Anomalous Casimir effect in axion electrodynamics

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We study the Casimir effect in axion electrodynamics. A finite θ-term affects the energy dispersion relation of photon if θ is time and/or space dependent. We focus on a special case with linearly inhomogeneous θ along the z-axis. Then we demonstrate that the Casimir force between two parallel plates perpendicular to the z-axis can be either attractive or repulsive, dependent on the gradient of θ. We call this repulsive component in the Casimir force induced by inhomogeneous θ the anomalous Casimir effect.

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I. INTRODUCTION

The Casimir effect [1] refers to a physical force resulting from the quantum fluctuations in the vacuum restricted by boundaries. One can alternatively interpret it as a relativistic extension of the van der Waals force mediated by the vacuum polarization [2]. In the original study of the Casimir effect, an attractive force between two parallel plates of perfect conductors emerges from the vacuum of Maxwell electrodynamics. The presence of boundaries discretizes the momenta, causing a finite difference in the vacuum energy, which is mathematically represented by the Abel-Plana formula in the simple case with parallel plates. The calculation machinery is quite analogous to the imaginary-time formalism of finite-temperature field theory (see Ref. [3] for discussions on temperature inversion symmetry in the Casimir effect). A pioneering experimental test for the Casimir force in the original scenario started out more than a half century ago [4], while the more accurate measurement is established after decades of development [5–8]. Theoretical generalizations of the original study include the dynamical Casimir effect [9–13] and the fermionic Casimir effect [14–17]. In a more interdisciplinary sense, in view of modern nuclear and high-energy physics, the Casimir effect shows great significance in the chiral bag model of hadrons [18–20], serves as a hypothetical candidate for the dark energy origin from QCD [21–23], and also has important relevance to the researches of strings, branes, and gravity [24–26]. Remarkably, the latest numerical simulations study the Casimir effect in Yang-Mills theory and relate it to nonperturbative mass generation [27]. Apart from such theoretical interests, moreover, the Casimir effect has considerable applications in the manufacture of microelectromechanical systems in nanotechnology [28–32].

Among various aspects of the study on the Casimir effect, one intriguing issue is the sign of the Casimir force. In fact, it has been demonstrated that the Casimir force can be flipped from attractive to repulsive via nontrivial geometry of the boundaries [33–35]. The sign flip of the Casimir effect may also be caused by special arrangements of objects and media with different permittivity or permeability [36–40]. Interestingly, gathering substantial related efforts have given rise to a famous "no-go" theorem: the Casimir force between two bodies with reflection symmetry is always attractive [41].

However, this no-go theorem can be circumvented in consideration that the "vacuum" in quantum field theory is not always trivial, but can have rich structures. For such theories, even if boundaries maintain reflection symmetry, nontrivial vacuum properties may produce a repulsion. Indeed, it has been argued that the sign flip of the Casimir force exists in the vacuum of the chiral Gross-Neveu model [42] or its scalar cousin, i.e., the CP^{N-1} model [43]. It is worth mentioning that these consequences have been numerically validated by first-principle simulations of lattice field theory in Ref. [44]. Lately, in a simpler setup even without interaction effects, a tunable Casimir force that can oscillate between attractive and repulsive has been derived by Ref. [45]. In their scenario, the key point lies in that the chiral media, i.e., optically active or gyrotropic media, endow the vacuum with an intrinsic breaking of spatial parity $\mathcal{P}$ and/or time reversal $\mathcal{T}$ symmetries. In this way, without breaking the reflection symmetry geometrically, a repulsive Casimir force is allowed, which indicates a specific mechanism to bypass the no-go theorem. Actually, materials exhibiting such intrinsic $\mathcal{P}$ and/or $\mathcal{T}$ symmetries breaking are familiar in condensed matter physics, and e.g., Refs. [46–49] have reported the repulsive Casimir force between two topological insulators, Hall materials, and Weyl semimetals, respectively.

Along these lines, we are pursuing a kindred mechanism for a repulsive Casimir force in axion electrodynamics also known as the Maxwell-Chern-Simons theory [50–52]. The Lagrangian density of axion electrodynamics contains an ordinary electromagnetic part and a $(3+1)$-dimensional topological term parametrized by the background $\theta(x)$ which can be interpreted as a background axion field. We note that our work should be distinguished from those in $(2+1)$-dimensional Chern-Simons electrodynamics [53–56]. Given that a constant $\theta(x)$ would not affect the equation of motion, we consider a linearly inhomogeneous background axion field; $\theta(x) = b_\mu x^\mu$ with a constant four-vector b_μ. This specific choice is also motivated by related works about the realization of quantum anomaly in condensed matter physics as discussed in Refs. [57–60] where a similar form of $\theta(x)$ is assumed. Axion electrodynamics is a useful theory to account for anomaly induced phenomena in chiral media, e.g., the Witten effect [61] the chiral magnetic effect [62–64], the anomalous Hall effect [65–68], etc. Nowadays, a Weyl semimetal, topological insulator, and axion crystal provide us with real-world playgrounds of axion electrodynamics, promoting cross-disciplinary studies and experimental searches for anomalous chiral phenomena [69–75]. In addition to activities in condensed matter physics, one can see recent reviews [76–78] for applications of chiral transport phenomena in the high-energy nuclear experiments.

There are preceding efforts on the Casimir effect in the framework of axion electrodynamics. In Ref. [79] the topological Casimir effect was proposed as a possible probe to detect the background θ angle and the QCD axion. Thereby, a mixing coupling between electric and magnetic fields plays an important role, which is often referred to as the magnetoelectric effect in condensed matter physics. More relevant to our present study is the work of Ref. [80] where the Casimir effect with a pure timelike $b_\mu = (b_0, \mathbf{0})$ was analyzed, leading to the conclusion that no repulsive Casimir force is found in that case. Our present work is a natural extension to the situation with a pure spacelike $b_\mu = (0, \boldsymbol{b})$, and as we would argue later, we discover a repulsive component of Casimir force.

For the above-mentioned purpose, we carry out an analytical computation of the zero-point oscillation energy and the associated Casimir force in the presence of $\theta(x)$. We adopt the original straightforward method by Casimir [1] as well as some technical implementation similar to the case with $b_\mu = (b_0, \mathbf{0})$ in Ref. [80]. In the Appendix, we provide calculations using alternative methodology based on scattering theory and the Lifshitz formula [81], which looks superficially different from Casimir's method but yields an equivalent result.

The structure of this paper is organized as follows. In Sec. II we present the definition of the theory we are interested in and the physical setup to idealize the anomalous Casimir effect in the presence of $b_\mu = (0, \boldsymbol{b})$. We proceed to concrete calculations of the vacuum energy in Sec. III. We figure out the energy dispersion relations and quantify the zero-point oscillation energy there. Section IV is devoted to our central results, i.e., the analytical expression of the anomalous Casimir effect and the numerical plot illustrating a repulsive region. Finally, we conclude our discussions in Sec. V.

II. AXION ELECTRODYNAMICS

We briefly introduce the axion electrodynamics and then expound our physical scenario for a repulsive Casimir force. We model the effect of chiral medium on the Casimir force using the axion electrodynamics, that is, the U(1) electrodynamics with a topological θ term defined by the following Lagrangian density:

$$\mathcal{L}_{\text{axion}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{4} \theta F_{\mu\nu} \tilde{F}^{\mu\nu}. \tag{1}$$

In the above expression $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\tilde{F}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$, where A_μ represents the U(1) gauge field. For space-time dependent $\theta(x)$, the topological θ term modifies the equations of motion. Here, for later convenience, let us denote its derivatives as

$$b_0(x) \equiv \partial_t \theta(x), \quad \boldsymbol{b}(x) \equiv -\boldsymbol{\nabla} \theta(x), \tag{2}$$

or equivalently $b_\mu(x) = \partial_\mu \theta(x)$ in the covariant notation. Nonzero b_0 and/or $\boldsymbol{b}$ add $\mathcal{CP}$-odd terms to the equations of motion. The Euler-Lagrange equations from Eq. (1),

$$\partial_\mu F^{\mu\nu} = b_\mu \tilde{F}^{\mu\nu}, \tag{3}$$

and the Bianchi identity, $\partial_\mu \tilde{F}^{\mu\nu} = 0$, comprise the Maxwell-Chern-Simons equations in the absence of source. The explicit forms read [82]

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = -\boldsymbol{b} \cdot \boldsymbol{B}, \tag{4}$$

$$\boldsymbol{\nabla} \times \boldsymbol{B} - \frac{\partial \boldsymbol{E}}{\partial t} = b_0 \boldsymbol{B} + \boldsymbol{b} \times \boldsymbol{E}, \tag{5}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0, \tag{6}$$

$$\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0. \tag{7}$$

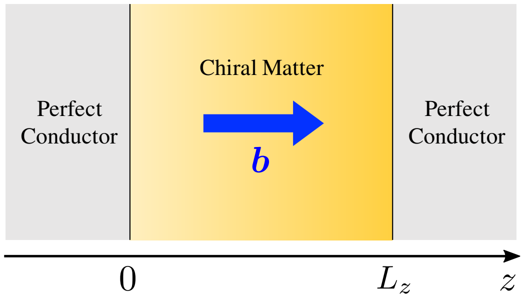

FIG. 1. Schematic illustration for the physical setup. Two perfect conductor plates at $z = 0$ and $z = L_z$ constitute the transverse planes coordinated by $\hat{x}$ and $\hat{y}$. The space between two plates is filled with chiral matter represented by the axion electrodynamics.

The first equation (4) implies an extra charge $-\boldsymbol{b}\cdot\boldsymbol{B}$, which is commonly called the Witten effect [61]. We can regard the right-hand side of Eq. (5) together with Maxwell's displacement current from the left-hand side as the current source for the magnetic field. Then, we find an extra current term, $\boldsymbol{j}_{\mathrm{CME}} = b_0\boldsymbol{B}$, which can be understood as the chiral magnetic effect with the identification of b_0 as the chiral chemical potential μ_5. Another extra current, $\boldsymbol{j}_{\mathrm{AHE}} = \boldsymbol{b}\times\boldsymbol{E}$, represents the anomalous Hall effect which exists even without the magnetic field.

These anomalous charge and currents are induced by the $\mathcal{CP}$-violating modifications on the vacuum in the axion electrodynamics. Since the vacuum properties are such changed, we can naturally anticipate noticeable impacts on other physical observables related to them. In this work, specifically, we explore such possibility in terms of the Casimir force. For such a purpose, as illustrated in Fig. 1, we install two plates of perfect conductors parallel to each other upright to the z-axis. The interval distance between two plates is L_z and the size of each transverse plate is L_xL_y. For simplicity, we assume constant b_0 and $\boldsymbol{b}$.

It is known that a timelike b_μ may incur tachyonic instabilities at long wavelength, which would impede the covariant quantization of the electromagnetic fields [83–85]. Also, we point out that the Casimir effect with constant $b_0 \neq 0$ but $\boldsymbol{b} = 0$ has been addressed in Ref. [80], where no sign flip of the Casimir force was observed. Thus, we focus on the situation with $b_0 = 0$ and $\boldsymbol{b} \neq 0$ in the present work. For transverse symmetry, we postulate $\boldsymbol{b} = b\hat{z}$, that is, $\boldsymbol{b}$ is directed perpendicular to the two plates. In our setup with such $\boldsymbol{b} \neq 0$, the reflection symmetry is explicitly broken, which suggests that there may arise a repulsive component in the Casimir force. Indeed, we will confirm this with concrete calculations.

III. VACUUM ENERGY

We impose the Dirichlet boundary condition, $A_\mu = 0$, at $z = 0$ and $z = L_z$, which is consistent with the properties of perfect conductors. Moreover, we take the limit $L_{x,y} \to \infty$. Then, we discretize the electromagnetic wave vector as $\boldsymbol{k} = (k_x, k_y, k_z = n\pi/L_z)$ with $n \in \mathbb{Z}$.

A canonical quantization scheme for A_μ with covariant gauge was proposed in Refs. [86–88], in which a tiny photon mass was introduced. Instead, here we adopt a path integral quantization with ghost fields. The Lagrangian density with the gauge fixing term parametrized by ξ, and the ghost fields c and $\bar{c}$, reads

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{\mathrm{photon}} + \mathcal{L}_{\mathrm{ghost}} \\ &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}bA_\nu\tilde{F}^{z\nu} + \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \frac{1}{2}\partial_\mu\bar{c}\partial^\mu c. \end{aligned} \tag{8}$$

The vacuum energy density ε is obtained from the generating functionals as follows:

$$VT\varepsilon = i\log Z_{\mathrm{photon}} + i\log Z_{\mathrm{ghost}}. \tag{9}$$

Here $V = L_xL_yL_z$ is the volume of the vacuum region between two plates and T is the time interval in the path integral. We keep them finite in the intermediate calculations and take the limits of $L_{x,y} \to \infty$ and $T \to \infty$ in the end. Beginning with the calculation of the photon part, we rewrite the photon Lagrangian as a bilinear form of A_μ in momentum space,

$$\mathcal{L}_{\mathrm{photon}} = -\frac{1}{2}A_\mu G^{-1}_{\mu\nu}A_\nu, \tag{10}$$

where

$$G^{-1}_{\mu\nu} = g_{\mu\nu}k^2 + i\epsilon_{\mu\nu\alpha\beta}b^\alpha k^\beta - \left(1 - \frac{1}{\xi}\right)k_\mu k_\nu. \tag{11}$$

Then we have

$$i\log Z_{\mathrm{photon}} = -\frac{i}{2}\log \mathrm{Det}[G^{-1}_{\mu\nu}(k)], \tag{12}$$

where Det represents the determinant with respect to the momentum index k and the Lorentz indices μ, ν. We firstly calculate the determinant over Lorentz indices as

$$\mathrm{Det}[G^{-1}_{\mu\nu}(k)] = \prod_k \xi^{-1}(k^2)^2[-(k^2)^2 + b^2(k^2 + k_z^2)]. \tag{13}$$

For further calculations, we employ the following notation for the energy dispersion relations determined from the on shell condition [89]:

$$\omega_{1,2}^2 = \boldsymbol{k}^2, \tag{14}$$

$$\omega_\pm^2 = k_x^2 + k_y^2 + \left(\sqrt{k_z^2 + \frac{b^2}{4}} \pm \frac{b}{2}\right)^2. \tag{15}$$

We note that $\omega_{1,2}$ are zeros of $(k^2)^2$ in Eq. (13). Since $(k^2)^2$ appears from the longitudinal and the scalar polarizations, the modes with $\omega_{1,2}$ are unphysical and their contributions to vacuum energy are canceled by the ghosts. The physical modes $\omega_\pm$ are zeros of $-(k^2)^2 + b^2(k^2 + k_z^2)$, and they correspond to the right- and left-handed photons. With these dispersion relations, we express the vacuum energy contributed from the photon as

$$i \log Z_{\text{photon}} = -\sum_{i=1,2,\pm}\sum_{k} \frac{i}{2}\log\left[k_0^2 - \omega_i^2(\boldsymbol{k})\right], \quad (16)$$

where we have dropped an irrelevant constant ξ^{-1}. By a similar computation for the ghost, we acquire

$$i \log Z_{\text{ghost}} = 2\sum_{k}\frac{i}{2}\log\left(k_0^2 - \boldsymbol{k}^2\right). \quad (17)$$

Notably Eq. (17) cancels the contribution from the unphysical modes with $i = 1, 2$ in Eq. (16). Summing the photon and the ghost contributions up, we get

$$VT\varepsilon = -\frac{i}{2}\sum_{\pm}\sum_{k}\log\left[k_0^2 - \omega_\pm^2(\boldsymbol{k})\right]. \quad (18)$$

Now, we take the limits of L_x, L_y, $T \to \infty$, which replace the phase space sum over $k = (k_0, k_x, k_y, n\pi/L_z)$ as

$$\frac{1}{VT}\sum_{k} \to \frac{1}{L_z}\sum_{n=0}^{\infty}\int\frac{dk_0 dk_x dk_y}{(2\pi)^3}. \quad (19)$$

Here, let us briefly explain how to compute the k_0-integral. Differentiating the integral with respect to $\omega_\pm$, we find two poles on the real k_0 axis. We deform the poles by the standard $i\epsilon$ prescription and carry out the k_0-integration. After further integrating over $\omega_\pm$, we extract a finite $\omega_\pm$-dependent piece, dropping an irrelevant divergent part,

$$\int\frac{dk_0}{2\pi}\log\left(k_0^2 - \omega_\pm^2 + i\epsilon\right) = i\omega_\pm + (\text{const}). \quad (20)$$

Eventually, we attain the vacuum energy per unit transverse area, $\mathcal{E} = V\varepsilon/L_x L_y$, given by

$$\mathcal{E} = \sum_{\pm}\sum_{n=0}^{\infty}\int\frac{dk_x dk_y}{(2\pi)^2}\frac{\omega_\pm(\boldsymbol{k})}{2}. \quad (21)$$

Equation (21) is nothing but the sum of the zero-point oscillation energy, sharing the same structure as the conventional Casimir effect except for the energy dispersion relations.

IV. CASIMIR FORCE

Based on the vacuum energy achieved in the last section, we will quantify the Casimir force in this section. We bring in a new notation to express the energy dispersion relation,

$$\omega_\pm^2(\boldsymbol{k}) = k_x^2 + k_y^2 + \frac{\pi^2}{L_z^2}\mu_\pm^2(n), \quad (22)$$

where

$$\mu_\pm(n) \equiv \sqrt{n^2 + \bar{b}^2} \pm \bar{b}. \quad (23)$$

Here, $\bar{b}$ denotes the dimensionless b defined by $\bar{b} = bL_z/(2\pi)$. We rescale $k_{x,y} \to \tilde{k}_{x,y} \equiv (L_z/\pi\mu_\pm)k_{x,y}$, to perform the transverse momentum integration as

$$\mathcal{E} = \frac{\pi^3}{L_z^3}\sum_{\pm}\sum_{n=0}^{\infty}\mu_\pm^3(n)\int^{\tilde{\Lambda}_\pm}\frac{d\tilde{k}_x d\tilde{k}_y}{(2\pi)^2}\frac{1}{2}\sqrt{\tilde{k}_x^2 + \tilde{k}_y^2 + 1}, \quad (24)$$

where we introduced an ultraviolet cutoff $\Lambda_\pm$ so that a step function, $\Theta(\Lambda_\pm - \omega_\pm(\boldsymbol{k}))$, is convoluted in the integrand. With the rescaled dimensionless cutoff, $\tilde{\Lambda}_\pm \equiv (L_z/\pi\mu_\pm)\Lambda_\pm$, we further evaluate the integral in Eq. (24) as

$$\begin{aligned}&\int^{\tilde{\Lambda}_\pm}\frac{d\tilde{k}_x d\tilde{k}_y}{(2\pi)^2}\sqrt{\tilde{k}_x^2 + \tilde{k}_x^2 + 1}\\ &\quad = \frac{1}{2\pi}\int_0^\infty dk_r k_r\sqrt{k_r^2 + 1}\,\Theta\left(\tilde{\Lambda}_\pm - \sqrt{k_r^2 + 1}\right)\\ &\quad = -\frac{1}{6\pi} + \frac{\tilde{\Lambda}_\pm^3}{6\pi}. \end{aligned} \quad (25)$$

After inserting Eq. (25) into Eq. (24), the second term proportional to $\tilde{\Lambda}_\pm^3$ leads to an irrelevant constant independent of L_z. We therefore safely leave this term out and reduce the expression of $\mathcal{E}$ to

$$\mathcal{E} = -\frac{\pi^2}{12L_z^3}\sum_{\pm}\sum_{n=0}^{\infty}\mu_\pm(n)^3. \quad (26)$$

Taking the sum over $\pm$, we further simplify the above expression into

$$\mathcal{E} = -\frac{\pi^2}{12L_z^3}\left[S\left(-\frac{3}{2}, \bar{b}\right) + 3\bar{b}^2 S\left(-\frac{1}{2}, \bar{b}\right)\right] - \frac{b^3}{24\pi}, \quad (27)$$

where we defined a function,

$$S(s, \bar{b}) \equiv \sum_{n=-\infty}^{\infty}(n^2 + \bar{b}^2)^{-s}, \quad (28)$$

for which we note that the sum runs from $n = -\infty$ to ∞. The last term in Eq. (27), coming from $n = 0$, is independent of L_z and hence gives no contribution to the Casimir force. Therefore we safely drop this term hereafter. Thus, our problem boils down to the calculation of $S(s,\bar{b})$. We rewrite Eq. (30) by multiplying it with the integral form of $\Gamma(s)$ and then divide by $\Gamma(s)$ as

$$S(s,\bar{b}) = \sum_{n=-\infty}^{\infty} \frac{1}{\Gamma(s)} \int_0^\infty (n^2+\bar{b}^2)^{-s} u^{s-1} e^{-u} du. \qquad (29)$$

We change the integration variable from u to $v = u/(n^2+\bar{b}^2)$ so that the integral becomes

$$\begin{aligned} S(s,\bar{b}) &= \sum_{n=-\infty}^{\infty} \frac{1}{\Gamma(s)} \int_0^\infty v^{s-1} e^{-n^2 v - \bar{b}^2 v} dv \\ &= \sum_{m=-\infty}^{\infty} \frac{\sqrt{\pi}}{\Gamma(s)} \int_0^\infty v^{s-3/2} e^{-\pi^2 m^2/v - \bar{b}^2 v} dv, \end{aligned} \qquad (30)$$

where we used Poisson's summation formula. The term with $m=0$ yields the Gamma function, while the terms with $m \neq 0$ take the form of the integral representation for the modified Bessel function of the second kind. We, therefore, arrive at

$$S(s,\bar{b}) = \frac{\sqrt{\pi}\bar{b}^{1-2s}}{\Gamma(s)} \left[\Gamma\left(s-\frac{1}{2}\right) + 4 \sum_{m=1}^{\infty} \frac{K_{\frac{1}{2}-s}(2\pi m \bar{b})}{(\pi m \bar{b})^{\frac{1}{2}-s}} \right]. \qquad (31)$$

Plugging this to the Casimir energy (27), we get

$$\begin{aligned} \mathcal{E} &= \mathcal{E}_\infty + \mathcal{E}_{\rm reg} \\ &= \mathcal{E}_\infty + \frac{b^4 L_z}{16\pi^2} \sum_{m=1}^{\infty} \left[\frac{K_1(mbL_z)}{mbL_z} - \frac{K_2(mbL_z)}{(mbL_z)^2} \right]. \end{aligned}$$

Here, the energy per unit transverse area, $\mathcal{E}$, includes a divergent portion,

$$\mathcal{E}_\infty = -\frac{5b^4 L_z}{512\pi^3} \Gamma(0). \qquad (32)$$

But the corresponding energy density $\mathcal{E}_\infty/L_z$ is independent of L_z. Thus, we can harmlessly subtract this energy density irrelevant to the Casimir force, by shifting a reference level of the energy density.

Finally, the Casimir force per unit transverse area is given by the derivative of $\mathcal{E}_{\rm reg}$ with respect to L_z, that is,

$$\begin{aligned} F(b) &= -\frac{\partial \mathcal{E}_{\rm reg}}{\partial L_z} \\ &= -\frac{b^4}{16\pi^2} \sum_{m=1}^{\infty} \left[\frac{3K_2(mbL_z)}{(mbL_z)^2} - K_0(mbL_z) \right]. \end{aligned} \qquad (33)$$

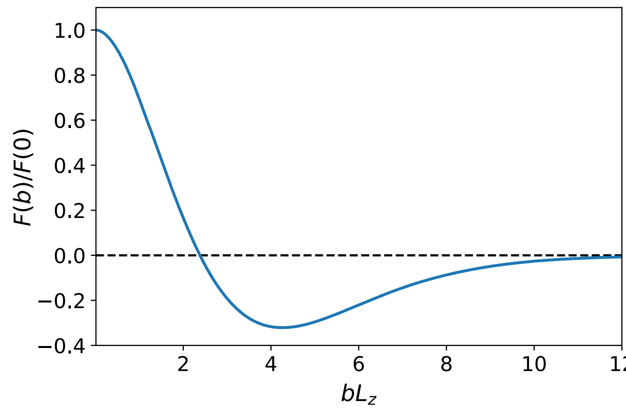

FIG. 2. Casimir force as a function of the dimensionless distance scaled with b.

This is our central result. We note that the limiting behaviors $K_2(x) \to 2x^{-2}$ and $K_0(x) \to \log x$ for $x \to 0$ result in

$$F(0) \equiv \lim_{b\to 0} F(b) = -\frac{3}{8\pi^2 L_z^4} \sum_{m=1}^{\infty} \frac{1}{m^4} = -\frac{\pi^2}{240 L_z^4}, \qquad (34)$$

which retrieves the well-known result within the Maxwell electrodynamics.

The b-dependence of the Casimir force (33) is shown in Fig. 2. One can observe that the Casimir force is repulsive when $bL_z > 2.38$. By tuning the distance between two plates while keeping bL_z larger than 2.38, the strength of the repulsive Casimir force is, in principle, arbitrarily tunable. The ratio $F(b)/F(0)$ takes the minimum value -0.32 for $bL_z = 4.26$. In the physical units, this extremal value of repulsive force is estimated as $3.95 \times 10^{-5} (b^4 [\mu m^4])$ dyn/cm^2. We note that our results qualitatively match Ref. [45] for $bL_z \ll 1$. In the Appendix, we present an alternative approach developed in Ref. [45] to reproduce exactly the same numerical result of $F(b)$ as in Fig. 2. Such an independent calculation based on different subtraction procedures serves as a double check for our results and a confirmation for our scheme to subtract infinities in Eqs. (20), (25), and (32).

V. CONCLUSION

We demonstrated a repulsive component of the Casimir force in axion electrodynamics by formulating its explicit expression in an analytically closed form. We circumvented the no-go theorem which tends to forbid the repulsive Casimir force between two objects with reflection symmetry. Our underlying idea consists in the intrinsic parity symmetry breaking in the chiral vacuum between the plates, which is quite analogous to a recent proposal in Ref. [45].

Our next step is to seek for experimental realization of our theoretical consequence. Our physical setup, in which the θ-angle has a spatial gradient perpendicular to plates, would be realized through topological materials. For instance, a Weyl semimetal with the separation between Weyl nodes features the gradient of the θ-angle in the electromagnetic effective action. Besides, it has been proposed that the periodically stacked structure of trivial and topological insulators also generates the gradient of the θ-angle [72]. Another promising proposal to engender the gradient of the θ-angle is to utilize an external rotating electric field supplied by a circularly polarized laser to irradiate Dirac semimetal [90]. These examples are feasible candidates for realizing the repulsive Casimir force revealed in the present paper.

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APPENDIX: SCATTERING FORMALISM FOR THE CASIMIR FORCE

We here supply an alternative methodology to derive the result (33), referring to the scattering theory approach developed in Ref. [45]. The scenario in Ref. [45] considers a situation of inserting nontrivial electromagnetic material in between two perfect conductor plates. Concretely speaking, their material features the birefringence parametrized by a constant shift in the z-component of the wave vector, δk_z, with the dispersion relation following from classical Maxwell electrodynamics, i.e.,

$$\omega_0^2 = k_x^2 + k_y^2 + (\bar{k}_z \pm \delta k_z)^2. \tag{A1}$$

As demonstrated by Eq. (21), all possible modifications on the Casimir energy in the vacuum of axion electrodynamics are encoded in the nontrivial dispersion relation. Then, despite physical distinction between the chirality origins in their and our studies, the formula derived in Ref. [45] is directly applicable for our current setup. To this end, we just need to replace their dispersion relation (A1) with ours in Eq. (15).

Hence, we quote their expression for the Casimir energy,

$$\mathcal{E}_{\mathrm{reg}} = \int_0^\infty \frac{d\zeta}{2\pi} \int_{-\infty}^\infty \frac{dk_x dk_y}{(2\pi)^2} \log\det(\mathbb{I} - R_1 U_{12} R_2 U_{21}), \tag{A2}$$

where $\zeta = -i\omega$ is the imaginary frequency, R_1 and R_2 refer to the reflection matrix of plate 1 at $z = 0$ and plate 2 at $z = L_z$, respectively, and U_{12} and U_{21} stand for the translation matrix from plate 1 to 2 and from 2 to 1. We note that above $\mathcal{E}_{\mathrm{reg}}$ may have an L_z independent discrepancy from Eq. (32), which would make no difference in the force. For a chiral medium between two planes, the translation matrices in helicity basis read

$$U_{12} = \begin{pmatrix} e^{ik_z^+ L_z} & 0 \\ 0 & e^{ik_z^- L_z} \end{pmatrix}, \qquad U_{21} = \begin{pmatrix} e^{ik_z^- L_z} & 0 \\ 0 & e^{ik_z^+ L_z} \end{pmatrix}, \tag{A3}$$

in accordance with the plane wave ansatz in Euclidean geometry. Here, the dispersion relations (15) are expressed with $k_z^\pm$ given in terms of ζ as

$$k_z^\pm \equiv i\sqrt{\zeta^2 + \boldsymbol{k}_\perp^2 \pm ib\sqrt{\zeta^2 + \boldsymbol{k}_\perp^2}}. \tag{A4}$$

Meanwhile, for two identical perfect conductor plates, the reflection matrix can be ideally presumed as

$$R_1 = R_2 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}. \tag{A5}$$

We insert Eqs. (A3) to (A5) into Eq. (A2) to obtain the Casimir energy $\mathcal{E}_{\mathrm{reg}}$. The spatial derivative finally yields the Casimir force,

$$F = -\frac{d\mathcal{E}_{\mathrm{reg}}}{dL_z} = \int_0^\infty \frac{d\zeta}{2\pi} \int_{-\infty}^\infty \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} \times \frac{2i(k_z^+ e^{2ik_z^+ L_z} + k_z^- e^{2ik_z^- L_z}) - 2i(k_z^+ + k_z^-) e^{2i(k_z^+ + k_z^-)L_z}}{1 - e^{2ik_z^+ L_z} - e^{2ik_z^- L_z} + e^{2i(k_z^+ + k_z^-)L_z}}. \tag{A6}$$

It can be proved that Eq. (A6) is equivalent to Eq. (33), and the numerical plot matches Fig. 2 perfectly.

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