Strongly coupled $\mathcal{N}=4$ supersymmetric Yang-Mills plasma on the Coulomb branch. II. Transport coefficients and hard probe parameters

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We study $\mathcal{N}=4$ super Yang-Mills theory on the Coulomb branch (cSYM) by using its Type IIB supergravity dual. We compute the transport coefficients and hard probe parameters of $\mathcal{N}=4$ cSYM at finite temperature T. We use the rotating black 3-brane solution of Type IIB supergravity with a single nonzero rotation parameter r_0 after analytically continuing $r_0 \to -ir_0$, and in an ensemble where the Hawking temperature T and a scalar condensate $\langle\mathcal{O}\rangle \sim r_0^4$ are held fixed. We find that the bulk viscosity to entropy density ratio of the large black hole branch decreases with temperature and has a maxima around the critical temperature T_c, while, for the small black hole branch, it increases with temperature. The other transport coefficients and parameters of hard probes, such as the conductivity, jet quenching parameter, drag force, and momentum diffusion coefficients of the large black hole branch increase with temperature and asymptote to their conformal value, while, for the small black hole branch, they decrease with temperature.

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I. INTRODUCTION

The AdS/CFT correspondence [1–3] is an important tool to compute the hydrodynamic transport coefficients and hard probe parameters of a strongly coupled plasma [4–11].

In this paper, we use the AdS/CFT correspondence to study a strongly coupled $\mathcal{N}=4$ super Yang-Mills (SYM) plasma on the Coulomb branch. In this branch, a scale Λ is generated dynamically through the Higgs mechanism where the scalar particles Φ_i $(i=1...6)$ of $\mathcal{N}=4$ SYM acquire a nonzero vacuum expectation value (VEV) that breaks the conformal symmetry, and the gauge symmetry $SU(N_c)$ to its subgroup $U(1)^{N_c-1}$, but preserves $\mathcal{N}=4$ supersymmetry and the gauge coupling is not renormalized [12].

The thermodynamics of $\mathcal{N}=4$ super Yang-Mills on the Coulomb (cSYM) is investigated in some detail in [13]. In this paper, we will study its hydrodynamic transport coefficients and hard probes by using its dual geometry given by a rotating black 3-brane solution of Type IIB supergravity with a single nonzero rotation parameter r_0 [12,14–18], after analytically continuing $r_0 \to -ir_0$, and in an ensemble where the Hawking temperature T and a scalar condensate $\langle\mathcal{O}\rangle \sim r_0^4$ are held fixed [13].

So far, the computations of the transport coefficients of the rotating black 3-brane have been limited to the grand canonical ensemble (where temperature T and angular velocity Ω or chemical potential μ are held fixed) and canonical ensemble (where temperature T and angular momentum density J or charge density $\rho = \langle J^0\rangle$ are held fixed), see [17–20]. In [21,22], it was shown that for planar rotating black 3-branes the two ensembles have different thermodynamics; for example, there is Hawking-Page transition in the canonical ensemble but not in the grand canonical ensemble.

The outline of this paper is as follows: In Sec. II, we write down the 5-dimensional Type IIB supergravity action and its rotating black 3-brane or R-charged black hole solution.

In Sec. III, we compute the hydrodynamic transport coefficients, such as shear viscosity, bulk viscosity and conductivity of the rotating black 3-brane solution dual to $\mathcal{N}=4$ super Yang-Mills on the Coulomb branch (cSYM) at strong coupling.

In Sec. IV, we calculate the drag force, momentum diffusion coefficient, and jet quenching parameter on the rotating black 3-brane solution.

II. TYPE IIB SUPERGRAVITY ACTION AND BACKGROUND SOLUTION

The action for the $U(1)^3$ consistent truncation of Type IIB supergravity on S^5 is given by [23,24], see also [25,26],

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$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g_5}\mathcal{L}_{\rm bulk}, \tag{2.1}$$

where

$$\mathcal{L}_{\rm bulk} = (\mathcal{R} - V) - \frac{1}{2}\sum_{I=1}^{2}(\partial\varphi_I)^2 - \frac{1}{4}R^2\sum_{a=1}^{3}X_a^{-2}(F^a)^2,$$
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu, \qquad V = -\frac{4}{R^2}\sum_{a=1}^{3}X_a^{-1},$$
$$X_1 = e^{-\frac{1}{\sqrt{6}}\varphi_1 - \frac{1}{\sqrt{2}}\varphi_2}, \qquad X_2 = e^{-\frac{1}{\sqrt{6}}\varphi_1 + \frac{1}{\sqrt{2}}\varphi_2}, \qquad X_3 = e^{\frac{2}{\sqrt{6}}\varphi_1}. \tag{2.2}$$

We have dropped the Chern-Simons term from the action (2.1) since it does not play any role in our discussion below.

In this paper, we compute the hydrodynamic and hard probe transport coefficients the following rotating black 3-brane solution of the above action (2.1) [19,27]

$$ds^2_{(5)} = \frac{r^2}{R^2}H^{1/3}(-f dt^2 + dx^2 + dy^2 + dz^2) + \frac{H^{-2/3}}{\frac{r^2}{R^2}f}dr^2, \tag{2.3}$$

where

$$f = 1 - \frac{r_h^4}{r^4}\frac{H(r_h)}{H(r)}, \qquad H = 1 - \frac{r_0^2}{r^2}, \tag{2.4}$$

$$\varphi_1 = \frac{1}{\sqrt{6}}\ln H, \qquad \varphi_2 = \frac{1}{\sqrt{2}}\ln H,$$
$$A^1_t = i\frac{r_0}{R^2}\frac{r_h^2\sqrt{H(r_h)}}{r^2 H(r)},$$
$$r_h^2 = \frac{1}{2}\left(r_0^2 + \sqrt{r_0^4 + 4m}\right), \tag{2.5}$$

$\kappa = \frac{r_0^2}{r_h^2}$, and $A^2_t = A^3_t = 0$. Our metric (2.3) is equivalent to the metric used in [19] after analytically continuing $r_0 \to -i\sqrt{q}$. Note that having an imaginary gauge potential does not lead to any inconsistencies in the 5-dimensional bulk spacetime, since all physical quantities in the bulk are given in terms of $(\partial_r A^1_t)^2$. In the field theory side, having a finite imaginary chemical potential μ means that we are exploring the phase diagram of $\mathcal{N} = 4$ cSYM at finite temperature T and imaginary chemical potential μ; see [28–31] for the study of the QCD phase diagram on the lattice at finite imaginary chemical potential without any inconsistencies.

The Hawking temperature T of the rotating black 3-brane solution (2.3) is given by

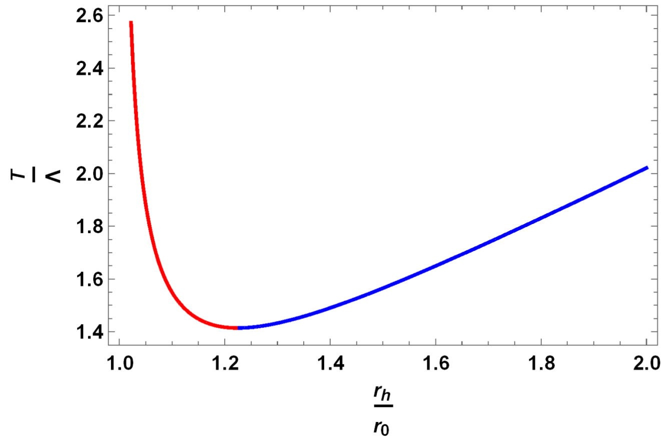

FIG. 1. Hawking temperature $\frac{T}{\Lambda}$ vs the radius of the horizon $\frac{r_h}{r_0}$ (2.6), normalized by the energy scale Λ and rotation parameter r_0, respectively.

$$\frac{T}{\Lambda} = \frac{1 - \frac{1}{2}\kappa}{\sqrt{\kappa - \kappa^2}}, \tag{2.6}$$

where $T_0 = \frac{r_h}{\pi R^2}$, $\Lambda = \frac{r_0}{\pi R^2}$, and $\kappa = \frac{r_0^2}{r_h^2} = \frac{\Lambda^2}{T_0^2}$. We have plotted $\frac{T}{\Lambda}$ in Fig. 1. We can also invert (2.6) to find

$$\kappa = \frac{1 + \frac{T^2}{\Lambda^2}(1 \mp \sqrt{\frac{T^2}{\Lambda^2} - 2})}{\frac{1}{2} + 2\frac{T^2}{\Lambda^2}}, \tag{2.7}$$

and

$$\frac{T_0^2}{T^2} = \frac{2 + \frac{1}{2}\frac{\Lambda^2}{T^2}}{1 + \frac{T^2}{\Lambda^2}(1 \mp \sqrt{\frac{T^2}{\Lambda^2} - 2})}. \tag{2.8}$$

Note that in (2.7) and (2.8) "−" corresponds to large black hole branch and "+" corresponds to small black hole branch.

The entropy density $s(T, \Lambda)$, for our ensemble where T and Λ are held fixed, is given by

$$s(T,\Lambda) = \frac{A_H}{4G_5 V_3} = \frac{1}{4G_5}\sqrt{g_{xx}(r_h)g_{yy}(r_h)g_{zz}(r_h)}$$
$$= \frac{\pi^2 N_c^2 T_0^3}{2}(1-\kappa)^{1/2}, \tag{2.9}$$

where $G_5 = \pi R^3/2N_c^2$, and V_3 is the three-dimensional volume.

III. HYDRODYNAMIC TRANSPORT COEFFICIENTS OF $\mathcal{N} = 4$ CSYM PLASMA

The transverse metric fluctuation $h_{xy}(t, z, r)$ decouples from other fluctuations; hence the shear viscosity for a general background metric $g_{\mu\nu}$ is given by [32]

$$\eta = \frac{1}{16\pi G_5}\sqrt{g_{xx}(r_h)g_{yy}(r_h)g_{zz}(r_h)}\frac{g_{xx}(r_h)}{g_{yy}(r_h)} = \frac{s}{4\pi}\frac{g_{xx}(r_h)}{g_{yy}(r_h)}. \quad (3.1)$$

Since, for our background metric (2.3) $g_{xx}=g_{yy}$, the shear viscosity η of $\mathcal{N}=4$ cSYM is simply

$$\frac{\eta}{s} = \frac{1}{4\pi}. \quad (3.2)$$

Bulk viscosity ζ can be computed by closely following [11]. To this end, we first replace $\varphi_1 \to \frac{1}{2}\tilde{\varphi}_1$ followed by $\varphi_2 \to \frac{\sqrt{3}}{2}\tilde{\varphi}_1$, to bring the Einstein-Maxwell-scalar part of our action (2.1) in the same form as the action used in [11], i.e.,

$$(16\pi G_5)\frac{\mathcal{L}}{\sqrt{-g_5}} = (\mathcal{R} - \tilde{V}(\tilde{\varphi}_1)) - \frac{1}{2}(\partial\tilde{\varphi}_1)^2 + \cdots, \quad (3.3)$$

where

$$\tilde{V}(\tilde{\varphi}_1) = -\frac{4}{R^2}\left(e^{\frac{2}{\sqrt{6}}\tilde{\varphi}_1}\left(1+\frac{\kappa(1-\kappa)}{2\kappa^3}(e^{-\frac{3}{\sqrt{6}}\tilde{\varphi}_1}-1)^3\right) + 2e^{-\frac{1}{\sqrt{6}}\tilde{\varphi}_1}\right). \quad (3.4)$$

In the $\tilde{r}=\varphi_1(r)$ gauge, the bulk viscosity ζ up to a constant is [11]

$$\frac{\zeta}{s} \propto \frac{1}{4\pi}\frac{\tilde{V}'(\tilde{r}_h)^2}{\tilde{V}(\tilde{r}_h)^2}, \quad (3.5)$$

where $'$ denotes the derivative with respect to $\tilde{r}=\tilde{\varphi}_1(r)$. Note that, in the gauge $\tilde{r}=\tilde{\varphi}_1(r)=\frac{2}{\sqrt{6}}\ln H(r)$, the horizon of the black hole is located at $\tilde{r}=\tilde{r}_h=\frac{2}{\sqrt{6}}\ln H(r_h)=\frac{2}{\sqrt{6}}\ln(1-\kappa)$ where κ is still given by (2.7). We have plotted $\frac{\zeta}{s}$ in Fig. 2

The conductivity σ_f of a $U(1)$ flavor charge can simply be computed using the general formula [5,33]

$$\sigma_f = \frac{1}{g_5^2}\sqrt{g_{xx}(r_h)g_{yy}(r_h)g_{zz}(r_h)}g^{xx}(r_h) = \frac{N_c N_f T_0}{4\pi}(1-\kappa)^{1/6}, \quad (3.6)$$

where we used $g_5^2=\frac{4\pi^2 R}{N_c N_f}$ and a bulk $U(1)$ flavor action of the form $S_f=-\frac{1}{4g_5^2}\int d^5x\sqrt{-g}F^2$ which can be derived from the low-energy limit of the Dirac-Born-Infeld action of probe N_f D7-branes [34]. Note that there is no mixing between the gravitational and flavor gauge field fluctuations. We have plotted σ_f in Fig. 3.

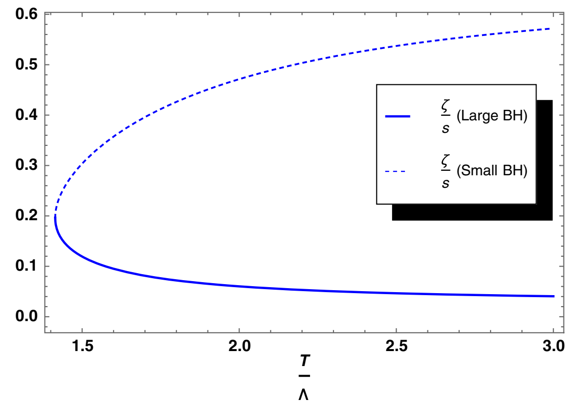

FIG. 2. The bulk viscosity to entropy density ratio $\frac{\zeta}{s}$ of $\mathcal{N}=4$ SYM plasma on the Coulomb branch for both large and small black holes (3.5).

The conductivity σ_{R} of a single R-charge can be computed by directly computing the two-point retarded correlation functions $G^{\mu\nu}$ of the spatial component of the R-current J^μ, in the presence background A_t^1 which results in mixing between the gravitational and gauge field fluctuations, and using Kubo formula, i.e.,

$$\sigma_R = \lim_{\omega\to 0} -\frac{1}{\omega}\mathrm{Im}G^{xx}(\omega,\mathbf{k}=0) = \frac{N_c^2 T_0}{32\pi}\frac{(2-\kappa)^2}{\sqrt{1-\kappa}}, \quad (3.7)$$

where in the last line we used $G^{xx}=\frac{-i(2-\kappa)^2 N_c^2 T_0\omega}{32\pi\sqrt{1-\kappa}}$ which is nothing but Eq. 4.34 of [19] after replacing $\kappa\to-\kappa$, and $G^{xx}\to\frac{1}{2}G^{xx}$ to compensate for the different normalization we have for the gauge fields. We have plotted σ_R in Fig. 3.

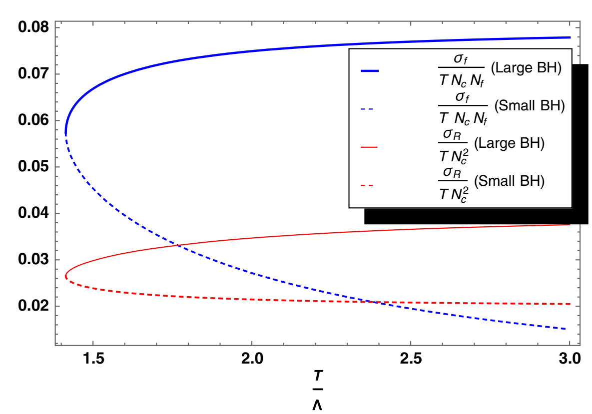

FIG. 3. The conductivity $\frac{\sigma_f}{TN_cN_f}$ of a $U(1)$ flavor charge (3.6), and $\frac{\sigma_R}{TN_c^2}$ of a single R-charge (3.7) of flavored and unflavored $\mathcal{N}=4$ SYM plasma, respectively, on the Coulomb branch for both large and small black holes.

IV. DRAG FORCE, MOMENTUM DIFFUSION AND JET QUENCHING IN $\mathcal{N}=4$ CSYM PLASMA

The Nambu-Goto (NG) action is

$$S_{\rm NG} = \int d\tau d\sigma \mathcal{L}(h_{ab}) = -\frac{1}{2\pi\alpha'}\int d\tau d\sigma \sqrt{-det\, h_{ab}}, \tag{4.1}$$

where the background induced metric on the string h_{ab} is given by

$$h_{ab} = g_{\mu\nu}\partial_a x^\mu(\tau,\sigma)\partial_b x^\nu(\tau,\sigma). \tag{4.2}$$

Using the embedding $(\tau,\sigma)\Rightarrow(t(\tau,\sigma),0,0,x(\tau,\sigma), r=\sigma)$, the background induced metric $h_{ab}(\dot{z},z')$ (4.2) becomes $(\cdot\equiv d/d\tau,'\equiv d/d\sigma)$

$$h_{ab}(\dot{x},x') = g_{tt}\partial_a t\partial_b t + g_{xx}\partial_a x\partial_b x + g_{rr}\partial_a r\partial_b r. \tag{4.3}$$

Using a particular Ansatz of the form $t(\tau,\sigma)=\tau+K(\sigma)$ and $z=v\tau+F(\sigma)$, which represents a "trailing string" configuration moving with velocity v, the background induced metric (4.3) becomes [35]

$$\begin{aligned} h_{\tau\tau}(v,x') &= g_{tt}+v^2 g_{xx},\\ h_{\sigma\sigma}(v,x') &= g_{tt}(K')^2 + g_{xx}(x')^2 + g_{rr},\\ h_{\tau\sigma}(v,x') &= g_{tt}K' + g_{xx}x'v. \end{aligned} \tag{4.4}$$

Finding the equation of motion from the action, we have

$$\partial_\sigma\left(\frac{g_{tt}g_{xx}(x'-vK')}{\sqrt{-det\, h_{ab}}}\right) = 0. \tag{4.5}$$

Requiring $h_{\tau\sigma}(v,x')=0$ to fix this gauge freedom, we have an additional constraint $K'=-\frac{g_{xx}}{g_{tt}}x'v$, which can be used to diagonalize (4.4) as [35]

$$\begin{aligned} h_{\tau\tau}(v,x') &= g_{tt}\left(1+v^2\frac{g_{xx}}{g_{tt}}\right),\\ h_{\sigma\sigma}(v,x') &= \left(1+v^2\frac{g_{xx}}{g_{tt}}\right)g_{xx}(x')^2+g_{rr}. \end{aligned} \tag{4.6}$$

Solving the equation of motion, in this gauge, for x', we find

$$(x')^2 = \frac{-C^2 g_{rr}}{g_{xx}^2 g_{tt}}\frac{1}{(1+v^2\frac{g_{xx}}{g_{tt}})(1+\frac{C^2}{g_{tt}g_{xx}})}, \tag{4.7}$$

where the integration constant C is related to the conjugate momenta $\Pi=\frac{\partial\mathcal{L}}{\partial x'}=-\frac{C}{2\pi\alpha'}$. Since the factor $1+v^2\frac{g_{xx}}{g_{tt}}$ in (4.7), for $v\neq 0$, vanishes when $-\frac{g_{tt}(r_s)}{g_{xx}(r_s)}=v^2$, requiring $(x')^2$ to be positive across $r=r_s$, the other factor $1+\frac{C^2}{g_{tt}g_{tt}}$ has to vanish at $r=r_s$ as well, which will fix the integration constant $C^2=-g_{tt}(r_s)g_{xx}(r_s)$ for $v\neq 0$.

So, the induced metric (4.6) for $v\neq 0$ becomes

$$\begin{aligned} h_{\tau\tau}(v,x') &= g_{tt}\left(1-\frac{g_{tt}(r_s)}{g_{tt}}\frac{g_{xx}}{g_{xx}(r_s)}\right),\\ h_{\sigma\sigma}(v,x') &= g_{rr}\left(\frac{1}{1-\frac{g_{xx}(r_s)g_{tt}(r_s)}{g_{xx}g_{tt}}}\right), \end{aligned} \tag{4.8}$$

which can be interpreted as a metric of a 2-dimensional black hole with a line element $ds^2_{(2)}$ given by

$$ds^2_{(2)} = h_{\tau\tau}d\tau^2 + h_{\sigma\sigma}d\sigma^2 = -g_{tt}(-\tilde{f}(r))d\tau^2 + \frac{1}{\tilde{p}(r)}d\sigma^2, \tag{4.9}$$

where $\tilde{f}(r)=1-\frac{g_{tt}(r_s)}{g_{tt}}\frac{g_{xx}}{g_{xx}(r_s)}$, $\tilde{p}(r)=g^{rr}(1-\frac{g_{xx}(r_s)g_{tt}(r_s)}{g_{xx}g_{tt}})$. The radius of the horizon r_s of the 2-dimensional black hole is found by solving the algebraic equation $-\frac{g_{tt}(r_s)}{g_{xx}(r_s)}=v^2$. And the Hawking temperature of the 2-dimensional black hole denoted as T_s is

$$T_s = \frac{1}{4\pi}\sqrt{-g_{tt}(r_s)\tilde{f}'(r_s)\tilde{p}'(r_s)}. \tag{4.10}$$

The drag force is given by [6,7], see also [35],

$$F_{\rm drag} = -\frac{C}{2\pi\alpha'} = -\frac{1}{2}\pi\sqrt{\lambda}T_0^2\gamma v Q(\kappa,\gamma), \tag{4.11}$$

where $Q(\kappa,\gamma)=\frac{\kappa}{2\gamma}\left(1+\sqrt{1+4\gamma^2\frac{1-\kappa}{\kappa^2}}\right)$ with the Lorentz factor $\gamma=\frac{1}{\sqrt{1-v^2}}$, and we have used $r_s^2=\gamma r_h^2 Q(\kappa,\gamma)$ which solves the algebraic equation $-\frac{g_{tt}(r_s)}{g_{xx}(r_s)}=v^2$. We have plotted $F_{\rm drag}$ in Fig. 4.

The velocity dependent transverse momentum diffusion constant per unit time $\kappa^\perp(v)$ is given by [35]

$$\kappa^\perp(v) = \frac{T_s}{\pi\alpha'}g_{xx}(r_s), \tag{4.12}$$

and the longitudinal momentum diffusion constant per unit time $\kappa^\parallel(v)$ is [36]

$$\kappa^\parallel(v) = \frac{T_s}{\pi\alpha'}\frac{1}{g_{xx}}\frac{(g_{tt}g_{xx})'}{(g_{tt}/g_{xx})'}\bigg|_{r=r_s}. \tag{4.13}$$

We have plotted $\kappa^\perp(0)$ and $\kappa^\parallel(0)$ in Fig. 5. Note from Fig. 5 that $\kappa^\perp(v)\neq\kappa^\parallel(v)$ even at $v=0$ in $\mathcal{N}=4$ cSYM plasma, even though they are equal to each other at $v=0$ in $\mathcal{N}=4$ SYM plasma. Also note that, as can be seen in Fig. 5, the

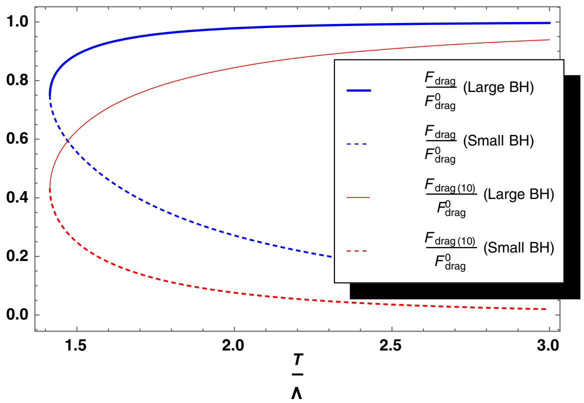

FIG. 4. The drag forces $\frac{F_{\text{drag}}}{F^0_{\text{drag}}}$ (4.11) and $\frac{F_{\text{drag}}(10)}{F^0_{\text{drag}}}$ of $\mathcal{N}=4$ SYM plasma on the Coulomb branch for both large and small black holes with $\gamma=1.0001$, normalized by the drag force $F^0_{\text{drag}}=-\frac{1}{2}\sqrt{\lambda}\pi T^2\gamma v$ of the conformal $\mathcal{N}=4$ SYM plasma [6,7].

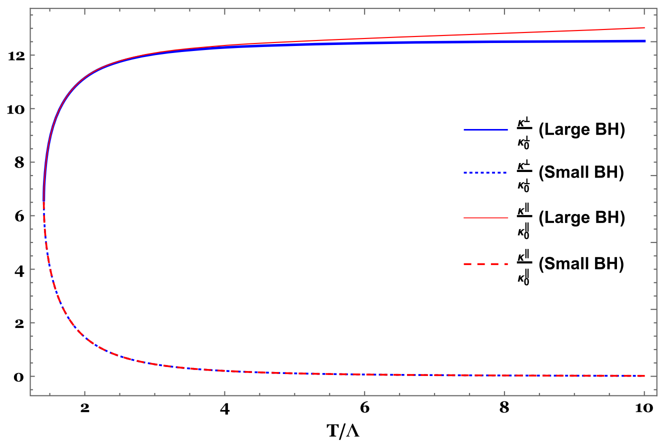

FIG. 5. The transverse and longitudinal momentum diffusion constants $\frac{\kappa^{\perp}(0)}{\kappa_0(0)}$ (4.12) and $\frac{\kappa^{\parallel}(0)}{\kappa_0(0)}$ (4.13), respectively, of $\mathcal{N}=4$ SYM plasma on the Coulomb branch for both large and small black holes, normalized by the momentum diffusion constant $\kappa_0(v)=\kappa_0(0)=\kappa_0^{\perp}(0)=\kappa_0^{\parallel}(0)=\sqrt{\lambda}\pi T^3$ of the conformal $\mathcal{N}=4$ SYM plasma [8,9]. Note that $\frac{\kappa^{\perp}(0)}{\kappa_0(0)}\simeq\frac{\kappa^{\parallel}(0)}{\kappa_0(0)}$ for the small black hole branch.

difference between $\kappa^{\perp}(v)$ and $\kappa^{\parallel}(v)$ gets enhanced with increasing T and v.

The 5-dimensional metric (2.3) can be uplifted to the full 10-dimensional metric as [12,14–18]

$$\begin{aligned} ds^2_{(10)} &= \frac{r^2}{R^2}\tilde{H}^{1/2}(-\tilde{f}dt^2+dx^2+dy^2+dz^2) \\ &\quad + R^2(\tilde{H}^{1/2}d\theta^2+H\tilde{H}^{-1/2}\sin^2\theta d\phi^2 \\ &\quad + \tilde{H}^{-1/2}\cos^2\theta d\Omega_3^2)+2A_t^1 H\tilde{H}^{-1/2}R^2\sin^2\theta dt d\phi \\ &\quad + \frac{\tilde{H}^{1/2}H^{-1}}{\frac{r^2}{R^2}f}dr^2, \end{aligned} \tag{4.14}$$

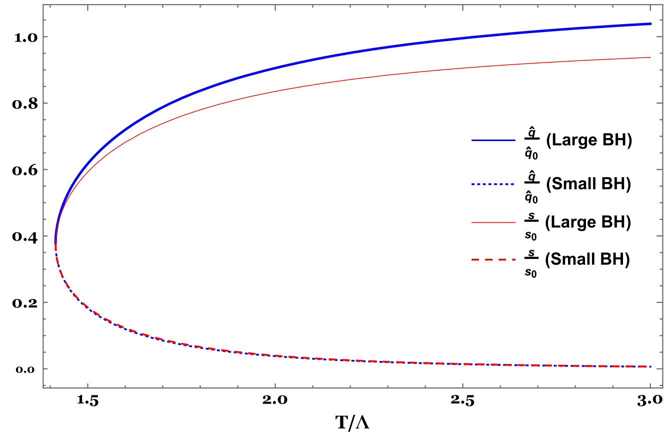

FIG. 6. The jet quenching parameter $\frac{\hat{q}}{\hat{q}_0}$ (4.17) and entropy density $\frac{s}{s_0}$ of $\mathcal{N}=4$ SYM plasma on the Coulomb branch for both large and small black holes, normalized by the jet quenching parameter $\hat{q}_0=\frac{\pi^{3/4}\Gamma(3/4)}{\sqrt{2}\Gamma(5/4)}\sqrt{\lambda}T^3$ and entropy density $s_0=\frac{1}{2}\pi^2N_c^2T^3$ of the conformal $\mathcal{N}=4$ SYM plasma. Note that $\frac{\hat{q}}{\hat{q}_0}\simeq\frac{s}{s_0}$ for the small black hole branch.

where

$$\tilde{H}=\sin^2\theta+H\cos^2\theta,\quad\text{and}\quad\tilde{f}=1-\frac{r_h^4}{r^4}\frac{H(r_h)}{\tilde{H}(r)},\tag{4.15}$$

f and H are the same as in (2.3). Our 10-dimensional metric (4.14) is equivalent to Eq. 2.21 of [18] after analytically continuing the rotation parameter $r_0\to -ir_0$, and rewriting (4.14) in terms of $\mu\equiv m^{1/4}$. Note that the $g_{t\phi}$ component of (4.14) is imaginary and one could make it real by analytically continuing $t\to -it$ as in [14,37]. However, since we are interested in real-time dynamics, such as computation of transport coefficients, we refrain from analytically continuing $t\to -it$, and we treat our 10-dimensional metric (4.14) as a complex saddle point. Also note that $g_{t\phi}=A_t^1=0$ in the extremal limit $r_h=r_0$.

In [17] the drag force was studied using the 10-dimensional metric (4.14), and it was shown that the drag force $F_{\text{drag}(10)}$ is (shown below after rewriting it in terms of κ, and making the analytic continuation $r_0\to -ir_0$ which is equivalent to replacing $\kappa\to -\kappa$)

$$F_{\text{drag}(10)}=-\frac{1}{2}\sqrt{\lambda}\pi T_0^2\sqrt{1-\kappa}\gamma v.\tag{4.16}$$

Note that (4.16) is equivalent to the $\gamma\to\infty$ limit of (4.11), and it has similar $\sqrt{1-\kappa}$ dependence as the entropy density (2.9) indicating that the drag force (4.16) could be the measure of the color degrees of freedom of the plasma [38]. We have plotted (4.16) in Fig. 4.

And, in [18], it was shown that the jet quenching parameter $\hat{q}$, studied using the 10-dimensional metric (4.14), is (shown below after rewriting it in terms of κ, and making the analytic continuation $r_0 \to -ir_0$ which is equivalent to replacing $\kappa \to -\kappa$)

$$\frac{\hat{q}}{\hat{q}_0} = \frac{\mathbf{K}(1/\sqrt{2})}{\mathbf{K}(n)} (2n^2)^2 (2n'^2)^{1/2}, \qquad (4.17)$$

where $\mathbf{K}(n)$ is the complete elliptic integral of the first kind, $n^2 = \frac{1-\kappa}{2-\kappa}$, $n' = \sqrt{1-n^2}$, and $\hat{q}_0 = \frac{\pi^{3/4}\Gamma(3/4)}{\sqrt{2}\Gamma(5/4)}\sqrt{\lambda}T^3$ [10,38]. In Mathematica, the complete elliptic integral of the first kind is implemented using EllipticK$[n^2] \equiv \mathbf{K}(n)$. We have plotted $\hat{q}$ in Fig. 6.

V. CONCLUSION

We have studied the transport coefficients of the non-extremal rotating black 3-brane dual to strongly coupled $\mathcal{N} = 4$ cSYM plasma, such as bulk viscosity to entropy density ratio $\frac{\zeta}{s}$ (3.5), and conductivity σ (3.6)(3.7), see Fig. 2 and Fig. 3, respectively. We have found that the bulk viscosity of the large black hole has a maxima around T_c, and its conductivity σ asymptotes to its conformal value starting from below it.

We have also computed the hard probe parameters of $\mathcal{N} = 4$ cSYM plasma. We have shown that the drag force F_{drag}, momentum diffusion coefficient κ, and jet quenching parameter $\hat{q}$ increase with temperature for the large black hole, see Figs. 4–6.

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[1] J. M. Maldacena, The large N limit of superconformal field theories and supergravity, Adv. Theor. Math. Phys. **2**, 231 (1998).

[2] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Gauge theory correlators from noncritical string theory, Phys. Lett. B **428**, 105 (1998).

[3] E. Witten, Anti-de Sitter space and holography, Adv. Theor. Math. Phys. **2**, 253 (1998).

[4] G. Policastro, D. T. Son, and A. O. Starinets, From AdS/CFT correspondence to hydrodynamics, J. High Energy Phys. 09 (2002) 043.

[5] N. Iqbal and H. Liu, Universality of the hydrodynamic limit in AdS/CFT and the membrane paradigm, Phys. Rev. D **79**, 025023 (2009).

[6] S. S. Gubser, Drag force in AdS/CFT, Phys. Rev. D **74**, 126005 (2006).

[7] C. P. Herzog, A. Karch, P. Kovtun, C. Kozcaz, and L. G. Yaffe, Energy loss of a heavy quark moving through $N = 4$ supersymmetric Yang-Mills plasma, J. High Energy Phys. 07 (2006) 013.

[8] S. S. Gubser, Momentum fluctuations of heavy quarks in the gauge-string duality, Nucl. Phys. **B790**, 175 (2008).

[9] J. Casalderrey-Solana and D. Teaney, Heavy quark diffusion in strongly coupled $N = 4$ Yang-Mills, Phys. Rev. D **74**, 085012 (2006).

[10] H. Liu, K. Rajagopal, and U. A. Wiedemann, Calculating the Jet Quenching Parameter from AdS/CFT, Phys. Rev. Lett. **97**, 182301 (2006).

[11] S. S. Gubser, S. S. Pufu, and F. D. Rocha, Bulk viscosity of strongly coupled plasmas with holographic duals, J. High Energy Phys. 08 (2008) 085.

[12] P. Kraus, F. Larsen, and S. P. Trivedi, The Coulomb branch of gauge theory from rotating branes, J. High Energy Phys. 03 (1999) 003.

[13] K. A. Mamo, preceding paper, Strongly coupled $\mathcal{N} = 4$ supersymmetric Yang-Mills plasma on the Coulomb branch. I. Thermodynamics, Phys. Rev. D **100**, 066010 (2019).

[14] A. Brandhuber and K. Sfetsos, Wilson loops from multicenter and rotating branes, mass gaps and phase structure in gauge theories, Adv. Theor. Math. Phys. **3**, 851 (1999).

[15] S. S. Gubser, Thermodynamics of spinning D3-branes, Nucl. Phys. **B551**, 667 (1999).

[16] M. Cvetic and S. S. Gubser, Phases of R charged black holes, spinning branes and strongly coupled gauge theories, J. High Energy Phys. 04 (1999) 024.

[17] E. Caceres and A. Guijosa, Drag force in charged $N = 4$ SYM plasma, J. High Energy Phys. 11 (2006) 077.

[18] S. D. Avramis and K. Sfetsos, Supergravity and the jet quenching parameter in the presence of R-charge densities, J. High Energy Phys. 01 (2007) 065.

[19] D. T. Son and A. O. Starinets, Hydrodynamics of r-charged black holes, J. High Energy Phys. 03 (2006) 052.

[20] O. DeWolfe, S. S. Gubser, and C. Rosen, Dynamic critical phenomena at a holographic critical point, Phys. Rev. D **84**, 126014 (2011).

[21] R. G. Cai and K. S. Soh, Critical behavior in the rotating D-branes, Mod. Phys. Lett. A **14**, 1895 (1999).

[22] X. Wu, Holographic entanglement entropy and thermodynamic instability of planar R-charged black holes, Phys. Rev. D **90**, 066008 (2014).

[23] M.Cvetič, M. J. Duff, P. Hoxha, J. T. Liu, H. Lü, J. X. Lu, R. Martinez-Acosta, C. N. Pope, H. Sati, and T. A. Tran, Embedding AdS black holes in ten-dimensions and eleven-dimensions, Nucl. Phys. **B558**, 96 (1999).

[24] M. Cvetic, H. Lu, C. N. Pope, A. Sadrzadeh, and T. A. Tran, Consistent SO(6) reduction of type IIB supergravity on S^5, Nucl. Phys. **B586**, 275 (2000).

[25] A. Donos, J. P. Gauntlett, and C. Pantelidou, Spatially modulated instabilities of magnetic black branes, J. High Energy Phys. 01 (2012) 061.

[26] K. A. Mamo and H. U. Yee, Thermalization of quark-gluon plasma in magnetic field at strong coupling, Phys. Rev. D **92**, 105005 (2015).

[27] K. Behrndt, M. Cvetic, and W. A. Sabra, Nonextreme black holes of five-dimensional $N = 2$ AdS supergravity, Nucl. Phys. **B553**, 317 (1999).

[28] A. Roberge and N. Weiss, Gauge theories with imaginary chemical potential and the phases of QCD, Nucl. Phys. **B275**, 734 (1986).

[29] M. G. Alford, A. Kapustin, and F. Wilczek, Imaginary chemical potential and finite fermion density on the lattice, Phys. Rev. D **59**, 054502 (1999).

[30] M. D'Elia and M. P. Lombardo, Finite density QCD via imaginary chemical potential, Phys. Rev. D **67**, 014505 (2003).

[31] P. de Forcrand and O. Philipsen, The QCD phase diagram for small densities from imaginary chemical potential, Nucl. Phys. **B642**, 290 (2002).

[32] K. A. Mamo, Holographic RG flow of the shear viscosity to entropy density ratio in strongly coupled anisotropic plasma, J. High Energy Phys. 10 (2012) 070.

[33] K. A. Mamo, Enhanced thermal photon and dilepton production in strongly coupled $N = 4$ SYM plasma in strong magnetic field, J. High Energy Phys. 08 (2013) 083.

[34] D. Mateos and L. Patino, Bright branes for strongly coupled plasmas, J. High Energy Phys. 11 (2007) 025.

[35] S. Li, K. A. Mamo, and H. U. Yee, Jet quenching parameter of the quark-gluon plasma in a strong magnetic field: Perturbative QCD and AdS/CFT correspondence, Phys. Rev. D **94**, 085016 (2016).

[36] S. I. Finazzo, R. Critelli, R. Rougemont, and J. Noronha, Momentum transport in strongly coupled anisotropic plasmas in the presence of strong magnetic fields, Phys. Rev. D **94**, 054020 (2016).

[37] M. Cvetic and S. S. Gubser, Thermodynamic stability and phases of general spinning branes, J. High Energy Phys. 07 (1999) 010.

[38] H. Liu, K. Rajagopal, and U. A. Wiedemann, Wilson loops in heavy ion collisions and their calculation in AdS/CFT, J. High Energy Phys. 03 (2007) 066.