Nonuniversal $U(1)_X$ extension to the MSSM with three families

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We propose a supersymmetric extension of the anomalyfree and three families nonuniversal $U(1)$ model, with the inclusion of four Higgs doublets and four Higgs singlets. The quark sector is extended by adding three exotic quark singlets, while the lepton sector includes two exotic charged lepton singlets, three right-handed neutrinos and three sterile Majorana neutrinos to obtain the fermionic mass spectrum. By implementing an additional $\mathbb{Z}_2$ symmetry, the Yukawa coupling terms are suited in such a way that the fermion mass hierarchy is obtained without fine-tuning. The effective mass matrix for SM neutrinos is fitted to current neutrino oscillation data to check the consistency of the model with experimental evidence, obtaining that the normal-ordering scheme is preferred over the inverse ones. The electron and up, down and strange quarks are massless at tree level, but they get masses through radiative correction at one loop level coming from the sleptons and Higgsinos contributions. We show that the model predicts a like-Higgs SM mass at electroweak scale by using the VEV according to the symmetry breaking and fermion masses.

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I. INTRODUCTION

Regardless the success of the Standard Model of electroweak interactions (SM) [1] in explaining the experimental data, it is considered an incomplete model since some features remain satisfactorily unexplained. Among them, there is the fermion mass hierarchy problem as well as naturalness problem; both have motivated many extensions of the SM, or even complete new theories. In the case of supersymmetry, it is the model which best explains the Higgs mass naturalness thanks to the exact cancellation of the quadratic divergences between contributions of particles and superpartners in the Higgs mass radiative corrections, providing a finite mass value for the particle.

When considering the minimal supersymmetric Standard Model (MSSM) superpotential, there exists a masslike parameter for the bilinear superfields coupling called μ, which is responsible of the Higgs and Higgsino masses. This parameter is expected to be at the order of SUSY breaking scale to provide Higgsino masses. However, the lightest Higgs mass is at the electroweak scale. Thus, it can not provide the correct neutralino masses and a phenomenological Higgs mass in accordance with the data presented by ATLAS and CMS experiments [2]. Additional to the above, the unexplained origin of this kind of coupling is what constitutes the μ-problem [3].

The next to minimal supersymmetric models (NMSSM) present an elegant solution to this problem [4] by introducing new scalar singlet field. Consequently, trilinear couplings $\hat{\chi}\hat{\phi}\hat{\phi}$ can be generated in such a way that a bilinear term, $\mu\hat{\phi}\hat{\phi}$, arises when the singlet scalar field acquires a vacuum expectation value (VEV) at the SUSY breaking scale. On the other hand, when including new fields in the theory, the Higgs and Higgsinos mass matrices are changed through new coupling constants, allowing the explanation of these masses in accordance to experimental data or collider constraints.

Looking back to the MSSM, it is known that the lightest Higgs mass can be approximated to $m_h^2 \approx m_Z^2 \cos^2 2\beta + \Delta m_h^2$, where Δm_h^2 comes from the 1-loop corrections due to the top quark and stops contributions [5]. Then, if we consider a big value for $\tan\beta$, Δm_h^2 must be at the same order of the tree level contribution, and stop particles should have a big mass values in order to get a 125 GeV Higgs mass. Nevertheless, in extensions of the MSSM the radiative corrections due to stop particles would not be necessary for explaining the Δm_h^2 term. This may come from a seesaw mechanism that creates an explicit dependence on the scalar singlet VEV [6]. Furthermore, if the scalar singlets come as part of a $U(1)_X$ extended gauge symmetry (USSM) [7], the respective D-term may give a new contribution to the lightest mass at tree level, causing

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the new mass eigenstate to be sharing the functional form of a SM-like Higgs boson.

The extensions of MSSM have other motivations. There are more scalar particles which can be searched in collider experiments, for instance a dark matter candidate [8]. Likewise, they can explain the small deviation of Higgs couplings to fermions, which turn out to be proportional to particle masses, as it has been found in experiments for tau lepton, top, and bottom quarks.

The MSSM is a two Higgs doublets anomalyfree theory, where the different hypercharge values allows to each Higgs doublet to couple with different kind of quarks, forbidding flavor changing neutral currents (FCNC). In order to extend the scalar doublets content without inducing any chiral anomaly, the minimum amount of them would be four. However, while a Yukawa linear combination can be diagonalized through a rotation, making it proportional to the particle mass, the other linear combination would not be diagonal, generating then the FCNC [9]. On the other hand, from the LHC it is known the upper bound for the tch vertex [10] which can be explained, as new physics at tree level, from a model with multiple Higgs doublets. For this reason a SUSY theory with FNCN is still phenomenological relevant.

In the present work, it is done a SUSY extension of the three family $U(1)_X$ anomaly free model [11]. The nonsupersymmetric model can explain the fermion mass hierarchy, as well as mixing angles for the Cabibbo-Kobayashi-Maskawa [12] and Pontecorvo-Maki-Nakagawa-Sakata matrices [13] just by using two Higgs doublets and a scalar singlet field which breaks the $U(1)_X$ symmetry giving masses to exotic particles. A singlet scalar field without vacuum expectation value (VEV) is required for giving masses to light fermions. Then for the corresponding SUSY extension, the scalar sector has to be doubled with different X-charge in such a way that the anomaly induced by Higgsinos are canceled and the model would be anomalyfree. After symmetry breaking, the mass matrices for the scalar, vector, and fermion sector are constructed. Furthermore, from the scalar CP-even mass matrix it is found that the theory is compatible with a 125 GeV mass for the lightest scalar particle, which we identify as the discovered Higgs boson in LHC. When considering the scalar CP-odd mass matrix, two would-be Goldstone bosons associated to the Z and Z' particles are found. The rest of mass eigenstates are found to be above the electroweak scale. Likewise, from the charged scalar bosons another would-be Goldstone boson is found, associated to the $W^{\pm}$ gauge bosons. In the present model, the masses of electron, quark up, and quark down are zero. Then we consider the SUSY contributions to the self-energies in order to generate the masses at one loop level.

II. GENERAL REMARKS OF THE MODEL

The nonsupersymmetric version of the model gives a scenario for solving the fermion mass hierarchy problem (FMH) with no need of unpleasant fine tunings on the Yukawa coupling constants [14]. The way that such problem is addressed relies in having two Higgs doublets Φ_1 and Φ_2; the FMH is understood partially from the VEV hierarchy among the two doublets. Also, with the help of the set configuration of the $U(1)_X$ charges for all particles, the couplings allowed by the gauge symmetry give a natural scenario for exhibiting the FMH. The inclusion of a parity symmetry $\mathbb{Z}_2$ helps in avoiding Yukawa terms in the Lagrangian that spoil the natural scenario wanted [15]. The new gauge symmetry extension comes in general with chiral anomalies, which have to be canceled in order to guarantee the renormalizability of the theory. In the model found in the literature [11], it was done by choosing the set configuration of $U(1)_X$ charges for all fermions [16], such that the following anomaly equations were canceled:

$$
\begin{aligned}
[\mathrm{SU}(3)_C]^2\mathrm{U}(1)_X \to A_C &= \sum_Q X_{Q_L} - \sum_Q X_{Q_R} \\
[\mathrm{SU}(2)_L]^2\mathrm{U}(1)_X \to A_L &= \sum_\ell X_{\ell_L} + 3\sum_Q X_{Q_L} \\
[\mathrm{U}(1)_Y]^2\mathrm{U}(1)_X \to A_{Y^2} &= \sum_{\ell,Q}[Y^2_{\ell_L}X_{\ell_L} + 3Y^2_{Q_L}X_{Q_L}] - \sum_{\ell,Q}[Y^2_{\ell_R}X_{L_R} + 3Y^2_{Q_R}X_{Q_R}] \\
\mathrm{U}(1)_Y[\mathrm{U}(1)_X]^2 \to A_Y &= \sum_{\ell,Q}[Y_{\ell_L}X^2_{\ell_L} + 3Y_{Q_L}X^2_{Q_L}] - \sum_{\ell,Q}[Y_{\ell_R}X^2_{\ell_R} + 3Y_{Q_R}X^2_{Q_R}] \\
[\mathrm{U}(1)_X]^3 \to A_X &= \sum_{\ell,Q}[X^3_{\ell_L} + 3X^3_{Q_L}] - \sum_{\ell,Q}[X^3_{\ell_R} + 3X^3_{Q_R}] \\
[\mathrm{Grav}]^2\mathrm{U}(1)_X \to A_\mathrm{G} &= \sum_{\ell,Q}[X_{\ell_L} + 3X_{Q_L}] - \sum_{\ell,Q}[X_{\ell_R} + 3X_{Q_R}],
\end{aligned}
\tag{1}
$$

TABLE I. Scalar content of the model, nonuniversal X quantum number, $\mathbb{Z}_2$ parity, and hypercharge.

Higgs scalar doublets	$X^{\pm}$	Y	Higgs scalar singlets	$X^{\pm}$	Y
$\hat{\Phi}_1 = \begin{pmatrix} \hat{\phi}_1^+ \\ \frac{\hat{h}_1+v_1+i\hat{\eta}_1}{\sqrt{2}} \end{pmatrix}$	$+2/3^+$	$+1$	$\hat{\chi} = \frac{\hat{\xi}_\chi+v_\chi+i\hat{\zeta}_\chi}{\sqrt{2}}$	$-1/3^+$	0
$\hat{\Phi}_2 = \begin{pmatrix} \hat{\phi}_2^+ \\ \frac{\hat{h}_2+v_2+i\hat{\eta}_2}{\sqrt{2}} \end{pmatrix}$	$+1/3^-$	$+1$	$\sigma = \frac{\hat{\sigma}_\chi+i\hat{\zeta}_\sigma}{\sqrt{2}}$	$-1/3^-$	0
$\hat{\Phi}'_1 = \begin{pmatrix} \frac{\hat{h}'_1+v'_1+i\hat{\eta}'_1}{\sqrt{2}} \\ \hat{\phi}_1^{-\prime} \end{pmatrix}$	$-2/3^+$	-1	$\hat{\chi}' = \frac{\hat{\xi}'_\chi+v'_\chi+i\hat{\zeta}'_\chi}{\sqrt{2}}$	$+1/3^+$	0
$\hat{\Phi}'_2 = \begin{pmatrix} \frac{\hat{h}'_2+v'_2+i\hat{\eta}'_2}{\sqrt{2}} \\ \hat{\phi}_2^{-\prime} \end{pmatrix}$	$-1/3^-$	-1	$\sigma' = \frac{\hat{\xi}'_\sigma+i\hat{\zeta}'_\sigma}{\sqrt{2}}$	$+1/3^-$	0

TABLE II. Fermion content of the Abelian extension, nonuniversal X quantum number, and parity $\mathbb{Z}_2$.

Left-handed fermions	$X^{\pm}$	Right-handed fermions	$X^{\pm}$
	SM quarks		
$\hat{q}_L^1 = \begin{pmatrix} \hat{u}^1 \\ \hat{d}^1 \end{pmatrix}_L$	$+1/3^+$	$\hat{u}_L^{1c}$	$-2/3^+$
		$\hat{u}_L^{2c}$	$-2/3^-$
$\hat{q}_L^2 = \begin{pmatrix} \hat{u}^2 \\ \hat{d}^2 \end{pmatrix}_L$	0^-	$\hat{u}_L^{3c}$	$-2/3^+$
		$\hat{d}_L^{1c}$	$+1/3^-$
$\hat{q}_L^3 = \begin{pmatrix} \hat{u}^3 \\ \hat{d}^3 \end{pmatrix}_L$	0^+	$\hat{d}_L^{2c}$	$+1/3^-$
		$\hat{d}_L^{3c}$	$+1/3^-$
	SM leptons		
$\hat{\ell}_L^e = \begin{pmatrix} \hat{\nu}^e \\ \hat{e}^e \end{pmatrix}_L$	0^+	$\hat{\nu}_L^{ec}$	$-1/3^-$
		$\hat{\nu}_L^{\mu c}$	$-1/3^-$
$\hat{\ell}_L^\mu = \begin{pmatrix} \hat{\nu}^\mu \\ \hat{\mu}^\mu \end{pmatrix}_L$	0^+	$\hat{\nu}_L^{\tau c}$	$-1/3^-$
		$\hat{e}_L^{ec}$	$+4/3^-$
$\hat{\ell}_L^\tau = \begin{pmatrix} \hat{\nu}^\tau \\ \hat{\tau}^\tau \end{pmatrix}_L$	-1^+	$\hat{e}_L^{\mu c}$	$+1/3^-$
		$\hat{e}_L^{\tau c}$	$+4/3^-$
	Non-SM quarks		
$\hat{\mathcal{T}}_L$	$+1/3^-$	$\hat{\mathcal{T}}_L^c$	$-2/3^-$
$\mathcal{J}_L^1$	0^+	$\hat{\mathcal{J}}_L^{c\,1}$	$+1/3^+$
$\mathcal{J}_L^2$	0^+	$\hat{\mathcal{J}}_L^{c\,2}$	$+1/3^+$
	Non-SM leptons		
$\hat{E}_L$	-1^+	$\hat{E}_L^c$	$+2/3^+$
$\hat{\mathcal{E}}_L$	$-2/3^+$	$\hat{\mathcal{E}}_L^c$	$+1^+$
Majorana fermions		$\mathcal{N}_R^{1,2,3}$	0^-

where subscripts Q and l account for quarks and leptons, respectively. Moreover, subscripts L and R correspond to left-handed and right-handed chiralities, respectively. Exotic fermions were also included in the model for accomplishing a free anomaly model, in the way that new degrees of freedom enter into the Eq. (1). Thus, there is a bigger set of $U(1)_X$ charges than the SM particles for fulfilling both anomaly cancellation and FMH. In the quark sector, an up-like quark $\mathcal{T}$ and two down-like quarks, $\mathcal{J}^1$ and $\mathcal{J}^2$, come into the bargain. The additional particles in the lepton sector are two charged leptons, E and $\mathcal{E}$; three Dirac right handed neutrinos, ν_R^e, ν_R^μ, and ν_R^τ; and three Majorana neutrinos $\mathcal{N}_R^{1,2,3}$. The majorana particles do not contribute to the anomaly equations, but they were included for giving masses to neutrinos through inverse seesaw mechanism (ISS) [17], according to neutrino oscillation experiments which give information about squared mass differences and mixing angles. For breaking the new symmetry into the SM gauge symmetry, an scalar singlet χ was added with a VEV around the TeV scale. Therefore, the model contains the following spontaneous symmetry breaking chain:

$$\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \otimes \mathrm{U}(1)_X \xrightarrow{\chi}$$
$$\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \xrightarrow{\Phi} \mathrm{SU}(3)_C \otimes \mathrm{U}(1)_Q. \qquad (2)$$

Because the lightest fermions, electron, down quark, and up quark, did not acquire masses at tree level, another scalar singlet σ had to be included for giving masses to such particles through radiative corrections. However, in the SUSY version of the model this scalar field is still present but now the corresponding superpartners of the particles inside the loop represent an additional SUSY contribution to their masses.

For the minimal supersymmetric extension, all fields are upgraded to superfields; we denote a superfield with a hat symbol, as usual. The number of scalar particles has to be doubled in comparison to the non-SUSY version, otherwise the model would be anomalous due to Higgsinos. Therefore, new added fields are $\hat{\Phi}'_1$, $\hat{\Phi}'_2$, $\hat{\chi}'$, and σ'. The introduced scalar fields have the same hypercharge and X charges as the nonprimed partners, but with opposite sign to secure anomaly cancellation. For getting the right masses of the gauge bosons in the SM, the following condition must be imposed on the VEVs of the Higgs doublets,

$$\sqrt{v_1^2 + v_2^2 + v_1'^2 + v_2'^2} = v = 246\ \mathrm{GeV}. \qquad (3)$$

The bosonic and fermonic content of the model explored in this paper is shown in the Tables I and II, respectively.

III. SCALAR AND GAUGE BOSON SECTOR

The Lagrangian for the scalar sector that describes the minimal supersymetric extension to the $U(1)_X$ model in the literature is given by the addition of a F-terms potential, a D-terms potential and a soft-supersymetry breaking potential. The F-terms were obtained from the following superpotential:

$$W_\phi = -\mu_1 \hat{\Phi}'_1 \hat{\Phi}_1 - \mu_2 \hat{\Phi}'_2 \hat{\Phi}_2 - \mu_\chi \hat{\chi}' \hat{\chi} - \mu_\sigma \hat{\sigma}' \hat{\sigma}$$
$$+ \lambda_1 \hat{\Phi}'_1 \hat{\Phi}_2 \hat{\sigma}' + \lambda_2 \hat{\Phi}'_2 \hat{\Phi}_1 \sigma. \qquad (4)$$

which is obtained according to the symmetry properties of the scalar superfields given in Table I. Thus, the F-terms potential for scalar fields reads

$$\begin{aligned} V_F = {} & \mu_1^2(\Phi_1^\dagger\Phi_1 + \Phi_1'^\dagger\Phi_1') + \mu_2^2(\Phi_2^\dagger\Phi_2 + \Phi_2'^\dagger\Phi_2') + \mu_\chi^2(\chi^*\chi + \chi'^*\chi') + +\mu_\sigma^2(\sigma^*\sigma + \sigma'^*\sigma') \\ & + (\lambda_1^2|\epsilon_{ij}\Phi_1'^i\Phi_2^j|^2 + \lambda_2^2|\epsilon_{ij}\Phi_2'^i\Phi_1^j|^2 + \lambda_1^2(\Phi_2^\dagger\Phi_2 + \Phi_1'^\dagger\Phi_1'\sigma'^*\sigma' + \lambda_2^2(\Phi_1^\dagger\Phi_1 + \Phi_2'^\dagger\Phi_2')\sigma^*\sigma - \lambda_1\mu_1\Phi_1^\dagger\Phi_2\sigma' \\ & - \lambda_1\mu_2\Phi_2'^\dagger\Phi_1'\sigma' - \lambda_2\mu_1\Phi_1'^\dagger\Phi_2'\sigma - \lambda_2\mu_2\Phi_2^\dagger\Phi_1\sigma - \lambda_1\mu_\sigma\epsilon_{ij}\Phi_1'^i\Phi_2^j - \lambda_2\mu_\sigma\epsilon_{ij}\Phi_2'^i\Phi_1^j + \text{H.c.}) \end{aligned} \tag{5}$$

On the other hand the D-terms potential, consequence of gauge symmetry, turns out to be

$$\begin{aligned} V_D = {} & \frac{g^2}{2}[|\Phi_1^\dagger\Phi_2|^2 + |\Phi_1'^\dagger\Phi_2'|^2 + |\Phi_1'^\dagger\Phi_1|^2 + |\Phi_1'^\dagger\Phi_2|^2 + |\Phi_2'^\dagger\Phi_1|^2 + |\Phi_2'^\dagger\Phi_2|^2 - |\Phi_1|^2|\Phi_2|^2 - |\Phi_1'|^2|\Phi_2'|^2] \\ & + \frac{g^2+g'^2}{8}(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2 - \Phi_1'^\dagger\Phi_1' - \Phi_2'^\dagger\Phi_2')^2 \\ & + \frac{g_X^2}{2}\left[\frac{2}{3}(\Phi_1^\dagger\Phi_1 - \Phi_1'^\dagger\Phi_1') + \frac{1}{3}(\Phi_2^\dagger\Phi_2 - \Phi_2'^\dagger\Phi_2') - \frac{1}{3}(\chi^*\chi - \chi'^*\chi') - \frac{1}{3}(\sigma^*\sigma - \sigma'^*\sigma')\right]^2 \end{aligned} \tag{6}$$

where the last term corresponds to the D-term associated to the $U(1)_X$ gauge symmetry and has a very important role for giving the lightest Higgs boson a mass around 125 GeV. This will be treated later. Finally, the soft supersymmetry breaking potential turns out to be

$$\begin{aligned} V_{\text{soft}} = {} & -m_1^2\Phi_1^\dagger\Phi_1 - m_1'^2\Phi_1'^\dagger\Phi_1' - m_2^2\Phi_2^\dagger\Phi_2 - m_2'^2\Phi_2'^\dagger\Phi_2' - m_\chi^2\chi^\dagger\chi - m_\chi'^2\chi'^\dagger\chi' - m_\sigma^2\sigma^\dagger\sigma - m_\sigma'^2\sigma'^\dagger\sigma' \\ & + [\mu_{11}^2\epsilon_{ij}(\Phi_1'^i\Phi_1^j) + \mu_{22}^2\epsilon_{ij}(\Phi_2'^i\Phi_2^j) + \mu_{\chi\chi}^2(\chi\chi') + \mu_{\sigma\sigma}^2(\sigma\sigma') - \tilde{\lambda}_1\Phi_1'^\dagger\Phi_2\sigma' - \tilde{\lambda}_2\Phi_2'^\dagger\Phi_1\sigma \\ & + \frac{2\sqrt{2}}{9}(k_1\Phi_1^\dagger\Phi_2\chi' - k_2\Phi_1^\dagger\Phi_2\chi^* + k_3\hat{\Phi}_1'^\dagger\hat{\Phi}_2'\chi - k_4\hat{\Phi}_1'^\dagger\hat{\Phi}_2'\chi'^*) + \text{H.c.}] \end{aligned} \tag{7}$$

where the last terms, proportional to the coupling constants named k_1, k_2, k_3, and k_4, break also softly the parity symmetry. These trilinear terms avoid the massless feature of some scalar particles, as we will show later. It is important to mention that the soft supersymmetry breaking potential also includes bilinear terms for sfermions and gauginos. Nonetheless, since those terms are not required for our calculations we have decided that it is not necessary to present them in the current work. Then, by adding all contributions, the scalar potential for the Higgs bosons reads

$$V_H = V_F + V_D + V_{\text{soft}}. \tag{8}$$

When considering the potential V_F it can be seen that, before including soft SUSY breaking, particles within the fields Φ_i and Φ_i', $i = 1, 2$, are expected to have the same mass μ_i due to the absence of mixing terms. Then, with the inclusion of a soft breaking potential, m_i^2, $m_i'^2$ terms arise and the diagonal entries change according to the effective parameters $m_{Hk}^2 = m_k^2 + \mu_k^2$ and $m_{Hk}'^2 = m_k'^2 + \mu_k^2$, ensuring that different Higgs doublets have now different mass eigenvalues. This is also exhibited by the scalar singlets, where diagonal entries are written in terms of the effective parameters $M_\chi^2 = m_\chi^2 + \mu_\chi^2$ and $M_\chi'^2 = m_\chi'^2 + \mu_\chi^2$. The following minima conditions for the Higgs potential have to be fulfilled:

$$\begin{aligned} & m_{H1}^2 + \frac{1}{8}(g^2+g'^2)C_{\text{EW}} + \frac{g_X^2}{9}c_X - \mu_{11}\frac{v_1'}{v_1} + \frac{\lambda_2^2}{2}v_2'^2 = 0 \\ & m_{H1}'^2 - \frac{1}{8}(g^2+g'^2)C_{\text{EW}} - \frac{g_X^2}{9}c_X - \mu_{11}\frac{v_1}{v_1'} + \frac{\lambda_1^2}{2}v_1'^2 = 0 \\ & m_{H2}^2 + \frac{1}{8}(g^2+g'^2)C_{\text{EW}} + \frac{g_X^2}{18}c_X - \mu_{22}\frac{v_2'}{v_2} + \frac{\lambda_1^2}{2}v_2^2 = 0 \\ & m_{H2}'^2 - \frac{1}{8}(g^2+g'^2)C_{\text{EW}} - \frac{g_X^2}{18}c_X - \mu_{22}\frac{v_2}{v_2'} + \frac{\lambda_2^2}{2}v_1^2 = 0 \\ & M_\chi^2 - \frac{g_X^2}{18}c_X - \mu_{\chi\chi}\frac{v_\chi'}{v_\chi} = 0 \\ & M_\chi'^2 + \frac{g_X^2}{18}c_X - \mu_{\chi\chi}\frac{v_\chi}{v_\chi'} = 0, \end{aligned} \tag{9}$$

where we also defined $C_{\text{EW}} = v_1^2 + v_2^2 - v_1'^2 - v_2'^2$ and $C_X = 2v_1^2 + v_2^2 - 2v_1'^2 - v_2'^2 + v_\chi'^2 - v_\chi^2$.

A. *CP*-even masses

Taking the VEV for all scalar fields, we get the mass matrices for the different Higgs boson particles, that also respect the minima conditions, Eq. (9). The 125 GeV Higgs boson is a *CP*-even scalar, and it must be obtained from the diagonalization of the following 6×6 mass matrix in the $(h_1, h_1', h_2, h_2', \xi_\chi, \xi_\chi', \xi_\sigma, \xi_\sigma')$ basis:

$$\frac{1}{2}M_h^2 = \begin{pmatrix} M_{hh} & M_{h\xi} \\ M_{h\xi}^T & M_{\xi\xi} \end{pmatrix}. \tag{10}$$

M_{hh} is a 4×4 matrix accounting for the mixings among the CP-even fields of the doublets in the model, and it reads

$$M_{hh} = \begin{pmatrix} f_{4g}v_1^2 - \frac{v_2 f_{1k}}{9v_1} + \frac{v_1'\mu_{11}^2}{2v_1} & -f_{4g}v_1 v_1' - \frac{\mu_{11}^2}{2} & f_{2g}v_1 v_2 + \frac{f_{1k}}{9} & -f_{2g}v_1 v_2' + \frac{1}{2}\lambda_2^2 v_1 v_2' \\ * & f_{4g}v_1'^2 - \frac{v_2' f_{2k}}{9v_1'} + \frac{v_1\mu_{11}^2}{2v_1'} & -f_{2g}v_1' v_2 + \frac{1}{2}\lambda_1^2 v_2 v_1' & f_{2g}v_1' v_2' + \frac{f_{2k}}{9} \\ * & * & f_{1g}v_2^2 - \frac{v_1 f_{1k}}{9v_2} + \frac{v_2'\mu_{22}^2}{2v_2} & -f_{1g}v_2 v_2' - \frac{\mu_{22}^2}{2} \\ * & * & * & f_{1g}v_2'^2 - \frac{v_1' f_{2k}}{9v_2'} + \frac{v_2\mu_{22}^2}{2v_2'} \end{pmatrix}. \tag{11}$$

The mixings between scalar doublets and singlets are written in the 4×4 $M_{h\xi}$ matrix and it is given by:

$$M_{h\xi} = \begin{pmatrix} \frac{1}{9}(k_2 v_2 - g_X^2 v_1 v_\chi) & \frac{1}{9}(-k_1 v_2 + g_X^2 v_1 v_\chi') & \frac{1}{2\sqrt{2}}(\widetilde{\lambda}_2 v_2' - \lambda_2\mu_2 v_2) & -\frac{1}{2\sqrt{2}}(\lambda_1\mu_1 v_2 + \lambda_2\mu_\sigma v_2') \\ \frac{1}{9}(-k_3 v_2' + g_X^2 v_1' v_\chi) & \frac{1}{9}(k_4 v_2' - g_X^2 v_1' v_\chi') - & -\frac{1}{2\sqrt{2}}(\lambda_2\mu_1 v_2' + \lambda_1\mu_\sigma v_2) & \frac{1}{2\sqrt{2}}(\widetilde{\lambda}_1 v_2 - \lambda_1\mu_2 v_2') \\ \frac{1}{9}(k_2 v_1 - \frac{1}{2}g_X^2 v_2 v_\chi) & \frac{1}{9}(-k_1 v_1 + \frac{1}{2}g_X^2 v_2 v_\chi') & -\frac{1}{2\sqrt{2}}(\lambda_2\mu_2 v_1 + \lambda_1\mu_\sigma v_1') & \frac{1}{2\sqrt{2}}(\widetilde{\lambda}_1 v_1' - \lambda_1\mu_1 v_1) \\ \frac{1}{9}(-k_3 v_1' + \frac{1}{2}g_X^2 v_2' v_\chi) & \frac{1}{9}(k_4 v_1' - \frac{1}{2}g_X^2 v_2' v_\chi') & \frac{1}{2\sqrt{2}}(\widetilde{\lambda}_2 v_1 - \lambda_2\mu_1 v_1') & -\frac{1}{2\sqrt{2}}(\lambda_1\mu_2 v_1' + \lambda_2\mu_\sigma v_1) \end{pmatrix} \tag{12}$$

Finally, the mixing matrix between Higgs singlets, $M_{\xi\xi}$, is written in the following equation

$$M_{\xi\xi} = \begin{pmatrix} \frac{g_X^2}{18}v_\chi^2 + \frac{v_\chi'\mu_{\chi\chi}^2}{2v_\chi} - \frac{k_{23}}{9v_\chi} & -\frac{g_X^2}{18}v_\chi v_\chi' - \frac{\mu_{\chi\chi}^2}{2} & 0 & 0 \\ * & \frac{g_X^2}{18}v_\chi'^{\,2} + \frac{v_\chi\mu_{\chi\chi}^2}{2v_\chi'} - \frac{k_{14}}{9v_\chi'} & 0 & 0 \\ * & * & M_\sigma^2 + \frac{\lambda_2^2}{4}(v_1^2 + v_2'^2) & -\frac{\mu_{\sigma\sigma}}{2} \\ * & * & * & M_\sigma'^2 + \frac{\lambda_1^2}{4}(v_2^2 + v_1'^2) \end{pmatrix} \tag{13}$$

Aimed in giving shorter expressions we have defined the coefficients $f_{ng} = \frac{g^2+g'^2}{8} + \frac{n}{18}g_X^2$ with n an integer, $f_{1k} = k_2 v_\chi - k_1 v_\chi'$, $f_{2k} = -k_3 v_\chi + k_4 v_\chi'$, $k_{23} = k_2 v_1 v_2 - k_3 v_1' v_2'$ and $k_{14} = -k_1 v_1 v_2 + k_4 v_1' v_2'$ and the sigma masses $M_\sigma = \frac{1}{2}(\mu_\sigma^2 + m_\sigma^2) - \frac{g_X^2}{36}(2v_1^2 + v_2^2 - 2v_1'^2 - v_2'^2 - v_\chi^2 + v_\chi'^2)$ and $M_\sigma' = \frac{1}{2}(\mu_\sigma^2 + m_\sigma'^2) + \frac{g_X^2}{36}(2v_1^2 + v_2^2 - 2v_1'^2 - v_2'^2 - v_\chi^2 + v_\chi'^2)$.

In order to diagonalize the CP-even Higgs mass matrix given in Eq. (10) we make use of perturbation theory by implementing a seesaw mechanism which requires then to specify a hierarchy among the parameters present in the different blocks. First, we suppose their order of magnitude such that they obey $\mathcal{O}(M_{\xi\xi}) \gg \mathcal{O}(M_{h\xi}) \gg \mathcal{O}(M_{hh})$ which implies a hierarchy among the different parameters in the potentials [Eq. (8)]. To have a correct phenomenological theory we assume $\mu_{\chi\chi}, \mu_{\sigma\sigma}, M_\sigma, M_\sigma' \gg \mu_{11}, \mu_{22} \gg k_i v_j \gg g_X^2 v_\chi v_j, g_X^2 v_\chi' v_j, g_X^2 v_\chi v_j', g_X^2 v_\chi' v_j'$, where $i = 1,\ 2,\ 3,\ 4$ and $j = 1,\ 2$. Therefore, the conditions for implementing a seesaw mechanism are fulfilled which leads to a block diagonal mass matrix made of two independent 4×4 matrices. Additionally, as the rank of M_h is 8, all mass eigenstates are massive. Considering that a Higgs singlet has not been observed in the experiments, it would be expected for them to acquire mass at high energy scale, supporting our hierarchy choice and implying that v_χ and v_χ' should be at least at the TeV scale, thus they satisfy v_χ, $v_\chi' \gg v_j, v_j'$, where $j = 1, 2$. Additional to our assumptions, a small program in C++ was written to find the parameter region in which a 125(GeV) value match the lightest eigenvalue using a simple Monte Carlo exploration. It was found that $k_i \sim 10^3$, $0 < \lambda_i, \tilde{\lambda}_i < 10^3$, $\mu_{11},\ \mu_{22} > 10^4$, $\mu_{\chi\chi}, \mu_{\sigma\sigma}, M_\sigma, M_\sigma' > 10^8$, and $v_\chi,\ v_\chi' > 10^3$, which in fact satisfy the considered parameter hierarchy. The singlet VEVs lower bound are fixed in such a way that guarantees a lightest eigenvalue coming from a scalar doublets mixture.

$$\tilde{M}_h \approx \begin{pmatrix} \tilde{M}_{hh} & 0 \\ 0 & M_{\xi\xi} \end{pmatrix},$$
$$\tilde{M}_{hh} = M_{hh} - M_{h\xi}M_{\xi\xi}^{-1}M_{h\xi}^T \approx M_{hh}. \tag{14}$$

Since the matrix rank for M_{hh} is 4, all Higgs particles are massive and acquire their tree level masses through M_{hh}, then they receive small contributions coming from the seesaw rotation ($M_{h\xi}M_{\xi\xi}^{-1}M_{h\xi}^T$), which can be neglected due to the parameters' order of magnitude. As a result we can assume that $\tilde{M}_{hh} \approx M_{hh}$. In this way it is possible to get an expression for the heaviest singlet Higgs particles by the diagonalization of the 4×4 decoupled matrix which in fact is a 2×2 block diagonal one since the singlets does not mix among them. We are considering that the heaviest particles must depend on $\mu_{\chi\chi}$, $\mu_{\sigma\sigma}$, M_σ, and M'_σ, which are the biggest parameters in the theory; while for the two lightest of these singlets depend then on v_χ, v'_χ, k_i, and λ_1 with the condition that the sum must reconstruct the original trace. In this way we can get the tree level approximation of these eigenvalues which can be written as:

$$m_{h5}^2 \approx \frac{g_X^2}{9}(v_\chi^2 + v_\chi'^2) - \frac{2}{9}\frac{v_1 v_2(k_2 v'_\chi - k_1 v_\chi) + v'_1 v'_2(k_4 v_\chi - k_3 v'_\chi)}{v_\chi v'_\chi},$$
$$m_{h6}^2 \approx \mu_{\chi\chi}^2 \frac{v_\chi^2 + v_\chi'^2}{v_\chi v'_\chi}, \tag{15}$$

$$m_{h7}^2 = (M_\sigma^2 + M_\sigma'^2) + \frac{1}{4}[\lambda_2^2(v_1^2 + v_2'^2) + \lambda_1^2(v_2^2 + v_1'^2)] - \sqrt{\mu_{\sigma\sigma}^4 - \left((M_\sigma^2 + M_\sigma'^2) + \frac{1}{4}[\lambda_2^2(v_1^2 + v_2'^2) - \lambda_1^2(v_2^2 + v_1'^2)]\right)^2},$$
$$m_{h8}^2 = (M_\sigma^2 + M_\sigma'^2) + \frac{1}{4}[\lambda_2^2(v_1^2 + v_2'^2) + \lambda_1^2(v_2^2 + v_1'^2)] + \sqrt{\mu_{\sigma\sigma}^4 - \left((M_\sigma^2 + M_\sigma'^2) + \frac{1}{4}[\lambda_2^2(v_1^2 + v_2'^2) - \lambda_1^2(v_2^2 + v_1'^2)]\right)^2} \tag{16}$$

Since the singlet σ acquire a null vacuum expectation value there is not a minimum condition which relates M_σ, M'_σ, and $\mu_{\sigma\sigma}$ as it happened with the other scalar particles. In fact, there is more freedom for making a hierarchy choice among them. However, this particles are expected to a reside at an unreachable energy scale for the current experiments and does not represent the main focus of this work; even though they have an important role in fermion mass generation as it will be shown later. For finding corresponding M_ϕ eigenvalues we took the following approach. We consider that at least one of the Higgs particles must be proportional to the electroweak VEVs such that it can be identified as the SM Higgs particle. Therefore, the other three must be heavy and functions of the SSB parameters and μ_{11}, μ_{22} and k_i; reason for which we took a small doublet's VEV limit (v_1, v_2, v'_1, $v'_2 \to 0$), and it is found that the matrix rank decreases to 3. In fact, it means that there is an eigenvalue which depends entirely on the EW VEVs and it can be identified as a SM Higgs particle. Furthermore, when considering this limit we are able to get the next two heavy states by discarding the additive terms proportional to those VEV. As a result, the matrix M_ϕ reduces to the following form:

$$M_\phi(v_i, v'_i \to 0) = \begin{pmatrix} \frac{\mu_{11}^2}{2}\frac{v'_1}{v_1} & -\frac{\mu_{11}^2}{2} & 0 & 0 \\ * & \frac{\mu_{11}^2}{2}\frac{v_1}{v'_1} & 0 & 0 \\ * & * & \frac{\mu_{22}^2}{2}\frac{v'_2}{v_2} & -\frac{\mu_{22}^2}{2} \\ * & * & * & \frac{\mu_{22}^2}{2}\frac{v_2}{v'_2} \end{pmatrix} \tag{17}$$

Therefore, we find two decoupled 2×2 matrices with determinant equal to zero, arising the two heaviest states. Then, in the tree level the eigenvalues are given by:

$$m_{h3}^2 \approx \mu_{11}^2 \frac{v_1^2 + v_1'^2}{v_1 v'_1}, \qquad m_{h4}^2 \approx \mu_{22}^2 \frac{v_2^2 + v_2'^2}{v_2 v'_2}. \tag{18}$$

At this point, there are only two CP-even particles for which one of them must be the like-Higgs SM. Returning to the original mass matrix M_{hh} (11), it is known that eigenvalues are roots of the characteristic polynomial. In our case, the polynomial is a fourth degree one. It can be solved analytically by using a general solution given by Ferrari's method [18]. It is worth to notice that by taking the small VEV limit in the corresponding two general solutions, accounting for the two heavy eigenstates, the eigenvalues in Eq. (18) are reproduced. On the other hand, the smaller value acquired by the general solution would correspond to the SM-like Higgs particle but the resulting expression for the latter is too complicated by using this method, requiring then a different approach for obtain it. However, by taking again the small VEV limit mentioned before and taking into account the chosen hierarchy, now the dominant terms are proportional to k_i and it results in the following expression.

$$m_{h2}^2 \approx \frac{2v^2(v_1 v_2(k_1 v'_\chi - k_2 v_\chi) + v'_1 v'_2(k_3 v_\chi - k_4 v'_\chi))}{9(v_1^2 + v_1'^2)(v_2^2 + v_2'^2)}. \tag{19}$$

Finally we focus on the lightest CP-even Higgs particle, which so far in our approximation became massless, but must match the 125 GeV observed one. For starting, we considered the determinant of the 4×4 matrix given by Eq. (11) only taking into account the dominant terms which are proportional to $\mu_{11}^2\mu_{22}^2$. It reads:

$$Det(\tilde{M}_{hh}) \approx \frac{\mu_{11}^2\mu_{22}^2}{2592}\Bigg[(k_3 v_\chi - k_4 v'_\chi)\Bigg((9(g^2+g'^2)+16g_X^2)\frac{(v_1^2-v_1'^2)^2}{v_1 v_2} + (9(g^2+g'^2)+4g_X^2)\frac{(v_2^2-v_2'^2)^2}{v_1 v_2} + 2(9(g^2+g'^2)+8g_X^2)\frac{(v_1^2-v_1'^2)(v_2^2-v_2'^2)}{v_1 v_2}\Bigg) + (k_1 v'_\chi - k_2 v_\chi)\Bigg((9(g^2+g'^2)+16g_X^2)\frac{(v_1^2-v_1'^2)^2}{v'_1 v'_2} + (9(g^2+g'^2)+4g_X^2)\frac{(v_2^2-v_2'^2)^2}{v'_1 v'_2} + 2(9(g^2+g'^2)+8g_X^2)\frac{(v_1^2-v_1'^2)(v_2^2-v_2'^2)}{v'_1 v'_2}\Bigg)\Bigg]. \tag{20}$$

The lightest Higgs mass eigenvalue is found by dividing this determinant by the other three found mass eigenvalues given by Eqs. (18)–(19), which can be written as

$$m_{h1}^2 \approx \frac{g_X^2(2v_1^2+v_2^2-2v_1'^2-v_2'^2)^2}{9(v_1^2+v_2^2+v_1'^2+v_2'^2)} + \frac{(g^2+g'^2)(v_1^2+v_2^2-v_1'^2-v_2'^2)^2}{4(v_1^2+v_2^2+v_1'^2+v_2'^2)} \tag{21}$$

Now, if we define the angles $\tan^2\tilde{\beta} = \frac{v_1^2+v_2^2}{v_1'^2+v_2'^2}$, $\tan\beta_1 = \frac{v_1}{v'_1}$, and $\tan\beta_2 = \frac{v_2}{v'_2}$ then the lightest scalar mass acquires the following form

$$m_{h1}^2 = m_Z^2\left(\cos^2 2\tilde{\beta} + \frac{4}{9}\frac{g_X^2}{g^2+g'^2}(\cos 2\beta_1 + \cos 2\beta_2)^2\right) \approx m_Z^2\cos^2 2\tilde{\beta} + \Delta m_h^2. \tag{22}$$

So in fact, when we consider the theory with additional scalar singlets and D-term, the correction term Δm_h^2 can be at the same order of tree level, and its experimental value can be explained with NMSSM and USSM. An interesting fact arises from the approximated expression for the lightest CP-even particle. Its tree level mass does not depend on the new physic's energy scale, given by v_χ and v'_χ. Additionally the μ_{11} and μ_{22} factors canceled out due to the seesaw mechanism, making an eigenvalue depending only on the electroweak scale VEV's, as it is expected.

As it will be shown later and has been already mentioned, the VEV should fulfill $v_1^2+v_1'^2+v_2^2+v_2'^2 = 246^2\ \mathrm{GeV}^2$. Then, for instance, it is found that a 125 GeV Higgs boson can be reached for values of $g_X = 1.06g$, and a large list of possible values for v_i and v'_i. For example considering $v_1 = 195$, $v_2 = 138$, $v'_1 = 52$, $v'_2 = 20$, and $g_X = 0.71$ a 125 GeV Higgs boson is found.

In Fig. 1 we plot v'_1 vs g_X by using the expression in the Eq. (21) for the like-Higgs SM at 95% of C.L. with 125.3 ± 0.4 GeV. We take v_1 proportional to the top quark mass, v'_2 at an intermediate value between the bottom quark and tau lepton masses, and $v_2 = \sqrt{v^2 - v_1^2 - v_1^{2\prime} - v_2'^2}$. This was done for v'_1 since it is not restricted directly by the fermion mass hierarchy (FMH). The exploration in the parameter space was done by using the Monte Carlo method for generating randomly the values of v'_1, v'_2, and g_X. For addressing the fermion mass hierarchy, the domains of v_1 and v'_2 were [170 GeV, 200 GeV] and [3 GeV, 7 GeV], respectively. A VEV is determined by the condition (3), thus $v'_1 = \sqrt{v^2 - v_1^2 - v_2^2 - v_2'^2}$. The remaining VEV could run freely, which means the its domain was $0 < v_2 < 246$ GeV. Last by not least, the coupling parameter g_X was explored in the domain [0, 1].

A similar plot is given in the Fig. 2, where the parameter space of v_2 vs g_X is explored within the experimental constraints at 95% of confidence level. This is shown because v_2 is also not constrained by FMH.

A clear dependence of the mass of the lightest CP-even particle with

$$\cos 2\tilde{\beta} = \frac{v_1'^2 + v_2'^2 - {v_1}^2 - {v_2}^2}{v^2}$$

is found in the Eq. (22). In the Fig. 3, the parameter space $\cos 2\tilde{\beta}$ vs g_X is explored. Negative values are found for

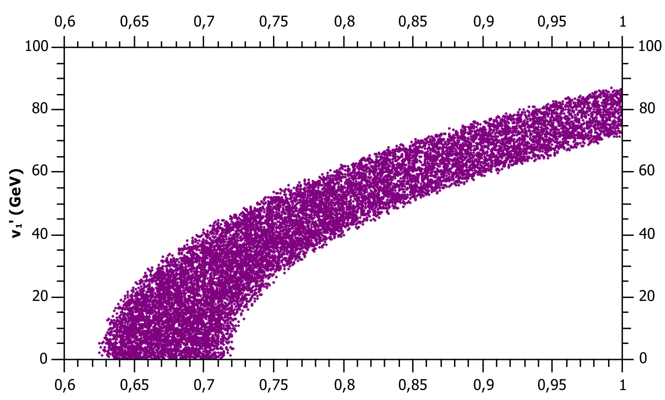

FIG. 1. Region in the parameter space v'_1 vs g_X with a Higgs mass of 125.3 ± 0.4 GeV at 95% of C.L.

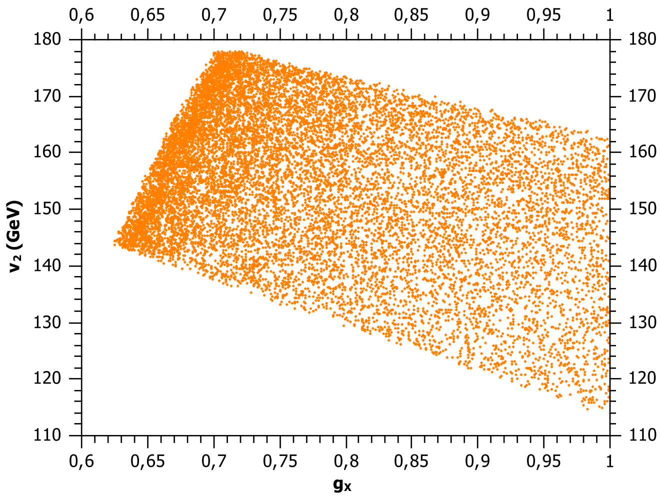

FIG. 2. Region in the parameter space v_2 vs g_X with a Higgs mass of 125.3 ± 0.4 GeV at 95% of C.L.

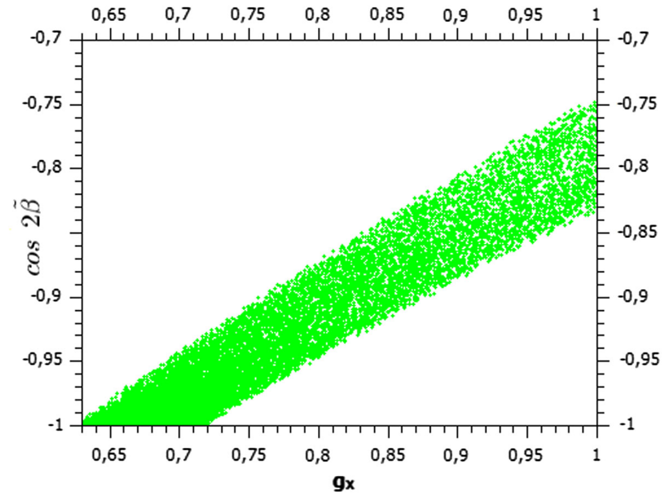

FIG. 3. Region in the parameter space $\cos 2\tilde{\beta}$ vs g_X with a Higgs mass of 125.3 ± 0.4 GeV at 95% of C.L.

$\cos 2\tilde{\beta}$ because nonprimed VEV happened to be greater than the primed ones. This behavior lies in two causes: One of them is the imposition of the FMH, where the top quark mass ($\approx v_1$) is bigger then the bottom quark mass ($\approx v_2'$) then $v_1 > v_2'$. On the other hand, all along the mass expression [better seen on Eq. (21)] there are quadratic differences between nonprimed and primed VEVs. For obtaining considerable contributions to get the 125 GeV mass, those differences have to be relatively big. Thus $v_1, v_2 > v_1', v_2'$ is preferred. As g_X gets bigger, smaller differences on the VEVs are needed. Thus $|\cos 2\tilde{\beta}|$ approaches to smaller values.

B. *CP*-odd masses

The mass matrix for the *CP*-odd particles must contain the would-be Goldstone bosons to give masses to the Z_μ and Z'_μ. Such matrix in the basis $(\eta_1, \eta_1', \eta_2, \eta_2', \zeta_\chi, \zeta_\chi', \zeta_\sigma, \zeta_\sigma')$ is given in the next equation:

$$\frac{1}{2}\mathcal{M}_\eta = \begin{pmatrix} \mathcal{M}_{\eta\eta} & \mathcal{M}_{\eta\zeta} \\ \mathcal{M}_{\eta\zeta}^T & \mathcal{M}_{\zeta\zeta} \end{pmatrix}. \tag{23}$$

$\mathcal{M}_{\eta\eta}$ contains the mixings between the *CP*-odd part of doublets and it can be written as

$$\mathcal{M}_{\eta\eta} = \begin{pmatrix} \frac{\mu_{11}^2 v_1'}{2v_1} - \frac{f_{1k} v_2}{9v_1} & \frac{\mu_{11}^2}{2} & \frac{f_{1k}}{9} & 0 \\ * & \frac{\mu_{11}^2 v_1}{2v_1'} - \frac{f_{2k} v_2'}{9v_1'} & 0 & \frac{f_{2k}}{9} \\ * & * & \frac{\mu_{22}^2 v_2'}{2v_2} - \frac{f_{1k} v_1}{9v_2} & \frac{\mu_{22}^2}{2} \\ * & * & * & \frac{\mu_{22}^2 v_2}{2v_2'} - \frac{f_{2k} v_1'}{9v_2'} \end{pmatrix}. \tag{24}$$

The matrix accounting for the mixings between the *CP*-odd parts of the doublets and the singlets is $\mathcal{M}_{\eta\zeta}$, and it turns out to be

$$\mathcal{M}_{\eta\chi} = \frac{1}{9}\begin{pmatrix} -k_2 v_2 & -k_1 v_2 & \frac{1}{2\sqrt{2}}(\lambda_2\mu_2 v_2 - \tilde{\lambda}_2 v_2') & -\frac{1}{2\sqrt{2}}(\lambda_1\mu_1 v_2 + \lambda_2\mu_\sigma v_2') \\ -k_3 v_2' & -k_4 v_2' & -\frac{1}{2\sqrt{2}}(\lambda_2\mu_1 v_2' + \lambda_1\mu_\sigma v_2) & \frac{1}{2\sqrt{2}}(\lambda_1\mu_2 v_2' - \tilde{\lambda}_1 v_2) \\ k_2 v_1 & k_1 v_1 & -\frac{1}{2\sqrt{2}}(\lambda_2\mu_2 v_1 + \lambda_1\mu_\sigma v_1') & \frac{1}{2\sqrt{2}}(\lambda_1\mu_1 v_1 - \tilde{\lambda}_1 v_1') \\ k_3 v_1' & k_4 v_1' & \frac{1}{2\sqrt{2}}(\lambda_2\mu_1 v_1' - \tilde{\lambda}_2 v_1) & -\frac{1}{2\sqrt{2}}(\lambda_1\mu_2 v_1' + \lambda_2\mu_\sigma v_1) \end{pmatrix}. \tag{25}$$

The mixings between the *CP*-odd part of the singlets, which are responsible of the would-be Goldstone boson due to the $U(1)_X$ symmetry breaking, are given by:

$$\mathcal{M}_{\zeta\zeta} = \begin{pmatrix} \frac{v'_\chi \mu^2_{\chi\chi}}{2v_\chi} - \frac{k_2 v_1 v_2 - k_3 v'_1 v'_2}{9v_\chi} & \frac{\mu^2_{\chi\chi}}{2} & 0 & 0 \\ * & \frac{v_\chi \mu^2_{\eta 2}}{2v'_\chi} - \frac{-k_1 v_1 v_2 + k_4 v'_1 v'_2}{9v'_\chi} & 0 & 0 \\ * & * & M^2_\sigma + \frac{\lambda^2_2}{4}(v^2_1 + v'^2_2) & \frac{\mu_{\sigma\sigma}}{2} \\ * & * & * & M'^2_\sigma + \frac{\lambda^2_1}{4}(v^2_2 + v'^2_1) \end{pmatrix}. \tag{26}$$

Being M^2_σ and M'^2_σ the same coefficients defined in the scalar CP-even mass matrix. The rank of the matrix $\mathcal{M}_{\eta\eta}$ turns out to be 4, so we can be sure that there are two null eigenvalues, corresponding to the would-be Goldstone bosons needed. It is important to notice that structure of this matrix is different from M_h, but it preserves the same scale structure. That is to say, $\mathcal{M}_{\zeta\zeta} \sim \mu_{\chi\chi}, \mu_{\sigma\sigma}, v_\chi, v'_\chi$ and $\mathcal{M}_{\eta\eta} \sim \mu_{ii}, \mu_{\sigma\sigma}, v_i, v'_i$ which fulfill the conditions for a seesaw mechanism, $\mathcal{M}_{\zeta\zeta} \gg \mathcal{M}_{\eta\eta}, \mathcal{M}_{\eta\zeta}$. When the rotation is done, the matrix reads:

$$\tilde{\mathcal{M}}_\eta \approx \begin{pmatrix} \tilde{\mathcal{M}}_{\eta\eta} & 0 \\ 0 & \mathcal{M}_{\zeta\zeta} \end{pmatrix}, \tag{27}$$

$$\tilde{\mathcal{M}}_{\eta\eta} = \mathcal{M}_{\eta\eta} - \mathcal{M}_{\eta\zeta}\mathcal{M}^{-1}_{\zeta\zeta}\mathcal{M}^T_{\eta\zeta}. \tag{28}$$

It is worth noticing that this matrix has a big dependence on the parity breaking terms k_i, $i = 1, 2, 3, 4$.

If we consider the limit in which these couplings go to zero, $k_i \to 0$ the mixing matrix $\mathcal{M}_{\eta\zeta}$ vanishes resulting in a 2×2 isolated singlet mixing matrix $\mathcal{M}_{\zeta\zeta}(k_i \to 0)$ containing the $U(1)_X$ would-be Goldstone boson and the heaviest pseudoscalar particle predicted by the model. The two mass eigenstates are written as:

$$m^2_{\eta 5} = 0, \qquad m^2_{\eta 6} \approx \mu^2_{\chi\chi}\frac{v^2_\chi + v'^2_\chi}{v_\chi v'_\chi}.$$

$$m^2_{\eta 7} = (M^2_\sigma + M'^2_\sigma) + \frac{1}{4}[\lambda^2_2(v^2_1 + v'^2_2) + \lambda^2_1(v^2_2 + v'^2_1)] - \sqrt{\mu^4_{\sigma\sigma} - \left((M^2_\sigma + M'^2_\sigma) + \frac{1}{4}[\lambda^2_2(v^2_1 + v'^2_2) - \lambda^2_1(v^2_2 + v'^2_1)]\right)^2},$$

$$m^2_{\eta 8} = (M^2_\sigma + M'^2_\sigma) + \frac{1}{4}[\lambda^2_2(v^2_1 + v'^2_2) + \lambda^2_1(v^2_2 + v'^2_1)] + \sqrt{\mu^4_{\sigma\sigma} - \left((M^2_\sigma + M'^2_\sigma) + \frac{1}{4}[\lambda^2_2(v^2_1 + v'^2_2) - \lambda^2_1(v^2_2 + v'^2_1)]\right)^2} \tag{29}$$

In contrast, the 4×4 matrix has a rank of 3 ensuring the existence of the would-be Goldstone boson due to $SU(2) \times U(1)$ symmetry breaking. The scheme for getting an expression for the eigenvalues is the same we used for the CP-even mass matrix. First, by considering the small VEV limit we can write the matrix as:

$$\tilde{\mathcal{M}}_{\eta\eta}(v_i, v'_i \to 0) = \begin{pmatrix} \frac{\mu^2_{11}}{2}\frac{v'_1}{v_1} & \frac{\mu^2_{11}}{2} & 0 & 0 \\ * & \frac{\mu^2_{11}}{2}\frac{v_1}{v'_1} & 0 & 0 \\ * & * & \frac{\mu^2_{22}}{2}\frac{v'_2}{v_2} & \frac{\mu^2_{22}}{2} \\ * & * & * & \frac{\mu^2_{22}}{2}\frac{v_2}{v'_2} \end{pmatrix} \tag{30}$$

leading to 2 heavy eigenstates which at tree level can be written as:

$$m^2_{\eta 3} \approx \mu^2_{11}\frac{v^2_1 + v'^2_1}{v_1 v'_1}, \qquad m^2_{\eta 4} \approx \mu^2_{22}\frac{v^2_2 + v'^2_2}{v_2 v'_2}. \tag{31}$$

Finally for the lightest massive CP-odd particle it was used a small VEV limit for the general solution of the quartic equation given by Ferrari's method [18]. Having a massless state allows us to reduce the characteristic polynomial to a third grade one. Ferrari's solutions implies the cubic general solution, and through this general expressions the lighest CP-odd Higgs particles can be written as

$$m^2_{\eta 2} \approx \frac{2v^2(v_1 v_2(k_1 v'_\chi - k_2 v_\chi) + v'_1 v'_2(k_3 v_\chi - k_4 v'_\chi))}{9(v^2_1 + v'^2_1)(v^2_2 + v'^2_2)}. \tag{32}$$

The other mass corresponds to the would-be Goldstone boson associated with Z^μ

$$m^2_{\eta 1} = 0. \tag{33}$$

C. Charged scalar bosons

In the case of the charged components of the scalar fields, the corresponding mass matrix must contain a would-be Goldstone boson that gives mass to the $W^{\mu\pm}$ gauge boson. The mass matrix in the basis $(\phi_1^+, \phi_1'^+, \phi_2^+, \phi_2'^+)$ reads

$$\mathcal{M}_C = \begin{pmatrix} m_{11}^C + \frac{v_1'}{v_1}\mu_{11}^2 - \frac{1}{2}\lambda_2^2 v_2'^2 & \frac{g^2}{4}v_1 v_1' + \mu_{11}^2 & \frac{g^2}{4}v_1 v_2 + \frac{2}{9}f_{1k} & \frac{g^2}{4}v_1 v_2' - \frac{1}{2}\lambda_2^2 v_1 v_2' \\ * & m_{11}^{C\prime} + \frac{v_1}{v_1'}\mu_{11}^2 - \frac{1}{2}\lambda_1^2 v_2^2 & \frac{g^2}{4}v_1' v_2 - \frac{1}{2}\lambda_1^2 v_2 v_1' & \frac{g^2}{4}v_1' v_2' + \frac{2}{9}f_{2k} \\ * & * & m_{22}^C + \frac{v_2'}{v_2}\mu_{22}^2 - \frac{1}{2}\lambda_1^2 v_1'^2 & \frac{g^2}{4}v_2 v_2' + \mu_{22}^2 \\ * & * & * & m_{22}^{C\prime} + \frac{v_2}{v_2'}\mu_{22}^2 - \frac{1}{2}\lambda_2^2 v_1^2 \end{pmatrix}, \quad (34)$$

where we have defined

$$m_{11}^C = \frac{g^2}{4}(v_1'^2 + v_2'^2 - v_2^2) - \frac{2v_2}{9v_1}f_{1k}, \qquad m_{11}^{C\prime} = \frac{g^2}{4}(v_1'^2 + v_2^2 - v_2'^2) - \frac{2v_2'}{9v_1'}f_{2k},$$
$$m_{22}^C = \frac{g^2}{4}(v_1'^2 + v_2'^2 - v_1^2) - \frac{2v_1}{9v_2}f_{1k}, \qquad m_{22}^{C\prime} = \frac{g^2}{4}(v_1^2 + v_2^2 - v_1'^2) - \frac{2v_1'}{9v_2'}f_{2k}. \quad (35)$$

The rank of the mass matrix for charged Higgs bosons is 3, so there is one would-be Goldstone boson that gives masses to the $W^{\pm\mu}$ gauge boson. The procedure for obtaining the mass eigenvalues is straightforward. We perform a small VEV limit to get the heavy eigenvalues. The would-be Goldstone boson is ensured by the vanishing determinant and the lightest massive eigenvalue is found by taking a small VEV approximation in Ferrari's general solution for a cubic polynomial, giving the following expressions:

$$m^2_{G_W^\pm} = 0,$$
$$m^2_{H_1^\pm} \approx \frac{2v^2(v_1 v_2(k_1 v_\chi' - k_2 v_\chi) + v_1' v_2'(k_3 v_\chi - k_4 v_\chi'))}{9(v_1^2 + v_1'^2)(v_2^2 + v_2'^2)},$$
$$m^2_{H_2^\pm} \approx \mu_{11}^2 \frac{v_1^2 + v_1'^2}{v_1 v_1'},$$
$$m^2_{H_3^\pm} \approx \mu_{22}^2 \frac{v_2^2 + v_2'^2}{v_2 v_2'}. \quad (36)$$

D. Gauge boson masses

As consequence of the inclusion of the symmetry $U(1)_X$, there is a new gauge boson B'_μ which has to be included in the covariant derivate.

$$D_\mu = \partial_\mu - igW_\mu^a T_a - ig'\frac{Y}{2}B_\mu - ig_X B'_\mu. \quad (37)$$

Therefore the gauge boson masses are determined by the interaction terms, which are present in the scalar kinetic terms of the scalar fields. On one hand, the charged bosons $W_\mu^\pm = (W_\mu^1 \mp W_\mu^2)/\sqrt{2}$ acquire standard model like masses $M_W = \frac{gv}{2}$. The neutral gauge bosons (W_μ^3, B_μ, B'_μ) make up a squared-mass matrix after SSB given by:

$$M_0^2 = \frac{1}{4}\begin{pmatrix} g^2 v^2 & -gg'v^2 & -\frac{2}{3}gg_X v^2(1+\cos^2\beta) \\ * & g'^2 v^2 & \frac{2}{3}g'g_X v^2(1+\cos^2\beta) \\ * & * & \frac{4}{9}g_X^2[V_\chi^2 + (1+3\cos^2\beta)v^2] \end{pmatrix},$$

where we have defined:

$$v^2 = v_1^2 + v_2^2 + v_1'^2 + v_2'^2 \quad (38)$$

$$\tan\beta = \frac{\sqrt{v_2^2 + v_2'^2}}{\sqrt{v_1^2 + v_1'^2}} \equiv \frac{V_2}{V_1} \quad (39)$$

$$V_\chi^2 \equiv v_\chi^2 + v_\chi'^2. \quad (40)$$

Despite in this model we have four Higgs doublets, it recreates the same mass structure found in [11] when adopting the definitions Eqs. (38)–(40). Nevertheless, it means that the neutral boson mass eigenvalues had been already determined, and they are given by

$$M_\gamma = 0, \quad (41)$$

$$M_Z \approx \frac{gv}{2\cos\theta_W}, \quad (42)$$

$$M_{Z'} \approx \frac{g_X V_\chi}{3}, \quad (43)$$

where $\tan\theta_W = \frac{g'}{g}$, as it was defined in the Standard Model. A mixing between the three neutral gauge bosons exist and it is exhibited by the following expression for the mass eigenstates:

$$\begin{pmatrix} A_\mu \\ Z_{1\mu} \\ Z_{2\mu} \end{pmatrix} = \begin{pmatrix} \sin\theta_W & \cos\theta_W & 0 \\ \cos\theta_W\cos\theta_Z & -\sin\theta_W\cos\theta_Z & \sin\theta_Z \\ -\cos\theta_W\sin\theta_Z & \sin\theta_W\sin\theta_Z & \cos\theta_Z \end{pmatrix} \times \begin{pmatrix} W_\mu^3 \\ B_\mu \\ B'_\mu \end{pmatrix}, \tag{44}$$

where θ_Z is a small mixing angle between the Z and Z' bosons such that in the limit $\theta_Z \to 0$ the standard model gauge bosons are recovered with an isolated Z' boson as $Z_1 \to Z$ and $Z_2 \to Z'$, such mixing angle is approximately given by:

$$\sin\theta_Z = (1+\cos^2\beta)\frac{2g_X\cos\theta_W}{3g}\left(\frac{M_Z}{M_{Z'}}\right)^2. \tag{45}$$

1. LHC constraints on the Z' mass

From the interaction terms of standard model fermions with the B'_μ gauge boson and using the mixing between the Z and Z' particles, one can derive the Feynman rules for fermions interacting with the massive boson Z'_μ. Taking into account the charge quantities of particles shown in Table II, the $U(1)_X$ interaction sector of standard model fermions is shown in the following lines. For quarks we have

$$\mathcal{L}_{\mathrm{int},QB'} = \frac{g_X}{3}\bar{u}^1\gamma^\mu P_L u^1 B'_\mu + \frac{2g_X}{3}\bar{u}^i\gamma^\mu P_R u^i B'_\mu + \frac{g_X}{3}\bar{d}^1\gamma^\mu P_L d^1 B'_\mu - \frac{g_X}{3}\bar{d}^i\gamma^\mu P_R d^i B'_\mu,$$

where Einstein notation convention is adopted with $i = 1, 2, 3$. We also recall that $P_{L,R} = \frac{1\mp\gamma^5}{2}$ are the chirality projectors. In the case of the charged leptons, we have the following interaction Lagrangian:

$$\mathcal{L}_{\mathrm{int},eB'} = -\frac{4g_X}{3}\bar{e}^e\gamma^\mu P_R e^e B'_\mu - \frac{g_X}{3}\bar{e}^\mu\gamma^\mu P_R e^\mu B'_\mu - g_X\bar{e}^\tau\gamma^\mu P_L e^\tau B'_\mu - \frac{4g_X}{3}\bar{e}^\tau\gamma^\mu P_R e^\tau B'_{\mu.}$$

Once we have the corresponding Feynman rules, the cross section for the $pp \to Z' \to l^+l^-$ process was calculated and it is given by:

$$\frac{d\sigma}{dMdy} = \frac{K(M)}{24\pi M^3}\sum_q PG_q^\dagger \tag{46}$$

where $M = M_{ff}$ is the final state invariant mass, y is the rapidity, $K(M) \approx 1.3$ resumes all leading order QED corrections and NLO QCD corrections, $P = s^2/[(s - M_{Z'}^2)^2 + M_{Z'}^2\Gamma_{Z'}^2]$ with $\sqrt{s}$ the collider CM energy together with $M_{Z'}$ and $\Gamma_{Z'}$ standing for the Z' mass and total decay width respectively and $G_q^\dagger = x_A x_B[f_{q/A}(x_A)f_{q/B}(x_B) + f_{q/B}(x_B)f_{q/A}(x_A)]$ contains the parton distribution functions (PDF) $f(x)$ being x the momentum fraction. In the simulation, the ratio $\Gamma_{Z'}/M_{Z'}$ was set to 0.01 which allows us to consider the narrow width approximation (NWA) for approximating the cross section to:

$$\sigma(pp \to f\bar{f}) = \sigma(pp \to Z')\mathrm{BR}(Z' \to f\bar{f}). \tag{47}$$

In the case of leptons, the pp collision is based on a CM energy of $\sqrt{s} = 14$ TeV and a 36.1 fb^{-1} luminosity in agreement with ATLAS detector parameters. Furthermore, leptons have to be isolated inside a cone of angular radius $\Delta R = 0.5$ in addition to a required transverse energy $E_T > 20$ GeV and a $|\eta| < 2.5$ pseudorapidity. The results of the total cross section for the l^+l^- pair production as a function of $M_{Z'}$ are shown in Fig. 4.

Once the Z' is discovered at LHC through the DY process, it is of great importance to establish the $Z - Z'$ mixing which turns out to be model dependent. The current analysis is based on pp collisions with CM energy of $\sqrt{s} = 13$ TeV collected by ATLAS (36.1 fb^{-1}) and CMS (35.9 fb^{-1}) at LHC. In particular, the process $pp \to Z' \to W^+W^-$ is considered where the coupling $Z'W^+W^-$ is only possible by the $\sin\theta_Z$ mixing of $Z - Z'$. Consequently, by considering again the NWA the diboson production can be approximated to

$$\sigma(pp \to W^+W^-) = \sigma(pp \to Z')\mathrm{BR}(Z' \to W^+W^-) \tag{48}$$

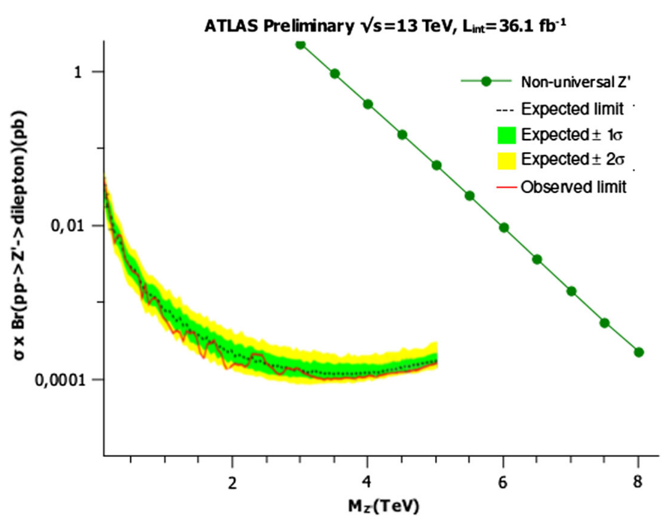

FIG. 4. Observed and expected 90% C.L upper limits for total cross section of dilepton production with ATLAS data [19]. It is compared with calculated cross section for Z' production times the branching ratio of the indicated decay.

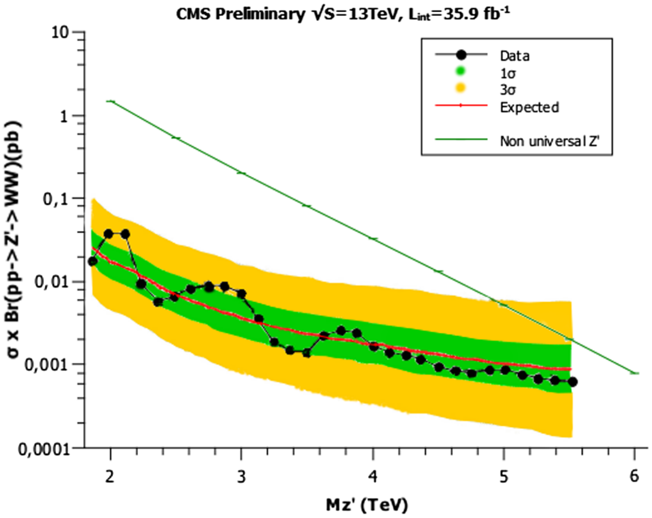

FIG. 5. Observed and expected 95% C.L upper limits for total cross section of diboson production with CMS data. It is compared with calculated cross section for Z' production times the branching ratio of the indicated decay.

where the expression for the partial width of $Z' \to W^+W^-$ is given by

$$\Gamma^{\mu\nu}_{Z'} = \frac{\alpha}{48}\cot^2\theta_W M_{Z'}\left(\frac{M_{Z'}}{M_W}\right)^4\left(1-4\frac{M_W^2}{M_{Z'}^2}\right)^{3/2} \times\left[1+20\left(\frac{M_W}{M_{Z'}}\right)^2+12\left(\frac{M_W}{M_{Z'}}\right)^4\right]\sin^2\theta_Z \quad (49)$$

with $\sin\theta_Z$ defined in Eq. (45).

Therefore, the model was implemented in MADGRAPH5_aMC@NLO together with PYTHIA 6 and Delphes 3 for studying the process at leading order by assuming the relevant parameters as $g_X = 0.63$ and $\cos\beta = 0.81$. Both of them correspond nearly to the smallest values found in the Monte Carlo simulation; specially required for the mixing angle θ_Z to soften experimental bounds on the Z' mass. A comparison among the total cross section of the process and the available CMS [20] and ATLAS [21] data on W^+W^- pair production is shown in Figs. 5 and 6.

The intercept between the curve obtained for the non-universal Z' decaying into W^+W^- and the 3σ upper limit given by CMS data shown in Fig. 5, indicates a exclusion limit for a new massive boson of $M_{Z'} > 5$ TeV, in contrast with ATLAS upper limits in Fig. 6, which provides a stronger constraint of $M_{Z'} > 5.9$ TeV

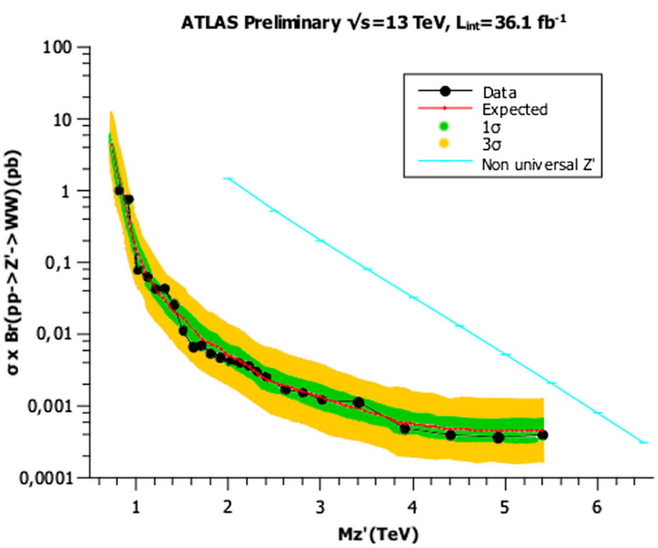

FIG. 6. Observed and expected 95% C.L upper limits for total cross section of diboson production with ATLAS data [22]. It is compared with calculated cross section for Z' production times the branching ratio of the indicated decay.

IV. FERMION SECTOR

A. Quark sector

According to the $SU(2)_L \otimes U(1)_Y \otimes U(1)_X \otimes Z_2$ symmetry, the most general Yukawa superpotencial for the quark superfields is given by:

$$\begin{aligned} W_Q = {} & \hat{q}^1_L\hat{\Phi}_2 h^{12}_{2u}\hat{u}^{2c}_L + \hat{q}^2_L\hat{\Phi}_1 h^{22}_{1u}\hat{u}^{2c}_L + \hat{q}^3_L\hat{\Phi}_1 h^{3k}_{1u}\hat{u}^{kc}_L \\ & - \hat{q}^3_L\hat{\Phi}'_2 h^{3j}_{2d}\hat{d}^{jc}_L + \hat{q}^1_L\hat{\Phi}_2 h^1_{2T}\hat{\mathcal{T}}^c_L + \hat{q}^2_L\hat{\Phi}_1 h^2_{1T}\hat{\mathcal{T}}^c_L \\ & - \hat{q}^1_L\hat{\Phi}'_2 1 h^{1a}_{1J}\hat{\mathcal{J}}^{ac}_L - \hat{q}^2_L\hat{\Phi}'_2 h^{2a}_{2J}\hat{\mathcal{J}}^{ac}_L + \hat{\mathcal{T}}_L\hat{\chi}' h^T_{\chi'}\hat{\mathcal{T}}^c_L \\ & - \hat{\mathcal{J}}^a_L\hat{\chi} h^{Jab}_{\chi}\hat{\mathcal{J}}^{bc}_L + \hat{\mathcal{T}}_L\hat{\chi}' h^2_{\chi' u}\hat{u}^{2c}_L + \hat{\mathcal{J}}^a_L\hat{\sigma} h^{Jaj}_{\sigma}\hat{d}^{jc}_L \\ & + \hat{\mathcal{T}}_L\hat{\sigma}' h^{Tk}_{\sigma'}\hat{u}^{kc}_L \end{aligned} \quad (50)$$

where $j = 1, 2, 3$ labels the down type singlet quarks, $k = 1, 3$ labels the first and third quark doublets, and $a = 1, 2$ is the index of the exotic $\mathcal{J}^a_L$ and $\mathcal{J}^{ca}_L$ quarks. It can be seen that this general superpotential match the nonsupersymmetrical one given in Ref. [11] if we promote the conjugate Higgs fields $\tilde{\phi}_i = i\sigma_2\phi^*_i$ to the independent ones Φ'_i required for a suitable anomaly cancellation. As a consequence, taking the VEV of all scalar fields, the quark mass matrices at tree level have an identical structure to its non-SUSY counterpart, as it can be seen in the following equations. For the up quark sector has

$$\mathcal{M}_{\mathcal{U}} = \begin{pmatrix} M_U & M_{UT} \\ M_{TU} & M_T \end{pmatrix} \quad (51)$$

where

$$M_U = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & h^{12}_{2u}v_2 & 0 \\ 0 & h^{22}_{1u}v_1 & 0 \\ h^{31}_{1u}v_1 & 0 & h^{33}_{1u}v_1 \end{pmatrix},$$

$$M_{UT} = \frac{1}{\sqrt{2}}\begin{pmatrix} h^1_{2T}v_2 \\ h^2_{1T}v_1 \\ 0 \end{pmatrix}, \quad (52)$$

$$M_{TU} = \frac{v'_\chi}{\sqrt{2}}\begin{pmatrix} 0 & h^2_{\chi' u} & 0 \end{pmatrix}, \qquad M_T = \frac{v'_\chi}{\sqrt{2}} g_{\chi' T}, \tag{53}$$

and for the down quark, the matrices can be written as

$$\mathcal{M}_{\mathcal{D}} = \begin{pmatrix} M_D & M_{DJ} \\ M_{JD} & M_J \end{pmatrix} \tag{54}$$

where

$$M_D = \frac{v'_2}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ h^{31}_{2d} & h^{32}_{2d} & h^{33}_{2d} \end{pmatrix},$$
$$M_{DJ} = \frac{1}{\sqrt{2}}\begin{pmatrix} h^{11}_{1J} v'_1 & h^{12}_{1J} v'_1 \\ h^{21}_{2J} v'_2 & h^{22}_{2J} v'_2 \\ 0 & 0 \end{pmatrix},$$
$$M_{JD} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}, \qquad M_J = \frac{v_\chi}{\sqrt{2}}\begin{pmatrix} g^{11}_{\chi J} & g^{12}_{\chi J} \\ g^{21}_{\chi J} & g^{22}_{\chi J} \end{pmatrix}. \tag{55}$$

It is worthwhile to notice that uplike quarks acquire mass from Φ_i Higgs fields while the downlike quarks do it from the Φ'_i ones. Since the matrix structure is identical to its non-SUSY counterpart, the same analysis and eigenvalues gotten in [11] can be done, taking care now that the downlike eigenvalues are coupled to primed Higgs VEV. The mass expressions can be approximated to:

$$m^2_u = 0, \qquad m^2_c = \frac{1}{2} v^2_1 \frac{[h^{22}_{1u} g_{\chi' T} - h^2_{1T} h^2_{\chi' u}]^2}{(g_{\chi' T})^2 + (h^2_{\chi' u})^2},$$
$$m^2_t = \frac{1}{2} v^2_1 [(h^{31}_{1u})^2 + (h^{33}_{1u})^2],$$
$$m^2_T = \frac{1}{2} {v'_\chi}^2 [(g_{\chi' T})^2 + (h^2_{\chi' u})^2]. \tag{56}$$

The hierarchy between top and charm masses comes from the seesaw rotation with the heavy $\mathcal{T}$ quark, which can be observed from the Yukawa coupling differences for the charm quark mass. From the charm and top quarks physical masses it is known that $\frac{m_c}{m_t} \approx 7 \times 10^{-3}$ which in terms of Yukawa couplings reads:

$$7 \times 10^{-3} \approx \frac{h^{22}_{1u} g_{\chi' T} - h^2_{1T} h^2_{\chi' u}}{\sqrt{(g_{\chi' T})^2 + (h^2_{\chi' u})^2}\sqrt{(h^{31}_{1u})^2 + (h^{33}_{1u})^2}} \tag{57}$$

To get an estimate of the above expression considering the mass hierarchy we can consider Yukawa couplings $(g_{\chi' T})^2$, $(h^2_{\chi' u})^2$, $(h^{31}_{1u})^2$, and $(h^{33}_{1u})^2$ of order 1 which implies a phenomenologically viable relation among Yukawas:

$$1.4 \times 10^{-2} \approx h^{22}_{1u} g_{\chi' T} - h^2_{1T} h^2_{\chi' u}. \tag{58}$$

For the down-quarks, the mass expressions can be approximated to:

$$m^2_d = 0, \qquad m^2_s = 0,$$
$$m^2_b = \frac{1}{2} v'^2_2 [(h^{31}_{2d})^2 + (h^{32}_{2d})^2 + (h^{33}_{2d})^2],$$
$$m^2_{J^1} = \frac{1}{2} v^2_\chi (g^{11}_{\chi J})^2, \qquad m^2_{J^2} = \frac{1}{2} v^2_\chi (g^{22}_{\chi J})^2. \tag{59}$$

The m^2_u, m^2_d, and m^2_s masses are equal to zero but they can be obtained by radiative corrections taking into account the SUSY contribution due to the respective superpartners as we will show later. According to the above expressions, it is the scalar particle ϕ_1 which gives mass to the top and charm quarks through v_1 and similarly v'_2 provides a mass value for the bottom quark. However the mass difference between charm and top quarks is fully dependent on the Yukawa couplings values which allows us to assume $v_1 \approx m_t$ and $v'_2 \approx m_b$. Looking at the exotic sector, they are governed entirely by the values of v_χ and v'_χ whose values are expected to be at least in the TeV scale. Furthermore, it agrees with recent experimental results which exclude exotic quarks with masses bellow 800 GeV [23].

B. Lepton sector

Analogously, the lepton superpotential corresponds to the non-SUSY Yukawa Lagrangian; the fields are promoted to superfields and the conjugate Higgs fields promoted to the primed ones. Then the superpotential reads as follows

$$\begin{aligned} W_L = {} & \hat{\ell}^p_L \hat{\Phi}_2 h^{pq}_{2\nu} \hat{\nu}^{qc}_L - \hat{\ell}^p_L \hat{\Phi}'_2 h^{p\mu}_{2e} \hat{e}^{\mu c}_L - \hat{\ell}^\tau_L \hat{\Phi}'_2 h^{\tau r}_{2e} \hat{e}^{rc}_L \\ & - \hat{\ell}^p_L \hat{\Phi}'_1 h^p_{1E} \hat{E}^c_L + \hat{E}_L \hat{\chi}' g_{\chi' E} \hat{E}^c_L \\ & - \hat{E}_L \mu_E \hat{\mathcal{E}}^c_L + \hat{\mathcal{E}}_L \hat{\chi} g_{\chi \mathcal{E}} \hat{\mathcal{E}}^c_L - \hat{\mathcal{E}}_L \mu_{\mathcal{E}} \hat{E}^c_L \\ & + \hat{\nu}^{jc}_L \hat{\chi}' h'^{Nij}_\chi \hat{N}^{ic}_L + \frac{1}{2} \hat{N}^{ic}_L M_{ij} \hat{N}^{jc}_L \\ & + \hat{E}_L \hat{\sigma} h^{e^c p}_\sigma \hat{e}^{cr}_L + \hat{\mathcal{E}}_L \hat{\sigma}' h^{e^c \mu}_{\sigma'} \hat{e}^{\mu c}_L, \end{aligned} \tag{60}$$

where $p = e, \mu$, $q = e, \mu, \tau$, $r = e, \tau$ and i, j label the right handed and Majorana neutrinos. The superpotential presented in the Eq. (60) generates the same mass matrix structure as well for the neutral and charged leptons when the VEV is taken by the fields, compared to the non-SUSY model.

1. Charged leptons masses at tree level

The mass matrix for the charged leptons follows also the same structure as the one obtained in the nonsupersymmetrical model. It is shown right ahead:

$$\mathcal{M}_E = \frac{1}{\sqrt{2}}\left(\begin{array}{ccc|cc} 0 & h_{2e}^{e\mu}v_2' & 0 & h_{1e}^{E}v_1' & 0 \\ 0 & h_{2e}^{\mu\mu}v_2' & 0 & h_{1\mu}^{E}v_1' & 0 \\ h_{2e}^{\tau e}v_2' & 0 & h_{2e}^{\tau\tau}v_2' & 0 & 0 \\ \hline 0 & 0 & 0 & g_{\chi' E}v_\chi' & -\mu_E \\ 0 & 0 & 0 & -\mu_{\mathcal{E}} & g_{\chi\mathcal{E}}v_\chi \end{array}\right) \tag{61}$$

Just as it happened in the old model, the electron remains massless at tree level. Therefore radiative corrections must be employed to explain the mass feature of such particle. The mass eigenvalues at leading order are given by:

$$m_e^2 = 0, \qquad m_\mu^2 = \frac{1}{2}v_2'^2[(h_{2e}^{e\mu})^2 + (h_{2e}^{\mu\mu})^2],$$
$$m_\tau^2 = \frac{1}{2}v_2'^2[(h_{2e}^{\tau e})^2 + (h_{2e}^{\tau\tau})^2]$$
$$m_E^2 = \frac{1}{2}g_{\chi' E}^2 v_\chi'^2, \qquad m_{\mathcal{E}}^2 = \frac{1}{2}g_{\chi\mathcal{E}}^2 v_\chi^2. \tag{62}$$

Despite V_2' also gives mass to the bottom quark, it is a good result that it also gives mass to the charged leptons since they are of the same scale order. So, in agreement with the particle physical mass values the ratio between μ and τ lepton masses is approximately 0.14, leaving us with the following relation among Yukawa couplings:

$$0.14 \approx \frac{\sqrt{(h_{2e}^{e\mu})^2 + (h_{2e}^{\mu\mu})^2}}{\sqrt{(h_{2e}^{\tau e})^2 + (h_{2e}^{\tau\tau})^2}} \tag{63}$$

2. Neutrino masses at tree level

As for neutrinos, the mass matrix involves the Dirac and Majorana terms in order to provide a mass spectrum via the inverse seesaw mechanism (ISS). In the basis $(\nu_L^q, \nu_L^{q\,C}, N_L^{i\,C})$, such matrix reads:

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & M_D^T \\ 0 & M_D & M_M \end{pmatrix}, \tag{64}$$

where the block matrices constituting the neutrino mass matrix are given by:

$$m_D = \frac{v_2}{\sqrt{2}}\begin{pmatrix} h_{2\nu}^{ee} & h_{2\nu}^{e\mu} & h_{2\nu}^{e\tau} \\ h_{2\nu}^{e\mu} & h_{2\nu}^{\mu\mu} & h_{2\nu}^{\mu\tau} \\ 0 & 0 & 0 \end{pmatrix}, \quad (M_D)^{ij} = \frac{v_\chi'}{\sqrt{2}}(h_\chi'^{\nu})^{ij},$$
$$(M_M)_{ij} = \frac{1}{2}M_{ij}. \tag{65}$$

For the ISS to work, the assumption on small Majorana coupling constants is made, $m_D \gg M_M$. Therefore, it can be shown that the matrix $\mathbb{M}_\nu$ can be approximately block diagonalized:

$$\mathbb{V}_{SS}^T \mathcal{M}_\nu \mathbb{V}_{SS} \approx \begin{pmatrix} m_{\mathrm{light}} & 0 \\ 0 & m_{\mathrm{heavy}} \end{pmatrix}, \tag{66}$$

where $m_{\mathrm{light}} = m_D^T (M_D)^{-1} M_M (M_D)^{-1} m_D$ is the 3×3 mass matrix for the light neutrinos and it must explain the observed mixing parameters in the PMNS matrix. The rotation matrix $\mathbb{V}_S$ can be calculated by:

$$\mathbb{V}_{SS} = \begin{pmatrix} I & \Theta_\nu \\ -\Theta_\nu^T & I \end{pmatrix}, \qquad \Theta_\nu = \begin{pmatrix} 0 & M_D^T \\ M_D & M_M \end{pmatrix}^{-1}\begin{pmatrix} m_D \\ 0 \end{pmatrix}. \tag{67}$$

Lastly, the m_{heavy} matrix involves the mixings of the exotic neutral leptons, and it is given by:

$$m_{\mathrm{heavy}} \approx \begin{pmatrix} 0 & M_D^T \\ M_D & M_M \end{pmatrix}. \tag{68}$$

Since the same structure as the non-SUSY model is followed also by the sector of neutral leptons, the same constraints are applied for the Yukawa parameters for explaining the quadratic difference of masses Δm^2 and mixing angles in the neutrino oscillation [24]. The parameter values are then shown in [11].

C. Fermion masses at one loop level

As it was seen in the previous section, the electron and the up, down and strange quarks turned out to be massless at tree level. However, since the physical mass of these particles is considerably small in comparison with the other particles and the model energy scale, they are expected to acquire a finite mass value through radiative corrections. In fact, it is performed via $\hat{\sigma}$ and $\hat{\sigma}'$ superfields couplings, as it is shown in Figs. 7, 9, and 8, which allows the transition between SM fermions and the exotic ones resulting in a nonzero value for their masses.

Considering first the electron mass, the vertices are generated by the following parts of the superpotential and the soft-breaking potential terms:

$$W_\phi \Rightarrow \lambda_1 \Phi_1' \Phi_2 \sigma' - \mu_\sigma \hat{\sigma}' \hat{\sigma}$$
$$W_L \Rightarrow \hat{E}_L \hat{\sigma} h_\sigma^{e^c r} \hat{e}_L^{rc} + \hat{\ell}_L^e \hat{\Phi}'_1 h_{1E}^e \hat{E}_L^c, \tag{69}$$

where $r = e$, τ and the couplings λ_1, $h_\sigma^{e^c k}$, and h_{1E}^e are dimensionless Yukawa coupling constants, but $\tilde{\lambda}_1$ and μ_σ are mass unit parameters from the scalar potential. Considering the Fig. 7, the non-SUSY contributions are given by:

$$v_2 \Sigma_{11(13)}^{NS} = \frac{-1}{16\pi^2}\frac{v_2}{\sqrt{2}}\frac{\lambda_1 \mu_\sigma h_\sigma^{ece(\tau)} h_{1E}^e}{M_E} C_0\left(\frac{m_{h1}'}{M_E}, \frac{m_\sigma'}{M_E}\right) \tag{70}$$

with

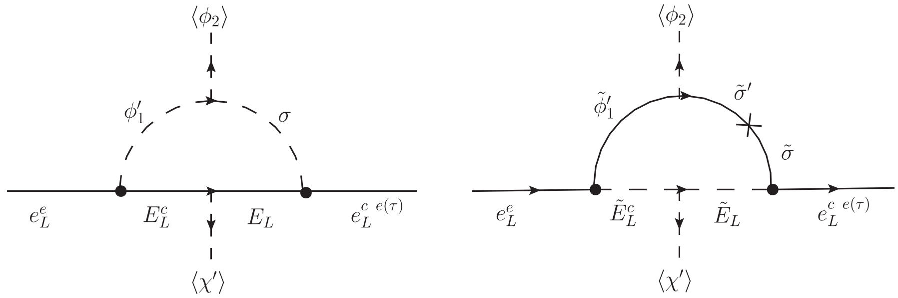

FIG. 7. One loop corrections to the leptons due to exotic fermions, sfermions, and Higgsinos.

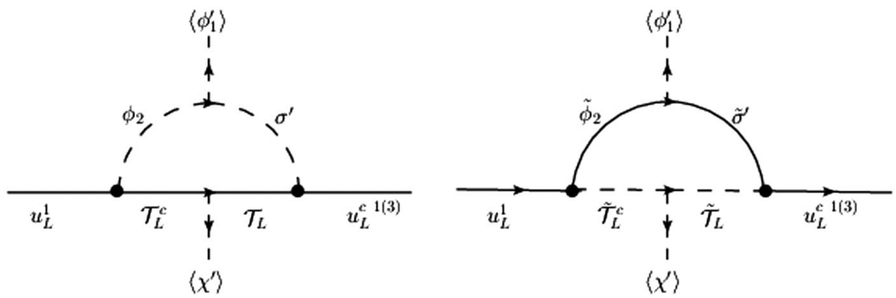

FIG. 8. One loop corrections to the quark up due to scalar singlets, exotic quarks, squarks, and Higgsinos.

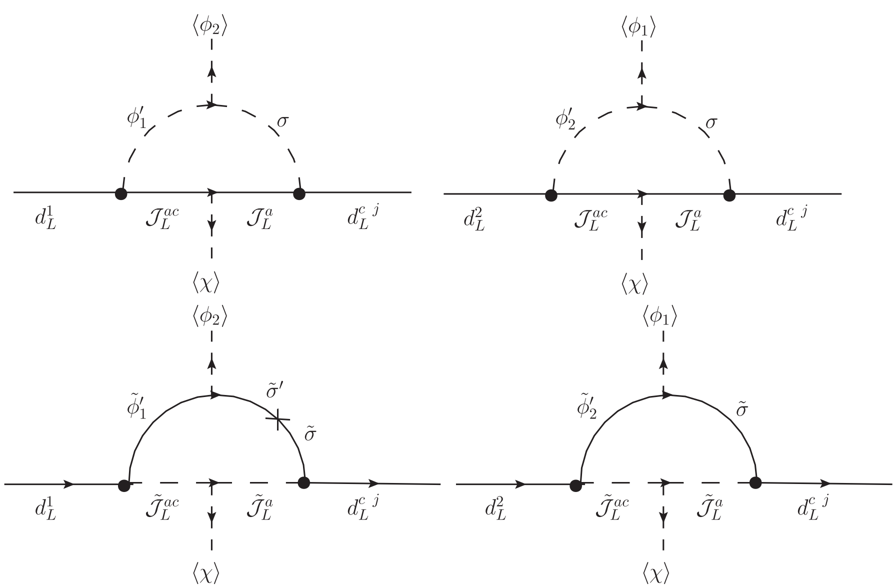

FIG. 9. One loop corrections to the quarks down and strange due to scalar singlets, exotic quarks, squarks, and Higgsinos.

$$C_0(\hat{m}_1,\hat{m}_2) = \frac{1}{(1-\hat{m}_1^2)(1-\hat{m}_2^2)(m_1^2-\hat{m}_2^2)} \times \left[\hat{m}_1^2\hat{m}_2^2 Ln\left(\frac{\hat{m}_1^2}{\hat{m}_2^2}\right) + \hat{m}_2^2 Ln(\hat{m}_2^2) - \hat{m}_1^2 Ln(\hat{m}_1^2)\right], \tag{71}$$

where M_E is the exotic charged fermion mass, m'_{h1} is the corresponding neutral scalar field mass related with the $\hat{\Phi}_1'$ superfield, m'_σ is the scalar field mass corresponding to the $\hat{\sigma}$ superfield and C_0 is the Veltmann-Passarino function evaluated for $p^2 = 0$ given by Eq. (71). In the radiative correction calculation, a rotation to the mass eigenstate basis for the exotic fermion was not done by assuming a mixing angle which suppresses the mixing among them, even though, in the SUSY contribution the rotation matrix is written explicitly because those zero tree level mass terms prevent the implementation of a renormalization scheme. It implies the presence of divergences in the B_0 Veltmann-Passarino function which are cancelled out by the super GIM mechanism resulting then in finite mass terms and a renormalizable theory. Therefore, the supersymmetric contributions to the radiative correction are given by:

$$v_2\Sigma^S_{11(13)}(p^2=0) = -\frac{1}{32\pi^2}\frac{v_2}{\sqrt{2}}\sum_{n=1}^{10}\sum_{k=1}^{2} Z_L^{\tilde{E}_L^c n} Z_L^{\tilde{E}_L n} Z_L^{\sigma k} Z_L^{\sigma' k}\lambda_1\mu_\sigma h_\sigma^{ece(\tau)} h^e_{1E} \times\left[\frac{(\tilde{m}_{\sigma k}+\tilde{m}'_{h_1})^2}{\tilde{M}^2_{L_n}} C_0\left(\frac{\tilde{m}'_{h1}}{\tilde{M}_{L_n}},\frac{\tilde{m}_{\sigma k}}{\tilde{M}_{L_n}}\right) + \tilde{m}'^2_{h1}B_0(0,\tilde{m}'_\sigma,\tilde{M}_{L_n}) + \tilde{m}^2_{\sigma k}B_0(0,\tilde{m}'_{h1},\tilde{M}_{L_n})\right] \quad (72)$$

where $\tilde{L}_n$ are the charged left sleptons mass eigenstates, $Z_L^{\sigma(\sigma')k}$ is the rotation matrix that connects σ (σ') with its mass eigenstates with eigenvalues $m_{\sigma k}$ which are running inside the loop. $Z_L^{\tilde{E}_L^c n}$ and $Z_L^{\tilde{E}_L n}$ are the rotation matrices which connect the exotic sleptons $\tilde{E}_L$, $\tilde{E}_L^c$ with the slepton mass eigenstates $\tilde{L}_n$ inside the loop. Σ^{NS}, Σ^S are defined as dimensionless parameters. Additional contributions may come from charged currents into the loop, which involves charginos and neutral sleptons.

The up quark radiative correction is analogous to the electron case but the Yukawa couplings indices and particle masses inside the loop have to be fixed in both SUSY and non-SUSY correction. In accordance to Fig. 8, the following terms coming from the superpotential and the soft breaking potential has to be considered:

$$W_Q \Rightarrow \hat{T}_L\hat{\sigma}' h^{Tj}_{\sigma'}\hat{u}^{kc}_L + \hat{q}^1_L\hat{\Phi}_2 h^1_{2T}\hat{T}^c_L$$
$$V_{\rm soft} \Rightarrow \tilde{\lambda}_1\Phi'^{\dagger}_1\Phi_2\sigma' + \text{H.c.} \qquad W_\phi \Rightarrow \lambda_1\hat{\phi}'_1\hat{\phi}_2\hat{\sigma}'. \quad (73)$$

According to the Fig. 8 the SUSY and non-SUSY contributions are given in Eqs. (74) and (75), respectively:

$$v'_1\Sigma^{NS}_{1k}(p^2=0) = \frac{-1}{16\pi^2}\frac{v'_1}{\sqrt{2}}\frac{\tilde{\lambda}_1 h^{Tk}_{\sigma'}h^1_{2T}}{M_T} C_0\left(\frac{m'_{h2}}{M_T},\frac{m'_\sigma}{M_T}\right). \quad (74)$$

$$v'_1\Sigma^S_{1k}(p^2=0) = -\frac{1}{32\pi^2}\frac{v'_1}{\sqrt{2}}\sum_{m=1}^{8} Z_U^{\tilde{T}_L^c m} Z_L^{\tilde{T}_L m}\lambda_1 h^{Tk}_{\sigma'}h^1_{2T} \times\left[\frac{(\tilde{m}'_\sigma+\tilde{m}'_{h_2})^2}{\tilde{M}^2_{T_m}} C_0\left(\frac{\tilde{m}'_{h1}}{\tilde{M}_{T_m}},\frac{\tilde{m}'_\sigma}{\tilde{M}_{T_m}}\right) + \tilde{m}'^2_{h2}B_0(0,\tilde{m}'_\sigma,\tilde{M}_{T_m}) + \tilde{m}'^2_\sigma B_0(0,\tilde{m}'_{h2},\tilde{M}_{T_m})\right] \quad (75)$$

where M_T is the T exotic quark mass and $k=1,3\ldots$ $\tilde{T}_m$ are the squark mass eigenstates, $\tilde{m}_{T_m}$ the corresponding mass eigenvalue, and Z_U the associated rotation matrix which is relating the states $\tilde{T}_L$ and $\tilde{T}^c_L$ with the mass eigenstates. Same as the electron radiative correction, a suppression angle is considered in such a way that the rotation matrix is not written explicitly in the non-SUSY correction and likewise is assumed for SUSY contributions due to the quadratic divergences.

Finally, for the downlike quarks it is needed to write corrections which provide the down and strange quarks masses given that the mass matrix which only provides a tree level mass to the bottom quark. Hence, the Feynman rules are constructed by considering the following terms coming from the superpotential and the soft breaking potential according to Fig. 9.

$$W_Q \Rightarrow \hat{\mathcal{J}}^a_L\hat{\sigma}h^{Jaj}_\sigma\hat{d}^{jc}_L + \hat{q}^1_L\hat{\Phi}'_1 h^{1a}_{1J}\hat{\mathcal{J}}^{ac}_L + \hat{q}^2_L\hat{\phi}'_2 h^{2a}_{2J}\hat{\mathcal{J}}^{ac}_L \quad (76)$$

$$W_\phi \Rightarrow \lambda_1\hat{\Phi}'_1\hat{\Phi}_2\hat{\sigma}' - \mu_\sigma\hat{\sigma}'\hat{\sigma} \quad (77)$$

$$V_{\rm soft} \Rightarrow \tilde{\lambda}_2\Phi'_2\Phi_1\sigma \quad (78)$$

Therefore, the SUSY and non-SUSY corrections can be written as:

$$v'_l\Sigma^{NS}_{lj}(p^2=0) = \frac{-1}{16\pi^2}\frac{v'_l}{\sqrt{2}}\frac{\lambda(l)h^{Jaj}_\sigma h^l_{lJ}}{M_{J^a}} C_0\left(\frac{m'_{hl}}{M_{J^a}},\frac{m'_\sigma}{M^a_J}\right). \quad (79)$$

$$v_l'\Sigma_{lj}^S(p^2=0) = -\frac{1}{32\pi^2}\frac{v_l'}{\sqrt{2}}\sum_{q=1}^{10}\sum_{k=1}^{2} Z_D^{\tilde{J}_L^{ac}q} Z_D^{\tilde{J}_L^{a}q}(1-\delta_{l1}(1-Z_L^{\sigma k}Z_L^{\sigma' k}\mu_\sigma))\lambda_l h_\sigma^{J^a j} h_{lJ}^{la}$$
$$\times\left[\frac{(\tilde{m}_{\sigma k}+\tilde{m}'_{h_l})^2}{\tilde{M}^2_{D_q}} C_0\left(\frac{\tilde{m}'_{hl}}{\tilde{M}_{D_q}},\frac{\tilde{m}_{\sigma k}}{\tilde{M}_{D_q}}\right) + \tilde{m}_{h_{l+1\mathrm{mod}2}} B_0(0,\tilde{m}_{\sigma k},\tilde{M}_{D_q}) + \tilde{m}_{\sigma k} B_0(0,\tilde{m}_{h_{l+1\mathrm{mod}2}},\tilde{M}_{D_q})\right]. \quad (80)$$

The module function is used in the index l because when $l=1$, the scalar particle at the top of the diagram is ϕ_2 but when $l=2$ the required scalar particle is ϕ_1, which corresponds to a $l+1\mathrm{mod}(2)$ function, returning the value "1" as required. δ_{l1} is the usual Kronecker delta with $l=1$, 2 denoting the Higgs(ino) particle inside the loop, $j=1$, 2, 3 runs for the left-handed quarks u_L^j, J_L^a indicates the exotic down-like quarks contribution which in this case the index a can take the values $a=1$, 2. $\lambda(l)$ is a function such that $\lambda(1)=\lambda_1$ and $\lambda(2)=\tilde{\lambda}_2$. Additionally, m_{hl} correspond to the scalar mass particle associated with the superfields $\hat{\Phi}_1$, and $\hat{\Phi}_2$ for $l=1$, 2, respectively and D_q are the right-handed down-like squarks mass eigenstates. When $l=1$ the radiative corrections Σ_{1j} generates the matrix elements in the down mass matrix which produces the down quark mass, similarly the case $l=2$ produces the strange quark mass.

V. DISCUSSION AND CONCLUSIONS

The model studied here is the supersymmetric extension of the three families $U(1)_X$ model [11]. The SUSY extension requires four Higgs doublets and four scalar singlets in order to not induce chiral anomalies and giving mass to quarks, charged leptons and neutrinos. Additionally, the singlets generate the mass for exotic fermions and break the $U(1)_X$ gauge symmetry. An interesting prediction of this extension is that there are tree level flavor changing neutral currents.

In both versions of the model, SUSY and non SUSY, the lightest particles (electron and up, down and strange quarks) are massless at tree level. However, in the supersymmetric model they acquire a mass value via radiative corrections through inert singlets into the loop after $U(1)_X$ and electroweak symmetry breaking.

By implementing a seesaw mechanism among the singlet and doublet Higgs fields, together with the D-terms corresponding to the $U(1)_X$, a Δm_h^2 is found at tree level and it turns out to be at the order of the MSSM contribution. The lightest scalar particle is identified as the Higgs boson and its mass is obtained at the order of 125 GeV. In fact, we show in Figs. 1 and 2 the region in the parameter space v_1' vs g_X and v_2 vs g_X which is compatible with a 125.3 GeV Higgs boson at 95% of C.L., where the VEVs v_1 and v_2' are fixed around the mass of the top quark and the bottom quark, respectively. As a result, we found that the coupling constant g_X, regarding to the $U(1)_X$ symmetry, takes values between 0.63 and 1 for $112 < v_2(\mathrm{GeV}) < 180$ and $0 < v_1'(\mathrm{GeV}) < 81$. Thus, values below 0.63 for g_X are excluded. Even more, if we take the LHC exclusion bounds on dilepton production, which gave $m_{Z'} > 8$ TeV, it implies from the expression for this gauge boson mass [Eq. (43)] that the VEVs $v_\chi \approx v_\chi'$ should be greater than 37 TeV.

The parameter space $\cos 2\tilde{\beta}$ vs g_X was explored and negative values are found for $\cos 2\tilde{\beta}$ because nonprimed VEV happened to be greater than the primed ones. This behavior lies in the fact that top quark mass ($\approx v_1$) is bigger than the bottom quark mass ($\approx v_2'$). On the other hand, in the mass expression for m_h, Eq. (21), there are quadratic differences between nonprimed and primed VEVs so v_1, $v_2 > v_1'$, v_2' is preferred. Thus the allowed region is $0.38 < \cos\tilde{\beta} < 1$.

Last but not less important, the model also predicts five CP-even, four CP-odd and three charged Higgs particles with a mass above the TeV scale. The would-be Goldstone bosons corresponding to Z_μ, $W_\mu^\pm$ and Z_μ' are also found.

APPENDIX: SCHEME FOR OBTAINING THE SCALAR PARTICLE MASS EXPRESSIONS

For the CP-even particles an additional first step was made, which is to perform a seesaw like rotation, taking into account that the mixing in $M_{h\xi}$ are small compared to the ones in $M_{\xi\xi}$. With that approximation, the matrix M_h is transformed to a block diagonal form:

$$M_h \to (\mathbb{V}_{\mathrm{SS}}^h)^T M_h \mathbb{V}_{\mathrm{SS}}^h \approx \begin{pmatrix} \tilde{M}_{hh} & 0 \\ 0 & M_{\xi\xi} \end{pmatrix}, \quad (\mathrm{A1})$$

where $\tilde{M}_{hh} = M_{hh} - M_{h\xi}M_{\xi\xi}^{-1}M_{h\xi}^T$. $\mathbb{V}_{\mathrm{SS}}^h$ rotates the matrix,

$$\mathbb{V}_{\mathrm{SS}}^h = \begin{pmatrix} 1 & \Theta^{h\dagger} \\ -\Theta^h & 1, \end{pmatrix} \quad (\mathrm{A2})$$

with $\Theta^h = M_{h\xi}(M_{\xi\xi})^{-1}$. The heavy remaining block component, $M_{\xi\xi}$, is a block diagonal 4×4 matrix, and therefore its eigenvalues can be obtained trivially.

[1] S. L. Glashow, Nucl. Phys. **22**, 579 (1961); A. Salam, *Elementary Particle Physics (Nobel Symp. No. 8)*, edited by N. Svartholm (Almquist and Wiksells, Stockholm, 1968), p. 367; S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967).

[2] G. Aad *et al.*, Phys. Lett. B **716**, 1 (2012); S. Chatrchyan *et al.*, Phys. Lett. B **716**, 30 (2012).

[3] J. E. Kim and H. Nilles, Phys. Lett. **138B**, 150 (1984).

[4] T. Elliott, S. F. King, and P. L. White, Phys. Rev. D **49**, 2435 (1994); D. Miller, R. Nevzorov, and P. Zerwas, Nucl. Phys. **B681**, 3 (2004); S. F. King and P. L. White, Phys. Rev. D **53**, 4049 (1996); B. A. Dobrescu and K. T. Matchev, J. High Energy Phys. 09 (2000) 031; S. King, M. Mühlleitner, and R. Nevzorov, Nucl. Phys. **B860**, 207 (2012).

[5] H. E. Haber and R. Hempfling, Phys. Rev. Lett. **66**, 1815 (1991); M. Yamaguchi, T. Yanagida, and Y. Okada, Prog. Theor. Phys. **85**, 101 (1991); J. Ellis, G. Ridolfi, and F. Zwirner, Phys. Lett. B **262**, 477 (1991); M. Carena, J. Espinosa, M. Quirós, and C. Wagner, Phys. Lett. B **355**, 209 (1995); M. Carena, M. Quirós, and C. Wagner, Nucl. Phys. **B461**, 407 (1996); H. E. Haber, R. Hempfling, and A. H. Hoang, Z. Phys. C **75**, 539 (1997); S. Heinemeyer, W. Hollik, and G. Weiglein, Phys. Rev. D **58**, 091701 (1998); Phys. Lett. B **440**, 296 (1998); Europhys. J. C **9**, 343 (1999); J. R. Espinosa and R.-J. Zhang, J. High Energy Phys. 03 (2000) 026; M. Carena, H. E. Haber, S. Heinemeyer, W. Hollik, C. E. M. Wagner, and G. Weiglein, Nucl. Phys. **B580**, 29 (2000); S. P. Martin, Phys. Rev. D **67**, 095012 (2003).

[6] P. Batra, A. Delgado, D. E. Kaplan, and T. M. Tait, J. High Energy Phys. 02 (2004) 043; 06 (2004) 032; Bellazzini, C. Csáki, A. Delgado, and A. Weiler, Phys. Rev. D **79**, 095003 (2009).

[7] H. Goldberg, Phys. Rev. Lett. **50**, 1419 (1983); J. Ellis, D. Nanopoulos, and K. Tamvakis, Phys. Lett. **121B**, 123 (1983); R. Harnik, D. T. Larson, H. Murayama, and M. Thormeier, Nucl. Phys. **B706**, 372 (2005); H.-C. Cheng, B. A. Dobrescu, and K. T. Matchev, Phys. Lett. B **439**, 301 (1998); M. Aoki and N. Oshimo, Phys. Rev. Lett. **84**, 5269 (2000).

[8] H. Pagels and J. R. Primack, Phys. Rev. Lett. **48**, 223 (1982); H. Goldberg, Phys. Rev. Lett. **50**, 1419 (1983).

[9] S. King, S. Moretti, and R. Nevzorov, Phys. Lett. B **634**, 278 (2006); S. F. King, S. Moretti, and R. Nevzorov, Phys. Rev. D **73**, 035009 (2006).

[10] C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C **40**, 100001 (2016).

[11] S. Mantilla, R. Martinez, and F. Ochoa, Phys. Rev. D **95**, 095037 (2017).

[12] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[13] M. C. Gonzalez-Garcia, M. Maltoni, and T. Schwetz, Nucl. Phys. **B908**, 199 (2016); H. Fritzsch and S. Zhou, Phys. Lett. B **718**, 1457 (2013).

[14] H. Georgi, Nucl. Phys. **B156**, 126 (1979); J. E. Kim, Phys. Rev. Lett. **45**, 1916 (1980); P. H. Frampton, Phys. Lett. **89B**, 352 (1980); S. Barr, Phys. Rev. D **21**, 1424 (1980); R. Barbieri and D. V. Nanopoulos, Phys. Lett. **91B**, 369 (1980); L. E. Ibañez, Nucl. Phys. **B193**, 317 (1981); F. Wilczek and A. Zee, Phys. Rev. Lett. **42**, 421 (1979); A. Davidson, K. C. Wali, and P. D. Manheim, Phys. Rev. Lett. **45**, 1135 (1980); H. Fritzsch, Phys. Lett. B **70**, 436 (1977); **73**, 317 (1978); Nucl. Phys. **B155**, 189 (1979); H. Fritzsch and J. Plankl, Phys. Rev. D **35**, 1732 (1987); C. D. Froggatt and H. B. Nielsen, Nucl. Phys. **B147**, 277 (1979); K. S. Babu and E. Ma, Phys. Rev. Lett. **61**, 674 (1988); E. Ma, Phys. Rev. Lett. **62**, 1228 (1989); **63**, 104 (1989); **64**, 2866 (1990); E. Ma, Phys. Rev. D **80**, 013013 (2009); S. M. Barr, Phys. Rev. D **21**, 1424 (1980); B. Balakrishna, Phys. Rev. Lett. **60**, 1602 (1988); B. Balakrishna, A. Kagan, and R. Mohapatra, Phys. Lett. B **205**, 345 (1988); Y. Koide, Phys. Rev. D **28**, 252 (1983); Z.-Z. Xing, Nuovo Cimento A (1971–1996) **109**, 115 (1996); N. Arkani-Hamed and M. Schmaltz, Phys. Rev. D **61**, 033005 (2000); K. Yoshioka, Mod. Phys. Lett. A **15**, 29 (2000); E. A. Mirabelli and M. Schmaltz, Phys. Rev. D **61**, 113011 (2000); L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).

[15] H. Fritzsch, Phys. Lett. **70B**, 436 (1977); **73B**, 317 (1978); Nucl. Phys. **B155**, 189 (1979); T. P. Cheng and M. Sher, Phys. Rev. D **35**, 3484 (1987); D. Du and Z. Z. Xing, Phys. Rev. D **48**, 2349 (1993); H. Fritzsch and Z. Z. Xing, Phys. Lett. **555**, 63 (2003); J. L. Diaz-Cruz, R. Noriega Papaqui, and A. Rosado, Phys. Rev. D **71**, 015014 (2005); K. Matsuda and H. Nishiura, Phys. Rev. D **74**, 033014 (2006); A. Carcamo, R. Martinez, and J.-A. Rodriguez, Eur. Phys. J. C **50**, 935 (2007); M. Fukugita, M. Tanimoto, and T. Yanagida, Prog. Theor. Phys. **89**, 263 (1993); M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986).

[16] P. Langacker and M. Plumacher, Phys. Rev. D **62**, 013006 (2000); K. Leroux and D. London, Phys. Lett. B **526**, 97 (2002); S. Baek, J. H. Jeon, and C. S. Kim, Phys. Lett. B **641**, 183 (2006); E. Ma, Phys. Lett. B **380**, 286 (1996); V. Barger, P. Langacker, and H.-S. Lee, Phys. Rev. D **67**, 075009 (2003); S. F. King, S. Moretti, and R. Nevzorov, Phys. Rev. D **73**, 035009 (2006); T. Hur, H.-S. Lee, and S. Nasri, Phys. Rev. D **77**, 015008 (2008); G. Belanger, A. Pukhov, and G. Servant, J. Cosmol. Astropart. Phys. 01 (2008) 009; L. Basso and S. Moretti, J. High Energy Phys. 08 (2011) 122; E. Accomando, C. Coriano, L. Delle Rose, J. Fiaschi, C. Marzo, and S. Moretti, J. High Energy Phys. 07 (2016) 086; E. Accomando, L. Delle Rose, S. Moretti, E. Olaiya, and C. H. Shepherd-Themistocleous, J. High Energy Phys. 04 (2017) 081; P. Langacker, Rev Mod. Phys. **81**, 1199 (2009); A. Leike, Phys. Rep. **317**, 143 (1999); J. Erler, P. Langacker, and T. J. Li, Phys. Rev. D **66**, 015002 (2002); S. Hesselbach, F. Franke, and H. Fraas, Eur. Phys. J. C **23**, 149 (2002); R. Martinez, J. Nisperuza, F. Ochoa, and J. Rubio, Phys. Rev. D **90**, 095004 (2014); R. Martinez, J. Nisperuza, F. Ochoa, J. P. Rubio, and C. F. Sierra, Phys. Rev. D **92**, 035016 (2015); R. Martinez and F. Ochoa, J. High Energy Phys. 05 (2016) 113; R. Martınez, J. Nisperuza, F. Ochoa, and J. P. Rubio, Phys. Rev. D **89**, 056008 (2014); C. E. Díaz, S. F Mantilla, and R. Martínez, Phys. Rev. D **98**, 015038 (2018); A. Das, S. Oda, N. Okada, and D. s. Takahashi, Phys. Rev. D **93**, 115038 (2016); A. Das, N. Okada, and D. Raut, Phys. Rev. D **97**, 115023 (2018); A. Das, N. Okada, and D. Raut, Eur. Phys. J. C **78**, 696 (2018).

[17] R. N. Mohapatra, Phys. Rev. Lett. **56**, 561 (1986); R. N. Mohapatra and J. W. Valle, Phys. Rev. D **34**, 1642 (1986); E. Catano, R. Martinez, and F. Ochoa, Phys. Rev. D **86**,

073015 (2012); P. S. B. Dev and A. Pilaftsis, Phys. Rev. D **86**, 035002 (2012).

[18] M. Abramowitz and I. Stegun, Solutions of Quartic Equations, in *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables* (Dover, New York, 1972), pp. 17–18.

[19] M. Aaboud, G. Aad *et al.* (ATLAS Collaboration), J. High Energy Phys. 10 (2017) 182.

[20] CMS Collaboration, Report No. CMS-PAS-B2G-17-001.

[21] ATLAS Collaboration, Report No. ATLAS-CONF-2017-051.

[22] P. Osland, A. A. Pankov, and A. V. Tsytrinov, Phys. Rev. D **96**, 055040 (2017).

[23] L. Serkin, arXiv:1901.01765.

[24] R. Davis, Jr., D. S. Harmer, and K. C. Hoffman, Phys. Rev. Lett. **20**, 1205 (1968); J. N. Abdurashitov *et al.*, Phys. Rev. C **80**, 015807 (2009); F. Kaether, W. Hampel, G. Heusser, J. Kiko, and T. Kirsten, Phys. Lett. B **685**, 47 (2010); B. T. Cleveland *et al.*, Astrophys. J. **496**, 505 (1968); B. Aharmim *et al.*, Phys. Rev. C **88**, 025501 (2013); G. Bellini *et al.* (Borexino Collaboration), Phys. Rev. D **82**, 033006 (2010); G. Bellini *et al.* (Borexino Collaboration), Nature (London) **512**, 383 (2014); J. Hosaka *et al.*, Phys. Rev. D **73**, 112001 (2006); J. Hosaka *et al.*, Phys. Rev. D **74**, 032002 (2006); J. P. Cravens and K. Abe *et al.*, Phys. Rev. D **78**, 032002 (2008); M. G. Aartsen *et al.*, Phys. Rev. D **91**, 072004 (2015); A. Gando *et al.* Phys. Rev. D **83**, 052002 (2011); M. Apollonio *et al.*, Phys. Lett. B **466**, 415 (1999); A. Piepke *et al.* (Palo Verde Collaboration), Prog. Part. Nucl. Phys. **48**, 113 (2002); F. P. An *et al.* (Daya Bay Collaboration), Phys. Rev. Lett. **115**, 111802 (2015); S. B. Kim *et al.* (RENO Collaboration), Nucl. Phys. **B908**, 94 (2016); J. Kopp *et al.* J. High Energy Phys. 5 (2013) 50; P. Adamson *et al.* Phys. Rev. Lett. **110**, 251801 (2013); P. Adamson *et al.* Phys. Rev. Lett. **110**, 171801 (2013); K. Abe *et al.* (T2K Collaboration), Phys. Rev. Lett. **112**, 061802 (2014); K. Iwamoto (T2K Collaboration), Proc. Sci. ICHEP2016 (**2017**) 517; J. Bian (NOνA Collaboration), Proc. Sci. ICHEP2016 (2016) 516; P. Adamson *et al.* (NOνA Collaboration), Phys. Rev. Lett. **116**, 151806 (2016).