$N^*(1535) \to N$ transition form-factors due to the axial current

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The form-factors for the transition $N^*(1535) \to N$ induced by isovector and isoscalar axial currents within the framework of light cone QCD sum rules by using the most general form of the interpolating current are calculated. In numerical calculations, we use two sets of values of input parameters. It is observed that the Q^2 dependence of the form-factor G_A can be described by the dipole form. Moreover, the form-factors $G_P^{(S)}$ are found to be highly sensitive to the variations in the auxiliary parameter β. The numerical calculations show that the form factor $G_T^{(S)}$ at all values of Q^2 and β for both sets of values of the parameters are smaller than that one for $G_A^{(S)}$ and $G_S^{(S)}$.

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I. INTRODUCTION

Nucleon form-factors are fundamental quantities for understanding the inner structure of hadrons at low energies. The electromagnetic form-factors of nucleons are studied in a wide range of momentum transfer squares (see [1]). The electromagnetic form-factors within the light cone sum rules (LCSR) were comprehensively studied in many works (see, e.g., [2–6]). Unlike the electromagnetic form-factors, those induced by isovector and isoscalar axial currents are not measured. Only the nucleon axial charge is experimentally well determined from the neutron β decay, and the latest value is $g_A = 1.2724$ [7]. The determination of the axial form-factors can give very useful information on the flavor structure and spin content of nucleon resonances. Therefore, the study of these form-factors receives special attention for understanding the structure of nucleon resonances.

The LCSR results for the nucleon axial form-factors and for tensor factors are studied in [8,9] and [10,11], respectively. The axial form-factors of the nucleon within the holography approach are investigated in [12]. In the lattice QCD, the momentum-transfer dependence of the axial form-factor is studied in [13–17]. The study of the momentum transfer dependence of the axial form-factor is possible only in pion electroproduction and neutrino-induced charged-current reactions.

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The experiments planned at Jefferson Laboratory (JLab), MAMI, are aimed to study the properties of baryon resonances in photo- and electroproduction reactions [18].

Motivated by the prospective experiments at JLab, MAMI, we aim to study the form-factors for the transition $N^*(1535) \to N$ induced by isovector and isoscalar axial currents in the framework of the LCSR. Note that the electromagnetic form-factors for the transition $N^*(1535) \to N$ within the LCSR are calculated in [19].

This paper is structured as follows. In Sec. II, we introduce the relevant correlation function and derive the LCSR for the form-factors induced by an axial quark current. Our numerical analysis for axial form-factors for the transition $N^*(1535) \to N$ is presented in Sec. III. This section also contains our discussion and conclusion.

II. SUM RULES FOR THE TRANSITION $N^*(1535) \to N$ FORM-FACTORS INDUCED BY AXIAL CURRENT

In this section, we derive the LSCR for the form-factors for the transition $N^*(1535) \to N$ induced by isovector and isoscalar axial currents. To this end, we introduce the following vacuum to $N^*(1535)$ correlation function:

$$\Pi_\mu(p,q) = i\int d^4x\, e^{iq\cdot x}\langle 0|T\{\eta(0)A_\mu^{(S)}(x)\}|N^*(p)\rangle. \tag{1}$$

In Eq. (1), η is the interpolating current for the nucleon and $A_\mu^{(S)}$ is the isovector (isoscalar) axial vector current,

$$A_\mu^{(S)} = \bar{u}\gamma_\mu\gamma_5 u \mp \bar{d}\gamma_\mu\gamma_5 d. \tag{2}$$

The nucleon interpolating current, in general, can be written as

$$\eta = 2\varepsilon^{abc}\sum_{\ell=1}^{2}(u^{aT}CA_1^\ell d^b)A_2^\ell u^c, \tag{3}$$

where a, b, and c are color indices, C is the charge-conjugation operator, $A_1^1 = I$, $A_2^1 = \gamma_5$, $A_1^2 = \gamma_5$, and $A_2^2 = \beta I$. The case $\beta = -1$ corresponds to the Ioffe current.

The LCSR for the relevant quantities is obtained by calculating the correlation function from the hadronic and QCD sides and then matching the result of the two representations. From the hadronic side, the result can be obtained by inserting the total set of the nucleon states between the nucleon interpolating current and the axial current $A_\mu^{(S)}$, and separating the contribution of the ground-state nucleon.

The standard procedure for obtaining QCD sum rules is the calculation of the correlation function from hadronic and QCD sides. We begin with the calculation of the hadronic side of the correlation function (1).

Now we will give a few details of this procedure. We explicitly keep the contribution of the state $N(940)$ in the hadronic dispersion relation and, according to the quark-hadron duality ansätz, represent higher states by a dispersion integral, starting from some threshold, s_0.

After separating the contribution from the ground-state N in the hadronic part, we get

$$\Pi_\mu = \frac{\langle 0|\eta|N\rangle\langle N|A_\mu^{(S)}|N^*\rangle}{m^2 - p'^2} + \cdots, \tag{4}$$

where $\cdots$ denotes the contributions from higher states and the continuum. The coupling of N to the interpolating current η, that is, the decay constant or the residue, is defined as

$$\langle 0|\eta|N(p)\rangle = \lambda_N u_N(p). \tag{5}$$

The matrix element of the axial current between N and N^* in terms of three form-factors is given by

$$\langle N(p')|A_\mu|N^*(p)\rangle = \bar{u}_N(p')\left[\gamma_\mu\gamma_5 G_A(q^2) + \frac{q_\mu}{m_N + m_{N^*}}\gamma_5 G_P(q^2) + i\sigma_{\mu\nu}\frac{q_\nu}{m_N + m_{N^*}}\gamma_5 G_T(q^2)\right]\gamma_5 u_{N^*}(p), \tag{6}$$

where G_A, G_P, and G_T are the axial, induced pseudoscalar, and induced tensor form-factors, respectively. The matrix element of the isoscalar axial-vector current between N and N^* is obtained from (6) by replacing $A_\mu \to A_\mu^S$, $G_A \to G_A^S$, $G_P \to G_P^S$, and $G_T \to G_T^S$. At this point, we would like to make the following remark. If initial and final state baryons are the same, the tensor form factors $G_T^{(S)}$ vanish as a consequence of the isospin symmetry and the G-parity invariance. In our problem, the initial and final states are different; hence, we need to calculate all the form factors. Taking into account this fact and putting Eqs. (6) and (5) into (4) for the hadronic contributions to the correlation function, we obtain

$$\Pi_\mu = \frac{\lambda_N}{m_N^2 - p'^2}(\not{p} - \not{q} + m_N)\left[\gamma_\mu(G_A - G_T) + \frac{q_\mu}{m_N + m_{N^*}}(G_P - G_T) + \frac{2p_\mu}{m_N + m_{N^*}}G_T\right]u_{N^*}(p) + \cdots \tag{7}$$

Using the equation of motion, namely $\not{p}u_{N^*}(p) = m_{N^*}u_{N^*}(p)$, in the correlation function from hadronic part, we get

$$\Pi_\mu = \frac{\lambda_N}{m_N^2 - p'^2}\{(G_A - G_T)((m_N - m_{N^*})\gamma_\mu + 2p_\mu - \not{q}\gamma_\mu) + \frac{(G_P - G_T)q_\mu}{m_{N^*} + m_N}(m_{N^*} + m_N - \not{q}) + \frac{2G_T p_\mu}{m_{N^*} + m_N}(m_{N^*} + m_N - \not{q})\} + \cdots \tag{8}$$

where $p' = p - q$. From this expression, we see that the correlator function can be decomposed into the following Lorentz structures:

$$\Pi_\mu(p,q) = \Pi_1 p_\mu + \Pi_2\gamma_\mu + \Pi_3\not{q}\gamma_\mu + \Pi_4 q_\mu + \Pi_5 q_\mu\not{q} + \Pi_6 p_\mu\not{q}. \tag{9}$$

Now, let us turn our attention to the calculation of the correlation function from the QCD side. For p'^2, $q^2 \ll 0$, the product of the two currents in Eq. (1) can be expanded around the light cone, $x^2 \sim 0$. The result of the operator-product expansion (OPE) is obtained by a sum over the distribution amplitudes (DA's) of N^*, with an increasing twist multiplied by their corresponding coefficient functions.

At this point, we would like to present some details of the calculations. Using the forms of the interpolating current and axial current given above, we get

$$(\Pi_\mu)_\rho = \frac{1}{2}\int d^4x e^{iq\cdot x}\sum_{\ell=1}^{2}\{(CA_1^\ell)_{\alpha\gamma}[A_2^\ell S_u(-x)\gamma_\mu\gamma_5]_{\rho\beta} + (A_2^\ell)_{\rho\alpha}[(CA_1^\ell)^T S_u(-x)\gamma_\mu\gamma_5]_{\gamma\beta} \mp (A_2^\ell)_{\rho\beta}[CA_1^\ell S_u(-x)\gamma_\mu\gamma_5]_{\alpha\gamma}\}4\varepsilon^{abc} \times \langle 0|u_\alpha^a(0)u_\beta^b(x)d_\gamma^c(0)|N^*(p)\rangle. \tag{10}$$

In Eq. (10), the upper (lower) sign in the last line corresponds to the isovector (isoscalar) axial current, and $S_q(x)$ is the light-quark propagator. In calculation, we used the free quark propagator; i.e.,

$$S_q(x) = \frac{i\not{x}}{2\pi^2 x^4}. \tag{11}$$

The matrix element of the three quarks between the vacuum and the states N^*, defined in terms of the DA's of N^* with increasing twist, is given in [19]. For the sake of completeness, we present these DA's in the Appendix.

Using the explicit expressions of the DA's of N^*, performing the integration over x, and selecting the coefficients of the structures $\not{q}\gamma_\mu$, $\not{q}q_\mu$, and $p_\mu\not{q}$ in both representations, we get

$$-\frac{\lambda_N}{m_N^2-p'^2}G_A^{(S)} = \Pi_3^{(S)} + \frac{m_N+m_{N^*}}{2}\Pi_6^{(S)}, -\frac{\lambda_N}{m_N^2-p'^2}\frac{G_P^{(S)}}{m_{N^*}+m_N} = \Pi_5^{(S)} + \frac{1}{2}\Pi_6^{(S)}, -\frac{\lambda_N}{m_N^2-p'^2}\frac{G_T^{(S)}}{m_{N^*}+m_N} = \frac{1}{2}\Pi_6^{(S)}. \tag{12}$$

In Eq. (12), $\Pi_5^{(S)}$, $\Pi_3^{(S)}$, and $\Pi_6^{(S)}$ are given by

$$\Pi_5^{(S)} = m_{N^*}^2\int_0^1 dx_2\left\{\frac{(1-\beta)N_1(x_2)+(1+\beta)N_2(x_2)}{\Delta^2(x_2)} + \frac{(1+\beta)N_3(x_2)}{\Delta^3(x_2)}\right\} \mp m_{N^*}^2\int_0^1 dx_3\left\{\frac{(1-\beta)N_4(x_3)+(1+\beta)N_5(x_3)}{\Delta^2(x_3)} + \frac{(1+\beta)N_6(x_3)}{\Delta^3(x_3)}\right\}, \tag{13}$$

$$\Pi_3^{(S)} = m_{N^*}\int_0^1 dx_2\left\{\frac{(1-\beta)N_7(x_2)+(1+\beta)N_8(x_2)}{\Delta(x_2)} + \frac{(1-\beta)N_9(x_2)+(1+\beta)N_{10}(x_2)}{\Delta^2(x_2)}\right\} \mp m_{N^*}\int_0^1 dx_3\left\{\frac{(1-\beta)N_{11}(x_3)+(1+\beta)N_{12}(x_3)}{\Delta(x_3)} + \frac{(1-\beta)N_{13}(x_3)+(1+\beta)N_{14}(x_3)}{\Delta^2(x_3)}\right\}, \tag{14}$$

and

$$\Pi_6^{(S)} = \int_0^1 dx_2\left\{\frac{(1-\beta)N_{15}(x_2)}{\Delta(x_2)} + m_{N^*}^2\frac{(1-\beta)N_{16}(x_2)+(1+\beta)N_{17}(x_2)}{\Delta^2(x_2)} + m_{N^*}^2\frac{(1+\beta)N_{18}(x_2)}{\Delta^3(x_2)}\right\} \mp \int_0^1 dx_3\left\{\frac{(1-\beta)N_{19}(x_3)}{\Delta(x_3)} + m_{N^*}^2\frac{(1-\beta)N_{20}(x_3)+(1+\beta)N_{21}(x_3)}{\Delta^2(x_3)} + m_{N^*}^2\frac{(1+\beta)N_{22}(x_3)}{\Delta^3(x_3)}\right\}, \tag{15}$$

where

$$\begin{aligned}
N_1(x) &= -\tilde{A}_{123}(x)+\tilde{A}_{1345}(x)-2\tilde{A}_{34}(x)-\tilde{V}_{123}(x)+\tilde{V}_{34}(x),\\
N_2(x) &= -\tilde{P}_{12}(x)+\tilde{S}_{12}(x)-2\tilde{T}_{123}(x)-3\tilde{T}_{127}(x)\\
&\quad -5\tilde{T}_{158}(x)-10\tilde{T}_{78}(x)+\frac{2}{x}\tilde{\tilde{T}}_{234578}(x),\\
N_3(x) &= \frac{2}{x}(Q^2+m_{N^*}^2x^2)\tilde{\tilde{T}}_{234578}(x),\\
N_4(x) &= \tilde{A}_{123}(x)+\tilde{A}_{34}(x)+\tilde{V}_{123}(x)-\tilde{V}_{34}(x),\\
N_5(x) &= -\tilde{P}_{12}(x)+\tilde{S}_{12}(x)-\tilde{T}_{127}(x)-\tilde{T}_{158}(x)\\
&\quad -2\tilde{T}_{78}(x)+\frac{2}{x}\tilde{\tilde{T}}_{234578}(x),\\
N_6(x) &= \frac{2}{x}(Q^2+m_{N^*}^2x^2)\tilde{\tilde{T}}_{234578}(x),\\
N_7(x) &= -A_3(x)+V_1(x)-V_3(x)-\frac{1}{2x}[\tilde{A}_{123}(x)+\tilde{V}_{123}(x)],
\end{aligned} \tag{16}$$

$$\begin{aligned}
N_8(x) &= P_1(x)+S_1(x)+2T_1(x)-4T_7(x)\\
&\quad -\frac{1}{2x}[\tilde{T}_{123}(x)+3\tilde{T}_{127}(x)],\\
N_9(x) &= -\frac{1}{2x}(Q^2+m_{N^*}^2x^2)[\tilde{A}_{123}(x)+\tilde{V}_{123}(x)]\\
&\quad -m_{N^*}^2[\tilde{\tilde{A}}_{123456}(x)-\tilde{\tilde{V}}_{123456}(x)],\\
N_{10}(x) &= -\frac{1}{2x}(Q^2+m_{N^*}^2x^2)[\tilde{T}_{123}(x)+3\tilde{T}_{127}(x)]\\
&\quad +m_{N^*}^2\tilde{\tilde{T}}_{234578}(x),\\
N_{11}(x) &= \frac{1}{2}[A_1(x)-V_1(x)],\\
N_{12}(x) &= \frac{1}{2}[P_1(x)+S_1(x)-T_1(x)+2T_7(x)]\\
&\quad +\frac{1}{4x}[\tilde{T}_{123}(x)+\tilde{T}_{127}(x)],\\
N_{13}(x) &= -m_{N^*}^2[\tilde{\tilde{A}}_{123456}(x)+\tilde{\tilde{V}}_{123456}(x)],
\end{aligned}$$

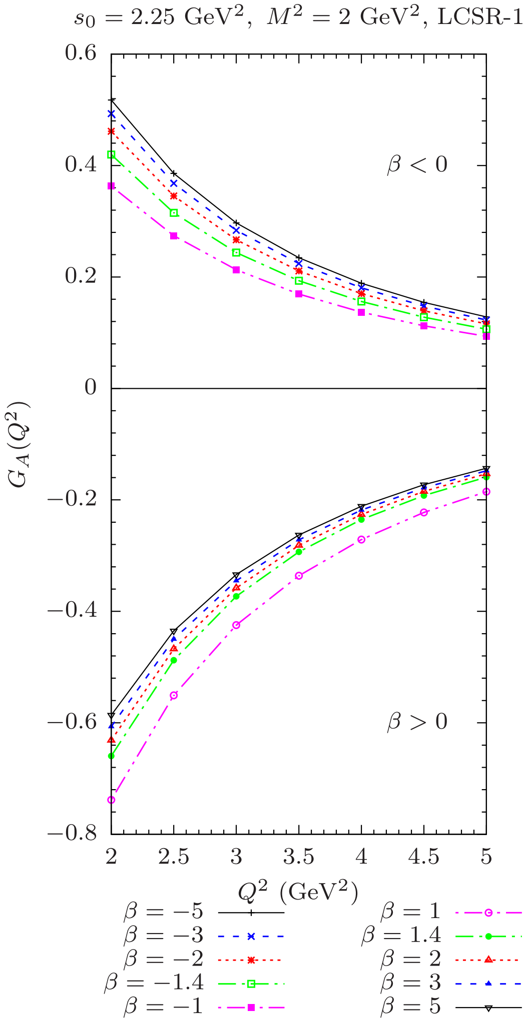

FIG. 1. The dependence of the form-factor $G_A(Q^2)$ on Q^2 for various values of β with the data set LCSR-1.

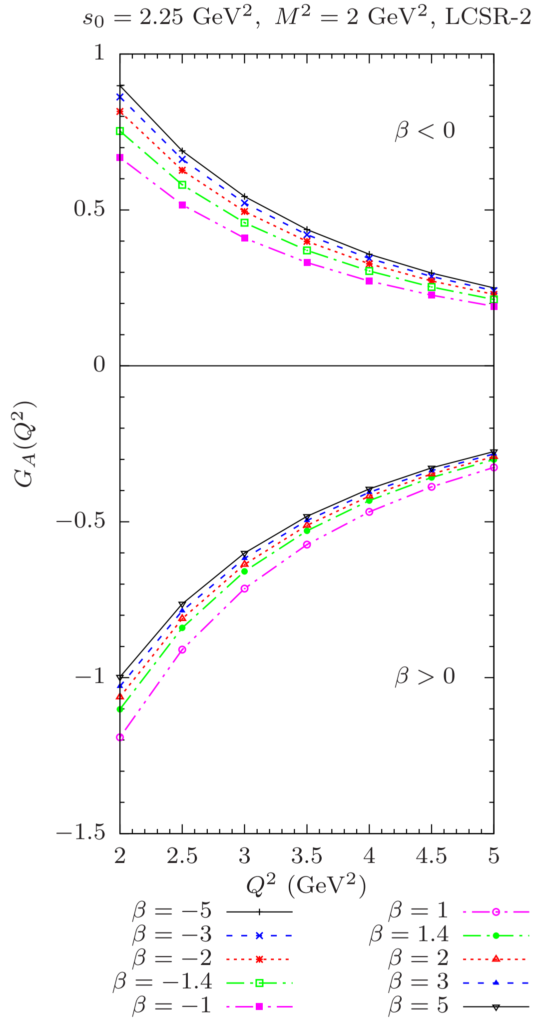

FIG. 2. The same as Fig. 1, but with the LCSR-2.

$$
\begin{aligned}
N_{14}(x) &= \frac{1}{4x}(Q^2+m_{N^*}^2x^2)[\tilde{T}_{123}(x)+\tilde{T}_{127}(x)] \\
&\quad +\frac{m_{N^*}^2}{4}\tilde{\tilde{T}}_{234578}(x), \\
N_{15}(x) &= -A_1(x)-V_1(x), \\
N_{16}(x) &= x[\tilde{A}_{123}(x)-\tilde{A}_{1345}(x)+2\tilde{A}_{34}(x)+\tilde{V}_{123}(x)-\tilde{V}_{34}(x)], \\
N_{17}(x) &= x[\tilde{P}_{12}(x)-\tilde{S}_{12}(x)+2\tilde{T}_{123}(x)+3\tilde{T}_{127}(x) \\
&\quad +5\tilde{T}_{158}(x)+10\tilde{T}_{78}(x)]-2\tilde{\tilde{T}}_{234578}(x), \\
N_{18}(x) &= -2(Q^2+m_{N^*}^2x^2)\tilde{\tilde{T}}_{234578}(x), \\
N_{19}(x) &= -A_1(x)+V_1(x), \\
N_{20}(x) &= x[-\tilde{A}_{123}(x)-\tilde{A}_{34}(x)-\tilde{V}_{123}(x)+\tilde{V}_{34}(x)], \\
N_{21}(x) &= x[\tilde{P}_{12}(x)-\tilde{S}_{12}(x)+\tilde{T}_{127}(x)+\tilde{T}_{158}(x) \\
&\quad +2\tilde{T}_{78}(x)]+2\tilde{\tilde{T}}_{234578}(x), \\
N_{22}(x) &= -2(Q^2+m_{N^*}^2x^2)\tilde{\tilde{T}}_{234578}(x),
\end{aligned}
\tag{17}
$$

and

$$
\begin{aligned}
&\tilde{S}_{12}=\tilde{S}_1-\tilde{S}_2, \quad \tilde{P}_{12}=\tilde{P}_2-\tilde{P}_1, \quad \tilde{V}_{123}=\tilde{V}_1-\tilde{V}_2-\tilde{V}_3, \\
&\tilde{V}_{1345}=-2\tilde{V}_1+\tilde{V}_3+\tilde{V}_4+2\tilde{V}_5, \quad \tilde{V}_{34}=\tilde{V}_4-\tilde{V}_3, \\
&\tilde{\tilde{V}}_{123456}=-\tilde{\tilde{V}}_1+\tilde{\tilde{V}}_2+\tilde{\tilde{V}}_3+\tilde{\tilde{V}}_4+\tilde{\tilde{V}}_5-\tilde{\tilde{V}}_6, \\
&\tilde{A}_{123}=-\tilde{A}_1+\tilde{A}_2-\tilde{A}_3, \quad \tilde{A}_{1345}=-2\tilde{A}_1-\tilde{A}_3-\tilde{A}_4+2\tilde{A}_5, \\
&\tilde{A}_{34}=\tilde{A}_3-\tilde{A}_4, \quad \tilde{\tilde{A}}_{123456}=\tilde{\tilde{A}}_1-\tilde{\tilde{A}}_2+\tilde{\tilde{A}}_3+\tilde{\tilde{A}}_4-\tilde{\tilde{A}}_5+\tilde{\tilde{A}}_6, \\
&\tilde{T}_{123}=\tilde{T}_1+\tilde{T}_2-2\tilde{T}_3, \quad \tilde{T}_{127}=\tilde{T}_1-\tilde{T}_2-2\tilde{T}_7, \\
&\tilde{T}_{158}=-\tilde{T}_1+\tilde{T}_5+2\tilde{T}_8, \\
&\tilde{\tilde{T}}_{234578}=2\tilde{\tilde{T}}_2-2\tilde{\tilde{T}}_3-2\tilde{\tilde{T}}_4+2\tilde{\tilde{T}}_5+2\tilde{\tilde{T}}_7+2\tilde{\tilde{T}}_8, \\
&\tilde{T}_{78}=\tilde{T}_7-\tilde{T}_8, \\
&\tilde{\tilde{T}}_{125678}=-\tilde{\tilde{T}}_1+\tilde{\tilde{T}}_2+\tilde{\tilde{T}}_5-\tilde{\tilde{T}}_6+2\tilde{\tilde{T}}_7+2\tilde{\tilde{T}}_8.
\end{aligned}
\tag{18}
$$

The generic expressions for the DA's with and without tildes are given as

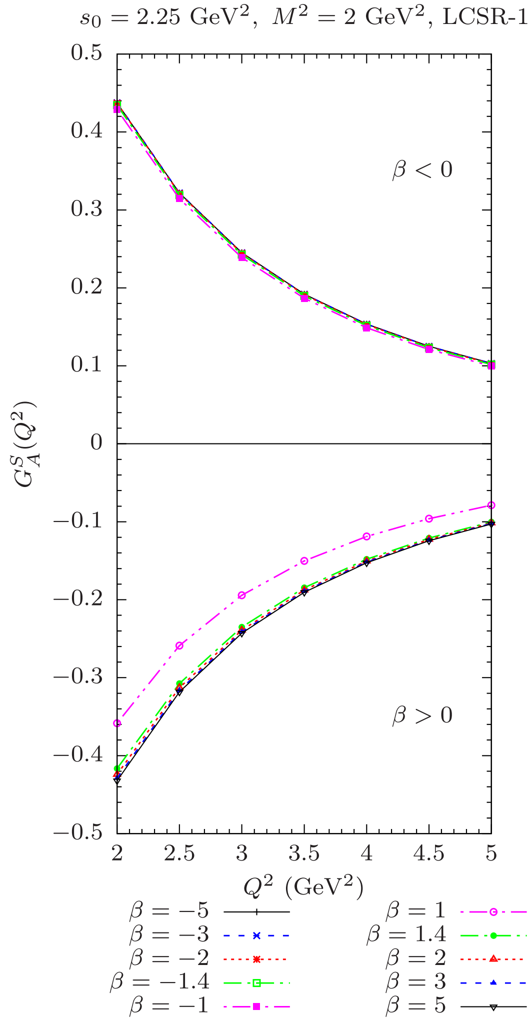

FIG. 3. The same as Fig. 1, but for $G_A^S(Q^2)$.

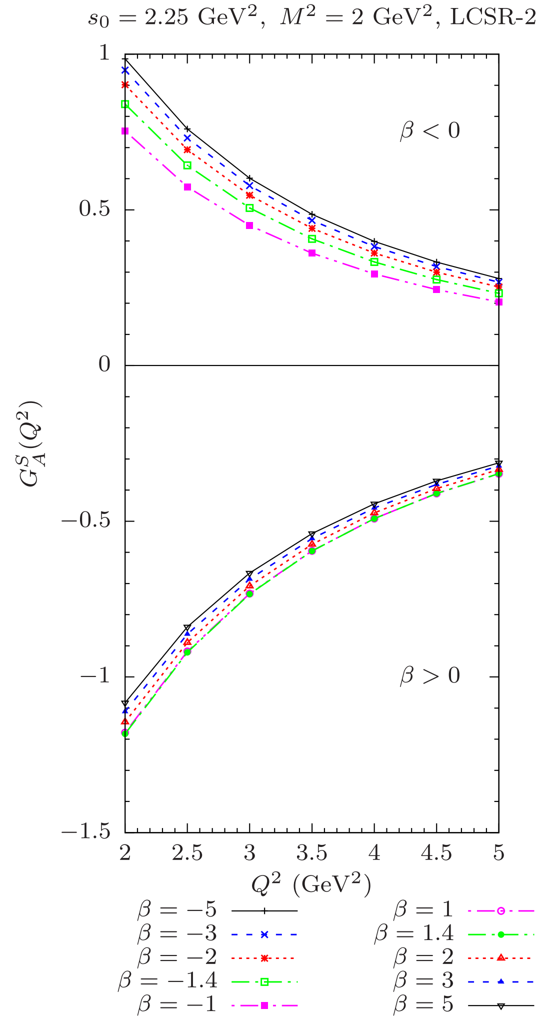

FIG. 4. The same as Fig. 3, but with LCSR-2.

$$
\begin{aligned}
F(x_2) &= \int_0^{1-x_2} dx_1\, F(x_1, x_2, 1-x_1-x_2),\\
\tilde{F}(x_2) &= \int_1^{x_2} dx_2' \int_0^{1-x_2'} dx_1\, F(x_1, x_2', 1-x_1-x_2'),\\
\tilde{\tilde{F}}(x_2) &= \int_1^{x_2} dx_2' \int_1^{x_2'} dx_2'' \int_0^{1-x_2''} dx_1\, F(x_1, x_2'', 1-x_1-x_2'').
\end{aligned} \tag{19}
$$

A similar set of expressions holds true when the argument is x_3.

Finally, for the derivation of the sum rules for the relevant form-factors, we perform a Borel transformation in the variable $-(q-p)^2$ in order to suppress the contributions from higher states and the continuum and to enhance the contribution of the ground state. This can be achieved with the help of the following subtraction rules:

$$
\begin{aligned}
\int dx \frac{\rho(x)}{\Delta(x)} &\to -\int_{x_0}^1 \frac{dx}{x} \rho(x) e^{-s(x)/M^2},\\
\int dx \frac{\rho(x)}{\Delta^2(x)} &\to \frac{1}{M^2} \int_{x_0}^1 \frac{dx}{x^2} \rho(x) e^{-s(x)/M^2} + \frac{\rho(x_0) e^{-s_0/M^2}}{x_0^2 m_{N^*}^2 + Q^2},\\
\int dx \frac{\rho(x)}{\Delta^3(x)} &\to -\frac{1}{2M^4} \int_{x_0}^1 \frac{dx}{x^3} \rho(x) e^{-s(x)/M^2}\\
&\quad - \frac{1}{2x_0} \frac{\rho(x_0) e^{-s_0/M^2}}{(Q^2 + x_0^2 m_{N^*}) M^2}\\
&\quad + \frac{1}{2} \frac{x_0^2 e^{-s_0/M^2}}{Q^2 + x_0^2 m_{N^*}^2} \frac{d}{dx_0} \left[\frac{1}{x_0} \frac{\rho(x_0)}{Q^2 + x_0^2 m_{N^*}^2} \right].
\end{aligned} \tag{20}
$$

In Eq. (20), the denominator on the left-hand side is defined as

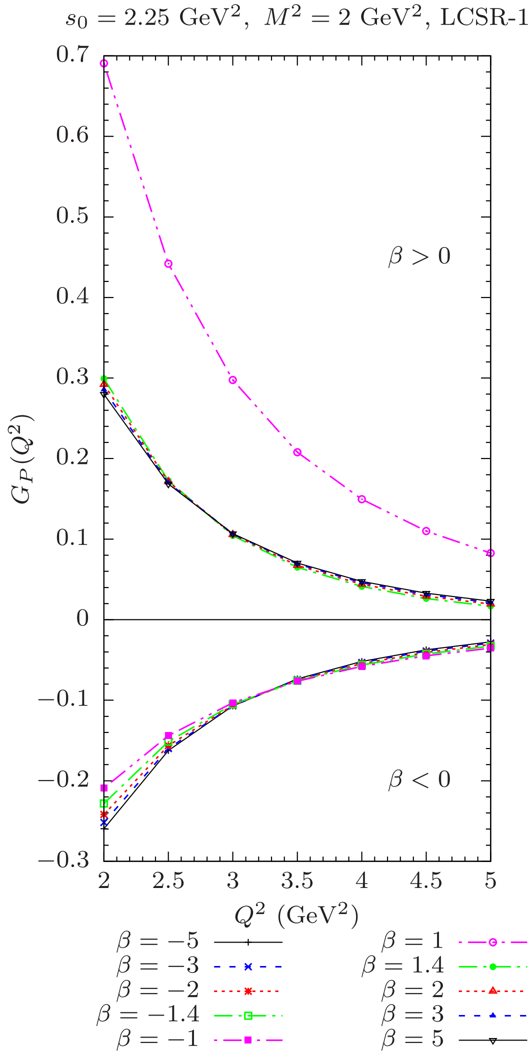

FIG. 5. The same as Fig. 1, but for $G_P(Q^2)$.

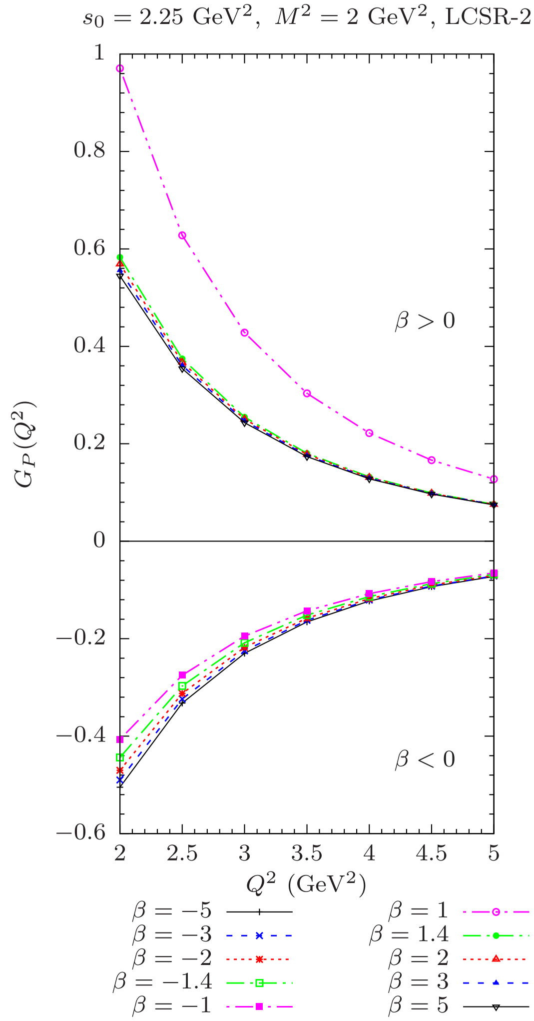

FIG. 6. The same as Fig. 5, but with LCSR-2.

$$\Delta(x) \coloneqq (xp - q)^2 = x(p - q)^2 - \bar{x}Q^2 - x\bar{x}m_{N^*}^2,$$

where $\bar{x} = 1 - x$. x_0 is the solution of $s(x) = s_0$, where

$$s(x) = \frac{\bar{x}Q^2 + x\bar{x}m_{N^*}^2}{x}.$$

III. NUMERICAL ANALYSIS

Having the explicit expressions of the form-factors $G_A^{(S)}$, $G_P^{(S)}$, and $G_T^{(S)}$, now we perform the numerical analysis of the sum rules for them.

The main nonperturbative input ingredients of the LCSR for the form-factors $G_A^{(S)}$, $G_P^{(S)}$, and $G_T^{(S)}$ are the DA's of N^*. They are presented in [19]. The values of the parameters appearing in the DA's are also given [19]. In numerical calculations, we use two set of values of parameters, i.e., LCSR-1 and LCSR-2.

The sum rules for the form-factors $G_A^{(S)}$, $G_P^{(S)}$, and $G_T^{(S)}$ contain three auxiliary parameters, i.e., the Borel mass squared, M^2, the continuum threshold, s_0, and the parameter β. The form-factors should be independent of these parameters. Therefore, the primary aim of any sum rules analysis is to find the appropriate domains of these auxiliary parameters for which the form-factors exhibit good stability to their variations.

The working regions of M^2 and s_0 are determined from the standard criterion that the contributions from higher states and the continuum as well as higher-twist terms should be suppressed. Our analysis shows that the working regions of M^2 and s_0, which satisfy the aforementioned criterion are $1\ \text{GeV}^2 \le M^2 \le 3\ \text{GeV}^2$ and $s_0 = (2.25 \pm 0.25)\ \text{GeV}^2$. In [20], the mass and residues of octet baryons are analyzed, and in the present work, we use the result for the residue of the nucleon given in the same reference. The uncertainties of the input parameters and intrinsic uncertainties carried by the LCSR method (namely the factorization scale, M^2, higher-twist corrections) can introduce an

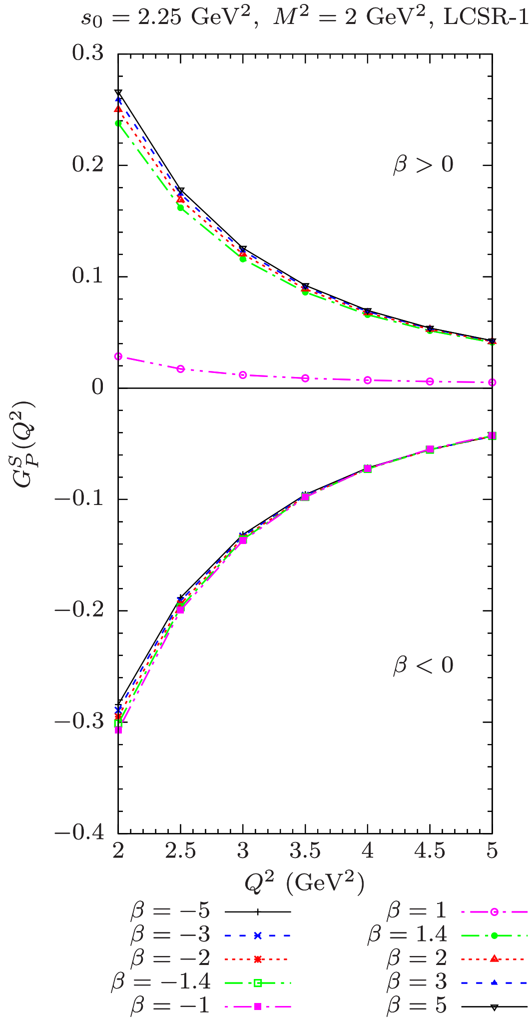

FIG. 7. The same as Fig. 1, but for $G_P^S(Q^2)$.

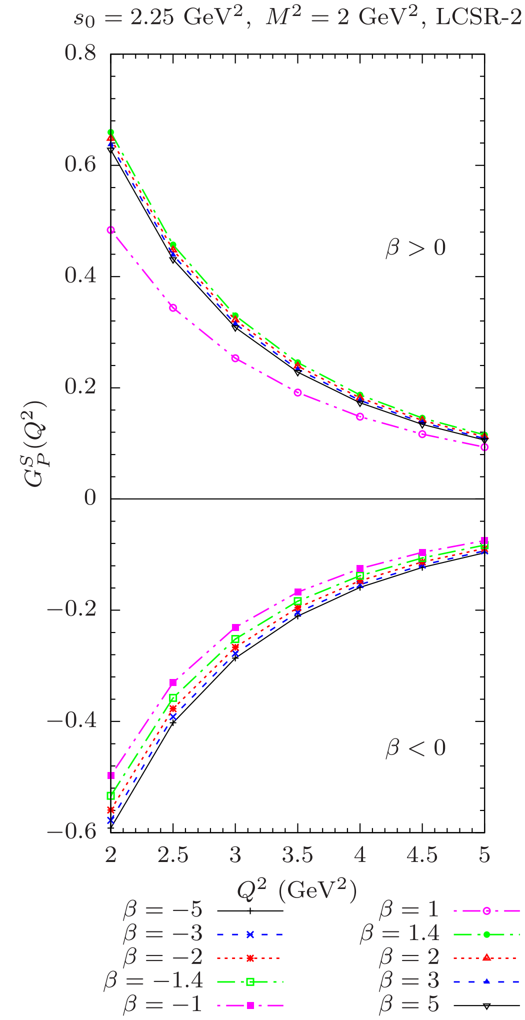

FIG. 8. The same as Fig. 7, but with LCSR-2.

uncertainty about $(15 \pm 5)\%$. Therefore, we expect that the sum rules for the form-factors work effectively in the domain,

$$2\ \text{GeV}^2 \leq Q^2 \leq 6\ \text{GeV}^2. \qquad (21)$$

Note that the LCSR approach is not dependable for $Q^2 \leq 1$ GeV^2. It is because the mass corrections are proportional to m_N^2/Q^2, which becomes very large for $Q^2 < 1$ GeV^2, and hence, the LCSR becomes unreliable.

IV. RESULTS AND DISCUSSION

In Figs. 1–12, we depict the dependence of the form-factors $G_A^{(S)}$, $G_P^{(S)}$, and $G_T^{(S)}$ on Q^2 for various values of β in its working region and at the fixed values of $M^2 = 2$ GeV^2 and $s_0 = 2.25$ GeV^2 for two sets of values of input parameters entering the DA's.

From the figures, we make the following observations:

- (i) The values of $G_A^{(S)}$ and G_T^S are positive (negative) for negative (positive) values of β, whereas G_P and G_P^S have the same sign as β for both sets of parameters, LCSR-1 and LCSR-2.
- (ii) The moduli of the form-factors G_A, G_A^S, G_P, and G_P^S for the second set of values are larger than those for the first set of parameters, and G_T and G_T^S have close values for both set of parameters.
- (iii) The form-factors G_P and G_P^S are sensitive to the variations in β when $\beta > 0$ for both sets of input parameters. For example, the values of the form-factors G_P and G_P^S at $\beta = 1$ are nearly twice as large as the corresponding values of these form-factors for other values of $\beta > 0$.
- (iv) All the form-factors considered display a similar dependence on Q^2 for both sets (LCSR-1 and LCSR-2) and for all values of β.

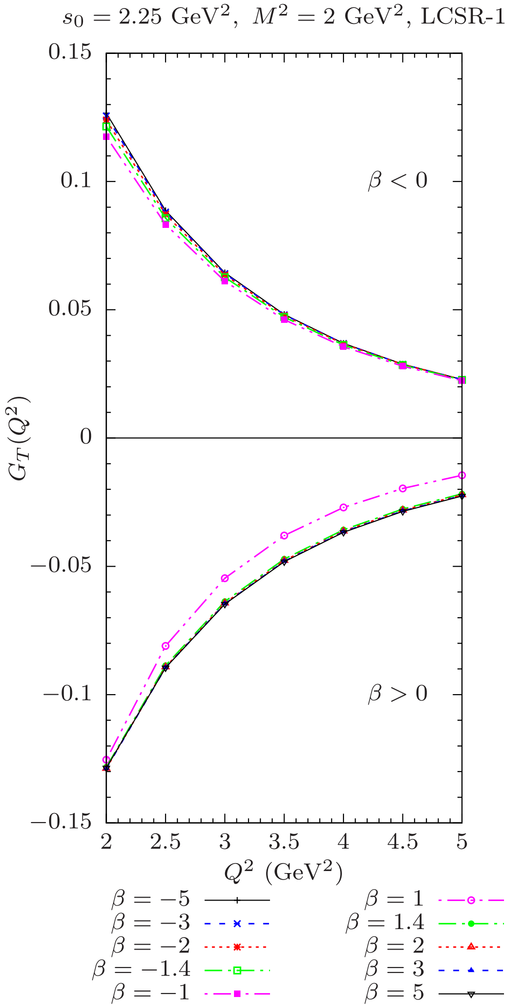

FIG. 9. The same as Fig. 1, but for $G_T(Q^2)$.

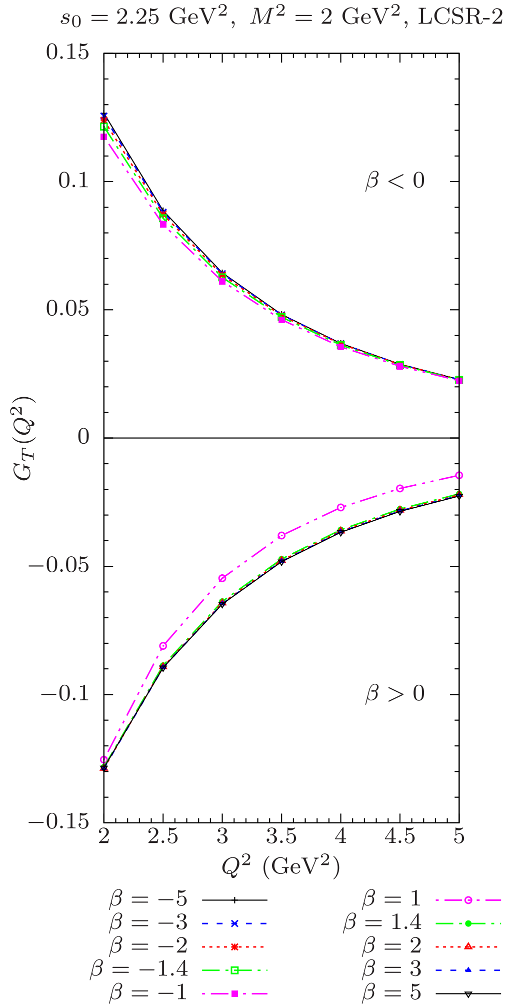

FIG. 10. The same as Fig. 9, but with LCSR-2.

(v) The moduli of the form factor G_P at $\beta > 0$ are approximately twice larger for the $\beta < 0$ case for both set of parameters.

(vi) The values of G_T and G_T^S for both set of parameters and any values of β are smaller than the values of the form-factors $G_A^{(S)}$ and $G_P^{(S)}$.

From our results, it follows that the form-factor $G_A(Q^2)$ can be parametrized by the dipole form,

$$G_A(Q^2) = \frac{g_A}{(1 + Q^2/m_A^2)^2}, \tag{22}$$

TABLE I. The average values of g_A and m_A in Eq. (22).

	g_A		m_A (GeV)	
	LCSR-1	LCSR-2	LCSR-1	LCSR-2
$\beta < 0$	1.76 ± 0.40	1.76 ± 0.40	1.52 ± 0.06	1.52 ± 0.06
$\beta > 0$	-3.07 ± 0.27	-3.07 ± 0.27	1.41 ± 0.03	1.41 ± 0.03

where we tabulated the average of the values of g_A and m_A for both parameter sets and for all the values of β that we considered in Table I. From Table I, it follows that the values of g_A and m_A for the LCSR-2 case are slightly larger than the ones for the LCSR-1 one for $\beta < 0$. The magnitude of g_A for the case $\beta > 0$ is larger than that for $\beta < 0$. Note that for $\beta = -1$ (the case of the Ioffe current), for the values of g_A and m_A, we find

$$g_A = 1.20 \quad \text{and} \quad m_A = 1.61 \text{ GeV} \tag{23}$$

for both LCSR-1 and LCSR-2. We observe that the value of m is practically insensitive to the variations of both positive and negative values of β.

When the experimental results will be obtained, there may appear the possibility to compare our predictions with experimental data.

Finally, we would like to note that our results can be improved by taking into account radiative corrections and

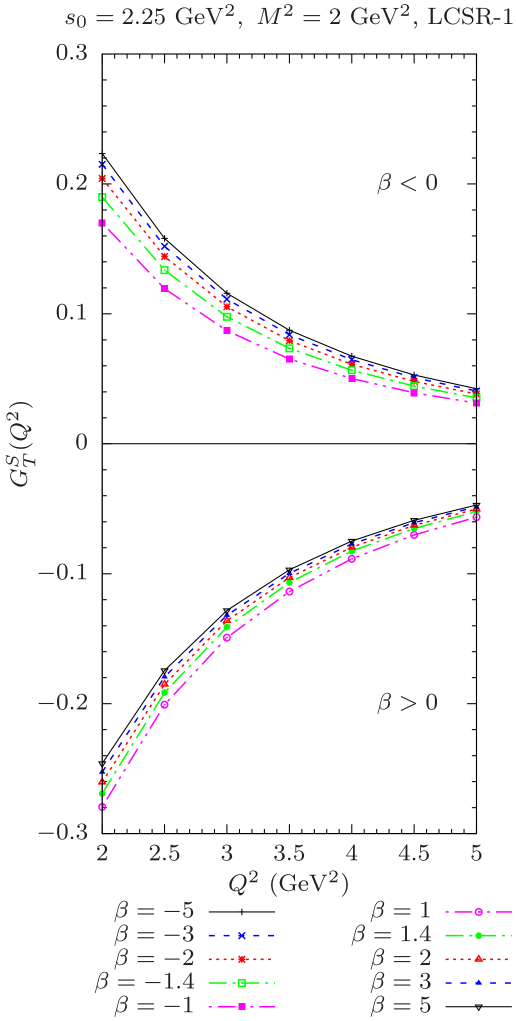

FIG. 11. The same as Fig. 1, but for $G_T^S(Q^2)$.

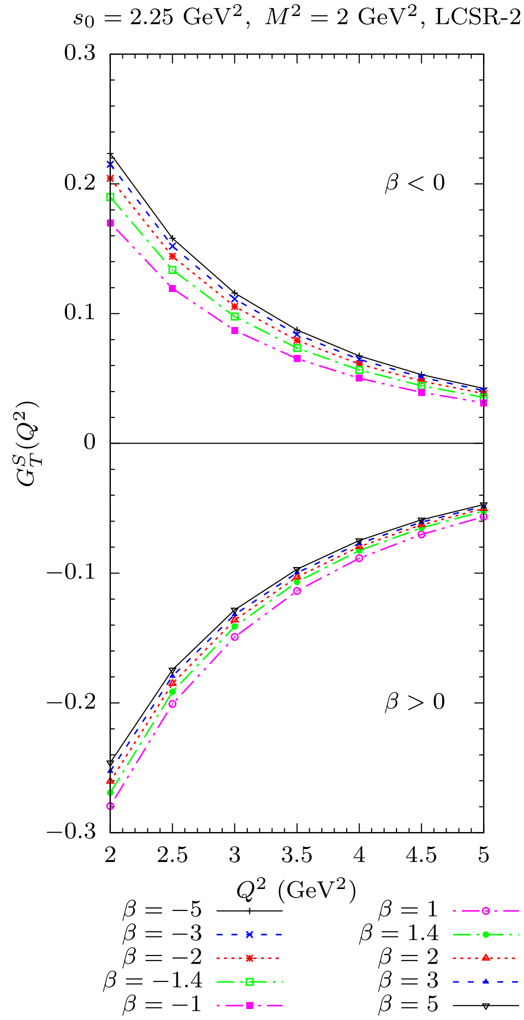

FIG. 12. The same as Fig. 11, but with LCSR-2.

the contributions of four and five particles as well as more precise determination of the input parameters.

V. CONCLUSION

In the present work, we calculated the $N^*(1535) \to N$ transition form-factors induced by isovector and isoscalar axial currents within the light cone QCD sum rules using the general form of the interpolating current. In performing numerical analysis, we used two sets of values of input parameters. We observed that the form-factor $G_A(Q^2)$ exhibits the dipole form dependence on Q^2, and we find the values of relevant parameters, namely g_A and m_A. It was also observed that $G_P^{(S)}$ shows a strong dependence on the variations of the auxiliary parameter β. The form factor $G_T^{(S)}$ at all values of Q^2 and β are smaller than the form factors $G_A^{(S)}$ and $G_P^{(S)}$.

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APPENDIX: N^* DISTRIBUTION AMPLITUDES

In this Appendix, we present the N^* DA's, which are necessary to calculate the $\Lambda \to N^*$ transition form-factors. The DA's of the N^* baryon are defined from the matrix element $\langle 0|\epsilon^{abc} u_\alpha^a(a_1 x) d_\beta^b(a_2 x) d_\gamma^c(a_3 x)|N^*(p)\rangle$. The general decomposition of the nucleon DA's involve 24 invariant functions that were firstly derived in [21]. Using this result, the N^* DA's were obtained in [19], and their expressions are given below,

$$
\begin{aligned}
&4\langle 0|\epsilon^{abc}u_\alpha^a(a_1x)d_\beta^b(a_2x)d_\gamma^c(a_3x)|N^*(p)\rangle \\
&= \mathcal{S}_1 m_{N^*} C_{\alpha\beta} N_\gamma^* - \mathcal{S}_2 m_{N^*}^2 C_{\alpha\beta}(\not{x}N^*)_\gamma + \mathcal{P}_1 m_{N^*}(\gamma_5 C)_{\alpha\beta}(\gamma_5 N^*)_\gamma + \mathcal{P}_2 m_{N^*}^2(\gamma_5 C)_{\alpha\beta}(\gamma_5 \not{x} N^*)_\gamma \\
&\quad - \left(\mathcal{V}_1 + \frac{x^2 m_{N^*}^2}{4}\mathcal{V}_1^M\right)(\not{p}C)_{\alpha\beta}N_\gamma^* + \mathcal{V}_2 m_{N^*}(\not{p}C)_{\alpha\beta}(\not{x}N^*)_\gamma + \mathcal{V}_3 m_{N^*}(\gamma_\mu C)_{\alpha\beta}(\gamma^\mu N^*)_\gamma - \mathcal{V}_4 m_{N^*}^2(\not{x}C)_{\alpha\beta}N_\gamma^* \\
&\quad - \mathcal{V}_5 m_{N^*}^2(\gamma_\mu C)_{\alpha\beta}(i\sigma^{\mu\nu}x_\nu N^*)_\gamma + \mathcal{V}_6 m_{N^*}^3(\not{x}C)_{\alpha\beta}(\not{x}N^*)_\gamma - \left(\mathcal{A}_1 + \frac{x^2 m_{N^*}^2}{4}\mathcal{A}_1^M\right)(\not{p}\gamma_5 C)_{\alpha\beta}(\gamma N^*)_\gamma \\
&\quad + \mathcal{A}_2 m_{N^*}(\not{p}\gamma_5 C)_{\alpha\beta}(\not{x}\gamma_5 N^*)_\gamma + \mathcal{A}_3 m_{N^*}(\gamma_\mu\gamma_5 C)_{\alpha\beta}(\gamma^\mu\gamma_5 N^*)_\gamma - \mathcal{A}_4 m_{N^*}^2(\not{x}\gamma_5 C)_{\alpha\beta}(\gamma_5 N^*)_\gamma \\
&\quad - \mathcal{A}_5 m_{N^*}^2(\gamma_\mu\gamma_5 C)_{\alpha\beta}(i\sigma^{\mu\nu}x_\nu\gamma_5 N^*)_\gamma + \mathcal{A}_6 m_{N^*}^3(\not{x}\gamma_5 C)_{\alpha\beta}(\not{x}\gamma_5 N^*)_\gamma - \left(\mathcal{T}_1 + \frac{x^2 m_{N^*}^2}{4}\mathcal{T}_1^M\right)(i\sigma_{\mu\nu}p_\nu C)_{\alpha\beta}(\gamma^\mu N^*)_\gamma \\
&\quad + \mathcal{T}_2 m_{N^*}(i\sigma_{\mu\nu}x^\mu p^\nu C)_{\alpha\beta}N_\gamma^* + \mathcal{T}_3 m_{N^*}(\sigma_{\mu\nu}C)_{\alpha\beta}(\sigma^{\mu\nu}N^*)_\gamma + \mathcal{T}_4 m_{N^*}(\sigma_{\mu\nu}p^\nu C)_{\alpha\beta}(\sigma^{\mu\rho}x_\rho N^*)_\gamma \\
&\quad - \mathcal{T}_5 m_{N^*}^2(i\sigma_{\mu\nu}x^\nu C)_{\alpha\beta}(\gamma^\mu N^*)_\gamma - \mathcal{T}_6 m_{N^*}^2(i\sigma_{\mu\nu}x^\mu p^\nu C)_{\alpha\beta}(\not{x}N^*)_\gamma - \mathcal{T}_7 m_{N^*}^2(\sigma_{\mu\nu}C)_{\alpha\beta}(\sigma^{\mu\nu}\not{x}N^*)_\gamma \\
&\quad + \mathcal{T}_8 m_{N^*}^3(\sigma_{\mu\nu}x^\nu C)_{\alpha\beta}(\sigma^{\mu\rho}x_\rho N^*)_\gamma.
\end{aligned}
$$

The functions labeled with calligraphic letters in the above expression do not possess definite twists, but they can be written in terms of the N^* distribution amplitudes (DA's) with definite and increasing twists via the scalar product $p\cdot x$ and the parameters a_i, $i = 1, 2, 3$. The relations between the two sets of DA's for the N^*, and for the scalar, pseudo scalar, vector, axial-vector, and tensor DA's for nucleons are

$$
\begin{gathered}
\mathcal{S}_1 = S_1 \qquad 2(p\cdot x)\mathcal{S}_2 = S_1 - S_2 \qquad \mathcal{P}_1 = P_1 \qquad 2(p\cdot x)\mathcal{P}_2 = P_2 - P_1 \qquad \mathcal{V}_1 = V_1 \\
2(p\cdot x)\mathcal{V}_2 = V_1 - V_2 - V_3 \qquad 2\mathcal{V}_3 = V_3 \qquad 4(p\cdot x)\mathcal{V}_4 = -2V_1 + V_3 + V_4 + 2V_5 \qquad 4(p\cdot x)\mathcal{V}_5 = V_4 - V_3 \\
4(p\cdot x)^2\mathcal{V}_6 = -V_1 + V_2 + V_3 + V_4 + V_5 - V_6 \qquad \mathcal{A}_1 = A_1 \qquad 2(p\cdot x)\mathcal{A}_2 = -A_1 + A_2 - A_3 \qquad 2\mathcal{A}_3 = A_3 \\
4(p\cdot x)\mathcal{A}_4 = -2A_1 - A_3 - A_4 + 2A_5 4(p\cdot x)\mathcal{A}_5 = A_3 - A_4 \qquad 4(p\cdot x)^2\mathcal{A}_6 = A_1 - A_2 + A_3 + A_4 - A_5 + A_6 \\
\mathcal{T}_1 = T_1 \qquad 2(p\cdot x)\mathcal{T}_2 = T_1 + T_2 - 2T_3 \qquad 2\mathcal{T}_3 = T_7 \qquad 2(p\cdot x)\mathcal{T}_4 = T_1 - T_2 - 2T_7 \\
2(p\cdot x)\mathcal{T}_5 = -T_1 + T_5 + 2T_8 \qquad 4(p\cdot x)^2\mathcal{T}_6 = 2T_2 - 2T_3 - 2T_4 + 2T_5 + 2T_7 + 2T_8 \qquad 4(p\cdot x)\mathcal{T}_7 = T_7 - T_8 \\
4(p\cdot x)^2\mathcal{T}_8 = -T_1 + T_2 + T_5 - T_6 + 2T_7 + 2T_8,
\end{gathered}
$$

where the terms in x^2, $\mathcal{V}_1^M$, $\mathcal{A}_1^M$, and $\mathcal{T}_1^M$ are left aside.

The distribution amplitudes $F[a_i(p\cdot x)] = S_i,\ P_i,\ V_i,\ A_i,\ T_i$ can be represented as

$$
F[a_i(p\cdot x)] = \int dx_1 dx_2 dx_3 \delta(x_1 + x_2 + x_3 - 1) e^{ip\cdot x\Sigma_i x_i a_i} F(x_i),
$$

where, x_i with $i = 1$, 2, and 3 are longitudinal momentum fractions carried by the participating quarks.

The explicit expressions for the Λ DA's up to twist 6 are given as Twist–3 DA's,

$$
\begin{aligned}
V_1(x_i,\mu) &= 120 x_1 x_2 x_3[\phi_3^0(\mu) + \phi_3^+(\mu)(1 - 3x_3)], \\
A_1(x_i,\mu) &= 120 x_1 x_2 x_3(x_2 - x_1)\phi_3^-(\mu), \\
T_1(x_i,\mu) &= 120 x_1 x_2 x_3\left[\phi_3^0(\mu) - \frac{1}{2}(\phi_3^+ - \phi_3^-)(\mu)(1 - 3x_3)\right].
\end{aligned}
$$

Twist–4 DA's,

$$
\begin{aligned}
V_2(x_i,\mu) &= 24x_1x_2[\phi_4^0(\mu) + \phi_4^+(\mu)(1-5x_3)],\\
A_2(x_i,\mu) &= 24x_1x_2(x_2-x_1)\phi_4^-(\mu),\\
T_2(x_i,\mu) &= 24x_1x_2[\xi_4^0(\mu) + \xi_4^+(\mu)(1-5x_3)],\\
V_3(x_i,\mu) &= 12x_3[\psi_4^0(\mu)(1-x_3) + \psi_4^+(\mu)(1-x_3-10x_1x_2) + \psi_4^-(\mu)(x_1^2+x_2^2-x_3(1-x_3))],\\
A_3(x_i,\mu) &= 12x_3(x_2-x_1)[(\psi_4^0+\psi_4^+)(\mu) + \psi_4^-(\mu)(1-2x_3)],\\
T_3(x_i,\mu) &= 6x_3[(\phi_4^0+\psi_4^0+\xi_4^0)(\mu)(1-x_3) + (\phi_4^+ + \psi_4^+ + \xi_4^+)(\mu)(1-x_3-10x_1x_2)\\
&\quad + (\phi_4^- - \psi_4^- + \xi_4^-)(\mu)(x_1^2+x_2^2-x_3(1-x_3))],\\
T_7(x_i,\mu) &= 6x_3[(\phi_4^0+\psi_4^0-\xi_4^0)(\mu)(1-x_3) + (\phi_4^+ + \psi_4^+ - \xi_4^+)(\mu)(1-x_3-10x_1x_2)\\
&\quad + (\phi_4^- - \psi_4^- - \xi_4^-)(\mu)(x_1^2+x_2^2-x_3(1-x_3))],\\
S_1(x_i,\mu) &= 6x_3(x_2-x_1)[(\phi_4^0+\psi_4^0+\xi_4^0+\phi_4^++\psi_4^++\xi_4^+)(\mu) + (\phi_4^- - \psi_4^- + \xi_4^-)(\mu)(1-2x_3)],\\
P_1(x_i,\mu) &= 6x_3(x_1-x_2)[(\phi_4^0+\psi_4^0-\xi_4^0+\phi_4^++\psi_4^+-\xi_4^+)(\mu) + (\phi_4^- - \psi_4^- - \xi_4^-)(\mu)(1-2x_3)].
\end{aligned}
$$

Twist–5 DA's,

$$
\begin{aligned}
V_4(x_i,\mu) &= 3[\psi_5^0(\mu)(1-x_3) + \psi_5^+(\mu)(1-x_3-2(x_1^2+x_2^2)) + \psi_5^-(\mu)(2x_1x_2-x_3(1-x_3))],\\
A_4(x_i,\mu) &= 3(x_2-x_1)[-\psi_5^0(\mu) + \psi_5^+(\mu)(1-2x_3) + \psi_5^-(\mu)x_3],\\
T_4(x_i,\mu) &= \frac{3}{2}[(\phi_5^0+\psi_5^0+\xi_5^0)(\mu)(1-x_3) + (\phi_5^++\psi_5^++\xi_5^+)(\mu)(1-x_3-2(x_1^2+x_2^2))\\
&\quad + (\phi_5^- - \psi_5^- + \xi_5^-)(\mu)(2x_1x_2-x_3(1-x_3))],\\
T_8(x_i,\mu) &= \frac{3}{2}[(\phi_5^0+\psi_5^0-\xi_5^0)(\mu)(1-x_3) + (\phi_5^++\psi_5^+-\xi_5^+)(\mu)(1-x_3-2(x_1^2+x_2^2))\\
&\quad + (\phi_5^- - \psi_5^- - \xi_5^-)(\mu)(2x_1x_2-x_3(1-x_3))],\\
V_5(x_i,\mu) &= 6x_3[\phi_5^0(\mu) + \phi_5^+(\mu)(1-2x_3)],\\
A_5(x_i,\mu) &= 6x_3(x_2-x_1)\phi_5^-(\mu),\\
T_5(x_i,\mu) &= 6x_3[\xi_5^0(\mu) + \xi_5^+(\mu)(1-2x_3)],\\
S_2(x_i,\mu) &= \frac{3}{2}(x_2-x_1)[-(\phi_5^0+\psi_5^0+\xi_5^0)(\mu) + (\phi_5^++\psi_5^++\xi_5^+)(\mu)(1-2x_3) + (\phi_5^- - \psi_5^- + \xi_5^-)(\mu)x_3],\\
P_2(x_i,\mu) &= \frac{3}{2}(x_2-x_1)[-(-\phi_5^0-\psi_5^0+\xi_5^0)(\mu) + (-\phi_5^+-\psi_5^++\xi_5^+)(\mu)(1-2x_3) + (-\phi_5^- + \psi_5^- + \xi_5^-)(\mu)x_3].
\end{aligned}
$$

Twist-6,

$$
\begin{aligned}
V_6(x_i,\mu) &= 2[\phi_6^0(\mu) + \phi_6^+(\mu)(1-3x_3)],\\
A_6(x_i,\mu) &= 2(x_2-x_1)\phi_6^-,\\
T_6(x_i,\mu) &= 2\left[\phi_6^0(\mu) - \frac{1}{2}(\phi_6^+ - \phi_6^-)(1-3x_3)\right].
\end{aligned}
$$

Finally, the x^2 corrections to the corresponding expressions $\mathcal{V}_1^M$, $\mathcal{A}_1^M$, $\mathcal{T}_1^M$ for the leading twist DA's V_1, A_1, and T_1 in the momentum fraction space are given as

$$
\mathcal{V}_1^M(x_2) = \int_0^{1-x_2} dx_1 V_1^M(x_1,x_2,1-x_1-x_2) = \frac{x_2^2}{24}[f_{N^*}C_f^u(x_2) + \lambda_1^{N^*}C_\lambda^u(x_2)],
$$

where

$$C_f^u(x_2) = (1-x_2)^3[113 + 495x_2 - 552x_2^2 - 10A_1^u(1-3x_2) + 2V_1^d(113 - 951x_2 + 828x_2^2)],$$
$$C_\lambda^u(x_2) = -(1-x_2)^3[13 - 20f_1^d + 3x_2 + 10f_1^u(1-3x_2)].$$

The expression for the axial–vector function $\mathcal{A}_1^{M(u)}(x_2)$ is given as

$$\mathcal{A}_1^{M(u)}(x_2) = \int_0^{1-x_2} dx_1 A_1^M(x_1, x_2, 1-x_1-x_2),$$
$$= \frac{x_2^2}{24}(1-x_2)^3[f_{N^*}D_f^u(x_2) + \lambda_1^{N^*}D_\lambda^u(x_2)],$$

with

$$D_f^u(x_2) = 11 + 45x_2 - 2A_1^u(113 - 951x_2 + 828x_2^2) + 10V_1^d(1 - 30x_2),$$
$$D_\lambda^u(x_2) = 29 - 45x_2 - 10f_1^u(7 - 9x_2) - 20f_1^d(5 - 6x_2).$$

Similarly, we get for the function $\mathcal{T}_1^{M(u)}(x_2)$,

$$\mathcal{T}_1^{M(u)}(x_2) = \int_0^{1-x_2} dx_1 T_1^M(x_1, x_2, 1-x_1-x_2), \qquad = \frac{x^2}{48}[f_{N^*}E_f^u(x_2) + \lambda_1^{N^*}E_\lambda^u(x_2)],$$

where

$$E_f^u(x_2) = -\{(1-x_2)[3(439 + 71x_2 - 621x_2^2 + 587x_2^3 - 184x_2^4) + 4A_1^u(1-x_2)^2(59 - 483x_2 + 414x_2^2)$$
$$- 4V_1^d(1301 - 619x_2 - 769x_2^2 + 1161x_2^3 - 414x_2^4)]\} - 12(73 - 220V_1^d)\ln[x_2],$$
$$E_\lambda^u(x_2) = -\{(1-x_2)[5 - 211x_2 + 281x_2^2 - 111x_2^3 + 10(1 + 61x_2 - 83x_2^2 + 33x_2^3)f_1^d$$
$$- 40(1-x_2)^2(2 - 3x_2)f_1^u]\} - 12(3 - 10f_1^d)\ln[x_2].$$

The following functions are encountered to the above amplitudes, and they can be defined in terms of the eight independent parameters, namely f_{N^*}, λ_1, λ_2, and f_1^u, f_1^d, f_2^d, A_1^u, V_1^d:

$$\phi_3^0 = \phi_6^0 = f_{N^*} \qquad \phi_4^0 = \phi_5^0 = \frac{1}{2}(f_{N^*} + \lambda_1^{N^*}) \qquad \xi_4^0 = \xi_5^0 = \frac{1}{6}\lambda_2^{N^*} \qquad \psi_4^0 = \psi_5^0 = \frac{1}{2}(f_{N^*} - \lambda_1^{N^*}),$$
$$\phi_3^- = \frac{21}{2}f_{N^*}A_1^u, \qquad \phi_3^+ = \frac{7}{2}f_{N^*}(1 - V_1^d), \qquad \phi_4^+ = \frac{1}{4}[f_{N^*}(3 - 10V_1^d) + \lambda_1^{N^*}(3 - 10f_1^d)],$$
$$\phi_4^- = -\frac{5}{4}[f_{N^*}(1 - 2A_1^u) - \lambda_1^{N^*}(1 - 2f_1^d - 4f_1^u)], \qquad \psi_4^+ = -\frac{1}{4}[f_{N^*}(2 + 5A_1^u - 5V_1^d) - \lambda_1^{N^*}(2 - 5f_1^d - 5f_1^u)],$$
$$\psi_4^- = \frac{5}{4}[f_{N^*}(2 - A_1^u - 3V_1^d) - \lambda_1^{N^*}(2 - 7f_1^d + f_1^u)], \qquad \xi_4^+ = \frac{1}{16}\lambda_2^{N^*}(4 - 15f_2^d), \qquad \xi_4^- = \frac{5}{16}\lambda_2^{N^*}(4 - 15f_2^d),$$
$$\phi_5^+ = -\frac{5}{6}[f_{N^*}(3 + 4V_1^d) - \lambda_1^{N^*}(1 - 4f_1^d)], \qquad \phi_5^- = -\frac{5}{3}[f_{N^*}(1 - 2A_1^u) - \lambda_1^{N^*}(f_1^d - f_1^u)],$$
$$\psi_5^+ = -\frac{5}{6}[f_{N^*}(5 + 2A_1^u - 2V_1^d) - \lambda_1^{N^*}(1 - 2f_1^d - 2f_1^u)], \qquad \psi_5^- = \frac{5}{3}[f_{N^*}(2 - A_1^u - 3V_1^d) + \lambda_1^{N^*}(f_1^d - f_1^u)],$$
$$\xi_5^+ = \frac{5}{36}\lambda_2^{N^*}(2 - 9f_2^d), \qquad \xi_5^- = -\frac{5}{4}\lambda_2^{N^*}f_2^d, \qquad \phi_6^+ = \frac{1}{2}[f_{N^*}(1 - 4V_1^d) - \lambda_1^{N^*}(1 - 2f_1^d)],$$
$$\phi_6^- = \frac{1}{2}[f_{N^*}(1 + 4A_1^d) + \lambda_1^{N^*}(1 - 4f_1^d - 2f_1^u)],$$

where the parameters A_1^u, V_1^d, f_1^d, f_1^u, and f_2^d are defined as [19]

TABLE II. Parameters of the DA's for the $N^*(1535)$ baryon at $\mu^2 = 2\ \text{GeV}^2$.

Model	$\lambda_1^{N^*}/\lambda_1^N$	$f_{N^*}/\lambda_1^{N^*}$	φ_{10}	φ_{11}	φ_{20}	φ_{21}	φ_{22}	η_{10}	η_{11}
LCSR–1	0.633	0.027	0.36	−0.95	0	0	0	0	0.94
LCSR–2	0.633	0.027	0.37	−0.96	0	0	0	−0.29	0.23

$$A_1^u = \varphi_{10} + \varphi_{11}, \qquad V_1^d = \frac{1}{3} - \varphi_{10} + \frac{1}{3}\varphi_{11}, \qquad f_1^u = \frac{1}{10} - \frac{1}{6}\frac{f_{N^*}}{\lambda_1^{N^*}} - \frac{3}{5}\eta_{10} - \frac{1}{3}\eta_{11},$$

$$f_1^d = \frac{3}{10} - \frac{1}{6}\frac{f_{N^*}}{\lambda_1^{N^*}} + \frac{1}{5}\eta_{10} - \frac{1}{3}\eta_{11}, \qquad f_2^d = \frac{4}{15} + \frac{2}{5}\xi_{10}.$$

The numerical values of the parameters φ_{10}, φ_{11}, φ_{20}, φ_{21}, φ_{22}, η_{10}, η_{11}, and $f_{N^*}/\lambda_1^{N^*}$, and $\lambda_1^{N^*}/\lambda_1^N$ are presented in Table II (this Table is taken from [19]; see also [22]).

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