


Strongly coupled $\mathcal{N} = 4$ supersymmetric Yang-Mills plasma on the Coulomb branch. I. Thermodynamics

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 (Received 14 June 2019; published 9 September 2019)

We study $\mathcal{N} = 4$ super Yang-Mills theory on the Coulomb branch (cSYM) in the strong coupling limit by using the AdS/CFT correspondence. The dual geometry is the rotating black 3-brane Type IIB supergravity solution with a single nonzero rotation parameter r_0 which sets a fixed mass scale corresponding to the scalar condensate $\langle \mathcal{O} \rangle \sim r_0^4$ in the Coulomb branch. We introduce a new ensemble where T and $\langle \mathcal{O} \rangle$ are held fixed; i.e., the free energy $F(T, \langle \mathcal{O} \rangle)$ is a function of T and $\langle \mathcal{O} \rangle$. We compute the equation of state (EoS) of $\mathcal{N} = 4$ cSYM at finite T , as well as the heavy quark-antiquark potential and the quantized mass spectrum of the scalar and spin-2 glueballs at $T = 0$. By computing the Wilson loop (minimal surface) at $T = 0$, we determine the heavy quark-antiquark potential $V(L)$ to be the Cornell potential, which is confining for large separation L . At $T \neq 0$, we find two black hole branches: the large black hole and small black hole branches. For the large black hole branch, that has positive specific heat, we find qualitatively similar EoS to that of pure Yang-Mills theory on the lattice. For the small black hole branch, that has negative specific heat, we find an EoS where the entropy and energy densities decrease with T . We also find a second-order phase transition between the large and small black hole branches with critical temperature $T_c = T_{\min}$.

DOI: [10.1103/PhysRevD.100.066010](https://doi.org/10.1103/PhysRevD.100.066010)

I. INTRODUCTION

The AdS/CFT correspondence [1–3] has opened a new window to the strongly coupled regime of gauge theories such as $\mathcal{N} = 4$ super Yang-Mills (SYM). Unfortunately, so far, we lack an exact string theory dual to QCD even though there are various works which explored different non-conformal deformations of $\mathcal{N} = 4$ SYM both through bottom-up approaches (where the details of the deformation of $\mathcal{N} = 4$ SYM and its string theory dual are unknown) [4–12], and top-down approaches (where both the details of the deformation of $\mathcal{N} = 4$ SYM and its string theory dual are known) [13–25].

In $\mathcal{N} = 4$ SYM on the Coulomb branch (cSYM) at zero temperature, a scale is introduced dynamically through the Higgs mechanism where the scalar particles Φ_i ($i = 1 \dots 6$) of $\mathcal{N} = 4$ SYM acquire a nonzero vacuum expectation value (VEV) that breaks the conformal symmetry, and the gauge symmetry $SU(N_c)$ to its subgroup $U(1)^{N_c-1}$ without breaking the supersymmetry, and without resulting in a

running of the coupling constant [19]. At finite temperature, the mechanism is the same except the fact that supersymmetry will be broken as well.

The string theory dual for $\mathcal{N} = 4$ cSYM at zero temperature is well known. Among various Type IIB supergravity background solutions that are dual to the strongly coupled $\mathcal{N} = 4$ cSYM at zero temperature [19–22], in this paper, we will study a Type IIB supergravity background solution that describes nonextremal rotating black 3-branes (with mass parameter m and single rotational parameter r_0) which, in the extremal limit, i.e., $r_0 \gg m^{1/4}$, is dual to $\mathcal{N} = 4$ SYM on the Coulomb branch at zero temperature that arises from N_c D3-branes distributed uniformly in the angular direction, inside a 3-sphere of radius r_0 [20].

So far the studies of the nonextremal rotating black 3-brane supergravity backgrounds have been limited to the grand canonical ensemble (which is described by fixed temperature T and angular velocity Ω or chemical potential μ , i.e., the Gibbs free energy $G(T, \mu)$ is a function of T and μ), and canonical ensemble [which is described by fixed temperature T and angular momentum density J or charge density $\langle J^0 \rangle = \rho$, i.e., the Helmholtz free energy $F(T, \langle J^0 \rangle)$ is a function of T and $\langle J^0 \rangle$]; see [26–36]. The two ensembles have different physics; for example, in planar rotating black 3-branes, Hawking-Page phase transition does not exist in the grand canonical ensemble even though it does exist in the canonical ensemble [30,35].

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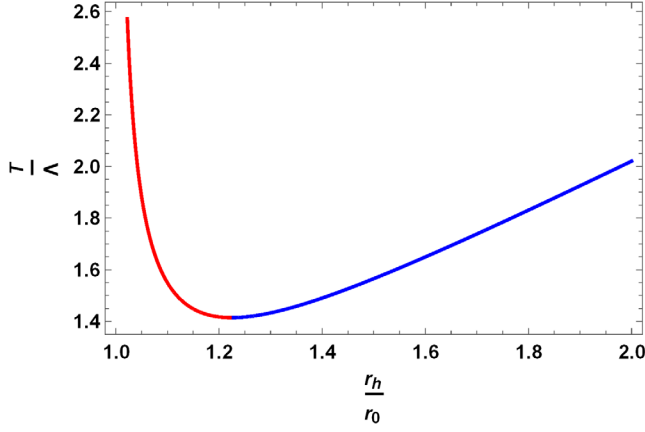


FIG. 1. Hawking temperature (2.4).

In this paper, we will introduce a new ensemble which is described by a fixed temperature T and a scalar condensate $\langle \mathcal{O} \rangle$; i.e., the Helmholtz free energy $F(T, \langle \mathcal{O} \rangle)$ is a function of T and $\langle \mathcal{O} \rangle$. The scalar condensate is the expectation value of dimension 4 operator $\mathcal{O} = Tr\Phi_{i_1}\Phi_{i_2}\Phi_{i_3}\Phi_{i_4}$, that is, $\langle \mathcal{O} \rangle \sim \lim_{r \rightarrow \infty} \sqrt{-\bar{g}} g^{rr} \partial_r h \sim \Lambda^4$ of the massless metric fluctuation $h = \bar{g}^{\mu\nu} h_{\mu\nu} = 1 - \bar{g}^{\mu\nu} g_{\mu\nu}$, where $\bar{g}_{\mu\nu}$ is the metric component of pure $AdS_5 \times S^5$ space while $g_{\mu\nu}$ is our 10-dimensional metric (4.1) [19], and $\Lambda \equiv \frac{r_0}{\pi R^2}$ with R the radius of the AdS_5 space.

Therefore, in our ensemble, the variation of the Helmholtz free energy $F(T, \langle \mathcal{O} \rangle)$ can be written as

$$dF(T, \langle \mathcal{O} \rangle) = -SdT + h_0 d\langle \mathcal{O} \rangle, \quad (1.1)$$

where the source $h_{(0)} = h(r \rightarrow \infty)$. One can compare the variation of the free energy in our ensemble (1.1) to the variation in the canonical ensemble

$$dF(T, \langle J^0 \rangle) = -SdT + A_i^{(0)} d\langle J^0 \rangle, \quad (1.2)$$

where the source $A_i^{(0)} = A_i(r \rightarrow \infty) = \mu$, and grand canonical ensemble

$$dG(T, \mu) = -SdT - \langle J^0 \rangle d\mu. \quad (1.3)$$

The outline of this paper is as follows: In Sec. II, we study the thermodynamics of rotating black 3-brane solution where a single rotation parameter r_0 is turned on. In Sec. III, we compute the heavy quark-antiquark potential $V(L)$ of $\mathcal{N} = 4$ cSYM. In Sec. IV, we study the mass spectrum of glueballs in $\mathcal{N} = 4$ cSYM.

II. THERMODYNAMICS OF $\mathcal{N} = 4$ CSYM PLASMA

The rotating black 3-brane solution of the 5-dimensional Einstein-Maxwell-scalar action found from the $U(1)^3$

consistent truncation of Type IIB supergravity on S^5 [37,38], see also [39–41], is given by

$$ds_{(5)}^2 = \frac{r^2}{R^2} H^{1/3} (-f dt^2 + dx^2 + dy^2 + dz^2) + \frac{H^{-2/3}}{\frac{r^2}{R^2} f} dr^2, \quad (2.1)$$

where

$$f = 1 - \frac{r_h^4 H(r_h)}{r^4 H(r)}, \quad H = 1 - \frac{r_0^2}{r^2}, \quad (2.2)$$

$$\varphi_1 = \frac{1}{\sqrt{6}} \ln H, \quad \varphi_2 = \frac{1}{\sqrt{2}} \ln H,$$

$$A_i^1 = i \frac{r_0 r_h^2 \sqrt{H(r_h)}}{R^2 r^2 H(r)},$$

$$r_h^2 = \frac{1}{2} \left(r_0^2 + \sqrt{r_0^4 + 4m} \right), \quad (2.3)$$

$\kappa = \frac{r_0^2}{r_h^2}$, m is the mass parameter, and $A_i^2 = A_i^3 = 0$. Note that our metric (2.1) is equivalent to the metric used in [33] after analytically continuing $r_0 \rightarrow -i\sqrt{q}$.

We should also note that having an imaginary gauge potential, in our ensemble, does not lead to any inconsistencies, since all physical quantities in the 5-dimensional spacetime are given in terms of $(\partial_r A_i^1)^2$. From the field theory side, having an imaginary gauge potential or imaginary chemical potential μ means that we are studying the phase diagram of $\mathcal{N} = 4$ cSYM at finite T and imaginary chemical potential, which is similar to studying the phase diagram of QCD at finite T and imaginary chemical potential, which is well known in that it does not lead to any inconsistencies; see [42–45] for the study of lattice QCD at finite imaginary chemical potential.

Note that we have turned off the other two gauge potentials A_i^2 and A_i^3 , but in a future publication [46], we will address what the phase diagram of the confining $\mathcal{N} = 4$ cSYM (studied in this paper) looks like at finite chemical potentials induced by A_i^2 and/or A_i^3 .

The Hawking temperature T of the black hole (rotating black 3-brane) solution (2.1) is given by

$$\frac{T}{\Lambda} = \frac{1 - \frac{1}{2}\kappa}{\sqrt{\kappa - \kappa^2}}, \quad (2.4)$$

where $T_0 = \frac{r_h}{\pi R^2}$, $\Lambda = \frac{r_0}{\pi R^2}$, and $\kappa = \frac{r_0^2}{r_h^2} = \frac{\Lambda^2}{T_0^2}$. We have plotted $\frac{T}{\Lambda}$ in Fig. 1. We can also invert (2.4) to find

$$\kappa = \frac{1 + \frac{T^2}{\Lambda^2} \left(1 \mp \sqrt{\frac{T^2}{\Lambda^2} - 2} \right)}{\frac{1}{2} + 2 \frac{T^2}{\Lambda^2}}. \quad (2.5)$$

Note that in (2.5) “−” corresponds to a large black hole branch and “+” corresponds to a small black hole branch.

The entropy density $s(T, \Lambda)$, for our ensemble where T and Λ are held fixed, is given by

$$\begin{aligned} s(T, \Lambda) &= \frac{A_H}{4G_5 V_3} = \frac{1}{4G_5} \sqrt{g_{xx}(r_h)g_{yy}(r_h)g_{zz}(r_h)} \\ &= \frac{\pi^2 N_c^2 T_0^3}{2} (1 - \kappa)^{1/2}, \end{aligned} \quad (2.6)$$

where $G_5 = \pi R^3/2N_c^2$, and V_3 is the three-dimensional volume. And, using (1.1) for fixed $\langle \mathcal{O} \rangle \sim \Lambda^4$, the corresponding free energy density $f(T, \Lambda)$ of our ensemble can be determined by integrating the entropy density $s(T, \Lambda)$ as [9,12]

$$\begin{aligned} f(T, \Lambda) &= - \int_{r_{h\min}}^{r_h} \frac{dT'}{dr'_h} s(r'_h, \Lambda) dr'_h \\ &= - \frac{\pi^2 N_c^2 T_0^4}{8} \left(1 - \kappa - \frac{3}{4} \kappa^2 - \kappa^2 \log\left(\frac{2}{\kappa} - 2\right) \right), \end{aligned} \quad (2.7)$$

where we choose $r_{h\min} = \sqrt{\frac{3}{2}} r_0$, and set the integration constant $f(T_{\min}, \Lambda) = 0$. Also note that $T_{\min} = T(r_h = r_{h\min}) = T_c$. We have plotted the free energy density $f(T, \Lambda)$ (2.7) in Fig. 2.

The other thermodynamic quantities can be determined from the free energy density $f(T, \Lambda)$ (2.7) as: pressure $p = -f$, energy density $\epsilon = p + Ts$, specific heat $C_\Lambda = T(\frac{\partial s}{\partial T})_\Lambda$, and speed of sound $c_s^2 = \frac{\partial p}{\partial \epsilon} = \frac{s}{C_\Lambda}$. We have plotted the thermodynamics quantities in Figs. 4–7. To compare our results with pure Yang-Mills theory on the lattice and improved holographic QCD see Fig. 5–9 in [12].

As a comparison to $\mathcal{N} = 4$ cSYM, we have also plotted, see Fig. 3, the free energy density of $\mathcal{N} = 4$ SYM on sphere f_{sphere} , which is given by [16], see also [47],

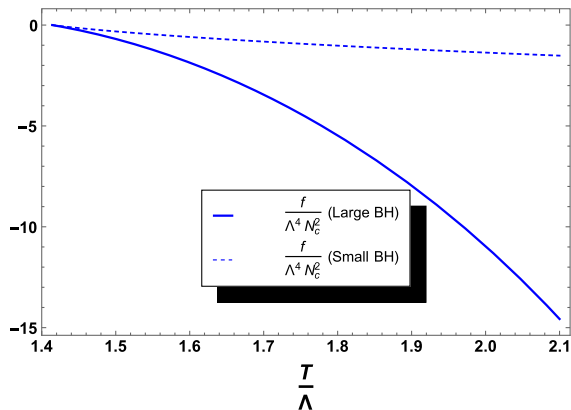


FIG. 2. The free energy density $\frac{f}{\Lambda^4 N_c^2}$ of $\mathcal{N} = 4$ cSYM plasma (2.7) for the large and small black holes. Note that $T_{\min} = T_c$.

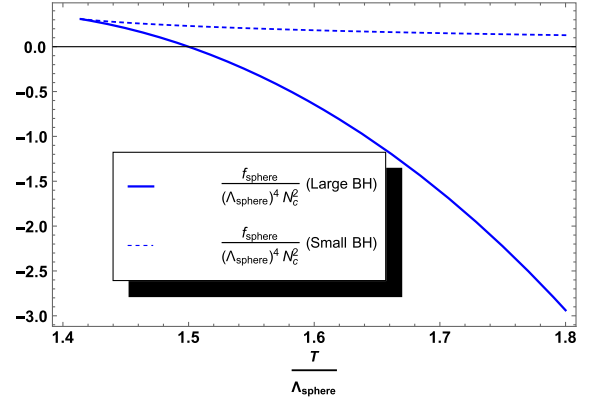


FIG. 3. The free energy density $\frac{f_{\text{sphere}}}{(\Lambda_{\text{sphere}})^4 N_c^2}$ of $\mathcal{N} = 4$ SYM plasma on 3-sphere of radius R (2.8) for the large and small black holes. Note that $T_{\min} < T_c = \frac{3}{2} \Lambda_{\text{sphere}}$.

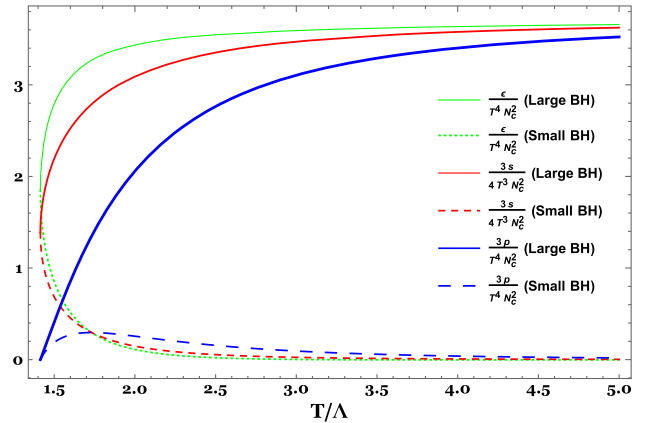


FIG. 4. The energy density $\frac{\epsilon}{T^4}$, entropy density $\frac{3}{4} \frac{s}{T^3 N_c^2}$, and pressure $\frac{3p}{T^3 N_c^2}$ of $\mathcal{N} = 4$ cSYM plasma for the large and small black holes.

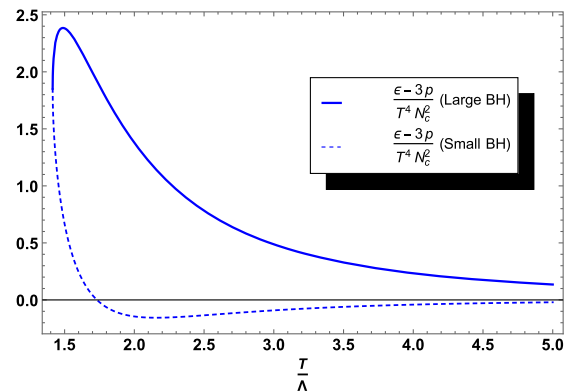


FIG. 5. The trace anomaly $\frac{\epsilon - 3p}{T^4 N_c^2}$ of $\mathcal{N} = 4$ cSYM plasma for the large and small black holes.

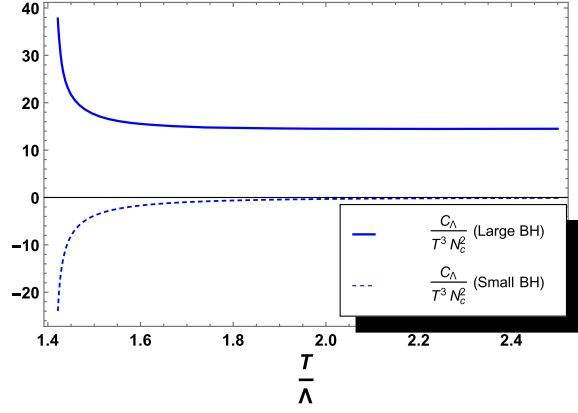


FIG. 6. The specific heat C_Λ of $\mathcal{N} = 4$ cSYM plasma for the large and small black holes.

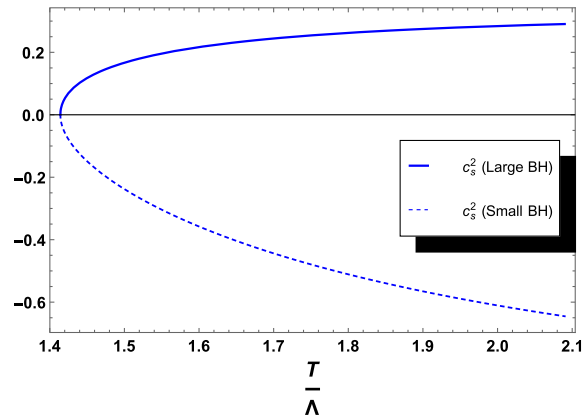


FIG. 7. The speed of sound c_s^2 of $\mathcal{N} = 4$ cSYM plasma for the large and small black holes.

$$f_{\text{sphere}} = \frac{F_{\text{sphere}}}{V_3} = -\frac{\pi^2 N_c^2 T_0^4}{8} (1 - \kappa_{\text{sphere}}), \quad (2.8)$$

where $\kappa_{\text{sphere}} = \frac{R^2}{r_h^2} = \frac{\Lambda_{\text{sphere}}^2}{T_0^2}$ with $\Lambda_{\text{sphere}} = \frac{1}{\pi R}$ and $T_0 = \frac{r_h}{\pi R^2}$, and the Hawking temperature $\frac{T}{\Lambda_{\text{sphere}}} = \frac{1 + \frac{1}{2}\kappa_{\text{sphere}}}{\sqrt{\kappa_{\text{sphere}}}}$.

Comparing Fig. 2 and Fig. 3, one can see that we have a second-order phase transition for $\mathcal{N} = 4$ cSYM at $T_c = T_{\text{min}}$ [it is second-order since the second-derivative of our order parameter (the free energy) or its specific heat capacity is discontinuous, see Fig. 6, while the first derivative of its free energy or the entropy is continuous as one goes from the large black hole to small black hole, see Fig. 4; also see [34,36] for similar second-order phase transitions between large and small black holes in $\mathcal{N} = 4$ SYM at finite-chemical potential]. And we have a first-order (Hawking-Page) phase transition in $\mathcal{N} = 4$ SYM on sphere at $T_c = \frac{3}{2}\Lambda_{\text{sphere}} = \frac{3}{2\pi R}$ (it is first-order since the first derivative of the free energy or its entropy changes

discontinuously as one goes from the large black hole with $s \sim N_c^2$ to the thermal-AdS with $s \sim 0$).

III. CORNELL POTENTIAL IN $\mathcal{N} = 4$ CSYM

The Nambu-Goto (NG) action is

$$S_{NG} = \int d\tau d\sigma \mathcal{L}(h_{ab}) = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{-deth_{ab}}, \quad (3.1)$$

where the background induced metric on the string $h_{ab} = g_{\mu\nu} \partial_a x^\mu(\tau, \sigma) \partial_b x^\nu(\tau, \sigma)$. Using the embedding $(\tau, \sigma) \Rightarrow (t(\tau, \sigma), 0, 0, x(\tau, \sigma), r = \sigma)$, the background induced metric $h_{ab}(x')$ becomes ($t \equiv d/d\sigma$)

$$\begin{aligned} h_{\tau\tau}(x') &= g_{tt}, \\ h_{\sigma\sigma}(x') &= g_{rr} \left(\frac{1}{1 + \frac{C^2}{g_{xx}g_{tt}}} \right), \end{aligned} \quad (3.2)$$

where we used

$$(x')^2 = \frac{-C^2 g_{rr}}{g_{xx}g_{tt}} \frac{1}{\left(1 + \frac{C^2}{g_{tt}g_{xx}}\right)}, \quad (3.3)$$

which is the solution of the NG equation of motion, and the integration constant C is related to the conjugate momenta $\Pi = \frac{\partial \mathcal{L}}{\partial x'} = -\frac{C}{2\pi\alpha'}$.

Considering a string configuration where a heavy quark is attached to each end of the string, we can extract the potential energy $V(L)$, of the two quarks separated by length L , from the on-shell Nambu-Goto action S_{NG} as

$$V(L) = \frac{-2S_{NG}}{T}, \quad (3.4)$$

where

$$\begin{aligned} -\frac{2\pi\alpha'}{T} S_{NG} &= \int_{r_m}^{\infty} dr \left(\sqrt{-deth_{ab}(x')} - \sqrt{-deth_{ab}(0)} \right) \\ &\quad - \int_{r_h}^{r_m} dr \sqrt{-deth_{ab}(0)}, \end{aligned} \quad (3.5)$$

and r_m is related to L through the boundary condition $\frac{L}{2} = \int_{r_m}^{\infty} x' dr$, and we also fix the integration constant C by demanding $x'|_{r=r_m} \rightarrow \infty$ which is satisfied only when $C^2 = -g_{tt}(r_m)g_{xx}(r_m)$. Note that we have a factor of 2 in (3.4) because our gauge covers only half of the full string configuration which accounts to only half of the full potential energy between the quarks; see [47] for discussion on how to compute $V(L)$ in the $x(r)$ gauge instead of the widely used $r(x)$ gauge of [48].

For $r \gg r_m$, after approximating $h_{\sigma\sigma}(x') \cong h_{\sigma\sigma}(0) = g_{rr}$,

$$\begin{aligned} V(L) &\simeq -\frac{1}{\pi\alpha'} \int_{r_0}^{r_m} dr \sqrt{-deth_{ab}(0)} \\ &\simeq -\frac{2\sqrt{\lambda}}{3\pi} \frac{1}{L} + \frac{\pi\sqrt{\lambda}\Lambda^2}{4} L + \frac{5\sqrt{\lambda}\Lambda}{6} + \mathcal{O}(r_0^4), \end{aligned} \quad (3.6)$$

where we used $\frac{L}{2} = \int_{r_m}^{\infty} x' dr \cong \frac{1}{3} \frac{R^2}{r_m}$ with $x' \cong \frac{g_{xx}(r_m)}{g_{xx}} \sqrt{\frac{g_{rr}}{g_{xx}}} \cong \frac{r_m^2 R^2}{r^4}$ for $r \gg r_m$, and we have set $r_h = r_0$ and $f = 1$ in the extremal limit.

In [20], the heavy quark-antiquark potential energy $V(L)$ was computed for the 10-dimensional background metric (4.1) after analytically continuing $t \rightarrow -it$ and in the extremal limit where $r_h = r_0$ or $\tilde{f} = f = 1$ case. The authors have shown that, for $\theta = \frac{\pi}{2}$, $V(L)$ smoothly interpolates between a Coulombic potential $V(L) = -\frac{2\Gamma(3/4)^2\sqrt{\lambda}}{\Gamma(1/4)^2} \frac{1}{L}$ for small L and a confining potential $V(L) = \frac{\pi\sqrt{\lambda}\Lambda^2}{2} L$ for large L . See curve (b) in Fig. 5 of [20]. Their numerical result agrees qualitatively with our analytic result (3.6) on the 5-dimensional metric (2.1).

IV. GLUEBALLS IN $\mathcal{N} = 4$ CSYM

It can easily be shown that bulk fluctuations in the 5-dimensional metric (2.1), at least in the near boundary limit where the metric is essentially AdS₅ space with IR cut-off at $r = r_0$, have a mass-gap and quantized mass spectrum proportional to $\Lambda = \frac{r_0}{\pi R^2}$.

In [20], it was shown that a scalar bulk fluctuation in a 10-dimensional metric [which is the 10-dimensional uplift of (2.1)]

$$\begin{aligned} ds_{(10)}^2 &= \frac{r^2}{R^2} \tilde{H}^{1/2} (-\tilde{f} dt^2 + dx^2 + dy^2 + dz^2) \\ &\quad + \frac{\tilde{H}^{1/2} H^{-1}}{\frac{r^2}{R^2} f} dr^2 + R^2 (\tilde{H}^{1/2} d\theta^2 + H \tilde{H}^{-1/2} \sin^2 \theta d\phi^2 \\ &\quad + \tilde{H}^{-1/2} \cos^2 \theta d\Omega_3^2) + 2A_t^1 H \tilde{H}^{-1/2} R^2 \sin^2 \theta dt d\phi, \end{aligned} \quad (4.1)$$

where

$$\tilde{H} = \sin^2 \theta + H \cos^2 \theta, \quad \text{and} \quad \tilde{f} = 1 - \frac{r_h^4 H(r_h)}{r^4 \tilde{H}(r)}, \quad (4.2)$$

f and H are the same as in (2.1), after analytically continuing $t \rightarrow -it$, indeed has mass gap proportional to Λ and a quantized mass spectrum $M_n^2 = 4\pi^2 \Lambda^2 n(n+1)$, see Eq. 54 in [20]. Since a scalar bulk fluctuation in (4.1) has the same 5-dimensional bulk equation of motion as in [20], which is the Jacobi equation, we can use this result to calculate the mass spectrum of glueballs in $\mathcal{N} = 4$ cSYM.

The transverse gravitational tensor fluctuation $h_{ij}^x(t, z, r)$ in the 10-dimensional metric (4.1), which is a source to dimension 4 stress-energy tensor operator T_x^y , also has the same 5-dimensional bulk equation of motion as the scalar field which is the Jacobi equation. Therefore, we can infer that the operator T_x^y which corresponds to spin-2 glueballs of $J^{PC} = 2^{++}$ [49] has mass spectrum given by $M_n^2 = 4\pi^2 \Lambda^2 n(n+1)$ for $n = 1, 2, \dots$

The real and imaginary parts of the bulk fluctuation of a massless complex scalar field $\Phi = e^{-\phi} + i\chi$, in the 10-dimensional metric (4.1), are sources to the dimension 4 scalar operators $\mathcal{O}_4 = Tr F^2$ and $\tilde{\mathcal{O}}_4 = Tr F \wedge F$, respectively [50], and their 5-dimensional bulk equation of motion is the Jacobi equation. Therefore, \mathcal{O}_4 and $\tilde{\mathcal{O}}_4$ which correspond to the scalar glueballs of $J^{PC} = 0^{++}$ and $J^{PC} = 0^{-+}$, respectively, have a degenerate mass spectrum given by $M_n^2 = 4\pi^2 \Lambda^2 n(n+1)$ for $n = 1, 2, \dots$

V. CONCLUSION

We have shown that the large black hole branch of the nonextremal rotating black 3-brane background solution (2.1) has a pure Yang-Mills-like equation of state: the pressure p vanishes at critical temperature $T_c = T_{\min} = \sqrt{2}\Lambda$, see Fig. 4; the trace anomaly $\epsilon - 3p$ has a maxima around T_c and vanishes at very high temperature, see Fig. 5; and the speed of sound c_s^2 approaches its conformal limit $1/3$ from below. In order to compare our results with pure Yang-Mills theory on the lattice and improved holographic QCD see Fig. 5–9 in [12].

Note that we have investigated the hydrodynamic transport coefficients and hard probe parameters of the strongly coupled $\mathcal{N} = 4$ cSYM plasma in [41].

ACKNOWLEDGMENTS

The author thanks Ho-Ung Yee for stimulating discussions and helpful comments on the draft, Pablo Morales, Andrey Sadofyev, and Yi Yin for discussions. This work was in part supported by the U.S. Department of Energy under Contract No. DE-FG-88ER40388.

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