

Radiative corrections to neutron and nuclear beta decays revisited

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The universal radiative corrections common to neutron and superallowed nuclear beta decays (also known as “inner” corrections) are revisited in light of a recent dispersion relation study that found $+2.467(22)\%$, i.e., about 2.4σ larger than the previous evaluation. For comparison, we consider several alternative computational methods. All employ an updated perturbative QCD four-loop Bjorken sum rule defined QCD coupling supplemented with a nucleon form factor based Born amplitude to estimate axial-vector induced hadronic contributions. In addition, we now include hadronic contributions from low Q^2 loop effects based on duality considerations and vector meson resonance interpolators. Our primary result, $2.426(32)\%$, corresponds to an average of a light-front holographic QCD approach and a three-resonance interpolator fit. It reduces the dispersion relation discrepancy to approximately 1.1σ and thereby provides a consistency check. Consequences of our new radiative correction estimate, along with that of the dispersion relation result, for Cabibbo-Kobayashi-Maskawa unitarity are discussed. The neutron lifetime- g_A connection is updated and shown to suggest a shorter neutron lifetime less than 879 s. We also find an improved bound on exotic, non-Standard Model, neutron decays or oscillations of the type conjectured as solutions to the neutron lifetime problem, $\text{BR}(n \rightarrow \text{exotics}) < 0.16\%$.

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I. INTRODUCTION

Precision tests of the Standard Model (SM) require accurate calculations of electroweak radiative corrections (RCs) [1–7]. For example, unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix leads to orthonormal relationships among row and column matrix elements and provides a means to search for indications of “new physics” via departures from SM expectations. However, for those searches to be credible, strong interaction effects must be reliably evaluated.

Consider the precise CKM first-row unitarity condition

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1. \quad (1)$$

Employing the particle data group 2018 average based on superallowed $0^+ \rightarrow 0^+$ nuclear beta decays [8,9],

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$$|V_{ud}| = 0.97420(10)_{NP}(18)_{RC}, \quad (2)$$

as extracted by Hardy and Towner [9], using a universal electroweak radiative correction [10] (also known as the “inner” correction),

$$\Delta_R^V = 0.02361(38), \quad (3)$$

along with the $K_{\mu 2}/\pi_{\mu 2}$ and K_{l3} weighted averages [8] of $|V_{us}| = 0.2253(7)$ and $|V_{us}| = 0.2231(8)$, respectively,

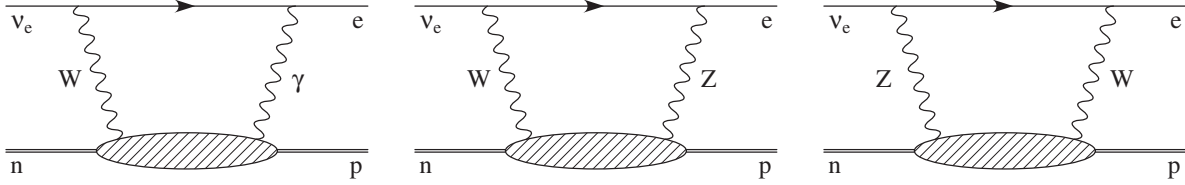
$$|V_{us}| = 0.2243(9) \quad (4)$$

(where the uncertainty has been increased by a scale factor $S = 1.8$ to account for $K_{\mu 2}$ and K_{l3} inconsistencies), and negligible $|V_{ub}|^2 \sim O(10^{-5})$ implies

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9994(4)_{ud}(4)_{us}. \quad (5)$$

Alternatively, one may employ the updated [11] $K_{\mu 2}/\pi_{\mu 2}$ constraint $|V_{us}|/|V_{ud}| = 0.2313(5)$ to derive the unitarity condition [12],

$$|V_{ud}| = 0.97427(11) \text{ unitarity condition from } K_{\mu 2}/\pi_{\mu 2}. \quad (6)$$

FIG. 1. γW and ZW box corrections to neutron decay.

Both Eqs. (2) and (5) are in good accord with those SM expectations. However, that confirmation has recently been questioned. A new analysis of the universal radiative corrections to neutron and superallowed nuclear beta decays based on a dispersion relations (DRs) study of hadronic effects by Seng *et al.* [13] finds a roughly 0.1% larger

$$\Delta_R^V = 0.02467(22), \quad (7)$$

with reduced uncertainty. It leads to a smaller, more precise [13]

$$|V_{ud}| = 0.97370(14) \quad \text{DR result [13]}, \quad (8)$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9984(3)(4). \quad (9)$$

Both Eqs. (8) and (9) exhibit an apparent roughly 3.2σ violation of unitarity. Taken literally, it could be interpreted as a strong hint of new physics. However, nuclear structure effects and other corrections to V_{ud} and V_{us} are still being investigated [14,15]. [We note that the average V_{us} and its uncertainty in Eq. (4) depend very mildly on the value of V_{ud} extracted from superallowed nuclear beta decays. For simplicity, we keep the constraint in Eq. (4) fixed throughout our presentation.]

Although the use of DRs for such an analysis represents a major advancement in the calculation of electroweak radiative corrections, it is important to reexamine the input leading to Eq. (7) and compare with other computational approaches. In that way, one can better assess their consistency and individual reliabilities. Close examination may reveal issues with the RCs or other inputs. For that reason, we update here an alternative study of the radiative corrections to neutron and superallowed nuclear beta decays, estimate hadronic uncertainties, and discuss various possible implications.

Before going into detail, let us briefly preview our study. We first recall the lowest-order one-loop universal radiative corrections to neutron and superallowed nuclear beta decay rates in the framework of the SM. Leading-log QED effects, beyond one-loop order, controlled by the renormalization group are included. Overall, they increase the RC by about 0.1%. However, some care must be exercised in examining compound effects, particularly since the DRs result to be compared with differs from the earlier calculations by a similar approximately 0.1%. That difference

could be offset by smaller changes in several other contributions to the decay rates.

Consider the weak vector amplitude stemming from tree- and loop-level effects. At very low momentum transfer, vector-current induced effects are protected from strong interactions by vector-current conservation. Hadronic effects, nevertheless, enter the vector amplitude via γW box diagrams (and to a lesser extent ZW box diagrams), see Fig. 1, where the operator product expansion of quark axial and vector currents can produce a vector amplitude. In that way, short-distance QCD and long-distance hadronic structure dependence are induced by the nonconserved axial current.

Up until 2006 [10], only the lowest-order, $\mathcal{O}(\alpha_s)$, QCD perturbative correction to the box diagrams was considered [16–18]. Nonperturbative long-distance hadronic corrections were estimated by evaluating a Born amplitude parametrized by inserting axial and vector nucleon dipole form factors in Fig. 1, an approach introduced in Ref. [19].

Those $\mathcal{O}(\alpha)$ (with $\alpha \simeq 1/137.036$) vector and axial-vector induced corrections, universal to all beta decays, were estimated to be

$$\Delta_R^V = \frac{\alpha}{2\pi} \left\{ 3 \ln \frac{m_Z}{m_p} + \ln \frac{m_Z}{m_A} + 2C_{\text{Born}} + A_g \right\}. \quad (10)$$

The $3 \ln(m_Z/m_p)$ short-distance vector-current induced contribution is free of QCD corrections, while the remaining terms, due to the axial-vector current, exhibit strong interaction effects. In Eq. (10), $m_A \sim 1.2$ GeV is a hadronic short-distance cutoff as employed in Ref. [20], $A_g \sim -0.34$ represents its perturbative QCD corrections, and $C_{\text{Born}} \sim 0.85$ denotes the Born (elastic) amplitude contribution. All three terms depend on hadronic structure and/or perturbative QCD. Collectively, those axial-vector induced loop contributions increase the decay rate by about $2.9\alpha/\pi \sim 6.7 \times 10^{-3}$. Although such contributions represent a relatively minor part of the full one-loop universal radiative corrections, they carry most of the theoretical uncertainty.

A strategy for improving strong interaction effects emerged, when it was shown [10] that the perturbative QCD corrections to beta decays and the Bjorken sum rule (BjSR) [21,22], the latter now known to four-loop QCD order [23–25], are identical in the chiral + isospin symmetry limits modulo small singlet contributions that we do not consider [26,27]. Therefore, one can make use of theoretical and experimental BjSR results to define an

effective physical QCD coupling [28] that spans the perturbative and (as more recently argued) nonperturbative loop momentum domains and is continuous throughout the Q^2 transition region (see, for example, Refs. [29–34]). An identical perturbative situation arises for the Gross-Llewellyn-Smith (GLS) nonsinglet sum rule [35], which is employed in the DR approach. In fact, that sum rule is closer in structure to the γW box diagram and the leading-twist term in its operator product expansion. However, accessing relevant low Q^2 data in that case is less straightforward.

Employing the known BjSR or equivalent GLS nonsinglet sum rule four-loop QCD corrections as input allows a precise evaluation of the perturbative QCD corrections to the γW box diagrams for loop momentum above the demarcation scale Q_0^2 [see Eq. (12) and discussion below], $Q_0^2 < Q^2 < \infty$, with little uncertainty. Below $Q^2 = Q_0^2$, a nonperturbative evaluation of hadronic loop effects is required. For that purpose, we depend primarily on a nucleon-based form factor Born amplitude contribution. In addition, one of our new approaches employs a somewhat speculative analytic extension of the BjSR coupling based on light-front holographic QCD (LFHQCD) [36,37] (see also Ref. [38] for a pedagogical introduction). Our use of that nonperturbative interpolator represents a novel application and fundamental test of that approach. It introduces a nonperturbative $\alpha_{g_1}(Q^2)$ given by

$$\frac{\alpha_{g_1}(Q^2)}{\pi} = \exp(-Q^2/Q_0^2) \quad \text{for } 0 < Q^2 < Q_0^2, \quad (11)$$

where g_1 designates its dependence on the polarized structure function $g_1(x, Q^2)$ from which it is derived [see Eq. (20)]. The transition scale we use,

$$Q_0^2 = 1.10(10) \text{ GeV}^2, \quad (12)$$

is fixed by matching nonperturbative and perturbative couplings [39,40] using $\alpha_s(m_Z) = 0.1181(10)$ and the four-loop QCD code in Refs. [39,40]. The matching is quite smooth and leads to additional $Q^2 < Q_0^2$ nonperturbative loop effects that were neglected in 2006 [10] under the assumption that they were included via the Born amplitude. However, as demonstrated by the DR study [13] such effects are distinct and should be separately included. Fortunately, they are relatively small. Nevertheless, they are a source of some uncertainty, and estimates of their magnitude represent the main difference between distinct calculations. In that regard, the DR uses the GLS nonsinglet sum rule data at low Q^2 for guidance, while our method follows ideas developed from BjSR studies [10]. Both have the same perturbative QCD corrections modulo singlet contributions (although the DR approach applies only the first three loops of the known four-loop effects [13]) and include similar estimates of the Born amplitude but differ in the low Q^2 evaluation of other hadronic effects. In addition to the anti-de Sitter (AdS-)based

LFHQCD approach, we also evaluate hadronic effects using a three-resonance interpolator function fixed by boundary conditions. Consistency of the two approaches reinforces their individual credibility. The results are subsequently averaged to give our current best estimate of the radiative corrections.

After presenting our updated evaluation of the RC to neutron decay, we take this opportunity to discuss its implications for our recent analysis of the neutron lifetime- g_A connection [41] in light of the new very precise Perkeo III [42,43] experimental result

$$g_A = 1.2764(6) \text{ Perkeo III (2018) [42, 43]}, \quad (13)$$

which increases the average of post-2002 experiments to

$$g_A^{\text{ave}} = 1.2762(5). \quad (14)$$

That value, taken together with the average trap neutron lifetime, $\tau_n^{\text{trap}} = 879.4(6)$ s, is used to (conservatively) improve our previous bound on exotic neutron decays from less than 0.27% to less than 0.16%. It actually suggests, as we later discuss, that one should probably anticipate a future reduction in the neutron lifetime to the range 878–879 s or a decrease in the value of g_A .

II. RADIATIVE CORRECTIONS TO NEUTRON DECAY

We begin by reviewing the electroweak radiative corrections for neutron decay and then isolate a subset that is also universal to superallowed Fermi decays called Δ_R^V . The inclusive neutron decay rate or inverse lifetime τ_n^{-1} in the SM is predicted to be

$$\frac{1}{\tau_n} = \frac{G_\mu^2 |V_{ud}|^2}{2\pi^3} m_e^5 (1 + 3g_A^2) (1 + \text{RC}) f, \quad (15)$$

where $G_\mu = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}$ is the Fermi constant obtained from the muon lifetime and g_A is the axial-current coupling obtained from the neutron decay asymmetry, $A_0 = 2g_A(1 - g_A)/(1 + 3g_A^2)$. $f = 1.6887(1)$ is a phase space factor that includes the Fermi function, a relatively large roughly +3% final state enhancement due to Coulomb interactions. RC stands for radiative corrections, which have been taken, up until recently, to be +0.03886(38) based on a study [10] in 2006. The more recent DR approach [13] found +0.03992(22), a significant increase outside of the error budgets. In the case of neutron decay, RCs are computed explicitly for the vector-current amplitude, and g_A is defined via Eq. (15) so that g_A^2 , g_V^2 , and f have the same and factorable RCs [20]. That g_A as defined via Eq. (15) is measured in neutron decay asymmetry studies, after correcting for residual recoil, weak magnetism, and small $\mathcal{O}(\alpha)$ corrections as discussed

by Wilkinson [44] and Shann [45]. Corrections to the asymmetry reduce its magnitude by about 1% and correspondingly decrease g_A by about 0.25% [42,43].

The purpose of this paper is to update and improve the 2006 RC calculation approach [10], compare it to the recent DR result [13], and try to understand any remaining difference. It is predicated in part by the DR finding that nonperturbative low Q^2 effects not covered by the Born contribution are present and should be included along with a post-2006 four-loop calculation of perturbative QCD corrections to the nonsinglet BjSR [25] and GLS sum rule.

The factorized components of the lowest-order RC to neutron decay are given by

$$\text{RC} = \frac{\alpha}{2\pi} \left[\bar{g}(E_m) + 3 \ln \frac{m_Z}{m_p} + \ln \frac{m_Z}{m_A} + 2C_{\text{Born}} + A_g \right], \quad (16)$$

where $\bar{g}(E_m)$ represents long-distance loop corrections as well as bremsstrahlung effects averaged over the neutron beta decay electron spectrum and $E_m = 1.292581$ MeV is the end point electron energy specific to neutron decay. We find its updated value is slightly shifted to

$$\frac{\alpha}{2\pi} \bar{g}(E_m) = 0.015035, \quad (17)$$

which reduces RC by 2×10^{-5} . That contribution to the neutron decay RCs in Eq. (17) is specific to the neutron spectrum and is not maintained for other beta decays, although the function $g(E)$ used to derive it is universal to all beta decays (see, however, Ref. [15]). It, along with the $3 \ln(m_Z/m_p)$ term in Eq. (10), is independent of strong interaction effects [46]. The rest of that RC expression represents axial current induced effects that are dependent on strong interactions. They provide the main focus for this paper.

III. AXIAL CURRENT LOOP CONTRIBUTIONS TO RC

The complete radiative corrections to neutron decay in Eq. (10), including axial-current induced and QED leading-log summation effects, can be written to a good approximation as [20]

$$\text{RC} = 0.03186 + 1.017A_{NP} + 1.08A_P, \quad (18)$$

where the +0.03186 corresponds to the pure vector-current induced part of the RC including higher-order effects. A_{NP} and A_P represent lowest-order α long-distance nonperturbative (NP) and short-distance perturbative (P) contributions to RC from axial-current effects in γW and ZW box diagrams (see Fig. 1). Coefficients of the $\mathcal{O}(\alpha)$ contributions in Eq. (18) follow from QED leading-log enhancements and interference with other parts of the vector-current

induced RC. The short-distance parts in our approach correspond to loop momentum $Q^2 > Q_0^2$ [see Eq. (12)], while long-distance parts correspond to $Q^2 < Q_0^2$.

For the universal Δ_R^V used by Hardy and Towner in their analysis of superallowed beta decays [9,47], one finds a corresponding approximate relationship:

$$\Delta_R^V = 0.01671 + 1.022A_{NP} + 1.065A_P. \quad (19)$$

The terms in Eqs. (18) and (19) were derived using the leading-log QED summation described in Appendix 1 of Ref. [20].

We subsequently employ Eqs. (18) and (19) to present updated radiative corrections for the neutron and superallowed beta decays. Note that when applied to the DR $\mathcal{O}(\alpha)$ corrections Eqs. (18) and (19) give somewhat larger effects than those reported in Ref. [13]. However, for the most part, whenever we refer to DR results, values cited correspond to the original literature [13].

The short-distance axial-current γW box diagram is the primary source of A_P . It is well described using an effective QCD coupling $\alpha_{g_1}(Q^2)$ defined for $Q^2 > Q_0^2$ via the isovector BjSR,

$$\int_0^1 dx [g_1^p(x, Q^2) - g_1^n(x, Q^2)] = \frac{g_A}{6} \left(1 - \frac{\alpha_{g_1}(Q^2)}{\pi} \right), \quad (20)$$

where g_1 is the polarized structure function at Bjorken x . That prescription incorporates the leading $\mathcal{O}(\alpha)$ axial-current induced amplitude from the γW box diagram, given by

$$\text{Box}(\gamma W)_{VA} = \frac{\alpha}{8\pi} \int_0^\infty dQ^2 \frac{m_W^2}{Q^2 + m_W^2} F(Q^2), \quad (21)$$

where the asymptotic behavior of $F(Q^2)$,

$$F(Q^2) \rightarrow \frac{1}{Q^2} \left(1 - \frac{\alpha_{g_1}(Q^2)}{\pi} \right), \quad (22)$$

will be called the Bjorken (Bj) function and α_{g_1} is defined to be the sum of the four-loop (or more if known) QCD corrections to the BjSR [25],

$$\begin{aligned} \frac{\alpha_{g_1}(Q^2)}{\pi} &= a_s + (4.583 - 0.3333n_f)a_s^2 \\ &+ (41.44 - 7.607n_f + 0.1775n_f^2)a_s^3 \\ &+ (479.4 - 123.4n_f + 7.697n_f^2 - 0.1037n_f^3)a_s^4, \end{aligned} \quad (23)$$

where $a_s = \frac{\alpha_{\overline{MS}}(Q^2)}{\pi}$ and n_f denotes the number of (effectively massless) quark flavors. That expression defines a coupling α_{g_1} which is valid perturbatively for $Q^2 > Q_0^2$.

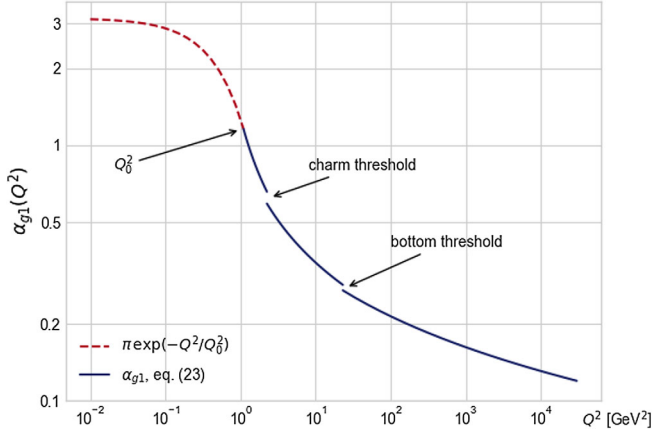


FIG. 2. Effective coupling α_{g_1} [33] as a function of Q^2 . The nonperturbative exponential form of Eq. (24) is used for low Q^2 (dashed red), and the perturbative QCD expression (23) is used for high Q^2 (solid blue). Note the remarkably smooth matching between the two regimes. The discontinuities are caused by decoupling of heavy quarks [39,40].

The behavior of α_{g_1} with Q^2 is shown in Fig. 2. Discontinuities in that plot are caused by changes in the number of active flavors in Eq. (23); we change n_f when $\sqrt{Q^2}$ crosses a quark decoupling threshold. Note, however, that Eq. (23) is derived in massless QCD. There are very small singlet contributions [26,27] to the BjSR that enter the four-loop QCD corrections. However, at the level of precision we consider, their effect is negligible.

In our first LFHQCD approach, for the nonperturbative domain $Q^2 < Q_0^2$, we employ the following prescription, which is supported by low Q^2 experimental studies of the BjSR (down to about $Q^2 = 0.2 \text{ GeV}^2$) [48]. We continue to use the expression in Eq. (22) but with [33]

$$\frac{\alpha_{g_1}}{\pi} = \exp(-Q^2/Q_0^2) \quad \text{for } Q^2 < Q_0^2, \quad (24)$$

and $F(Q_0^2 = 1.10(10) \text{ GeV}^2) = 0.575(50) \text{ GeV}^{-2}$, based on matching with the perturbative prediction obtained from $\alpha_s^{\overline{\text{MS}}}(m_Z^2)$, evolved using a five-loop beta function [39]. Its functional exponential form is consistent with low Q^2 BjSR data and normalization $\alpha_{g_1}(Q^2 = 0) = \pi$ as suggested by AdS duality studies [33,49]. The Born (elastic) contributions to the hadronic corrections are computed separately using form factors for loop momenta $Q^2 < Q_0^2$.

In 2006 [10], when only three-loop QCD corrections to the BjSR were known and considered, the transition Q_0^2 turned out to be close to 0.7 GeV^2 . Our current use of a four-loop based coupling α_{g_1} along with charm and bottom quark threshold masses, as well as improved low Q^2 data, have increased the transition Q_0^2 [see Eq. (12)] and now better establish the AdS duality interpretation, features central to our update.

We can use the Bj function defined by Eqs. (22)–(24) to evaluate the integral over Q^2 in Eq. (21) for the different domains of the γW box diagram using $\alpha_s(m_Z^2) = 0.1181(10)$, $m_c = 1.5 \text{ GeV}$, $m_b = 4.8 \text{ GeV}$ (as decoupling thresholds of heavy quarks; see Ref. [50] for an up-to-date discussion of quark masses), and $m_t = 173.2 \text{ GeV}$ with the results

$$I_1 = 0.199 \frac{\alpha}{\pi} \quad 0 < Q^2 < Q_0^2, \quad (25)$$

$$I_2 = 1.965(21) \frac{\alpha}{\pi} \quad Q_0^2 < Q^2 < \infty, \quad (26)$$

where $I_i = 2 \times$ the γW integrated box amplitude contribution, as appropriate for radiative corrections to the decay rates. To those loop effects, we must add the ZW box diagram contribution [7] see Eq. (21),

$$I_{ZW} = 0.060 \frac{\alpha}{\pi}, \quad (27)$$

and the Born [10] integral

$$I_{\text{Born}} = 0.85 \frac{\alpha}{\pi} \quad 0 < Q^2 < Q_0^2. \quad (28)$$

In these calculations and in the figures, we disregard the error in Q_0^2 , since its effect is negligible in comparison with the uncertainties assigned to the electroweak corrections. Contributions of QED vacuum polarization are incorporated via the coefficients in Eqs. (18) and (19).

Our first estimate, which follows the 2006 evaluation [10] but with a four-loop BjSR coupling definition and higher Q_0^2 value, does not include the contribution in Eq. (25). The Born contribution in Eq. (28) leads to

$$A_{NP} = 0.85 \frac{\alpha}{\pi} = 1.97 \times 10^{-3}, \quad (29)$$

while the sum of Eqs. (26) and (27) gives

$$A_P = 2.025(21) \frac{\alpha}{\pi} = 4.70(5) \times 10^{-3}. \quad (30)$$

Plugging those values into Eqs. (18) and (19) gives

$$\text{RC} = 0.03895, \text{ first approximation}, \quad (31)$$

$$\Delta_R^V = 0.02374, \text{ first approximation}, \quad (32)$$

for our updated first approximation radiative corrections to neutron and superallowed nuclear beta decays.

In our next, more complete AdS-motivated [33] approach, we retain the nonperturbative low Q^2 contribution from Eq. (25) and find $A_{NP} = 2.44 \times 10^{-3}$, which leads to

$$\text{RC} = 0.03942(32) \text{ AdS BjSR approach}, \quad (33)$$

$$\Delta_R^V = 0.02421(32) \text{ AdS BjSR approach}. \quad (34)$$

The two methods differ by 0.00047, with the latter about midway between our first approximation and the DR results [13]. The generic error attached to Eqs. (33) and (34) as well as to later alternative approaches, $\pm 3.2 \times 10^{-4}$, results from an error budget of roughly $\pm 20\%$ of I_1 , $\pm 10\%$ of I_{Bom} and $\pm 5\%$ of the perturbative contribution ($I_2 + I_{\text{ZW}}$), all added in quadrature. The $\pm 20\%$ uncertainty in I_1 corresponds to a contribution $I_1 = (0.20 \pm 0.04)\alpha/\pi$, where the error spans the range of values considered in our analysis. In the case of the I_{Bom} , $\pm 10\%$ uncertainty covers the corresponding DR calculation [13,14] as well as a very large range of the integration limit values Q_0^2 . The perturbative part includes QCD effects as well as uncalculated two-loop electroweak corrections. Further reduction of that error is likely to require a first principle's lattice calculation along with a more complete two-loop electroweak comparison between neutron beta decay and muon decay.

The values and uncertainties given above should be compared with the DR results [13],

$$\text{RC} = 0.03992(22) \text{ DR result [13]}, \quad (35)$$

$$\Delta_R^V = 0.02467(22). \quad (36)$$

It is interesting to contrast the AdS-based (34) value,

$$V_{\text{ud}} = 0.97391(18) \text{ AdS BjSR approach}, \quad (37)$$

with

$$V_{\text{ud}} = 0.97370(14) \text{ DR result [13]}. \quad (38)$$

We note that the value of V_{ud} in Eq. (37) has moved closer to unitarity expectations (approximately 0.9742). An additional shift of about -0.0006 in the universal radiative corrections to superallowed decays or an equivalent change in another part of those studies would fully restore unitarity.

To examine the sensitivity of our estimate to the specific Bj function interpolator used to integrate through the nonperturbative $Q^2 < Q_0^2$ region, we consider the resonance sum interpolator approach introduced in 2006 [10] but with somewhat modified matching conditions used to determine $F(Q^2)$ in the low-momentum domain. The new conditions allow us to better specify the nonperturbative constraints implied by the very precise perturbative requirements. In that way, we can match the nonperturbative and perturbative values of $F(Q^2)$.

The underlying model of our next interpolator is large N for $\text{SU}(N)$ QCD, which predicts $F(Q^2)$ should correspond to an infinite sum of vector and axial-vector resonances. For an approximation to that model, we can use a finite sum of resonances with residues set by the boundary and matching conditions. We can then integrate over Q^2 between 0 and Q_0^2 , i.e., including the nonperturbative domain in an approximation to that model. For that purpose, we chose

the sum of three resonances with residues determined using three matching or boundary conditions. We impose conditions:

- (i) We require that in the domain $Q_0^2 \leq Q^2 < \infty$ the three-resonance interpolator leads to the same perturbative corrections to the decay rates as the BjSR approach. This implies that the three-resonance $F(Q^2)$ function satisfies the integral condition $\int_{Q_0^2}^{\infty} \frac{m_W^2}{Q^2 + m_W^2} F(Q^2) dQ^2 = 7.86$, four times the coefficient of α/π in I_2 [cf. Eq. (26)]. We apply a condition on the integral rather than asymptotic matching in order to better reflect the effect of perturbative QCD.
- (ii) There should be no $1/Q^4$ terms in the expansion of $F(Q^2)$ for large Q^2 or [see Eq. (39)] $m_1^2 A + m_2^2 B + m_3^2 C = 0$. This condition enforces chiral symmetry asymptotically.
- (iii) We employ $F(0) = A/m_1^2 + B/m_2^2 + C/m_3^2$ with $F(0)$ arbitrary until we consider two possible ways to fix its value, i.e., using either the perturbative value of $F(Q_0^2)$ or the AdS value of $F(0)$ as a normalization condition for the three-resonance interpolator.

These conditions are similar to those imposed in 2006 [10] with some improvements. Because of the larger Q_0^2 employed, the integral in condition 1 is extended down to Q_0^2 . More importantly, as pointed out in the DR analysis, the condition $F(0) = 0$ used in 2006 was not justified. Instead, we use the perturbative value of $F(Q_0^2)$ to normalize the three-resonance interpolator and determine its underlying uncertainty.

After solving the three coupled condition equations, one finds [for the three-resonance form of $F(Q^2)$ with given vector and axial-vector masses]

$$F(Q^2) = \frac{A}{Q^2 + m_1^2} + \frac{B}{Q^2 + m_2^2} + \frac{C}{Q^2 + m_3^2}, \quad (39)$$

$$\begin{aligned} m_1 &= 0.776 \text{ GeV}, & m_2 &= 1.230 \text{ GeV}, \\ m_3 &= 1.465 \text{ GeV}, \end{aligned} \quad (40)$$

$$\begin{aligned} A &= -1.511(9) + 1.422(3)F(0), \\ B &= 6.951(40) - 3.533(10)F(0), \\ C &= -4.476(21) + 2.092(7)F(0). \end{aligned} \quad (41)$$

That interpolator, integrated over $0 < Q^2 < Q_0^2$, leads to

$$I_1(\text{three resonance}) = [0.094(9) + 0.103(3)F(0)] \frac{\alpha}{\pi}. \quad (42)$$

With that change in I_1 , the radiative corrections become a function of $F(0)$,

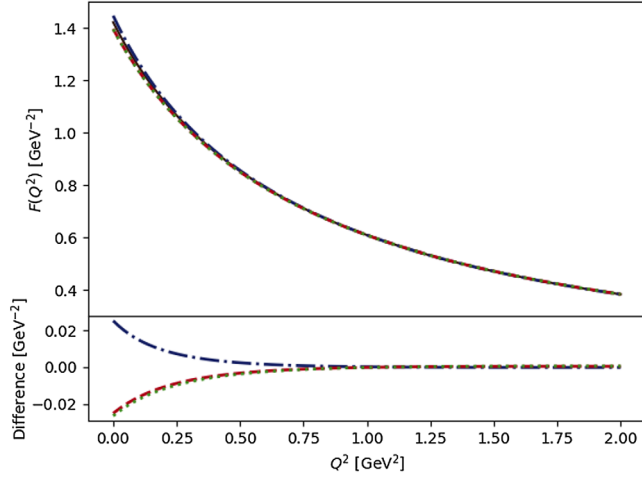


FIG. 3. Interpolators as in Eq. (39), with $m_{1,2,3}$ as in Eq. (40) (solid brown line), and with m_i decreased by 5%: $i = 1$ (blue, dashed-dotted), $i = 2$ (red, dashed), and $i = 3$ (green, dotted). Lower panel: differences between an interpolator with a decreased value of m_i and the interpolator with mass values given in Eq. (40) (the same line styles as in the upper panel).

$$\begin{aligned} \text{RC} &= 0.03917 + 2.43 \times 10^{-4} F(0), \\ \Delta_R^V &= 0.02396 + 2.45 \times 10^{-4} F(0). \end{aligned} \quad (43)$$

If we match the interpolator in Eq. (39) with the perturbative value $F(Q_0^2) = 0.575(50)$, we find $F(0) = 1.42(15)$ and the radiative corrections, which we adopt as the three-resonance solution,

$$\left. \begin{aligned} \text{RC} &= 0.03952(32) \\ \Delta_R^V &= 0.02431(32) \end{aligned} \right\} \text{three-resonance solution,} \quad (44)$$

where a small uncertainty, $\pm 4 \times 10^{-5}$, is accounted for in the $\pm 32 \times 10^{-5}$ overall errors. To test the sensitivity of our results to the specific resonance mass scales employed, we have redone the three-resonance interpolator with each of $m_{1,2,3}$ reduced by 5%, one m_i at a time. Although the values of A , B , and C are significantly modified, the different interpolators, value of I_1 , and radiative corrections are essentially unchanged, as illustrated in Fig. 3. Indeed, our results are rather insensitive to reasonable changes in the m_i values.

For an alternative prescription, we evaluate the radiative corrections resulting from the three-resonance interpolator for the AdS boundary condition $F(0) = \frac{1}{Q_0^2} = 0.91 \text{ GeV}^{-2}$ and find

$$\begin{aligned} \text{RC} &= 0.03939(32), \\ \Delta_R^V &= 0.02418(32). \end{aligned} \quad (45)$$

Those values are in very good agreement with the AdS BjSR results in Eqs. (33) and (34). They provide a nice consistency check on the AdS BjSR approach. We do not

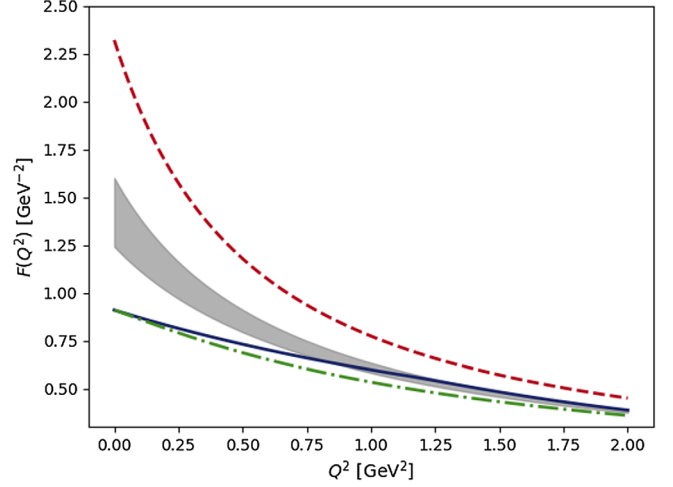


FIG. 4. Examples of $F(Q^2)$ at low Q^2 : $\frac{1}{Q^2}(1 - \frac{\alpha_{g1}(Q^2)}{\pi})$ (solid blue line); three-resonance interpolator based on $F(Q^2)$ in Eq. (39) with two choices of $F(0)$: $F(0) = 1.42(15)$ (gray band) and $F(0) = 1/Q_0^2$ (dash-dotted, green); and two-resonance interpolator from Eq. (46) (dashed red line), meant to approximate the low Q^2 DR findings modulo the Born contribution. The area under each curve is proportional to the size of the radiative corrections.

consider them as independent since both employ the same $F(0)$ boundary condition.

The Q^2 dependences of the various interpolators are illustrated in Fig. 4. The band surrounding the $F(0) = 1.42(15)$ curve corresponds to the uncertainty associated with the error in $\alpha_s(m_Z^2) = 0.1181(10)$. Similar bands (not shown) exist for the other curves as well, but all are small in comparison with our overall uncertainty, $\pm 32 \times 10^{-5}$ for the radiative corrections. The good agreement between the AdS and three-resonance solution for $F(0) = 0.91 \text{ GeV}^{-2}$ helps validate the AdS approach. In all cases, the radiative corrections are proportional to areas under the curves.

The dashed curve in Fig. 4 corresponds to an example of a two-resonance interpolating function given by

$$F_2(Q^2) = \frac{1.66}{Q^2 + m_1^2} - \frac{0.66}{Q^2 + m_2^2}, \quad (46)$$

which exhibits the following features:

$$\begin{aligned} F_2(0) &= 2.32, & F_2(Q_0^2) &= 0.732, \\ F_2(2 \text{ GeV}^2) &= 0.450. \end{aligned} \quad (47)$$

It roughly represents our approximation of an effective DR interpolator for $0 < Q^2 < 2 \text{ GeV}^2$. Integrating $\frac{\alpha}{4\pi} F_2(Q^2)$ over that domain leads to $0.47\alpha/\pi$, in good agreement with the $0.48(7)\alpha/\pi$ found in the DR study [13]. Those contributions are to be compared with the roughly $0.35\alpha/\pi$ coming from our three-resonance interpolator when integrated over that same Q^2 domain. That $0.13\alpha/\pi$ difference combined with the $0.06\alpha/\pi$ Born difference = $0.19\alpha/\pi$ is

TABLE I. Universal and neutron-specific radiative corrections.

Line number	Δ_R^V	RC	Source
1	0.02467(22)	0.03992(22)	[13] DR result
2	0.02361(38)	0.03886(38)	[10] 2006 result
3	0.02421(32)	0.03942(32)	AdS BjSR approach, Eqs. (33) and (34)
4	0.02431(32)	0.03952(32)	Three-resonance interpolator, Eq. (44)
5	0.02426(32)	0.03947(32)	Average of lines 3 and 4

responsible for about a 4×10^{-4} difference (in the $0 < Q^2 < 2 \text{ GeV}^2$ domain) between the DR result and our three-resonance interpolator finding.

The 1.1σ difference between the DR and our results may therefore be traced primarily to our use of a larger $\alpha_s(m_Z^2)$, four loops rather than three in the QCD sum rule corrections, different perturbative-nonperturbative matching, and three rather than two vector/axial-vector poles in the interpolator. More specifically, to reproduce the DR, low Q^2 contribution requires an interpolator with an $F(0)$ central value near 2.3 GeV^{-2} , while our matching conditions and interpolator imply $F(0) = 1.42(15) \text{ GeV}^{-2}$. From that perspective, it would be interesting if a more first principles method, such as lattice QCD, could be employed to directly compute the value of $F(0)$.

In Table I, we compare the universal and neutron-specific radiative corrections obtained from a dispersion relation approach (line 1) with an earlier calculation from 2006 (line 2) as well as the AdS BjSR result (line 3), three-resonance interpolator (line 4), and the average of lines 3 and 4 in line 5.

We take the average on line 5 as representative of our study and use it in discussing implications. Although it is somewhat smaller than the earlier DR result [13], both are fairly consistent. In fact, the agreement can be viewed as a validation of the LFHQCD and three-resonance interpolator approaches.

Although we are consistent with the DR results at about the 1.1σ level, i.e., approximately 4×10^{-4} , the remaining difference is important for interpreting CKM unitarity and making predictions for neutron decay. In that regard, we speculate on the basis of our analysis that the central value difference may decrease if the DR approach is extended to include four-loop QCD corrections, low Q^2 corrections are parametrized using three rather than two vector meson mass scales, and perturbation theory matching is extended below 2 GeV^2 .

We also note that the radiative corrections on lines 3, 4, and 5 of Table I are reduced somewhat if larger values of $\alpha_s(m_Z^2)$ are employed as input. For example, using $\alpha_s(m_Z^2) = 0.1200$, a preferred value for some experimental inputs into the world average [8] leads to a reduction by roughly 1×10^{-4} , which increases our V_{ud} , as currently

extracted from superallowed Fermi decays [9], from $0.97389(18)$ to $0.97394(18)$.

IV. IMPLICATIONS OF LARGER RADIATIVE CORRECTIONS

For our discussion of implications from larger radiative corrections, we employ our averages given in line 5 of Table I, $\Delta_R^V = 0.02426(32)$ and $\text{RC} = 0.03947(32)$. That scenario leads to $V_{ud} = 0.97389(18)$, which comes closer to CKM unitarity expectations than the DR value $V_{ud} = 0.97370(14)$. Combined with $|V_{us}| = 0.2243(9)$ from Eq. (4) and $|V_{ub}|^2 = 1.6 \times 10^{-5}$, they correspond to roughly 2.3 and 3.2σ deviations, respectively. The 3.2σ deviation is large enough to start taking new physics extensions of the SM seriously [51], while the 2.3σ effect is more suggestive of missing SM effects. For example, additional nuclear physics quenching of the Born corrections to superallowed beta decays has been suggested as a way of increasing V_{ud} by about 0.00022 [14].

More specifically, if the quenching correction, as evaluated in Ref. [14], is applied to our $V_{ud} = 0.97389(18)$ result, it leads to $V_{ud}^Q = 0.97414(28)$, where the increased error is due to an additional nuclear quenching uncertainty. Using it together with $V_{us} = 0.2243(9)$, one finds $|V_{ud}^Q|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.00074(68)$, so the first CKM row sum is consistent with unity at close to the 1σ level.

If instead we employ the precise relation [11,12]

$$|V_{us}|/|V_{ud}^Q| = 0.2313(5), \quad (48)$$

the deviation from unity is further reduced to $-0.00028(62)$, or -0.45σ , in excellent agreement with CKM unitarity.

In a further application, we consider the RC to neutron decay. Using $1 + \text{RC} = 1.03947(32)$ in the neutron lifetime formula [41], one finds a master formula relating $|V_{ud}|$, τ_n , and g_A ,

$$|V_{ud}|^2 \tau_n (1 + 3g_A^2) = 4905.7(1.7) \text{ s}. \quad (49)$$

Employing $\tau_n^{\text{trap}} = 879.4(6) \text{ s}$ and post-2002 average $g_A = 1.2762(5)$ leads to

$$V_{ud} = 0.9735(5). \quad (50)$$

The uncertainty in V_{ud} from neutron decay measurements is starting to become competitive in accuracy with superallowed beta decay determinations. In addition, its central value may also be indicating a deviation from unitarity. A central value shift to unitarity and $V_{ud} \sim 0.9742$ would require a reduction in either τ_n or g_A . Given the recent precision of Perkeo III, we consider g_A fixed at the new post-2002 average $1.2762(5)$, which then suggests a $\tau_n < 879 \text{ s}$.

An alternate interpretation of the apparent violation of CKM unitarity in Eq. (9) resulting from larger universal radiative corrections, consistent with $|V_{us}|/|V_{ud}| = 0.2313$

from $K_{\mu 2}/\pi_{\mu 2}$, suggests the solution $V_{ud} = 0.9735$ and $V_{us} = 0.2252$, which requires new physics. One possible explanation could be the existence of a 0.1% increase in the muon decay rate from new physics, which shifts G_{μ} to a value larger than the real G_F . Alternatively, it could stem from an opposite-sign effect in nuclear beta decay. That solution agrees with the current central value in Eq. (50). Of course, such a scenario would be very exciting. It will also be well tested by the next generation of precise τ_n and g_A measurements.

Recently, we discussed [41] a resolution of the neutron lifetime problem [the beam $\tau_n^{\text{beam}} = 888.0(2.0)$ s and trap $\tau_n^{\text{trap}} = 879.4(0.6)$ s lifetime discrepancy] based on a precise connection between τ_n and g_A , the axial coupling measured in neutron decay asymmetries. We note that a shift in the universal beta decay radiative corrections alone makes a negligible change in the relationship

$$\tau_n(1 + 3g_A^2) = 5172.0(1.1) \text{ s}, \quad (51)$$

used for that study due to a cancelation of uncertainties and common shifts between superallowed and neutron beta decay rates. Similarly, a change in G_F will not change Eq. (51). However, a shift in the nuclear theory corrections as suggested in Ref. [14] will modify it. For example, a shift in V_{ud} by +0.0002 by further quenching of the Born contribution would lower the 5172.0 s in Eq. (51) to 5169.9 s. More important is the recent increase in the post-2002 g_A^{average} from 1.2755(11) to 1.2762(5) with the addition of the Perkeo III result [42,43] in Eq. (13). That shift reduces the predicted neutron lifetime from 879.5(1.3) s to

$$\tau_n = 878.7(0.6) \text{ s (prediction based on } g_A = 1.2762(5)). \quad (52)$$

That prediction is further reduced if V_{ud} were to increase to respect CKM unitarity. Indeed, one would expect τ_n closer to 878 s.

We conclude by noting that the new post-2002 g_A average in Eq. (14) can be used in the analysis of Ref. [41] to reduce the bound on exotic neutron decays (such as $n \rightarrow$ dark particles [52,53]) from $< 0.27\%$ to

$$\text{BR}(\text{exotic neutron decays}) < 0.16\% (95\% \text{ one-sided CL}), \quad (53)$$

where we have not allowed for negative exotic branching ratios in the statistical distribution. That bound leaves little

chance for a 1% dark particle decay as the solution to the neutron lifetime problem (unless one modifies the neutron asymmetry with new physics, e.g., Ref. [54]).

We have presented an updated analysis of the radiative corrections to neutron and superallowed nuclear beta decays. It extends the BjSR function into the nonperturbative low loop momentum region, incorporating four-loop QCD effects as well as LFHQCD ideas and their confirmation by low-energy experimental data. The value obtained was averaged with a slightly larger three-resonance result. On the basis of our considerations, we advocate the universal value $\Delta_R^V = +2.426(32)\%$ as a competitive result about midway between earlier estimates [10] and the recent dispersion relation result [13]. Further study of the remaining small difference is warranted. Tests of both approaches will result from the next generation of the neutron lifetime and g_A asymmetry measurements that aim for 10^{-4} sensitivity. Lattice calculations of $F(Q^2)$ may be possible [55]. Will CKM unitarity be violated and new physics be uncovered? Time will tell.

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