Relating the Cabibbo angle to $\tan\beta$ in a two Higgs-doublet model

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In a two Higgs-doublet model with D_4 flavor symmetry we establish a relation between $\tan\beta$ and the Cabibbo angle. Due to a small number of parameters, the quark Yukawa sector of the model is very predictive. The flavor changing neutral currents are small enough to allow for relatively light nonstandard scalars to pass through the flavor constraints.

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Employing flavor symmetries to understand the apparent arbitrariness of the quark masses and mixings in the Standard Model (SM) is an exercise continuing for decades. The Yukawa Lagrangian of the SM also contains many redundant parameters, which is not a very attractive feature of the model. Therefore, theoretical constructions beyond the SM (BSM) that attempt to address these issues in a minimalistic manner should deserve some attention. To this end, we notice that the quark masses and mixings adhere to the following approximate pattern,

$$m_u \approx 0, \qquad m_d \approx 0,$$
$$V_{\rm CKM} = \begin{pmatrix} \cos\theta_C & \sin\theta_C & 0 \\ -\sin\theta_C & \cos\theta_C & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{1}$$

where $V_{\rm CKM}$ stands for the Cabibbo–Kobayashi–Maskawa (CKM) matrix with only the Cabibbo block retained. The quantity $\sin\theta_C \approx 0.22$ appearing in Eq. (1) denote the Cabibbo mixing parameter. In this approximate scenario, we note that there are only five nonzero parameters in the quark sector, namely, four quark masses (m_c, m_s, m_t, m_b) and the Cabibbo parameter itself. Thus, a flavor-model that contains five or fewer parameters in its quark Yukawa Lagrangian, might have a better aesthetic appeal than the SM in the sense that many of the redundant parameters have been erased by the flavor symmetry leaving behind only the relevant ones. The model can be even more attractive, if the zeros in Eq. (1) emerge naturally as a consequence of the Yukawa textures imposed by the flavor symmetry. As we will demonstrate, these objectives can be achieved in the simple framework of a two Higgs-doublet model (2HDM) [1,2] with a D_4 flavor symmetry.

The discrete symmetry group D_4 has five irreducible representations: $\mathbf{1}_{++}$, $\mathbf{1}_{+-}$, $\mathbf{1}_{-+}$, $\mathbf{1}_{--}$ and $\mathbf{2}$ [3,4]. We pick a basis such that the generators in the $\mathbf{2}$ representation are given by

$$a = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \qquad b = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{2}$$

Note that a is of order 4, whereas b is of order 2. The rest of the elements can be obtained by taking products of powers of these two elements. In this basis, the relevant tensor products are obtained as

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{\mathbf{2}} \otimes \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}_{\mathbf{2}} = [x_1y_1 + x_2y_2]_{\mathbf{1}_{++}} \oplus [x_1y_2 - x_2y_1]_{\mathbf{1}_{--}}$$
$$\oplus [x_1y_2 + x_2y_1]_{\mathbf{1}_{-+}} \oplus [x_1y_1 - x_2y_2]_{\mathbf{1}_{+-}}, \tag{3a}$$

$$\mathbf{1}_{rs} \otimes \mathbf{1}_{r's'} = \mathbf{1}_{r''s''}, \tag{3b}$$

where $r'' = r \cdot r'$ and $s'' = s \cdot s'$. The quark fields are assumed to transform under D_4 in the following way:

$$\mathbf{2}\colon \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}, \qquad \begin{bmatrix} p_{1R} \\ p_{2R} \end{bmatrix}, \qquad \begin{bmatrix} n_{1R} \\ n_{2R} \end{bmatrix}, \tag{4a}$$

$$\mathbf{1}_{++}\colon Q_3, \qquad \mathbf{1}_{--}\colon p_{3R}, \qquad \mathbf{1}_{-+}\colon n_{3R}, \tag{4b}$$

where the Q_A's $(A = 1, 2, 3)$ are the usual left-handed SU(2) quark doublets, whereas the p_{AR}'s and n_{AR}'s are the right-handed up-type and down-type quark fields, respectively, which are singlets of the SU(2) part of the gauge symmetry. Note that the square brackets in Eqs. (2), (3), and (4) as well as in the subsequent text, denote the representations of D_4 and has nothing to do with the representation

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of the enclosed fields under SU(2). In the Higgs sector there are two SU(2) doublets ϕ_k $(k=1,2)$ and their transformation under the D_4 symmetry is as follows:

$$\mathbf{2}\colon \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}. \tag{5}$$

The most general Yukawa Lagrangian for the quarks that is consistent with the gauge and D_4 symmetries can be written as

$$\begin{aligned} -\mathcal{L}_Y = {} & A_u(\bar{Q}_1\tilde{\phi}_2 - \bar{Q}_2\tilde{\phi}_1)p_{3R} + B_u\bar{Q}_3(\tilde{\phi}_1 p_{1R} + \tilde{\phi}_2 p_{2R}) \\ & + A_d(\bar{Q}_1\phi_2 + \bar{Q}_2\phi_1)n_{3R} + B_d\bar{Q}_3(\phi_1 n_{1R} + \phi_2 n_{2R}) \\ & + \mathrm{H.c.}, \end{aligned} \tag{6}$$

where, we have used the standard abbreviation $\tilde{\phi}_k = i\sigma_2\phi_k^*$. The complex phases of the Yukawa couplings can be absorbed in the quark field redefinitions. Thus, the D_4 symmetry reduces the number of Yukawa couplings drastically to the extent that we are left with only five unknown parameters in Eq. (6), namely, four Yukawa couplings and the ratio of the two vacuum expectation values (VEVs), $\tan\beta \equiv v_2/v_1$. Quite remarkably, these are just enough to reproduce the five nonzero parameters in the quark sector when Eq. (1) holds. Therefore, at this leading order, using a D_4 flavor symmetry we have successfully removed all the unnecessary parameters from the quark Yukawa Lagrangian. The mass matrices that follow from Eq. (6) are given by

$$M_u = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & A_u v_2 \\ 0 & 0 & -A_u v_1 \\ B_u v_1 & B_u v_2 & 0 \end{pmatrix},$$
$$M_d = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & A_d v_2 \\ 0 & 0 & A_d v_1 \\ B_d v_1 & B_d v_2 & 0 \end{pmatrix}, \tag{7}$$

where $\langle\phi_k\rangle = v_k/\sqrt{2}$ represents the VEV of ϕ_k. The diagonal mass matrices can be obtained via the following biunitary transformations:

$$D_u = V_L \cdot M_u \cdot V_R^\dagger = \mathrm{diag}(m_u, m_c, m_t), \tag{8a}$$

$$D_d = U_L \cdot M_d \cdot U_R^\dagger = \mathrm{diag}(m_d, m_s, m_b). \tag{8b}$$

The matrices, V and U relate the quark fields in the gauge basis to those in the mass basis as follows:

$$u_L = V_L p_L, \qquad u_R = V_R p_R, \tag{9a}$$

$$d_L = U_L n_L, \qquad d_R = U_R n_R, \tag{9b}$$

where, u and d denote the physical up and down type quarks respectively. The CKM matrix is then given by

$$V_{\mathrm{CKM}} = V_L \cdot U_L^\dagger. \tag{10}$$

The matrices, V_L and U_L can be obtained by diagonalizing $M_u M_u^\dagger$ and $M_d M_d^\dagger$ respectively, which can be calculated from Eq. (7) as follows:

$$M_u M_u^\dagger = \frac{1}{2}\begin{pmatrix} A_u^2 v_2^2 & -A_u^2 v_1 v_2 & 0 \\ -A_u^2 v_1 v_2 & A_u^2 v_1^2 & 0 \\ 0 & 0 & B_u^2 v^2 \end{pmatrix},$$
$$M_d M_d^\dagger = \frac{1}{2}\begin{pmatrix} A_d^2 v_2^2 & A_d^2 v_1 v_2 & 0 \\ A_d^2 v_1 v_2 & A_d^2 v_1^2 & 0 \\ 0 & 0 & B_d^2 v^2 \end{pmatrix}, \tag{11}$$

where, $v = \sqrt{v_1^2 + v_2^2}$ is the total electroweak VEV. To diagonalize the above matrices, we introduce the matrix,

$$U_\beta = \begin{pmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{12}$$

One can easily check that

$$D_u^2 = U_\beta \cdot (M_u M_u^\dagger) \cdot U_\beta^\dagger = \mathrm{diag}(0, A_u^2 v^2/2, B_u^2 v^2/2), \tag{13a}$$

$$D_d^2 = U_\beta^\dagger \cdot (M_d M_d^\dagger) \cdot U_\beta = \mathrm{diag}(0, A_d^2 v^2/2, B_d^2 v^2/2). \tag{13b}$$

Thus, we can identify the masses of the physical quarks as

$$m_{u,d}^2 = 0, \qquad m_{c,s}^2 = \frac{1}{2}A_{u,d}^2 v^2, \qquad m_{t,b}^2 = \frac{1}{2}B_{u,d}^2 v^2. \tag{14}$$

Also, comparing with the definitions in Eq. (8), we can conclude

$$V_L = U_\beta, \qquad U_L = U_\beta^\dagger. \tag{15}$$

Using Eq. (10) we can now easily calculate the CKM matrix as follows:

$$V_{\mathrm{CKM}} = U_\beta \cdot U_\beta = \begin{pmatrix} \cos 2\beta & \sin 2\beta & 0 \\ -\sin 2\beta & \cos 2\beta & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{16}$$

Therefore, comparing with Eq. (1), one can identify the Cabibbo mixing angle as

$$\sin\theta_C = \sin 2\beta \approx 0.22. \tag{17}$$

This relation between the Cabibbo parameter and $\tan\beta$ is the key result of our analysis. Note that the relation of Eq. (17) is purely a consequence of the Yukawa textures in Eq. (7) which are dictated by the D_4 symmetry. Therefore, this relation should be stable under quantum corrections.

At this stage, it is reasonable to ask whether such a value of $\tan\beta$ will be allowed from the scalar sector. As we will see, the value of $\tan\beta$ can be quite arbitrary if we allow for terms that softly break the D_4 symmetry in the scalar sector. Keeping these in mind, we write the scalar potential as

$$\begin{aligned} V = & -\mu_1^2(\phi_1^\dagger\phi_1) - \mu_2^2(\phi_2^\dagger\phi_2) - \mu_{12}^2(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1) \\ & + \lambda_1(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2)^2 + \lambda_2(\phi_1^\dagger\phi_2 - \phi_2^\dagger\phi_1)^2 \\ & + \lambda_3(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1)^2 + \lambda_4(\phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2)^2. \end{aligned} \tag{18}$$

Note that, in the limit $\mu_1^2 = \mu_2^2$, $\mu_{12}^2 = 0$ the D_4 symmetry will be exact in the scalar potential. However, in this case one can easily verify that the minimization conditions will enforce $v_1 = v_2$, i.e., $\tan\beta = 1$ which will be incompatible with Eq. (17). Therefore, we decide to proceed with the potential of Eq. (18) containing the most general bilinear terms. The minimization conditions, in this case, can be used to solve for the bilinear parameters μ_1^2 and μ_2^2 as follows:

$$\mu_1^2 = (\lambda_1 + 2\lambda_3 - \lambda_4)v_2^2 + (\lambda_1 + \lambda_4)v_1^2 + \mu_{12}^2\frac{v_2}{v_1}, \tag{19a}$$

$$\mu_2^2 = (\lambda_1 + 2\lambda_3 - \lambda_4)v_1^2 + (\lambda_1 + \lambda_4)v_2^2 + \mu_{12}^2\frac{v_1}{v_2}. \tag{19b}$$

After the spontaneous symmetry breaking, we expand the scalar doublets as

$$\phi_k = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}w_k^+ \\ v_k + h_k + iz_k \end{pmatrix} \quad (k = 1, 2). \tag{20}$$

The massless unphysical scalars $\omega^\pm$ and ζ in the charged and the pseudoscalar sectors respectively, can be extracted using the following rotation:

$$\begin{aligned} \begin{pmatrix} \omega^\pm \\ H^\pm \end{pmatrix} &= \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}\begin{pmatrix} w_1^\pm \\ w_2^\pm \end{pmatrix}, \\ \begin{pmatrix} \zeta \\ A \end{pmatrix} &= \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}. \end{aligned} \tag{21}$$

In the above equation, $H^\pm$ and A stand for physical charged scalar and pseudoscalar respectively, whose masses can be calculated as

$$m_+^2 = \frac{2\mu_{12}^2}{\sin 2\beta} - 2\lambda_3 v^2, \tag{22a}$$

$$m_A^2 = \frac{2\mu_{12}^2}{\sin 2\beta} - 2(\lambda_2 + \lambda_3)v^2. \tag{22b}$$

The mass squared matrix in the scalar sector is given by

$$V_S^{\text{mass}} = (\,h_1 \quad h_2\,)\frac{\mathcal{M}_S^2}{2}\begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \tag{23a}$$

$$\text{with,} \quad \mathcal{M}_S^2 = \begin{pmatrix} 2(\lambda_1 + \lambda_4)v_1^2 + \mu_{12}^2\frac{v_2}{v_1} & 2(\lambda_1 + 2\lambda_3 - \lambda_4)v_1v_2 - \mu_{12}^2 \\ 2(\lambda_1 + 2\lambda_3 - \lambda_4)v_1v_2 - \mu_{12}^2 & 2(\lambda_1 + \lambda_4)v_2^2 + \mu_{12}^2\frac{v_1}{v_2} \end{pmatrix}. \tag{23b}$$

The diagonalization of $\mathcal{M}_S^2$ will lead to two physical CP-even scalars H and h which are obtained via the following rotation

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} h_1 \\ h_2 \end{pmatrix}. \tag{24}$$

This diagonalization will then entail the following relations:

$$m_H^2\cos^2\alpha + m_h^2\sin^2\alpha = 2(\lambda_1 + \lambda_4)v_1^2 + \mu_{12}^2\frac{v_2}{v_1}, \tag{25a}$$

$$m_H^2\sin^2\alpha + m_h^2\cos^2\alpha = 2(\lambda_1 + \lambda_4)v_2^2 + \mu_{12}^2\frac{v_1}{v_2}, \tag{25b}$$

$$(m_H^2 - m_h^2)\sin\alpha\cos\alpha = 2(\lambda_1 + 2\lambda_3 - \lambda_4)v_1v_2 - \mu_{12}^2. \tag{25c}$$

We note that the potential of Eq. (18) contains seven parameters among which two of the bilinear parameters, μ_1^2 and μ_2^2, have been traded in favor of v_1 and v_2 (or equivalently v and $\tan\beta$) using Eq. (19). The remaining five parameters (four lambdas and μ_{12}^2) can then be exchanged for four physical masses (m_+, m_A, m_H and m_h) and the mixing angle, α using Eqs. (22) and (25). On top of this, putting $\alpha = \beta - \pi/2$ [5] will ensure that h possesses exact SM-like couplings at the tree-level, so that it can be identified with the 125 GeV scalar discovered at the LHC. In this *alignment limit* [6,7], Eq. (25) can be rearranged to obtain simpler expressions for m_h and m_H as follows:

$$m_h^2 = 2(\lambda_1 + \lambda_3)v^2, \tag{26a}$$

$$m_H^2 = \frac{2\mu_{12}^2}{\sin 2\beta} + 2(\lambda_4 - \lambda_3)v^2. \tag{26b}$$

From Eqs. (22) and (26) we can see that, in the limit $\mu_{12}^2 \gg v^2$, only the SM-like Higgs scalar, h, remains at the EW scale while the other nonstandard scalars are quasi-degenerate and super heavy. Considering the absence of any convincing hints of new physics at the collider experiments, such a spectrum of the scalar masses might be desirable. Moreover, in the limit $m_+ \approx m_H \approx m_A \gg m_h$, the bound from the electroweak T-parameter can be easily avoided [8,9].

For the sake of completeness we now discuss the scalar mediated flavor changing neutral currents (FCNCs) in our model. Comparing Eq. (6) with the general 2HDM Yukawa Lagrangian

$$\mathcal{L}_Y = -\sum_{k=1}^{2} [\bar{Q}\Gamma_k \phi_k n_R + \bar{Q}\Delta_k \tilde{\phi}_k p_R] + \text{H.c.}, \tag{27}$$

we can write,

$$\Delta_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -A_u \\ B_u & 0 & 0 \end{pmatrix}, \qquad \Delta_2 = \begin{pmatrix} 0 & 0 & A_u \\ 0 & 0 & 0 \\ 0 & B_u & 0 \end{pmatrix},$$
$$\Gamma_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & A_d \\ B_d & 0 & 0 \end{pmatrix}, \qquad \Gamma_2 = \begin{pmatrix} 0 & 0 & A_d \\ 0 & 0 & 0 \\ 0 & B_d & 0 \end{pmatrix}. \tag{28}$$

Note that in writing Eq. (27), we have suppressed the generation indices. The matrices, N_u and N_d, which control the FCNC couplings in the up and down sectors respectively, are given by [1]

$$N_u = \frac{1}{\sqrt{2}} V_L(\Delta_1 v_2 - \Delta_2 v_1)V_R^\dagger, \tag{29a}$$

$$N_d = \frac{1}{\sqrt{2}} U_L(\Gamma_1 v_2 - \Gamma_2 v_1)U_R^\dagger. \tag{29b}$$

As an explicit example, N_u and N_d will get involved in the FCNC couplings in the physical CP-even sector as follows

$$\mathcal{L}_Y^{\text{CP even}} = -\frac{h}{v}(\bar{u}D_u u + \bar{d}D_d d) - \frac{H}{v}[\bar{u}(N_u P_R + N_u^\dagger P_L)u + \bar{d}(N_d P_R + N_d^\dagger P_L)d], \tag{30}$$

where, we have suppressed again the generation indices and imposed the alignment limit. To calculate the expressions for N_u and N_d using Eq. (29), we need to know V_R and U_R which can be obtained by diagonalizing $M_u^\dagger M_u$ and $M_d^\dagger M_d$ respectively. In this way, we find

$$V_R = U_R = \begin{pmatrix} -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \\ \cos\beta & \sin\beta & 0 \end{pmatrix}. \tag{31}$$

Now we can easily compute N_u and N_d as follows:

$$N_u = -\begin{pmatrix} 0 & m_c & 0 \\ 0 & 0 & 0 \\ m_t & 0 & 0 \end{pmatrix}, \qquad N_d = -\begin{pmatrix} 0 & m_s & 0 \\ 0 & 0 & 0 \\ m_b & 0 & 0 \end{pmatrix}. \tag{32}$$

Clearly, due to small number of parameters in the Yukawa sector, the FCNC couplings are completely determined in terms of the known physical parameters. One should keep in mind that Eq. (32) represents the FCNC couplings at the leading order, i.e., when the CKM matrix is block-diagonal and the first generation quark masses are zero. In a more complete theory, these FCNC couplings are expected to receive small corrections. But it is still encouraging to note that already at this leading order, the FCNCs in the down sector are suppressed at least by m_b/v, which means they are quite small in this model. Consequently, the lower limits on the nonstandard scalar masses are brought down to about 3 TeV as opposed to about 100 TeV for $\mathcal{O}(1)$ FCNC couplings [1,10]. This makes our model testable at the collider experiments.

A more interesting scenario arises if one considers slight departure from the exact alignment limit by turning on small values of $\cos(\beta - \alpha)$. But one should keep in mind that FCNC couplings mediated by the SM-like Higgs boson, h, will start to seep in via such a misalignment. However, as mentioned earlier, since the FCNC couplings are already suppressed, $|\cos(\beta - \alpha)| \lesssim 3\%$ will still be consistent with the flavor data. Such a deviation from the alignment limit can, in principle, be sensed as tiny deficits in the Higgs signal strengths because the tree-level couplings of h are suppressed by $\sin(\beta - \alpha)$. Now, we combine this with the measurement of the trilinear Higgs coupling via Higgs pair production which probes the following quantity,

$$\kappa_\lambda = \frac{\lambda_{hhh}}{(\lambda_{hhh})^{\text{SM}}} = \frac{\sin(\beta - \alpha)}{2m_h^2 \sin 4\beta}[(m_H^2 - m_h^2)\sin 4\alpha + (m_H^2 + m_h^2)\sin 4\beta + 2(m_H^2 - m_h^2)\sin 2(\alpha + \beta)]. \tag{33}$$

One can easily check that $\kappa_\lambda = 1$, as expected in the limit $\alpha = \beta - \pi/2$. Currently the bound on κ_λ is quite

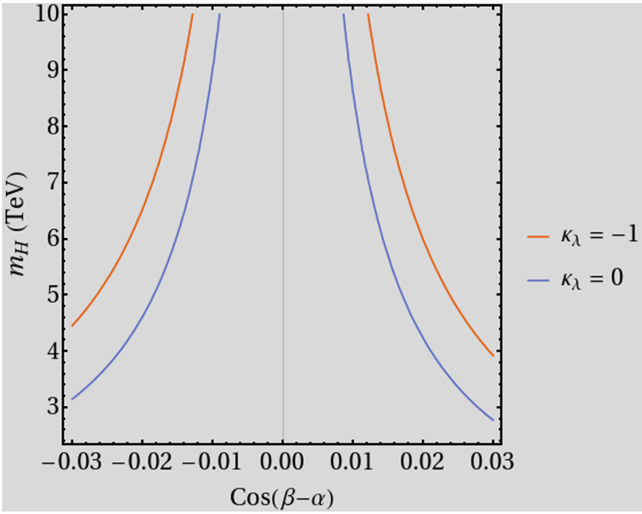

FIG. 1. Contours of κ_λ in the $\cos(\beta - \alpha)$-m_H plane. In drawing this plot, we have assumed $\sin 2\beta = 0.22$ and $m_h = 125$ GeV.

weak [11,12]. Note that, the values of m_h and β, appearing on the right-hand side of Eq. (33), are known in our model. Therefore, assuming that $\cos(\beta - \alpha)$ and κ_λ settle for some nonstandard values, we can *predict* the value of m_H. This feature has been illustrated in Fig. 1, where we can see that if, for example, the values of $\cos(\beta - \alpha)$ and κ_λ are found to be 0.025 and -1 respectively, then we can conclude that there should be a heavy neutral scalar appearing at around 5 TeV. Although probing such a tiny value of $\cos(\beta - \alpha)$ might be an ambitious task for the near future, our model, nevertheless, exemplifies how the Higgs precision measurements can play a crucial role in pinning down the scale of new physics.

To summarize, in this article we have pointed out an intriguing possibility that there might be a connection between the Cabibbo angle and $\tan\beta$ in a 2HDM. To our knowledge, such a possibility has not been emphasized earlier in the context of 2HDMs. We accomplish this in a 2HDM with a D_4 symmetry which is only softly broken in the scalar potential. Because of the small number of Yukawa parameters, all the FCNC couplings are completely determined in our model. Additionally, the FCNC couplings are sufficiently small so that relatively light scalars accessible at the colliders can successfully negotiate the flavor constraints. Although the complete CKM matrix and the exact nonzero masses of the first generation of quarks have not been reproduced in our minimalistic scenario, we believe that the interesting features of this model outweigh the dissatisfaction with the small parameters in the quark sector. Perhaps the present framework can be taken as the first step toward a more complete theoretical construction which can address the full structure of the quark masses and mixings. Finally, it should be noted that, although there are quite a few previous examples of the use of D_4 symmetry to understand the leptonic sector [13–18], instances where D_4 symmetry has been employed to explain the quark masses and mixings are rare [16,19] and use four Higgs-doublets. Therefore, the current paper should be considered as a simpler alternative and an interesting addition to the existing literature on model building using D_4 flavor symmetry.

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