# Testing the generalized uncertainty principle with macroscopic mechanical oscillators and pendulums

P. A. Bushev,[1*] J. Bourhill,[2] M. Goryachev,[2] N. Kukharchyk,[1] E. Ivanov,[2] S. Galliou,[3] M. E. Tobar,[2] and S. Danilishin[4]

[1]*Experimentalphysik, Universität des Saarlandes, D-66123 Saarbrücken, Germany*
[2]*ARC Centre of Excellence for Engineered Quantum Systems, University of Western Australia, Crawley, Western Australia 6009, Australia*
[3]*FEMTO-ST Institute, Université Bourgogne Franche-Comté, CNRS, ENSMM, 25000 Besançon, France*
[4]*Institut für Theoretische Physik, Leibniz Universität Hannover and Max-Planck Institut für Gravitationsphysik (Albert-Einstein-Institut), 30167 Hannover, Germany*

(Received 21 March 2019; published 20 September 2019)

Recent progress in observing and manipulating mechanical oscillators at quantum regime provides new opportunities of studying fundamental physics, for example to search for low energy signatures of quantum gravity. For example, it was recently proposed that such devices can be used to test quantum gravity effects, by detecting the change in the $[\hat{x}, \hat{p}]$ commutation relation that could result from quantum gravity corrections. We show that such a correction results in a dependence of a resonant frequency of a mechanical oscillator on its amplitude, which is known as the amplitude-frequency effect. By implementing this new method we measure the amplitude-frequency effect for a 0.3 kg ultra-high-Q sapphire split-bar mechanical resonator and for an $\sim 10^{-5}$ kg quartz bulk acoustic wave resonator. Our experiments with a sapphire resonator have established the upper limit on a quantum gravity correction constant of $\beta_0$ to not exceed $5.2 \times 10^6$, which is a factor of 6 better than previously measured. The reasonable estimates of $\beta_0$ from experiments with quartz resonators yields $\beta_0 < 4 \times 10^4$. The datasets of 1936 measurements of a physical pendulum period by Atkinson [E. C. Atkinson, Proc. Phys. Soc. London **48**, 606 (1936).] could potentially lead to significantly stronger limitations on $\beta_0 \ll 1$. Yet, due to the lack of proper pendulum frequency stability measurement in these experiments the exact upper bound on $\beta_0$ cannot be reliably established. Moreover, pendulum based systems only allow one to test a specific form of the modified commutator that depends on the mean value of momentum. The electromechanical oscillators to the contrary enable testing of any form of generalized uncertainty principle directly due to a much higher stability and a higher degree of control.

DOI: 10.1103/PhysRevD.100.066020

## I. INTRODUCTION

At present, one of the grandest challenges of physics is to unite its two most successful theories, quantum mechanics (QM) and general relativity (GR), into a single unified mathematical framework. Attempting this unification has challenged theorists and mathematicians for several decades and numerous works have highlighted the seeming incompatibility between QM and GR [1]. It was generally supposed that this required energies at the Planck scale and therefore beyond the reach of current laboratory technology [2]. However, in a relatively recent publication, Pikovsky *et al.* [3] proposed a new method of testing a set of quantum gravity (QG) theories [4–8] by using an ingenious interferometric measurement of an optomechanical system. The prediction of most of the QG theories (such as, e.g., string theory) and the physics of black holes lead to the existence of the minimum measurable length set by the Planck length $L_p = \sqrt{\hbar G/c^3} \simeq 1.6 \times 10^{-35}$ m [4,7,8]. This results in the modification of the *Heisenberg uncertainty principle* in such a way as to prohibit the coordinate uncertainty, $\Delta x \sim \hbar/\Delta p$, from tending to zero as $\Delta p \to \infty$ [9–13]. The modified uncertainty relation, known as *generalized uncertainty principle* (GUP), is model independent and can be written for a single degree of freedom of a quantum system as

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left[1 + \beta_0 \frac{\Delta p^2 + \langle p \rangle^2}{M_p^2 c^2}\right], \tag{1}$$

where $\beta_0$ is a dimensionless model parameter, $M_p = \sqrt{\hbar c/G} \simeq 2.2 \times 10^{-8}$ kg is the Planck mass, and $\langle p \rangle$ is the quantum ensemble average of the momentum of the system. The dependence of minimum uncertainty of

*pavel.bushev@physik.uni-saarland.de

 

coordinate on average momentum is questionable but some theories [6,8] imply that it reflects the connection of spacetime curvature and the density of energy and matter manifested in Einstein's equations of general relativity.

Another more intuitive form of the GUP, e.g.,

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left[1+\gamma_0\left(\frac{\Delta p}{M_p c}\right)^2\right], \tag{2}$$

which depends only on the uncertainties of the canonical variables of the particle but not on their mean values is predicted in [7]. To test this theory one needs to either measure the deviation of the oscillator's ground-state energy $E_{\min}$ with respect to its unperturbed value $\hbar\Omega_0/2$ [14], or to test QG corrections to the dynamics of the quantum uncertainty of the mechanical degree of freedom using the pulsed measurement procedure proposed in [3], which requires a quantum level of precision.

The lowest modal energies measured in large mechanical systems such as an AURIGA detector with effective mass of the mode $m_{\mathrm{eff}} \simeq 1000$ kg [14] and in a dumbbell sapphire oscillator with $m_{\mathrm{eff}} \simeq 0.3$ kg [15] set the limit on the QG model parameter $\gamma_0 \lesssim 3\times 10^{33}$, which is still too large compared to the predicted values of the order of unity [16].

## II. THEORY

From the GUP (1) one can derive the new canonical commutation relation,

$$[\hat{x},\hat{p}]_{\beta_0} = i\hbar\left[1+\beta_0\left(\frac{\hat{p}}{M_p c}\right)^2\right], \tag{3}$$

that is deformed by the QG correction defined by the model parameter $\beta_0$. As shown by Kempf *et al.* [6], parameter $\beta_0$ defines the scale of the absolutely smallest coordinate uncertainty $\Delta x_{\min} = \hbar\sqrt{\beta_0}/(M_p c)$. In this work, we experimentally set an upper limit on the value of the model parameter $\beta_0$ using the dynamical implications of the contorted commutator on the oscillations of a high-Q mechanical resonator of mass $m$ and (unperturbed) resonance frequency $\Omega_0$.

We start our consideration with the simple premise that the modification of the fundamental commutator for a harmonic oscillator is equivalent to the nonlinear modification of the Hamiltonian by means of the perturbative transformation of momentum, $\hat{p} \to \hat{p} - \beta_0\hat{p}^3/(3M_p^2c^2)$, which restores the canonic commutator, $[\hat{x},\hat{p}] = i\hbar$, at the expense of adding the nonlinear term to the Hamiltonian of the resonator: $\hat{H} \to \hat{H}_0 + \Delta\hat{H} = (\hat{p}^2/2m + m\Omega_0^2\hat{x}^2/2) + \beta_0\hat{p}^4/(3m(M_p c)^2)$. Such nonlinear correction results in the dependence of the oscillator resonance frequency on its energy [6,8,17]. The dynamics of the system can be described by a well-known Duffing oscillator model characterized by amplitude dependence of the resonance frequency, i.e., the so-called amplitude-frequency effect [18,19]. The necessary frequency resolution in order to sense subtle QG effects can be estimated by using the following expression:

$$\delta\Omega(A)/\Omega_0 = \beta_0(m_{\mathrm{eff}}\Omega_0 A/M_p c)^2, \tag{4}$$

where $\delta\Omega = \Omega(A) - \Omega_0$ is the deviation of the amplitude-dependent resonance frequency $\Omega(A)$ from the unperturbed value $\Omega_0$, $m_{\mathrm{eff}}$ is the effective mass of the mode and $A$ is the oscillation amplitude. So, the experimentally measured dependence of the resonance frequency on the amplitude, particularly its null result, may be used to set an upper limit for the model parameter $\beta_0$.

The above-mentioned theoretical considerations do not specify which degree of freedom is subject to the QG corrections. If one considers a center of mass mode, then the scale of perturbation is strongly enhanced for the heavier than the Planck mass oscillators, as compared to individual atoms and molecules in the lattice. For instance, the precise measurement of the Lamb shift in hydrogen yielded an upper bound for the model parameter $\beta_0 < 10^{36}$ [16]. However, recent experiments with microscopic high-Q oscillators with effective masses ranging from $10^{-11}$ to $10^{-5}$ kg established a new upper bound for $\beta_0 < 3\times 10^7$ [19]. The intrinsic acoustic nonlinearity of micro-oscillators prevented testing of quantum gravity corrections with greater precision.

In the following we describe an experiment with the subkilogram split-bar (SB) sapphire mechanical oscillator, where we demonstrate improvement for the upper value of the correction parameter $\beta_0$ compared to the previous work with intermediate range mechanical oscillators [19] by nearly an order of magnitude. In addition to that, we provide reasonable estimates of $\beta_0$ from experiments with bulk acoustic wave (BAW) quartz resonators yielding the limit of $4\times 10^4$. As a consequence of the mean value entered on the right-hand side of Eq. (1) the systems with higher mass and larger amplitude are preferred. As an example one may take measurements of the period of the physical pendulum in 1936 [20], where a much lower upper bound of $\beta_0 \lesssim 10^{-4}$ can be established from the deviation of the period dependence of amplitude from the well-known Bernoulli nonlinearity. However, due to the absence of the evaluation of the pendulum frequency stability the exact upper bound on $\beta_0$ cannot be obtained.

## III. MEASUREMENTS OF CORRECTION STRENGTH $\beta_0$ WITH SAPPHIRE DUMBBELL OSCILLATOR

Microwave oscillators based on electromagnetic whispering gallery mode (WGM) sapphire crystals offer excellent short- and middle-term frequency stability [21] due to

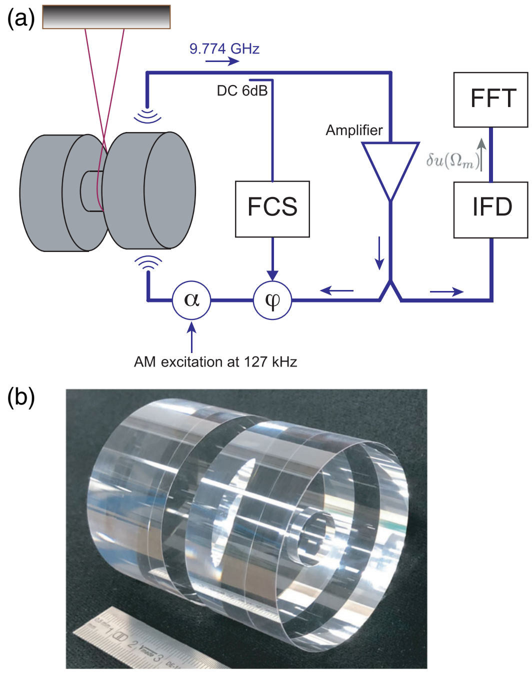


FIG. 1. (a) Simplified experiment schematic. See the text for details. (b) Picture of the sapphire SB resonator. The ruler shows the scale of the system.

WGM high quality factors exceeding $10^8$ and the existence of frequency-temperature turnover points. For these reasons these devices found applications in fundamental tests [22–24]. The mechanical modes of sapphire resonators may attain $Q_M \simeq 10^8$–$10^9$ [25–27]. The resonance frequencies of WGMs are very sensitive to changes in circumference and the height of the cylinder resonator and to strain in the crystal lattice thus yielding the necessary coupling between mechanical and electromagnetic degrees of freedom for the observation of mechanical motion. Yet, no acoustic nonlinearities have been detected for the large sapphire mechanical resonators, making these devices an excellent platform for QG tests.

The experimental setup, shown in Fig. 1(a), is based on a cylindrical dumbbell shape or SB sapphire resonator, which is fabricated out of a single crystal HEMEX-grade sapphire fabricated by GT Advanced Technologies, Inc., USA. The rotation symmetry axis of the resonator is parallel to the c axis of the crystal. The SB resonator consists of two bars with diameter 55 mm and height 28 mm, which are separated by a neck of diameter 15 mm and length 8 mm, see Fig. 1(b). Two electromagnetic WGM resonators are formed in each bar and undergo the same mechanical motion, i.e., they oscillate in phase for the breathing mode, which is similar to the fundamental longitudinal mode of the conventional cylindrical resonator of the same length and diameter. The resonance frequency of this mode is $\Omega_0/2\pi = 127.071$ kHz and its effective mass is calculated by using finite element modeling $m_{\mathrm{eff}} = 0.3$ kg. In order to maximize the mechanical Q-factor, the resonator is suspended via a niobium wire loop around the neck. The whole construction is placed inside a temperature stabilized vacuum chamber at 300 K. The vacuum chamber in turn is placed on a vibration isolation platform and kept at a pressure of $\sim 10^{-2}$ mbar.

A parametric transducer is used to detect the mechanical vibrations of the SB resonator, see Ref. [15] for details. For that purpose, the WGM sapphire resonator serves as a dispersive element inside a closed electronic loop, which together with an amplifier and a phase shifter constitutes a microwave oscillator operating at the resonance frequency of the chosen WGM mode [28]. An interferometric frequency control system (FCS) suppresses spurious phase fluctuations and locks the microwave oscillator to the frequency of the $\mathrm{WGE}_{15,1,1}$ mode at $\omega_{\mathrm{WGE}}/2\pi \simeq 9.774$ GHz [29]. The in-loop voltage-controlled attenuator $\alpha$ is used for the parametric excitation of the mechanical vibrations at 127 kHz. Approximately half of the generated power inside the microwave sapphire oscillator is diverted to the interferometric frequency discriminator (IFD). The output signal of the IFD is a linear function of its input frequency and is measured with an HP 89410A spectrum analyzer. All instruments are time referenced to the hydrogen maser frequency standard VCH-103.

The spatial overlap between microwave and mechanical modes results in an interaction between these degrees of freedom, which can be described by the standard optomechanical Hamiltonian $\hat{H}_{\mathrm{int}} = -\hbar g_0 \hat{a}^\dagger \hat{a}\, \hat{x}$, where $g_0$ is a single photon optomechanical coupling, $\hat{a}^\dagger, \hat{a}$ are raising and lowering operators for the WGM, and $\hat{x}$ is the canonical position operator for the center of mass mechanical motion [30]. The microwave signal modulated at the resonance frequency of mechanical mode $\Omega_0$ induces radiation-pressure force which drives mechanical vibrations. The calibration of the amplitude of the center of mass motion is made by using the standard expression

$$\delta u(\Omega) = \delta x(\Omega)(du/df)(df/dx), \tag{5}$$

where $df/dx$ is determined from the amplitude of the output IFD signal $\delta u(\Omega)$. That signal is proportional to the applied modulated power $\delta P$,

$$\delta u(\Omega) = \chi \left(\frac{du}{df}\right)\left(\frac{df}{dx}\right)^2 \delta P, \tag{6}$$

where $\chi$ is the constant describing electromagnetic coupling of the signal and mechanical property of the oscillator [15]. The transduction constant is calculated to be $\delta x/\delta u = 526$ nm/mV.

The mechanical response of the SB resonator to the acoustic excitation in the vicinity of the resonance frequency is shown in Fig. 2(a). The applied excitation signal at 127 kHz is relatively weak, resulting in the maximal

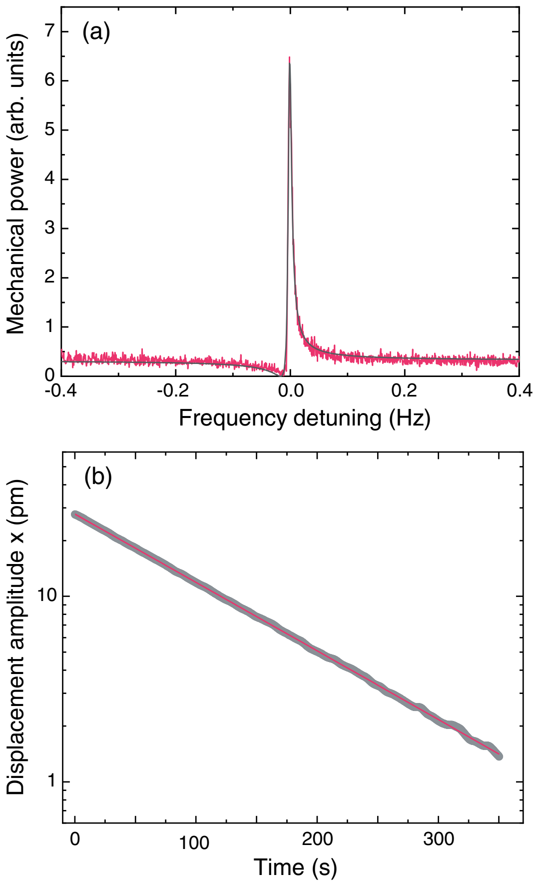


FIG. 2. (a) Mechanical response of the SB resonator to the acoustic excitation of the in-phase breathing mode around 127 kHz. The solid curve shows the fit to the quadrature signal due to double-balanced mixing. (b) Typical ringdown measurement of the in-phase breathing mode. The solid curve shows the fit to the exponential decay.

amplitude of mechanical vibrations of 6 pm. The output signal is measured by using a phase-sensitive interferometric setup which results in superposition of dispersive and absorptive quadrature components. The solid curve displays the fit of the experimental data to such a composite absorptive-dispersive response and yields the resonance frequency of the mechanical resonator $\Omega_0/2\pi = 127070.9695 \pm 0.0003$ Hz, its FWHM linewidth $\Gamma_M/2\pi = 3.5$ mHz, and the 55° mismatch between the arms of the IFD.

The ringdown measurements of the mechanical vibrations are made in two steps. In the first step the resonance frequency of mechanical vibrations $\Omega_0/2\pi$ is determined. For that purpose, the radiation-pressure force is applied to the resonator for the time sufficient to settle the mechanical vibrations (several minutes). Then, the output signal $\delta u(\Omega)$ is measured for every frequency point in the scanning range of 1 Hz. The resonance frequency corresponds to the point which yields the maximal IFD response $\delta u(\Omega)$. This procedure is repeated for the different excitation amplitudes (20–35 pm) in every single experimental run and detected no resonance frequency shift within an accuracy of 10 mHz determined by the resolution bandwidth of a FFT analyzer. In the second step, after the mechanical resonance frequency is located, the amplitude modulation (AM) excitation is turned off, and then the mechanical vibrations are measured as they decrease due to acoustic losses. The amplitude and frequency of the decaying vibrations, i.e., the amplitude and the frequency of the spectral peak, are then tracked and recorded every 0.2 s. For that purpose a marker is placed on the maximum voltage value in the spectra, and its frequency and amplitude are recorded for every time bin. The frequency accuracy of such measurements is determined by the resolution bandwidth of the FFT analyzer, which is set to 5 Hz.

The typical ringdown measurement is presented in Fig. 2(b). For this particular example, the resonant frequency is $\Omega_0/2\pi = 127070.97$ Hz. The solid curve shows the fit of the experimental data to the exponential decay with characteristic time constant $\tau = 173$ s, which yields the mechanical quality factor $Q_M = \Omega_0\tau/2 = 3.4 \times 10^7$ and the same FWHM linewidth $\Gamma_M$ which is found in mechanical response measurements. In addition to the extracting of the parameters of the exponential decay, the Duffing equation was numerically solved in order to attain the best fit parameters for the ringdown amplitudes. Following this procedure we extract the upper limit for the QG model parameter to be $\beta_0 < 6 \times 10^{11}$.

The frequency measurements yield much more stringent limit on $\beta_0$. In all measured ringdown series, there is no evidence of any detectable frequency shift up to the maximum amplitude of mechanical displacement of 75 pm. The null-frequency shift measured in the experiment corresponds to an accuracy of $\delta\Omega/\Omega_0 = 3.9 \times 10^{-5}$ and accordingly Eq. (4) yields the upper limit for the QG model parameter $\beta_0 < 5.2 \times 10^6$.

The sapphire SB resonator demonstrates a large potential for an even more stringent test of $\beta_0 \ll 1$. Here, we propose two possible ways to improve the experiment. Firstly, the mechanical response [Fig. 2(a)] could be measured for much larger input power $\delta P$. It is possible to excite vibrations in a sapphire resonator with an amplitude of several nanometers [27]. That would result in a much higher signal-to-noise ratio and as a consequence would improve the accuracy of determination of the mechanical resonance frequency $\Omega_0$ to better than 0.1 mHz. Together with an increase of the oscillation amplitude up to 0.1–1 nm scale, the upper limit on the correction strength may potentially be improved by at least 8 orders of magnitude arriving at the level $\beta_0 \lesssim 10^{-2}$. Secondly, one can implement an electromechanical sapphire oscillator by closing a feedback loop with the IFD output signal $\delta u(\Omega_M)$. In that case the uncertainty in determination of frequency shift will be decreased with the integration time $T$ as $1/\sqrt{\Omega_0 T}$. Assuming a driving amplitude of the SB resonator of $A \simeq 1$ nm and an average time of $T = 1$ h, the testable limit $\beta_0 \ll 1$ is within experimental accuracy.

## IV. ESTIMATION OF THE CORRECTION STRENGTH $\beta_0$ WITH BAW

Another mechanical system, namely a quartz bulk BAW resonator, also constitutes a fruitful platform for precise tests of quantum gravity. This system exhibits a high resonance frequency $\Omega_0/2\pi \simeq 10$ MHz, a milligram scale of the effective mass of oscillating modes [31], a large Q-factor close to $10^{10}$ at low temperatures [32], and a high-frequency stability of electromechanical oscillators reaching a level of $5 \times 10^{-14}$. The above listed features of quartz BAW are very attractive for fundamental tests such as Lorentz symmetry [33]. However, we note that quartz crystals possess their own quite strong elastic nonlinearities that can mimic the quantum gravity effect. These nonlinearities lead to a similar frequency shift, quadratic in amplitude and known as the amplitude-frequency effect or isochronism, see Ref. [34], p. 245. This effect can be made to nearly vanish by means of an optimal choice of the cut angle of the crystal, known as the LD-cut [18,35]. The QG correction strength can be estimated from Eq. (4) and by using the experimental parameters $m_{\mathrm{eff}} = 5$ mg, $\Omega_0/2\pi = 10$ MHz, $\delta\Omega/2\pi \simeq 1$ mHz, $A \simeq 1$ nm. Our estimation yields $\beta_0 \lesssim 4 \times 10^4$, which is still limited by elastic nonlinearity. In order to single out the quantum gravity frequency from such nonlinearity, the amplitude-frequency shift shall be measured in dependence on the effective mass of the resonating mode. We also believe that experimenting with kilogram-scale quartz BAW [36] will result in a much more stringent test of the quantum gravity model parameter in regime $\beta_0 \lesssim 0.1$, because of weaker nonlinearity due to the lower acoustic energy density and much larger effective mass.

## V. ESTIMATION OF THE CORRECTION STRENGTH $\beta_0$ WITH PHYSICAL PENDULUMS

At present, the most stringent limit on correction strength $\beta_0$ is arguably set by the experiments with physical pendulums, e.g., by the 1936 Atkinson measurements of the period of a kilogram-scale physical pendulum as a function of its amplitude. The combination of relatively large angular amplitudes, $\theta_0 \sim 1°$, and large mass, $\sim 1$ kg, makes the pendulum an ideal object for testing the generalized commutator described by Eq. (1). Since the physical pendulum possesses an intrinsic softening nonlinearity, leading to a quadratic (in the first order) dependence of the oscillation period on the angular amplitude, the QG correction can be considered an additional quadratic nonlinearity that can be determined from the accurate measurement of the period-amplitude dependence and comparison of the result against the well-known formula, see Ref. [37]. Using a Legendre polynomial approximation of the elliptic integral in the exact dependence of the pendulum period on angular amplitude $\theta_0$, one can rewrite Eq. (4) for the physical pendulum as follows:

$$\frac{\delta T(\theta_0)}{T_0} = \left[\frac{1}{16} - \beta_0 \left(\frac{2\pi m L}{M_p c T_0}\right)^2\right]\theta_0^2 + \frac{11}{3072}\theta_0^4, \qquad (7)$$

where $\delta T(\theta_0)$ is the amplitude-dependent deviation of the period of the pendulum $T_0$, $m$ is the mass of the pendulum, $\theta_0 \simeq A/L$ is the angular amplitude of the pendulum, and $L$ is its length. The numeric terms describe the intrinsic nonlinearity of physical pendulum. The dependence between the rate and arc ($\theta_0$) for the free pendulum was measured already in 1936 [20], using a pendulum with length $L = 1$ m and mass $m \simeq 6$ kg. Using the data in this work, we estimate $\beta_0 \lesssim 10^{-4}$. Other experiments with different kinds of pendulums carried at different times show no evidence of the deviation of the oscillation period from the conventional theory of a physical pendulum [38,39], and result in a similar estimate on the upper limit for $\beta_0$ [40].

In general, a mechanical pendulum cannot serve as an accurate clock, for its resonance frequency (or period) fluctuates over time due to various environmental factors: temperature, humidity, pressure, ageing, local gravity variation, seismic excitation, etc. At best such a system provides fractional frequency stability at the $10^{-5}$ level at 1 s, compared to $10^{-14}$ for the best quartz BAW. Spurious modes of the pendulum can also lead to significant cross talk between the modes, leading to unaccounted nonlinearities. The lack of data on such crucial metrological characteristics of the experiment as Allan deviation, i.e., the frequency stability of the oscillations, the Q-factor of a system, and the absence of any information on systematic errors ensuing from the design of the suspension point, allows only a qualitative conclusion that $\beta_0 \ll 1$. For a proper measurement of $\beta_0$ one has to repeat the experiments with pendulums or with metrological systems such as SB sapphire resonators or BAWs in a systematic metrological way.

## VI. CONCLUSION

In this work, we designed an experiment to test the modification of dynamics of a mechanical oscillator following from the generalized uncertainty principle predicted by some phenomenological theories of quantum gravity. We set an upper limit on the QG correction strength $\beta_0$ to the canonical commutator (3), using an ultra-high-Q mechanical sapphire resonator with a subkilogram mass of the resonating mode. In the original work [3] an experiment was proposed where the sequence of light pulses separated by a quarter of a mechanical oscillation period was reflected off an oscillator four times before measuring its phase, which should depend on $\beta_0$. Our

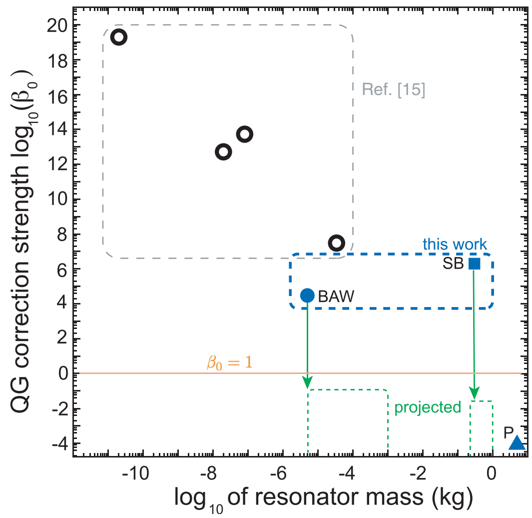


FIG. 3. The correction strength $\beta_0$ versus mass of mechanical oscillator determined in various experiments. The open circles correspond to $\beta_0$ reported in Ref.[19]. The closed circle is the estimated $\beta_0$ for the quartz BAW at LD-cut, see Ref. [18]. The square is the upper limit for $\beta_0$ obtained with a sapphire split-bar resonator. The triangle shows the estimate of the correction strength from the measurements of the period of the physical pendulum, see Ref. [20].

results show that the same goal can be attained in a simpler and more reliable way using continuous rf measurements of frequency of the electromechanical oscillator that can be measured with higher precision than any other physical parameter.

The overview experimental tests of correction strength $\beta_0$ using mechanical oscillators is presented in Fig. 3, where the measured $\beta_0$ is plotted as a function of effective mass of the mechanical mode. Evidently, heavier oscillators result in more stringent limits on correction strength. Although massive mechanical pendulums could be used to set a limit on $\beta_0 \ll 1$ in the variant of GUP relation described by Eq. (1), the high Q-factor and unprecedented frequency stability of the state-of-the-art quartz BAW resonators and SB sapphire resonators are more promising for precise tests of a more intuitive form of the GUP described by Eq. (2) that follows from the minimal length scale conjecture of quantum gravity theories [41] as it requires a quantum level of sensitivity in frequency measurement. Having said that, the perspective of utilizing low-frequency ($<1$ Hz) mechanical oscillators or pendulums in this context remains unclear compared to the high-frequency systems (kHz–GHz), where the near quantum regime [42] or even the quantum limit [43–45] has already been reached.

## ACKNOWLEDGMENTS

The research was supported by Australian Research Council Centre of Excellence for Engineered Quantum Systems Grant No. CE170100009. P. A. B. thanks M. Plenio, R. Blatt, F. Scardigli, A. Vikman, and P. Bosso for valuable discussions. S. D. would like to thank the Lower Saxonian Ministry of Science and Culture, which supported his research within the frame of the program Research Line (Forschungslinie) QUANOMET Quantum- and Nano-Metrology. The authors are also very thankful to Y. Chen and the members of the MQM discussion group for insightful conversations that inspired this work.

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