New approach to the Sivers effect in the collinear twist-3 formalism

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The single-transverse spin asymmetry for hadron production in transversely polarized proton scattering receives a major contribution from the Sivers effect, which can be systematically described within the collinear twist-3 factorization framework in various processes. The conventional method in the evaluation of the Sivers effect known as pole calculation is technically quite different from nonpole calculation, which is another method used in evaluating the final state twist-3 effect. In this paper, we extend the nonpole technique to the Sivers effect and show consistency with the conventional method through an explicit calculation of the $\mathcal{O}(\alpha_s)$ correction in semi-inclusive deep inelastic scattering. As a result, we clarify that the conventional pole calculation is implicitly using the equation of motion and the Lorentz invariant relations whose importance became widely known in the nonpole calculation. We also clarify some technical advantages in using the new nonpole method.

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I. INTRODUCTION

The origin of the single transverse-spin asymmetries (SSAs) in high-energy hadron scatterings has been a long-standing mystery for over 40 years since strikingly large asymmetries were observed in the mid-1970s [1,2]. The Relativistic Heavy Ion Collider experiment has provided many data of the SSAs for various hadron productions in the past decade [3–7] and motivated a lot of theoretical work on the development of the perturbative QCD framework. Much theoretical effort has been devoted to develop a reliable QCD-based theory in order to deal with the given experimental data. The twist-3 framework in the collinear factorization approach was established as a rigorous framework which can provide a systematic description of the large SSAs.

It is commonly known that there are two major effects which lead to the large SSAs observed in the experiment, i.e., the initial state Sivers effect and final state Collins effect. The Sivers effect is essentially a twist-3 contribution generated from a transversely polarized hadron in the initial state. Starting from the pioneering work by Efremov and Teryaev [8], more systematic techniques were developed in a series of studies around 2000 [9–12]. A solid theoretical foundation for the calculation of the Sivers effect was finally laid in Ref. [12]. We will show the calculation technique in detail in the next section, and here we just give a brief introduction. The Sivers effect of the transversely polarized proton can be expressed by the dynamical twist-3 function defined by a Fourier transformed proton matrix element $T_{q,F} \simeq \mathcal{F}.T.\langle pS_\perp|\bar{\psi}gF^{+-}\psi|pS_\perp\rangle$, and the cross section in deep inelastic scattering (DIS) can be derived as

$$d\sigma = iT_{q,F} \otimes D \otimes d\hat{\sigma}, \tag{1}$$

where D represents the usual twist-2 fragmentation function and $d\hat{\sigma}$ is a hard partonic cross section. Because all the nonperturbative functions are real in this equation, the partonic cross section has to give an imaginary contribution in order to cancel i in the coefficient. This imaginary contribution can be given by the pole part of a propagator in the partonic scattering. In the quantum field theory, the propagator is defined by the time-ordered product of two fields, and it has an $i\epsilon$ term in the denominator. The imaginary contribution can emerge from a residue of contour integration. This is a basic mechanism of the pole calculation for the Sivers-type contribution. Next, we turn to the twist-3 fragmentation effect of a spin-0 hadron, which is known as the Collins effect. The cross section formula for the twist-3 fragmentation contribution was completed in a pp collision [13] and DIS [14] in a formal way. The dynamical twist-3 fragmentation function can be defined as $\hat{D}_{q,F} \simeq \mathcal{F}.T.\langle 0|gF^{+-}\psi|hX\rangle\langle hX|\bar{\psi}|0\rangle$, and the cross section in DIS is expressed by the same form as Eq. (1), just replacing $T_{q,F}$ with $\hat{D}_{q,F}$ and D with the usual twist-2 parton distribution function, respectively. The main

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difference is that the fragmentation function $\hat{D}_{q,F}$ is complex, and, therefore, it gives the imaginary contribution. The pole contribution from the hard cross section is no longer needed. This is a mechanism of the nonpole contribution from the Collins effect. The origin of generating the imaginary phase results in the main technical difference between the calculations for the Sivers and the Collins effects. The cross section for the pole contribution depends only on the dynamical function, while the result for the nonpole contribution is expressed in terms of three types of nonperturbative functions: the dynamical, intrinsic, and kinematical functions. In general, the hard cross sections in the nonpole calculation are not gauge and Lorentz invariant; because they are not physical observables, only their sum leads to a physical result as measured by experiment. This problem is solved by using two types of the relations among the nonperturbative functions which are called the equation of motion relation and Lorentz invariant relation [15].

As discussed above, the calculations for the pole contribution and the nonpole contribution are technically different from each other. Although the calculation techniques for those contributions are important basics of the higher twist calculation, not so many theorists are familiar with both of them because of the technical differences. In this paper, we revisit the result of the pole calculation from the viewpoint of the nonpole calculation in order to understand two calculations in a unified way. We show that the nonpole calculation has several technical advantages and, thus, should be extended to more complicated calculations like next-to-leading-order calculation and twist-4 calculation.

The remainder of the paper is organized as follows: In Sec. II, we introduce the notation and review the conventional pole calculation in detail. In Sec. III, we show the nonpole calculation method for the twist-3 contribution in order to reexamine the pole contributions. Finally, in Sec. IV, we summarize the achievements in this paper and make some comments on possible applications of the new nonpole method.

II. CONVENTIONAL POLE CALCULATION AT TWIST-3

The conventional collinear expansion framework at twist 3 has been developed in Refs. [8–17]. We review here the pole calculation for semi-inclusive deep inelastic scattering (SIDIS) in order to clarify the difference from the new method of nonpole calculation that we will propose in the next section. SIDIS is a suitable process to check the consistency between the two methods, because the twist-3 cross section has been already completed in Ref. [12] based on the conventional method. In addition, SIDIS is a relatively easier process than pp collision because of some technical issue.

We consider the process of polarized SIDIS

$$e(l) + p^{\uparrow}(p, S_{\perp}) \to e(l') + h(P_h) + X, \tag{2}$$

where the initial proton is transversely polarized. l and l' are, respectively, the momenta of the incoming and outgoing electrons. p and $S_{\perp}$ are the momentum and the transverse spin of the beam proton, respectively, and P_h is the momentum of the final state hadron. In this paper, we focus on the one-photon exchange process with the photon invariant mass $q^2 = (l - l')^2 = -Q^2$, and the extension to a charged current interaction is straightforward. The polarized cross section for SIDIS is given by

$$\frac{d^4\Delta\sigma}{dx_B dy dz_h dP_{h\perp}} = \frac{\alpha_{em}^2}{32\pi^2 z_h x_B^2 S_{ep}^2 Q^2} L^{\mu\nu} W_{\mu\nu}, \tag{3}$$

where the standard Lorentz invariant variables in SIDIS are defined as

$$S_{ep} = (p + l)^2, \qquad x_B = \frac{Q^2}{2p\cdot q},$$
$$z_h = \frac{p\cdot P_h}{p\cdot q}, \qquad y = \frac{p\cdot q}{p\cdot l}. \tag{4}$$

The leptonic tensor is defined as follows:

$$L^{\mu\nu} = 2\left(l^{\mu}l'^{\nu} + l^{\nu}l'^{\mu} - \frac{Q^2}{2}g^{\mu\nu}\right). \tag{5}$$

In order to simplify the discussion, we will mainly consider the metric part $L^{\mu\nu} \to -Q^2 g^{\mu\nu}$. We will discuss the result with the full leptonic tensor (5) in the end of Sec. III. The SSA in SIDIS can be generated by both the initial state and final state twist-3 contributions. In this paper, we focus on the contribution from initial state twist-3 distribution functions of the transversely polarized proton, and then the polarized differential cross section can be written as

$$\frac{d^4\Delta\sigma}{dx_B dy dz_h dP_{h\perp}} = \frac{\alpha_{em}^2}{32\pi^2 z_h x_B^2 S_{ep}^2 Q^2} \sum_i \int \frac{dz}{z^2} W_i D_{i\to h}(z), \tag{6}$$

where $D_{i\to h}(z)$ is the twist-2 unpolarized fragmentation function. The hadronic part W_i describes a scattering of the virtual photon on the transversely polarized proton, with the leptonic metric part contracted. We will make the subscript i implicit in the rest part of this paper for simplicity.

In the conventional pole calculation, one needs to consider diagrams as shown in Fig. 1, in which the hadronic part reads

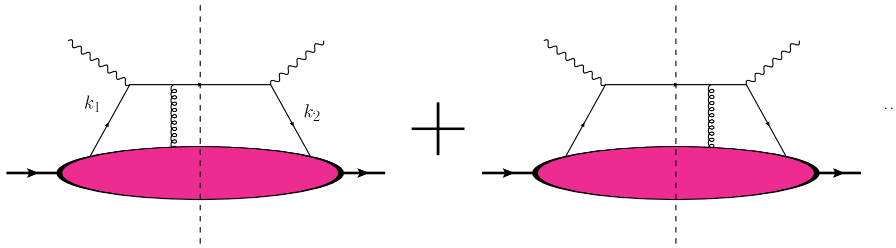

FIG. 1. Diagrammatic description of Eq. (7).

$$W_{\rm pole} = \int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\int d^4y_1 \times \int d^4y_2 e^{ik_1\cdot y_1}e^{i(k_2-k_1)\cdot y_2} \times \langle pS_\perp|\bar\psi_j(0)gA^\alpha(y_2)\psi_i(y_1)|pS_\perp\rangle H^{\rm pole}_{ji,\alpha}(k_1,k_2). \tag{7}$$

The twist-3 contribution is generated by the pole term, which comes from the imaginary part of the quark propagator

$$\frac{1}{k^2+i\epsilon} = P\left(\frac{1}{k^2}\right) - i\pi\delta(k^2); \tag{8}$$

thus, only $-i\pi\delta(k^2)$ is considered in the conventional pole method. We perform collinear expansion $k_i \to x_i p$ for the hard part $H^{\rm pole}_{ji,\alpha}(k_1,k_2)$. There are three types of pole contributions at the leading order with respect to QCD coupling constant α_s: soft-gluon-pole (SGP, $x_2-x_1=0$), soft-fermion-pole (SFP, $x_1=0$ or $x_2=0$, $x_1\neq x_2$), and hard-pole (HP, $x_1=x_B, x_2\neq x_B$ or $x_2=x_B, x_1\neq x_B$) contributions. Full diagrams for each pole contribution are shown in Figs. 2(a)–2(c). It is known that there is another type of contribution given by diagrams with two quark lines in the same side of the cut [18]. This contribution is relatively easier to calculate and, thus, will not be discussed in this paper.

One can factor out the δ functions in $H^{\rm pole}_{ji,\alpha}(k_1,k_2)$ for the three pole contributions:

$$\begin{aligned} H^{\rm pole}_{ji,\alpha}(k_1,k_2) &= H^{\rm SGP}_{Lji,\alpha}(k_1,k_2)\{-i\pi\delta[(p_c-(k_2-k_1))^2]\}(2\pi)\delta[(k_2+q-p_c)^2] \\ &+ H^{\rm SFP}_{Lji,\alpha}(k_1,k_2)\{-i\pi\delta[(p_c-k_2+k_1-q)^2]\}(2\pi)\delta[(k_2+q-p_c)^2] \\ &+ H^{\rm HP}_{Lji,\alpha}(k_1,k_2)\{-i\pi\delta[(k_1+q)^2]\}(2\pi)\delta[(k_2+q-p_c)^2] + (\text{complex conjugate diagrams}), \end{aligned} \tag{9}$$

where the factor $(2\pi)\delta[(k_2+q-p_c)^2]$ represents the on-shell condition of the unobserved parton and p_c is the four-momentum of the final state fragmenting parton.

A systematic way to calculate the pole contributions was developed in Ref. [12]. We confirmed that Ward-Takahashi identity (WTI) shown in Ref. [12] is valid for the diagrams in Figs. 2(a)–2(c) as

$$(k_2-k_1)^\alpha H^{\rm pole}_{ji,\alpha}(k_1,k_2) = 0. \tag{10}$$

Considering k_1 and k_2 derivatives, we can derive relations

$$\begin{aligned} (x_2-x_1)\frac{\partial}{\partial k_1^\beta}H^{\rm pole}_{ji,p}(k_1,k_2)\Big|_{k_i=x_ip} &= H^{\rm pole}_{ji,\beta}(x_1p,x_2p), \\ (x_2-x_1)\frac{\partial}{\partial k_2^\beta}H^{\rm pole}_{ji,p}(k_1,k_2)\Big|_{k_i=x_ip} &= -H^{\rm pole}_{ji,\beta}(x_1p,x_2p), \end{aligned} \tag{11}$$

where $H^{\rm pole}_{ji,p}(k_1,k_2) = p^\beta H^{\rm pole}_{ji,\beta}(k_1,k_2)$. Thus, we can derive the following useful relations for SFP and HP:

$$\begin{aligned} \frac{\partial}{\partial k_1^\beta}H^{\rm SFP(HP)}_{ji,p}(k_1,k_2)\Big|_{k_i=x_ip} &= \frac{1}{x_2-x_1}H^{\rm SFP(HP)}_{ji,\beta}(x_1p,x_2p), \\ \frac{\partial}{\partial k_2^\beta}H^{\rm SFP(HP)}_{ji,p}(k_1,k_2)\Big|_{k_i=x_ip} &= -\frac{1}{x_2-x_1}H^{\rm SFP(HP)}_{ji,\beta}(x_1p,x_2p), \end{aligned} \tag{12}$$

which lead to

$$-\frac{\partial}{\partial k_1^\beta}H^{\rm SFP(HP)}_{ji,p}(k_1,k_2)\Big|_{k_i=x_ip} = \frac{\partial}{\partial k_2^\beta}H^{\rm SFP(HP)}_{ji,p}(k_1,k_2)\Big|_{k_i=x_ip}. \tag{13}$$

However, we cannot derive the same relation with Eqs. (12) and (13) for the SGP diagrams directly from WTI, because $H^{\rm SGP}_{Lji,\alpha}(x_1p,x_2p)$ contains $\delta(x_2-x_1)$. So far, the only way to derive the relations is to calculate all the relevant diagrams explicitly, which is annoying in higher-order perturbative QCD calculations. In SIDIS at $\mathcal{O}(\alpha_s)$, the authors of Ref. [12] have checked explicitly that the above relation also holds true for $H^{\rm SGP}_{ji,p}$:

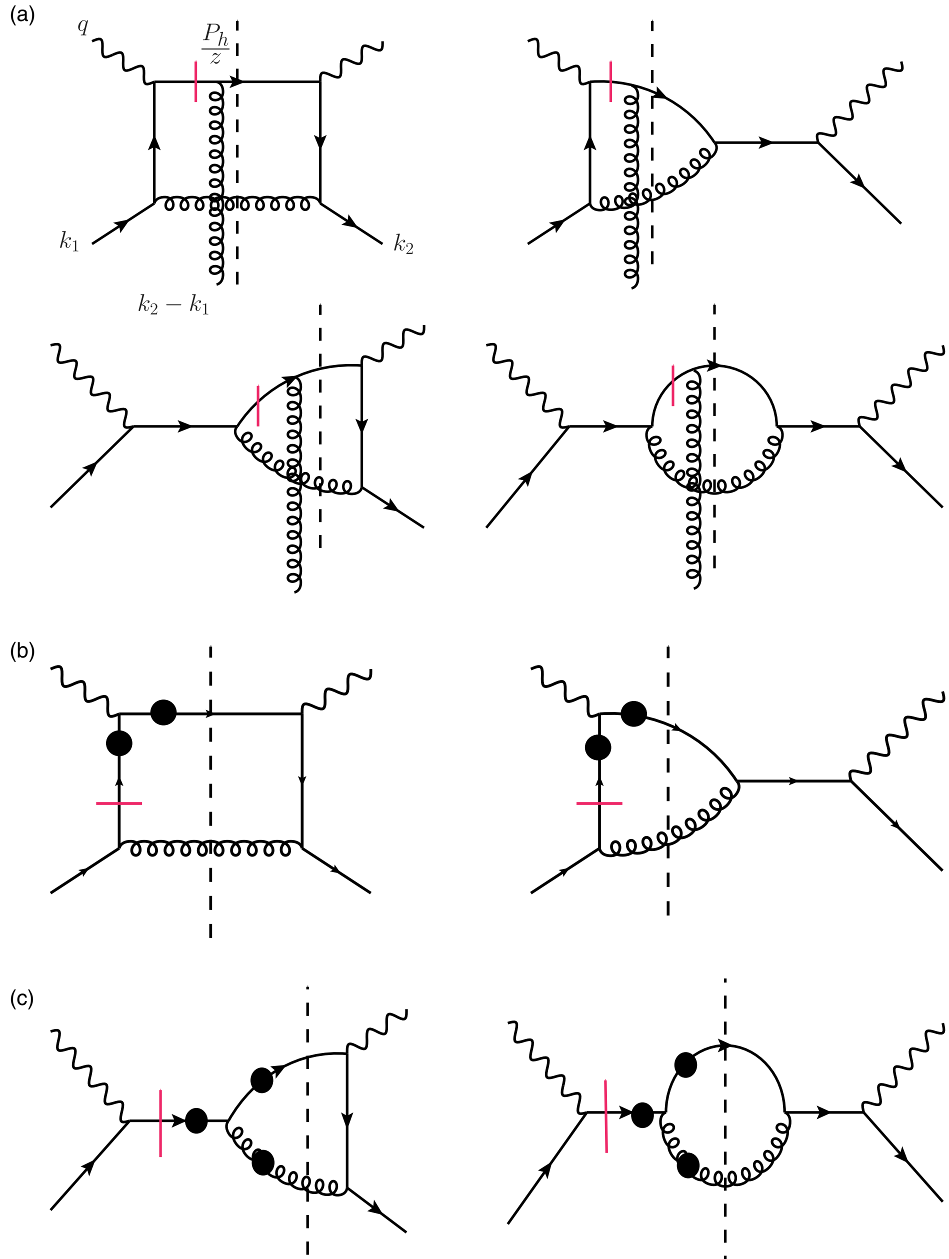

FIG. 2. (a) The diagrams for the SGP contribution. The separation of the pole (8) is carried out for the red barred propagators. The complex conjugate diagrams also need to be considered. (b) The diagrams for the SFP contribution. The gluon line with momentum $k_2 - k_1$ attaches to each black dot. (c) The diagrams for the HP contribution.

$$-\frac{\partial}{\partial k_1^\beta} H_{ji,p}^{\mathrm{SGP}}(k_1, k_2)\Big|_{k_i = x_i p} = \frac{\partial}{\partial k_2^\beta} H_{ji,p}^{\mathrm{SGP}}(k_1, k_2)\Big|_{k_i = x_i p}. \tag{14}$$

Now we can perform collinear expansion of the hard part:

$$\begin{aligned} H_{ji,\rho}^{\mathrm{pole}}(k_1, k_2) &\simeq H_{ji,\rho}^{\mathrm{pole}}(x_1 p, x_2 p) + \frac{\partial}{\partial k_1^\alpha} H_{ji,\rho}^{\mathrm{pole}}(k_1, k_2)\Big|_{k_i = x_i p} \omega_\beta^\alpha k_1^\beta + \frac{\partial}{\partial k_2^\alpha} H_{ji,\rho}^{\mathrm{pole}}(k_1, k_2)\Big|_{k_i = x_i p} \omega_\beta^\alpha k_2^\beta \\ &= H_{ji,\rho}^{\mathrm{pole}}(x_1 p, x_2 p) + \frac{\partial}{\partial k_2^\alpha} H_{ji,\rho}^{\mathrm{pole}}(k_1, k_2)\Big|_{k_i = x_i p} \omega_\beta^\alpha (k_2 - k_1)^\beta, \end{aligned} \tag{15}$$

where the projection tensor is defined as $\omega_\beta^\alpha = g_\beta^\alpha - \bar{n}^\alpha n_\beta$ with the unit vectors taken as $\bar{n} = [1, 0, 0]$ and $n = [0, 1, 0]$. We work in the hadron frame, and $p^\mu = p^+ \bar{n}^\mu$. We can neglect the k^- component at twist-3 accuracy and identify $\omega_\beta^\alpha k_i^\beta \simeq k_{i\perp}^\alpha$.

We use the same identification for all vectors projected by ω^α_β below. The next step is to decompose the components of the gluon field A^α into longitudinal and transverse as

$$A^\alpha \simeq \frac{A^n}{p^+} p^\alpha + A^\alpha_\perp, \tag{16}$$

where $A^n = A \cdot n$ and we neglected the A^- component. Substituting Eqs. (15) and (16) into Eq. (7), we can extract the twist-3 contribution:

$$\begin{aligned} W_{\rm pole} =& \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \int d^4y_1 \int d^4y_2 e^{ik_1\cdot y_1} e^{i(k_2-k_1)\cdot y_2} \langle pS_\perp|\bar\psi_j(0) gA^n(y_2)\psi_i(y_1)|pS_\perp\rangle \\ &\times \frac{1}{p^+}\frac{\partial}{\partial k_2^\alpha} H^{\rm pole}_{ji,p}(k_1,k_2)\Big|_{k_i=(k_i\cdot n)p} (k_{2\perp}-k_{1\perp})^\alpha \\ &+ \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \int d^4y_1 \int d^4y_2 e^{ik_1\cdot y_1} e^{i(k_2-k_1)\cdot y_2} \langle pS_\perp|\bar\psi_j(0) gA^\alpha_\perp(y_2)\psi_i(y_1)|pS_\perp\rangle H^{\rm pole}_{ji,\alpha}(x_1p,x_2p) \\ =& \, ip^+ \int dx_1 \int dx_2 \int \frac{dy_1^-}{2\pi} \int \frac{dy_2^-}{2\pi} e^{ix_1p^+y_1^-} e^{i(x_2-x_1)p^+y_2^-} \langle pS_\perp|\bar\psi_j(0) g[\partial^\alpha_\perp A^n(y_2^-) - \partial^n A^\alpha_\perp(y_2^-)]\psi_i(y_1^-)|pS_\perp\rangle \\ &\times \frac{\partial}{\partial k_2^\alpha} H^{\rm pole}_{ji,p}(k_1,k_2)\Big|_{k_i=x_ip} + (p^+)^2 \int dx_1 \int dx_2 \int \frac{dy_1^-}{2\pi} \int \frac{dy_2^-}{2\pi} e^{ix_1p^+y_1^-} e^{i(x_2-x_1)p^+y_2^-} \langle pS_\perp|\bar\psi_j(0) gA^\alpha_\perp(y_2^-)\psi_i(y_1^-)|pS_\perp\rangle \\ &\times [(x_2-x_1)\frac{\partial}{\partial k_2^\alpha} H^{\rm pole}_{ji,p}(k_1,k_2)\Big|_{k_i=x_ip} + H^{\rm pole}_{ji,\alpha}(x_1p,x_2p)]. \end{aligned} \tag{17}$$

The last term in Eq. (17) can be eliminated by using the relation Eq. (11), and the first term can be rewritten as

$$W_{\rm pole} = i\int dx_1 \int dx_2 {\rm Tr}\left[M^\alpha_F(x_1,x_2) \frac{\partial}{\partial k_2^\alpha} H^{\rm pole}_p(k_1,k_2)\Big|_{k_i=x_ip}\right], \tag{18}$$

where $M^\alpha_F(x_1,x_2)$ is the F-type dynamical function which can be further expanded as

$$M^\alpha_{ij,F}(x_1,x_2) = p^+ \int \frac{dy_1^-}{2\pi} \int \frac{dy_2^-}{2\pi} e^{ix_1p^+y_1^-} e^{i(x_2-x_1)p^+y_2^-} \langle pS_\perp|\bar\psi_j(0) gF^{\alpha n}(y_2^-)\psi_i(y_1^-)|pS_\perp\rangle = -\frac{M_N}{2}\epsilon^{\alpha\bar n n S_\perp}(\not p)_{ij} T_{q,F}(x_1,x_2) + \cdots, \tag{19}$$

with the nucleon mass M_N and the field strength tensor defined as $F^{\alpha n}(y_2^-) = \partial^\alpha_\perp A^n(y_2^-) - \partial^n A^\alpha_\perp(y_2^-)$; notice that the nonlinear gluon term in the field strength tensor has been omitted, because it comes from Feynman diagrams with linked gluons more than one and, therefore, does not show in Eq. (18). $T_{q,F}(x_1,x_2)$ is the well-known Qiu-Sterman function, defined as the following:[1]

$$T_{q,F}(x_1,x_2) = \left(\frac{g}{2\pi M_N}\right) \int \frac{dy_1^- dy_2^-}{4\pi} e^{ix_1p^+y_1^-} e^{i(x_2-x_1)p^+y_2^-} \langle pS_\perp|\bar\psi(0)\not n \epsilon^{\alpha\bar n n S_\perp} F^n_\alpha(y_2^-)\psi(y_1^-)|pS_\perp\rangle. \tag{20}$$

Using Eq. (12) for SFP and HP, one can evaluate the derivative of the hard part with respect to k_2 in Eq. (18). For SGP, we need to rely on the master formula [19]

$$\frac{\partial}{\partial k_2^\alpha} H^{\rm SGP}_{ji,p}(k_1,k_2)\Big|_{k_i=x_ip} = \frac{1}{2NC_F}[i\pi\delta(x_2-x_1)]\left(\frac{\partial}{\partial p_c^\alpha} - \frac{p_{c\alpha}p^\mu}{p_c\cdot p}\frac{\partial}{\partial p_c^\mu}\right) H_{ji}(x_1p), \tag{21}$$

where $H_{ji}(x_1p)$ is the usual $2\to 2$ $\gamma^* q \to qg$ scattering cross section without the extra gluon line attached. Combining the three pole contributions together, we obtain the final result based on the conventional pole calculation:

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[1]We rescaled the function as $T_{q,F}(x_1,x_2) \to (g/2\pi M_N)T_{q,F}(x_1,x_2)$ from the original definition in Ref. [9] for convenience. Our definition of $T_{q,F}(x_1,x_2)$ is the same with $F^q_{FT}(x_2,x_1)$ in Ref. [15].

$$\frac{d^4\Delta\sigma}{dx_B dy dz_h dP_{h\perp}} = \frac{\pi M_N \alpha_{em}^2 \alpha_s}{8 z_h x_B^2 S_{ep}^2 Q^2} \sum_q e_q^2 \int \frac{dz}{z^2} D_{q\to h}(z) \int \frac{dx}{x} \delta[(xp+q-p_c)^2]((\hat{s}+Q^2)\epsilon^{p_c \bar{n} n S_\perp} + \hat{t}\epsilon^{q\bar{n}nS_\perp})$$
$$\times \left[x\frac{d}{dx} T_{q,F}(x,x)\hat{\sigma}_D + T_{q,F}(x,x)\hat{\sigma}_{ND} + T_{q,F}(0,x)\hat{\sigma}_{\rm SFP} + T_{q,F}(x_B,x)\hat{\sigma}_{\rm HP} \right], \tag{22}$$

where all hard cross sections are listed below:

$$\begin{aligned}
\hat{\sigma}_D &= \frac{1}{2N}\frac{16Q^2[(\hat{s}+\hat{t})^2+(\hat{t}+\hat{u})^2]}{\hat{s}\,\hat{t}\,\hat{u}^2},\\
\hat{\sigma}_{ND} &= \frac{1}{2N}\frac{16Q^2[2Q^6+\hat{t}^3-4Q^2\hat{s}\,\hat{u}+3\hat{t}^2\hat{u}+4\hat{t}\hat{u}^2+2\hat{u}^3+Q^4(3\hat{t}+\hat{u})]}{\hat{s}^2\hat{t}\hat{u}^2},\\
\hat{\sigma}_{\rm SFP} &= -\frac{1}{2N}\frac{16Q^2[2Q^4+\hat{u}^2+Q^2(\hat{t}+3\hat{u})]}{\hat{s}\,\hat{t}\,\hat{u}^2},\\
\hat{\sigma}_{\rm HP} &= \left(\frac{1}{2N}\frac{1}{\hat{t}} - C_F\frac{1}{\hat{s}+Q^2}\right)\frac{16Q^2[Q^6+3Q^4\hat{s}+\hat{s}^3+Q^2(3\hat{s}^2+\hat{t}^2)]}{\hat{s}^2\hat{u}^2},
\end{aligned} \tag{23}$$

with the standard Mandelstam variables defined as

$$\hat{s} = (xp+q)^2, \qquad \hat{t} = (xp-p_c)^2, \qquad \hat{u} = (q-p_c)^2. \tag{24}$$

In the next section, we show that the new nonpole method can reproduce these hard cross sections. We would like to make a comment on the relation Eq. (14) for the SGP diagrams; this relation is required to construct the gauge-invariant matrix for the dynamical function. However, there is no simple way to prove this relation. We have to check if it is correct diagram by diagram. This is a frustrating point of the conventional pole calculation. We will show that the new method can avoid such complexity and, thus, is a more flexible calculation technique.

III. THE NEW NONPOLE CALCULATION FOR SIVERS EFFECT

We introduce the new nonpole calculation method in this section. The main difference between the pole and the nonpole methods is on the decomposition of the propagator shown in Eq. (8). In the new method, we directly perform the contour integrations and never carry out the decomposition for any propagators. The new method is expected to remove the mathematical complexity that lies in the validity of the decomposition. In this sense, the new method can be regarded as a more flexible approach and can be easily extended to more complicated cases. Removal of the workload on Eq. (14) is one of the important consequences.

A. General formalism

In the new method we propose here, the hadronic part should be written as a sum of all the diagrams, i.e., $W=\sum_i W^{(i)}$, where i denotes the number of gluon attachment. Let us start with the diagram in Fig. 3 without any gluon attached; the hadronic part is given by

$$W^{(0)} = \int \frac{d^4k}{(2\pi)^4} \int d^4y_1 e^{ik\cdot y_1} \langle pS_\perp|\bar{\psi}_j(0)\psi_i(y_1)|pS_\perp\rangle H_{ji}(k). \tag{25}$$

The twist-3 contribution from the diagrams without a gluon attached can be obtained by performing collinear expansion of the hard part:

$$H_{ji}(k) \simeq H_{ji}(xp) + \frac{\partial}{\partial k^\alpha} H_{ji}(k)\Big|_{k=xp} k_\perp^\alpha. \tag{26}$$

Then Eq. (25) can be expanded into two parts:

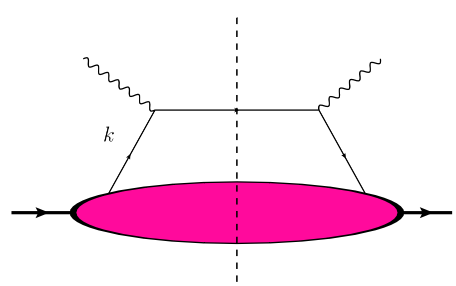

FIG. 3. Diagrammatic description of Eq. (25).

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\begin{aligned}
W^{(0)} &= \int \frac{d^4k}{(2\pi)^4} \int d^4y_1 e^{ik\cdot y_1} \langle pS_\perp | \bar{\psi}_j(0)\psi_i(y_1)|pS_\perp\rangle \left[H_{ji}(xp) + \frac{\partial}{\partial k^\alpha} H_{ji}(k)|_{k=xp} k_\perp^\alpha \right] \\
&= p^+ \int dx \int \frac{dy_1^-}{2\pi} e^{ixp^+y_1^-} \langle pS_\perp | \bar{\psi}_j(0)\psi_i(y_1^-)|pS_\perp\rangle H_{ji}(xp) \\
&\quad + ip^+ \int dx \int \frac{dy_1^-}{2\pi} e^{ixp^+y_1^-} \langle pS_\perp | \bar{\psi}_j(0)\partial_\perp^\alpha \psi_i(y_1^-)|pS_\perp\rangle \frac{\partial}{\partial k^\alpha} H_{ji}(k)|_{k=xp}.
\end{aligned} \tag{27}
$$

In general, the first term could give the twist-3 contribution when the hard part gives transverse component $H_{ji}(xp) \sim (\gamma^\perp)_{ji}$. Next, we consider the diagrams with one gluon attachment as shown in Fig. 1 which were also considered in the conventional method. Here, we need to consider a set of diagrams $H_{ji,\alpha}(k_1,k_2)$ shown in Fig. 4 and their complex conjugate. We call them nonpole diagrams because we do not separate the pole term for any propagators. The nonpole contribution to the hadronic part reads

$$
W^{(1)} = \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \int d^4y_1 \int d^4y_2 e^{ik_1\cdot y_1} e^{i(k_2-k_1)\cdot y_2} \langle pS_\perp | \bar{\psi}_j(0) gA^\alpha(y_2)\psi_i(y_1)|pS_\perp\rangle H_{ji,\alpha}(k_1,k_2). \tag{28}
$$

Similar to the strategy in dealing with the diagrams without a gluon attachment, the first step to extract the twist-3 contribution from one gluon attached diagrams is to perform collinear expansion of the hard part:

$$
H_{ji,\rho}(k_1,k_2) = H_{ji,\rho}(x_1p, x_2p) + \frac{\partial H_{ji,\rho}(k_1,k_2)}{\partial k_1^\alpha}\Big|_{k_i=x_ip} k_{1\perp}^\alpha + \frac{\partial H_{ji,\rho}(k_1,k_2)}{\partial k_2^\alpha}\Big|_{k_i=x_ip} k_{2\perp}^\alpha. \tag{29}
$$

One also needs to decompose the gluon field A^α into longitudinal and transverse components as in Eq. (16). Then Eq. (28) can be expanded as follows:

$$
\begin{aligned}
W^{(1)} &= \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \int d^4y_1 \int d^4y_2 e^{ik_1\cdot y_1} e^{i(k_2-k_1)\cdot y_2} \langle pS_\perp | \bar{\psi}_j(0) gA^n(y_2)\psi_i(y_1)|pS_\perp\rangle \\
&\quad \times \frac{1}{p^+} \left[H_{ji,p}(x_1p,x_2p) + \frac{\partial}{\partial k_1^\alpha} H_{ji,p}(k_1,k_2)\Big|_{k_i=x_ip} k_{1\perp}^\alpha + \frac{\partial}{\partial k_2^\alpha} H_{ji,p}(k_1,k_2)\Big|_{k_i=x_ip} k_{2\perp}^\alpha \right] \\
&\quad + \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \int d^4y_1 \int d^4y_2 e^{ik_1\cdot y_1} e^{i(k_2-k_1)\cdot y_2} \langle pS_\perp | \bar{\psi}_j(0) gA_\perp^\alpha(y_2)\psi_i(y_1)|pS_\perp\rangle H_{ji,\alpha}(x_1p,x_2p);
\end{aligned} \tag{30}
$$

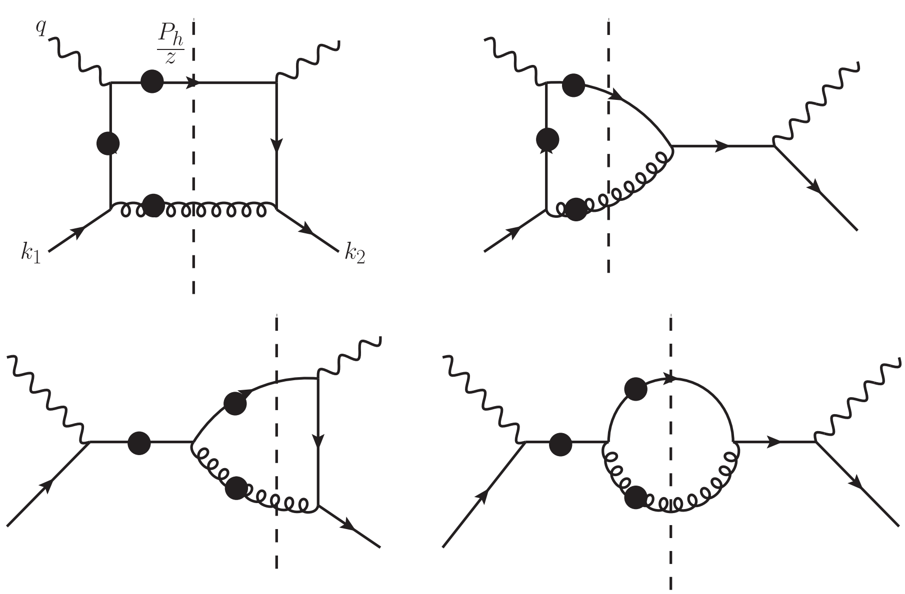

FIG. 4. The diagrams for $H_{ji,\alpha}(k_1,k_2)$. The gluon line with momentum $k_2 - k_1$ attaches to each black dot.

notice that other terms in the combination of Eqs. (16) and (29) contribute to higher twist. The hard part shown in the above equation can be further simplified by using the WTI relations. It is straightforward to derive the counterpart of Eq. (10) for the nonpole diagrams [14]:

$$(k_2 - k_1)^\alpha H_{ji,\alpha}(k_1, k_2) = H_{ji}(k_2) - H_{ji}(k_1). \tag{31}$$

The nonpole hard part does not have the delta function $\delta(x_2 - x_1)$; therefore, we can derive the following useful relations:

$$\begin{aligned}
H_{ji,p}(x_1 p, x_2 p) &= \frac{1}{x_2 - x_1 - i\epsilon}[H_{ji}(x_2 p) - H_{ji}(x_1 p)], \\
\frac{\partial}{\partial k_1^\beta} H_{ji,p}(k_1, k_2)\Big|_{k_i = x_i p} &= \frac{1}{x_2 - x_1 - i\epsilon}\left[H_{ji,\beta}(x_1 p, x_2 p) - \frac{\partial}{\partial k_1^\beta} H_{ji}(k_1)\Big|_{k_1 = x_1 p}\right], \\
\frac{\partial}{\partial k_2^\beta} H_{ji,p}(k_1, k_2)\Big|_{k_i = x_i p} &= -\frac{1}{x_2 - x_1 - i\epsilon}\left[H_{ji,\beta}(x_1 p, x_2 p) - \frac{\partial}{\partial k_2^\beta} H_{ji}(k_2)\Big|_{k_2 = x_2 p}\right],
\end{aligned} \tag{32}$$

where the sign of $i\epsilon$ was determined by the fact that only the final state interaction exists in SIDIS. If a process has both the initial and the final interactions as in the case of a pp collision, the rhs of Eq. (32) could have both $\pm i\epsilon$ and the sign cannot be uniquely determined from the WTI (31). We need more consideration on this point when we apply the same technique to a pp collision.

We would like to emphasize the validity of WTI to higher-order diagrams. The WTI is a consequence of the gauge invariance in QCD, and, therefore, we can use the same Eqs. (31) and (32) to higher-order diagrams as long as we use an appropriate regularization scheme like dimensional regularization. By using these useful relations derived from WTI, the hard part terms $H_{ji,\alpha}(k_1, k_2)$ and $H_{ji,p}(k_1, k_2)$ contained in Eq. (30) are, respectively, given by

$$\begin{aligned}
H_{ji,\alpha}(x_1 p, x_2 p) &= -(x_2 - x_1)\frac{\partial}{\partial k_2^\alpha} H_{ji,p}(k_1, k_2)\Big|_{k_i = x_i p} + \frac{\partial}{\partial k_2^\alpha} H_{ji}(k_2)\Big|_{k_2 = x_2 p} \\
&= (x_2 - x_1)\left[\frac{1}{x_2 - x_1 - i\epsilon} H_{ji,\alpha}(x_1 p, x_2 p) - \frac{1}{x_2 - x_1 - i\epsilon}\frac{\partial}{\partial k_2^\alpha} H_{ji}(k_2)\Big|_{k_2 = x_2 p}\right] + \frac{\partial}{\partial k_2^\alpha} H_{ji}(k_2)\Big|_{k_2 = x_2 p}, \\
H_{ji,p}(x_1 p, x_2 p) &+ \frac{\partial}{\partial k_2^\alpha} H_{ji,p}(k_1, k_2)\Big|_{k_i = x_i p} k_{2\perp}^\alpha + \frac{\partial}{\partial k_1^\alpha} H_{ji,p}(k_1, k_2)\Big|_{k_i = x_i p} k_{1\perp}^\alpha \\
&= \frac{1}{x_2 - x_1 - i\epsilon}[H_{ji}(x_2 p) - H_{ji}(x_1 p)] + \frac{\partial}{\partial k_2^\alpha} H_{ji,p}(k_1, k_2)\Big|_{k_i = x_i p}(k_{2\perp} - k_{1\perp})^\alpha \\
&\quad + \left(\frac{\partial}{\partial k_1^\alpha} H_{ji,p}(k_1, k_2)\Big|_{k_i = x_i p} + \frac{\partial}{\partial k_2^\alpha} H_{ji,p}(k_1, k_2)\Big|_{k_i = x_i p}\right) k_{1\perp}^\alpha \\
&= \frac{1}{x_2 - x_1 - i\epsilon}\left\{[H_{ji}(x_2 p) - H_{ji}(x_1 p)] - H_{ji,\alpha}(x_1 p, x_2 p)(k_{2\perp} - k_{1\perp})^\alpha + \frac{\partial}{\partial k_2^\alpha} H_{ji}(k_2)\Big|_{k_2 = x_2 p}(k_{2\perp} - k_{1\perp})^\alpha \right. \\
&\quad \left. + \left[\frac{\partial}{\partial k_2^\alpha} H_{ji}(k_2)\Big|_{k_2 = x_2 p} - \frac{\partial}{\partial k_1^\alpha} H_{ji}(k_1)\Big|_{k_1 = x_1 p}\right] k_{1\perp}^\alpha\right\}.
\end{aligned} \tag{33}$$

We iteratively used the third relation in Eq. (32) for the first equation. Substituting Eq. (33) into Eq. (30), we obtain the final result

$$
\begin{aligned}
W^{(1)} = {} & p^+ \int dx \int \frac{dy_1}{2\pi} e^{ixp^+y_1^-} \langle pS_\perp|\bar\psi_j(0)\left[ig\int_{y_1^-}^0 dy_2^- A^n(y_2^-)\right]\psi_i(y_1^-)|pS_\perp\rangle H_{ji}(xp) \\
& - ip^+ \int dx_1 \int dx_2 \int \frac{dy_1^-}{2\pi}\int \frac{dy_2^-}{2\pi} e^{ix_1p^+y_1^-} e^{i(x_2-x_1)p^+y_2^-} \langle pS_\perp|\bar\psi_j(0) gF^{\alpha n}(y_2^-)\psi_i(y_1^-)|pS_\perp\rangle \frac{H_{ji,\alpha}(x_1p,x_2p)}{x_2-x_1-i\epsilon} \\
& + ip^+ \int dx \int \frac{dy_1^-}{2\pi} e^{ixp^+y_1^-} \langle pS_\perp|\bar\psi_j(0)\left[ig\int_{y_1^-}^0 dy_2^- A^n(y_2^-)\right]\partial_\perp^\alpha\psi_i(y_1^-)|pS_\perp\rangle \frac{\partial}{\partial k^\alpha}H_{ji}(k)\Big|_{k=xp} \\
& + ip^+ \int dx \int \frac{dy_1^-}{2\pi} e^{ixp^+y_1^-} \langle pS_\perp|\bar\psi_j(0) ig\int_{y_1^-}^\infty dy_2^- F^{\alpha n}(y_2^-)\psi_i(y_1^-)|pS_\perp\rangle \frac{\partial}{\partial k^\alpha}H_{ji}(k)\Big|_{k=xp} \\
& + ip^+ \int dx \int \frac{dy_1^-}{2\pi} e^{ixp^+y_1^-} \langle pS_\perp|\bar\psi_j(0)[-igA_\perp^\alpha(y_1^-)]\psi_i(y_1^-)|pS_\perp\rangle \frac{\partial}{\partial k^\alpha}H_{ji}(k)\Big|_{k=xp}.
\end{aligned} \tag{34}
$$

Summing over all twist-3 contributions in the diagrams in Figs. 1 and 3, represented by Eqs. (27) and (34), respectively, we can construct the gauge-invariant expression

$$
W = \int dx \mathrm{Tr}[M(x)H(xp)] + i\int dx \mathrm{Tr}\left[M_\partial^\alpha(x)\frac{\partial}{\partial k^\alpha}H(k)\Big|_{k=xp}\right] - i\int dx_1\int dx_2 \frac{1}{x_2-x_1-i\epsilon}\mathrm{Tr}[M_F^\alpha(x_1,x_2)H_\alpha(x_1p,x_2p)], \tag{35}
$$

where the matrices are given by

$$
M_{ij}(x) = p^+\int \frac{dy_1^-}{2\pi} e^{ixp^+y_1^-}\langle pS_\perp|\bar\psi_j(0)\psi_i(y_1^-)|pS_\perp\rangle, \tag{36}
$$

$$
\begin{aligned}
M_{ij,\partial}^\alpha(x) &= p^+\int \frac{dy_1^-}{2\pi} e^{ixp^+y_1^-}\langle pS_\perp|\bar\psi_j(0)D_\perp^\alpha(y_1^-)\psi_i(y_1^-)|pS_\perp\rangle \\
&\quad + p^+\int \frac{dy_1^-}{2\pi} e^{ixp^+y_1^-}\langle pS_\perp|\bar\psi_j(0)ig\left[\int_{y_1^-}^\infty dy_2^- F^{\alpha n}(y_2^-)\right]\psi_i(y_1^-)|pS_\perp\rangle \\
&= -i\frac{M_N}{2}\epsilon^{\alpha\bar n n S_\perp}(\not p)_{ij} f_{1T}^{\perp(1)}(x) + \cdots,
\end{aligned} \tag{37}
$$

where the operator definition of $f_{1T}^{\perp(1)}(x)$ is

$$
f_{1T}^{\perp(1)}(x) = \left(\frac{-i}{2M_N}\right)\int \frac{dy_1^-}{2\pi} e^{ixp^+y_1^-}\langle pS_\perp|\bar\psi(0)\not n\epsilon^{\alpha\bar n n S_\perp}\left(D_{\perp\alpha}(y_1^-) + ig\left[\int_{y_1^-}^\infty dy_2^- F_\alpha^n(y_2^-)\right]\right)\psi(y_1^-)|pS_\perp\rangle. \tag{38}
$$

The definition of $M_F^\alpha(x_1,x_2)$ and its decomposition are already introduced in Eq. (19). In the present case, the first term in Eq. (35) cannot give a twist-3 contribution, because the spin projection $\gamma_\alpha\epsilon^{\alpha\bar n n S_\perp}$ is forbidden by PT invariance. Therefore, we can eliminate the first term in Eq. (35) and rewrite the twist-3 hadronic part as

$$
W = \frac{M_N}{2}\epsilon^{\alpha\bar n n S_\perp}\left\{\int dx f_{1T}^{\perp(1)}(x)\mathrm{Tr}\left[\not p\frac{\partial}{\partial k^\alpha}H(k)\Big|_{k=xp}\right] + i\int dx_1\int dx_2 T_{q,F}(x_1,x_2)\frac{1}{x_2-x_1-i\epsilon}\mathrm{Tr}[\not p H_\alpha(x_1p,x_2p)]\right\}. \tag{39}
$$

In the new method presented above, we needed only the well-defined relations Eq. (32) to construct the gauge-invariant matrix elements. We find that the difficulty associated with the relation Eq. (14) in the conventional calculation was removed. This is one of the advantages in the new method. Another advantage is that, by using Eq. (39) and the discussion in Appendix B, we do not need to calculate the derivative of the hard part over the momentum $k(k_i)$; this will significantly reduce the complexity of twist-3 calculation, in particular, for higher-order calculations.

B. SIDIS at $\mathcal{O}(\alpha_S)$

In this subsection, we show in detail the calculation of the hadronic part for SIDIS at $\mathcal{O}(\alpha_s)$. We factor out the on-shell δ function from the hard partonic part:

$$H(k) = \bar{H}(k)(2\pi)\delta[(k+q-p_c)^2], \tag{40}$$

$$H_\alpha(x_1p, x_2p) = \bar{H}^L_\alpha(x_1p, x_2p)(2\pi)\delta[(x_2p+q-p_c)^2] + \bar{H}^R_\alpha(x_1p, x_2p)(2\pi)\delta[(x_1p+q-p_c)^2], \tag{41}$$

where $\bar{H}^L_\alpha(x_1p, x_2p)$ is given by a sum of 12 diagrams in Fig. 4 and $\bar{H}^R_\alpha(x_1p, x_2p)$ is its complex conjugate. The derivative of $H(k)$ over k can be converted to that over the standard Mandelstam variables $\hat{s}$, $\hat{t}$, and $\hat{u}$. For details, see Appendix B. Then we can calculate Eq. (39) as

$$\begin{aligned} W = \pi M_N \int \frac{dx}{x}\delta[(xp+q-p_c)^2]\Bigg\{ & x\frac{df_{1T}^{\perp(1)}(x)}{dx}(\epsilon^{q\bar{n}nS_\perp} - \epsilon^{p_c\bar{n}nS_\perp})\frac{2}{\hat{u}}\hat{\sigma}(\hat{s},\hat{t},\hat{u}) + f_{1T}^{\perp(1)}(x)\Bigg[((\hat{s}+Q^2)\epsilon^{p_c\bar{n}nS_\perp} + \hat{t}\epsilon^{q\bar{n}nS_\perp}) \\ & \times\frac{2}{\hat{u}}\left(\frac{\partial}{\partial\hat{t}} - \frac{\partial}{\partial\hat{s}}\right)\hat{\sigma}(\hat{s},\hat{t},\hat{u}) - (\epsilon^{q\bar{n}nS_\perp} - \epsilon^{p_c\bar{n}nS_\perp})\frac{2}{\hat{u}}\hat{\sigma}(\hat{s},\hat{t},\hat{u}) - \epsilon^{\alpha\bar{n}nS_\perp}\mathrm{Tr}[\gamma_\alpha\bar{H}(xp)]\Bigg] \\ & + i\epsilon^{\alpha\bar{n}nS_\perp}\int dx' T_{q,F}(x',x)\left[\frac{1}{x-x'-i\epsilon}\mathrm{Tr}[x\not{p}\bar{H}^L_\alpha(x'p,xp)] - \frac{1}{x-x'+i\epsilon}\mathrm{Tr}[x\not{p}\bar{H}^R_\alpha(xp,x'p)]\right]\Bigg\}, \end{aligned} \tag{42}$$

where $\hat{\sigma}(\hat{s},\hat{t},\hat{u})$ is the $2\to 2$ partonic cross section in SIDIS. For the $q\gamma^*\to qg$ channel, it reads

$$\hat{\sigma}(\hat{s},\hat{t},\hat{u}) \equiv \mathrm{Tr}[x\not{p}\bar{H}(xp)] = -8C_FQ^2\frac{(\hat{s}+\hat{t})^2 + (\hat{t}+\hat{u})^2}{\hat{s}\,\hat{u}}. \tag{43}$$

Notice that, for convenience, we have changed the notation $x_1\to x', x_2\to x$ in $\bar{H}^L(x'p,xp)$ and $x_1\to x, x_2\to x'$ in $\bar{H}^R(x'p,xp)$ in order to factor out the common delta function $\delta[(xp+q-p_c)^2]$. We discuss the gauge and Lorentz invariances of the hard cross sections associated with $f_{1T}^{\perp(1)}(x)$. The hard cross section with the nonderivative function $f_{1T}^{\perp(1)}(x)$ is not apparently gauge invariant because of the term $\epsilon^{\alpha pnS_\perp}\mathrm{Tr}[\gamma_\alpha\bar{H}(xp)]$. The gauge invariance requires the unpolarized spin projection $x\not{p}$ with $\bar{H}(xp)$ like Eq. (43). On the other hand, the hard cross section associated with the derivative function $\frac{d}{dx}f_{1T}^{\perp(1)}(x)$ is not Lorentz invariant. The vector n in the parametrization (19) satisfies $\bar{n}\cdot n = 1$ and $n^2 = 0$. These conditions are not enough to uniquely determine the form of n, and there are two possible choices in SIDIS:

$$n^\alpha = \frac{p^+}{p\cdot p_c}p_c^\alpha \quad\text{or}\quad n^\alpha = \frac{p^+}{p\cdot p_c}p_c^\alpha + \frac{2p^+p_c\cdot q}{2(p_c\cdot q)(p\cdot q) + Q^2(p\cdot p_c)}\left(q^\alpha - \frac{p\cdot q}{p\cdot p_c}p_c^\alpha\right). \tag{44}$$

We can check that the coefficient $(\epsilon^{q\bar{n}nS_\perp} - \epsilon^{p_c\bar{n}nS_\perp})$ of $\frac{d}{dx}f_{1T}^{\perp(1)}(x)$ depends on the choice of n. This ambiguity of the cross section is physically interpreted as the frame dependence, because the spatial component of n is determined so that it has the opposite direction of the momentum p as $\vec{n} = -\vec{p}/p^+$. From the requirement of the frame independence, the cross section has to be proportional to the factor $[(\hat{s}+Q^2)\epsilon^{p_c\bar{n}nS_\perp} + \hat{t}\epsilon^{q\bar{n}nS_\perp}]$ as already shown in the cross section (22) derived by the conventional pole method. We will show later that the gauge and Lorenz invariances of the cross section are guaranteed by using the relations

$$f_{1T}^{\perp(1)}(x) = \pi T_{q,F}(x,x), \tag{45}$$

$$\frac{d}{dx}f_{1T}^{\perp(1)}(x) = \pi\frac{d}{dx}T_{q,F}(x,x), \tag{46}$$

which enable us to express the cross section only in terms of $T_{q,F}(x',x)$ as in the case of the conventional calculation. One can find the derivation of these relations in Appendix A.

Now we show how to calculate the hard partonic part $\bar{H}^L(x'p,xp)$. There are four types of x' dependence in the Feynman gauge. Figure 5 shows typical diagrams including x'-dependent propagators. Each propagator can be calculated as follows:

$$\begin{aligned} &\text{propagator (1)}: \frac{\not{p}_c-(x-x')\not{p}^2}{[p_c-(x-x')p]^2+i\epsilon} = -\frac{1}{\hat{t}}x\not{p} + \frac{x}{x-x'-i\epsilon}\frac{1}{\hat{t}}\not{p}_c, \\ &\text{propagator (2)}: \frac{\not{p}_c-(x-x')\not{p}-\not{q}}{[p_c-(x-x')p-q]^2+i\epsilon} = \frac{1}{\hat{u}}x\not{p} - \frac{x}{x'-i\epsilon}\frac{1}{\hat{u}}(x\not{p}+\not{q}-\not{p}_c), \\ &\text{propagator (3)}: \frac{x'\not{p}+\not{q}}{[x'p+q]^2+i\epsilon} = \frac{1}{\hat{s}+Q^2}x\not{p} + \frac{x}{x'-x_B+i\epsilon}\frac{1}{\hat{s}+Q^2}[x_B\not{p}+\not{q}], \\ &\text{propagator (4)}: \frac{V_{\alpha\rho\tau}((x-x')p, -xp-q+p_c, x'p+q-p_c)}{[x'p+q-p_c]^2+i\epsilon} = \frac{1}{\hat{u}}(xp_\tau g_{\alpha\rho} + xp_\alpha g_{\rho\tau} - 2xp_\rho g_{\alpha\tau}) \\ &\qquad + \frac{x}{x-x'-i\epsilon}\frac{1}{\hat{u}}[(xp+q-p_c)_\tau g_{\alpha\rho} - 2(xp+q-p_c)_\alpha g_{\rho\tau} + (xp+q-p_c)_\rho g_{\alpha\tau}], \end{aligned} \tag{47}$$

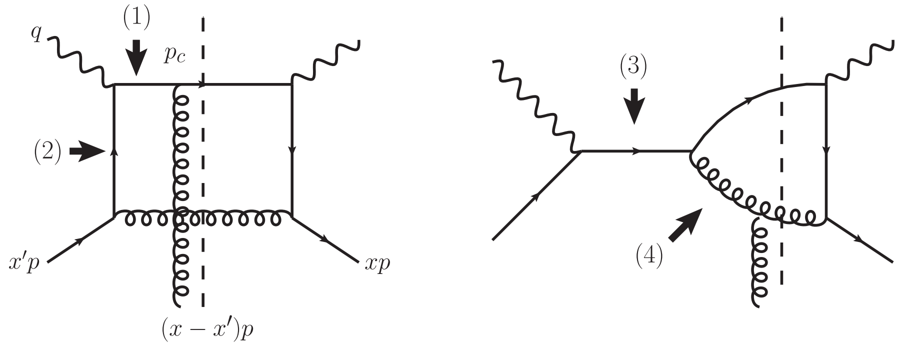

FIG. 5. Typical diagrams including x'-dependent propagators. Calculation for the propagators (1)–(4) are shown in Eq. (47).

where $V_{\alpha\rho\tau}$ comes from the three-gluon vertex. We can find that all x' dependences appear only in the denominators, $x-x'-i\epsilon$, $x'-i\epsilon$, and $x'-x_B+i\epsilon$. Products of two denominators can be disentangled as

$$\frac{x}{x-x'-i\epsilon}\frac{x}{x'-i\epsilon}=\frac{x}{x-x'-i\epsilon}+\frac{x}{x'-i\epsilon},$$
$$\frac{x}{x-x'-i\epsilon}\frac{x}{x'-x_B+i\epsilon}=\frac{\hat{s}+Q^2}{\hat{s}}\left(\frac{x}{x-x'-i\epsilon}+\frac{x}{x'-x_B+i\epsilon}\right). \tag{48}$$

From the above discussion, we can conclude that the part of the cross section with $\bar{H}^L_\alpha(x'p,xp)$ is given by

$$\epsilon^{\alpha\bar{n}nS_\perp}\int dx' T_{q,F}(x',x)\frac{1}{x-x'-i\epsilon}\mathrm{Tr}[x\not{p}\bar{H}^L_\alpha(x'p,xp)]$$
$$=\int dx' T_{q,F}(x',x)\left[\frac{1}{x-x'-i\epsilon}H_{F1}+\frac{x}{(x-x'-i\epsilon)^2}H_{F2}+\frac{1}{x'-i\epsilon}H_{F3}+\frac{1}{x'-x_B+i\epsilon}H_{F4}\right]. \tag{49}$$

All the hard parts H_{Fi} are independent of x'. We can repeat the same discussion on $\bar{H}^R_\alpha(xp,x'p)$. Then we can calculate each hard partonic cross section and obtain the following result for the hadronic part:

$$W=\pi M_N\int\frac{dx}{x}\delta[(xp+q-p_c)^2]\left\{x\frac{df_{1T}^{\perp(1)}(x)}{dx}(\epsilon^{q\bar{n}nS_\perp}-\epsilon^{p_c\bar{n}nS_\perp})\frac{2}{\hat{u}}\hat{\sigma}(\hat{s},\hat{t},\hat{u})+[(\hat{s}+Q^2)\epsilon^{p_c\bar{n}nS_\perp}+\hat{t}\epsilon^{q\bar{n}nS_\perp}]f_{1T}^{\perp(1)}(x)\hat{\sigma}_{ND'}\right.$$
$$+i\int dx' T_{q,F}(x',x)\left[\left(\frac{1}{x-x'-i\epsilon}-\frac{1}{x-x'+i\epsilon}\right)H_{F1}+\left(\frac{x}{(x-x'-i\epsilon)^2}-\frac{x}{(x-x'+i\epsilon)^2}\right)H_{F2}+\left(\frac{1}{x'-i\epsilon}-\frac{1}{x'+i\epsilon}\right)H_{F3}\right.$$
$$\left.\left.+\left(\frac{1}{x'-x_B+i\epsilon}-\frac{1}{x'-x_B-i\epsilon}\right)H_{F4}\right]\right\}, \tag{50}$$

where the hard cross sections are given by

$$\hat{\sigma}_{ND'}=16C_FQ^2\frac{Q^2\hat{t}-\hat{t}^2-\hat{t}\,\hat{u}-\hat{u}^2}{\hat{s}^2\hat{u}^2},$$
$$H_{F1}=[(\hat{s}+Q^2)\epsilon^{p_c\bar{n}nS_\perp}+\hat{t}\epsilon^{q\bar{n}nS_\perp}]\left(-\frac{1}{2}\hat{\sigma}_{ND}+\frac{1}{2}\hat{\sigma}_{ND'}\right),$$
$$H_{F2}=[(\hat{s}+Q^2)\epsilon^{p_c\bar{n}nS_\perp}+\hat{t}\epsilon^{q\bar{n}nS_\perp}]\hat{\sigma}_D-(\epsilon^{q\bar{n}nS_\perp}-\epsilon^{p_c\bar{n}nS_\perp})\frac{2}{\hat{u}}\hat{\sigma}(\hat{s},\hat{t},\hat{u}),$$
$$H_{F3}=-\frac{1}{2}[(\hat{s}+Q^2)\epsilon^{p_c\bar{n}nS_\perp}+\hat{t}\epsilon^{q\bar{n}nS_\perp}]\hat{\sigma}_{\mathrm{SFP}},$$
$$H_{F4}=\frac{1}{2}[(\hat{s}+Q^2)\epsilon^{p_c\bar{n}nS_\perp}+\hat{t}\epsilon^{q\bar{n}nS_\perp}]\hat{\sigma}_{\mathrm{HP}}. \tag{51}$$

$\hat{\sigma}_{ND}$, $\hat{\sigma}_D$, $\hat{\sigma}_{\mathrm{SFP}}$, and $\hat{\sigma}_{\mathrm{HP}}$ can be found in Eq. (23). Since H_{Fi} are all independent of x', the x' integration involves only $T_{q,F}(x',x)$ and the propagators. Then we can perform x' integration in Eq. (50) as

$$\int dx'\left(\frac{1}{x-x'-i\epsilon}-\frac{1}{x-x'+i\epsilon}\right)T_{q,F}(x',x)=2\pi iT_{q,F}(x,x),$$
$$\int dx'\left[\frac{x}{(x-x'-i\epsilon)^2}-\frac{x}{(x-x'+i\epsilon)^2}\right]T_{q,F}(x',x)=-\pi ix\frac{d}{dx}T_{q,F}(x,x),$$
$$\int dx'\left(\frac{1}{x'-i\epsilon}-\frac{1}{x'+i\epsilon}\right)T_{q,F}(x',x)=2\pi iT_{q,F}(0,x),$$
$$\int dx'\left(\frac{1}{x'-x_B+i\epsilon}-\frac{1}{x'-x_B-i\epsilon}\right)T_{q,F}(x',x)=-2\pi iT_{q,F}(x_B,x), \tag{52}$$

where we have used the symmetric property of Qiu-Sterman function $T_{q,F}(x',x) = T_{q,F}(x,x')$ in the integration of the double pole coefficient. Substituting these relations into Eq. (50) and using Eqs. (45) and (46), we can finally derive the transverse polarized cross section in SIDIS based on the new method as

$$\begin{aligned}\frac{d^4\Delta\sigma}{dx_B dy dz_h dP_{h\perp}} &= \frac{\pi M_N \alpha_{em}^2 \alpha_s}{8 z_h x_B^2 S_{ep}^2 Q^2} \sum_q e_q^2 \int \frac{dz}{z^2} D_{q\to h}(z) \\ &\times \int \frac{dx}{x} \delta[(xp+q-p_c)^2]((\hat{s}+Q^2)\epsilon^{p_c \bar{n} n S_\perp} + \hat{t}\epsilon^{q\bar{n} n S_\perp}) \\ &\times \left[x\frac{d}{dx} T_{q,F}(x,x)\hat{\sigma}_D + T_{q,F}(x,x)\hat{\sigma}_{ND} \right. \\ &\left. + T_{q,F}(0,x)\hat{\sigma}_{\rm SFP} + T_{q,F}(x_B,x)\hat{\sigma}_{\rm HP} \right]. \end{aligned} \quad (53)$$

This is exactly the same with the result of the conventional calculation (22). We would like to emphasize that the cross section is never gauge and Lorentz invariant if the kinematical function $f_{1T}^{\perp(1)}(x)$ and Qiu-Sterman function $T_{q,F}(x,x)$ are independent of each other. The relation between them is needed for the physically acceptable result.

In the end, we make a comment on the generality of our result. We considered only the metric part $L^{\mu\nu} \simeq -Q^2 g^{\mu\nu}$ in our calculation so that one can easily follow the calculation and clearly see the difference between two calculation methods. It is a natural question whether the consistency holds when we consider the full leptonic tensor shown in Eq. (5). The conventional way to calculate the cross section in SIDIS is that we expand the hadronic tensor in terms of orthogonal bases. The symmetric part of the tensor $W^{\mu\nu}$ has ten independent components, and one of them is fixed by the condition $q_\mu W^{\mu\nu} = 0$. Then $W^{\mu\nu}$ can be expanded by nine independent bases as

$$W^{\mu\nu} = \sum_{i=1}^{9} (W^{\rho\sigma}\tilde{\mathcal{V}}_{i\rho\sigma})\mathcal{V}_i^{\mu\nu}. \quad (54)$$

One can find the explicit forms of $\mathcal{V}_{i\rho\sigma}$ and $\tilde{\mathcal{V}}_i^{\mu\nu}$ in Ref. [20]. Then the contracted form with $L^{\mu\nu}$ is rewritten as

$$L^{\mu\nu}W_{\mu\nu} = \sum_{i=1}^{9} (L^{\mu\nu}\mathcal{V}_{i\mu\nu})(W^{\rho\sigma}\tilde{\mathcal{V}}_{i\rho\sigma}). \quad (55)$$

This equation means that the calculation with the full leptonic tensor $L^{\mu\nu}$ results in the calculation of the hard cross sections $W^{\rho\sigma}\tilde{\mathcal{V}}_{i\rho\sigma}$. Three tensors $\tilde{\mathcal{V}}_{5,6,7}^{\mu\nu}$ are irrelevant to our study, because they are pure imaginary. We verified that the consistency between the two methods holds for all six hard cross sections ($i = 1, 2, 3, 4, 8, 9$). This result shows that the consistency holds for the full leptonic tensor and enhances the generality of our result.

IV. SUMMARY

We proposed the new nonpole calculation method for the Sivers effect in the twist-3 cross section and confirmed the consistency with the conventional pole calculation. We found out that the relation $f_{1T}^{\perp(1)}(x) = \pi T_{q,F}(x,x)$ is very important to guarantee the gauge and Lorentz invariances of the final result. We reproduced this relation without introducing the definition of the transverse momentum dependent (TMD) Sivers function. The importance of Eq. (45) has been mainly discussed in the context of the matching between the TMD factorization and the collinear twist-3 factorization frameworks [21,22]. Our calculation showed that this was also important for the gauge and Lorentz invariances of the twist-3 physical observables for the Sivers effect. This result provides a new perspective on the relation. The same technique can be also applied to the gluon Sivers function and the twist-3 gluon distribution functions [16]. The relation between them is relatively nontrivial compared to the quark functions. From the requirement of the gauge and Lorentz invariances of the twist-3 cross section, we can derive a similar relation with Eq. (45) for the gluon distribution functions.

One of the advantages in the new nonpole calculation method is that we do not need to prove Eq. (14) for the SGP contribution as required in the conventional pole method, which can be checked only through diagram by diagram calculation. It is known that this relation may not be hold when the description of the fragmentation part is changed to another framework such as NRQCD for heavy quarkonium production. In the new method, we never separate the pole contributions, and then no singularity arises from the relation associated with WTI. Our new method will extend the applicability of the collinear twist-3 framework.

In the new method, one does not need to perform derivatives over the initial parton's transverse momentum in the calculation of Feynman diagrams. We can anticipate that a lot of propagators depend on the initial parton's momentum in higher-order diagrams. The direct operation of the derivatives is a highly complicated task. Our method could significantly reduce this complexity as discussed in Sec. III. As mentioned just below Eq. (32), the WTI does not change for the higher-order diagrams as long as the gauge invariance is preserved. Most of our results are available without change for the higher-order cross section in SIDIS. A set of equations derived in this paper could be useful to derive the first next-leading-order cross section for the SSA in an ep collision which could be measured at an electron-ion collider in the near future.

We expect the new method presented in this manuscript can be extended to higher-twist calculation, which becomes one of the standard methods to investigate the nontrivial nuclear effect in heavy ion collisions [23–27]. As we do not need to perform derivatives over the initial parton's transverse momentum in the new nonpole method, we expect the new approach will be of great use in performing the next-to-leading-order calculation at higher twist, in which the conventional collinear expansion caused ambiguity in setting up the initial parton's kinematics [28,29]; this ambiguity can be resolved in the new nonpole method.

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APPENDIX A: TWIST-3 QUARK-GLUON CORRELATION FUNCTIONS

1. Definition of the twist-3 functions

We introduce the definition of all relevant twist-3 functions for the transversely polarized proton [15,30].

a. D-type dynamical function

$$\begin{aligned}M^{\alpha}_{ij,D}(x_1,x_2) &= (p^+)^2\int\frac{dy_1^-}{2\pi}\int\frac{dy_2^-}{2\pi}e^{ix_1p^+y_1^-}e^{i(x_2-x_1)p^+y_2^-}\\&\quad\times\langle pS_\perp|\bar\psi_j(0)[0,y_2^-]\\&\quad\times D_\perp^\alpha(y_2^-)[y_2^-,y_1^-]\psi_i(y_1^-)|pS_\perp\rangle\\&= -\frac{M_N}{2}\epsilon^{\alpha\bar n n S_\perp}(\not p)_{ij}T_{q,D}(x_1,x_2)+\cdots, \qquad (A1)\end{aligned}$$

where $D_\perp^\alpha(y_2^-) = \partial_\perp^\alpha - igA_\perp^\alpha(y_2^-)$ and $[0,y_2^-]$ is the Wilson line

$$[0,y_2^-] = P\exp\left(ig\int_{y_2^-}^0 dy^- A^n(y^-)\right). \qquad (A2)$$

The D-type function $T_{q,D}(x_1,x_2)$ is real and antisymmetric $T_{q,D}(x_1,x_2) = -T_{q,D}(x_2,x_1)$.

b. Kinematical function

$$\begin{aligned}M^{\alpha}_{ij,\partial}(x) &= p^+\int\frac{dy_1^-}{2\pi}e^{ixp^+y_1^-}\langle pS_\perp|\bar\psi_j(0)[0,y_1^-]D_\perp^\alpha(y_1^-)\psi_i(y_1^-)|pS_\perp\rangle\\&\quad+p^+\int\frac{dy_1^-}{2\pi}e^{ixp^+y_1^-}\langle pS_\perp|\bar\psi_j(0)ig\left[\int_{y_1^-}^\infty dy_2^-[0,y_2^-]F^{\alpha n}(y_2^-)[y_2^-,y_1^-]\right]\psi_i(y_1^-)|pS_\perp\rangle,\\&= -i\frac{M_N}{2}\epsilon^{\alpha\bar n n S_\perp}(\not p)_{ij}f_{1T}^{\perp(1)}(x)+\cdots. \qquad (A3)\end{aligned}$$

By using the translation invariance [15]

$$\begin{aligned}&\langle pS_\perp|\bar\psi_j(0)\overleftarrow{D}_\perp^\alpha(0)[0,y_1^-]\psi_i(y_1^-)|pS_\perp\rangle+\langle pS_\perp|\bar\psi_j(0)[0,y_1^-]D_\perp^\alpha(y_1^-)\psi_i(y_1^-)|pS_\perp\rangle\\&\quad+\int_{y_1^-}^0\frac{dy_2^-}{2\pi}\langle pS_\perp|\bar\psi_j(0)[0,y_2^-]igF^{\alpha n}(y_2^-)[y_2^-,y_1^-]\psi_i(y_1^-)|pS_\perp\rangle = 0, \qquad (A4)\end{aligned}$$

we can show $M_\partial^*(x) = -M_\partial(x)$ and, therefore, $f_{1T}^{\perp(1)}(x)$ is a real function. The kinematical function $f_{1T}^{\perp(1)}(x)$ has another definition using the quark TMD correlator. Here, we recall the definition of the quark Sivers function [31]:

$$\begin{aligned}M_{ij}(x,p_T) &= \int\frac{dy^-}{2\pi}\int\frac{d^2\xi_T}{2\pi}e^{ixp^+y^-}e^{ip_T\cdot\xi_T}\langle pS_\perp|\bar\psi_j(0)[0,\infty^-][\infty^-,\infty^-+\xi_T][\infty^-+\xi_T,y^-+\xi_T]\psi_i(y^-+\xi_T)|pS_\perp\rangle\\&= -\frac{1}{2M_N}f_{1T}^\perp(x,p_T)\epsilon^{p_T\bar n\mu S_\perp}\gamma_\mu+\cdots. \qquad (A5)\end{aligned}$$

We can find a relation between the first moment of $M(x,p_T)$ and the correlator of the kinematical function $M_\partial^\alpha(x)$:

$$\begin{aligned}\int d^2p_T p_T^\alpha M_{ij}(x,p_T) &= \int\frac{dy^-}{2\pi}\int\frac{d^2\xi_T}{2\pi}e^{ixp^+y^-}\left(-i\frac{\partial}{\partial\xi_{T\alpha}}\right)e^{ip_T\cdot\xi_T}\langle pS_\perp|\bar\psi_j(0)[0,\infty^-][\infty^-,\infty^-+\xi_T]\\&\quad\times[\infty^-+\xi_T,y^-+\xi_T]\psi_i(y^-+\xi_T)|pS_\perp\rangle\\&= i\int\frac{dy^-}{2\pi}e^{ixp^+y^-}\langle pS_\perp|\bar\psi_j(0)D_\perp^\alpha(y^-)\psi_i(y^-)|pS_\perp\rangle\\&\quad+i\int\frac{dy^-}{2\pi}e^{ixp^+y^-}\langle pS_\perp|\bar\psi_j(0)ig\left[\int_{y^-}^\infty dy_2^-F^{\alpha n}(y_2^-)\right]\psi_i(y^-)|pS_\perp\rangle\\&= \frac{i}{p^+}M_{ij,\partial}^\alpha(x).\end{aligned}\tag{A6}$$

Then $f_{1T}^{\perp(1)}(x)$ can be expressed by the first moment of the quark Sivers function [32,33]:

$$f_{1T}^{\perp(1)}(x)=\int d^2p_T\frac{|p_T|^2}{2M_N^2}f_{1T}^\perp(x,p_T).\tag{A7}$$

The matching between TMD functions and collinear functions itself is an active research subject in perturbative QCD phenomenology. One can find recent developments in Refs. [21,22], and references therein.

c. F-type dynamical function

$$\begin{aligned}M_{ij,F}^\alpha(x_1,x_2) &= p^+\int\frac{dy_1^-}{2\pi}\int\frac{dy_2^-}{2\pi}e^{ix_1p^+y_1^-}e^{i(x_2-x_1)p^+y_2^-}\langle pS_\perp|\bar\psi_j(0)[0,y_2^-]gF^{\alpha n}(y_2^-)[y_2^-,y_1^-]\psi_i(y_1^-)|pS_\perp\rangle\\&= -\frac{M_N}{2}\epsilon^{\alpha\bar nnS_\perp}(\not p)_{ij}T_{q,F}(x_1,x_2)+\cdots,\end{aligned}\tag{A8}$$

where the F-type function $T_{q,F}(x_1,x_2)$ is real and symmetric $T_{q,F}(x_1,x_2)=T_{q,F}(x_2,x_1)$.

2. Relation among the functions

We can derive an operator identity among the three types of correlators [30]. In order to derive the relation, we use the identity for the $D_\perp^\alpha(y_2^-)[y_2^-,y_1^-]$ in $M_D^\alpha(x_1,x_2)$:

$$\begin{aligned}D_\perp^\alpha(y_2^-)[y_2^-,y_1^-] &= [y_2^-,y_1^-]D_\perp^\alpha(y_1^-)+i\int_{y_1^-}^{y_2^-}dy_3^-[y_2^-,y_3^-]gF^{\alpha n}(y_3^-)[y_3^-,y_1^-]\\&= [y_2^-,y_1^-]D_\perp^\alpha(y_1^-)+i\int_{y_1^-}^\infty dy_3^-[y_2^-,y_3^-]gF^{\alpha n}(y_3^-)[y_3^-,y_1^-]-i\int_{y_2^-}^\infty dy_3^-[y_2^-,y_3^-]gF^{\alpha n}(y_3^-)[y_3^-,y_1^-]\\&= [y_2^-,y_1^-]D_\perp^\alpha(y_1^-)+i\int_{y_1^-}^\infty dy_3^-[y_2^-,y_3^-]gF^{\alpha n}(y_3^-)[y_3^-,y_1^-]-i\int_{-\infty}^\infty dy_3^-\theta(y_3^--y_2^-)[y_2^-,y_3^-]gF^{\alpha n}(y_3^-)[y_3^-,y_1^-],\end{aligned}\tag{A9}$$

where we used the step function

$$\theta(y_3^--y_2^-)=\int\frac{dx}{2\pi i}\frac{e^{i(y_3^--y_2^-)x}}{x-i\epsilon}.\tag{A10}$$

We calculate each term in the rhs of (A9) below.

(1) First term:

$$(p^+)^2 \int \frac{dy_1^-}{2\pi} \int \frac{dy_2^-}{2\pi} e^{ix_1 p^+ y_1^-} e^{i(x_2 - x_1)p^+ y_2^-} \langle pS_\perp | \bar{\psi}_j(0)[0, y_1^-] D_\perp^\alpha(y_1^-) \psi_i(y_1^-) | pS_\perp \rangle$$
$$= \delta(x_2 - x_1) \left[p^+ \int \frac{dy_1^-}{2\pi} e^{ix_1 p^+ y_1^-} \langle pS_\perp | \bar{\psi}_j(0)[0, y_1^-] D_\perp^\alpha(y_1^-) \psi_i(y_1^-) | pS_\perp \rangle \right]. \tag{A11}$$

(2) Second term:

$$(p^+)^2 \int \frac{dy_1^-}{2\pi} \int \frac{dy_2^-}{2\pi} \int_{y_1^-}^{\infty} dy_3^- e^{ix_1 p^+ y_1^-} e^{i(x_2 - x_1)p^+ y_2^-} \langle pS_\perp | \bar{\psi}_j(0)[0, y_3^-] igF^{\alpha n}(y_3^-)[y_3^-, y_1^-] \psi_i(y_1^-) | pS_\perp \rangle$$
$$= \delta(x_2 - x_1) \left[p^+ \int \frac{dy_1^-}{2\pi} \int_{y_1^-}^{\infty} dy_3^- e^{ix_1 p^+ y_1^-} \langle pS_\perp | \bar{\psi}_j(0)[0, y_3^-] igF^{\alpha n}(y_3^-)[y_3^-, y_1^-] \psi_i(y_1^-) | pS_\perp \rangle \right]. \tag{A12}$$

(3) Third term:

$$-(p^+)^2 \int \frac{dy_1^-}{2\pi} \int \frac{dy_2^-}{2\pi} \int dy_3^- \theta(y_3^- - y_2^-) e^{ix_1 p^+ y_1^-} e^{i(x_2 - x_1)p^+ y_2^-} \langle pS_\perp | \bar{\psi}_j(0)[0, y_3^-] igF^{\alpha n}(y_3^-)[y_3^-, y_1^-] \psi_i(y_1^-) | pS_\perp \rangle$$
$$= -\int \frac{dy_1^-}{2\pi} \int \frac{dy_2^-}{2\pi} \int \frac{dy_3^-}{2\pi} \int dx \frac{(p^+)^2}{x - i\epsilon} e^{ix_1 p^+ y_1^-} e^{iy_3^- x} e^{i(\{x_2 - x_1\}p^+ - x)y_2^-} \langle pS_\perp | \bar{\psi}_j(0)[0, y_3^-] gF^{\alpha n}(y_3^-)[y_3^-, y_1^-] \psi_i(y_1^-) | pS_\perp \rangle$$
$$= \frac{1}{x_1 - x_2 + i\epsilon} \left[p^+ \int \frac{dy_1^-}{2\pi} \int \frac{dy_3^-}{2\pi} e^{ix_1 p^+ y_1^-} e^{i(x_2 - x_1)p^+ y_3^-} \langle pS_\perp | \bar{\psi}_j(0)[0, y_3^-] gF^{\alpha n}(y_3^-)[y_3^-, y_1^-] \psi_i(y_1^-) | pS_\perp \rangle \right]. \tag{A13}$$

Combining (A11)–(A13), we can show $M_D^\alpha(x_1, x_2) = \frac{1}{x_1 - x_2 + i\epsilon} M_F^\alpha(x_1, x_2) + \delta(x_2 - x_1) M_\partial^\alpha(x_1)$, and then the relation among the twist-3 functions is given by

$$T_{q,D}(x_1, x_2) = \frac{1}{x_1 - x_2 + i\epsilon} T_{q,F}(x_1, x_2) + i\delta(x_2 - x_1) f_{1T}^{\perp(1)}(x_1). \tag{A14}$$

Using the interchange symmetry $x_1 \leftrightarrow x_2$, we can rewrite the above relation as

$$0 = \left(\frac{1}{x_1 - x_2 + i\epsilon} - \frac{1}{x_1 - x_2 - i\epsilon} \right) T_{q,F}(x_1, x_2) + 2i\delta(x_2 - x_1) f_{1T}^{\perp(1)}(x_1). \tag{A15}$$

From the operator definition (A8), one can find that $T_{q,F}(x_1, x_2)$ contains the factor $e^{ix_1 p^+ (y_1^- - y_2^-)}$. We can perform x_1 integration:

$$\int dx_1 \left(\frac{1}{x_1 - x_2 + i\epsilon} - \frac{1}{x_1 - x_2 - i\epsilon} \right) e^{ix_1 p^+ (y_1^- - y_2^-)} = -2\pi i \left(\theta(y_2^- - y_1^-) + \theta(y_1^- - y_2^-) \right) e^{ix_2 p^+ (y_1^- - y_2^-)} = -2\pi i e^{ix_2 p^+ (y_1^- - y_2^-)}, \tag{A16}$$

and then

$$\int dx_1 \left(\frac{1}{x_1 - x_2 + i\epsilon} - \frac{1}{x_1 - x_2 - i\epsilon} \right) T_{q,F}(x_1, x_2) = -2\pi i T_{q,F}(x_2, x_2). \tag{A17}$$

After the integration of (A15) with respect to x_1, we can derive the relation

$$f_{1T}^{\perp(1)}(x) = \pi T_{q,F}(x, x), \tag{A18}$$

which is nothing but the relation (45). This is the well-known relation between the first moment of the Sivers function $f_{1T}^{\perp(1)}(x)$ and the Qiu-Sterman function $T_{q,F}(x, x)$ [32,34]. The same relation can be derived as we performed here in a simple way. One can easily show the relation (46) by the derivative of (A18) with respect to x.

APPENDIX B: CALCULATION OF THE DERIVATIVE TERM $\frac{\partial}{\partial k^\alpha} H(k)|_{k=xp}$

We show how to calculate the hard part $\frac{\partial}{\partial k^\alpha} H(k)|_{k=xp}$ in Eq. (39) without direct operation of the k derivative. We can calculate the part of the kinematical function as

$$i\int dx \mathrm{Tr}\left[M_\partial^\alpha(x)\frac{\partial}{\partial k^\alpha}H(k)\Big|_{k=xp}\right] = \frac{M_N}{2}\int\frac{dx}{x}\epsilon^{\alpha\bar{n}nS_\perp}f_{1T}^{\perp(1)}(x)\mathrm{Tr}\left[x\not{p}\frac{\partial}{\partial k^\alpha}H(k)\Big|_{k=xp}\right]$$
$$= \frac{M_N}{2}\int\frac{dx}{x}\epsilon^{\alpha\bar{n}nS_\perp}f_{1T}^{\perp(1)}(x)\left\{\frac{\partial}{\partial k^\alpha}\mathrm{Tr}[\not{k}H(k)]\Big|_{k=xp} - \mathrm{Tr}[\gamma^\alpha H(xp)]\right\}. \tag{B1}$$

We focus on the first term in the parentheses. Because $H(k)$ carries the information about k, q, and p_c, it can be written by all possible Lorentz invariant variables:

$$\mathrm{Tr}[\not{k}H(k)] = \hat{\sigma}(k^2,\tilde{s},\tilde{t},\hat{u},Q^2)(2\pi)\delta(\tilde{s}+\tilde{t}+\hat{u}+Q^2-k^2), \tag{B2}$$

where we defined the variables

$$\tilde{s} = (k+q)^2, \qquad \tilde{t} = (k-p_c)^2. \tag{B3}$$

We can set $k^2 = 0$, because $\frac{\partial}{\partial k^\alpha}k^2|_{k=xp}\frac{\partial}{\partial k^2} = 2xp^\alpha\frac{\partial}{\partial k^2}$ is canceled with $\epsilon^{\alpha\bar{n}nS_\perp}$. We find that $\hat{\sigma}(k^2,\tilde{s},\tilde{t},\hat{u},Q^2)$ coincides with $\hat{\sigma}(\hat{s},\hat{t},\hat{u})$ in Eq. (43) in the collinear limit $k = xp$. Then the k derivative is converted into $\hat{s}$ and $\hat{t}$ derivatives:

$$\frac{\partial}{\partial k^\alpha}[\hat{\sigma}(k^2,\tilde{s},\tilde{t},\hat{u},Q^2)\delta(\tilde{s}+\tilde{t}+\hat{u}+Q^2-k^2)]|_{\tilde{s}=\hat{s},\tilde{t}=\hat{t}} = \left(2q^\alpha\frac{\partial}{\partial\hat{s}} - 2p_c^\alpha\frac{\partial}{\partial\hat{t}}\right)[\hat{\sigma}(\hat{s},\hat{t},\hat{u})\delta(\hat{s}+\hat{t}+\hat{u}+Q^2)]. \tag{B4}$$

We calculate the k-derivative term in Eq. (B1) as

$$\begin{aligned}
&\frac{M_N}{2}\int\frac{dx}{x}\epsilon^{\alpha\bar{n}nS_\perp}f_{1T}^{\perp(1)}(x)\frac{\partial}{\partial k^\alpha}\mathrm{Tr}[\not{k}H(k)]\Big|_{k=xp}\\
&= \pi M_N\int\frac{dx}{x}\epsilon^{\alpha\bar{n}nS_\perp}f_{1T}^{\perp(1)}(x)\left(2q^\alpha\frac{\partial}{\partial\hat{s}} - 2p_c^\alpha\frac{\partial}{\partial\hat{t}}\right)\hat{\sigma}(\hat{s},\hat{t},\hat{u})\delta(\hat{s}+\hat{t}+\hat{u}+Q^2)\\
&= \pi M_N\int\frac{dx}{x}\epsilon^{\alpha\bar{n}nS_\perp}f_{1T}^{\perp(1)}(x)\left\{\delta(\hat{s}+\hat{t}+\hat{u}+Q^2)\left(2q^\alpha\frac{\partial}{\partial\hat{s}} - 2p_c^\alpha\frac{\partial}{\partial\hat{t}}\right)\hat{\sigma}(\hat{s},\hat{t},\hat{u})\right.\\
&\quad\left. + \left(\frac{2q^\alpha - 2p_c^\alpha}{2p\cdot q - 2p\cdot p_c}\right)\hat{\sigma}(\hat{s},\hat{t},\hat{u})\frac{\partial}{\partial x}\delta(\hat{s}+\hat{t}+\hat{u}+Q^2)\right\}\\
&= \pi M_N\int\frac{dx}{x}\delta(\hat{s}+\hat{t}+\hat{u}+Q^2)\left\{x\frac{d}{dx}f_{1T}^{\perp(1)}(x)(\epsilon^{q\bar{n}nS_\perp} - \epsilon^{p_c\bar{n}nS_\perp})\frac{2}{\hat{u}}\hat{\sigma}(\hat{s},\hat{t},\hat{u})\right.\\
&\quad\left. + f_{1T}^{\perp(1)}(x)\left[\left(2\epsilon^{q\bar{n}nS_\perp}\frac{\partial}{\partial\hat{s}} - 2\epsilon^{p_c\bar{n}nS_\perp}\frac{\partial}{\partial\hat{t}}\right)\hat{\sigma}(\hat{s},\hat{t},\hat{u}) + (\epsilon^{q\bar{n}nS_\perp} - \epsilon^{p_c\bar{n}nS_\perp})\frac{2}{\hat{u}}\left(x\frac{\partial}{\partial x}\hat{\sigma}(\hat{s},\hat{t},\hat{u}) - \hat{\sigma}(\hat{s},\hat{t},\hat{u})\right)\right]\right\}.
\end{aligned} \tag{B5}$$

We can calculate x derivative of $\hat{\sigma}(\hat{s},\hat{t},\hat{u})$ as

$$x\frac{\partial}{\partial x}\hat{\sigma}(\hat{s},\hat{t},\hat{u}) = \left((\hat{s}+Q^2)\frac{\partial}{\partial\hat{s}} + \hat{t}\frac{\partial}{\partial\hat{t}}\right)\hat{\sigma}(\hat{s},\hat{t},\hat{u}). \tag{B6}$$

Finally, we combine the second term in Eq. (B1) and obtain the result in Eq. (42):

$$\begin{aligned}
&\pi M_N\int\frac{dx}{x}\delta(\hat{s}+\hat{t}+\hat{u}+Q^2)\left\{x\frac{d}{dx}f_{1T}^{\perp(1)}(x)(\epsilon^{q\bar{n}nS_\perp} - \epsilon^{p_c\bar{n}nS_\perp})\frac{2}{\hat{u}}\hat{\sigma}(\hat{s},\hat{t},\hat{u})\right.\\
&\left.+f_{1T}^{\perp(1)}(x)\left[\left((\hat{s}+Q^2)\epsilon^{p_c\bar{n}nS_\perp} + \hat{t}\epsilon^{q\bar{n}nS_\perp}\right)\frac{2}{\hat{u}}\left(\frac{\partial}{\partial\hat{t}} - \frac{\partial}{\partial\hat{s}}\right)\hat{\sigma}(\hat{s},\hat{t},\hat{u}) - (\epsilon^{q\bar{n}nS_\perp} - \epsilon^{p_c\bar{n}nS_\perp})\frac{2}{\hat{u}}\hat{\sigma}(\hat{s},\hat{t},\hat{u}) - \epsilon^{\alpha\bar{n}nS_\perp}\mathrm{Tr}[\gamma_\alpha\bar{H}(xp)]\right]\right\}.
\end{aligned} \tag{B7}$$

The derivative over the Mandelstam variable can be carried out after the calculation of the diagrams, which is much easier than the direct k derivative of $H(k)$.

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