Heavy quark dynamics in a hot magnetized QCD medium

Manu Kurian,[1,2,*] Santosh K. Das,[3,†] and Vinod Chandra[1,‡]
[1] *Indian Institute of Technology Gandhinagar, Gandhinagar-382355, Gujarat, India*
[2] *Department of Physics, McGill University, 3600 University Street, Montreal, Quebec, H3A 2T8, Canada*
[3] *School of Physical Science, Indian Institute of Technology Goa, Ponda-403401, Goa, India*

(Received 23 July 2019; revised manuscript received 6 September 2019; published 4 October 2019)

The heavy quark drag and momentum diffusion have been investigated in a hot magnetized quark-gluon plasma, along the directions parallel and perpendicular to the magnetic field. The analysis is done within the framework of Fokker-Planck dynamics by considering the heavy quark scattering with thermal quarks and gluons at the leading order in the coupling constant. An extended quasiparticle model is adopted to encode the thermal QCD medium interactions in the presence of a magnetic field. Further, the higher Landau level effects on the temperature behavior of the parallel and perpendicular components of the drag force and diffusion coefficients have studied. It has been observed that both the equation of state and the magnetic field play key roles in the temperature dependence of the heavy quark dynamics.

DOI: 10.1103/PhysRevD.100.074003

I. INTRODUCTION

The heavy ion-collision experiments at Relativistic Heavy Ion Collider (RHIC) and at the Large Hadron Collider (LHC) set the stage to investigate the deconfined state of the nuclear matter called quark-gluon plasma (QGP), as a near-ideal fluid [1–3]. Recent studies revealed the presence of ultraintense magnetic field in the noncentral asymmetric collisions [4–7]. Inclusion of the magnetic field to the theoretical investigations regarding the QGP/hot QCD matter is seen to affect its transport and thermodynamic properties in a significant way [8–17]. In particular, the novel phenomena such as chiral magnetic effect [18–20], chiral vortical effect [21–23], chiral charge separation [24], magnetic catalysis [25], and more recently the realization of global $\Lambda -$ hyperon polarization in the RHIC [26,27] opens up new directions in the investigation on magnetized QGP.

Heavy quarks (HQs), mainly charm and bottom, are identified as effective probes to characterize the properties of the QGP [28–33]. The HQs are mostly created in the early stages of collision and propagate through the bulk medium (QGP) while interacting with its constituents (light quarks and gluons). The HQs are the witness of the entire space-time evolution of the bulk medium. There have been several attempts to investigate the HQ dynamics in QCD matter [34–50] and the related experimental observables such as nuclear suppression factor, elliptic flow and heavy baryon to meson ratio which serve as direct QGP probes [51–53].

Recently, it has been recognized that the strong electromagnetic fields created at early times of the heavy-ion collisions can affect the HQs dynamics. HQ directed flow (v_1) [54] is identified as a potential probe of the strong initial electromagnetic field created in heavy-ion collisions. This transient field can induce opposite v_1 for the charm and anticharm quarks due to their opposite charge. The v_1 for hadrons containing HQs is predicted to be several orders of magnitude larger than that for hadrons containing light quark; a prediction that appears to be vindicated by early experimental results at both RHIC and LHC energies [55–57]. However, recent calculations [54,58,59] on the HQ directed flow due to the electromagnetic field, within the Langevin dynamics, ignore the impact of the magnetic field on HQ transport coefficients. Hence, it is an interesting aspect to investigate the HQ drag and momentum diffusion in the presence of the magnetic field and explore its consequences on experimental observables.

The light quarks/antiquarks degree of freedom are affected by the magnetic field and follow the Landau level dynamics in the thermal equilibrium. Whereas the electrically neutral gluons indirectly coupled to the magnetic field through the self-energy via quark/antiquark loop. The transport coefficients of the magnetized QGP have been studied in the lowest Landau level (LLL) approximation such that the thermal occupation of the higher Landau levels (HLLs) is exponentially suppressed [11,60]. The validity of LLL approximation is questionable since HLL contributions are significant at the temperature range much above the transition temperature. The recent studies [61–63] revealed the significance of HLLs in the analysis by focusing on the more realistic regime $gT \ll \sqrt{|eB|}$. Several investigations on the properties of HQs and quarkonia in the presence of

*manu.kurian@iitgn.ac.in
†dsantoshphy@gmail.com
‡vchandra@iitgn.ac.in

magnetic field have been done in Refs. [64–70]. There are a few attempts within the holographic and conformal field theory description of the HQ transport in anisotropic strongly coupled theories [71,72] and with a strong magnetic field background [73–76]. Since the external magnetic field constraints the light quarks/antiquarks motion in preferred spatial direction (either parallel or antiparallel to the field), one needs to analyze the HQ dynamics in both parallel and perpendicular to the magnetic field. In the recent work [77], the authors have showed that the diffusion coefficient of HQ became anisotropic in the presence of a strong magnetic field in the LLL approximation.

The present article primarily focuses on the study of longitudinal and transverse components of the momentum diffusion and drag of the HQ in the strongly magnetized QGP the regime $gT \ll \sqrt{|eB|}$, incorporating the effects of hot QCD medium interactions and HLLs. To that end, the modeling of the local momentum distribution function of gluons and quarks are essential such that the realistic equation of state (EoS) of the QGP could be mimicked. At this juncture, a recently proposed effective fugacity quasiparticle model (EQPM) [78,79] has been employed in the analysis. The thermal QCD medium effects are reflected in the temperature dependence of the effective fugacity of the EQPM phase space distribution function. The quasiparticle description of HQ dynamics in the QGP has been investigated in the absence of the magnetic field in Refs. [80,81].

For the quantitative description of the HQ dynamics in the magnetized QGP, the relativistic Boltzmann equation needs to be solved by embedding the proper collision integral in the presence of external magnetic field background. The motion of HQ can be analyzed as a Brownian motion while considering their perturbative interaction, and the large HQ mass allows us to assume for the low momentum transfer scattering [34]. Under such constraints, the relativistic transport equation can be reduced to the Fokker-Planck equation. This assumption has been widely employed in the investigation of HQ propagation in the medium [35,36,45]. The current investigation is done within the framework of the Fokker-Planck dynamics by analyzing the scattering of HQs with thermal gluons and quarks separately in the presence of the magnetic field. At the leading order in the coupling constant α_s, the HQ scattering with gluons and quarks are mediated by a one-gluon exchange. The HLLs contribution to the HQ dynamics in the magnetized QGP has also been estimated.

The article is organized as follows. Section II is devoted to the mathematical formulation of the gluonic and quark contributions to the HQ drag and diffusion within the framework of the Fokker-Planck equation followed by the quasiparticle modeling of the magnetized QGP. The discussions on the effect of thermal QCD medium and HLLs on the temperature behavior of the HQ dynamics are presented in Sec. III. Section IV contains the conclusions and outlook of the article.

II. HEAVY QUARK DRAG AND DIFFUSION IN MAGNETIZED QGP

HQs are subjected to random motion at the finite temperature due to the scattering with thermally excited quarks and gluons. Since the kinematics of quarks and gluons are different in the presence of the magnetic field, the HQ scattering with quarks and gluons need to be calculated separately [82]. Note that the mass of HQ, M_{HQ}, is assumed to follow $M_{\mathrm{HQ}} \gg \sqrt{q_f eB}$, where $q_f e$ is the fractional charge of the quark of flavor f, such that HQ motion is not directly affected by the magnetic field. In the Ref. [54], the authors have included the Lorentz force in the analysis of the direct flow of the charm quark. The HQ dynamics can be understood in terms of the phase space distribution function with the prescription of transport theory.

A. Thermal gluon contribution to HQ transport

In this section, we are considering the gluonic contribution to the HQ dynamics while traveling in the QGP in the presence of the strong magnetic field $\mathbf{B} = B\hat{z}$. Note that the magnetic field affects the gluon dynamics through the self-energy and the Debye screening mass in the system [13]. The dynamics of HQ can be understood in terms of the drag and momentum diffusion in the medium.

1. Formalism of HQ drag and diffusion

The evolution of the HQ momentum distribution function f_{HQ} in the QGP can be described by the Boltzmann equation as [34],

$$p^{\mu}\partial_{\mu} f_{\mathrm{HQ}} = \left(\frac{\partial f_{\mathrm{HQ}}}{\partial t}\right)_{\mathrm{col}}. \tag{1}$$

The term $(\frac{\partial f_{\mathrm{HQ}}}{\partial t})_{\mathrm{col}}$ is the relativistic collision integral that quantifies the rate of change in the HQ distribution function due to the interactions with thermal gluons in the medium. For the two-body collision, the collision term takes the following form [34],

$$\left(\frac{\partial f_{\mathrm{HQ}}}{\partial t}\right)_{\mathrm{col}} = \int d^3\mathbf{q}[\omega(\mathbf{p}+\mathbf{q},\mathbf{q}) f_{\mathrm{HQ}}(\mathbf{p}+\mathbf{q}) - \omega(\mathbf{p},\mathbf{q}) f_{\mathrm{HQ}}(\mathbf{p})]. \tag{2}$$

The quantity $\omega(\mathbf{p},\mathbf{q})$ is the rate of collisions with gluons that change the HQ momentum from $\mathbf{p}$ to $\mathbf{p}-\mathbf{q}$ and defined as,

$$\omega(\mathbf{p},\mathbf{q}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} f_g(\mathbf{k}) v \sigma_{\mathbf{p},\mathbf{k}\to\mathbf{p}-\mathbf{q},\mathbf{k}+\mathbf{q}}, \tag{3}$$

where the interaction cross-section $\sigma_{\mathbf{p},\mathbf{k}\to\mathbf{p}-\mathbf{q},\mathbf{k}+\mathbf{q}}$ is related to the matrix element $\mathcal{M}_{\mathrm{HQ},g}$ of the HQ scattering process with gluons. Here, $f_g(\mathbf{k})$ is the momentum distribution of gluons and v is the relative velocity between the colliding particles. Note that in the integrand of Eq. (2), the first term constitutes the gain term through the scattering whereas the second term represents the loss out of the volume element around the HQ momentum $\mathbf{p}$.

The integral operator in the Boltzmann equation can be simplified by employing the Landau approximation which assumes that most of the HQ-qluon scattering is soft with small momentum transfer. Hence, we can expand $\omega(\mathbf{p}+\mathbf{q},\mathbf{q})f_{\mathrm{HQ}}(\mathbf{p}+\mathbf{q})$ up to the second order of the momentum transfer $\mathbf{q}$ as,

$$\omega(\mathbf{p}+\mathbf{q},\mathbf{q})f_{\mathrm{HQ}}(\mathbf{p}+\mathbf{q}) \approx \omega(\mathbf{p},\mathbf{q})f_{\mathrm{HQ}}(\mathbf{p}) + \mathbf{q}\cdot\frac{\partial}{\partial\mathbf{p}}[\omega f_{\mathrm{HQ}}] + \frac{1}{2}q_i q_j \frac{\partial^2}{\partial p_i \partial p_j}[\omega f_{\mathrm{HQ}}]. \tag{4}$$

The relativistic Boltzmann equation can be reduced to the Fokker-Planck equation by employing the Eqs. (2)–(4) in the Eq. (1) and takes the form as follows,

$$\frac{\partial f_{\mathrm{HQ}}}{\partial t} = \frac{\partial}{\partial p_i}\left[A_i(\mathbf{p})f_{\mathrm{HQ}} + \frac{\partial}{\partial p_j}[B_{ij}(\mathbf{p})f_{\mathrm{HQ}}]\right], \tag{5}$$

where A_i and B_{ij} measure the drag force and momentum diffusion of the HQs, respectively. For the process, $HQ(p) + g(k) \to HQ(p') + g(k')$, where g stands for gluons in the magnetized thermal medium, the HQ drag and momentum diffusion takes the following forms [34,35],

$$\begin{aligned} A_i &= \frac{1}{d_{\mathrm{HQ}}}\frac{1}{2E_p}\int\frac{d^3\mathbf{k}}{(2\pi)^3E_k}\int\frac{d^3\mathbf{p}'}{(2\pi)^3E_{p'}}\int\frac{d^3\mathbf{k}'}{(2\pi)^3E_{k'}} \\ &\quad\times|\mathcal{M}_{\mathrm{HQ},g}|^2(2\pi)^4\delta^4(p+k-p'-k')f_g(\mathbf{k}) \\ &\quad\times(1+f_g(\mathbf{k}'))(\mathbf{p}-\mathbf{p}')_i \\ &\equiv \langle\langle(\mathbf{p}-\mathbf{p}')_i\rangle\rangle, \end{aligned} \tag{6}$$

and

$$\begin{aligned} B_{ij} &= \frac{1}{2d_{\mathrm{HQ}}}\frac{1}{2E_p}\int\frac{d^3\mathbf{k}}{(2\pi)^3E_k}\int\frac{d^3\mathbf{p}'}{(2\pi)^3E_{p'}}\int\frac{d^3\mathbf{k}'}{(2\pi)^3E_{k'}} \\ &\quad\times|\mathcal{M}_{\mathrm{HQ},g}|^2(2\pi)^4\delta^4(p+k-p'-k')f_g(\mathbf{k}) \\ &\quad\times(1+f_g(\mathbf{k}'))(\mathbf{p}-\mathbf{p}')_i(\mathbf{p}-\mathbf{p}')_j \\ &\equiv \langle\langle(\mathbf{p}-\mathbf{p}')_i(\mathbf{p}-\mathbf{p}')_j\rangle\rangle, \end{aligned} \tag{7}$$

respectively. Here, d_{HQ} is the statistical degeneracy of the HQ. The Eq. (6) indicates that the HQ drag is the measure of the thermal average of the momentum transfer $\mathbf{q} = \mathbf{p}-\mathbf{p}'$ due to the scattering of HQ with the thermal particles. Whereas the momentum diffusion in Eq. (7) measures the thermal average of square of the momentum transfer. In the static limit $\mathbf{p}\to 0$, we can consider $B_{ij}\to K\delta_{ij}$ [34] with K is the HQ diffusion coefficient. The magnetic field provides the preferred spatial direction and we need to consider the HQ motion parallel and perpendicular to the magnetic field. The longitudinal and transverse components of HQ drag and diffusion in the presence of magnetic field ($\mathbf{B} = B\hat{z}$) defines as the thermal average of the corresponding components and its square of the momentum transfer due to the HQ scattering process with thermal particles. The HQ drag force components can be defined as,

$$A_{\parallel} = \langle\langle q_z\rangle\rangle, \qquad A_{\perp} = \frac{1}{\sqrt{2}}\langle\langle q_{\perp}\rangle\rangle, \tag{8}$$

where longitudinal and transverse components quantitatively measure the anisotropy in the drag force in the presence of the magnetic field. Here, $q_{\perp} = |\mathbf{q}_{\perp}|$ is the transverse component of the momentum transfer. Similarly, the components of the HQ momentum diffusion coefficients in the presence of the magnetic field within the static limit can be defined as,

$$K_{\parallel} = \langle\langle q_z^2\rangle\rangle, \qquad K_{\perp} = \frac{1}{2}\langle\langle q_{\perp}^2\rangle\rangle. \tag{9}$$

Employing the fluctuation-dissipation theorem, we can define the longitudinal and transverse drag coefficients respectively as,

$$\eta_{\parallel} = \frac{K_{\parallel}}{2TM_{\mathrm{HQ}}}, \qquad \eta_{\perp} = \frac{K_{\perp}}{2TM_{\mathrm{HQ}}}. \tag{10}$$

We intend to compute the HQ drag force components and diffusion coefficients in the longitudinal and transverse directions considering the thermal medium effects of the magnetized QGP in the current analysis. The EoS effects can enter through both the parton momentum distribution function and the Debye screening mass (effective coupling) while defining the scattering amplitude of the interaction. Proper modeling of the hot magnetized QGP is essential to incorporate the effects of QCD medium interactions.

2. *Modeling of hot magnetized QGP*

The effective modeling of the QGP by encoding the thermal QCD medium effects have been studied in several means such as self-consistent quasiparticle model [83], models based on the Gribov-Zwanziger quantization [84–86], Nambu-Jona-Lasinio (NJL) and Polyakov-loop-extended NJL based quasiparticle models [87], effective mass quasiparticle model with Polyakov loop [88] and effective fugacity quasiparticle model (EQPM) [78,79]. The present analysis is based on the EQPM in which the realistic EoS can be interpreted in terms of temperature dependent quasigluon and quasiquark/antiquark fugacities, z_g and z_q, respectively. We consider the recent $(2+1)$-flavor lattice QCD EoS (LEoS) [89] from the lattice QCD simulations. Within the framework of the EQPM, the thermal QCD medium constitutes of effective gluonic sector and matter sector (light quarks). We have studied the EQPM for the magnetized QGP in the Ref. [60]. The EQPM quark distribution function in the presence of the magnetic field ($\mathbf{B} = B\hat{z}$) has the following form,

$$f_q^l = \frac{z_q \exp(-\beta E_k^l)}{1 + z_q \exp(-\beta E_k^l)}. \tag{11}$$

Here, $E_k^l = \sqrt{k_z^2 + m^2 + 2l|q_f eB|}$ is the Landau energy eigenvalue in which $l = 0, 1, 2, \ldots$ is the order of the Landau levels. Note that the effective fugacity parameter for quark and antiquark is same, i.e., $z_q = z_{\bar{q}}$ and hence the momentum distribution of quarks and antiquarks is identical (in the case of vanishing chemical potential). The dispersion relation of electrically chargeless gluon remains intact in the magnetic field and the distribution function has the form,

$$f_g = \frac{z_g \exp(-\beta|\mathbf{k}|)}{1 - z_g \exp(-\beta|\mathbf{k}|)}. \tag{12}$$

We choose the units $k_B = 1$, $c = 1$, $\hbar = 1$ and $\beta = \frac{1}{T}$. The physical significance of the effective fugacity parameter can be interpreted from the nontrivial dispersion relation which encodes the quasiparton collective excitations and takes the forms,

$$\omega_q^l = \sqrt{k_z^2 + m^2 + 2l|q_f eB|} + T^2 \partial_T \ln(z_q), \tag{13}$$

and

$$\omega_g = |\mathbf{k}| + T^2 \partial_T \ln(z_g), \tag{14}$$

for quarks and gluons, respectively. The effective Boltzmann equation and mean field terms are well investigated within the framework of the EQPM both in the presence and absence of magnetic field [62,90].

It is important to note that the EQPM is motivated from the charge renormalization in the QCD medium as the effective mass models are based on the mass renormalization. Both the dispersion relation and effective coupling are sensitive to the magnetic field in the medium. The magnetic field dependence on the temperature behavior of the effective coupling can be estimated from of the Debye screening mass of the magnetized QGP. The Debye screening mass in the QGP can be defined in terms of the EQPM momentum distribution function as [60],

$$m_D^2 = -4\pi\alpha_s \int d\Upsilon \frac{d}{d\mathbf{k}} (2N_c f_g + 2N_f f_q^l), \tag{15}$$

where the integration phase factor $d\Upsilon = \frac{d^3\mathbf{k}}{(2\pi)^3}$ for gluons and $d\Upsilon = \frac{|q_f eB|}{2\pi} \sum_{l=0}^{\infty} \frac{dk_z}{2\pi} (2 - \delta_{0l})$ for quarks in the presence of the magnetic field. Here, $\alpha_s(T)$ is the QCD running coupling constant at finite temperature [91]. The Eq. (15) can be solved separately for gluonic and light quark sector and takes the following form,

$$m_D^2 = \frac{24\alpha_s T^2}{\pi} PolyLog[2, z_g] + \frac{4\alpha_s}{T} \sum_f \frac{|q_f eB|}{\pi} \int_0^\infty \sum_{l=0}^\infty dk_z (2 - \delta_{l0}) f_q^l (1 - f_q^l). \tag{16}$$

The term expressed in terms of $PolyLog$ function over the effective gluon fugacity parameter in the Eq. (16) represents the gluonic contribution to the screening mass. Whereas the second term constitutes the contributions of quarks incorporating the HLL effects. For the system of ultrarelativistic noninteracting quarks and gluons (ideal EoS), $z_{q,g} = 1$, the Debye mass takes form,

$$m_D^{2\ \mathrm{ideal}} = 4\pi\alpha_s(T) \left[T^2 + \sum_f \Lambda_f(|q_f eB|, T) \right], \tag{17}$$

in which the quark (with flavor f) contribution in the ideal case ($z_q = 1$) can be defined as,

$$\Lambda_f = \frac{1}{T} \frac{|q_f eB|}{\pi^2} \int_0^\infty \sum_{l=0}^\infty dk_z (2 - \delta_{l0}) n_q^l (1 - n_q^l), \tag{18}$$

where $n_q^l = \frac{1}{\exp(\beta E_k^l) + 1}$ is the Fermi-Dirac distribution function. Defining the effective running coupling constant $\alpha_{\mathrm{eff}}^l(T, z_q, z_g, |eB|)$ as, $m_D^2 = \frac{\alpha_{\mathrm{eff}}^l}{\alpha_s} m_D^{2\ \mathrm{ideal}}$ and employing Eqs. (16) and (17), we obtain,

$$\frac{\alpha_{\mathrm{eff}}^l}{\alpha_s} = \frac{6T^2 PolyLog[2, z_g]}{\pi^2 (T^2 + \sum_f \Lambda_f(|q_f eB|, T))} + \frac{\sum_f |q_f eB| \int_0^\infty \sum_{l=0}^\infty dk_z (2 - \delta_{l0}) f_q^l (1 - f_q^l)}{T\pi^2 (T^2 + \sum_f \Lambda_f(|q_f eB|, T))}. \tag{19}$$

Note that the hot QCD medium effects are entering through the quasiparton momentum distribution functions and the scattering amplitude. The effective coupling constant, which is a dynamical input of the HQ transport, is incorporated through the HQ scattering with thermal gluons and quarks. We use these concepts in the estimation of HQ drag and diffusion in the presence of the magnetic field in the next sections.

3. *Thermal gluon contribution to HQ drag and diffusion*

The gluon contribution to the HQ transport coefficients is incorporated through the HQ scattering with thermal gluons in the hot medium. The $2 \leftrightarrow 2$ HQ-gluon scattering is dominated by t-channel gluon exchange [36] and the matrix element $\mathcal{M}_{\mathrm{HQ},g}$ in the static limit has the following form [77],

$$\mathcal{M}^{bc}_{\mathrm{HQ},g} = -i8\pi\alpha_{\mathrm{eff}} M_{\mathrm{HQ}} f^{abc} t_R{}^a \frac{1}{(q^2+s(q_\perp))} \times (|\mathbf{k}|+|\mathbf{k}'|)\epsilon.\bar{\epsilon}^*. \tag{20}$$

Here, $q \equiv |\mathbf{q}|$ and α_{eff} is the effective coupling in medium. Here, (b,ϵ) and $(c,\bar{\epsilon})$ are the representation of color and polarization of the incoming and outgoing gluons. The quantity f^{abc} is the structure constant and t^a_R is the generator of the group $SU(N_c)$. The quantity $s(q_\perp)$ defines from the leading order real part of the retarded self-energy, $s(q_\perp) = \underset{\omega\to 0}{\mathrm{Re}}\,\Pi^{00}_{R\,\mathrm{Fermion}}(q)$, where ω is the energy transfer in the soft scattering process, in the regime $gT \ll \sqrt{|eB|}$ and takes the following form,

$$s(q_\perp) = 4\pi\alpha_{\mathrm{eff}} \sum_f \Lambda_f \exp\left(\frac{-q_\perp^2}{2|q_f eB|}\right), \tag{21}$$

where Λ_f is defined in the Eq. (18) such that $s(q_\perp = 0)$ gives the quark contribution to the Debye mass in the magnetized medium as $m_D^2 = \underset{\omega\to 0}{\mathrm{Re}}\,\Pi^{00}_{R\,\mathrm{Fermion}}(q\to 0)$. Here, we consider the static limit with dominant soft momentum transfer $\sim gT$ such that the Landau level of the quark does not changes with the interaction, i.e., $l = l'$. In the LLL approximation, the Eq. (21) reduced to the form $s(q_\perp) = \alpha_s \sum_f \frac{|q_f eB|}{\pi}\exp(\frac{-q_\perp^2}{2|q_f eB|})$ for the noninteracting case. Note that the authors of Ref. [61] have showed the transverse components of the gluon self-energy vanishes (negligible) up to one-loop order, i.e., transverse current vanishes, within the regime $gT \ll \sqrt{|eB|}$. This can be understood from the Landau quantization of transverse motion of the quarks in the regime of the current focus. We can compute $|\mathcal{M}_{\mathrm{HQ},g}|^2$ from Eq. (20) in the static limit ($|\mathbf{k}| = |\mathbf{k}'|$) by employing the polarization sum, $\sum_{\epsilon,\bar{\epsilon}^*}|\epsilon.\bar{\epsilon}^*|^2 = 1 + \cos^2\theta_{\mathbf{k}\mathbf{k}'}$, where $\theta_{\mathbf{k}\mathbf{k}'}$ is the angle between $\mathbf{k}$ and $\mathbf{k}'$ and the color summation, $\sum f^{abc} f^{a'bc} t^a_R t^{a'}_R = N_c C^{\mathrm{HQ}}_R I$, in which C^{HQ}_R is the color Casimir of the HQ. Incorporating all these arguments, we can define the gluon contribution to the longitudinal HQ drag component by substituting the Eq. (20) in the Eq. (6) as,

$$A^{\mathrm{gluon}}_{\parallel} = \frac{4}{\pi}\alpha^2_{\mathrm{eff}} N_c C^{\mathrm{HQ}}_R \frac{1}{d_{\mathrm{HQ}}}\int_0^\infty dq\, q^2 \frac{1}{(q^2+s(q_\perp))^2} \times \int_0^\infty d|\mathbf{k}||\mathbf{k}|^2(1+\cos^2\theta_{\mathbf{k}\mathbf{k}'})\delta(|\mathbf{k}|-|\mathbf{k}-\mathbf{q}|) \times f_g(1+f_g)q_z. \tag{22}$$

Following the prescriptions in [77], we can further simplify the Eq. (22) by considering the rotational symmetry such that $q_z^2 = \frac{q^2}{3}$ and employing $\delta(|\mathbf{k}|-|\mathbf{k}-\mathbf{q}|) = q^{-1}\delta(\cos\theta_{\mathbf{kq}} - \frac{q}{2|\mathbf{k}|})\Theta(|\mathbf{k}|-\frac{q}{2})$ and $\cos\theta_{\mathbf{k}\mathbf{k}'} = 1-\frac{q^2}{2|\mathbf{k}|^2}$. Finally, Eq. (22) becomes,

$$A^{\mathrm{gluon}}_{\parallel} = \frac{4}{\sqrt{3}\pi}\alpha^2_{\mathrm{eff}} N_c C^{\mathrm{HQ}}_R \frac{1}{d_{\mathrm{HQ}}}\int_0^\infty dq \frac{q^2}{(q^2+s(q_\perp))^2} \times \int_{q/2}^\infty d|\mathbf{k}||\mathbf{k}|^2\left[1+\left(1-\frac{q^2}{2|\mathbf{k}|^2}\right)^2\right] f_g(1+f_g). \tag{23}$$

Similarly, we can calculate the quark contribution to longitudinal HQ momentum diffusion by substituting Eq. (20) in Eq. (7) and has the following form,

$$K^{\mathrm{gluon}}_{\parallel} = \frac{4}{3\pi}\alpha^2_{\mathrm{eff}} N_c C^{\mathrm{HQ}}_R \frac{1}{d_{\mathrm{HQ}}}\int_0^\infty dq\, q^2 \frac{1}{(q^2+s(q_\perp))^2} \times \int_0^\infty d|\mathbf{k}||\mathbf{k}|^2(1+\cos^2\theta_{\mathbf{k}\mathbf{k}'})\delta(|\mathbf{k}|-|\mathbf{k}-\mathbf{q}|) \times f_g(1+f_g)q^2. \tag{24}$$

Performing the q integral and dropping out the terms subleading to m_D/T by assuming the contribution from hard thermal gluons with $|\mathbf{k}| \gtrsim T$ as in the described in the Refs. [36,92], the leading order $K^{\mathrm{gluon}}_{\parallel}$ in the presence of magnetic field takes the following form,

$$K^{\mathrm{gluon}}_{\parallel} = \frac{8\pi}{9}\alpha^2_{\mathrm{eff}} N_c C^{\mathrm{HQ}}_R \frac{1}{d_{\mathrm{HQ}}} T^3\left[\log\left(\frac{2T}{m_D}\right)+2\xi\right], \tag{25}$$

where $\xi = \frac{1}{2} - \gamma_E + \frac{\zeta'(2)}{\zeta(2)}$, in which γ_E and ζ are the Euler's constant and Riemann zeta function, respectively. The effects of hot QCD medium interactions and HLLs in the temperature behavior of the HQ transport coefficients are entering through momentum distribution function and effective coupling in the medium. For the noninteracting case ($z_{q,g} = 1$), for the LLL quarks we have, $\log(\frac{2T}{m_D}) \approx [\log(\frac{1}{\alpha_s}) - \log(\frac{\sum_f |q_f eB|}{T^2\pi})]$. The form of LLL quark contribution to the longitudinal momentum diffusion in the ideal EoS is consistent with the observations in the recent work [77]. The gluonic contribution is isotropic up to the leading order of m_D/T. The general form of the gluonic contribution to the transverse component of the momentum diffusion can be obtained from Eq. (9) and has the form,

$$K^{\mathrm{gluon}}_{\perp} = \frac{2}{\pi}\alpha^2_{\mathrm{eff}} N_c C^{\mathrm{HQ}}_R \frac{1}{d_{\mathrm{HQ}}}\int_0^\infty dq\, q^2 \frac{q_\perp^2}{(q^2+s(q_\perp))^2} \times \int_0^\infty d|\mathbf{k}||\mathbf{k}|^2(1+\cos^2\theta_{\mathbf{k}\mathbf{k}'})\delta(|\mathbf{k}|-|\mathbf{k}-\mathbf{q}|) \times f_g(1+f_g). \tag{26}$$

The quark contributions to the HQ dynamics are highly anisotropic in the magnetized medium and are discussed in the next section.

B. Leading order quark contributions to HQ drag and diffusion

To incorporate the quark contribution to the HQ dynamics, we consider the process $HQ(p) + l(k) \rightarrow HQ(p') + l(k')$, where l represents the light quarks in the thermal medium. The quark dynamics is significantly affected by the strong magnetic field and follow the $1+1$-dimensional Landau level dynamics. The HQ motion is subjected to the scattering with the quarks both in the direction parallel and perpendicular to the magnetic field.

1. HQ longitudinal and transverse drag

The longitudinal and transverse components of HQ drag for the HQ-quark scattering process can be described in the static limit by following the same prescription as that of the gluonic case as in Eq. (6) and takes the forms as,

$$A_{\parallel}^{\text{quark}} = \int d^3\mathbf{q} \frac{d\Gamma(\mathbf{q})}{d^3\mathbf{q}} q_z, \tag{27}$$

and

$$A_{\perp}^{\text{quark}} = \frac{1}{\sqrt{2}} \int d^3\mathbf{q} \frac{d\Gamma(\mathbf{q})}{d^3\mathbf{q}} q_{\perp}. \tag{28}$$

The quantity $\frac{d\Gamma(\mathbf{q})}{d^3\mathbf{q}}$ defines the HQ scattering rate with quarks per unit volume of momentum transfer via one gluon exchange and has the following form,

$$\frac{d\Gamma}{d^3\mathbf{q}} = \frac{1}{d_{\text{HQ}}} \frac{1}{2M_{\text{HQ}}} \frac{1}{(2\pi)^3 2E_q} \int \frac{d\Upsilon}{E_k^l} \int \frac{d\Upsilon'}{E_{k'}^l} |\mathcal{M}_{\text{HQ},q}|^2 \\ \times (2\pi)^2 \delta^2(p + k - p' - k') f_q^l(k_z)(1 - f_q^l(k_z')), \tag{29}$$

with $f_q^l(k_z)$ and E_k^l are the EQPM distribution function and Landau level energy eigenvalue of the quark, respectively. Here, $|\mathcal{M}_{\text{HQ},q}|$ is the matrix element of the HQ scattering with the thermal quarks. In the recent work [77], the authors have estimated the HQ-quark scattering rate from the retarded gluon correlator by employing the real time Schwinger-Keldysh formalism and has the following form,

$$\frac{d\Gamma}{d^3\mathbf{q}} = \frac{\alpha_{\text{eff}} T}{\pi} N_c C_R^{\text{HQ}} \frac{s(q_\perp)}{(q^2 + s(q_\perp))^2} \delta(q_z), \tag{30}$$

in which α_{eff} is the effective coupling as described in the Eq. (19). The $1+1$-dimensional fermionic dynamics is unaffected (i.e., transverse current is negligible) by the HLLs within the regime $gT \ll \sqrt{|eB|}$, as clearly seen in the leading order results of self-energy (up to one-loop order), energy-momentum tensor and current density as described in the Refs. [61,62]. The delta function can be understood from the $1+1$-dimensional Landau level kinematic of the quarks. The Boltzmann equation reduces to the Fokker-Planck dynamics within the Landau approximation, which can be physically motivated as most of the scattering processes are soft with small momentum transfer. Since, we are focusing on the limit of soft momentum transfer, the interactions/scattering process does not change the Landau levels of the quarks in the static limit as $\Delta\epsilon \gg \omega$, where $\Delta\epsilon \sim \sqrt{q_f eB}$ is the energy gap associated with adjacent Landau levels and ω is the energy transfer in the scattering process. Hence, the energy-momentum transfer of the quark $(\Delta E, \Delta k_z) = (\omega, q_z)$ satisfy the $\omega \simeq \pm q_z$ as in the case of LLL in Ref. [77]. The static limit $\omega \rightarrow 0$ imposes the vanishing longitudinal momentum transfer, denoted as $\delta(q_z)$. The two-loop and higher order corrections beyond the static limit requires a more general collision term that takes account change of Landau levels due to collisions, gives the nonvanishing the quark contribution to the longitudinal component of the drag and diffusion, which is beyond the scope of the current analysis.

The contribution from the quarks with LLL and the HLLs next to $l = 0$ state constitute the leading order quark contribution to the HQ drag and diffusion in the strong magnetic field background. We conclude from Eqs. (27) and (30) that in the strong magnetic field, the quark contribution to the longitudinal drag component goes to zero. By substituting Eq. (30) in Eq. (28), the leading order transverse HQ drag takes the following form,

$$A_{\perp}^{\text{quark}} = 2T\alpha_{\text{eff}}^2 N_c C_R^{\text{HQ}} \frac{\sqrt{|eB|}}{\pi} \\ \times \int_0^\infty dx \frac{\sqrt{x} N(T,x)}{(x + 2\frac{\alpha_{\text{eff}}}{\pi} N(T,x))^2}, \tag{31}$$

where $x = \frac{q_\perp^2}{2|eB|}$ and $N(T,x)$ is defined as,

$$N = \frac{1}{T} \sum_f |q_f| e^{-\frac{x}{|q_f|}} \int_0^\infty \sum_{l=0}^\infty dk_z (2 - \delta_{l0}) n_q^l (1 - n_q^l). \tag{32}$$

The thermal quark contribution to the HQ drag is in the transverse direction and have a dependence on the HLLs and nonideal EoS.

2. HQ diffusion

Similar to the HQ momentum described in Eq. (7) due to the HQ-gluon scattering, the quark contribution to the longitudinal and diffusion coefficients take the forms as follows,

$$K_{\parallel}^{\text{quark}} = \int d^3\mathbf{q} \frac{d\Gamma(\mathbf{q})}{d^3\mathbf{q}} q_z^2, \tag{33}$$

and

$$K_{\perp}^{\text{quark}} = \frac{1}{2}\int d^3\mathbf{q}\frac{d\Gamma(\mathbf{q})}{d^3\mathbf{q}}q_{\perp}^2. \quad (34)$$

By substituting the definition of HQ-quark scattering rate in Eq. (34), we obtain the leading order quark contribution to the transverse momentum diffusion as follows,

$$K_{\perp}^{\text{quark}} = 4T\alpha_{\text{eff}}^2 N_c C_R^{\text{HQ}}\frac{|eB|}{2\pi} \times \int_0^{\infty} dx \frac{xN(T,x)}{(x+2\frac{\alpha_{\text{eff}}}{\pi}N(T,x))^2}, \quad (35)$$

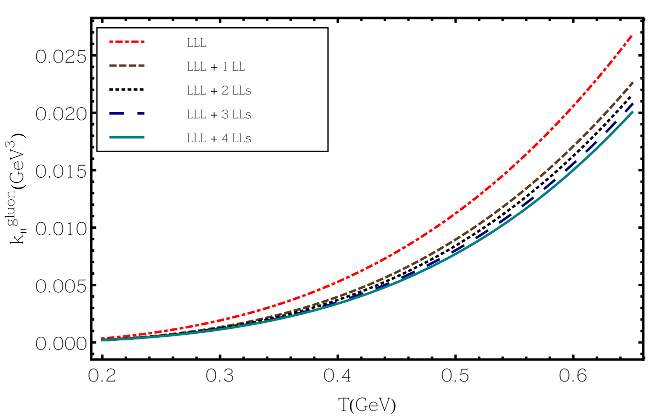
where $N(T,x)$ is defined in the Eq. (32). The longitudinal component of HQ diffusion due to scattering with quarks vanishes in the strong magnetic field and can be understood from the Eqs. (30) and (33). Similar to the drag force, the quark contribution to the HQ momentum diffusion in the magnetic field is anisotropic in nature. In the ideal EoS ($z_{q,g}=1$), our results for LLL can be reduced to that in the Ref. [77].

III. RESULTS AND DISCUSSION

We initiate the discussion with the effects of HLLs on the temperature dependence of the HQ momentum diffusion and drag in the magnetized QGP. Since the thermal quarks and gluons follow different dynamics in the magnetic field, we have considered HQ scattering with quarks and gluons separately, along the direction parallel and perpendicular to the magnetic field. The gluonic contribution to the HQ drag force and momentum diffusion is incorporated to the HQ-thermal gluon scattering process. The temperature behavior of the $K_{\parallel}^{\text{gluon}}$ and $A_{\parallel}^{\text{gluon}}$ at $|eB| = 15m_{\pi}^2$ is depicted in the Fig. 1. The HLLs effects to the gluonic contributions can enter through the Debye screening mass as described in the Eqs. (23) and (25). For the numerical estimation of the effect of HLLs, we plotted with different Landau levels. We truncate the Landau levels at $l=4$ and the HLL contributions beyond $l=4$ seems to be negligible in the chosen range of temperature. We observe that the HLL contribution quantitatively reduces the longitudinal HQ momentum diffusion and drag, and the effect is more pronounced in the high temperature regime. The gluonic contribution to the HQ drag and diffusion is isotropic up to the leading order of m_D/T.

We have incorporated the $(1+1)$-dimensional Landau level kinematics for quarks while estimating the quark contribution to the HQ transport from the HQ-thermal quark scattering in the presence of the magnetic field. The effect of HLLs on the temperature behavior of the $K_{\perp}^{\text{quark}}$ and $A_{\perp}^{\text{quark}}$ are shown in the Fig. 2. We observe that HLL corrections enhance the quark contribution to the transverse components of the drag force and diffusion coefficient. The HLLs effect is quite significant in the higher temperature regime. The quark contribution in the longitudinal direction vanishes in the leading order and can be understood from Eqs. (27), (30) and (33). Our observation on the anisotropic nature of the HQ drag and diffusion is qualitatively consistent with the results in the Ref. [77].

The electrically neutral gluons are indirectly affected by the magnetic field through the quark loop while defining the self-energy where the quark/antiquark loop contributes. The Debye screening mass can be defined from the real part of the gluon self-energy. The dominant contribution to screening mass of the thermal gluons (and effective coupling) in the strong magnetic field is from the LLL quarks and HLLs gives the further correction to the mass. In the strong field limit, $T^2 \ll eB$, the contributions from HLLs are negligible (proportional to the Boltzmann factor, $\exp(-\sqrt{eB}/T)$). But in the more realistic regime $gT \ll \sqrt{|eB|}$, the HLL contribution enhances the screening mass and this affects the scattering rate and momentum diffusion, Eqs. (20), (24) respectively. For the quark case, in addition to the screening mass, the magnetic field enters through the momentum distribution via Landau level dispersion relation. The Boltzmann factor is non-negligible



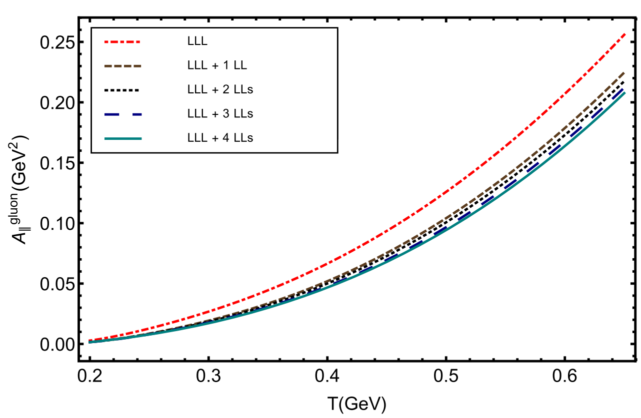


FIG. 1. The HLLs effect in the temperature behavior of the $K_{\parallel}^{\text{gluon}}$ (left panel) and $A_{\parallel}^{\text{gluon}}$ (right panel)in the static limit at $|eB| = 15m_{\pi}^2$.

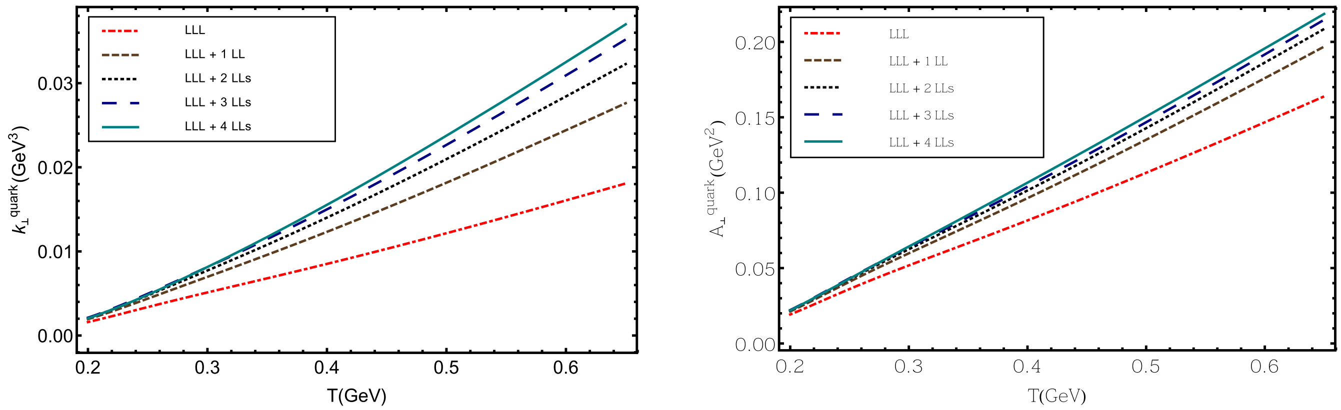


FIG. 2. The HLLs effect in the temperature dependence of the $K_\perp^{\mathrm{quark}}$ (left panel) and $A_\perp^{\mathrm{quark}}$ (right panel) at $|eB| = 15m_\pi^2$.

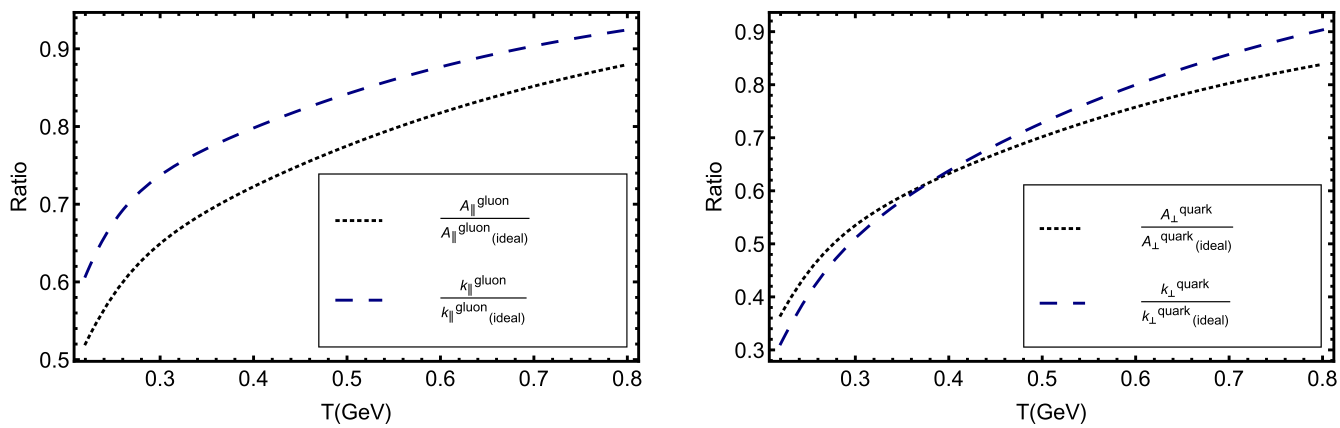


FIG. 3. The EoS dependence on the longitudinal (left panel) and transverse (right panel) HQ momentum diffusion and drag at $|eB| = 15m_\pi^2$.

in the regime and hence we incorporated the collision process of HLL quark (in addition to LLL quark) with the HQs in the magnetized medium. The dependence of the quark distribution on the momentum diffusion is shown in Eq. (35).

The thermal medium dependence of the HQ drag force and momentum diffusion is governed by the quasiparticle momentum distribution function and effective coupling. We plotted the EoS dependence of the drag force and diffusion coefficient for both gluonic and quark contributions at $|eB| = 15m_\pi^2$ in the Fig. 3. The EoS dependence in HQ transport is more visible in the temperature regime near to the transition temperature, $T_c = 0.17$ GeV. Asymptotically, the ratio tends to unity, which implies that the quasipartons will behave like free particles at very high temperature regime.

In Fig. 4, we depicted the effects of HLLs and EoS in the anisotropy in the HQ momentum diffusion by estimating the temperature behavior of the ratio $\frac{K_\parallel}{K_\perp}$. The quantities $K_\parallel$ and $K_\perp$ represent the total contribution from the quark and gluonic sector to the longitudinal and transverse diffusion coefficient in the presence of the magnetic field. We observe that the HLLs enhance the anisotropy in the HQ momentum diffusion, especially in the higher temperature region. For the LLL case, we have $K_\perp \gg K_\parallel$ in the temperature regime near to T_c. This observation for the LLL case is in line with the results of the recent work [77].

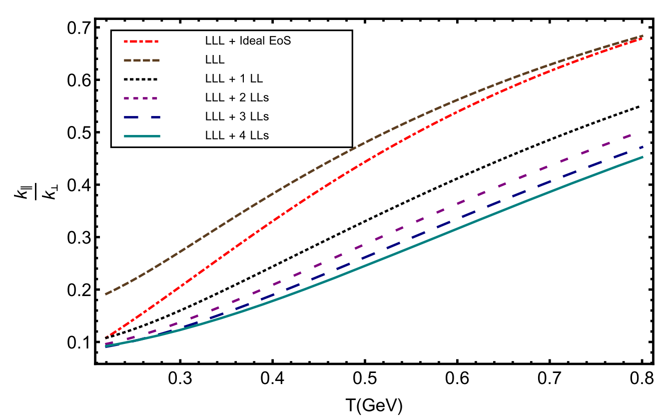


FIG. 4. The effects of HLLs and EoS in the magnetic field induced anisotropy in the HQ momentum diffusion.

The anisotropy in the HQ drag coefficients can be understood from the Eq. (10) in the hot QCD medium.

IV. CONCLUSION AND OUTLOOK

In conclusion, we have computed the temperature dependence of the longitudinal and transverse components of the HQ drag force and momentum diffusion in the presence of magnetic field by considering the thermal gluonic and quark contributions, separately. We have employed the Fokker-Planck dynamics to describe HQ transport in the hot magnetized medium. The HQ drag and diffusion are influenced by the magnetic field and hot QCD medium. We observed that the inclusion of HLLs is essential for the description of drag force and momentum diffusion of the HQs at the higher temperature regime far from the transition temperature. Notably, the HLLs quantitatively suppresses the gluonic contribution to the HQ drag and diffusion, whereas the quark contribution to the transverse components gets enhanced at the high temperature regime. The magnetic fields effects are embedded through the Landau levels in the quasiquark momentum distribution functions and in the respective energy dispersion relations. On the other hand, the gluon kinematics is coupled with the magnetic field through the effective coupling. Thermal medium effects are incorporated through the quasiparticle description of the magnetized medium. Furthermore, we have studied the anisotropy in the HQ momentum diffusion that induced from the $(1+1)$-dimensional Landau dynamics of the thermal quarks in the presence of the magnetic field. Finally, both the EoS and HLLs are seen to have a significant impact on the HQ momentum diffusion anisotropy in the magnetized medium.

HQ directed flow [54] is considered as a sensitive probe to the creation and characterization of the magnetic field in the heavy-ion collisions. The anisotropic momentum diffusion and drag coefficients of the HQs due to the magnetic field can affect the HQ directed flow measured recently both at RHIC and LHC energies [55–57]. HQ elliptic flow is another observable which can be affected by this anisotropic HQ transport coefficients. The estimation of the HQ transport coefficients beyond the static limit for an arbitrary magnetic field is another important task. We intend to work on these interesting aspects of the HQ dynamics in the near future.

ACKNOWLEDGMENTS

We are highly thankful to Charles Gale for useful comments and suggestions. M. K. would like to acknowledge the hospitality of McGill University and the IIT Gandhinagar Overseas Research Experience Fellowship to visit McGill University, where a part of this work is completed. V. C. would like to acknowledge SERB for the Early Career Research Award (ECRA/2016), and DST, Govt. of India for INSPIRE-Faculty Fellowship (IFA-13/PH-55). S. K. D. acknowledges the support by the National Science Foundation of China (Grants No. 11805087 and No. 11875153). We are indebted to the people of India for their generous support for the research in basic sciences.

APPENDIX: HEAVY QUARK-THERMAL QUARK SCATTERING

The gluon self-energy in the real-time Keldysh formalism within the regime $gT \ll \sqrt{|eB|}$ can be defined as,

$$\Pi^{\mu\nu}_{R\,\mathrm{Fermion}}(Q) = i4\pi\alpha_s T_R \langle J^\mu_r(Q) J^\nu_a(-Q)\rangle, \quad \text{(A1)}$$

where $Q=(\omega, \mathbf{q})$ and current operator $J^\mu = (J^0, j^i)$ can be relate to the electromagnetic current density j_{em} and energy-momentum tensor $T^{\mu\nu}$ of the magnetized medium, where the dominant contribution is from the longitudinal components of the macroscopic quantities [61,62]. The authors of the Refs. [61,62] have showed that $1+1$-dimensional fermionic dynamics is unaffected by the HLLs, i.e., transverse current vanishes, within the regime $gT \ll \sqrt{|eB|}$. Hence, we can decouple the transverse dynamics from the longitudinal dynamics as in the case of strong field limit and has the following form,

$$\Pi^{\mu\nu}_{R\,\mathrm{Fermion}}(Q) = \pi s(q_\perp)\Pi^{\mu\nu}_{R1+1}(\omega, q_z), \quad \text{(A2)}$$

where $s(q_\perp)$ is defined in Eq. (21) for the static limit with soft momentum exchange $\sim gT$, in the current regime of focus. We consider, $l=l'$, for the soft interactions. The general calculation of gluon self-energy and the screening mass with all Landau levels is described in the Ref. [93]. The retarded self-energy in the $1+1$-dimension $\Pi^{\mu\nu}_{R\,1+1}(\omega, q_z)$ has the following form,

$$\Pi^{\mu\nu}_{R\,1+1}(\omega, q_z) = i\langle J^\mu_r(q_\parallel) J^\nu_a(-q_\parallel)\rangle. \quad \text{(A3)}$$

Here, $\langle J^\mu_r(q_\parallel) J^\nu_a(-q_\parallel)\rangle$ is the $1+1$-dimensional retarded current-current correlator. Note that the degeneracy factor $\frac{|q_f eB|}{2\pi}(2-\delta_{l0})$ for the Landau levels is incorporated in the quantity $s(q_\perp)$ along with the effects of HLLs. Following the same technique of bosonization as in the case of strong field limit as described in [77], we obtain $\langle J^\mu_r(q_\parallel) J^\nu_a(-q_\parallel)\rangle = \frac{1}{i\pi[(\omega+i\epsilon)^2-q_z^2]}(q^2_\parallel g^{\mu\nu}_\parallel - q^\mu_\parallel g^\nu_\parallel)$, where ϵ is a small parameter, $q^\mu_\parallel = (\omega, 0, 0, q_z)$ and $g^{\mu\nu}_\parallel = \mathrm{diag}(1, 0, 0, -1)$.

In the case of HLLs, the general form of energy transfer ω due to quark scattering (massless limit) defines as, $\omega = \sqrt{k'^2_z + 2l'|q_f eB|} - \sqrt{k^2_z + 2l|q_f eB|}$. Here, we consider most of the scattering processes are soft with small momentum transfer (Landau approximation). In the limit of soft momentum transfer $\sim gT$, the scattering processes do not change the Landau levels of the quarks in the static limit within the regime $gT \ll \sqrt{|eB|}$. Hence, the energy-momentum transfer of the quark $(\Delta E, \Delta k_z) = (\omega, q_z)$ satisfy the $\omega \simeq \pm q_z$ by neglecting the term $\mathcal{O}(1/eB^2)$ as in the case of LLL in Ref. [77]. Incorporating these arguments we have,

$$\mathrm{Re}\Pi^{00}_{R\,\mathrm{Fermion}}(\omega, \mathbf{q}) = -\frac{s(q_\perp)q_z^2}{q_\parallel^2}, \tag{A4}$$

$$\mathrm{Im}\Pi^{00}_{R\,\mathrm{Fermion}}(\omega, \mathbf{q}) = \frac{\pi s(q_\perp)\omega}{2}(\delta(\omega - q_z) + \delta(\omega + q_z)). \tag{A5}$$

It is important to note that the two-loop and higher order corrections and the weak or moderate magnetic field beyond the regime $gT \ll \sqrt{|eB|}$ gives nonvanishing transverse currents in the system and is beyond the scope of the current analysis.

Now, we can define the scattering rate from the gluon retarded propagator as,

$$\frac{d\Gamma}{d^3\mathbf{q}} = -\frac{8\pi\alpha_{\mathrm{eff}}C_R^{\mathrm{HQ}}}{(2\pi)^3}\lim_{\omega\to 0}\frac{T}{\omega}\mathrm{Im}[G_R^{00}(\omega, \mathbf{q})], \tag{A6}$$

where,

$$-\lim_{\omega\to 0}\frac{\mathrm{Im}G_R^{00}}{\omega} = \lim_{\omega\to 0}\frac{1}{\omega} \times \left[\frac{\mathrm{Im}\Pi^{00}_{R\,\mathrm{Fermion}}}{(Q^2 - \mathrm{Re}\Pi^{00}_{R\,\mathrm{Fermion}})^2 + (\mathrm{Im}\Pi^{00}_{R\,\mathrm{Fermion}})^2}\right]. \tag{A7}$$

Substituting Eqs. (A4), (A5) and (A7) to Eq. (A6), we obtain

$$\frac{d\Gamma}{d^3\mathbf{q}} = \frac{\alpha_{\mathrm{eff}}T}{\pi}N_c C_R^{\mathrm{HQ}}\frac{s(q_\perp)}{(q^2 + s(q_\perp))^2}\delta(q_z). \tag{A8}$$

It is clear that the limit $\omega \to 0$ imposes the vanishing longitudinal momentum transfer, denoted as $\delta(q_z)$, from the Eq. (A5) as $\lim_{\omega\to 0}\frac{\mathrm{Im}\Pi^{00}_{R\,\mathrm{Fermion}}}{\omega} = \pi s(q_\perp)\delta(q_z)$. One needs to include more general collision terms beyond the static limit to incorporate the change in Landau levels of quarks due to the interactions.

[1] Adams *et al.* (STAR Collaboration), Nucl. Phys. **A757**, 102 (2005); K. Adcox *et al.* (PHENIX Collaboration), Nucl. Phys. **A757**, 184 (2005); B. B. Back *et al.* (PHOBOS Collaboration), Nucl. Phys. **A757**, 28 (2005); A. Arsence *et al.* (BRAHMS Collaboration), Nucl. Phys. **A757**, 1 (2005).
[2] K. Aamodt *et al.* (ALICE Collaboration), Phys. Rev. Lett. **105**, 252301 (2010).
[3] U. W. Heinz, J. Phys. A **42**, 214003 (2009).
[4] V. Skokov, A. Y. Illarionov, and V. Toneev, Int. J. Mod. Phys. A **24**, 5925 (2009).
[5] Y. Zhong, C. B. Yang, X. Cai, and S. Q. Feng, Adv. High Energy Phys. **2014**, 193039 (2014).
[6] V. Roy, S. Pu, L. Rezzolla, and D. H. Rischke, Phys. Rev. C **96**, 054909 (2017).
[7] W.-T. Deng and Xu-Guang Huang, Phys. Rev. C **85**, 044907 (2012).
[8] B. Karmakar, R. Ghosh, A. Bandyopadhyay, N. Haque, and M. G. Mustafa, Phys. Rev. D **99**, 094002 (2019).
[9] A. Bandyopadhyay, B. Karmakar, N. Haque, and M. G. Mustafa, Phys. Rev. D **100**, 034031 (2019).
[10] S. Koothottil and V. M. Bannur, Phys. Rev. C **99**, 035210 (2019).
[11] K. Hattori, X. G. Huang, D. H. Rischke, and D. Satow, Phys. Rev. D **96**, 094009 (2017).
[12] K. Hattori and D. Satow, Phys. Rev. D **94**, 114032 (2016).
[13] M. Kurian and V. Chandra, Phys. Rev. D **97**, 116008 (2018).
[14] S. Rath and B. K. Patra, Phys. Rev. D **100**, 016009 (2019).
[15] P. Mohanty, A. Dash, and V. Roy, Eur. Phys. J. A **55**, 35 (2019).
[16] A. Das, H. Mishra, and R. K. Mohapatra, Phys. Rev. D **99**, 094031 (2019).
[17] S. Ghosh, B. Chatterjee, P. Mohanty, A. Mukharjee, and H. Mishra, Phys. Rev. D **100**, 034024 (2019).
[18] K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys. Rev. D **78**, 074033 (2008), D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, Nucl. Phys. **A803**, 227 (2008); D. E. Kharzeev, Ann. Phys. (Amsterdam) **325**, 205 (2010); D. E. Kharzeev and D. T. Son, Phys. Rev. Lett. **106**, 062301 (2011).
[19] A. V. Sadofyev and M. V. Isachenkov, Phys. Lett. B **697**, 404 (2011); A. V. Sadofyev, V. I. Shevchenko, and V. I. Zakharov, Phys. Rev. D **83**, 105025 (2011).
[20] D. She, S. Q. Feng, Y. Zhong, and Z. B. Yin, Eur. Phys. J. A **54**, 48 (2018).
[21] D. E. Kharzeev, J. Liao, S. A. Voloshin, and G. Wang, Prog. Part. Nucl. Phys. **88**, 1 (2016).
[22] A. Avkhadiev and A. V. Sadofyev, Phys. Rev. D **96**, 045015 (2017).
[23] N. Yamamoto, Phys. Rev. D **96**, 051902 (2017).
[24] X. G. Huang, Y. Yin, and J. Liao, Nucl. Phys. **A956**, 661 (2016).
[25] V. P. Gusynin, V. A. Miransky, and I. A. Shovkovy, Nucl. Phys. **B462**, 249 (1996).
[26] L. Adamczyk *et al.* (STAR Collaboration), Nature (London) **548**, 62 (2017).

[27] F. Becattini, I. Karpenko, M. Lisa, I. Upsal, and S. Voloshin, Phys. Rev. C **95**, 054902 (2017).
[28] X. Dong and V. Greco, Prog. Part. Nucl. Phys. **104**, 97 (2019).
[29] F. Prino and R. Rapp, J. Phys. G **43**, 093002 (2016).
[30] R. Rapp *et al.*, Nucl. Phys. **A979**, 21 (2018).
[31] S. Cao *et al.*, Phys. Rev. C **99**, 054907 (2019).
[32] G. Aarts *et al.*, Eur. Phys. J. A **53**, 93 (2017).
[33] A. Andronic *et al.*, Eur. Phys. J. C **76**, 107 (2016).
[34] B. Svetitsky, Phys. Rev. D **37**, 2484 (1988).
[35] M. Golam Mustafa, D. Pal, and D. Kumar Srivastava, Phys. Rev. C **57**, 889 (1998); **57**, 3499 (1998).
[36] G. D. Moore and D. Teaney, Phys. Rev. C **71**, 064904 (2005).
[37] H. van Hees, V. Greco, and R. Rapp, Phys. Rev. C **73**, 034913 (2006).
[38] H. van Hees, M. Mannarelli, V. Greco, and R. Rapp, Phys. Rev. Lett. **100**, 192301 (2008).
[39] P. B. Gossiaux and J. Aichelin, Phys. Rev. C **78**, 014904 (2008).
[40] S. K. Das, J. e. Alam, and P. Mohanty, Phys. Rev. C **80**, 054916 (2009).
[41] W. M. Alberico, A. Beraudo, A. De Pace, A. Molinari, M. Monteno, M. Nardi, F. Prino, and M. Sitta, Eur. Phys. J. C **73**, 2481 (2013).
[42] J. Uphoff, O. Fochler, Z. Xu, and C. Greiner, Phys. Lett. B **717**, 430 (2012).
[43] C. Young, B. Schenke, S. Jeon, and C. Gale, Phys. Rev. C **86**, 034905 (2012).
[44] S. Cao, G. Y. Qin, and S. A. Bass, Phys. Rev. C **88**, 044907 (2013).
[45] S. K. Das, F. Scardina, S. Plumari, and V. Greco, Phys. Rev. C **90**, 044901 (2014).
[46] S. K. Das, F. Scardina, S. Plumari, and V. Greco, Phys. Lett. B **747**, 260 (2015).
[47] T. Song, H. Berrehrah, D. Cabrera, J. M. Torres-Rincon, L. Tolos, W. Cassing, and E. Bratkovskaya, Phys. Rev. C **92**, 014910 (2015).
[48] S. Cao, T. Luo, G. Y. Qin, and X. N. Wang, Phys. Rev. C **94**, 014909 (2016).
[49] F. Scardina, S. K. Das, V. Minissale, S. Plumari, and V. Greco, Phys. Rev. C **96**, 044905 (2017); S. Plumari, V. Minissale, S. K. Das, G. Coci, and V. Greco, Eur. Phys. J. C **78**, 348 (2018).
[50] B. Singh, A. Abhishek, S. K. Das, and H. Mishra, arXiv: 1812.05263.
[51] A. Adare *et al.* (PHENIX Collaboration), Phys. Rev. Lett. **98**, 172301 (2007).
[52] S. S. Adler *et al.* (PHENIX Collaboration), Phys. Rev. Lett. **96**, 032301 (2006).
[53] S. Acharya *et al.* (ALICE Collaboration), Phys. Lett. B **793**, 212 (2019).
[54] S. K. Das, S. Plumari, S. Chatterjee, J. Alam, F. Scardina, and V. Greco, Phys. Lett. B **768**, 260 (2017).
[55] F. Grosa (ALICE Collaboration), Proc. Sci. HardProbes2018 (**2018**) 138.
[56] S. Singha (STAR Collaboration), Nucl. Phys. **A982**, 671 (2019).
[57] J. Adam *et al.* (STAR Collaboration), arXiv:1905.02052.
[58] S. Chatterjee and P. Bozek, arXiv:1804.04893.
[59] G. Coci, L. Oliva, S. Plumari, S. K. Das, and V. Greco, Nucl. Phys. **A982**, 189 (2019).
[60] M. Kurian and V. Chandra, Phys. Rev. D **96**, 114026 (2017).
[61] K. Fukushima and Y. Hidaka, Phys. Rev. Lett. **120**, 162301 (2018).
[62] M. Kurian, S. Mitra, S. Ghosh, and V. Chandra, Eur. Phys. J. C **79**, 134 (2019).
[63] M. Kurian and V. Chandra, Phys. Rev. D **99**, 116018 (2019).
[64] C. S. Machado, F. S. Navarra, E. G. de Oliveira, J. Noronha, and M. Strickland, Phys. Rev. D **88**, 034009 (2013).
[65] C. Bonati, M. D'Elia, and A. Rucci, Phys. Rev. D **92**, 054014 (2015).
[66] M. Hasan, B. Chatterjee, and B. K. Patra, Eur. Phys. J. C **77**, 767 (2017).
[67] B. Singh, L. Thakur, and H. Mishra, Phys. Rev. D **97**, 096011 (2018).
[68] X. Guo, S. Shi, N. Xu, Z. Xu, and P. Zhuang, Phys. Lett. B **751**, 215 (2015).
[69] P. Gubler, K. Hattori, S. H. Lee, M. Oka, S. Ozaki, and K. Suzuki, Phys. Rev. D **93**, 054026 (2016).
[70] S. Li, K. A. Mamo, and H. U. Yee, Phys. Rev. D **94**, 085016 (2016).
[71] D. Giataganas and H. Soltanpanahi, J. High Energy Phys. 06 (2014) 047.
[72] D. Giataganas and H. Soltanpanahi, Phys. Rev. D **89**, 026011 (2014).
[73] S. I. Finazzo, R. Critelli, R. Rougemont, and J. Noronha, Phys. Rev. D **94**, 054020 (2016).
[74] K. Rajagopal and A. V. Sadofyev, J. High Energy Phys. 10 (2015) 018.
[75] E. Kiritsis and G. Pavlopoulos, J. High Energy Phys. 04 (2012) 096.
[76] K. A. Mamo, Phys. Rev. D **94**, 041901 (2016).
[77] K. Fukushima, K. Hattori, H. U. Yee, and Y. Yin, Phys. Rev. D **93**, 074028 (2016).
[78] V. Chandra and V. Ravishankar, Phys. Rev. D **84**, 074013 (2011).
[79] V. Chandra, R. Kumar, and V. Ravishankar, Phys. Rev. C **76**, 054909 (2007).
[80] S. K. Das, V. Chandra, and J. e. Alam, J. Phys. G **41**, 015102 (2014).
[81] V. Chandra and S. K. Das, Phys. Rev. D **93**, 094036 (2016).
[82] K. Hattori, Nucl. Phys. **A982**, 551 (2019).
[83] V. M. Bannur, Phys. Rev. C **75**, 044905 (2007); Phys. Lett. B **647**, 271 (2007); J. High Energy Phys. 09 (2007) 046.
[84] N. Su and K. Tywoniuk, Phys. Rev. Lett. **114**, 161601 (2015).
[85] W. Florkowski, R. Ryblewski, N. Su, and K. Tywoniuk, Phys. Rev. C **94**, 044904 (2016); Acta Phys. Pol. B **47**, 1833 (2016).
[86] A. Bandyopadhyay, N. Haque, M. G. Mustafa, and M. Strickland, Phys. Rev. D **93**, 065004 (2016).
[87] A. Dumitru and R. D. Pisarski, Phys. Lett. B **525**, 95 (2002); K. Fukushima, Phys. Lett. B **591**, 277 (2004); S. K. Ghosh, T. K. Mukherjee, M. G. Mustafa, and R. Ray, Phys. Rev. D **73**, 114007 (2006); H. Abuki and K. Fukushima, Phys. Lett. B **676**, 57 (2009); H. M. Tsai and B. Muller, J. Phys. G **36**, 075101 (2009).
[88] M. D'Elia, A. Di Giacomo, and E. Meggiolaro, Phys. Lett. B **408**, 315 (1997); Phys. Rev. D **67**, 114504 (2003);

P. Castorina and M. Mannarelli, Phys. Rev. C **75**, 054901 (2007); Phys. Lett. B **644**, 336 (2007).
[89] M. Cheng *et al.*, Phys. Rev. D **77**, 014511 (2008).
[90] S. Mitra and V. Chandra, Phys. Rev. D **97**, 034032 (2018).
[91] M. Laine and Y. Schroder, J. High Energy Phys. 03 (2005) 067.
[92] S. Caron-Huot and G. D. Moore, Phys. Rev. Lett. **100**, 052301 (2008).
[93] S. Ghosh and V. Chandra, Phys. Rev. D **98**, 076006 (2018).