\tilde{D}^2\tilde{g}_{ij} - \tilde{D}_\mu\tilde{\phi}^k \tilde{D}^\mu \tilde{\phi}^l \tilde{R}_{kilj} - \alpha_W g_W^2 \tilde{g}_{lj}\tilde{g}_{ki}\tilde{w}_a^l \tilde{w}_a^k - \alpha_Y g_Y^2 \tilde{g}_{lj}\tilde{g}_{ki}\tilde{y}^l\tilde{y}^k - \tilde{V}_{;ij})\xi^j \\ &+ 2g_W \mathcal{W}_\mu^a (\tilde{g}_{jk}(\tilde{w}_a^k)_{;i}\tilde{D}^\mu\tilde{\phi}^j)\xi^i + 2g_Y \mathcal{B}_\mu(\tilde{g}_{jk}(\tilde{y}^k)_{;i}\tilde{D}^\mu\tilde{\phi}^j)\xi^i.\end{aligned} \tag{C16}$$

We also need to introduce the FP action

$$\begin{aligned}\mathcal{L}_{\mathrm{FP}} := &\, i g_W \bar{c}_W^a \frac{\delta G_W^a}{\delta\theta_W^b} c_W^b + i g_Y \bar{c}_Y \frac{\delta G_Y}{\delta\theta_Y} c_Y + i g_Y \bar{c}_W^a \frac{\delta G_W^a}{\delta\theta_Y} c_Y \\ &+ i g_W \bar{c}_Y \frac{\delta G_Y}{\delta\theta_W^b} c_W^b\end{aligned} \tag{C17}$$

associated with the gauge-fixing term, where

$$\begin{aligned}\frac{\delta G_W^a}{\delta\theta_W^c} := &-\frac{1}{g_W}[(\partial^\mu\delta^{ad} - g_W\varepsilon^{abd}\tilde{W}^{b\mu})(\partial_\mu\delta^{dc} - g_W\varepsilon^{dec}\tilde{W}_\mu^e) \\ &+ g_W^2\alpha_W\tilde{g}_{ij}\tilde{w}_a^i\tilde{w}_c^j] + \mathcal{O}(\xi)\end{aligned} \tag{C18}$$

$$\frac{\delta G_W^a}{\delta\theta_Y} := -g_W\alpha_W\tilde{g}_{ij}\tilde{w}_a^i\tilde{y}^j + \mathcal{O}(\xi), \tag{C19}$$

$$\frac{\delta G_Y}{\delta\theta_W^b} := -g_Y\alpha_Y\tilde{g}_{ij}\tilde{y}^i\tilde{w}_b^j + \mathcal{O}(\xi), \tag{C20}$$

$$\frac{\delta G_Y}{\delta\theta_Y} := -\frac{1}{g_Y}(\partial^2 + g_Y^2\alpha_Y\tilde{g}_{ij}\tilde{y}^i\tilde{y}^j) + \mathcal{O}(\xi). \tag{C21}$$

$\mathcal{L}_{\mathrm{FP}}$ is expanded as

$$\begin{aligned}\mathcal{L}_{\mathrm{FP}} = &\, i((\tilde{D}^\mu\bar{c}_W^a)\tilde{D}_\mu c_W^a - g_W^2\alpha_W\tilde{g}_{ij}\tilde{w}_a^i\tilde{w}_b^j\bar{c}_W^a c_W^b) \\ &+ i((\partial^\mu\bar{c}_Y)\partial_\mu c_Y - g_Y^2\alpha_Y\tilde{g}_{ij}\tilde{y}^i\tilde{y}^j\bar{c}_Y c_Y) \\ &- i g_W g_Y \alpha_W \bar{c}_W^a\tilde{g}_{ij}\tilde{w}_a^i\tilde{y}^j c_Y \\ &- i g_W g_Y \alpha_Y \bar{c}_Y\tilde{g}_{ij}\tilde{y}^i\tilde{w}_a^j c_W^a + \cdots,\end{aligned} \tag{C22}$$

where

$$\tilde{D}_\mu c_W^a := \partial_\mu c_W^a - g_W\varepsilon^{abc}\tilde{W}_\mu^b c_W^c, \tag{C23}$$

$$\tilde{D}_\mu \bar{c}_W^a := \partial_\mu \bar{c}_W^a - g_W\varepsilon^{abc}\tilde{W}_\mu^b \bar{c}_W^c. \tag{C24}$$

In Eq. (C22) we only show the quadratic terms of the fluctuation fields.

The one-loop corrections to the electroweak gauge boson vacuum polarization functions $\Pi_A(p^2)$ can be evaluated by using the quadratic Lagrangian, $\mathcal{L}^{(2)} + \mathcal{L}_{\mathrm{GF}} + \mathcal{L}_{\mathrm{FP}}$. In Sec. V A we calculate the one-loop diagrams where the internal lines are the fluctuation fields or FP ghosts.

APPENDIX D: $f\bar{f} \to \varphi^i\varphi^j$ AMPLITUDE

The four-point scalar boson scattering amplitudes are described by the Riemann curvature tensor $\bar{R}_{ijkl}$ and the covariant derivatives of the potential $\bar{V}_{;ij}$, $\bar{V}_{;ijk}$, and $\bar{V}_{;ijkl}$ at the vacuum in the nonlinear sigma model, as we show in Sec. III. Therefore, these tensors can be measured through the measurements of the scalar boson scattering cross sections.

When we consider a gauged nonlinear sigma model, the derivative $\partial_\mu\phi^i$ is replaced by the covariant one $(D_\mu\phi)^i$,

$$(D_\mu\phi)^i := \partial_\mu\phi^i + g_V V_\mu v^i(\phi), \tag{D1}$$

with V_μ and $v^i(\phi)$ being a gauge field and its corresponding Killing vector. The gauge coupling strength is denoted by g_V in Eq. (D1). If the Killing vector $v^i(\phi)$ does not vanish at the vacuum,

$$\bar{v}^i := v^i(\phi)\Big|_{\phi=\bar{\phi}} \neq 0, \tag{D2}$$

this implies that the gauge symmetry is spontaneously broken, and the gauge boson V_μ acquires a mass,

$$M_V^2 = g_V^2 \bar{g}_{ij}(\bar{v}^i)(\bar{v}^j), \tag{D3}$$

with

$$\bar{g}_{ij} := g_{ij}(\phi)\Big|_{\phi=\bar{\phi}}. \tag{D4}$$

The magnitude of the Killing vector at the vacuum, $\bar{g}_{ij}(\bar{v}^i)(\bar{v}^j)$, can therefore be determined by measuring the gauge boson's mass.

How can we measure the first covariant derivative of the Killing vector

$$(\bar{v}^i)_{;j} := (v^i)_{;j}\Big|_{\phi=\bar{\phi}} \tag{D5}$$

from experimental observables in the gauged nonlinear sigma model? We address the issue in this Appendix and show that the process $f\bar{f} \to V_\mu \to \varphi^i\varphi^j$ can be used to determine $(\bar{v}^i)_{;j}$. Here we introduce a spin-1/2 fermion multiplet f. It couples with the gauge field V_μ through its covariant derivative

$$D_\mu f := \partial_\mu f + g_V V_\mu T_V^{(f)} f, \tag{D6}$$

with $T_V^{(f)}$ being the charge matrix of the fermion multiplet f. Note that, in order to keep the Lagrangian gauge invariant, the fermion current

$$J_V^\mu := \bar{f}\gamma^\mu T_V^{(f)} f \tag{D7}$$

must be conserved,

$$0 = \partial_\mu J_V^\mu. \tag{D8}$$

In order to calculate the $f\bar{f} \to V_\mu \to \varphi^i\varphi^j$ amplitude, we consider the gauge interaction Lagrangian

$$\mathcal{L}_{V\phi} = g_V V_\mu g_{ij}(\phi)(\partial^\mu\phi^i)v^j(\phi), \tag{D9}$$

which can be derived from the nonlinear sigma model kinetic term,

$$\frac{1}{2}g_{ij}(\phi)(D_\mu\phi)^i(D^\mu\phi)^j \in \mathcal{L}. \tag{D10}$$

Expanding the scalar manifold metric $g_{ij}(\phi)$ and the Killing vector $v^j(\phi)$ by the dynamical excitation field φ^i, we obtain

$$g_{ij}(\phi) = \bar{g}_{ij} + \varphi^k\bar{g}_{ij,k} + \cdots, \tag{D11}$$

$$v^j(\phi) = \bar{v}^j + \varphi^k(\bar{v}^j)_{,k} + \cdots, \tag{D12}$$

with

$$\bar{g}_{ij,k} := \frac{\partial}{\partial\phi^k}g_{ij}\Big|_{\phi=\bar{\phi}}, \qquad (\bar{v}^j)_{,k} := \frac{\partial}{\partial\phi^k}v^j\Big|_{\phi=\bar{\phi}} \tag{D13}$$

and

$$\phi^i = \bar{\phi}^i + \varphi^i. \tag{D14}$$

The interaction Lagrangian (D9) can be expanded as

$$\begin{aligned}\mathcal{L}_{V\phi} &= g_V V_\mu\bar{g}_{ij}(\partial^\mu\varphi^i)(\bar{v}^j)\\ &\quad + g_V V_\mu(\partial^\mu\varphi^i)\varphi^k(\bar{g}_{ij}(\bar{v}^j)_{,k} + \bar{g}_{ij,k}(\bar{v}^j)) + \cdots.\end{aligned} \tag{D15}$$

Note that on-shell amplitudes are not affected by total derivative terms in the Lagrangian. The interaction Lagrangian (D15) can thus be replaced by

$$\begin{aligned}\mathcal{L}'_{V\phi} &= \mathcal{L}_{V\phi} - \frac{1}{2}\partial^\mu(g_V V_\mu\varphi^i\varphi^k(\bar{g}_{ij}(\bar{v}^j)_{,k} + \bar{g}_{ij,k}(\bar{v}^j)))\\ &= g_V V_\mu\bar{g}_{ij}(\partial^\mu\varphi^i)(\bar{v}^j) - g_V(\partial^\mu V_\mu)\varphi^i\varphi^k(\bar{g}_{ij}(\bar{v}^j)_{,k} + \bar{g}_{ij,k}(\bar{v}^j))\\ &\quad + \frac{1}{2}g_V V_\mu(\partial^\mu\varphi^i)\varphi^k(\bar{g}_{ij}(\bar{v}^j)_{,k} + \bar{g}_{ij,k}(\bar{v}^j) - \bar{g}_{kj}(\bar{v}^j)_{,i} - \bar{g}_{kj,i}(\bar{v}^j)) + \cdots.\end{aligned} \tag{D16}$$

On the other hand, it is straightforward to show that

$$\begin{aligned}g_{ij}(v^j)_{;k} - g_{kj}(v^j)_{;i} &= g_{ij}(v^j)_{,k} + g_{ij}\Gamma^j_{kl}v^l - g_{kj}(v^j)_{,i} + g_{kj}\Gamma^j_{il}v^l\\ &= g_{ij}(v^j)_{,k} + \frac{1}{2}[g_{il,k} + g_{ki,l} - g_{kl,i}]v^l - g_{kj}(v^j)_{,i} - \frac{1}{2}[g_{kl,i} + g_{ik,l} - g_{il,k}]v^l\\ &= g_{ij}(v^j)_{,k} + g_{il,k}(v^l) - g_{kj}(v^j)_{,i} - g_{kl,i}(v^l).\end{aligned} \tag{D17}$$

Here the affine connection Γ^j_{kl} is defined by

$$\Gamma^j_{kl} = \frac{1}{2} g^{jm}(g_{ml,k} + g_{km,l} - g_{kl,m}). \tag{D18}$$

It is now easy to see that

$$\mathcal{L}'_{V\phi} = g_V V_\mu \bar{g}_{ij}(\partial^\mu \varphi^i)(\bar{v}^j) - g_V(\partial^\mu V_\mu)\varphi^i \varphi^k (\bar{g}_{ij}(\bar{v}^j)_{,k} + \bar{g}_{ij,k}(\bar{v}^j)) + \frac{1}{2} g_V V_\mu (\partial^\mu \varphi^i)\varphi^k (\bar{g}_{ij}(\bar{v}^j)_{;k} - \bar{g}_{kj}(\bar{v}^j)_{;i}) + \cdots. \tag{D19}$$

Thanks to the fermion current conservation, the term proportional to $\partial^\mu V_\mu$ does not contribute to the $f\bar{f} \to V_\mu \to \varphi^i \varphi^j$ amplitude. It is now easy to show that

$$\mathcal{M}(f\bar{f} \to V_\mu \to \varphi^i \varphi^j) \propto (\bar{g}_{ik}(\bar{v}^k)_{;j} - \bar{g}_{jk}(\bar{v}^k)_{;i}) \frac{g_V^2 \delta^{ab}}{s - M_V^2}. \tag{D20}$$

The first covariant derivative of the Killing vector, $\bar{g}_{ik}(\bar{v}^k)_{;j}$, thus plays the role of the $V_\mu - \varphi^i - \varphi^j$ interaction vertex in the $f\bar{f} \to V_\mu \to \varphi^i \varphi^j$ amplitude.

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