Effects of a dense medium on parameters of doubly heavy baryons

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The spectroscopic properties of the doubly heavy spin-1/2 baryons $\Xi_{QQ'}$, $\Xi'_{QQ'}$, $\Omega_{QQ'}$, and $\Omega'_{QQ'}$, with heavy quarks Q and Q' being b or/and c, are investigated in cold nuclear matter. In particular, the behavior of the mass of these particles with respect to the density of the medium in the range $\rho \in [0, 1.4]\rho_{\text{sat}}$, with $\rho_{\text{sat}} = 0.11^3$ GeV^3 being the saturation density of nuclear matter, is investigated. From the shifts in the mass and vector self-energy of the states under consideration, it is obtained that $\Xi_{QQ'}$ and $\Xi'_{QQ'}$ baryons with two heavy quarks and one u or d quark are affected by the medium, considerably. It is also seen that the $\Omega_{QQ'}$ and $\Omega'_{QQ'}$ states, containing two heavy quarks and one s quark do not see the dense medium, at all. The value of mass for the Ξ_{cc} state obtained at $\rho \to 0$ limit is nicely consistent with the experimental data. Our results on parameters of other members can be useful in the search for these states. The obtained results may also shed light on the future in-medium experiments aiming to search for the behavior of the doubly heavy baryons under extreme conditions.

I. INTRODUCTION

Although the doubly heavy baryons have been predicted by the quark model many decades ago [1–3], only the spin−1/2 double-charmed baryon Ξ_{cc} has been experimentally observed so far. Many models and approaches were used before the first observation of the Ξ_{cc} state to understand the structure and spectrum of the doubly heavy baryons. For instance, potential model and several versions of the bag model were used to calculate the mass spectrum of baryons with two charmed quarks surrounded by an ordinary or strange quark in Ref. [4]. The SELEX Collaboration reported the first detection of Ξ_{cc}^+ state in the charge decay mode $\Xi_{cc}^+ \to \Lambda_c^+ K^- \pi^+$ in 2002 [5]. The measured mass for this state was 3519 ± 1 MeV/c^2. Then in 2005, the same collaboration confirmed the same state in the charged decay mode $\Xi_{cc}^+ \to pD^+ K^-$ [6]. The updouble ratios of sum rules (DRSR), mass-splittings of doubly heavy baryons were obtained [7]. Using QCD sum rules, the doubly heavy baryon states were analyzed in

Refs. [8–11]. The hypercentral constituent quark model (hCQM) was used in Refs. [12,13] to obtain the mass spectra of doubly heavy baryons. The mass spectra and radiative decays of doubly heavy baryons were investigated within the diquark picture in a relativized quark model in Ref. [14]. In Ref. [15], an extended chromomagnetic model by further considering the effect of color interaction was used to study the mass spectra of all the lowest S-wave doubly and triply heavy-quark baryons. In Refs. [16,17], the Bethe-Salpeter equation was applied for the mass spectra of the doubly heavy baryons. Discovery potentials of doubly heavy baryons and their weak decays were analyzed for instance in Refs. [18–21].

Among the results of theoretical studies, some of them were of great importance. These studies showed that the value of mass measured by SELEX Collaboration for the Ξ_{cc} state remains considerably below the theoretical predictions. Thus, in Ref. [8], the mass of this state was found as 3.72 ± 0.20 GeV, which its central value remains roughly 200 MeV above the experimental result. Motivated by these analyses the LHCb Collaboration started to study this state. In 2017, this collaboration announced the observation of a Ξ_{cc}^{++} state in $\Lambda_c^+ K^- \pi^+ \pi^-$ invariant mass, where the Λ_c^+ baryon was reconstructed in the decay mode $pK^- \pi^+$ [22]. The measured value for the mass of Ξ_{cc}^{++} state by LHCb Collaboration was $3621.40 \pm 72(\text{stat}) \pm 0.27(\text{syst}) \pm 14(\Lambda_c^+)$ MeV/c^2, where the last uncertainty was due to the limited knowledge of the Λ_c^+ baryon mass.

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As is seen, the result of LHCb Collaboration differs considerably from the SELEX data. This tension was the starting points of a rush theoretical investigations deciding to explain the existing discrepancy between the SELEX and LHCb results. In Ref. [23], the authors showed that the intrinsic heavy-quark QCD mechanism for the hadroproduction of heavy hadrons at large x_F can resolve the apparent conflict between measurements of double-charm baryons by the SELEX fixed-target experiment and the LHCb experiment at the LHC collider.

We hope that, by the development of experimental facilities, we will be able to detect other members of the doubly heavy baryons. The production mechanism of doubly heavy baryons has an important place in the literature [23–40]. Naturally, a doubly heavy baryon can be produced using a two-step procedure: (i) in a hard interaction, a double heavy diquark is produced perturbatively, (ii) and then it is transformed to the baryon within the soft hadronization process [38].

Understanding the hadronic properties at finite temperature/density and under extreme conditions are of great importance. Such investigations can help us in the understanding of the natures and internal structures of the dense astrophysical objects like neutron stars as well as in analyzing the results of the heavy ion collision and the in-medium experiments. The spectroscopic parameters of the light and single-heavy baryons in medium have been widely investigated (for instance see [41–48] and references therein). Although, the doubly heavy baryons have been widely studied in vacuum, the number of works devoted to the investigations of the properties of these baryons in a dense medium is very limited (for instance see Refs. [49,50]). In Ref. [50] We investigated the fate of the doubly heavy spin-3/2 $\Xi^*_{QQ'}$ and $\Omega^*_{QQ'}$ baryons in cold nuclear matter. The shifts on the physical parameters of these states due to nuclear medium were calculated at saturation medium density and compared with their vacuum values. In the present study, we investigate the doubly heavy spin-1/2 $\Xi_{QQ'}$, $\Xi'_{QQ'}$, $\Omega_{QQ'}$, and $\Omega'_{QQ'}$ baryons in dense medium by the technique of the in-medium QCD sum rule. In particular, we discuss the behavior of different parameters related to the states under consideration with respect to the changes in the medium density in the range $\rho \in [0, 1.4]\rho_{\text{sat}}$. We report the values of the masses and vector self energies of the spin-1/2 doubly heavy baryons at saturation nuclear matter density, $\rho_{\text{sat}} = 0.11^3$ GeV^3, and compare the obtained results for the masses with their vacuum values in order to determine the order of shifts in the masses due to the dense medium. The obtained results may shed light on the production and study of the in-medium properties of these baryons in future experiments. Production of the doubly heavy baryons in dense medium requires simultaneous production of two pairs of the heavy quark-antiquark. A heavy quark from one pair, then, needs to come together with the heavy quark of the other pair,

with the aim of forming a heavy diquark with the total spin 1 or 0. Meeting of the heavy diquark with a light quark forms a doubly heavy baryon in medium. These processes need that the quarks be in the vicinity of each other both in the ordinary and rapidity spaces.

The rest of the paper is organized as follows. In next section we derive the in-medium QCD sum rules for the masses and vector self-energies of the doubly heavy spin-1/2 $\Xi_{QQ'}$, $\Xi'_{QQ'}$, $\Omega_{QQ'}$ and $\Omega'_{QQ'}$ baryons. In Sec. III, using the input parameters, first, we fix the auxiliary parameters entering the sum rules by the requirements of the model. We discuss the behaviors of the physical quantities under consideration with respect to the changes in the density and calculate their values at saturation nuclear matter density. We compare the values of the masses obtained at $\rho \to 0$ limit with other theoretical predictions as well as the existing experimental result on the doubly-charmed Ξ_{cc} state. Section IV is devoted to the discussions and comments. We present the in-medium light and heavy quarks propagators used in the calculations together with their ingredients: in-medium quark, gluon and mixed condensates in Appendix A. We reserve the Appendix B to present the in-medium input parameters used in the numerical analyses.

II. SUM RULES FOR THE IN-MEDIUM PARAMETERS OF THE SPIN-1/2 DOUBLY HEAVY BARYONS

The aim of this section is to find the masses and vector self-energies of the doubly heavy spin-1/2 $\Xi_{QQ'}$, $\Xi'_{QQ'}$, $\Omega_{QQ'}$ and $\Omega'_{QQ'}$ baryons in terms of QCD degrees of freedom as well as the auxiliary parameters entering the calculations. To this end, we employ the in-medium QCD sum rule approach as one of the powerful and predictive nonperturbative methods in hadron physics. Here baryons without a prime refer to the symmetric states with respect to the exchange of two heavy-quark fields and those with a prime to the asymmetric states. For the classification of the ground state spin-1/2 and spin-3/2 baryons one can see for instance Ref. [50].

For the calculations of the physical parameters of the baryons under consideration the following in-medium correlation function is used:

$$\Pi^{S(A)}(p) = i \int d^4x e^{ip\cdot x} \langle \psi_0 | \mathcal{T}[J^{S(A)}(x)\bar{J}^{S(A)}(0)]|\psi_0\rangle, \quad (1)$$

where p is the external four-momentum of the double heavy baryons, $|\psi_0\rangle$ is the ground state of the nuclear medium and $\mathcal{T}$ is the time ordering operator. As we mentioned above, for the doubly heavy spin-1/2 baryons, the interpolating currents can be symmetric (J^S) or antisymmetric (J^A) with respect to the exchange of two heavy-quark fields. Considering the quantum numbers of the

TABLE I. The quark flavors of the doubly heavy spin–1/2 baryons.

Baryon	q	Q	Q'
$\Xi_{QQ'}$ or $\Xi'_{QQ'}$	u or d	b or c	b or c
$\Omega_{QQ'}$ or $\Omega'_{QQ'}$	s	b or c	b or c

doubly heavy spin-1/2 baryons, the symmetric and antisymmetric interpolating currents can be written as

$$J^S = \frac{1}{\sqrt{2}}\epsilon_{abc}\{(Q^{aT}Cq^b)\gamma_5 Q'^c + (Q'^{aT}Cq^b)\gamma_5 Q^c$$
$$+ \beta[(Q^{aT}C\gamma_5 q^b)Q'^c + (Q'^{aT}C\gamma_5 q^b)Q^c]\},$$
$$J^A = \frac{1}{\sqrt{6}}\epsilon_{abc}\{2(Q^{aT}CQ'^b)\gamma_5 q^c + (Q^{aT}Cq^b)\gamma_5 Q'^c$$
$$- (Q'^{aT}Cq^b)\gamma_5 Q^c + \beta[2(Q^{aT}C\gamma_5 Q'^b)q^c$$
$$+ (Q^{aT}C\gamma_5 q^b)Q'^c - (Q'^{aT}C\gamma_5 q^b)Q^c]\}, \tag{2}$$

where a, b and c are color indices, C is the charge conjugation operator, β is an arbitrary mixing parameter and q is a light quark field. In Table I, we present the quark flavors of the doubly heavy spin–1/2 baryons.

As an example, let us briefly explain how the current of the doubly heavy baryons in its antisymmetric form is obtained. Considering the quark content and spin of these baryons the current J^A can be decomposed as

$$J^A \sim \epsilon^{abc}\{(Q^{aT}C\Gamma Q'^b)\tilde{\Gamma}q^c + (Q^{aT}C\Gamma q^b)\tilde{\Gamma}Q'^c$$
$$- (Q'^{aT}C\Gamma q^b)\tilde{\Gamma}Q^c\}, \tag{3}$$

where Γ, $\tilde{\Gamma} = 1$, γ_5, γ_μ, $\gamma_5\gamma_\mu$, or $\sigma_{\mu\nu}$. Considering all quantum numbers of the states under study, we shall determine Γ and $\tilde{\Gamma}$. To this end, let us first consider the transpose of the quantity $\epsilon^{abc}(Q^{aT}C\Gamma Q'^b)$ from the first term in Eq. (3):

$$[\epsilon^{abc}Q^{aT}C\Gamma Q'^b]^T = -\epsilon^{abc}Q'^{bT}\Gamma^T C^{-1}Q^a$$
$$= \epsilon^{abc}Q'^{bT}C(C\Gamma^T C^{-1})Q^a, \tag{4}$$

where we used a simple theorem in the first line: If $A = BD$, where A, B, and D are matrices whose elements are Grassmann numbers, then $A^T = -D^T B^T$. In above equation, we also used $C^T = C^{-1}$ and $C^2 = -1$. The quantity, $C\Gamma^T C^{-1}$ is equal to Γ for $\Gamma = 1$, γ_5, or $\gamma_5\gamma_\mu$ and it is equal to $-\Gamma$ for $\Gamma = \gamma_\mu$ or $\sigma_{\mu\nu}$. After switching the color dummy indices, one obtains

$$[\epsilon^{abc}Q^{aT}C\Gamma Q'^b]^T = -\epsilon^{abc}Q'^{aT}C\Gamma Q^b, \tag{5}$$

for $\Gamma = 1$, γ_5 or $\gamma_5\gamma_\mu$ and

$$[\epsilon^{abc}Q^{aT}C\Gamma Q'^b]^T = \epsilon^{abc}Q'^{aT}C\Gamma Q^b, \tag{6}$$

for $\Gamma = \gamma_\mu$ or $\sigma_{\mu\nu}$. The right-hand side of last two equations are antisymmetric with respect to the replacement of two heavy quarks, $Q \leftrightarrow Q'$. Using this property, we get

$$[\epsilon^{abc}Q^{aT}C\Gamma Q'^b]^T = \epsilon^{abc}Q^{aT}C\Gamma Q'^b, \tag{7}$$

for $\Gamma = 1$, γ_5, or $\gamma_5\gamma_\mu$ and

$$[\epsilon^{abc}Q^{aT}C\Gamma Q'^b]^T = -\epsilon^{abc}Q^{aT}C\Gamma Q'^b, \tag{8}$$

for $\Gamma = \gamma_\mu$ or $\sigma_{\mu\nu}$. From other side, the transpose of a 1×1 matrix $\epsilon^{abc}Q^{aT}C\Gamma Q'^b$ should be equal to the same 1×1 matrix. Hence, we conclude that $\Gamma = 1$, γ_5, or $\gamma_5\gamma_\mu$.

The simplest way is to take the baryons interpolated by J^A to have the same total spin and spin projection as the light quark q. Therefore, the spin of the diquark formed by two heavy quarks is zero. This implies that $\Gamma = 1$ or γ_5. Thus, the two possible forms of the interpolating current J^A can be written as

$$j_1^A = \epsilon^{abc}(Q^{aT}CQ'^b)\tilde{\Gamma}_1 q^c$$
$$j_2^A = \epsilon^{abc}(Q^{aT}C\gamma_5 Q'^b)\tilde{\Gamma}_2 q^c. \tag{9}$$

The matrices $\tilde{\Gamma}_1$ and $\tilde{\Gamma}_2$ are determined using the Lorentz and parity considerations. As j_1^A and j_2^A are Lorentz scalars, one should have $\tilde{\Gamma}_1$, $\tilde{\Gamma}_2 = 1$, or γ_5. Finally, considering the parity transformation leads to $\tilde{\Gamma}_1 = \gamma_5$ and $\tilde{\Gamma}_2 = 1$. Thus,

$$j_1^A = \epsilon^{abc}(Q^{aT}CQ'^b)\gamma_5 q^c$$
$$j_2^A = \epsilon^{abc}(Q^{aT}C\gamma_5 Q'^b)q^c. \tag{10}$$

Obviously one uses their arbitrary linear combination, which leads to the form

$$j^A \sim \epsilon^{abc}[(Q^{aT}CQ'^b)\gamma_5 q^c + \beta(Q^{aT}C\gamma_5 Q'^b)q^c], \tag{11}$$

for the first term in Eq. (3), where we introduced the mixing parameter β. Using similar arguments for the second and third terms in Eq. (3), we get the current J^A used in the calculations. From a similar manner one can derive the symmetric current J^S. We will use the symmetric current J^S and the antisymmetric current J^A to interpolate the baryons without prime ($\Xi_{QQ'}$, $\Omega_{QQ'}$) and primed baryons ($\Xi'_{QQ'}$, $\Omega'_{QQ'}$), respectively (for details see for instance Ref. [8]). We should also remark that the currents J^S and J^A are some possible currents which are obtained using the quantum numbers of the doubly heavy baryons under consideration. To construct the most general operators one may go through a similar procedure as explained in Ref. [51] for the light baryons.

The correlation function in Eq. (1) can be calculated in two different ways: in terms of hadronic parameters called phenomenological (or physical) side and in terms of QCD parameters like quark masses as well as in-medium quark, gluon and mixed condensates called QCD (or theoretical) side. By matching the coefficients of the selected structures from both sides, one can get the sum rules for different physical observables. The calculations are started in x-space and then they are transferred to the momentum space. The Borel transformation is applied to both sides with the aim of suppressing the contributions of the higher states and continuum. As a last step, a continuum subtraction procedure is applied with accompany of the quark-hadron duality assumption.

On the phenomenological side, the correlation function is saturated with a complete set of the in-medium hadronic state carrying the same quantum numbers as the related interpolating current. By performing the integral over four$-x$, we get

$$\Pi^{S(A)}(p) = -\frac{\langle\psi_0|J^{S(A)}(0)|B(p^*,s)\rangle\langle B(p^*,s)|\bar{J}^{S(A)}(0)|\psi_0\rangle}{p^{*2}-m^{*2}}$$
$$+\cdots, \tag{12}$$

where dots are used to show the contributions of the higher states and continuum. The ket $|B(p^*,s)\rangle$ represents the doubly heavy spin-1/2 baryon state with spin s and the in-medium momentum p^*. Here m^* is the modified mass of the same state due to the dense medium. To proceed, the following matrix elements are defined:

$$\langle\psi_0|J^{S(A)}(0)|B(p^*,s)\rangle = \lambda^* u(p^*,s)$$
$$\langle B(p^*,s)|\bar{J}^{S(A)}(0)|\psi_0\rangle = \bar{\lambda}^* \bar{u}(p^*,s) \tag{13}$$

where λ^* is the modified coupling strength of the baryon to nuclear medium and $u(p^*,s)$ is the in-medium Dirac spinor. After inserting Eq. (13) into Eq. (12) and performing summation over spins, we get the following expression for the phenomenological side of the correlation function:

$$\Pi^{S(A)}(p) = -\frac{\lambda^{*2}(\not{p}^*+m^*)}{p^{*2}-m^{*2}}+\cdots = -\frac{\lambda^{*2}}{\not{p}^*-m^*}+\cdots$$
$$= -\frac{\lambda^{*2}}{(p^\mu-\Sigma_\upsilon^\mu)\gamma_\mu-m^*}+\cdots, \tag{14}$$

where Σ_υ^μ is written in terms of the vector self-energy the doubly heavy spin-1/2 baryonic state (Σ_υ) as: $\Sigma_\upsilon^\mu = \Sigma_\upsilon u^\mu + \Sigma_\upsilon' p^\mu$ with u^μ being the four-velocity of the nuclear medium and Σ_υ' is ignored because of its small value [52]. We shall work in the rest frame of the medium, $u^\mu = (1,0)$.

One can decompose the correlation function in terms of different structures as

$$\Pi^{S(A)}(p) = \Pi_{\not{p}}^{S(A)}(p^2,p_0)\not{p} + \Pi_{\not{u}}^{S(A)}(p^2,p_0)\not{u}$$
$$+ \Pi_U^{S(A)}(p^2,p_0)U + \cdots, \tag{15}$$

where U is the unit matrix and p_0 is the energy of the quasiparticle. The coefficients of different structures, i.e., the invariant amplitudes $\Pi_i^{S(A)}(p^2,p_0)$ with $i = \not{p}, \not{u}$ and U in above relation are obtained as

$$\Pi_{\not{p}}^{S(A)}(p^2,p_0) = -\lambda^{*2}\frac{1}{p^2-\mu^2},$$
$$\Pi_{\not{u}}^{S(A)}(p^2,p_0) = +\lambda^{*2}\frac{\Sigma_\upsilon}{p^2-\mu^2},$$
$$\Pi_U^{S(A)}(p^2,p_0) = -\lambda^{*2}\frac{m^*}{p^2-\mu^2}, \tag{16}$$

where $\mu^2 = m^{*2} - \Sigma_\upsilon^2 + 2p_0\Sigma_\upsilon$. After applying the Borel transformation with respect to p^2, we obtain

$$\hat{B}\Pi_{\not{p}}^{S(A)}(p^2,p_0) = \lambda^{*2}e^{-\mu^2/\mathcal{M}^2},$$
$$\hat{B}\Pi_{\not{u}}^{S(A)}(p^2,p_0) = -\lambda^{*2}\Sigma_\upsilon e^{-\mu^2/\mathcal{M}^2},$$
$$\hat{B}\Pi_U^{S(A)}(p^2,p_0) = \lambda^{*2}m^* e^{-\mu^2/\mathcal{M}^2}, \tag{17}$$

where $\mathcal{M}^2$ is the Borel mass parameter to be fixed later.

On the QCD side, we insert the explicit forms of the interpolating currents into the correlation function and contract the quark fields via the Wick theorem, as a result of which the following expressions for the symmetric and anti-symmetric parts are obtained in terms of the in-medium light quark (S_q^{ij}) and heavy quark (S_Q^{ij}) propagators [8]:

$$\Pi^S(p) = i\kappa\epsilon_{abc}\epsilon_{a'b'c'}\int d^4x e^{ip\cdot x}\{-\gamma_5 S_Q^{cb'}\tilde{S}_q^{ba'}S_{Q'}^{ac'}\gamma_5 - \gamma_5 S_{Q'}^{cb'}\tilde{S}_q^{ba'}S_Q^{ac'}\gamma_5 + \gamma_5 S_{Q'}^{cc'}\gamma_5\text{Tr}[S_Q^{ab'}\tilde{S}_q^{ba'}]$$

$$+\gamma_5 S_Q^{cc'}\gamma_5\text{Tr}[S_{Q'}^{ab'}\tilde{S}_q^{ba'}] + \beta(-\gamma_5 S_Q^{cb'}\gamma_5\tilde{S}_q^{ba'}S_{Q'}^{ac'} - \gamma_5 S_{Q'}^{cb'}\gamma_5\tilde{S}_q^{ba'}S_Q^{ac'} - S_Q^{cb'}\tilde{S}_q^{ba'}\gamma_5 S_{Q'}^{ac'}\gamma_5$$

$$-S_{Q'}^{cb'}\tilde{S}_q^{ba'}\gamma_5 S_Q^{ac'}\gamma_5 + \gamma_5 S_{Q'}^{cc'}\text{Tr}[S_Q^{ab'}\gamma_5\tilde{S}_q^{ba'}] + S_{Q'}^{cc'}\gamma_5\text{Tr}[S_Q^{ab'}\tilde{S}_q^{ba'}\gamma_5] + \gamma_5 S_Q^{cc'}\text{Tr}[S_{Q'}^{ab'}\gamma_5\tilde{S}_q^{ba'}]$$

$$+S_Q^{cc'}\gamma_5\text{Tr}[S_{Q'}^{ab'}\tilde{S}_q^{ba'}\gamma_5]) + \beta^2(-S_Q^{cb'}\gamma_5\tilde{S}_q^{ba'}\gamma_5 S_{Q'}^{ac'} - S_{Q'}^{cb'}\gamma_5\tilde{S}_q^{ba'}\gamma_5 S_Q^{ac'} + S_{Q'}^{cc'}\text{Tr}[S_Q^{ba'}\gamma_5\tilde{S}_{Q'}^{ab'}\gamma_5] + S_Q^{cc'}\text{Tr}[S_q^{ba'}\gamma_5\tilde{S}_{Q'}^{ab'}\gamma_5])\}_{\psi_0}, \tag{18}$$

$$\Pi^A(p) = \frac{i}{6}\epsilon_{abc}\epsilon_{a'b'c'}\int d^4x e^{ip\cdot x}\{2\gamma_5 S_Q^{cb'}\tilde{S}_{Q'}^{aa'}S_q^{bc'}\gamma_5 + \gamma_5 S_Q^{cb'}\tilde{S}_q^{ba'}S_{Q'}^{ac'}\gamma_5 - 2\gamma_5 S_{Q'}^{ca'}\tilde{S}_Q^{ab'}S_q^{bc'}\gamma_5$$

$$+ \gamma_5 S_{Q'}^{cb'}\tilde{S}_q^{ba'}S_Q^{ac'}\gamma_5 - 2\gamma_5 S_q^{ca'}\tilde{S}_Q^{ab'}S_{Q'}^{bc'}\gamma_5 + 2\gamma_5 S_q^{ca'}\tilde{S}_{Q'}^{bb'}S_Q^{ac'}\gamma_5 + 4\gamma_5 S_q^{cc'}\gamma_5 \text{Tr}[S_Q^{ab'}\tilde{S}_{Q'}^{ba'}]$$

$$+ \gamma_5 S_{Q'}^{cc'}\gamma_5\text{Tr}[S_Q^{ab'}\tilde{S}_q^{ba'}] + \gamma_5 S_Q^{cc'}\gamma_5\text{Tr}[S_{Q'}^{ab'}\tilde{S}_q^{ba'}] + \beta(2\gamma_5 S_Q^{cb'}\gamma_5\tilde{S}_{Q'}^{aa'}S_q^{bc'} + \gamma_5 S_Q^{cb'}\gamma_5\tilde{S}_q^{ba'}S_{Q'}^{ac'}$$

$$- 2\gamma_5 S_{Q'}^{ca'}\gamma_5\tilde{S}_Q^{ab'}S_q^{bc'} + \gamma_5 S_q^{cb'}\gamma_5\tilde{S}_q^{ba'}S_Q^{ac'} - 2\gamma_5 S_q^{ca'}\gamma_5\tilde{S}_Q^{ab'}S_{Q'}^{bc'} + 2\gamma_5 S_q^{ca'}\gamma_5\tilde{S}_{Q'}^{bb'}S_Q^{ac'}$$

$$+ 2S_Q^{cb'}\tilde{S}_{Q'}^{aa'}\gamma_5 S_q^{bc'}\gamma_5 + S_Q^{cb'}\tilde{S}_q^{ba'}\gamma_5 S_{Q'}^{ac'}\gamma_5 - 2S_{Q'}^{ca'}\tilde{S}_Q^{ab'}\gamma_5 S_q^{bc'}\gamma_5 + S_{Q'}^{cb'}\tilde{S}_q^{ba'}\gamma_5 S_Q^{ac'}\gamma_5$$

$$- 2S_q^{ca'}\tilde{S}_Q^{ab'}\gamma_5 S_{Q'}^{bc'}\gamma_5 + 2S_q^{ca'}\tilde{S}_{Q'}^{bb'}\gamma_5 S_Q^{ac'}\gamma_5 + 4\gamma_5 S_q^{cc'}\text{Tr}[S_Q^{ab'}\gamma_5\tilde{S}_{Q'}^{ba'}] + 4S_q^{cc'}\gamma_5\text{Tr}[S_Q^{ab'}\tilde{S}_{Q'}^{ba'}\gamma_5]$$

$$+ \gamma_5 S_{Q'}^{cc'}\text{Tr}[S_Q^{ab'}\gamma_5\tilde{S}_q^{ba'}] + S_{Q'}^{cc'}\gamma_5\text{Tr}[S_Q^{ab'}\tilde{S}_q^{ba'}\gamma_5] + \gamma_5 S_Q^{cc'}\text{Tr}[S_{Q'}^{ab'}\gamma_5\tilde{S}_q^{ba'}] + S_Q^{cc'}\gamma_5\text{Tr}[S_{Q'}^{ab'}\tilde{S}_q^{ba'}\gamma_5])$$

$$+ \beta^2(2S_Q^{cb'}\gamma_5\tilde{S}_{Q'}^{aa'}\gamma_5 S_q^{bc'} + S_Q^{cb'}\gamma_5\tilde{S}_q^{ba'}\gamma_5 S_{Q'}^{ac'} - 2S_{Q'}^{ca'}\gamma_5\tilde{S}_Q^{ab'}\gamma_5 S_q^{bc'} + S_{Q'}^{cb'}\gamma_5\tilde{S}_q^{ba'}\gamma_5 S_Q^{ac'}$$

$$- 2S_q^{ca'}\gamma_5\tilde{S}_Q^{ab'}\gamma_5 S_{Q'}^{bc'} + 2S_q^{ca'}\gamma_5\tilde{S}_{Q'}^{bb'}\gamma_5 S_Q^{ac'} + 4S_q^{cc'}\text{Tr}[S_{Q'}^{ba'}\gamma_5\tilde{S}_Q^{ab'}\gamma_5] + S_{Q'}^{cc'}\text{Tr}[S_q^{ba'}\gamma_5\tilde{S}_Q^{ab'}\gamma_5]$$

$$+ S_Q^{cc'}\text{Tr}[S_q^{ba'}\gamma_5\tilde{S}_{Q'}^{ab'}\gamma_5])\}_{\psi_0}, \tag{19}$$

where $\tilde{S} = CS^TC$; and $\kappa = 1$ for $Q = Q'$ cases and $\kappa = \frac{1}{2}$ for the baryons with $Q \neq Q'$. The subindex ψ_0 represents that the calculations are done in a dense medium. The explicit expressions of the in-medium light and heavy quark propagators together with their ingredients including the in-medium quark, gluon, and mixed condensates are presented in Appendix A.

On QCD side, the invariant amplitudes $\Pi_i^{S(A)}(p^2, p_0)$ corresponding to different structures in Eq. (15) can be represented as the following dispersion integral:

$$\Pi_i^{S(A)}(p^2, p_0) = \int_{(m_Q+m_{Q'})^2}^{\infty} \frac{\rho_i^{S(A)}(s, p_0)}{s - p^2} ds, \tag{20}$$

where $\rho_i^{S(A)}(s, p_0)$ are the corresponding two-point spectral densities, which can be obtained from the imaginary parts of the correlation function. In the QCD side, the main goal is to calculate these spectral densities. To this end, we use the explicit forms of the in-medium light and heavy quarks propagators. By performing the integration over four−x, we transfer the calculations to the momentum space. But before that we use the following expression in order to rearrange the obtained expressions:

$$\frac{1}{(x^2)^m} = \int \frac{d^D k}{(2\pi)^D} e^{-ik\cdot x} i(-1)^{m+1} 2^{D-2m} \pi^{D/2}$$

$$\times \frac{\Gamma[D/2 - m]}{\Gamma[m]}\left(-\frac{1}{k^2}\right)^{D/2-m}, \tag{21}$$

which leads us to obtain expressions with three four-dimensional integrals. It is very straightforward to perform the integral over four–x leading to a Dirac delta function. The resultant Dirac delta function is used to perform the second four-integral. Finally, the remaining four-integral is performed using the Feynman parametrization tool, which leads to the following equality as an example:

$$\int d^4\ell \frac{(\ell^2)^m}{(\ell^2 + \Delta)^n} = \frac{i\pi^2(-1)^{m-n}\Gamma[m+2]\Gamma[n-m-2]}{\Gamma[2]\Gamma[n](-\Delta)^{n-m-2}}. \tag{22}$$

To be able to obtain the imaginary parts corresponding to different structures, the following equality is applied:

$$\Gamma\left[\frac{D}{2} - n\right]\left(-\frac{1}{\Delta}\right)^{D/2-n} = \frac{(-1)^{n-1}}{(n-2)!}(-\Delta)^{n-2}\ln[-\Delta], \tag{23}$$

where $\ln[-\Delta] = i\pi + \ln[\Delta]$ and the condition $\Delta > 0$ brings constraints on the limits of the integrals over the Feynman parameters. As examples, for the symmetric case of the correlation function, the spectral densities $\rho_i^S(s, p_0)$ corresponding to different structures are obtained as

$$\rho_{\not{p}}^S(s, p_0) = \frac{3}{32768\xi^6\pi^{12}} \int_0^1 dz \int_0^{1-z} dw \Big\{ \xi^2 m_Q^4 wz(w+z)[3(\beta(5\beta+2)+5)(w+z-1)(w+z) - 2\xi(\beta-1)^2]$$

$$+ 2\xi m_Q^2 sw^2z^2(w+z-1)[\xi(\beta-1)^2 - 4(\beta(5\beta+2)+5)(w+z-1)(w+z)]$$

$$+ 5[\beta(5\beta+2)+5]s^2w^3z^3(w+z-1)^3 - \frac{\pi^2}{2}\xi^2\Big\langle \frac{\alpha_s}{\pi}G^2 \Big\rangle_\rho wz(w+z-1)[(\beta(3\beta+2)+3)w^2$$

$$+ w(\beta(\beta(7z-3)+6z-2)+7z-3) + (\beta(3\beta+2)+3)(z-1)z] \Big\} \Theta[L(s, z, w)]$$

$$+ \frac{1}{384\pi^6} \int_{z_1}^{z_2} dz \Big\{ \frac{9}{2}(\beta^2-1)m_Q\langle \bar{u}u \rangle_\rho + (z-1)z[4(3\beta^2+\beta+3)m_q\langle \bar{u}u \rangle_\rho$$

$$- 3(\beta(3\beta+2)+3)p_0\langle u^\dagger u \rangle_\rho - (\beta(3\beta-2)+3)\langle u^\dagger iD_0u \rangle_\rho] \Big\}, \tag{24}$$

$$\rho_{\not{\psi}}^S(s, p_0) = -\frac{1}{384\pi^6} \int_{z_1}^{z_2} dz \Big\{ \frac{9}{2}(\beta^2-1)m_q m_Q\langle u^\dagger u \rangle_\rho - (z-1)z[3\beta^2 p_0(m_q\langle \bar{u}u \rangle_\rho - 4\langle u^\dagger iD_0u \rangle_\rho)$$

$$+ \beta(3\langle \bar{u}g_s\sigma Gu \rangle_\rho - 2m_q p_0\langle \bar{u}u \rangle_\rho + 8p_0\langle u^\dagger iD_0u \rangle_\rho - 6s\langle u^\dagger u \rangle_\rho) + 3p_0(m_q\langle \bar{u}u \rangle_\rho - 4\langle u^\dagger iD_0u \rangle_\rho)] \Big\}, \tag{25}$$

$$\rho_U^S(s, p_0) = \frac{9(\beta^2-1)}{16384\xi^4\pi^{12}} \int_0^1 dz \int_0^{1-z} dw \Big\{ \frac{2m_Q s^2w^2z^2(w+z-1)^2(w+z)}{\xi} - 3m_Q^3 swz(w+z-1)(w+z)^2$$

$$+ \xi m_Q^5(w+z)^3 - \frac{\pi^2}{18}\Big\langle \frac{\alpha_s}{\pi}G^2 \Big\rangle_\rho m_Q[z^2(\xi(8w-1)+2w^3) + 2wz(\xi(4w-3)+(w-1)w^2)$$

$$+ (w-1)w^2(\xi-4w^2) + z^3(\xi+2(w-1)w) + 2(w+2)z^4 - 4z^5] \Big\} \Theta[L(s, z, w)]$$

$$+ \frac{1}{4096\pi^6} \int_{z_1}^{z_2} dz \Big\{ -16(\beta^2-1)m_Q[3m_q\langle \bar{u}u \rangle_\rho - 2p_0\langle u^\dagger u \rangle_\rho] + 8(\beta-1)^2(z-1)z[3\langle \bar{u}g_s\sigma Gu \rangle_\rho$$

$$+ 8m_q p_0\langle u^\dagger u \rangle_\rho - 6s\langle \bar{u}u \rangle_\rho] + 48(\beta+1)^2 m_Q^2\langle \bar{u}u \rangle_\rho + (\beta-1)^2\langle \bar{u}g_s\sigma Gu \rangle_\rho \Big\}, \tag{26}$$

where $\Theta[L(s, z, w)]$ is the unit-step function and

$$L(s, z, w) = -\frac{(w-1)[m_Q^2 w\xi + z(m_Q^2\xi - sw(w+z-1))]}{\xi^2},$$

$$\xi = w^2 + w(z-1) + (z-1)z,$$

$$z_1 = \frac{s - \sqrt{s^2 - 4m_Q^2 s}}{2s},$$

$$z_2 = \frac{s + \sqrt{s^2 - 4m_Q^2 s}}{2s}. \tag{27}$$

After applying the Borel transformation on the variable p^2 to the QCD side and performing the continuum subtraction, we match the coefficients of different structures from the physical as well as the QCD sides of the correlation function. As a result, we get the following in-medium sum rules

$$\lambda^{*2}e^{-\mu^2/\mathcal{M}^2} = \int_{(m_Q+m_{Q'})^2}^{s_0^*} ds\rho_{\not{p}}^{S(A)}(s, p_0)e^{-s/\mathcal{M}^2},$$

$$-\lambda^{*2}\Sigma_v e^{-\mu^2/\mathcal{M}^2} = \int_{(m_Q+m_{Q'})^2}^{s_0^*} ds\rho_{\not{\psi}}^{S(A)}(s, p_0)e^{-s/\mathcal{M}^2},$$

$$\lambda^{*2}m^* e^{-\mu^2/\mathcal{M}^2} = \int_{(m_Q+m_{Q'})^2}^{s_0^*} ds\rho_U^{S(A)}(s, p_0)e^{-s/\mathcal{M}^2}, \tag{28}$$

where s_0^* is the in-medium continuum threshold. By simultaneous solving of these coupled sum rules, we get the physical quantities in terms of the QCD degrees of freedom as well as the in-medium auxiliary parameters.

III. NUMERCAL RESULTS

In this section, we numerically analyze the sum rules obtained in previous section in order to estimate the in-medium and vacuum mass as well as the vector self-energy
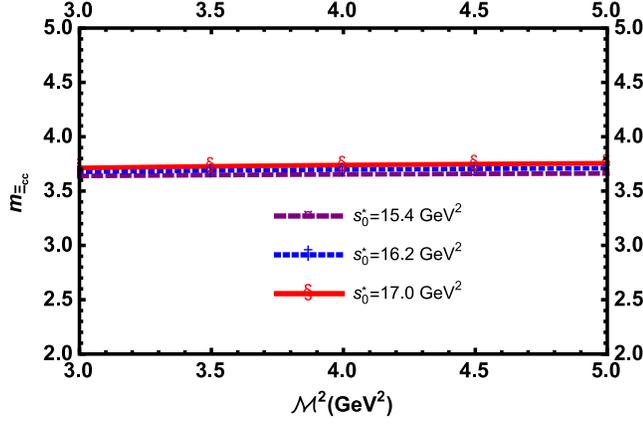
FIG. 1. $m_{\Xi_{cc}}$ as a function of $\mathcal{M}^2$ at three different values of in-medium continuum threshold.

of the doubly heavy spin-1/2, $\Xi_{QQ'}^{(\prime)}$, and $\Omega_{QQ'}^{(\prime)}$ baryons. To this end, we need numerical values of input parameters like quark masses and in-medium as well as vacuum condensates including quark, gluon, and mixed condensates of different dimensions, whose values are presented in Appendix B.

The sum rules for the physical quantities in Eq. (28) contain three auxiliary parameters: the Borel mass parameter $\mathcal{M}^2$, the in-medium continuum threshold s_0^*, and mixing parameter β entering the symmetric and antisymmetric spin-1/2 currents. We shall find their working regions according to the standard prescriptions of the method such that the dependence of the physical quantities on these parameters are mild at these regions. To this end, we require the pole dominance as well as the convergence of the series of the operator product expansion (OPE). In technique language, the upper band of the Borel mass parameter is determined by requiring that the pole contribution exceeds the contributions of the higher states and continuum, i.e.,

$$\frac{\int_{(m_Q+m_{Q'})^2}^{s_0^*} ds \rho_i^{S(A)}(s) e^{-s/M^2}}{\int_{(m_Q+m_{Q'})^2}^{\infty} ds \rho_i^{S(A)}(s) e^{-s/M^2}} > \frac{1}{2}, \qquad (29)$$

while the lower limit of $\mathcal{M}^2$ is obtained demanding that the perturbative part exceeds the nonperturbative contributions and the series of nonperturbative operators converge. The continuum threshold is not totally arbitrary but it depends on the energies of the first excited states in the channels under consideration. We have not experimental information about the masses of the excited states under study yet. Hence, we consider the interval $m_{QQ'} + E_1 \leq \sqrt{s_0} \leq m_{QQ'} + E_2$, where a energy from E_1 to E_2 is needed to excite the baryons, and demand that the Borel curves are most flat and the pole dominance and the OPE convergence conditions are satisfied. Our analyses show that choosing the window $m_{QQ'} + 0.3$ GeV $\leq \sqrt{s_0} \leq m_{QQ'} + 0.5$ GeV for the doubly heavy baryons satisfies

TABLE II.   Working regions of the Borel mass $\mathcal{M}^2$ and the in-medium continuum threshold s_0^* for different channels.

Channel	$\mathcal{M}^2$ (GeV^2)	s_0^* (GeV^2)
Ξ_{cc}, Ω_{cc}	[3–5]	[15.4–17.0],[16.2–17.9]
Ξ_{bc}, Ω_{bc}	[6–8]	[49.3–52.1],[49.7–52.6]
Ξ'_{bc}, Ω'_{bc}	[6–8]	[50.3–53.1],[50.4–53.3]
Ξ_{bb}, Ω_{bb}	[8–12]	[105.2–109.4],[105.5–109.6]

all these conditions. For the Ξ_{cc} baryon, as an example, the mass in the limit $\rho \to 0$ ($m_{\Xi_{cc}}$) shows a good stability with respect to $\mathcal{M}^2 \in [3$–$5]$ GeV^2 in the interval $s_0^* \in [15.4$–$17.0]$ GeV^2, which is obtained from the above restrictions (see Fig. 1). From this figure, it is also clear that the variations of mass with respect to the continuum threshold are minimal in the chosen window. For the Borel mass parameter and the in-medium continuum threshold the ranges presented in Table II for different channels fulfill all the requirements of the method.

For determination of the reliable region of the auxiliary parameter β, we plot the QCD side of the result obtained using the structure $\not{p}$ at Ξ_{cc} channel, as an example, as a function of x in Fig. 2, where we use $x = \cos\theta$ with $\theta = \arctan\beta$ to explore the whole region $-\infty < \beta < \infty$ by sweeping the region $-1 \leq x \leq 1$. From this figure, we obtain the following working intervals for x, where the results are roughly independent of x:

$$-1 \leq x \leq -0.60 \quad \text{and} \quad 0.60 \leq x \leq 1, \qquad (30)$$

for the vacuum and

$$-1 \leq x \leq -0.25 \quad \text{and} \quad 0.25 \leq x \leq 1, \qquad (31)$$

for the medium. Note that, the Ioffe current ($\beta = -1$) with $x = -0.71$ remains inside the reliable regions both for vacuum and in-medium cases. It is also clear that the
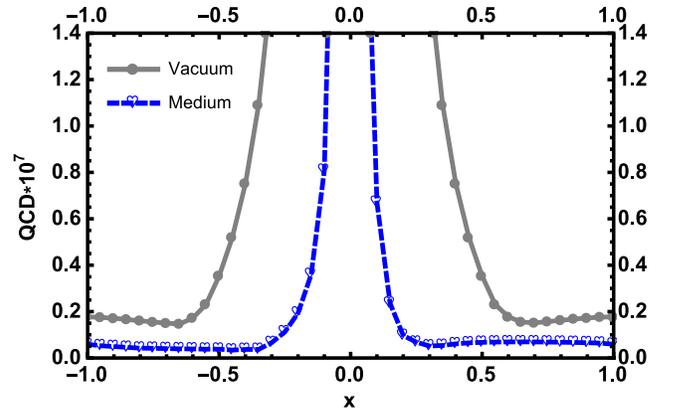
FIG. 2.   Variation of QCD side as a function of x obtained using the structure $\not{p}$ at Ξ_{cc} channel.
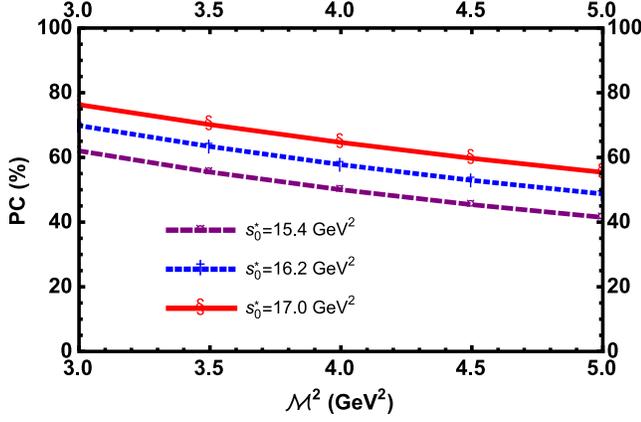
FIG. 3. PC with respect to $\mathcal{M}^2$ for the Ξ_{cc} channel and the structure $\not{p}$ at $\rho = \rho_{sat}$ and $x = 0.85$.

medium enlarge the reliable regions of β, considerably. This is one of the main results of the present study.

In order to check the pole contribution (PC), as an example for the Ξ_{cc} channel and the structure $\not{p}$, we plot PC as a function of $\mathcal{M}^2$ at three fixed values of the in-medium continuum threshold and at saturation nuclear matter density and $x = 0.85$ in Fig. 3. From this figure we obtain, in average, PC = 70% and PC = 49% at lower and higher limits of the Borel parameter, respectively. Our analyses show also that, with the above working windows for the auxiliary parameters, the series of sum rules converge, nicely.

We plot the ratio of the in-medium mass to vacuum mass, i.e., m^*/m, with respect to $\mathcal{M}^2$ for the doubly heavy $\Xi_{QQ'}$ and $\Omega_{QQ'}$ baryons at average value of the continuum threshold and at the saturation nuclear matter density in Fig. 4. This figure shows that in the selected windows for $\mathcal{M}^2$, m^*/m for all members show good stability against the variations of $\mathcal{M}^2$. It is also clear that the $\Omega_{QQ'}$ baryons are not affected by the medium at the saturation medium density, while the mass of $\Xi_{QQ'}$ baryons reduce to nearly 80% of their vacuum values at saturation nuclear matter density. Note that the vacuum masses are obtained from the in-medium calculations in the limit $\rho \to 0$.

The main goal of the present study is to investigate the behavior of the mass of the states under consideration with respect to the density of the medium. In this accordance, in Fig. 5, we depict the ratio m^*/m with respect to ρ/ρ_{sat} for the doubly heavy $\Xi_{QQ'}$ and $\Omega_{QQ'}$ baryons at average values of the continuum threshold, Borel mass parameter and considering the reliable regions of the mixing parameter β. We consider the range $\rho \in [0, 1.4]\rho_{sat}$ to investigate the behavior of the masses, where the previously presented value of ρ_{sat} is equivalent to roughly $1/5$ of the density of the neutron stars' core. From this figure we read that the $\Omega_{QQ'}$ baryons containing two heavy and one strange quarks do not see the dense medium at all. Similar behavior is the case for the doubly heavy $\Omega'_{QQ'}$ baryons. The doubly heavy
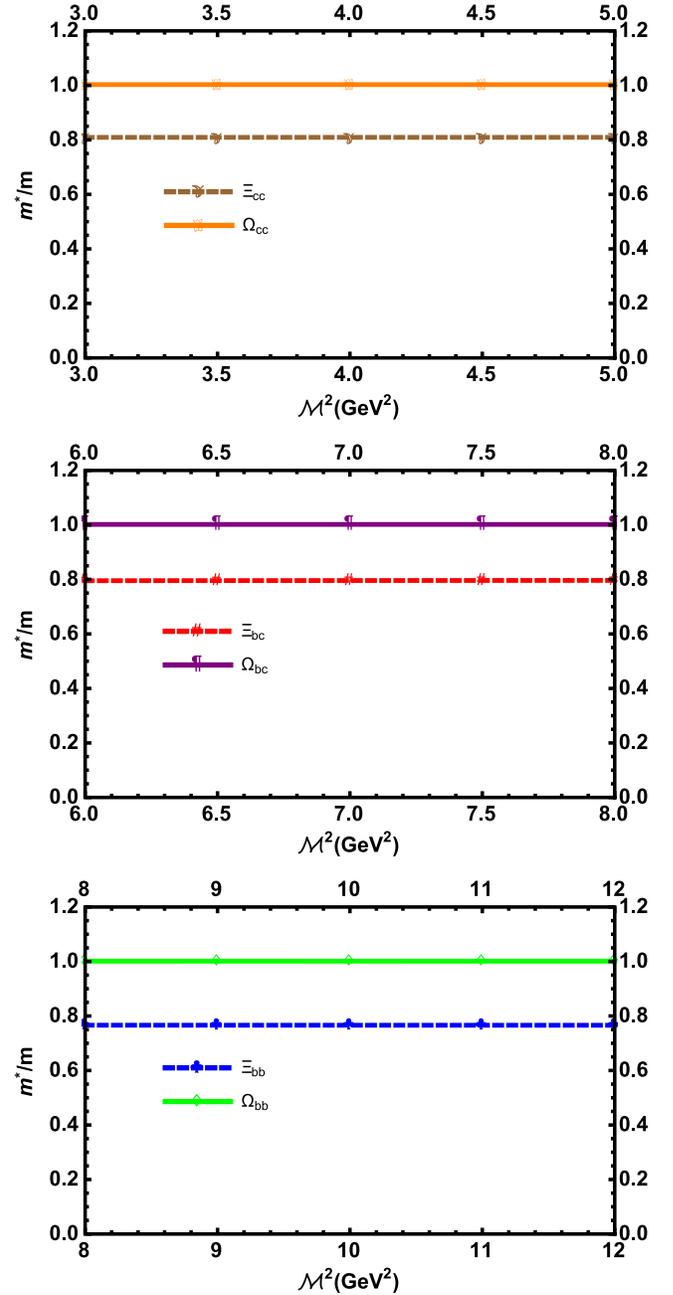
FIG. 4. The in-medium mass to vacuum mass ratio m^*/m with respect to Borel mass $\mathcal{M}^2$ for the spin-1/2 doubly heavy $\Xi_{QQ'}$ and $\Omega_{QQ'}$ baryons at average value of continuum threshold and at saturation nuclear matter density.

baryons $\Xi_{QQ'}^{(\prime)}$ with the quark contents of two heavy quarks and one up or down quark, however, are affected by the medium, considerably. Such that, as it is seen from Fig. 5, the mass of the baryons Ξ_{cc}, Ξ_{bc}, and Ξ_{bb} reach to 42%, 40%, and 24% of their vacuum values at $\rho/\rho_{sat} = 1.4$, respectively. At $\rho/\rho_{sat} = 1$, the in-medium mass to vacuum mass ratios for these baryons are obtained as 0.81, 0.79, and 0.77, respectively. The negative shifts on the masses due to

FIG. 5.   The in-medium mass to vacuum mass ratio m^*/m of the doubly heavy $\Xi_{QQ'}$ and $\Omega_{QQ'}$ baryons with respect to ρ/ρ_{sat} at average values of continuum threshold and Borel mass parameter.

TABLE III.   The average modified mass and vector self-energies of the doubly heavy $\Xi_{QQ'}^{(\prime)}$ and $\Omega_{QQ'}^{(\prime)}$ baryons in GeV at the saturation nuclear matter density together with their vacuum mass values.

	$m(\rho = 0)$	$m^*(\rho = \rho_{\mathrm{sat}})$	$\Sigma_v(\rho = \rho_{\mathrm{sat}})$
Ξ_{cc}	3.65 ± 0.05	2.96 ± 0.04	0.71 ± 0.10
Ω_{cc}	3.66 ± 0.09	3.66 ± 0.09	0.12 ± 0.03
Ξ_{bc}	6.49 ± 0.04	5.17 ± 0.03	1.07 ± 0.15
Ω_{bc}	6.51 ± 0.05	6.51 ± 0.05	0.14 ± 0.04
Ξ_{bb}	9.17 ± 0.06	6.97 ± 0.05	1.45 ± 0.20
Ω_{bb}	9.20 ± 0.06	9.19 ± 0.06	0.14 ± 0.04
Ξ'_{bc}	6.53 ± 0.09	5.01 ± 0.07	1.05 ± 0.14
Ω'_{bc}	7.01 ± 0.04	6.98 ± 0.04	0.13 ± 0.04

in the numerical results are due to the errors in the values of the input parameters as well as the uncertainties in determination of the working windows of the auxiliary parameters. It would be instructive to check the impact of some important input parameters like $\sigma_{\pi N}$ and its strange counterpart σ_{sN} on the finite density behavior of the studied hadrons. They appear as $y = \frac{2m_q}{m_s}\frac{\sigma_{sN}}{\sigma_{\pi N}}$ in the calculations. As we present in the Appendix B, we use the average of values obtained in Refs. [53,54] for this parameter. However, different methods and approaches obtain different values for y. In Ref. [55] the numerical values of σ_{sN} and $\sigma_{\pi N}$ are collected from different sources [53,56–62], which give y in the interval $[-0.05, 0.36]$ considering the corresponding errors. Taking into account this interval we see that the mass of, as an example Ω_{cc} state containing a strange quark, is changed maximally by 0.09% compared to the value considered in the Appendix B. Therefore, the effect of y on the parameters of the doubly heavy baryons is very weak. Our analyses show that the auxiliary parameters are sources of the main uncertainties in the presented results.

By comparison of the vacuum masses with the masses at saturation point, we see that $\Omega_{QQ'}^{(\prime)}$ baryons are not aware of the environment. The negative shifts on the masses of the $\Xi_{QQ'}^{(\prime)}$ baryons, however, refer to the strong scalar attractions of these states by the dense medium. The baryons $\Omega_{QQ'}^{(\prime)}$ gain small vector self-energies in dense medium compared to the $\Xi_{QQ'}^{(\prime)}$ baryons that receive large positive vector self-energies referring to the strong vector repulsion of these states by the nuclear medium.

At the end of this section, we would like to compare the vacuum mass values of the spin–1/2 doubly heavy $\Xi_{QQ'}^{(\prime)}$ and $\Omega_{QQ'}^{(\prime)}$ baryons obtained from the derived sum rules in the $\rho \to 0$ limit with the theoretical predictions as well as the existing experimental data in Ξ_{cc} channel. Table IV is presented in this respect. As seen from this table, the results obtained by using different approaches are over all consistent/close with/to each other within the errors. There are some channels that some predictions show considerable

the medium show that these baryons are attracted by the medium, considerably.

The saturation nuclear matter density is an important point that we would like to present the numerical values of the modified masses as well as the vector self-energies at all channels under study. To this end, in Table III, we collect the average values of these quantities at $\rho = \rho_{\mathrm{sat}}$ together with the vacuum masses of the doubly heavy $\Xi_{QQ'}^{(\prime)}$ and $\Omega_{QQ'}^{(\prime)}$ baryons obtained in the limit $\rho \to 0$. The uncertainties

TABLE IV.   Vacuum masses of the doubly heavy $\Xi_{QQ'}^{(\prime)}$ and $\Omega_{QQ'}^{(\prime)}$ baryons in GeV compared to other theoretical predictions. PS means present study.

$m(\rho=0)$	PS	[8]	[10]	[11]	[12]/[13]	[14]	[17]
Ξ_{cc}	3.65 ± 0.05	3.72(0.20)	$3.63^{+0.08}_{-0.07}$	4.26 ± 0.19	3.511	3.606	3.63 ± 0.02
Ω_{cc}	3.66 ± 0.09	3.73(0.20)	$3.75^{+0.08}_{-0.09}$	4.25 ± 0.20	3.650	3.715	3.73 ± 0.02
Ξ_{bc}	6.49 ± 0.04	6.72(0.20)	$\cdots$	6.75 ± 0.05	6.914	$\cdots$	6.99 ± 0.02
Ω_{bc}	6.51 ± 0.05	6.75(0.30)	$\cdots$	7.02 ± 0.08	7.136	$\cdots$	7.09 ± 0.01
Ξ_{bb}	9.17 ± 0.06	9.96(0.90)	$10.22^{+0.07}_{-0.07}$	9.78 ± 0.07	10.312	10.138	10.31 ± 0.01
Ω_{bb}	9.20 ± 0.06	9.97(0.90)	$10.33^{+0.07}_{-0.08}$	9.85 ± 0.07	10.446	10.230	10.37 ± 0.01
Ξ'_{bc}	6.53 ± 0.09	6.79(0.20)	$\cdots$	6.95 ± 0.08	$\cdots$	$\cdots$	7.01 ± 0.02
Ω'_{bc}	7.01 ± 0.04	6.80(0.30)	$\cdots$	7.02 ± 0.08	$\cdots$	$\cdots$	7.10 ± 0.01
$m(\rho=0)$	[64]	[65]	[63]	[66]	[67]	[35]	[68]
Ξ_{cc}	3.52–3.56	3.55	3.48	3.627 ± 0.012	3.660 ± 0.07	3.610	3.642
Ω_{cc}	3.62–3.65	3.73	3.59	$-$	3.740 ± 0.08	3.738	3.732
Ξ_{bc}	6.83–6.85	6.80	6.82	6.914 ± 0.013	6.990 ± 0.09	6.943	$\cdots$
Ω_{bc}	6.94–6.95	6.98	6.93	$-$	7.060 ± 0.09	6.998	$\cdots$
Ξ_{bb}	10.08–10.10	10.10	10.09	10.162 ± 0.012	10.340 ± 0.10	10.143	$\cdots$
Ω_{bb}	10.18–10.19	10.28	10.21	$-$	10.370 ± 0.10	10.273	$\cdots$
Ξ'_{bc}	$\cdots$	6.87	6.85	6.933 ± 0.012	7.040 ± 0.09	6.959	$\cdots$
Ω'_{bc}	$\cdots$	7.05	6.97	$-$	7.090 ± 0.09	7.032	$\cdots$

differences with other predictions. For instance in Ξ_{cc} channel, the results of Refs. [11,63] differ from the other predictions, considerably. The former has a prediction a bit larger and the later has the one a bit smaller than the other theoretical results. Our prediction on the mass of Ξ_{cc}, 3.65 ± 0.05, is in a nice agreement with the experimental result of LHCb collaboration, $3621.40 \pm 72(\text{stat}) \pm 0.27(\text{syst}) \pm 14(\Lambda_c^+)$ MeV/c^2 [22]. Our predictions on the mass of other members together with the predictions of other theoretical models can shed light on the future experiments aiming to hunt the doubly heavy baryons and measure their properties.

IV. CONCLUDING REMARKS

After the discovery of the Ξ_{cc}^{++} state, as a member of the doubly heavy spin–1/2 baryon's family, by LHCb collaboration in 2017 and the tension between the LHCb result with the previous SELEX data has put the subject of doubly heavy baryons at the center of interests in hadron physics. With the developments in experimental side, it is expected that other members of the family will be discovered in near future. Naturally, many theoretical studies try to report their predictions on the parameters of the doubly heavy baryons using variety of models and approaches. The studies are mainly done in the vacuum. The present study is the first comprehensive work discussing these baryons both in vacuum and medium with finite density. Thus, in the present work, we derive the masses and vector self energies of the doubly heavy baryons with both the symmetric and

antisymmetric currents in terms of the QCD degrees of freedom, density of the medium, in-medium nonperturbative operators of different dimensions as well as continuum threshold, Borel mass parameter, and mixing parameter β as the helping parameters entering the calculations. With the standard prescriptions of the in-medium QCD sum rules method we restricted the auxiliary parameters to find their reliable working window. We observed that the medium enlarges the working window of the mixing parameter β, considerably. Using the reliable working intervals of the helping parameters we extracted the masses and vector self-energies of the baryons under consideration at saturated nuclear matter density. It is observed that the $\Omega_{QQ'}^{(\prime)}$ baryons do not overall see the medium at all, while the parameters of $\Xi_{QQ'}^{(\prime)}$ baryons are affected by the medium considerably. Such that, at saturated nuclear matter density, the masses of the baryons Ξ_{cc}, Ξ_{bc}, and Ξ_{bb} reach to 0.81, 0.79, and 0.77 of their vacuum values, respectively. The negative shifts on the masses due to the medium show that these baryons are attracted (scalar self-energy attraction) by the medium. The $\Omega_{QQ'}^{(\prime)}$ baryons gain small vector self-energies at saturated nuclear matter density. The positive and large vector self-energies of the $\Xi_{QQ'}^{(\prime)}$ baryons indicate that these baryons endure strong vector repulsion from the medium.

We investigated the behavior of the m^*/m with respect to ρ/ρ_{sat} for the doubly heavy spin–1/2 baryons in the range $\rho \in [0, 1.4]\rho_{\text{sat}}$. We observed that the masses of the doubly

heavy $\Omega'_{QQ'}$ baryons remain unchanged even up to $\rho = 1.4\rho_{\text{sat}}$. While the doubly heavy baryons $\Xi^{(')}_{QQ'}$ are affected by the medium, considerably. Such that the masses of the baryons Ξ_{cc}, Ξ_{bc}, and Ξ_{bb} reach to 42%, 40%, and 24% of their vacuum values at the end point, respectively.

We extracted the masses of all members in $\rho \to 0$ limit as well and compared the results with other theoretical vacuum predictions. Our prediction on the mass of Ξ_{cc} is in a nice agreement with the experimental data of LHCb collaboration [22]. Our predictions on the vacuum masses of other members may help experimental groups in the search for these baryons, which are natural outcomes of the quark model. Our results on the in-medium masses and vector-self energies of the doubly heavy baryons may shed light on the future in-medium experiments and help physicists in analyzing the results of such experiments.

APPENDIX A: THE LIGHT AND HEAVY QUARKS PROPAGATORS AND THEIR IN-MEDIUM INGREDIENTS

In this Appendix, we present the explicit expressions of the in-medium quarks propagators including their ingredients: the in-medium quark, gluon, and mixed condensates. In the calculations, the light quark propagator is used in the fixed point gauge,

$$S_q^{ij}(x) = \frac{i}{2\pi^2}\delta^{ij}\frac{1}{(x^2)^2}\slashed{x} - \frac{m_q}{4\pi^2}\delta^{ij}\frac{1}{x^2} + \chi_q^i(x)\bar{\chi}_q^j(0)$$
$$- \frac{ig_s}{32\pi^2}F_{\mu\nu}^A(0)t^{ij,A}\frac{1}{x^2}[\slashed{x}\sigma^{\mu\nu} + \sigma^{\mu\nu}\slashed{x}] + \cdots, \quad \text{(A1)}$$

where χ_q^i and $\bar{\chi}_q^j$ are the Grassmann background quark fields, $F_{\mu\nu}^A$ are classical background gluon fields, and $t^{ij,A} = \frac{\lambda^{ij,A}}{2}$ with $\lambda^{ij,A}$ being the standard Gell-Mann matrices. The heavy quark propagator is given as

$$S_Q^{ij}(x) = \frac{i}{(2\pi)^4}\int d^4 k e^{-ik\cdot x}\left\{\frac{\delta_{ij}}{\slashed{k} - m_Q}\right.$$
$$- \frac{g_s F_{\mu\nu}^A(0)t^{ij,A}}{4}\frac{\sigma_{\mu\nu}(\slashed{k} + m_Q) + (\slashed{k} + m_Q)\sigma_{\mu\nu}}{(k^2 - m_Q^2)^2}$$
$$\left. + \frac{\pi^2}{3}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\delta_{ij}m_Q\frac{k^2 + m_Q\slashed{k}}{(k^2 - m_Q^2)^4} + \cdots\right\}. \quad \text{(A2)}$$

By replacing these explicit forms of the light and heavy quark propagators in the correlation function in Eqs. (18)–(19), the products of the Grassmann background quark fields and classical background gluon fields, which correspond to the ground-state matrix elements of the corresponding quark and gluon operators [52] are obtained,

$$\chi_{a\alpha}^q(x)\bar{\chi}_{b\beta}^q(0) = \langle q_{a\alpha}(x)\bar{q}_{b\beta}(0)\rangle_\rho,$$
$$F_{\kappa\lambda}^A F_{\mu\nu}^B = \langle G_{\kappa\lambda}^A G_{\mu\nu}^B\rangle_\rho,$$
$$\chi_{a\alpha}^q\bar{\chi}_{b\beta}^q F_{\mu\nu}^A = \langle q_{a\alpha}\bar{q}_{b\beta}G_{\mu\nu}^A\rangle_\rho, \quad \text{(A3)}$$

where, ρ is the medium density. The matrix elements in the right-hand sides of the equations in Eq. (A3) contain the in-medium quark, gluon, and mixed condensates, whose explicit forms are given as [52]:

(1) Quark condensate:

$$\langle q_{a\alpha}(x)\bar{q}_{b\beta}(0)\rangle_\rho = -\frac{\delta_{ab}}{12}\left[\left(\langle\bar{q}q\rangle_\rho + x^\mu\langle\bar{q}D_\mu q\rangle_\rho + \frac{1}{2}x^\mu x^\nu\langle\bar{q}D_\mu D_\nu q\rangle_\rho + \cdots\right)\delta_{\alpha\beta}\right.$$
$$\left. + \left(\langle\bar{q}\gamma_\lambda q\rangle_\rho + x^\mu\langle\bar{q}\gamma_\lambda D_\mu q\rangle_\rho + \frac{1}{2}x^\mu x^\nu\langle\bar{q}\gamma_\lambda D_\mu D_\nu q\rangle_\rho + \cdots\right)\gamma_{\alpha\beta}^\lambda\right], \quad \text{(A4)}$$

(2) Gluon condensate:

$$\langle G_{\kappa\lambda}^A G_{\mu\nu}^B\rangle_\rho = \frac{\delta^{AB}}{96}[\langle G^2\rangle_\rho(g_{\kappa\mu}g_{\lambda\nu} - g_{\kappa\nu}g_{\lambda\mu}) + O(\langle\mathbf{E}^2 + \mathbf{B}^2\rangle_\rho)], \quad \text{(A5)}$$

where the term $O(\langle\mathbf{E}^2 + \mathbf{B}^2\rangle_\rho)$ is neglected because of its small contribution.

(3) Quark-gluon mixed condensate:

$$\langle g_s q_{a\alpha}\bar{q}_{b\beta}G_{\mu\nu}^A\rangle_\rho = -\frac{t_{ab}^A}{96}\{\langle g_s\bar{q}\sigma\cdot Gq\rangle_\rho[\sigma_{\mu\nu} + i(u_\mu\gamma_\nu - u_\nu\gamma_\mu)\slashed{u}]_{\alpha\beta} + \langle g_s\bar{q}\slashed{u}\sigma\cdot Gq\rangle_\rho[\sigma_{\mu\nu}\slashed{u} + i(u_\mu\gamma_\nu - u_\nu\gamma_\mu)]_{\alpha\beta}$$
$$- 4(\langle\bar{q}u\cdot Du\cdot Dq\rangle_\rho + im_q\langle\bar{q}\slashed{u}u\cdot Dq\rangle_\rho)[\sigma_{\mu\nu} + 2i(u_\mu\gamma_\nu - u_\nu\gamma_\mu)\slashed{u}]_{\alpha\beta}\}, \quad \text{(A6)}$$

where $D_\mu = \frac{1}{2}(\gamma_\mu\slashed{D} + \slashed{D}\gamma_\mu)$. The modified in-medium different condensates in Eqs. (A4)–(A6) are presented as:

$$\langle \bar{q}\gamma_\mu q\rangle_\rho = \langle \bar{q}\slashed{u}q\rangle_\rho u_\mu,$$

$$\langle \bar{q}D_\mu q\rangle_\rho = \langle \bar{q}u \cdot Dq\rangle_\rho u_\mu = -im_q\langle \bar{q}\slashed{u}q\rangle_\rho u_\mu,$$

$$\langle \bar{q}\gamma_\mu D_\nu q\rangle_\rho = \frac{4}{3}\langle \bar{q}\slashed{u}u \cdot Dq\rangle_\rho \left(u_\mu u_\nu - \frac{1}{4}g_{\mu\nu}\right) + \frac{i}{3}m_q\langle \bar{q}q\rangle_\rho (u_\mu u_\nu - g_{\mu\nu}),$$

$$\langle \bar{q}D_\mu D_\nu q\rangle_\rho = \frac{4}{3}\langle \bar{q}u \cdot Du \cdot Dq\rangle_\rho \left(u_\mu u_\nu - \frac{1}{4}g_{\mu\nu}\right) - \frac{1}{6}\langle g_s\bar{q}\sigma \cdot Gq\rangle_\rho (u_\mu u_\nu - g_{\mu\nu}),$$

$$\langle \bar{q}\gamma_\lambda D_\mu D_\nu q\rangle_\rho = 2\langle \bar{q}\slashed{u}u \cdot Du \cdot Dq\rangle_\rho \left[u_\lambda u_\mu u_\nu - \frac{1}{6}(u_\lambda g_{\mu\nu} + u_\mu g_{\lambda\nu} + u_\nu g_{\lambda\mu})\right] - \frac{1}{6}\langle g_s\bar{q}\slashed{u}\sigma \cdot Gq\rangle_\rho (u_\lambda u_\mu u_\nu - u_\lambda g_{\mu\nu}). \quad \text{(A7)}$$

APPENDIX B: NUMERICAL INPUTS

In numerical calculations, the vacuum condensates are used at a renormalization scale of 1 GeV: $\rho_{sat} = 0.11^3\,\text{GeV}^3$, $\langle q^\dagger q\rangle_\rho = \frac{3}{2}\rho$, $\langle s^\dagger s\rangle_\rho = 0$ [52], $\langle \bar{q}q\rangle_0 = (-0.241)^3\,\text{GeV}^3$ [69], $\langle \bar{s}s\rangle_0 = 0.8\langle \bar{q}q\rangle_0$ [52], $\langle \bar{q}q\rangle_\rho = \langle \bar{q}q\rangle_0 + \frac{\sigma_{\pi N}}{2m_q}\rho$ [70], $m_q = \frac{m_u+m_d}{2} = 0.00345$ GeV [71], $\langle \bar{s}s\rangle_\rho = \langle \bar{s}s\rangle_0 + y\frac{\sigma_{\pi N}}{2m_q}\rho$, $y = 0.05 \pm 0.01$ (the average of values obtained in Refs. [53,54]), $\langle \frac{\alpha_s}{\pi}G^2\rangle_0 = (0.33 \pm 0.04)^4\,\text{GeV}^4$, $\langle \frac{\alpha_s}{\pi}G^2\rangle_\rho = \langle \frac{\alpha_s}{\pi}G^2\rangle_0 - (0.65 \pm 0.15)\,\text{GeV}\,\rho$, $\langle q^\dagger iD_0 q\rangle_\rho = 0.18\,\text{GeV}\,\rho$, $\langle s^\dagger iD_0 s\rangle_\rho = \frac{m_s\langle \bar{s}s\rangle_\rho}{4} + 0.02\,\text{GeV}\,\rho$, $\langle \bar{q}iD_0 q\rangle_\rho = \langle \bar{s}iD_0 s\rangle_\rho = 0$, $\langle \bar{q}g_s\sigma Gq\rangle_0 = m_0^2\langle \bar{q}q\rangle_0$, $\langle \bar{s}g_s\sigma Gs\rangle_0 = m_0^2\langle \bar{s}s\rangle_0$ [52], $m_0^2 = 0.8\,\text{GeV}^2$ [69], $\langle \bar{q}g_s\sigma Gq\rangle_\rho = \langle \bar{q}g_s\sigma Gq\rangle_0 + 3\,\text{GeV}^2\,\rho$, $\langle \bar{s}g_s\sigma Gs\rangle_\rho = \langle \bar{s}g_s\sigma Gs\rangle_0 + 3y\,\text{GeV}^2\,\rho$, $\langle q^\dagger g_s\sigma Gq\rangle_\rho = -0.33\,\text{GeV}^2\,\rho$, $\langle q^\dagger iD_0 iD_0 q\rangle_\rho = 0.031\,\text{GeV}^2\rho - \frac{1}{12}\langle q^\dagger g_s\sigma Gq\rangle_\rho$, $\langle s^\dagger g_s\sigma Gs\rangle_\rho = -0.33y\,\text{GeV}^2\,\rho$, and $\langle s^\dagger iD_0 iD_0 s\rangle_\rho = 0.031y\,\text{GeV}^2\rho - \frac{1}{12}\langle s^\dagger g_s\sigma Gs\rangle_\rho$ [52,70]. For the pion nucleon sigma term we use $\sigma_{\pi N} = 0.059$ GeV [72].

The light quark masses are used at a renormalization scale 1 GeV, as well: $m_u = 2.16^{+0.49}_{-0.26}$ MeV, $m_d = 4.67^{+0.48}_{-0.17}$ MeV, $m_s = 93^{+11}_{-5}$ MeV [71]. For the heavy quarks, we use the pole masses. The relation between the pole mass m_Q and $\overline{\text{MS}}$ mass $\bar{m}_Q$ for the heavy quarks in three loops is given as [73–76]

$$m_Q = \bar{m}_Q(\bar{m}_Q)\left\{1 + \frac{4\bar{\alpha}_s(\bar{m}_Q)}{3\pi} + \left[-1.0414\Sigma_k\left(1 - \frac{4}{3}\frac{\bar{m}_{Q_k}}{\bar{m}_Q}\right) + 13.4434\right]\left[\frac{\bar{\alpha}_s(\bar{m}_Q)}{\pi}\right]^2\right.$$
$$\left. + [0.6527N_L^2 - 26.655N_L + 190.595]\left[\frac{\bar{\alpha}_s(\bar{m}_Q)}{\pi}\right]^3\right\} \quad \text{(B1)}$$

where $\bar{\alpha}_s(\mu)$ is the strong interaction coupling constant in the $\overline{\text{MS}}$ scheme, and the sum over k extends over the N_L flavors Q_k lighter than Q. Using the $\overline{\text{MS}}$ mass values presented in PDG, one gets $m_b = 4.78 \pm 0.06$ GeV and $m_c = 1.67 \pm 0.07$ GeV for the bottom and charm pole masses, which are used in numerical calculations.

Note that, at dense medium, each condensate is expanded up to the first order in nucleon density as $\langle \hat{O}\rangle_\rho = \langle \hat{O}\rangle_0 + \langle \hat{O}\rangle_N\rho$, where $\langle \hat{O}\rangle_0$ is the vacuum expectation value of the operator $\hat{O}$ and $\langle \hat{O}\rangle_N$ is its expectation value between one-nucleon states [52,70,71].

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