Field theories with (2,0) AdS supersymmetry in $\mathcal{N}=1$ AdS superspace

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In three dimensions, it is known that field theories possessing extended (p,q) anti-de Sitter (AdS) supersymmetry with $\mathcal{N}=p+q\geq 3$ can be realized in (2,0) AdS superspace. Here we present a formalism to reduce every field theory with (2,0) AdS supersymmetry to $\mathcal{N}=1$ AdS superspace. As nontrivial examples, we consider supersymmetric nonlinear sigma models formulated in terms of $\mathcal{N}=2$ chiral and linear supermultiplets. The $(2,0)\to(1,0)$ AdS reduction technique is then applied to the off-shell massless higher-spin supermultiplets in (2,0) AdS superspace constructed in [1]. As a result, for each superspin value $\hat{s}$, integer ($\hat{s}=s$) or half-integer ($\hat{s}=s+\frac{1}{2}$), with $s=1,2,\ldots$, we obtain two off-shell formulations for a massless $\mathcal{N}=1$ superspin-$\hat{s}$ multiplet in AdS_3. These models prove to be related to each other by a superfield Legendre transformation in the flat superspace limit, but the duality is not lifted to the AdS case. Two out of the four series of $\mathcal{N}=1$ supersymmetric higher-spin models thus derived are new. The constructed massless $\mathcal{N}=1$ supersymmetric higher-spin actions in AdS_3 are used to formulate (i) higher-spin supercurrent multiplets in $\mathcal{N}=1$ AdS superspace, and (ii) new topologically massive higher-spin off-shell supermultiplets. Examples of $\mathcal{N}=1$ higher-spin supercurrents are given for models of a complex scalar supermultiplet. We also present two new off-shell formulations for a massive $\mathcal{N}=1$ gravitino supermultiplet in AdS_3.

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I. INTRODUCTION

In three spacetime dimensions, the AdS group is a product of two simple groups,

$$\mathrm{SO}(2,2)\cong(\mathrm{SL}(2,\mathbb{R})\times\mathrm{SL}(2,\mathbb{R}))/\mathbb{Z}_2, \qquad (1.1)$$

and so are its supersymmetric extensions $\mathrm{OSp}(p|2;\mathbb{R})\times\mathrm{OSp}(q|2;\mathbb{R})$.[1] This implies that $\mathcal{N}$-extended AdS supergravity exists in several incarnations [2], which are known as the (p,q) AdS supergravity theories, where the integers $p\geq q\geq 0$ are such that $\mathcal{N}=p+q$. The so-called (p,q) AdS superspace [3]

$$\mathrm{AdS}_{(3|p,q)}=\frac{\mathrm{OSp}(p|2;\mathbb{R})\times\mathrm{OSp}(q|2;\mathbb{R})}{\mathrm{SL}(2,\mathbb{R})\times\mathrm{SO}(p)\times\mathrm{SO}(q)} \qquad (1.2)$$

may be interpreted as a maximally symmetric solution of (p,q) AdS supergravity.[2] Within the off-shell formulation for $\mathcal{N}$-extended conformal supergravity which was first sketched in [4] and then fully developed in [5], $\mathrm{AdS}_{(3|p,q)}$ originates as a maximally symmetric supergeometry with covariantly constant torsion and curvature generated by a symmetric torsion $S^{IJ}=S^{JI}$, with the structure-group indices I, J taking values from 1 to $\mathcal{N}$. It turns out that S^{IJ} is nonsingular and can be brought to the form

$$S^{IJ}=\mathcal{S}\mathrm{diag}(\overbrace{+1,\ldots,+1}^{p},\overbrace{-1,\ldots,-1}^{q=\mathcal{N}-p}), \qquad (1.3)$$

for some positive parameter $\mathcal{S}$ of unit dimension. For $p=\mathcal{N}\geq 4$ and $q=0$, there exist more general AdS superspaces [3] than the conformally flat ones defined by (1.2).

In the extended $\mathcal{N}=p+q\geq 3$ case, general (p,q) supersymmetric field theories in AdS_3 can be realized in (2,0) AdS superspace, $\mathrm{AdS}_{(3|2,0)}$ [3,6].[3] Such realizations are often useful for applications, for instance, in order to study the target space geometry of supersymmetric

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[1] More general AdS supergroups exist for $\mathcal{N}\geq 4$.

[2] In the case of $\mathcal{N}=1$ AdS supersymmetry, both notations (1,0) and $\mathcal{N}=1$ are used in the literature. We will often use the notation $\mathrm{AdS}^{3|2}$ for $\mathcal{N}=1$ AdS superspace.

[3] General aspects of (2,0) supersymmetric field theory in AdS_3 were studied in [7].

nonlinear σ-models in AdS_3 [6]. It is worth elaborating on the σ-model story in some more detail. For the $\mathcal{N}=3$ and $\mathcal{N}=4$ choices, manifestly (p,q) supersymmetric formulations have been constructed [3] for the most general nonlinear σ-models in AdS_3 (these formulations make use of the curved superspace techniques developed in [5]). This manifestly supersymmetric setting is very powerful since it allows one to generate arbitrary nonlinear σ-models with (p,q) AdS supersymmetry. However, it also has a drawback that the hyperkähler geometry of the σ-model target space is hidden. In order to uncover this geometry, the formulation of the nonlinear σ-model in (2,0) AdS superspace becomes truly indispensable [6].[4]

This work is somewhat similar in spirit to [6,13]; however, our goals are quite different. Specifically, we develop a formalism to reduce every field theory with (2,0) AdS supersymmetry to $\mathcal{N}=1$ AdS superspace. This formalism is then applied to carry out the $(2,0)\to(1,0)$ AdS reduction of the off-shell massless higher-spin supermultiplets in $\mathrm{AdS}_{(3|2,0)}$ constructed in [1]. There are at least two motivations for pursuing such an application. First, certain theoretical arguments imply that there exist more general off-shell massless higher-spin $\mathcal{N}=1$ supermultiplets in AdS_3 than those described in [14]. Second, $\mathcal{N}=1$ supermultiplets of conserved higher-spin currents have never been constructed in AdS_3 (except for the superconformal multiplets of conserved currents in Minkowski superspace [15] which can readily be lifted to AdS_3). Both issues will be addressed below. In particular, we will derive new off-shell higher-spin $\mathcal{N}=1$ supermultiplets in AdS_3, which will be used to construct new topologically massive higher-spin supermultiplets.

The table of contents reflects the structure of the paper. Our notation and conventions follow [5].

II. $(2,0)\to(1,0)$ AdS SUPERSPACE REDUCTION

The aim of this section is to elaborate on the details of the procedure for reducing the field theories in (2,0) AdS superspace to $\mathcal{N}=1$ AdS superspace. Explicit examples of such a reduction are given by considering supersymmetric nonlinear σ-models.

[4]Analogous results exist in four dimensions. The most general $\mathcal{N}=2$ supersymmetric σ-model in AdS_4 was constructed [8,9] using a formulation in terms of $\mathcal{N}=1$ covariantly chiral superfields, as an extension of the earlier analysis in the super-Poincaré case [10,11]. One of the main virtues of the $\mathcal{N}=1$ formulation [8,9] is its geometric character; however the second supersymmetry is hidden. General off-shell $\mathcal{N}=2$ supersymmetric σ-models in AdS_4 were actually formulated a few years earlier [12] in $\mathcal{N}=2$ AdS superspace. The latter approach makes $\mathcal{N}=2$ supersymmetry manifest, but the hyperkähler geometry of the σ-model target space is hidden. The two σ-model formulations are related via the $\mathcal{N}=2\to\mathcal{N}=1$ AdS superspace reduction [13].

A. Geometry of (2,0) AdS superspace: Complex basis

We begin by briefly reviewing the key results concerning (2,0) AdS superspace; see [6,7] for the details. There are two ways to describe the geometry of (2,0) AdS superspace, which correspond to making use of either a real or a complex basis for the spinor covariant derivatives. We first consider the formulation in the complex basis.

The covariant derivatives of (2,0) AdS superspace are

$$\mathcal{D}_{\mathcal{A}}=(\mathcal{D}_a,\mathcal{D}_\alpha,\bar{\mathcal{D}}^\alpha)=E_{\mathcal{A}}+\Omega_{\mathcal{A}}+\mathrm{i}\Phi_{\mathcal{A}}J,\qquad E_{\mathcal{A}}=E_{\mathcal{A}}{}^{\mathcal{M}}\frac{\partial}{\partial z^{\mathcal{M}}},\tag{2.1}$$

where $z^{\mathcal{M}}=(x^m,\theta^\mu,\bar{\theta}_\mu)$ are local superspace coordinates, and J is the generator of the R-symmetry group, $\mathrm{U}(1)_R$. The generator J is defined to act on the covariant derivatives as follows:

$$[J,\mathcal{D}_\alpha]=\mathcal{D}_\alpha,\qquad [J,\bar{\mathcal{D}}^\alpha]=-\bar{\mathcal{D}}^\alpha,\qquad [J,\mathcal{D}_a]=0.\tag{2.2}$$

The Lorentz connection, $\Omega_{\mathcal{A}}$, can be written in several equivalent forms, which are

$$\Omega_{\mathcal{A}}=\frac{1}{2}\Omega_{\mathcal{A}}{}^{bc}M_{bc}=-\Omega_{\mathcal{A}}{}^{b}M_b=\frac{1}{2}\Omega_{\mathcal{A}}{}^{\beta\gamma}M_{\beta\gamma}.\tag{2.3}$$

The relations between the Lorentz generators with two vector indices ($M_{ab}=-M_{ba}$), one vector index (M_a) and two spinor indices ($M_{\alpha\beta}=M_{\beta\alpha}$) are given in the Appendix A.

The covariant derivatives of (2,0) AdS superspace obey the following graded commutation relations:

$$\{\mathcal{D}_\alpha,\mathcal{D}_\beta\}=0,\qquad \{\bar{\mathcal{D}}_\alpha,\bar{\mathcal{D}}_\beta\}=0,\tag{2.4a}$$

$$\{\mathcal{D}_\alpha,\bar{\mathcal{D}}_\beta\}=-2\mathrm{i}(\mathcal{D}_{\alpha\beta}-2\mathcal{S}M_{\alpha\beta})-4\mathrm{i}\varepsilon_{\alpha\beta}\mathcal{S}J,\tag{2.4b}$$

$$[\mathcal{D}_a,\mathcal{D}_\beta]=(\gamma_a)_\beta{}^\gamma\mathcal{S}\mathcal{D}_\gamma,\quad [\mathcal{D}_a,\bar{\mathcal{D}}_\beta]=(\gamma_a)_\beta{}^\gamma\mathcal{S}\bar{\mathcal{D}}_\gamma,\tag{2.4c}$$

$$[\mathcal{D}_a,\mathcal{D}_b]=-4\mathcal{S}^2M_{ab}.\tag{2.4d}$$

Here the parameter $\mathcal{S}$ is related to the AdS scalar curvature as $R=-24\mathcal{S}^2$.

There exists a universal formalism to determine isometries of curved superspace backgrounds in diverse dimensions [16,17]. This formalism was used in [7] to compute the isometries of (2,0) AdS superspace (as well as supersymmetric backgrounds in off-shell $\mathcal{N}=2$ supergravity theories [18]). The isometries of (2,0) AdS superspace are generated by the Killing supervector fields $\zeta^{\mathcal{A}}E_{\mathcal{A}}$, which are defined to solve the master equation

$$\left[\zeta + \frac{1}{2}l^{bc}M_{bc} + \mathrm{i}\tau J, \mathcal{D}_{\mathcal{A}}\right] = 0, \tag{2.5a}$$

where

$$\zeta = \zeta^{\mathcal{B}}\mathcal{D}_{\mathcal{B}} = \zeta^b\mathcal{D}_b + \zeta^\beta\mathcal{D}_\beta + \bar{\zeta}_\beta\bar{\mathcal{D}}^\beta, \quad \overline{\zeta^b} = \zeta^b, \tag{2.5b}$$

and τ and l^{bc} are some real $U(1)_R$ and Lorentz superfield parameters, respectively. It follows from Eq. (2.5) that the parameters ζ_α, τ and $l_{\alpha\beta}$ are uniquely expressed in terms of the vector parameter $\zeta_{\alpha\beta}$ as follows:

$$\zeta_\alpha = \frac{\mathrm{i}}{6}\bar{\mathcal{D}}^\beta\zeta_{\beta\alpha}, \qquad \tau = \frac{\mathrm{i}}{2}\mathcal{D}^\alpha\zeta_\alpha, \qquad l_{\alpha\beta} = 2(\mathcal{D}_{(\alpha}\zeta_{\beta)} - \mathcal{S}\zeta_{\alpha\beta}). \tag{2.6}$$

The vector parameter $\zeta_{\alpha\beta}$ satisfies the equation

$$\mathcal{D}_{(\alpha}\zeta_{\beta\gamma)} = 0. \tag{2.7}$$

This implies the standard Killing equation,

$$\mathcal{D}_a\zeta_b + \mathcal{D}_b\zeta_a = 0. \tag{2.8}$$

One may also prove the following relations:

$$\bar{\mathcal{D}}_\alpha\tau = \frac{\mathrm{i}}{3}\bar{\mathcal{D}}^\beta l_{\alpha\beta} = 4\mathcal{S}\zeta_\alpha, \qquad \bar{\mathcal{D}}_\alpha\zeta_\beta = 0, \qquad \mathcal{D}_{(\alpha}l_{\beta\gamma)} = 0. \tag{2.9}$$

The Killing supervector fields prove to generate the supergroup $\mathrm{OSp}(2|2;\mathbb{R}) \times \mathrm{Sp}(2,\mathbb{R})$, the isometry group of (2,0) AdS superspace. Rigid supersymmetric field theories in (2,0) AdS superspace are required to be invariant under the isometry transformations. An infinitesimal isometry transformation acts on a tensor superfield $\boldsymbol{U}$ (with suppressed indices) by the rule

$$\delta_\zeta\boldsymbol{U} = \left(\zeta + \frac{1}{2}l^{bc}M_{bc} + \mathrm{i}\tau J\right)\boldsymbol{U}. \tag{2.10}$$

B. Geometry of (2,0) AdS superspace: Real basis

Instead of dealing with the complex basis for the (2,0) AdS spinor covariant derivatives, Eq. (2.1), it is more convenient to switch to a real basis in order to carry out reduction to $\mathcal{N} = 1$ AdS superspace $\mathrm{AdS}^{3|2}$. Following [3], such a basis is introduced by replacing the complex operators $\mathcal{D}_\alpha$ and $\bar{\mathcal{D}}_\alpha$ with $\boldsymbol{\nabla}^I_\alpha = (\boldsymbol{\nabla}^{\underline{1}}_\alpha, \boldsymbol{\nabla}^{\underline{2}}_\alpha)$ defined as follows:

$$\mathcal{D}_\alpha = \frac{1}{\sqrt{2}}(\boldsymbol{\nabla}^{\underline{1}}_\alpha - \mathrm{i}\boldsymbol{\nabla}^{\underline{2}}_\alpha), \quad \bar{\mathcal{D}}_\alpha = -\frac{1}{\sqrt{2}}(\boldsymbol{\nabla}^{\underline{1}}_\alpha + \mathrm{i}\boldsymbol{\nabla}^{\underline{2}}_\alpha). \tag{2.11}$$

In a similar way we introduce real coordinates, $z^{\mathcal{M}} = (x^m, \theta^\mu_I)$, to parametrize (2,0) AdS superspace. Defining $\boldsymbol{\nabla}_a = \mathcal{D}_a$, the algebra of (2,0) AdS covariant derivatives (2.4) turns into[5]

$$\{\boldsymbol{\nabla}^I_\alpha, \boldsymbol{\nabla}^J_\beta\} = 2\mathrm{i}\delta^{IJ}\boldsymbol{\nabla}_{\alpha\beta} - 4\mathrm{i}\delta^{IJ}\mathcal{S}M_{\alpha\beta} + 4\varepsilon_{\alpha\beta}\varepsilon^{IJ}\mathcal{S}J, \tag{2.12a}$$

$$[\boldsymbol{\nabla}_a, \boldsymbol{\nabla}^J_\beta] = \mathcal{S}(\gamma_a)_\beta{}^\gamma\boldsymbol{\nabla}^J_\gamma, \quad [\boldsymbol{\nabla}_a, \boldsymbol{\nabla}_b] = -4\mathcal{S}^2M_{ab}. \tag{2.12b}$$

The action of the $\mathrm{U}(1)_R$ generator on the spinor covariant derivatives is given by

$$[J, \boldsymbol{\nabla}^I_\alpha] = -\mathrm{i}\varepsilon_{IJ}\boldsymbol{\nabla}^J_\alpha. \tag{2.13}$$

As may be seen from (2.12), the graded commutation relations for the operators $\boldsymbol{\nabla}_a$ and $\boldsymbol{\nabla}^{\underline{1}}_\alpha$ have the following properties:

(1) These (anti)commutation relations do not involve $\boldsymbol{\nabla}^{\underline{2}}_\alpha$,

$$\{\boldsymbol{\nabla}^{\underline{1}}_\alpha, \boldsymbol{\nabla}^{\underline{1}}_\beta\} = 2\mathrm{i}\boldsymbol{\nabla}_{\alpha\beta} - 4\mathrm{i}\mathcal{S}M_{\alpha\beta}, \tag{2.14a}$$

$$[\boldsymbol{\nabla}_a, \boldsymbol{\nabla}^{\underline{1}}_\beta] = \mathcal{S}(\gamma_a)_\beta{}^\gamma\boldsymbol{\nabla}^{\underline{1}}_\gamma, \qquad [\boldsymbol{\nabla}_a, \boldsymbol{\nabla}_b] = -4\mathcal{S}^2M_{ab}. \tag{2.14b}$$

(2) Relations (2.14) are identical to the algebra of the covariant derivatives of $\mathrm{AdS}^{3|2}$, see (2.4).

We thus see that $\mathrm{AdS}^{3|2}$ is naturally embedded in (2,0) AdS superspace as a subspace. The real Grassmann variables of (2,0) AdS superspace, $\theta^\mu_I = (\theta^\mu_{\underline{1}}, \theta^\mu_{\underline{2}})$, may be chosen in such a way that $\mathrm{AdS}^{3|2}$ corresponds to the surface defined by $\theta^\mu_{\underline{2}} = 0$. We also note that no $\mathrm{U}(1)_R$ curvature is present in the algebra of $\mathcal{N} = 1$ AdS covariant derivatives. These properties make possible a consistent $(2,0) \to (1,0)$ AdS superspace reduction.

Now we will recast the fundamental properties of the (2,0) AdS Killing supervector fields in the real representation (2.11). The isometries of (2,0) AdS superspace are described in terms of those first-order operators

$$\zeta := \zeta^{\mathcal{B}}\boldsymbol{\nabla}_{\mathcal{B}} = \zeta^b\boldsymbol{\nabla}_b + \zeta^\beta_J\boldsymbol{\nabla}^J_\beta, \qquad J = \underline{1}, \underline{2}, \tag{2.15a}$$

which solve the equation

$$\left[\zeta + \frac{1}{2}l^{bc}M_{bc} + \mathrm{i}\tau J, \boldsymbol{\nabla}_{\mathcal{A}}\right] = 0, \tag{2.15b}$$

for some real parameters τ and $l^{ab} = -l^{ba}$. Equation (2.15b) is equivalent to

[5] The antisymmetric tensors ε^{IJ} and ε_{IJ} are normalized as $\varepsilon^{\underline{12}} = \varepsilon_{\underline{12}} = 1$.

$$\nabla^I_\alpha \zeta^J_\beta = -\varepsilon_{\alpha\beta}\varepsilon^{IJ}\tau + \mathcal{S}\delta^{IJ}\zeta_{\alpha\beta} + \frac{1}{2}\delta^{IJ} l_{\alpha\beta}, \tag{2.16a}$$

$$\nabla^I_\alpha \zeta_b = 2\mathrm{i}\zeta^{\beta I}(\gamma_b)_{\alpha\beta}, \tag{2.16b}$$

$$\nabla^I_\alpha \tau = -4\mathrm{i}\mathcal{S}\varepsilon^{IJ}\zeta_{\alpha J}, \tag{2.16c}$$

$$\nabla^I_\alpha l_{\beta\gamma} = 8\mathrm{i}\mathcal{S}\varepsilon_{\alpha(\beta}\zeta^I_{\gamma)}, \tag{2.16d}$$

and

$$\nabla_a \zeta_b = l_{ab} = -l_{ba}, \tag{2.17a}$$

$$\nabla_a \zeta^\beta_I = -\mathcal{S}\zeta^\alpha_I(\gamma_a)_\alpha{}^\beta, \tag{2.17b}$$

$$\nabla_a \tau = 0, \tag{2.17c}$$

$$\nabla_a l^{bc} = 4\mathcal{S}^2(\delta^b_a\zeta^c - \delta^c_a\zeta^b). \tag{2.17d}$$

Some nontrivial implications of the above equations which will be important for our subsequent consideration are

$$\nabla^I_{(\alpha}\zeta_{\beta\gamma)} = 0, \qquad \nabla^I_{(\alpha}l_{\beta\gamma)} = 0, \tag{2.18a}$$

$$\nabla^I_{(\alpha}\zeta^J_{\beta)} = 2\mathcal{S}\delta^{IJ}\zeta_{\alpha\beta}, \qquad \nabla^{\gamma(I}\zeta^{J)}_\gamma = 0, \tag{2.18b}$$

$$\zeta^{I\alpha} = \frac{\mathrm{i}}{6}\nabla^I_\beta\zeta^{\alpha\beta} = \frac{\mathrm{i}}{12\mathcal{S}}\nabla^I_\beta l^{\alpha\beta} = -\frac{\mathrm{i}}{4\mathcal{S}}\varepsilon^{IJ}\nabla^\alpha_J\tau, \tag{2.18c}$$

$$\tau = -\frac{1}{4}\varepsilon_{IJ}\nabla^{\gamma I}\zeta^J_\gamma. \tag{2.18d}$$

Equation (2.17) implies that ζ_a is a Killing vector field,

$$\nabla_a\zeta_b + \nabla_b\zeta_a = 0, \tag{2.19}$$

while (2.17b) is a Killing spinor equation. The real parameter τ is constrained by

$$(\nabla^{\underline{2}})^2\tau = (\nabla^{\underline{1}})^2\tau = 8\mathrm{i}\mathcal{S}\tau, \qquad \nabla_a\tau = 0. \tag{2.20}$$

C. Reduction from (2,0) to $\mathcal{N}=1$ AdS superspace

Given a tensor superfield $U(x,\theta_I)$ on (2,0) AdS superspace, its $\mathcal{N}=1$ projection (or bar-projection) is defined by

$$U| := U(x,\theta_I)|_{\theta_{\underline{2}}=0} \tag{2.21}$$

in a *special coordinate system* to be specified below. By definition, $U|$ depends on the real coordinates $z^M = (x^m, \theta^\mu)$, with $\theta^\mu := \theta^\mu_{\underline{1}}$, which will be used to parametrize $\mathcal{N}=1$ AdS superspace $\mathrm{AdS}^{3|2}$. For the (2,0) AdS covariant derivative

$$\boldsymbol{\nabla}_\mathcal{A} = (\boldsymbol{\nabla}_a, \boldsymbol{\nabla}^I_\alpha) = E_\mathcal{A}{}^\mathcal{M}\frac{\partial}{\partial z^\mathcal{M}} + \frac{1}{2}\Omega_\mathcal{A}{}^{bc}M_{bc} + \mathrm{i}\Phi_\mathcal{A}J, \tag{2.22}$$

its bar-projection is defined as

$$\boldsymbol{\nabla}_\mathcal{A}| = E_\mathcal{A}{}^\mathcal{M}|\frac{\partial}{\partial z^\mathcal{M}} + \frac{1}{2}\Omega_\mathcal{A}{}^{bc}|M_{bc} + \mathrm{i}\Phi_\mathcal{A}|J. \tag{2.23}$$

We use the freedom to perform general coordinate, local Lorentz and $\mathrm{U}(1)_R$ transformations to choose the following gauge condition:

$$\boldsymbol{\nabla}_a| = \nabla_a, \qquad \boldsymbol{\nabla}^{\underline{1}}_\alpha| = \nabla_\alpha, \tag{2.24}$$

where

$$\nabla_A = (\nabla_a, \nabla_\alpha) = E_A{}^M\frac{\partial}{\partial z^M} + \frac{1}{2}\omega_A{}^{bc}M_{bc} \tag{2.25}$$

denotes the set of covariant derivatives for $\mathrm{AdS}^{3|2}$, which obey the following graded commutation relations:

$$\{\nabla_\alpha, \nabla_\beta\} = 2\mathrm{i}\nabla_{\alpha\beta} - 4\mathrm{i}\mathcal{S}M_{\alpha\beta}, \tag{2.26a}$$

$$[\nabla_a, \nabla_\beta] = \mathcal{S}(\gamma_a)_\beta{}^\gamma\nabla_\gamma, \quad [\nabla_a, \nabla_b] = -4\mathcal{S}^2M_{ab}. \tag{2.26b}$$

In such a coordinate system, the operator $\boldsymbol{\nabla}^{\underline{1}}_\alpha|$ contains no partial derivative with respect to $\theta_{\underline{2}}$. As a consequence, $(\boldsymbol{\nabla}^{\underline{1}}_{\alpha_1}\cdots\boldsymbol{\nabla}^{\underline{1}}_{\alpha_k}U)| = \nabla_{\alpha_1}\cdots\nabla_{\alpha_k}U|$, for any positive integer k, where U is a tensor superfield on (2,0) AdS superspace. Let us study how the $\mathcal{N}=1$ descendants of U defined by $U_{\alpha_1\ldots\alpha_k} := (\boldsymbol{\nabla}^{\underline{2}}_{\alpha_1}\cdots\boldsymbol{\nabla}^{\underline{2}}_{\alpha_k}U)|$ transform under the (2,0) AdS isometries, with k a non-negative integer.

We introduce the $\mathcal{N}=1$ projection of the (2,0) AdS Killing supervector field (2.15)

$$\zeta| = \xi^b\nabla_b + \xi^\beta\nabla_\beta + \epsilon^\beta\boldsymbol{\nabla}^{\underline{2}}_\beta|, \qquad \xi^b := \zeta^b|,$$
$$\xi^\beta := \zeta^\beta_{\underline{1}}|, \qquad \epsilon^\beta := \zeta^\beta_{\underline{2}}|. \tag{2.27}$$

We also introduce the $\mathcal{N}=1$ projections of the Lorentz and $\mathrm{U}(1)_R$ parameters in (2.15):

$$\lambda^{bc} := l^{bc}|, \qquad \epsilon := \tau|. \tag{2.28}$$

It follows from (2.15) that the $\mathcal{N}=1$ parameters $\xi^B = (\xi^b, \xi^\beta)$ and λ^{bc} obey the equation

$$\left[\xi + \frac{1}{2}\lambda^{bc}M_{bc}, \nabla_A\right] = 0, \qquad \xi = \xi^B\nabla_B = \xi^b\nabla_b + \xi^\beta\nabla_\beta, \tag{2.29}$$

which tells us that ξ^B is a Killing supervector field of $\mathcal{N}=1$ AdS superspace [3]. This equation is equivalent to

$$\nabla_{(\alpha}\xi_{\beta\gamma)}=0,\qquad \nabla_\beta\xi^{\beta\alpha}=-6\mathrm{i}\xi^\alpha, \tag{2.30a}$$

$$\nabla_\alpha\xi_\beta=\frac{1}{2}\lambda_{\alpha\beta}+\mathcal{S}\xi_{\alpha\beta}, \tag{2.30b}$$

$$\nabla_{(\alpha}\lambda_{\beta\gamma)}=0,\qquad \nabla_\beta\lambda^{\beta\alpha}=-12\mathrm{i}\mathcal{S}\xi^\alpha. \tag{2.30c}$$

These relations automatically follow from the (2,0) AdS Killing equations, Eqs. (2.16a)–(2.16d), upon $\mathcal{N}=1$ projection. Thus $(\xi^a,\xi^\alpha,\lambda^{ab})$ parametrize the infinitesimal isometries of $\mathrm{AdS}^{3|2}$ [3] (see also [14]).

The remaining parameters ϵ^α and ϵ generate the second supersymmetry and $\mathrm{U}(1)_R$ transformations, respectively. Using the Killing equations (2.18), it can be shown that they satisfy the following properties:

$$\epsilon_\alpha=\frac{\mathrm{i}}{4\mathcal{S}}\nabla_\alpha\epsilon,\qquad \epsilon=-\frac{1}{2}\nabla^\alpha\epsilon_\alpha, \tag{2.31a}$$

$$(\mathrm{i}\nabla^2+8\mathcal{S})\epsilon=0,\qquad \nabla_a\epsilon=0. \tag{2.31b}$$

These imply that the only independent components of ϵ are $\epsilon|_{\theta=0}$ and $\nabla_\alpha\epsilon|_{\theta=0}$. They correspond to the $\mathrm{U}(1)_R$ and second supersymmetry transformations, respectively.

Given a matter tensor superfield $\boldsymbol{U}$, its (2,0) AdS transformation law

$$\delta_\zeta\boldsymbol{U}=\left(\zeta+\frac{1}{2}l^{bc}M_{bc}+\mathrm{i}\tau J\right)\boldsymbol{U} \tag{2.32}$$

turns into

$$\delta_\zeta\boldsymbol{U}|=\delta_\xi\boldsymbol{U}|+\delta_\epsilon\boldsymbol{U}|, \tag{2.33a}$$

$$\delta_\xi\boldsymbol{U}|=\left(\xi^b\nabla_b+\xi^\beta\nabla_\beta+\frac{1}{2}\lambda^{bc}M_{bc}\right)\boldsymbol{U}|, \tag{2.33b}$$

$$\delta_\epsilon\boldsymbol{U}|=(\epsilon^\beta(\boldsymbol{\nabla}^2_\beta\boldsymbol{U})|+\mathrm{i}\epsilon J\boldsymbol{U}|). \tag{2.33c}$$

It follows from (2.15) and (2.33) that every $\mathcal{N}=1$ descendant $U_{\alpha_1\ldots\alpha_k}:=(\boldsymbol{\nabla}^2_{\alpha_1}\cdots\boldsymbol{\nabla}^2_{\alpha_k}\boldsymbol{U})|$ is a tensor superfield on $\mathrm{AdS}^{3|2}$,

$$\delta_\xi U_{\alpha_1\ldots\alpha_k}=\left(\xi^b\nabla_b+\xi^\beta\nabla_\beta+\frac{1}{2}\lambda^{bc}M_{bc}\right)U_{\alpha_1\ldots\alpha_k}. \tag{2.34}$$

For the ϵ-transformation we get

$$\begin{aligned}\delta_\epsilon U_{\alpha_1\ldots\alpha_k}&=\epsilon^\beta(\boldsymbol{\nabla}^2_\beta\boldsymbol{\nabla}^2_{\alpha_1}\cdots\boldsymbol{\nabla}^2_{\alpha_k}\boldsymbol{U})|+\mathrm{i}\epsilon(J\boldsymbol{\nabla}^2_{\alpha_1}\cdots\boldsymbol{\nabla}^2_{\alpha_k}\boldsymbol{U})|\\&=\epsilon^\beta U_{\beta\alpha_1\ldots\alpha_k}-\epsilon\sum_{l=1}^{k}\boldsymbol{\nabla}^2_{\alpha_1}\cdots\boldsymbol{\nabla}^2_{\alpha_{l-1}}\boldsymbol{\nabla}^1_{\alpha_l}\boldsymbol{\nabla}^2_{\alpha_{l+1}}\\&\quad\cdots\boldsymbol{\nabla}^2_{\alpha_k}\boldsymbol{U})|+\mathrm{i}q\epsilon U_{\alpha_1\ldots\alpha_k},\end{aligned} \tag{2.35}$$

where q is the $\mathrm{U}(1)_R$ charge of $\boldsymbol{U}$ defined by $J\boldsymbol{U}=q\boldsymbol{U}$. In the second term on the right, we have to push $\boldsymbol{\nabla}^1_{\alpha_l}$ to the far left through the $(l-1)$ factors of $\boldsymbol{\nabla}^2$'s by making use of the relation $\{\boldsymbol{\nabla}^1_\alpha,\boldsymbol{\nabla}^2_\beta\}=4\varepsilon_{\alpha\beta}\mathcal{S}J$ and taking into account the relation

$$(\boldsymbol{\nabla}^1_{\alpha_l}\boldsymbol{\nabla}^2_{\alpha_1}\cdots\boldsymbol{\nabla}^2_{\alpha_{l-1}}\boldsymbol{\nabla}^2_{\alpha_{l+1}}\cdots\boldsymbol{\nabla}^2_{\alpha_k}\boldsymbol{U})|=\nabla_{\alpha_l}U_{\alpha_1\ldots\alpha_{l-1}\alpha_{l+1}\ldots\alpha_k}. \tag{2.36}$$

As the next step, the $\mathrm{U}(1)_R$ generator J should be pushed to the right until it hits $\boldsymbol{U}$ producing on the way insertions of $\boldsymbol{\nabla}^1$. Then the procedure should be repeated. As a result, the variation $\delta_\epsilon U_{\alpha_1\ldots\alpha_k}$ is expressed in terms of the superfields $U_{\alpha_1\ldots\alpha_{k+1}},U_{\alpha_1\ldots\alpha_k},\cdots U_{\alpha_1},U$.

So far we have been completely general and discussed infinitely many descendants $U_{\alpha_1\ldots\alpha_k}$ of $\boldsymbol{U}$. However only a few of them are functionally independent. Indeed, Eq. (2.12a) tells us that

$$\{\boldsymbol{\nabla}^2_\alpha,\boldsymbol{\nabla}^2_\beta\}=2\mathrm{i}\boldsymbol{\nabla}_{\alpha\beta}-4\mathrm{i}\mathcal{S}M_{\alpha\beta}, \tag{2.37}$$

and thus every $U_{\alpha_1\ldots\alpha_k}$ for $k>2$ can be expressed in terms of U, U_α and $U_{\alpha_1\alpha_2}$. Therefore, it suffices to consider $k\le 2$.

Let us give two examples of matter superfields on (2,0) AdS superspace. We first consider a covariantly chiral scalar superfield $\boldsymbol{\phi}$, $\bar{\mathcal{D}}_\alpha\boldsymbol{\phi}=0$, with an arbitrary $\mathrm{U}(1)_R$ charge q defined by $J\boldsymbol{\phi}=q\boldsymbol{\phi}$. It transforms under the (2,0) AdS isometries as

$$\delta_\zeta\boldsymbol{\phi}=(\zeta+\mathrm{i}q\tau)\boldsymbol{\phi}. \tag{2.38}$$

When expressed in the real basis (2.11), the chirality constraint on $\boldsymbol{\phi}$ means

$$\boldsymbol{\nabla}^2_\alpha\boldsymbol{\phi}=\mathrm{i}\boldsymbol{\nabla}^1_\alpha\boldsymbol{\phi}. \tag{2.39}$$

As a result, there is only one independent $\mathcal{N}=1$ superfield upon reduction,

$$\varphi:=\boldsymbol{\phi}|. \tag{2.40}$$

We then get the following relations:

$$\boldsymbol{\nabla}^2_\alpha\boldsymbol{\phi}|=\mathrm{i}\nabla_\alpha\varphi, \tag{2.41a}$$

$$(\boldsymbol{\nabla}^2)^2\boldsymbol{\phi}|=-\nabla^2\varphi-8\mathrm{i}q\mathcal{S}\varphi. \tag{2.41b}$$

The ϵ-transformation (2.35) is given by

$$\delta_\epsilon \varphi = \mathrm{i}\epsilon^\beta \nabla_\beta \varphi + \mathrm{i}q\epsilon\varphi. \tag{2.42}$$

Our second example is a real linear superfield $\mathbb{L} = \bar{\mathbb{L}}, \bar{\mathcal{D}}^2 \mathbb{L} = 0$. The real linearity constraint relates the $\mathcal{N} = 1$ descendants of $\mathbb{L}$ as follows:

$$(\boldsymbol{\nabla}^{\underline{2}})^2 \mathbb{L} = (\boldsymbol{\nabla}^{\underline{1}})^2 \mathbb{L}, \tag{2.43a}$$

$$\boldsymbol{\nabla}^{\underline{1}\beta} \boldsymbol{\nabla}^{\underline{2}}_\beta \mathbb{L} = 0. \tag{2.43b}$$

Thus, $\mathbb{L}$ is equivalent to two independent, real $\mathcal{N} = 1$ superfields:

$$X \coloneqq \mathbb{L}|, \qquad W_\alpha \coloneqq \mathrm{i}\boldsymbol{\nabla}^{\underline{2}}_\alpha \mathbb{L}|. \tag{2.44}$$

Here X is unconstrained, while W_α obeys the constraint (2.43b)

$$\nabla^\alpha W_\alpha = 0, \tag{2.45}$$

which means that W_α is the field strength of an $\mathcal{N} = 1$ vector multiplet. Since $\mathbb{L}$ is neutral under the R-symmetry group $\mathrm{U}(1)_R$, $J\mathbb{L} = 0$, the second SUSY and $\mathrm{U}(1)_R$ transformation laws of the $\mathcal{N} = 1$ descendants of $\mathbb{L}$ are as follows:

$$\delta_\epsilon X = \delta_\epsilon \mathbb{L}| = \epsilon^\beta (\boldsymbol{\nabla}^{\underline{2}}_\beta \mathbb{L})| = -\mathrm{i}\epsilon^\beta W_\beta, \tag{2.46a}$$

$$\begin{aligned}\delta_\epsilon W_\alpha &= \mathrm{i}(\boldsymbol{\nabla}^{\underline{2}}_\alpha \delta_\epsilon \mathbb{L})| = \mathrm{i}\epsilon^\beta (\boldsymbol{\nabla}^{\underline{2}}_\beta \boldsymbol{\nabla}^{\underline{2}}_\alpha \mathbb{L})| - \epsilon[J, \boldsymbol{\nabla}^{\underline{2}}_\alpha]\mathbb{L}| \\ &= -\epsilon^\beta \nabla_{\alpha\beta} X - \frac{\mathrm{i}}{2}\epsilon_\alpha \nabla^2 X - \mathrm{i}\epsilon \nabla_\alpha X.\end{aligned} \tag{2.46b}$$

D. The (2,0) AdS supersymmetric actions in $\mathrm{AdS}^{3|2}$

Every rigid supersymmetric field theory in (2,0) AdS superspace may be reduced to $\mathcal{N} = 1$ AdS superspace. Here we provide the key technical details of the reduction.

In accordance with [5–7,18], there are two ways of constructing supersymmetric actions in (2,0) AdS superspace: (i) either by integrating a real scalar $\mathcal{L}$ over the full (2,0) AdS superspace,[6]

$$\begin{aligned} S &= \int \mathrm{d}^3x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} \boldsymbol{E} \mathcal{L} = \frac{1}{16}\int \mathrm{d}^3x e \mathcal{D}^2 \bar{\mathcal{D}}^2 \mathcal{L}|_{\theta=0} \\ &= \frac{1}{16}\int \mathrm{d}^3x e \bar{\mathcal{D}}^2 \mathcal{D}^2 \mathcal{L}|_{\theta=0} \\ &= \int \mathrm{d}^3x e \left(\frac{1}{16}\mathcal{D}^\alpha \bar{\mathcal{D}}^2 \mathcal{D}_\alpha + \mathrm{i}\mathcal{S}\bar{\mathcal{D}}^\alpha \mathcal{D}_\alpha\right)\mathcal{L}|_{\theta=0} \\ &= \int \mathrm{d}^3x e \left(\frac{1}{16}\bar{\mathcal{D}}_\alpha \mathcal{D}^2 \bar{\mathcal{D}}^\alpha + \mathrm{i}\mathcal{S}\mathcal{D}^\alpha \bar{\mathcal{D}}_\alpha\right)\mathcal{L}|_{\theta=0}, \end{aligned} \tag{2.47}$$

[6] The component inverse vierbein is defined as usual, $e_a{}^m(x) = E_a{}^m|_{\theta=0}$, with $e^{-1} = \det(e_a{}^m)$.

with $\boldsymbol{E}^{-1} = \mathrm{Ber}(E_\mathcal{A}{}^\mathcal{M})$; or (ii) by integrating a covariantly chiral scalar $\mathcal{L}_\mathrm{c}$ over the chiral subspace of the (2,0) AdS superspace,

$$S_\mathrm{c} = \int \mathrm{d}^3x \mathrm{d}^2\theta \mathcal{E} \mathcal{L}_\mathrm{c} = -\frac{1}{4}\int \mathrm{d}^3x e \mathcal{D}^2 \mathcal{L}_\mathrm{c}|_{\theta=0}, \quad \bar{\mathcal{D}}^\alpha \mathcal{L}_\mathrm{c} = 0, \tag{2.48}$$

with $\mathcal{E}$ being the chiral density. The superfield Lagrangians $\mathcal{L}$ and $\mathcal{L}_c$ are neutral and charged, respectively with respect to the group $\mathrm{U}(1)_R$:

$$J\mathcal{L} = 0, \qquad J\mathcal{L}_\mathrm{c} = -2\mathcal{L}_\mathrm{c}. \tag{2.49}$$

The two types of supersymmetric actions are related to each other by the rule

$$\int \mathrm{d}^3x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} \boldsymbol{E} \mathcal{L} = \int \mathrm{d}^3x \mathrm{d}^2\theta \mathcal{E} \mathcal{L}_\mathrm{c}, \qquad \mathcal{L}_\mathrm{c} \coloneqq -\frac{1}{4}\bar{\mathcal{D}}^2 \mathcal{L}. \tag{2.50}$$

Instead of reducing the above actions to components, in this paper we need their reduction to $\mathcal{N} = 1$ AdS superspace. We remind the reader that the supersymmetric action in $\mathrm{AdS}^{3|2}$ makes use of a real scalar Lagrangian L. The superspace and component forms of the action are

$$S = \int \mathrm{d}^{3|2}z E L = \frac{1}{4}\int \mathrm{d}^3x e (\mathrm{i}\nabla^2 + 8\mathcal{S}) L|_{\theta=0}. \tag{2.51}$$

For the action (2.47) we get

$$S = \int \mathrm{d}^3x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} \boldsymbol{E} \mathcal{L} = -\frac{\mathrm{i}}{4}\int \mathrm{d}^{3|2}z E (\boldsymbol{\nabla}^{\underline{2}})^2 \mathcal{L}|, \tag{2.52}$$

with $E^{-1} = \mathrm{Ber}(E_A{}^M)$. The chiral action (2.48) reduces to $\mathrm{AdS}^{3|2}$ as follows:

$$S_\mathrm{c} = \int \mathrm{d}^3x \mathrm{d}^2\theta \mathcal{E} \mathcal{L}_\mathrm{c} = 2\mathrm{i}\int \mathrm{d}^{3|2}z E \mathcal{L}_\mathrm{c}|. \tag{2.53}$$

Making use of the (2,0) AdS transformation law $\delta\mathcal{L} = \zeta\mathcal{L}$, $\delta\mathcal{L}_\mathrm{c} = (\zeta - 2\mathrm{i}\tau)\mathcal{L}_\mathrm{c}$, and the Killing equation (2.15b), it can be checked explicitly that the $\mathcal{N} = 1$ action defined by the right-hand side of (2.52), or (2.53) are invariant under the (2,0) AdS isometry transformations.

E. Supersymmetric nonlinear sigma models

To illustrate the $(2,0) \to (1,0)$ AdS superspace reduction described above, here we discuss two interesting examples.

Our first example is a general nonlinear σ-model with (2,0) AdS supersymmetry [6,7]. It is described by the action

$$S=\int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\boldsymbol{E}K(\phi^i,\bar{\phi}^{\bar{j}})+\left\{\int \mathrm{d}^3x\mathrm{d}^2\theta\mathcal{E}W(\phi^i)+\mathrm{c.c}\right\},$$
$$\bar{\mathcal{D}}_\alpha\phi^i=0, \tag{2.54}$$

where $K(\phi^i,\bar{\phi}^{\bar{j}})$ is the Kähler potential of a Kähler manifold and $W(\phi^i)$ is a superpotential. The $\mathrm{U}(1)_R$ generator is realized on the dynamical superfields ϕ^i and $\bar{\phi}^{\bar{i}}$ as

$$\mathrm{i}J=\mathfrak{J}^i(\phi)\partial_i+\bar{\mathfrak{J}}^{\bar{i}}(\bar{\phi})\partial_{\bar{i}}, \tag{2.55}$$

where $\mathfrak{J}^i(\phi)$ is a holomorphic Killing vector field such that

$$\mathfrak{J}^i(\phi)\partial_iK=-\frac{\mathrm{i}}{2}\mathfrak{D}(\phi,\bar{\phi}),\qquad \bar{\mathfrak{D}}=\mathfrak{D}, \tag{2.56}$$

for some Killing potential $\mathfrak{D}(\phi,\bar{\phi})$. The superpotential has to obey the condition

$$\mathfrak{J}^i(\phi)\partial_iW=-2\mathrm{i}W \tag{2.57}$$

in order for the action (2.54) to be invariant under the (2,0) AdS isometry transformations

$$\delta\phi^i=(\zeta+\mathrm{i}\tau J)\phi^i. \tag{2.58}$$

In the real representation (2.11), the chirality condition on ϕ^i turns into

$$\boldsymbol{\nabla}^2_\alpha\phi^i=\mathrm{i}\boldsymbol{\nabla}^1_\alpha\phi^i. \tag{2.59}$$

It follows that upon $\mathcal{N}=1$ reduction, ϕ^i leads to just one superfield,

$$\varphi^i\coloneqq\phi^i|. \tag{2.60}$$

In particular, we have the following relations:

$$\boldsymbol{\nabla}^2_\alpha\phi^i|=\mathrm{i}\nabla_\alpha\varphi^i, \tag{2.61a}$$

$$(\boldsymbol{\nabla}^2)^2\phi^i|=-\nabla^2\varphi^i-8\mathcal{S}\mathfrak{J}^i(\varphi). \tag{2.61b}$$

Using the reduction rules (2.52) and (2.53), we obtain

$$S=\int \mathrm{d}^{3|2}zE\{-\mathrm{i}K_{i\bar{j}}(\varphi,\bar{\varphi})\nabla^\alpha\varphi^i\nabla_\alpha\bar{\varphi}^{\bar{j}}+\mathcal{S}\mathfrak{D}(\varphi,\bar{\varphi})$$
$$+(2\mathrm{i}W(\varphi)+\mathrm{c.c.})\}, \tag{2.62}$$

where we have made use of the standard notation

$$K_{i_1\cdots i_p\bar{j}_1\cdots\bar{j}_q}\coloneqq\frac{\partial^{p+q}K(\varphi,\bar{\varphi})}{\partial\varphi^{i_1}\cdots\partial\varphi^{i_p}\partial\bar{\varphi}^{\bar{j}_1}\cdots\partial\bar{\varphi}^{\bar{j}_q}}. \tag{2.63}$$

The action (2.62) is manifestly $\mathcal{N}=1$ supersymmetric. One may explicitly check that it is also invariant under the second supersymmetry and R-symmetry transformations generated by a real scalar parameter ϵ subject to the constraints (2.31), which are

$$\delta_\epsilon\varphi^i=\mathrm{i}\epsilon^\alpha\nabla_\alpha\varphi^i+\epsilon\mathfrak{J}^i(\varphi). \tag{2.64}$$

The family of supersymmetric σ-models (2.54) includes a special subclass which is specified by the two conditions: (i) all ϕ's are neutral, $J\phi^i=0$; and (ii) no superpotential is present, $W(\phi)=0$. In this case no restriction on the Kähler potential is imposed by Eq. (2.56), and the action (2.54) is invariant under arbitrary Kähler transformations

$$K\to K+\Lambda+\bar{\Lambda}, \tag{2.65}$$

with $\Lambda(\phi^i)$ a holomorphic function. The corresponding action in $\mathcal{N}=1$ AdS superspace is obtained from (2.62) by setting $\mathfrak{D}(\varphi,\bar{\varphi})=0$ and $W(\varphi)=0$, and thus the action is manifestly Kähler invariant.

Let us also consider a supersymmetric nonlinear σ-model formulated in terms of several Abelian vector multiplets with action [7]

$$S=-2\int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\boldsymbol{E}F(\mathbb{L}^i),\qquad \bar{\mathcal{D}}^2\mathbb{L}^i=0,\qquad \bar{\mathbb{L}}^i=\mathbb{L}^i, \tag{2.66}$$

where $F(x^i)$ is a real analytic function of several variables, which is defined modulo linear inhomogeneous shifts

$$F(x)\to F(x)+b_ix^i+c, \tag{2.67}$$

with real parameters b_i and c. The real linear scalar $\mathbb{L}^i$ is the field strength of a vector multiplet. Upon reduction to $\mathcal{N}=1$ AdS superspace, $\mathbb{L}^i$ generates two different $\mathcal{N}=1$ superfields:

$$X^i\coloneqq\mathbb{L}^i|,\qquad W^i_\alpha\coloneqq\mathrm{i}\boldsymbol{\nabla}^2_\alpha\mathbb{L}^i|. \tag{2.68}$$

Here the real scalar X^i is unconstrained, while the real spinor W^i_α obeys the constraint

$$\nabla^\alpha W^i_\alpha=0, \tag{2.69}$$

which means that W^i_α is the field strength of an $\mathcal{N}=1$ vector multiplet. Reducing the action (2.66) to $\mathcal{N}=1$ AdS superspace gives

$$S=-\frac{\mathrm{i}}{2}\int \mathrm{d}^{3|2}zEg_{ij}(X)\{\nabla^\alpha X^i\nabla_\alpha X^j+W^{\alpha i}W^j_\alpha\}, \tag{2.70}$$

where we have introduced the target-space metric

$$g_{ij}(X)=\frac{\partial^2F(X)}{\partial X^i\partial X^j}. \tag{2.71}$$

The vector multiplets in (2.70) can be dualized into scalar ones, which gives

$$S_{\rm dual} = -\frac{\rm i}{2}\int {\rm d}^{3|2}z E\{g_{ij}(X)\nabla^\alpha X^i \nabla_\alpha X^j + g^{ij}(X)\nabla^\alpha Y_i \nabla_\alpha Y_j\}, \tag{2.72}$$

with $g^{ij}(X)$ being the inverse metric. Riemannian metrics of the type (2.71) appeared in the literature 20 years ago in the context of $\mathcal{N}=4$ supersymmetric quantum mechanics [19] and $\mathcal{N}=4$ superconformal mechanics [20].

III. MASSLESS HIGHER-SPIN MODELS: TYPE II SERIES

There exist two off-shell formulations for a massless multiplet of half-integer superspin $(s+\frac{1}{2})$ in (2,0) AdS superspace [1], with $s=2,3,\ldots,$ which are called the type II and type III series[7] by analogy with the terminology used in [7] for the linearized off-shell formulations for $\mathcal{N}=2$ supergravity ($s=1$). In this section we describe the $(2,0)\to(1,0)$ AdS superspace reduction of the type II theory. The reduction of the type III theory will be given in Sec. IV.

A. The type II theory

We fix an integer $s>1$. In accordance with [1], the massless type II multiplet of superspin $(s+\frac{1}{2})$ is described in terms of two unconstrained real tensor superfields

$$\mathcal{V}^{(\rm II)}_{(s+\frac{1}{2})} = \{\mathfrak{H}_{\alpha(2s)}, \mathfrak{L}_{\alpha(2s-2)}\}, \tag{3.1}$$

where $\mathfrak{H}_{\alpha(2s)} = \mathfrak{H}_{(\alpha_1\ldots\alpha_{2s})}$ and $\mathfrak{L}_{\alpha(2s-2)} = \mathfrak{L}_{(\alpha_1\ldots\alpha_{2s-2})}$ are symmetric in their spinor indices.

The dynamical superfields are defined modulo gauge transformations of the form

$$\delta_\lambda \mathfrak{H}_{\alpha(2s)} = \bar{\mathcal{D}}_{(\alpha_1}\lambda_{\alpha_2\ldots\alpha_{2s})} - \mathcal{D}_{(\alpha_1}\bar{\lambda}_{\alpha_2\ldots\alpha_{2s})} \equiv g_{\alpha(2s)} + \bar{g}_{\alpha(2s)}, \tag{3.2a}$$

$$\delta_\lambda \mathfrak{L}_{\alpha(2s-2)} = -\frac{\rm i}{2}(\bar{\mathcal{D}}^\beta \lambda_{\beta\alpha(2s-2)} + \mathcal{D}^\beta \bar{\lambda}_{\beta\alpha(2s-2)}), \tag{3.2b}$$

where the gauge parameter $\lambda_{\alpha(2s-1)}$ is unconstrained complex. Equation (3.2a) implies that the complex gauge parameter $g_{\alpha(2s)}$ is a covariantly longitudinal linear superfield,

$$g_{\alpha(2s)} \coloneqq \bar{\mathcal{D}}_{(\alpha_1}\lambda_{\alpha_2\ldots\alpha_{2s})}, \qquad \bar{\mathcal{D}}_{(\alpha_1}g_{\alpha_2\ldots\alpha_{2s+1})} = 0. \tag{3.3}$$

The gauge transformation of $\mathfrak{H}_{\alpha(2s)}$, Eq. (3.2a), corresponds to the superconformal gauge prepotential [21,22]. The prepotential $\mathfrak{L}_{\alpha(2s-2)}$ is a compensating multiplet. In addition to (3.2b), the compensator $\mathfrak{L}_{\alpha(2s-2)}$ also possesses its own gauge freedom of the form

$$\delta_\xi \mathfrak{L}_{\alpha(2s-2)} = \xi_{\alpha(2s-2)} + \bar{\xi}_{\alpha(2s-2)}, \quad \bar{\mathcal{D}}_\beta \xi_{\alpha(2s-2)} = 0, \tag{3.4}$$

with the gauge parameter $\xi_{\alpha(2s-2)}$ being covariantly chiral.

Associated with $\mathfrak{L}_{\alpha(2s-2)}$ is the real field strength

$$\mathbb{L}_{\alpha(2s-2)} = {\rm i}\mathcal{D}^\beta \bar{\mathcal{D}}_\beta \mathfrak{L}_{\alpha(2s-2)}, \qquad \mathbb{L}_{\alpha(2s-2)} = \bar{\mathbb{L}}_{\alpha(2s-2)}, \tag{3.5}$$

which is a covariantly linear superfield,

$$\mathcal{D}^2 \mathbb{L}_{\alpha(2s-2)} = 0 \Leftrightarrow \bar{\mathcal{D}}^2 \mathbb{L}_{\alpha(2s-2)} = 0. \tag{3.6}$$

It is inert under the gauge transformation (3.4), $\delta_\xi \mathbb{L}_{\alpha(2s-2)} = 0$. From (3.2b) we can read off the λ-gauge transformation of the field strength:

$$\begin{aligned}\delta_\lambda \mathbb{L}_{\alpha(2s-2)} &= \frac{1}{4}(\mathcal{D}^\beta \bar{\mathcal{D}}^2 \lambda_{\beta\alpha(2s-2)} - \bar{\mathcal{D}}^\beta \mathcal{D}^2 \bar{\lambda}_{\beta\alpha(2s-2)}),\\ &= -\frac{s}{2s+1}\mathcal{D}^\beta \bar{\mathcal{D}}^\gamma (g_{\beta\gamma\alpha(2s-2)} + \bar{g}_{\beta\gamma\alpha(2s-2)})\\ &\quad - \frac{2{\rm i}s}{2s+1}\mathcal{D}^{\beta\gamma}\bar{g}_{\beta\gamma\alpha(2s-2)}.\end{aligned} \tag{3.7}$$

The type II theory is described by the action

$$\begin{aligned} S^{(\rm II)}_{(s+\frac{1}{2})}[\mathfrak{H}_{\alpha(2s)}, \mathfrak{L}_{\alpha(2s-2)}] = \left(-\frac{1}{2}\right)^s \int {\rm d}^3x{\rm d}^2\theta{\rm d}^2\bar{\theta}\boldsymbol{E}\Big\{&\frac{1}{8}\mathfrak{H}^{\alpha(2s)}\mathcal{D}^\beta \bar{\mathcal{D}}^2 \mathcal{D}_\beta \mathfrak{H}_{\alpha(2s)} - \frac{s}{8}([\mathcal{D}_\beta, \bar{\mathcal{D}}_\gamma]\mathfrak{H}^{\beta\gamma\alpha(2s-2)})[\mathcal{D}^\delta, \bar{\mathcal{D}}^\rho]\mathfrak{H}_{\delta\rho\alpha(2s-2)}\\ &+ \frac{s}{2}(\mathcal{D}_{\beta\gamma}\mathfrak{H}^{\beta\gamma\alpha(2s-2)})\mathcal{D}^{\delta\rho}\mathfrak{H}_{\delta\rho\alpha(2s-2)} + 2{\rm i}s\mathcal{S}\mathfrak{H}^{\alpha(2s)}\mathcal{D}^\beta \bar{\mathcal{D}}_\beta \mathfrak{H}_{\alpha(2s)}\\ &- \frac{2s-1}{2}(\mathbb{L}^{\alpha(2s-2)}[\mathcal{D}^\beta, \bar{\mathcal{D}}^\gamma]\mathfrak{H}_{\beta\gamma\alpha(2s-2)} + 2\mathbb{L}^{\alpha(2s-2)}\mathbb{L}_{\alpha(2s-2)})\\ &- \frac{(s-1)(2s-1)}{4s}(\mathcal{D}_\beta \mathfrak{L}^{\beta\alpha(2s-3)}\bar{\mathcal{D}}^2\mathcal{D}^\gamma \mathfrak{L}_{\gamma\alpha(2s-3)} + {\rm c.c.}) - 4(2s-1)\mathcal{S}\mathfrak{L}^{\alpha(2s-2)}\mathbb{L}_{\alpha(2s-2)}\Big\}. \end{aligned} \tag{3.8}$$

[7]Type I series will be referred to as the longitudinal formulation for the gauge massless half-integer superspin multiplets in (1,1) AdS superspace [21] and Minkowski superspace [22]. The type I series and its dual are naturally related to the off-shell formulations for massless higher-spin $\mathcal{N}=1$ supermultiplets in four dimensions [23–25]. The type II and type III series have no four-dimensional counterpart.

It is invariant under the gauge transformations (3.2) and (3.4).

The structure $\mathcal{D}_\beta \mathfrak{L}^{\beta\alpha(2s-3)} \bar{\mathcal{D}}^2 \mathcal{D}^\gamma \mathfrak{L}_{\gamma\alpha(2s-3)}$ in (3.8) is not defined for $s = 1$. However it comes with the factor $(s-1)$ and therefore drops out from (3.8) for $s = 1$. The action (3.8) for $s = 1$ coincides with the linearized action for (2,0) AdS supergravity, which was originally derived in Sec. 10. 1 of [7].

B. Reduction of the gauge prepotentials to AdS$^{3|2}$

Let us turn to reducing the gauge prepotentials (3.1) to $\mathcal{N} = 1$ AdS superspace.[8] Our first task is to work out such a reduction for the superconformal gauge multiplet $\mathfrak{H}_{\alpha(2s)}$. In the real representation (2.11), the longitudinal linear constraint (3.3) takes the form

$$\boldsymbol{\nabla}^{\underline{2}}_{(\alpha_1} g_{\alpha_2\ldots\alpha_{2s+1})} = \mathrm{i}\boldsymbol{\nabla}^{\underline{1}}_{(\alpha_1} g_{\alpha_2\ldots\alpha_{2s+1})}. \tag{3.9}$$

It follows that $g_{\alpha(2s)}$ has two independent $\theta_{\underline{2}}$-components, which are

$$g_{\alpha(2s)}|, \qquad \boldsymbol{\nabla}^{\underline{2}\beta} g_{\alpha(2s-1)\beta}|. \tag{3.10}$$

The gauge transformation of $\mathfrak{H}_{\alpha(2s)}$, Eq. (3.2), allows us to choose two gauge conditions

$$\mathfrak{H}_{\alpha(2s)}| = 0, \qquad \boldsymbol{\nabla}^{\underline{2}\beta} \mathfrak{H}_{\alpha(2s-1)\beta}| = 0. \tag{3.11}$$

In this gauge we stay with the following unconstrained real $\mathcal{N} = 1$ superfields:

$$H_{\alpha(2s+1)} \coloneqq \mathrm{i}\boldsymbol{\nabla}^{\underline{2}}_{(\alpha_1} \mathfrak{H}_{\alpha_2\ldots\alpha_{2s+1})}|, \tag{3.12a}$$

$$H_{\alpha(2s)} \coloneqq \frac{\mathrm{i}}{4} (\boldsymbol{\nabla}^{\underline{2}})^2 \mathfrak{H}_{\alpha(2s)}|. \tag{3.12b}$$

There exists a residual gauge freedom which preserves the gauge conditions (3.11). It is described by unconstrained real $\mathcal{N} = 1$ superfields $\zeta_{\alpha(2s)}$ and $\zeta_{\alpha(2s-1)}$ defined by

$$g_{\alpha(2s)}| = -\frac{\mathrm{i}}{2}\zeta_{\alpha(2s)}, \qquad \bar{\zeta}_{\alpha(2s)} = \zeta_{\alpha(2s)}, \tag{3.13a}$$

$$\boldsymbol{\nabla}^{\underline{2}\beta} g_{\alpha(2s-1)\beta}| = \frac{2s+1}{2s}\zeta_{\alpha(2s-1)}, \qquad \bar{\zeta}_{\alpha(2s-1)} = \zeta_{\alpha(2s-1)}. \tag{3.13b}$$

The gauge transformation laws of the superfields (3.12) are given by

$$\delta H_{\alpha(2s+1)} = \mathrm{i}\nabla_{(\alpha_1}\zeta_{\alpha_2\ldots\alpha_{2s+1})}, \tag{3.14a}$$

$$\delta H_{\alpha(2s)} = \nabla_{(\alpha_1}\zeta_{\alpha_2\ldots\alpha_{2s})}. \tag{3.14b}$$

Our next step is to reduce the compensator $\mathfrak{L}_{\alpha(2s-2)}$ to $\mathcal{N} = 1$ AdS superspace. Making use of the representation (2.11), we observe that the chirality condition (3.4) reads

$$\boldsymbol{\nabla}^{\underline{2}}_\beta \xi_{\alpha(2s-2)} = \mathrm{i}\boldsymbol{\nabla}^{\underline{1}}_\beta \xi_{\alpha(2s-2)}. \tag{3.15}$$

The gauge transformation (3.4) allows us to impose a gauge condition

$$\mathfrak{L}_{\alpha(2s-2)}| = 0. \tag{3.16}$$

Thus, upon reduction to $\mathcal{N} = 1$ superspace, we have the following real superfields

$$\Psi_{\beta;\alpha(2s-2)} \coloneqq \mathrm{i}\boldsymbol{\nabla}^{\underline{2}}_\beta \mathfrak{L}_{\alpha(2s-2)}|, \tag{3.17a}$$

$$L_{\alpha(2s-2)} \coloneqq \frac{\mathrm{i}}{4}(\boldsymbol{\nabla}^{\underline{2}})^2 \mathfrak{L}_{\alpha(2s-2)}|. \tag{3.17b}$$

Here $\Psi_{\beta;\alpha(2s-2)}$ is a reducible superfield which belongs to the representation $\mathbf{2} \otimes (\mathbf{2s-1})$ of $\mathrm{SL}(2,\mathbb{R})$, $\Psi_{\beta;\alpha_1\ldots\alpha_{2s-2}} = \Psi_{\beta;(\alpha_1\ldots\alpha_{2s-2})}$. The condition (3.16) is preserved by the residual gauge freedom generated by a real unconstrained $\mathcal{N} = 1$ superfield $\eta_{\alpha(2s-2)}$ defined by

$$\xi_{\alpha(2s-2)}| = -\frac{\mathrm{i}}{2}\eta_{\alpha(2s-2)}, \qquad \bar{\eta}_{\alpha(2s-2)} = \eta_{\alpha(2s-2)}. \tag{3.18}$$

We may now determine how the η-transformation acts on the superfields (3.17a) and (3.17b). We obtain

$$\delta_\eta \Psi_{\beta;\alpha(2s-2)} = \mathrm{i}\nabla_\beta \eta_{\alpha(2s-2)}, \tag{3.19a}$$

$$\delta_\eta L_{\alpha(2s-2)} = 0, \tag{3.19b}$$

where we have used the chirality constraint (3.15) and the expression (3.18) for the residual gauge transformation.

Next, we analyze the λ-gauge transformation and reduce the $\mathcal{N} = 2$ field strength $\mathbb{L}_{\alpha(2s-2)}$ to AdS$^{3|2}$. In the real basis for the covariant derivatives, the real linearity constraint (3.6) is equivalent to two constraints:

$$(\boldsymbol{\nabla}^{\underline{2}})^2 \mathbb{L}_{\alpha(2s-2)} = (\boldsymbol{\nabla}^{\underline{1}})^2 \mathbb{L}_{\alpha(2s-2)}, \tag{3.20a}$$

$$\boldsymbol{\nabla}^{\underline{1}\beta}\boldsymbol{\nabla}^{\underline{2}}_\beta \mathbb{L}_{\alpha(2s-2)} = 0. \tag{3.20b}$$

These constraints imply that the resulting $\mathcal{N} = 1$ components of $\mathbb{L}_{\alpha(2s-2)}$ are given by

$$\mathbb{L}_{\alpha(2s-2)}|, \qquad \mathrm{i}\boldsymbol{\nabla}^{\underline{2}}_\beta \mathbb{L}_{\alpha(2s-2)}|, \tag{3.21}$$

of which the former is unconstrained and the latter is a constrained $\mathcal{N} = 1$ superfield that proves to be a gauge-invariant field strength, as we shall see below. The relation between $\mathbb{L}_{\alpha(2s-2)}$ and the prepotential $\mathfrak{L}_{\alpha(2s-2)}$ is given by (3.5), which can be expressed as

$$\mathbb{L}_{\alpha(2s-2)} = -\frac{\mathrm{i}}{2}\{(\boldsymbol{\nabla}^{\underline{1}})^2 + (\boldsymbol{\nabla}^{\underline{2}})^2\}\mathfrak{L}_{\alpha(2s-2)}. \tag{3.22}$$

[8] In the super-Poincaré case, the $\mathcal{N} = 2 \to \mathcal{N} = 1$ reduction of $\mathfrak{H}_{\alpha(2s)}$ has been carried out in [26].

We now compute the bar-projection of (3.22) in the gauge (3.16) and make use of the definition (3.17b) to obtain

$$\mathbb{L}_{\alpha(2s-2)}| = -2L_{\alpha(2s-2)}. \tag{3.23}$$

Making use of (3.22) and (3.17), the bar-projection of $\mathrm{i}\boldsymbol{\nabla}^2_\beta \mathbb{L}_{\alpha(2s-2)}$ leads to the $\mathcal{N}=1$ field strength

$$\mathcal{W}_{\beta;\alpha(2s-2)} := \mathrm{i}\boldsymbol{\nabla}^2_\beta \mathbb{L}_{\alpha(2s-2)}| = -\mathrm{i}(\nabla^\gamma\nabla_\beta - 4\mathrm{i}\mathcal{S}\delta_\beta^\gamma)\Psi_{\gamma;\alpha(2s-2)}. \tag{3.24}$$

Here $\mathcal{W}_{\beta;\alpha(2s-2)}$ is a real superfield, $\mathcal{W}_{\beta;\alpha(2s-2)} = \bar{\mathcal{W}}_{\beta;\alpha(2s-2)}$, and is a descendant of the real unconstrained prepotential $\Psi_{\beta;\alpha(2s-2)}$ defined modulo gauge transformation (3.19). The field strength proves to be gauge invariant under (3.19), and it satisfies the condition

$$\nabla^\beta \mathcal{W}_{\beta;\alpha(2s-2)} = 0, \tag{3.25}$$

as a consequence of (3.20b) and the identity (A7b). Let us express the gauge transformation of $\mathbb{L}_{\alpha(2s-2)}$, Eq. (3.7) in terms of the real basis for the covariant derivatives. This leads to

$$\delta\mathbb{L}_{\alpha(2s-2)} = \frac{\mathrm{i}s}{2s+1}\{\boldsymbol{\nabla}^{1\beta}\boldsymbol{\nabla}^{2\gamma}(g_{\beta\gamma\alpha(2s-2)} + \bar{g}_{\beta\gamma\alpha(2s-2)}) + \boldsymbol{\nabla}^{\beta\gamma}(g_{\beta\gamma\alpha(2s-2)} - \bar{g}_{\beta\gamma\alpha(2s-2)})\}, \tag{3.26}$$

In a similar way, one should also rewrite $\boldsymbol{\nabla}^2_\beta \delta\mathbb{L}_{\alpha(2s-2)}$ in the real basis. This allows us to derive the gauge transformations for $L_{\alpha(2s-2)}$ and $\mathcal{W}_{\beta;\alpha(2s-2)}$

$$\delta L_{\alpha(2s-2)} = -\frac{s}{2(2s+1)}\nabla^{\beta\gamma}\zeta_{\beta\gamma\alpha(2s-2)}, \tag{3.27a}$$

$$\delta\mathcal{W}_{\beta;\alpha(2s-2)} = \mathrm{i}(\nabla^\gamma\nabla_\beta - 4\mathrm{i}\mathcal{S}\delta_\beta^\gamma)\zeta_{\gamma\alpha(2s-2)}. \tag{3.27b}$$

We can then read off the transformation law for the prepotential $\Psi_{\beta;\alpha(2s-2)}$

$$\delta\Psi_{\beta;\alpha(2s-2)} = -\zeta_{\beta\alpha(2s-2)} + \mathrm{i}\nabla_\beta\eta_{\alpha(2s-2)}, \tag{3.27c}$$

where we have also taken into account the η-gauge freedom (3.19).

Applying the $\mathcal{N}=1$ reduction rule (2.52) to the type II action (3.8), we find that it becomes a sum of two actions,

$$S^{(\mathrm{II})}_{(s+\frac{1}{2})}[\mathfrak{H}_{\alpha(2s)}, \mathfrak{L}_{\alpha(2s-2)}] = S^{\parallel}_{(s+\frac{1}{2})}[H_{\alpha(2s+1)}, L_{\alpha(2s-2)}] + S^{\perp}_{(s)}[H_{\alpha(2s)}, \Psi_{\beta;\alpha(2s-2)}]. \tag{3.28}$$

Explicit expressions for these $\mathcal{N}=1$ actions will be given in the next subsection.

C. Massless higher-spin $\mathcal{N}=1$ supermultiplets in AdS$_3$

The gauge transformations (3.14a), (3.14b), (3.27a) and (3.27c) tell us that in fact we are dealing with two different $\mathcal{N}=1$ supersymmetric higher-spin gauge theories.

Given a positive integer $n>0$, we say that a supersymmetric gauge theory describes a multiplet of superspin $n/2$ if it is formulated in terms of a superconformal gauge prepotential $H_{\alpha(n)}$ and possibly a compensating multiplet. The gauge freedom of the real tensor superfield $H_{\alpha(n)}$ is

$$\delta_\zeta H_{\alpha(n)} = \mathrm{i}^n(-1)^{\lfloor n/2\rfloor}\nabla_{(\alpha_1}\zeta_{\alpha_2\ldots\alpha_n)}, \tag{3.29}$$

with the gauge parameter $\zeta_{\alpha(n-1)}$ being real but otherwise unconstrained.

1. Longitudinal formulation for massless superspin-$(s+\frac{1}{2})$ multiplet

One of the two $\mathcal{N}=1$ theories provides an off-shell formulation for the massless superspin-$(s+\frac{1}{2})$ multiplet. It is formulated in terms of the real unconstrained gauge superfields

$$\mathcal{V}^{\parallel}_{(s+\frac{1}{2})} = \{H_{\alpha(2s+1)}, L_{\alpha(2s-2)}\}, \tag{3.30}$$

which are defined modulo gauge transformations

$$\delta H_{\alpha(2s+1)} = \mathrm{i}\nabla_{(\alpha_1}\zeta_{\alpha_2\ldots\alpha_{2s+1})}, \tag{3.31a}$$

$$\delta L_{\alpha(2s-2)} = -\frac{s}{2(2s+1)}\nabla^{\beta\gamma}\zeta_{\beta\gamma\alpha(2s-2)}, \tag{3.31b}$$

where the parameter $\zeta_{\alpha(2s)}$ is unconstrained real. The gauge-invariant action is

$$\begin{aligned}
S^{\parallel}_{(s+\frac{1}{2})}[H_{\alpha(2s+1)}, L_{\alpha(2s-2)}] = \left(-\frac{1}{2}\right)^s \int \mathrm{d}^{3|2}z\,E\Big\{&-\frac{\mathrm{i}}{2}H^{\alpha(2s+1)}\mathbb{Q}H_{\alpha(2s+1)} - \frac{\mathrm{i}}{8}\nabla_\beta H^{\beta\alpha(2s)}\nabla^2\nabla^\gamma H_{\gamma\alpha(2s)}\\
&+\frac{\mathrm{i}s}{4}\nabla_{\beta\gamma}H^{\beta\gamma\alpha(2s-1)}\nabla^{\rho\delta}H_{\rho\delta\alpha(2s-1)} + (2s-1)L^{\alpha(2s-2)}\nabla^{\beta\gamma}\nabla^\delta H_{\beta\gamma\delta\alpha(2s-2)}\\
&+2(2s-1)\left(L^{\alpha(2s-2)}(\mathrm{i}\nabla^2 - 4\mathcal{S})L_{\alpha(2s-2)} - \frac{\mathrm{i}}{s}(s-1)\nabla_\beta L^{\beta\alpha(2s-3)}\nabla^\gamma L_{\gamma\alpha(2s-3)}\right)\\
&+\mathcal{S}\left(s\nabla_\beta H^{\beta\alpha(2s)}\nabla^\gamma H_{\gamma\alpha(2s)} + \frac{1}{2}(2s+1)H^{\alpha(2s+1)}(\nabla^2 - 4\mathrm{i}\mathcal{S})H_{\alpha(2s+1)}\right)\Big\},
\end{aligned} \tag{3.32}$$

where $\mathbb{Q}$ is the quadratic Casimir operator of the 3D $\mathcal{N}=1$ AdS supergroup (A9). The action (3.32) coincides with the off-shell $\mathcal{N}=1$ supersymmetric action for massless half-integer superspin in AdS in the form given in [14]. This supersymmetric gauge theory in AdS$^{3|2}$ was described in [14]. Its flat-superspace limit was presented earlier in [26]. In what follows, we will refer to the above theory as the longitudinal formulation for the massless superspin-$(s+\frac{1}{2})$ multiplet.

The structure $\nabla_\beta L^{\beta\alpha(2s-3)}\nabla^\gamma L_{\gamma\alpha(2s-3)}$ in (3.32) is not defined for $s=1$. However it comes with the factor $(s-1)$ and drops out from (3.32) for $s=1$. The resulting action

$$\begin{aligned}
&S^{\parallel}_{(\frac{3}{2})}[H_{\alpha(3)},L] \\
&= -\frac{1}{2}\int \mathrm{d}^{3|2}z\,E\Big\{-\frac{\mathrm{i}}{2}H^{\alpha(3)}\mathbb{Q}H_{\alpha(3)}-\frac{\mathrm{i}}{8}\nabla_\beta H^{\beta\alpha(2)}\nabla^2\nabla^\gamma H_{\gamma\alpha(2)} \\
&\quad +\frac{\mathrm{i}}{4}\nabla_{\beta\gamma}H^{\beta\gamma\alpha}\nabla^{\rho\delta}H_{\rho\delta\alpha}+L\nabla^{\beta\gamma}\nabla^\delta H_{\beta\gamma\delta}+2L(\mathrm{i}\nabla^2-4\mathcal{S})L \\
&\quad +\mathcal{S}\Big(\nabla_\beta H^{\beta\alpha(2)}\nabla^\gamma H_{\gamma\alpha(2)}+\frac{3}{2}H^{\alpha(3)}(\nabla^2-4\mathrm{i}\mathcal{S})H_{\alpha(3)}\Big)\Big\}
\end{aligned} \tag{3.33}$$

is the linearized action for $\mathcal{N}=1$ AdS supergravity. In the flat-superspace limit, the action is equivalent to the one given in [27].

2. Transverse formulation for massless superspin-s multiplet

The other $\mathcal{N}=1$ theory provides a formulation for the massless superspin-s multiplet. It is described by the unconstrained real superfields

$$\mathcal{V}^{\perp}_{(s)}=\{H_{\alpha(2s)},\Psi_{\beta;\alpha(2s-2)}\}, \tag{3.34}$$

which are defined modulo gauge transformations of the form

$$\delta H_{\alpha(2s)}=\nabla_{(\alpha_1}\zeta_{\alpha_2\ldots\alpha_{2s})}, \tag{3.35a}$$

$$\delta\Psi_{\beta;\alpha(2s-2)}=-\zeta_{\beta\alpha(2s-2)}+\mathrm{i}\nabla_\beta\eta_{\alpha(2s-2)}, \tag{3.35b}$$

where the gauge parameters $\zeta_{\alpha(2s-1)}$ and $\eta_{\alpha(2s-2)}$ are unconstrained real. The gauge-invariant action is given by

$$\begin{aligned}
S^{\perp}_{(s)}[H_{\alpha(2s)},\Psi_{\beta;\alpha(2s-2)}] &= \Big(-\frac{1}{2}\Big)^s\int \mathrm{d}^{3|2}z\,E\Big\{\frac{1}{2}H^{\alpha(2s)}(\mathrm{i}\nabla^2+8s\mathcal{S})H_{\alpha(2s)} \\
&\quad -\mathrm{i}s\nabla_\beta H^{\beta\alpha(2s-1)}\nabla^\gamma H_{\gamma\alpha(2s-1)}-(2s-1)\mathcal{W}^{\beta;\alpha(2s-2)}\nabla^\gamma H_{\gamma\beta\alpha(2s-2)} \\
&\quad -\frac{\mathrm{i}}{2}(2s-1)\Big(\mathcal{W}^{\beta;\alpha(2s-2)}\mathcal{W}_{\beta;\alpha(2s-2)}+\frac{s-1}{s}\mathcal{W}_{\beta;}{}^{\beta\alpha(2s-3)}\mathcal{W}^{\gamma;}{}_{\gamma\alpha(2s-3)}\Big) \\
&\quad -2\mathrm{i}(2s-1)\mathcal{S}\Psi^{\beta;\alpha(2s-2)}\mathcal{W}_{\beta;\alpha(2s-2)}\Big\},
\end{aligned} \tag{3.36a}$$

where $\mathcal{W}_{\beta;\alpha(2s-2)}$ denotes the field strength

$$\mathcal{W}_{\beta;\alpha(2s-2)}=-\mathrm{i}(\nabla^\gamma\nabla_\beta-4\mathrm{i}\mathcal{S}\delta^\gamma_\beta)\Psi_{\gamma;\alpha(2s-2)},\qquad \nabla^\beta\mathcal{W}_{\beta;\alpha(2s-2)}=0. \tag{3.36b}$$

The action (3.36) defines a new $\mathcal{N}=1$ supersymmetric higher-spin theory which did not appear in [14,21,26] even in the super-Poincaré case.

The structure $\mathcal{W}_{\beta;}{}^{\beta\alpha(2s-3)}\mathcal{W}^{\gamma;}{}_{\gamma\alpha(2s-3)}$ in (3.36) is not defined for $s=1$. However it comes with the factor $(s-1)$ and drops out from (3.36) for $s=1$. The resulting gauge-invariant action

$$\begin{aligned}
S^{\perp}_{(1)}[H_{\alpha(2)},\Psi_\beta] &= -\frac{1}{2}\int \mathrm{d}^{3|2}z\,E\Big\{\frac{1}{2}H^{\alpha(2)}(\mathrm{i}\nabla^2+8\mathcal{S})H_{\alpha(2)}-\mathrm{i}\nabla_\beta H^{\beta\alpha}\nabla^\gamma H_{\gamma\alpha} \\
&\quad -\mathcal{W}^\beta\nabla^\gamma H_{\gamma\beta}-\frac{\mathrm{i}}{2}\mathcal{W}^\beta\mathcal{W}_\beta-2\mathrm{i}\mathcal{S}\Psi^\beta\mathcal{W}_\beta\Big\}
\end{aligned} \tag{3.37}$$

provides an off-shell realization for a massless gravitino multiplet in AdS$_3$. In the flat-superspace limit, this model reduces to the one described in [26].

In the $s>1$ case, the gauge freedom of the prepotential $\Psi_{\beta;\alpha(2s-2)}$ (3.35) allows us to impose a gauge condition

$$\Psi_{(\alpha_1;\alpha_2\ldots\alpha_{2s-1})}=0\Leftrightarrow\Psi_{\beta;\alpha(2s-2)}=\sum_{k=1}^{2s-2}\varepsilon_{\beta\alpha_k}\varphi_{\alpha_1\ldots\hat{\alpha}_k\ldots\alpha_{2s-2}}, \tag{3.38}$$

for some field $\varphi_{\alpha(2s-3)}$. Since we gauge away the symmetric part of $\Psi_{\beta;\alpha(2s-2)}$, the two gauge parameters $\zeta_{\alpha(2s-1)}$ and $\eta_{\alpha(2s-2)}$ are related. The theory is now realized in terms of the following dynamical variables:

$$\{H_{\alpha(2s)}, \varphi_{\alpha(2s-3)}\}, \tag{3.39}$$

with the gauge freedom

$$\delta H_{\alpha(2s)} = -\nabla_{(\alpha_1\alpha_2}\eta_{\alpha_3\ldots\alpha_{2s})}, \tag{3.40a}$$

$$\delta\varphi_{\alpha(2s-3)} = \mathrm{i}\nabla^\beta\eta_{\beta\alpha(2s-3)}. \tag{3.40b}$$

It follows that in the flat-superspace limit, $\mathcal{S}=0$, and in the gauge (3.38), the action (3.36) coincides with Eq. (B.25) of [21]. The component structure of this model will be discussed in Appendix B 1.

IV. MASSLESS HIGHER-SPIN MODELS: TYPE III SERIES

In this section we carry out the $\mathcal{N}=1$ AdS superspace reduction of the type III theory [1] following the procedure employed in Sec. III.

A. The type III theory

We fix a positive integer $s>1$. In accordance with [1], the massless type III multiplet of superspin $(s+\frac{1}{2})$ is described in terms of two unconstrained real tensor superfields

$$\mathcal{V}^{(\mathrm{III})}_{(s+\frac{1}{2})} = \{\mathfrak{H}_{\alpha(2s)}, \mathfrak{V}_{\alpha(2s-2)}\}, \tag{4.1}$$

which are symmetric in their spinor indices, $\mathfrak{H}_{\alpha(2s)} = \mathfrak{H}_{(\alpha_1\ldots\alpha_{2s})}$ and $\mathfrak{V}_{\alpha(2s-2)} = \mathfrak{V}_{(\alpha_1\ldots\alpha_{2s-2})}$.

The dynamical superfields are defined modulo gauge transformations of the form

$$\delta_\lambda\mathfrak{H}_{\alpha(2s)} = \bar{\mathcal{D}}_{(\alpha_1}\lambda_{\alpha_2\ldots\alpha_{2s})} - \mathcal{D}_{(\alpha_1}\bar{\lambda}_{\alpha_2\ldots\alpha_{2s})} = g_{\alpha(2s)} + \bar{g}_{\alpha(2s)}, \tag{4.2a}$$

$$\delta_\lambda\mathfrak{V}_{\alpha(2s-2)} = \frac{1}{2s}(\bar{\mathcal{D}}^\beta\lambda_{\beta\alpha(2s-2)} - \mathcal{D}^\beta\bar{\lambda}_{\beta\alpha(2s-2)}), \tag{4.2b}$$

where the gauge parameter $\lambda_{\alpha(2s-1)}$ is unconstrained complex, and the longitudinal linear parameter $g_{\alpha(2s)}$ is defined as in (3.3). As in the type II case, $\mathfrak{H}_{\alpha(2s)}$ is the superconformal gauge multiplet, while $\mathfrak{V}_{\alpha(2s-2)}$ is a compensating multiplet. The only difference from the type II case occurs in the gauge transformation law for the compensator $\mathfrak{V}_{\alpha(2s-2)}$.

The compensator $\mathfrak{V}_{\alpha(2s-2)}$ also possesses its own gauge freedom of the form

$$\delta_\xi\mathfrak{V}_{\alpha(2s-2)} = \xi_{\alpha(2s-2)} + \bar{\xi}_{\alpha(2s-2)}, \quad \bar{\mathcal{D}}_\beta\xi_{\alpha(2s-2)} = 0, \tag{4.3}$$

with the gauge parameter $\xi_{\alpha(2s-2)}$ being covariantly chiral, but otherwise arbitrary.

Associated with $\mathfrak{V}_{\alpha(2s-2)}$ is the real field strength

$$\mathbb{V}_{\alpha(2s-2)} = \mathrm{i}\mathcal{D}^\beta\bar{\mathcal{D}}_\beta\mathfrak{V}_{\alpha(2s-2)}, \quad \mathbb{V}_{\alpha(2s-2)} = \bar{\mathbb{V}}_{\alpha(2s-2)}, \tag{4.4}$$

which is inert under (4.3), $\delta_\xi\mathbb{V}_{\alpha(2s-2)}=0$. It is not difficult to see that $\mathbb{V}_{\alpha(2s-2)}$ is covariantly linear,

$$\mathcal{D}^2\mathbb{V}_{\alpha(2s-2)} = 0 \Leftrightarrow \bar{\mathcal{D}}^2\mathbb{V}_{\alpha(2s-2)} = 0. \tag{4.5}$$

It varies under the λ-gauge transformation as

$$\begin{aligned}\delta_\lambda\mathbb{V}_{\alpha(2s-2)} &= \frac{\mathrm{i}}{4s}(\mathcal{D}^\beta\bar{\mathcal{D}}^2\lambda_{\beta\alpha(2s-2)} + \bar{\mathcal{D}}^\beta\mathcal{D}^2\bar{\lambda}_{\beta\alpha(2s-2)}).\\ &= -\frac{\mathrm{i}}{2s+1}\mathcal{D}^\beta\bar{\mathcal{D}}^\gamma(g_{\beta\gamma\alpha(2s-2)} - \bar{g}_{\beta\gamma\alpha(2s-2)})\\ &\quad -\frac{2}{2s+1}\mathcal{D}^{\beta\gamma}\bar{g}_{\beta\gamma\alpha(2s-2)}.\end{aligned} \tag{4.6}$$

Modulo normalization, there exists a unique action being invariant under the gauge transformations (4.2) and (4.3). It is given by

$$\begin{aligned}S^{(\mathrm{III})}_{(s+\frac{1}{2})}[\mathfrak{H}_{\alpha(2s)}, \mathfrak{V}_{\alpha(2s-2)}] = \left(-\frac{1}{2}\right)^s \int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\boldsymbol{E}\bigg\{&\frac{1}{8}\mathfrak{H}^{\alpha(2s)}\mathcal{D}^\beta\bar{\mathcal{D}}^2\mathcal{D}_\beta\mathfrak{H}_{\alpha(2s)}\\ &-\frac{1}{16}([\mathcal{D}_\beta,\bar{\mathcal{D}}_\gamma]\mathfrak{H}^{\beta\gamma\alpha(2s-2)})[\mathcal{D}^\delta,\bar{\mathcal{D}}^\rho]\mathfrak{H}_{\delta\rho\alpha(2s-2)}\\ &+\frac{1}{4}(\mathcal{D}_{\beta\gamma}\mathfrak{H}^{\beta\gamma\alpha(2s-2)})\mathcal{D}^{\delta\rho}\mathfrak{H}_{\delta\rho\alpha(2s-2)} + \mathrm{i}\mathcal{S}\mathfrak{H}^{\alpha(2s)}\mathcal{D}^\beta\bar{\mathcal{D}}_\beta\mathfrak{H}_{\alpha(2s)}\\ &-\frac{2s-1}{2}\left(\mathbb{V}^{\alpha(2s-2)}\mathcal{D}^{\beta\gamma}\mathfrak{H}_{\beta\gamma\alpha(2s-2)} + \frac{1}{2}\mathbb{V}^{\alpha(2s-2)}\mathbb{V}_{\alpha(2s-2)}\right)\\ &+\frac{1}{8}(s-1)(2s-1)(\mathcal{D}_\beta\mathfrak{V}^{\beta\alpha(2s-3)}\bar{\mathcal{D}}^2\mathcal{D}^\gamma\mathfrak{V}_{\gamma\alpha(2s-3)} + \mathrm{c.c.}) + 2s(2s-1)\mathcal{S}\mathfrak{V}^{\alpha(2s-2)}\mathbb{V}_{\alpha(2s-2)}\bigg\}.\end{aligned} \tag{4.7}$$

Although the structure $\mathcal{D}_\beta \mathfrak{B}^{\beta\alpha(2s-3)}\bar{\mathcal{D}}^2\mathcal{D}^\gamma \mathfrak{B}_{\gamma\alpha(2s-3)}$ in (4.7) is not defined for $s=1$, it comes with the factor $(s-1)$ and drops out from (4.7) for the $s=1$ case. In this case the action coincides with the type III supergravity action in (2,0) AdS superspace, which was originally derived in Sec. 10. 2 of [7].

B. Reduction of the gauge prepotentials to $\mathrm{AdS}^{3|2}$

The reduction of the superconformal gauge multiplet $\mathfrak{H}_{\alpha(2s)}$ to $\mathrm{AdS}^{3|2}$ has been carried out in the previous section. We saw that in the gauge (3.11), $\mathfrak{H}_{\alpha(2s)}$ is described by the two unconstrained real superfields $H_{\alpha(2s+1)}$ and $H_{\alpha(2s)}$ defined according to (3.12), with their gauge transformation laws given by Eqs. (3.14a) and (3.14b), respectively. Now it remains to reduce the prepotential $\mathfrak{B}_{\alpha(2s-2)}$ to $\mathcal{N}=1$ AdS superspace, following the same approach as outlined in the type II series. The gauge transformation (4.3) allows us to choose a gauge condition

$$\mathfrak{B}_{\alpha(2s-2)}| = 0. \tag{4.8}$$

The compensator $\mathfrak{B}_{\alpha(2s-2)}$ is then equivalent to the following real $\mathcal{N}=1$ superfields, which we define as follows:

$$\Upsilon_{\beta;\alpha(2s-2)} := \mathrm{i}\boldsymbol{\nabla}^{\underline{2}}_\beta \mathfrak{B}_{\alpha(2s-2)}|, \tag{4.9a}$$

$$V_{\alpha(2s-2)} := \frac{\mathrm{i}}{4}(\boldsymbol{\nabla}^{\underline{2}})^2 \mathfrak{B}_{\alpha(2s-2)}|. \tag{4.9b}$$

The residual gauge freedom, which preserves the gauge condition (4.8) is described by a real unconstrained $\mathcal{N}=1$ superfield $\eta_{\alpha(2s-2)}$ defined by

$$\xi_{\alpha(2s-2)}| = -\frac{\mathrm{i}}{2}\eta_{\alpha(2s-2)}, \qquad \bar{\eta}_{\alpha(2s-2)} = \eta_{\alpha(2s-2)}. \tag{4.10}$$

As a result, we may determine how (4.9a) and (4.9b) vary under η-transformation

$$\delta_\eta \Upsilon_{\beta;\alpha(2s-2)} = \mathrm{i}\nabla_\beta \eta_{\alpha(2s-2)}, \tag{4.11a}$$

$$\delta_\eta V_{\alpha(2s-2)} = 0. \tag{4.11b}$$

Next, we analyze the λ-gauge transformation and reduce the field strength $\mathbb{V}_{\alpha(2s-2)}$ to $\mathrm{AdS}^{3|2}$. In the real basis for the covariant derivatives, the real linearity constraint (4.5) turns into

$$(\boldsymbol{\nabla}^{\underline{2}})^2 \mathbb{V}_{\alpha(2s-2)} = (\boldsymbol{\nabla}^{\underline{1}})^2 \mathbb{V}_{\alpha(2s-2)}, \tag{4.12a}$$

$$\boldsymbol{\nabla}^{\underline{1}\beta}\boldsymbol{\nabla}^{\underline{2}}_\beta \mathbb{V}_{\alpha(2s-2)} = 0. \tag{4.12b}$$

This tells us that $\mathbb{V}_{\alpha(2s-2)}$ is equivalent to two real $\mathcal{N}=1$ superfields

$$\mathbb{V}_{\alpha(2s-2)}|, \qquad \mathrm{i}\boldsymbol{\nabla}^{\underline{2}}_\beta \mathbb{V}_{\alpha(2s-2)}|. \tag{4.13}$$

The relation between the field strength $\mathbb{V}_{\alpha(2s-2)}$ and the prepotential $\mathfrak{B}_{\alpha(2s-2)}$ is given by (4.4), which can be expressed as

$$\mathbb{V}_{\alpha(2s-2)} = -\frac{\mathrm{i}}{2}\{(\boldsymbol{\nabla}^{\underline{1}})^2 + (\boldsymbol{\nabla}^{\underline{2}})^2\}\mathfrak{B}_{\alpha(2s-2)}. \tag{4.14}$$

We now compute the bar-projection of (4.14) in the gauge (4.8) and make use of the definition (4.9b) to obtain

$$\mathbb{V}_{\alpha(2s-2)}| = -2V_{\alpha(2s-2)}. \tag{4.15}$$

The bar-projection of $\mathrm{i}\boldsymbol{\nabla}^{\underline{2}}_\beta \mathbb{V}_{\alpha(2s-2)}$ leads to the $\mathcal{N}=1$ field-strength

$$\begin{aligned}\Omega_{\beta;\alpha(2s-2)} &:= \mathrm{i}\boldsymbol{\nabla}^{\underline{2}}_\beta \mathbb{V}_{\alpha(2s-2)}| \\ &= -\mathrm{i}(\nabla^\gamma\nabla_\beta - 4\mathrm{i}\mathcal{S}\delta_\beta{}^\gamma)\Upsilon_{\gamma;\alpha(2s-2)},\end{aligned} \tag{4.16}$$

which is a real superfield, $\Omega_{\beta;\alpha(2s-2)} = \bar{\Omega}_{\beta;\alpha(2s-2)}$, and is a descendant of the real unconstrained prepotential $\Upsilon_{\beta;\alpha(2s-2)}$ defined modulo gauge transformation (4.11). One may check that the field strength is invariant under (4.11) and obeys the condition

$$\nabla^\beta \Omega_{\beta;\alpha(2s-2)} = 0. \tag{4.17}$$

Let us express the gauge transformation of $\mathbb{V}_{\alpha(2s-2)}$, Eq. (4.6) in terms of the real basis for the covariant derivatives. This leads to

$$\begin{aligned}\delta\mathbb{V}_{\alpha(2s-2)} = -\frac{1}{2s+1}\{&\boldsymbol{\nabla}^{\underline{1}\beta}\boldsymbol{\nabla}^{\underline{2}\gamma}(g_{\beta\gamma\alpha(2s-2)} - \bar{g}_{\beta\gamma\alpha(2s-2)}) \\ &+ \boldsymbol{\nabla}^{\beta\gamma}(g_{\beta\gamma\alpha(2s-2)} + \bar{g}_{\beta\gamma\alpha(2s-2)})\},\end{aligned} \tag{4.18}$$

One should also express its corollary $\boldsymbol{\nabla}^{\underline{2}}_\beta \delta\mathbb{V}_{\alpha(2s-2)}$ in the real basis for the covariant derivatives. We determine the gauge transformations law for $V_{\alpha(2s-2)}$ and $\Omega_{\beta;\alpha(2s-2)}$ to be

$$\delta V_{\alpha(2s-2)} = \frac{1}{2s}\nabla^\beta \zeta_{\beta\alpha(2s-2)}, \tag{4.19a}$$

$$\delta\Omega_{\beta;\alpha(2s-2)} = \frac{1}{2s+1}(\nabla^\gamma\nabla_\beta\nabla^\delta - 4\mathrm{i}\mathcal{S}\nabla^\delta\delta_\beta{}^\gamma)\zeta_{\delta\gamma\alpha(2s-2)}. \tag{4.19b}$$

From (4.19b) we read off the transformation law for the prepotential $\Upsilon_{\beta;\alpha(2s-2)}$:

$$\delta\Upsilon_{\beta;\alpha(2s-2)} = \frac{\mathrm{i}}{2s+1}(\nabla^\gamma \zeta_{\gamma\beta\alpha(2s-2)} + (2s+1)\nabla_\beta \eta_{\alpha(2s-2)}), \tag{4.20}$$

where we have also taken into account the η-gauge freedom (4.11).

Performing $\mathcal{N}=1$ reduction to the original type III action (4.7), we arrive at two decoupled $\mathcal{N}=1$ actions

$$S^{(\mathrm{III})}_{(s+\frac{1}{2})}[\mathfrak{H}_{\alpha(2s)},\mathfrak{V}_{\alpha(2s-2)}] = S^{\perp}_{(s+\frac{1}{2})}[H_{\alpha(2s+1)},\Upsilon_{\beta;\alpha(2s-2)}] + S^{\parallel}_{(s)}[H_{\alpha(2s)},V_{\alpha(2s-2)}]. \tag{4.21}$$

We will present the exact form of these actions in the next subsection.

C. Massless higher-spin $\mathcal{N}=1$ supermultiplets in AdS$_3$

Upon reduction to $\mathcal{N}=1$ superspace, the type III theory leads to two $\mathcal{N}=1$ supersymmetric gauge theories.

1. Longitudinal formulation for massless superspin-s multiplet

One of the two $\mathcal{N}=1$ theories provides an off-shell realisation for massless superspin-s multiplet described in terms of the real unconstrained superfields

$$\mathcal{V}^{\parallel}_{(s)} = \{H_{\alpha(2s)},V_{\alpha(2s-2)}\}, \tag{4.22}$$

which are defined modulo gauge transformations of the form

$$\delta H_{\alpha(2s)} = \nabla_{(\alpha_1}\zeta_{\alpha_2\ldots\alpha_{2s})}, \tag{4.23a}$$

$$\delta V_{\alpha(2s-2)} = \frac{1}{2s}\nabla^{\beta}\zeta_{\beta\alpha(2s-2)}, \tag{4.23b}$$

where the gauge parameter $\zeta_{\alpha(2s-1)}$ is unconstrained real. The gauge-invariant action is given by

$$\begin{aligned}S^{\parallel}_{(s)}[H_{\alpha(2s)},V_{\alpha(2s-2)}] = \left(-\frac{1}{2}\right)^{s}\int \mathrm{d}^{3|2}z E\Big\{&\frac{1}{2}H^{\alpha(2s)}(\mathrm{i}\nabla^2+4\mathcal{S})H_{\alpha(2s)}\\ &-\frac{i}{2}\nabla_{\beta}H^{\beta\alpha(2s-1)}\nabla^{\gamma}H_{\gamma\alpha(2s-1)} - (2s-1)V^{\alpha(2s-2)}\nabla^{\beta\gamma}H_{\beta\gamma\alpha(2s-2)}\\ &+(2s-1)\left(\frac{1}{2}V^{\alpha(2s-2)}(i\nabla^2+8s\mathcal{S})V_{\alpha(2s-2)} + (s-1)\nabla_{\beta}V^{\beta\alpha(2s-3)}\nabla^{\gamma}V_{\gamma\alpha(2s-3)}\right)\Big\}.\end{aligned} \tag{4.24}$$

Modulo an overall normalization factor, (4.24) coincides with the off-shell $\mathcal{N}=1$ supersymmetric action for massless superspin-s multiplet in the form given in [14]. In the flat-superspace limit it reduces to the action derived in [26].

Although the structure $\nabla_{\beta}V^{\beta\alpha(2s-3)}\nabla^{\gamma}V_{\gamma\alpha(2s-3)}$ in (4.24) is not defined for $s=1$, it comes with the factor $(s-1)$ and thus drops out from (4.24) for $s=1$. The resulting gauge-invariant action

$$\begin{aligned}S^{\parallel}_{(1)}[H_{\alpha(2)},V] = -\frac{1}{2}\int \mathrm{d}^{3|2}zE\Big\{&\frac{1}{2}H^{\alpha(2)}(\mathrm{i}\nabla^2+4\mathcal{S})H_{\alpha(2)} - \frac{\mathrm{i}}{2}\nabla_{\beta}H^{\beta\alpha}\nabla^{\gamma}H_{\gamma\alpha}\\ &-V\nabla^{\beta\gamma}H_{\beta\gamma}+\frac{1}{2}V(\mathrm{i}\nabla^2+8\mathcal{S})V\Big\}\end{aligned} \tag{4.25}$$

describes an off-shell massless gravitino multiplet in AdS$_3$. In the flat-superspace limit, it reduces to the gravitino multiplet model described in [28] (see also [26]).

2. Transverse formulation for massless superspin-$(s+\frac{1}{2})$ multiplet

The other theory provides an off-shell formulation for massless superspin-$(s+\frac{1}{2})$ multiplet. It is described by the unconstrained superfields

$$\mathcal{V}^{\perp}_{(s+\frac{1}{2})} = \{H_{\alpha(2s+1)},\Upsilon_{\beta;\alpha(2s-2)}\}, \tag{4.26}$$

which are defined modulo gauge transformations of the form

$$\delta H_{\alpha(2s+1)} = \mathrm{i}\nabla_{(\alpha_1}\zeta_{\alpha_2\ldots\alpha_{2s+1})}, \tag{4.27a}$$

$$\delta\Upsilon_{\beta;\alpha(2s-2)} = \frac{\mathrm{i}}{2s+1}(\nabla^{\gamma}\zeta_{\gamma\beta\alpha(2s-2)}+(2s+1)\nabla_{\beta}\eta_{\alpha(2s-2)}). \tag{4.27b}$$

The gauge-invariant action is

$$
\begin{aligned}
S^{\perp}_{(s+\frac{1}{2})}[H_{\alpha(2s+1)},\Upsilon_{\beta;\alpha(2s-2)}] = \left(-\frac{1}{2}\right)^{s}\int \mathrm{d}^{3|2}zE\Big\{&-\frac{\mathrm{i}}{2}H^{\alpha(2s+1)}\mathbb{Q}H_{\alpha(2s+1)} - \frac{\mathrm{i}}{8}\nabla_{\beta}H^{\beta\alpha(2s)}\nabla^{2}\nabla^{\gamma}H_{\gamma\alpha(2s)} \\
&+\frac{\mathrm{i}}{8}\nabla_{\beta\gamma}H^{\beta\gamma\alpha(2s-1)}\nabla^{\rho\delta}H_{\rho\delta\alpha(2s-1)} - \frac{\mathrm{i}}{4}(2s-1)\Omega^{\beta;\alpha(2s-2)}\nabla^{\gamma\delta}H_{\gamma\delta\beta\alpha(2s-2)} \\
&-\frac{\mathrm{i}}{8}(2s-1)(\Omega^{\beta;\alpha(2s-2)}\Omega_{\beta;\alpha(2s-2)} - 2(s-1)\Omega_{\beta;}{}^{\beta\alpha(2s-3)}\Omega^{\gamma;}{}_{\gamma\alpha(2s-3)}) \\
&+\mathcal{S}\left(H^{\alpha(2s+1)}(\nabla^{2}-4\mathrm{i}\mathcal{S})H_{\alpha(2s+1)} + \frac{1}{2}\nabla_{\beta}H^{\beta\alpha(2s)}\nabla^{\gamma}H_{\gamma\alpha(2s)}\right) \\
&+\mathrm{i}s(2s-1)\mathcal{S}\Upsilon^{\beta;\alpha(2s-2)}\Omega_{\beta;\alpha(2s-2)}\Big\},
\end{aligned}
\tag{4.28a}
$$

where $\Omega_{\beta;\alpha(2s-2)}$ denotes the real field strength

$$
\Omega_{\beta;\alpha(2s-2)} = -\mathrm{i}(\nabla^{\gamma}\nabla_{\beta} - 4\mathrm{i}\mathcal{S}\delta_{\beta}{}^{\gamma})\Upsilon_{\gamma;\alpha(2s-2)}, \qquad \nabla^{\beta}\Omega_{\beta;\alpha(2s-2)} = 0. \tag{4.28b}
$$

This action defines a new $\mathcal{N}=1$ supersymmetric higher-spin theory which did not appear in [14,21,26].

The structure $\Omega_{\beta;}{}^{\beta\alpha(2s-3)}\Omega^{\gamma;}{}_{\gamma\alpha(2s-3)}$ in (4.28a) is not defined for $s=1$. However it comes with the factor $(s-1)$ and hence drops out from (4.28a) for $s=1$. The resulting gauge-invariant action

$$
\begin{aligned}
S^{\perp}_{(\frac{3}{2})}[H_{\alpha(3)},\Upsilon_{\beta}] = -\frac{1}{2}\int \mathrm{d}^{3|2}zE\Big\{&-\frac{\mathrm{i}}{2}H^{\alpha(3)}\mathbb{Q}H_{\alpha(3)} - \frac{\mathrm{i}}{8}\nabla_{\beta}H^{\beta\alpha(2)}\nabla^{2}\nabla^{\gamma}H_{\gamma\alpha(2)} \\
&+\frac{\mathrm{i}}{8}\nabla_{\beta\gamma}H^{\beta\gamma\alpha}\nabla^{\rho\delta}H_{\rho\delta\alpha} - \frac{\mathrm{i}}{4}\Omega^{\beta}\nabla^{\gamma\delta}H_{\gamma\delta\beta} \\
&+\mathcal{S}\left(H^{\alpha(3)}(\nabla^{2}-4\mathrm{i}\mathcal{S})H_{\alpha(3)} + \frac{1}{2}\nabla_{\beta}H^{\beta\alpha(2)}\nabla^{\gamma}H_{\gamma\alpha(2)}\right) \\
&-\frac{\mathrm{i}}{8}\Omega^{\beta}\Omega_{\beta} + \mathrm{i}\mathcal{S}\Upsilon^{\beta}\Omega_{\beta}\Big\}
\end{aligned}
\tag{4.29}
$$

provides an off-shell formulation for a linearized supergravity multiplet in AdS_3. In the flat-superspace limit, it reduces to the linearised supergravity model proposed in [26].

V. ANALYSIS OF THE RESULTS

Let $s>0$ be a positive integer. For each superspin value, integer (s) or half-integer $(s+\frac{1}{2})$, we have constructed two off-shell formulations which have been called longitudinal and transverse. Now we have to explain this terminology.

Consider a field theory in $AdS^{3|2}$ that is described in terms of a real tensor superfield $V_{\alpha(n)}$. We assume the action to have the form

$$
S^{\parallel}[V_{\alpha(n)}] = \int \mathrm{d}^{3|2}zE\mathcal{L}(\mathrm{i}^{n+1}\nabla_{\beta}V_{\alpha(n)}). \tag{5.1}
$$

It is natural to call $\nabla_{\beta}V_{\alpha(n)}$ a longitudinal superfield, by analogy with a longitudinal vector field. This theory possesses a dual formulation that is obtained by introducing a first-order action

$$
S_{\text{first-order}} = \int \mathrm{d}^{3|2}zE\{\mathcal{L}(\Sigma_{\beta;\alpha(n)}) + \mathrm{i}^{n+1}\mathcal{W}^{\beta;\alpha(n)}\Sigma_{\beta;\alpha(n)}\}, \tag{5.2}
$$

where $\Sigma_{\beta;\alpha(n)}$ is unconstrained and the Lagrange multiplier is

$$
\mathcal{W}_{\beta;\alpha(n)} = \mathrm{i}^{n+1}(\nabla^{\gamma}\nabla_{\beta} - 4\mathrm{i}\mathcal{S}\delta_{\beta}^{\gamma})\Psi_{\gamma;\alpha(n)}, \;\; \nabla^{\beta}\mathcal{W}_{\beta;\alpha(n)} = 0, \tag{5.3}
$$

for some unconstrained prepotential $\Psi_{\gamma;\alpha(n)}$. Varying (5.2) with respect to $\Psi_{\gamma;\alpha(n)}$ gives

$$
\nabla^{\beta}\nabla_{\gamma}\Sigma_{\beta;\alpha(n)} - 4\mathrm{i}\mathcal{S}\Sigma_{\gamma;\alpha(n)} = 0 \Rightarrow \Sigma_{\beta;\alpha(n)} = \mathrm{i}^{n+1}\nabla_{\beta}V_{\alpha(n)}, \tag{5.4}
$$

and then $S_{\text{first-order}}$ reduces to the original action (5.1). On the other hand, we may start from $S_{\text{first-order}}$ and integrate $\Sigma_{\beta;\alpha(n)}$ out. This will lead to a dual action of the form

$$S^{\perp}[\Psi_{\gamma;\alpha(n)}] = \int \mathrm{d}^{3|2}z E \mathcal{L}_{\mathrm{dual}}(\mathcal{W}_{\beta;\alpha(n)}). \qquad (5.5)$$

This is a gauge theory since the action is invariant under gauge transformations

$$\delta\Psi_{\gamma;\alpha(n)} = \mathrm{i}^{n+1}\nabla_{\gamma}\eta_{\alpha(n)}. \qquad (5.6)$$

The gauge-invariant field strength $\mathcal{W}_{\beta;\alpha(n)}$ can be called a transverse superfield, due to the constraint (5.3) it obeys.

It is natural to call the dual formulations (5.1) and (5.5) as longitudinal and transverse, respectively.

Now, let us consider the transverse and longitudinal formulations for the massless superspin-s models, which are given by Eqs. (3.26) and (4.24), respectively. These actions depend parametrically on $\mathcal{S}$, the curvature of AdS superspace. We denote by $S^{\perp}_{(s)}[H_{\alpha(2s)},\Psi_{\beta;\alpha(2s-2)}]_{\mathrm{FS}}$ and $S^{\parallel}_{(s)}[H_{\alpha(2s)},V_{\alpha(2s-2)}]_{\mathrm{FS}}$ these actions in the limit $\mathcal{S}=0$, which corresponds to a flat superspace. The dynamical systems $S^{\perp}_{(s)}[H_{\alpha(2s)},\Psi_{\beta;\alpha(2s-2)}]_{\mathrm{FS}}$ and $S^{\parallel}_{(s)}[H_{\alpha(2s)},V_{\alpha(2s-2)}]_{\mathrm{FS}}$ prove to be related to each other by the Legendre transformation described above. Thus $S^{\perp}_{(s)}[H_{\alpha(2s)},\Psi_{\beta;\alpha(2s-2)}]_{\mathrm{FS}}$ and $S^{\parallel}_{(s)}[H_{\alpha(2s)},V_{\alpha(2s-2)}]_{\mathrm{FS}}$ are dual formulations of the same theory. This duality does not survive if $\mathcal{S}$ is nonvanishing.

The same feature characterizes the longitudinal and transverse formulations for the massless superspin-$(s+\frac{1}{2})$ multiplet, which are described by the actions (3.32) and (4.28), respectively. The flat-superspace counterparts of these higher-spin models, which we denote by $S^{\parallel}_{(s+\frac{1}{2})}[H_{\alpha(2s+1)},L_{\alpha(2s-2)}]_{\mathrm{FS}}$ and $S^{\perp}_{(s+\frac{1}{2})}[H_{\alpha(2s+1)},\Upsilon_{\beta;\alpha(2s-2)}]_{\mathrm{FS}}$, are dual to each other. However, this duality does not survive if we turn on a nonvanishing AdS curvature.

The above discussion can be illustrated by considering the model for linearized gravity in AdS$_3$. It is described by the action

$$\begin{aligned} S_{\mathrm{gravity}} = \frac{1}{8}\int \mathrm{d}^3x e\bigg\{&\mathfrak{h}^{\alpha(4)}\Box\mathfrak{h}_{\alpha(4)} - \nabla_{\beta(2)}\mathfrak{h}^{\beta(2)\alpha(2)}\nabla^{\gamma(2)}\mathfrak{h}_{\alpha(2)\gamma(2)} \\ &+\frac{1}{2}\nabla^{\alpha(2)}\mathfrak{y}\nabla^{\beta(2)}\mathfrak{h}_{\alpha(2)\beta(2)} - \frac{1}{4}\nabla^{\alpha(2)}\mathfrak{y}\nabla_{\alpha(2)}\mathfrak{y} \\ &+8\mathcal{S}^2\mathfrak{h}^{\alpha(4)}\mathfrak{h}_{\alpha(4)} + 6\mathcal{S}^2\mathfrak{y}^2\bigg\}, \end{aligned} \qquad (5.7)$$

which is invariant under gauge transformations

$$\delta_{\xi}\mathfrak{h}_{\alpha(4)} = \nabla_{(\alpha_1\alpha_2}\zeta_{\alpha_3\alpha_4)}, \qquad \delta_{\xi}\mathfrak{y} = \frac{2}{3}\nabla^{\alpha(2)}\zeta_{\alpha(2)}. \qquad (5.8)$$

In the flat-space limit, $\mathcal{S}=0$, the model possesses a dual formulation in which the scalar compensator $\mathfrak{y}$ is replaced with a gauge one-form.[9] However, such a duality transformation cannot be lifted to AdS$_3$.

VI. NONCONFORMAL HIGHER-SPIN SUPERCURRENTS

In the previous sections, we have shown that there exist two different off-shell formulations for the massless higher-spin $\mathcal{N}=1$ supermultiplets. Massless half-integer superspin theory can be realised in terms of the dynamical variables (3.30) and (4.26), while the models (3.34) and (4.22) define massless multiplet of integer superspin s, with $s>1$. These models lead to different $\mathcal{N}=1$ higher-spin supercurrent multiplets. Our aim in this section is to describe the general structure of $\mathcal{N}=1$ supercurrent multiplets in AdS.

A. $\mathcal{N}=1$ supercurrents: Half-integer superspin case

Our half-integer supermultiplet in the longitudinal formulation (3.30) can be coupled to external sources

$$S^{(s+\frac{1}{2})}_{\mathrm{source}} = \int \mathrm{d}^{3|2}z E\{\mathrm{i}H^{\alpha(2s+1)}J_{\alpha(2s+1)} + 4L^{\alpha(2s-2)}S_{\alpha(2s-2)}\}. \qquad (6.1)$$

The condition that the above action is invariant under the gauge transformations (3.31) gives the conservation equation

$$\nabla^{\beta}J_{\beta\alpha(2s)} = -\frac{2s}{(2s+1)}\nabla_{(\alpha_1\alpha_2}S_{\alpha_3\cdots\alpha_{2s})}. \qquad (6.2)$$

For the transverse theory (4.26) described by the prepotentials $\{H_{\alpha(2s+1)},\Upsilon_{\beta;\alpha(2s-2)}\}$, we construct an action functional of the form

$$\begin{aligned} S^{(s+\frac{1}{2})}_{\mathrm{source}} = \int \mathrm{d}^{3|2}z E\{&\mathrm{i}H^{\alpha(2s+1)}J_{\alpha(2s+1)} \\ &+ 2\mathrm{i}s\Upsilon^{\beta;\alpha(2s-2)}U_{\beta;\alpha(2s-2)}\}. \end{aligned} \qquad (6.3)$$

Requiring that the action is invariant under the gauge transformations (4.27) leads to

$$\nabla^{\beta}J_{\beta\alpha(2s)} = \frac{2s}{2s+1}\nabla_{(\alpha_1}U_{\alpha_2\cdots\alpha_{2s})}, \qquad \nabla^{\beta}U_{\beta;\alpha(2s-2)} = 0. \qquad (6.4)$$

From the above consideration, it follows that the most general conservation equation in the half-integer superspin case takes the form

[9] There is another dual realization in which $\mathfrak{h}_{\alpha(4)}$ turns into a gauge one-form $\mathfrak{h}_{b;\alpha(4)}$ with an additional gauge freedom.

$$\nabla^\beta J_{\beta\alpha(2s)} = \frac{2s}{2s+1}(\nabla_{(\alpha_1}U_{\alpha_2\cdots\alpha_{2s})} - \nabla_{(\alpha_1\alpha_2}S_{\alpha_3\cdots\alpha_{2s})}), \tag{6.5a}$$

$$\nabla^\beta U_{\beta;\alpha(2s-2)} = 0. \tag{6.5b}$$

B. $\mathcal{N}=1$ supercurrents: Integer superspin case

In complete analogy with the half-integer superspin case, we couple the prepotentials (4.22) in terms of which the integer superspin-s is described, to external sources

$$S^{(s)}_{\rm source} = \int d^{3|2}zE\{H^{\alpha(2s)}J_{\alpha(2s)} + 2sV^{\alpha(2s-2)}R_{\alpha(2s-2)}\}. \tag{6.6}$$

For such an action to be invariant under the gauge freedom (4.23), the sources must be conserved

$$\nabla^\beta J_{\beta\alpha(2s-1)} = \nabla_{(\alpha_1}R_{\alpha_2\cdots\alpha_{2s-1})}. \tag{6.7}$$

Next, we turn to the transverse formulation (3.34) characterized by the prepotentials $\{H_{\alpha(2s)}, \Psi_{\beta;\alpha(2s-2)}\}$ and construct an action functional

$$S^{(s)}_{\rm source} = \int d^{3|2}zE\{H^{\alpha(2s)}J_{\alpha(2s)} + {\rm i}\Psi^{\beta;\alpha(2s-2)}T_{\beta;\alpha(2s-2)}\}. \tag{6.8}$$

Demanding that the action be invariant under the gauge transformations (3.35), we derive the following conditions:

$$\nabla^\beta J_{\beta\alpha(2s-1)} = {\rm i}T_{\alpha(2s-1)}, \qquad \nabla^\beta T_{\beta;\alpha(2s-2)} = 0. \tag{6.9}$$

From the above consideration, the most general conservation equation for the multiplet of currents in the integer superspin case is given by

$$\nabla^\beta J_{\beta\alpha(2s-1)} = \nabla_{(\alpha_1}R_{\alpha_2\cdots\alpha_{2s-1})} + {\rm i}T_{\alpha(2s-1)}, \tag{6.10a}$$

$$\nabla^\beta T_{\beta;\alpha(2s-2)} = 0. \tag{6.10b}$$

C. From $\mathcal{N}=2$ supercurrents to $\mathcal{N}=1$ supercurrents

In our recent paper [1], we constructed the general conservation equation for the $\mathcal{N}=2$ higher-spin supercurrent multiplets in (2,0) AdS superspace, which takes the form

$$\bar{\mathcal{D}}^\beta \mathbb{J}_{\beta\alpha(2s-1)} = \bar{\mathcal{D}}_{(\alpha_1}(\mathbb{Y}_{\alpha_2\ldots\alpha_{2s-1})} + {\rm i}\mathbb{Z}_{\alpha_2\ldots\alpha_{2s-1})}). \tag{6.11}$$

Here $\mathbb{J}_{\alpha(2s)}$ denotes the higher-spin supercurrent, while the trace supermultiplets $\mathbb{Y}_{\alpha(2s-2)}$ and $\mathbb{Z}_{\alpha(2s-2)}$ are covariantly linear. The explicit form of this multiplet of currents was presented by considering simple $\mathcal{N}=2$ supersymmetric models for a chiral scalar superfield. Unlike in 4D $\mathcal{N}=1$ supergravity where every supersymmetric matter theory can be coupled to only one of the off-shell supergravity formulations (either old-minimal or new-minimal), here in the (2,0) AdS case our trace multiplets require both type II and type III compensators to couple to.

The general conservation equation (6.11) naturally gives rise to the $\mathcal{N}=1$ higher-spin supercurrent multiplets discussed in the previous subsection. One may show that in the real basis, (6.11) turns into

$$\boldsymbol{\nabla}^{\underline{1}\beta}\mathbb{J}_{\beta\alpha(2s-1)} = \boldsymbol{\nabla}^{\underline{1}}_{(\alpha_1}\mathbb{Y}_{\alpha_2\cdots\alpha_{2s-1})} - \boldsymbol{\nabla}^{\underline{2}}_{(\alpha_1}\mathbb{Z}_{\alpha_2\cdots\alpha_{2s-1})}, \tag{6.12a}$$

$$\boldsymbol{\nabla}^{\underline{2}\beta}\mathbb{J}_{\beta\alpha(2s-1)} = \boldsymbol{\nabla}^{\underline{1}}_{(\alpha_1}\mathbb{Z}_{\alpha_2\cdots\alpha_{2s-1})} + \boldsymbol{\nabla}^{\underline{2}}_{(\alpha_1}\mathbb{Y}_{\alpha_2\cdots\alpha_{2s-1})}, \tag{6.12b}$$

The real linearity constraints on the trace supermultiplets are equivalent to

$$(\boldsymbol{\nabla}^{\underline{2}})^2\mathbb{Y}_{\alpha(2s-2)} = (\boldsymbol{\nabla}^{\underline{1}})^2\mathbb{Y}_{\alpha(2s-2)}, \quad \boldsymbol{\nabla}^{\underline{1}\beta}\boldsymbol{\nabla}^{\underline{2}}_\beta\mathbb{Y}_{\alpha(2s-2)} = 0, \tag{6.13a}$$

$$(\boldsymbol{\nabla}^{\underline{2}})^2\mathbb{Z}_{\alpha(2s-2)} = (\boldsymbol{\nabla}^{\underline{1}})^2\mathbb{Z}_{\alpha(2s-2)}, \quad \boldsymbol{\nabla}^{\underline{1}\beta}\boldsymbol{\nabla}^{\underline{2}}_\beta\mathbb{Z}_{\alpha(2s-2)} = 0. \tag{6.13b}$$

It follows from (6.12) and (6.13) that $\mathbb{J}_{\alpha(2s)}$ contains two independent real $\mathcal{N}=1$ supermultiplets:

$$J_{\alpha(2s)} := \mathbb{J}_{\alpha(2s)}|, \tag{6.14a}$$

$$J_{\alpha(2s+1)} := {\rm i}\boldsymbol{\nabla}^{\underline{2}}_{(\alpha_1}\mathbb{J}_{\alpha_2\cdots\alpha_{2s+1})}|, \tag{6.14b}$$

while the independent real $\mathcal{N}=1$ components of $\mathbb{Y}_{\alpha(2s-2)}$ and $\mathbb{Z}_{\alpha(2s-2)}$ are defined by

$$R_{\alpha(2s-2)} := \mathbb{Y}_{\alpha(2s-2)}|, \qquad U_{\beta;\alpha(2s-2)} := {\rm i}\boldsymbol{\nabla}^{\underline{2}}_\beta\mathbb{Y}_{\alpha(2s-2)}|, \tag{6.15a}$$

$$S_{\alpha(2s-2)} := \mathbb{Z}_{\alpha(2s-2)}|, \qquad T_{\beta;\alpha(2s-2)} := {\rm i}\boldsymbol{\nabla}^{\underline{2}}_\beta\mathbb{Z}_{\alpha(2s-2)}|. \tag{6.15b}$$

Making use of (6.13), one may readily show that

$$\nabla^\beta U_{\beta;\alpha(2s-2)} = 0, \tag{6.16a}$$

$$\nabla^\beta T_{\beta;\alpha(2s-2)} = 0. \tag{6.16b}$$

On the other hand, Eq. (6.12) implies that the $\mathcal{N}=1$ superfields obey the following conditions:

$$\nabla^\beta J_{\beta\alpha(2s)} = \frac{2s}{2s+1}(\nabla_{(\alpha_1}U_{\alpha_2\cdots\alpha_{2s})} - \nabla_{(\alpha_1\alpha_2}S_{\alpha_3\cdots\alpha_{2s})}), \tag{6.17a}$$

$$\nabla^\beta J_{\beta\alpha(2s-1)} = \nabla_{(\alpha_1}R_{\alpha_2\cdots\alpha_{2s-1})} + \mathrm{i}T_{\alpha(2s-1)}. \tag{6.17b}$$

Indeed, the right-hand side of Eq. (6.17a) coincides with (6.5a). Therefore, Eqs. (6.16a) and (6.17a) define the $\mathcal{N}=1$ higher-spin current multiplets associated with the massless half-integer superspin formulations (3.30) and (4.26). In a similar way, it can be observed that Eqs. (6.16b) and (6.17b) correspond to the $\mathcal{N}=1$ higher-spin supercurrents for the two integer superspin models (3.34) and (4.22).

VII. EXAMPLES OF $\mathcal{N}=1$ HIGHER-SPIN SUPERCURRENTS

In this section we give an explicit realization of the $\mathcal{N}=1$ higher-spin supercurrent introduced earlier.

Consider a massless chiral scalar multiplet in (2,0) AdS superspace with action [1]

$$S = \int \mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\boldsymbol{E}\bar{\Phi}\Phi, \qquad \bar{\mathcal{D}}_\alpha\Phi = 0. \tag{7.1}$$

The chiral superfield is charged under the R-symmetry group $\mathrm{U}(1)_R$,

$$J\Phi = -r\Phi, \qquad r = \text{const}. \tag{7.2}$$

This action is a special case of the supersymmetric nonlinear sigma model studied in Sec. II D with a vanishing superpotential, $W(\Phi)=0$. Making use of (2.62), the reduction of the action (7.1) to $\mathcal{N}=1$ AdS superspace is given by

$$S = \int \mathrm{d}^{3|2}zE\{-\mathrm{i}\nabla^\alpha\bar{\varphi}\nabla_\alpha\varphi + 4r\mathcal{S}\bar{\varphi}\varphi\}, \tag{7.3}$$

where we have denoted $\varphi \coloneqq \Phi|$. This action is manifestly $\mathcal{N}=1$ supersymmetric. It also possesses hidden second supersymmetry and $\mathrm{U}(1)_R$ invariance. These transformations are

$$\delta_\epsilon\varphi = \mathrm{i}\epsilon^\alpha\nabla_\alpha\varphi - \mathrm{i}\epsilon r\varphi, \qquad \delta_\epsilon\bar{\varphi} = \mathrm{i}\epsilon^\alpha\nabla_\alpha\bar{\varphi} + \mathrm{i}\epsilon r\bar{\varphi}, \tag{7.4}$$

where ϵ^α is given in terms or ϵ according to (2.31), and the real parameter ϵ is constrained by (2.31b). It can be seen that φ and $\bar{\varphi}$ obey the equations of motion

$$(\mathrm{i}\nabla^2 + 4r\mathcal{S})\varphi = 0, \qquad (\mathrm{i}\nabla^2 + 4r\mathcal{S})\bar{\varphi} = 0. \tag{7.5}$$

When studying higher-spin supercurrents in the (2,0) and (1,0) AdS superspaces, it is advantageous to make use of a condensed notation employed in [1]. We introduce auxiliary real variables $\zeta^\alpha \in \mathbb{R}^2$ and associate with any tensor superfield $U_{\alpha(m)}$ the following index-free field:

$$U_{(m)}(\zeta) \coloneqq \zeta^{\alpha_1}\ldots\zeta^{\alpha_m}U_{\alpha_1\ldots\alpha_m}, \tag{7.6}$$

which is a homogeneous polynomial of degree m in ζ^α. Furthermore, we make use of the bosonic variables ζ^α and the corresponding partial derivatives $\partial/\partial\zeta^\alpha$ to convert the spinor and vector covariant derivatives into index-free operators. In the case of (2,0) AdS superspace, we introduce operators which increase the degree of homogeneity in ζ^α:

$$\mathcal{D}_{(1)} \coloneqq \zeta^\alpha\mathcal{D}_\alpha, \qquad \bar{\mathcal{D}}_{(1)} \coloneqq \zeta^\alpha\bar{\mathcal{D}}_\alpha, \qquad \mathcal{D}_{(2)} \coloneqq \mathrm{i}\zeta^\alpha\zeta^\beta\mathcal{D}_{\alpha\beta}. \tag{7.7}$$

We also introduce two operators that decrease the degree of homogeneity in ζ^α:

$$\mathcal{D}_{(-1)} \coloneqq \mathcal{D}^\alpha\frac{\partial}{\partial\zeta^\alpha}, \qquad \bar{\mathcal{D}}_{(-1)} \coloneqq \bar{\mathcal{D}}^\alpha\frac{\partial}{\partial\zeta^\alpha}. \tag{7.8}$$

The operators associated with the real spinor covariant derivatives, $\boldsymbol{\nabla}^I_\alpha$, may be defined in a similar way:

$$\boldsymbol{\nabla}^I_{(1)} \coloneqq \zeta^\alpha\boldsymbol{\nabla}^I_\alpha, \qquad \boldsymbol{\nabla}_{(2)} \coloneqq \mathrm{i}\zeta^\alpha\zeta^\beta\boldsymbol{\nabla}_{\alpha\beta}, \tag{7.9}$$

$$\boldsymbol{\nabla}^I_{(-1)} \coloneqq \boldsymbol{\nabla}^{I\alpha}\frac{\partial}{\partial\zeta^\alpha}. \tag{7.10}$$

Analogous operators are introduced in the case of $\mathcal{N}=1$ AdS superspace. They are

$$\nabla_{(1)} \coloneqq \zeta^\alpha\nabla_\alpha, \quad \nabla_{(2)} \coloneqq \mathrm{i}\zeta^\alpha\zeta^\beta\nabla_{\alpha\beta}, \tag{7.11}$$

$$\nabla_{(-1)} \coloneqq \nabla^\alpha\frac{\partial}{\partial\zeta^\alpha}. \tag{7.12}$$

It was shown in [1] that by using the massless equations of motion, $\mathcal{D}^2\Phi = 0$, the $\mathcal{N}=2$ higher-spin supercurrent multiplet associated with the theory (7.1) is described by the conservation equation

$$\mathcal{D}_{(-1)}\mathbb{J}_{(2s)} = \mathcal{D}_{(1)}\mathbb{T}_{(2s-2)}. \tag{7.13a}$$

Here the real supercurrent $\mathbb{J}_{(2s)} = \bar{\mathbb{J}}_{(2s)}$ is given by

$$\mathbb{J}_{(2s)} = \sum_{k=0}^{s}(-1)^k\left\{\frac{1}{2}\binom{2s}{2k+1}\mathcal{D}^k_{(2)}\bar{\mathcal{D}}_{(1)}\bar{\Phi}\mathcal{D}^{s-k-1}_{(2)}\mathcal{D}_{(1)}\Phi + \binom{2s}{2k}\mathcal{D}^k_{(2)}\bar{\Phi}\mathcal{D}^{s-k}_{(2)}\Phi\right\}, \tag{7.13b}$$

while the trace multiplet $\mathbb{T}_{(2s-2)}$ has the form

$$\mathbb{T}_{(2s-2)} = 2\mathrm{i}\mathcal{S}(1-2r)(2s+1)(s+1) \times \sum_{k=0}^{s-1} \frac{1}{2s-2k+1}(-1)^k \binom{2s}{2k+1} \times \mathcal{D}_{(2)}^k \bar{\Phi} \mathcal{D}_{(2)}^{s-k-1}\Phi. \tag{7.13c}$$

One may check that $\mathbb{T}_{(2s-2)}$ is covariantly linear,

$$\bar{\mathcal{D}}^2 \mathbb{T}_{(2s-2)} = 0, \qquad \mathcal{D}^2 \mathbb{T}_{(2s-2)} = 0. \tag{7.13d}$$

As is seen from (7.13c), $\mathbb{T}_{(2s-2)}$ vanishes for $r = 1/2$, in which case Φ is an $\mathcal{N} = 2$ superconformal multiplet.

The complex trace multiplet $\mathbb{T}_{(2s-2)}$ may be split into its real and imaginary parts:

$$\mathbb{T}_{(2s-2)} = \mathbb{Y}_{(2s-2)} - \mathrm{i}\mathbb{Z}_{(2s-2)}, \tag{7.14a}$$

with

$$\mathbb{Y}_{(2s-2)} = 2\mathrm{i}\mathcal{S}(1-2r)(2s+1)(s+1)\sum_{k=0}^{s-1}\frac{2k-s+1}{(2k+3)(2s-2k+1)}(-1)^k\binom{2s}{2k+1}\mathcal{D}_{(2)}^k\bar{\Phi}\mathcal{D}_{(2)}^{s-k-1}\Phi, \tag{7.14b}$$

$$\mathbb{Z}_{(2s-2)} = -2\mathcal{S}(1-2r)(2s+1)(s+1)(s+2)\sum_{k=0}^{s-1}\frac{1}{(2k+3)(2s-2k+1)}(-1)^k\binom{2s}{2k+1}\mathcal{D}_{(2)}^k\bar{\Phi}\mathcal{D}_{(2)}^{s-k-1}\Phi. \tag{7.14c}$$

In accordance with (6.14), the supercurrent $\mathbb{J}_{(2s)}$ reduces to two different multiplets upon projection to $\mathcal{N} = 1$ superspace:

$$J_{(2s)} \coloneqq \mathbb{J}_{(2s)}| = \sum_{k=0}^{s}(-1)^{k+1}\left\{\binom{2s}{2k+1}\nabla_{(2)}^k\nabla_{(1)}\bar{\varphi}\nabla_{(2)}^{s-k-1}\nabla_{(1)}\varphi - \binom{2s}{2k}\nabla_{(2)}^k\bar{\varphi}\nabla_{(2)}^{s-k}\varphi\right\}, \tag{7.15a}$$

$$\begin{aligned} J_{(2s+1)} &\coloneqq \mathrm{i}\boldsymbol{\nabla}_{(1)}^2\mathbb{J}_{(2s)}| = -\frac{1}{\sqrt{2}}(\mathcal{D}_{(1)} + \bar{\mathcal{D}}_{(1)})\mathbb{J}_{(2s)}|, \\ &= (2s+1)\sum_{k=0}^{s}\frac{1}{2s-2k+1}(-1)^{k+1}\binom{2s}{2k}\left\{\nabla_{(2)}^k\bar{\varphi}\nabla_{(2)}^{s-k}\nabla_{(1)}\varphi + (-1)^{s-1}\nabla_{(2)}^k\varphi\nabla_{(2)}^{s-k}\nabla_{(1)}\bar{\varphi}\right\}, \end{aligned} \tag{7.15b}$$

of which the former corresponds to the integer superspin current and the latter half-integer superspin current.

In the case of half-integer superspin, the conservation equation (6.5) is satisfied provided we impose (7.5):

$$\nabla_{(-1)}J_{(2s+1)} = \frac{2s}{2s+1}(\nabla_{(1)}U_{(2s-1)} + \mathrm{i}\nabla_{(2)}S_{(2s-2)}), \qquad \nabla^\beta U_{\beta;(2s-2)} = 0, \tag{7.16a}$$

with

$$\begin{aligned} S_{(2s-2)} &\coloneqq \mathbb{Z}_{(2s-2)}| \\ &= -2\mathcal{S}(1-2r)(2s+1)(s+1)(s+2)\sum_{k=0}^{s-1}\frac{1}{(2k+3)(2s-2k+1)}(-1)^k\binom{2s}{2k+1}\nabla_{(2)}^k\bar{\varphi}\nabla_{(2)}^{s-k-1}\varphi, \end{aligned} \tag{7.16b}$$

$$\begin{aligned} U_{\beta;(2s-2)} &\coloneqq -\frac{1}{\sqrt{2}}(\mathcal{D}_\beta + \bar{\mathcal{D}}_\beta)\mathbb{Y}_{(2s-2)}|, \\ &= -2\mathrm{i}\mathcal{S}(1-2r)(2s+1)(s+1)\sum_{k=0}^{s-1}\frac{2k-s+1}{(2k+3)(2s-2k+1)}(-1)^k\binom{2s}{2k+1} \\ &\quad\times\Big\{\nabla_{(2)}^k\bar{\varphi}\nabla_{(2)}^{s-k-1}\nabla_\beta\varphi + (-1)^{s+1}\nabla_{(2)}^k\varphi\nabla_{(2)}^{s-k-1}\nabla_\beta\bar{\varphi} \\ &\quad+ 2\mathrm{i}\mathcal{S}(s-k-1)\zeta_\beta(\nabla_{(2)}^k\bar{\varphi}\nabla_{(2)}^{s-k-2}\nabla_{(1)}\varphi + (-1)^{s+1}\nabla_{(2)}^k\varphi\nabla_{(2)}^{s-k-2}\nabla_{(1)}\bar{\varphi})\Big\}. \end{aligned} \tag{7.16c}$$

It may also be verified that the $\mathcal{N}=1$ supercurrent multiplet for integer superspin obeys the conditions (6.10) on-shell:

$$\nabla_{(-1)}J_{(2s)} = \nabla_{(1)}R_{(2s-2)} + \mathrm{i}T_{(2s-1)}, \qquad \nabla^{\beta}T_{\beta;(2s-2)} = 0 \tag{7.17a}$$

with

$$\begin{aligned} R_{(2s-2)} &:= \mathbb{Y}_{(2s-2)}| \\ &= 2\mathrm{i}\mathcal{S}(1-2r)(2s+1)(s+1)\sum_{k=0}^{s-1}\frac{2k-s+1}{(2k+3)(2s-2k+1)}(-1)^k\binom{2s}{2k+1}\nabla^k_{(2)}\bar{\varphi}\nabla^{s-k-1}_{(2)}\varphi, \end{aligned} \tag{7.17b}$$

$$\begin{aligned} T_{\beta;(2s-2)} &:= -\frac{1}{\sqrt{2}}(\mathcal{D}_\beta + \bar{\mathcal{D}}_\beta)\mathbb{Y}_{(2s-2)}|, \\ &= 2\mathcal{S}(1-2r)(2s+1)(s+1)(s+2)\sum_{k=0}^{s-1}\frac{1}{(2k+3)(2s-2k+1)}(-1)^k\binom{2s}{2k+1} \\ &\quad\times\Big\{\nabla^k_{(2)}\bar{\varphi}\nabla^{s-k-1}_{(2)}\nabla_\beta\varphi + (-1)^s\nabla^k_{(2)}\varphi\nabla^{s-k-1}_{(2)}\nabla_\beta\bar{\varphi} \\ &\quad + 2\mathrm{i}\mathcal{S}(s-k-1)\zeta_\beta(\nabla^k_{(2)}\bar{\varphi}\nabla^{s-k-2}_{(2)}\nabla_{(1)}\varphi + (-1)^s\nabla^k_{(2)}\varphi\nabla^{s-k-2}_{(2)}\nabla_{(1)}\bar{\varphi})\Big\}. \end{aligned} \tag{7.17c}$$

The above technique can also be used to construct $\mathcal{N}=1$ higher-spin supercurrents for the Abelian vector multiplets model described by the action (2.66). We will not elaborate on such a construction in the present work.

In four dimensions, various aspects of the higher-spin supercurrent multiplets were studied in [29–33] in the $\mathcal{N}=1$ super-Poincaré case and in [34] for $\mathcal{N}=1$ AdS supersymmetry. In particular, the general nonconformal higher-spin supercurrent multiplets for $\mathcal{N}=1$ supersymmetric field theories in Minkowski space were proposed in [31,32], and their AdS counterparts were formulated in [34]. Explicit realizations of the higher-spin supercurrents were derived in [34] for various $\mathcal{N}=1$ supersymmetric theories in AdS_4, including a model of N massive chiral scalar superfields with an arbitrary mass matrix.

VIII. APPLICATIONS AND OPEN PROBLEMS

Let us briefly summarize the main results obtained in this paper. In Sec. II we developed a formalism to reduce every field theory with (2,0) AdS supersymmetry to $\mathcal{N}=1$ AdS superspace. In Secs. III and IV we applied this reduction procedure to the off-shell massless higher-spin supermultiplets in $\mathrm{AdS}_{(3|2,0)}$ constructed in [1]. For each superspin value, integer (s) or half-integer $(s+\frac{1}{2})$, the reduction produced two off-shell gauge formulations, longitudinal and transverse, for massless $\mathcal{N}=1$ supermultiplets in AdS_3. The transverse formulations for massless higher-spin $\mathcal{N}=1$ supermultiplets in AdS_3 are new gauge theories. In Sec. V, we proved that for each superspin value the longitudinal and transverse theories are dually equivalent only in the flat superspace limit. In Sec. VI we formulated, for the first time, the nonconformal higher-spin supercurrent in $\mathcal{N}=1$ AdS superspace. In Sec. VII we provided the explicit examples of these supercurrents in simple models for a chiral scalar superfield.

There are several interesting applications of the results obtained in this paper. In particular, the massless higher-spin $\mathcal{N}=1$ supermultiplets in AdS_3, which were derived in Secs. III and IV, can be used to construct off-shell topologically massive supermultiplets in AdS_3 by extending the approaches advocated in [14,22,26]. Such a massive supermultiplet is described by a gauge-invariant action being the sum of massless and superconformal higher-spin actions, following the philosophy of topologically massive theories [28,35–37].

Given a positive integer n, the conformal superspin-$\frac{n}{2}$ action [14,26,38] is

$$S^{(n/2)}_{\mathrm{SCS}}[H_{\alpha(n)}] = -\frac{\mathrm{i}^n}{2^{\lfloor n/2\rfloor+1}}\int \mathrm{d}^{3|2}z\,E\,H^{\alpha(n)}W_{\alpha(n)}(H), \tag{8.1}$$

where $W_{\alpha(n)}(H)$ denotes the higher-spin super-Cotton tensor. The latter is a unique descendant of $H_{\alpha(n)}$ with the properties

$$W_{\alpha(n)}(\delta_\zeta H) = 0, \tag{8.2a}$$

$$\nabla^\beta W_{\beta\alpha(n-1)} = 0, \tag{8.2b}$$

where $\delta_\zeta H_{\alpha(n)}$ is the gauge transformation (3.29). These properties imply the gauge invariance of (8.1). In a flat superspace, $W_{\alpha(n)}$ has the form [38]

$$\mathcal{S}=0 \Rightarrow W_{\alpha_1\ldots\alpha_n} = \left(-\frac{\mathrm{i}}{2}\right)^n \nabla^{\beta_1}\nabla_{\alpha_1}\ldots\nabla^{\beta_n}\nabla_{\alpha_n}H_{\beta_1\ldots\beta_n}. \tag{8.3}$$

The construction of $W_{\alpha(n)}$ in arbitrary conformally flat backgrounds is described in [39].

Given a positive integer s, there are two off-shell gauge-invariant formulations for a topologically massive superspin-s multiplet in AdS_3. The corresponding actions are

$$S^{\|}_{(s)}[H_{\alpha(2s)}, V_{\alpha(2s-2)}|\mu] = S^{(s)}_{\mathrm{SCS}}[H_{\alpha(2s)}] + \mu^{2s-1} S^{\|}_{(s)}[H_{\alpha(2s)}, V_{\alpha(2s-2)}], \tag{8.4a}$$

$$S^{\perp}_{(s)}[H_{\alpha(2s)}, \Psi_{\beta;\alpha(2s-2)}|\mu] = S^{(s)}_{\mathrm{SCS}}[H_{\alpha(2s)}] + \mu^{2s-1} S^{\perp}_{(s)}[H_{\alpha(2s)}, \Psi_{\beta;\alpha(2s-2)}]. \tag{8.4b}$$

The dynamical system (8.4a) was introduced in [14], while its flat-superspace counterpart appeared earlier in [26]. The other theory, Eq. (8.4b), is a new formulation for massive superspin-s multiplet in AdS_3.

In the Minkowski superspace limit, the dynamical systems (8.4a) and (8.4b) are equivalent, since they are related to each other by the superfield Legendre transformation described in Sec. V. On the mass shell, dynamics can be recast in terms of the gauge-invariant field strength $W_{\alpha(2s)}$ which obeys the equations [26]

$$D^\beta W_{\beta\alpha_1\cdots\alpha_{2s-1}} = 0, \qquad -\frac{\mathrm{i}}{2} D^2 W_{\alpha(2s)} = m\sigma W_{\alpha(2s)}, \qquad \sigma = \pm 1 \tag{8.5}$$

where the mass m and helicity parameter σ are determined by μ.[10] It is an interesting open problem to understand whether the AdS models (8.4a) and (8.4b) lead to equivalent dynamics, modulo a redefinition of the mass parameter μ.

There are two off-shell gauge-invariant formulations for a topologically massive superspin-$(s+\frac{1}{2})$ multiplet in AdS_3. The corresponding actions are

$$S^{\|}_{(s+\frac{1}{2})}[H_{\alpha(2s+1)}, L_{\alpha(2s-2)}|\mu] = S^{(s+\frac{1}{2})}_{\mathrm{SCS}}[H_{\alpha(2s+1)}] + \mu^{2s-1} S^{\|}_{(s)}[H_{\alpha(2s+1)}, L_{\alpha(2s-2)}], \tag{8.6a}$$

$$S^{\perp}_{(s+\frac{1}{2})}[H_{\alpha(2s+1)}, \Upsilon_{\beta;\alpha(2s-2)}|\mu] = S^{(s+\frac{1}{2})}_{\mathrm{SCS}}[H_{\alpha(2s+1)}] + \mu^{2s-1} S^{\perp}_{(s+\frac{1}{2})}[H_{\alpha(2s+1)}, \Upsilon_{\beta;\alpha(2s-2)}]. \tag{8.6b}$$

[10] Equations (8.5) describe the irreducible massive multiplet of superhelicity $\kappa = (s+\frac{1}{4})\sigma$ [40], with the $\mathcal{N}=1$ superhelicity operator being defined according to [41].

The theory defined by (8.6a) was introduced in [14], while its flat-superspace counterpart appeared earlier in [26]. The other model, Eq. (8.6b), is a new formulation for a massive superspin-$(s+\frac{1}{2})$ multiplet in AdS_3.

In the Minkowski superspace limit, the dynamical systems (8.6a) and (8.6b) are equivalent, since they are related to each other by the superfield Legendre transformation described in Sec. V. It is also an interesting open problem to understand whether the models (8.6) and (8.6b) in AdS_3 generate equivalent dynamics.

We now present two off-shell formulations for the massive $\mathcal{N}=1$ gravitino supermultiplet in AdS_3 and analyze the corresponding equations of motion.[11] The massive extension of the longitudinal theory (4.25) is described by the action

$$S^{\|}_{(1),\mu} = -\frac{1}{2}\int \mathrm{d}^{3|2}z\, E\bigg\{\frac{\mathrm{i}}{2}H^{\alpha\beta}\nabla^2 H_{\alpha\beta} - \frac{\mathrm{i}}{2}\nabla_\beta H^{\alpha\beta}\nabla^\gamma H_{\gamma\alpha} - V\nabla^{\alpha\beta}H_{\alpha\beta} + \frac{\mathrm{i}}{2}V\nabla^2 V + (\mu+2\mathcal{S})H^{\alpha\beta}H_{\alpha\beta} - 2(\mu-2\mathcal{S})V^2\bigg\}, \tag{8.7}$$

with μ a real mass parameter. The massive gravitino action is thus constructed from the massless one by adding masslike terms. In the limit $\mu \to 0$, the action reduces to (4.25). The equations of motion for the dynamical superfields $H^{\alpha\beta}$ and V are

$$2\nabla^\gamma{}_{(\alpha}H_{\beta)\gamma} - \mathrm{i}\nabla^2 H_{\alpha\beta} - 2\nabla_{\alpha\beta}V - 4\mu H_{\alpha\beta} = 0, \tag{8.8a}$$

$$\nabla^{\alpha\beta}H_{\alpha\beta} = (\mathrm{i}\nabla^2 + 8\mathcal{S} - 4\mu)V. \tag{8.8b}$$

Multiplying (8.8a) by $\nabla^{\alpha\beta}$ and noting that $[\nabla_{\alpha\beta}, \nabla^2] = 0$ yields

$$-\mathrm{i}\nabla^2\nabla^{\alpha\beta}H_{\alpha\beta} + 4\Box V - 4\mu\nabla^{\alpha\beta}H_{\alpha\beta} = 0. \tag{8.9}$$

Substituting (8.8b) into (8.9) leads to

$$V = 0. \tag{8.10}$$

Now that $V=0$ on-shell, Eq. (8.8b) turns into

$$\nabla^{\alpha\beta}H_{\alpha\beta} = 0, \tag{8.11}$$

while (8.8) can equivalently be written as

$$-\mathrm{i}\nabla^\gamma\nabla_\alpha H_{\beta\gamma} - (2\mu + 4\mathcal{S})H_{\alpha\beta} = 0. \tag{8.12}$$

[11] The construction of the models (8.7) and (8.16) is similar to those used to derive the off-shell formulations for massive superspin-1 and superspin-3/2 multiplets in four dimensions [42–52].

Making use of the identity (A7b), it immediately follows from (8.12) that

$$\nabla^\alpha H_{\alpha\beta} = 0, \tag{8.13}$$

and then (8.12) is equivalent to

$$-\frac{\mathrm{i}}{2}\nabla^2 H_{\alpha\beta} = (\mu + 2\mathcal{S})H_{\alpha\beta}. \tag{8.14}$$

Therefore, we have demonstrated that the model (8.7) leads to the following conditions on the mass shell:

$$V = 0, \tag{8.15a}$$

$$\nabla^\alpha H_{\alpha\beta} = 0 \Rightarrow \nabla^{\alpha\beta} H_{\alpha\beta} = 0, \tag{8.15b}$$

$$-\frac{\mathrm{i}}{2}\nabla^2 H_{\alpha\beta} = (\mu + 2\mathcal{S})H_{\alpha\beta}. \tag{8.15c}$$

Such conditions are required to describe an irreducible on-shell massive gravitino multiplet in 3D $\mathcal{N}=1$ AdS superspace [40].

In the transverse formulation (3.37), the action for a massive gravitino multiplet is given by

$$\begin{aligned} S^{\perp}_{(1),\mu} = -\frac{1}{2}\int \mathrm{d}^{3|2}z E\bigg\{&\frac{\mathrm{i}}{2}H^{\alpha\beta}\nabla^2 H_{\alpha\beta} - \mathrm{i}\nabla_\beta H^{\alpha\beta}\nabla^\gamma H_{\gamma\alpha} \\ &- H^{\alpha\beta}\nabla_\alpha \mathcal{W}_\beta - \frac{\mathrm{i}}{2}\mathcal{W}^\alpha \mathcal{W}_\alpha + (\mu + 4\mathcal{S})H^{\alpha\beta}H_{\alpha\beta} \\ &- \mathrm{i}(\mu + 2\mathcal{S})(\Psi^\alpha \mathcal{W}_\alpha + 2\mu\Psi^\alpha\Psi_\alpha)\bigg\}. \end{aligned} \tag{8.16}$$

In the limit $\mu \to 0$, the action reduces to (3.37). One may check that the equations of motion for this model imply that

$$\Psi_\alpha = 0, \tag{8.17a}$$

$$\nabla^\alpha H_{\alpha\beta} = 0 \Rightarrow \nabla^{\alpha\beta} H_{\alpha\beta} = 0, \tag{8.17b}$$

$$-\frac{\mathrm{i}}{2}\nabla^2 H_{\alpha\beta} = (\mu + 4\mathcal{S})H_{\alpha\beta}. \tag{8.17c}$$

The actions (8.7) and (8.16) can be made into gauge-invariant ones using the Stueckelberg construction.

In the Minkowski superspace limit, the massive models (8.7) and (8.16) lead to the identical equations of motion described in terms of $H_{\alpha\beta}$:

$$D^\alpha H_{\alpha\beta} = 0, \qquad -\frac{\mathrm{i}}{2}D^2 H_{\alpha\beta} = \mu H_{\alpha\beta}. \tag{8.18}$$

In the AdS case, Eqs. (8.15) and (8.17) lead to equivalent dynamics modulo a redefinition of μ. It is an interesting open problem to understand whether there exists a duality transformation relating these models.

There exist alternative off-shell gauge-invariant formulations for massive higher-spin supermultiplets in AdS_3 proposed in [14] for $\mathcal{N}=1$ AdS supersymmetry and in [1] for (2,0) AdS supersymmetry. In the $\mathcal{N}=1$ case the corresponding action is

$$\begin{aligned} S^{(n/2)}_{\text{massive}}[H_{\alpha(n)}] = -\frac{\mathrm{i}^n}{2^{\lfloor n/2\rfloor+1}\mu}\int \mathrm{d}^{3|2}z E H^{\alpha(n)} \\ \times\left(\mu + \frac{\mathrm{i}}{2}\nabla^2\right)W_{\alpha(n)}(H), \end{aligned} \tag{8.19}$$

with $\mu \neq 0$ a real parameter. This action may be viewed as a deformation of the superconformal model (8.1). It is invariant under the gauge transformation (3.29) as a consequence of the condition (8.2b) and the identity (A7c).

In the flat superspace limit, the action (8.19) leads to the equation of motion

$$-\frac{\mathrm{i}}{2}D^2 W_{\alpha(n)} = \mu W_{\alpha(n)}. \tag{8.20}$$

Since $W_{\alpha(n)}$ is transverse, the equation of motion implies that $W_{\alpha(n)}$ describes a massive higher-spin supermultiplet, compare with (8.5). The (2,0) supersymmetric extension of the model (8.19) is presented in [1].

It should be pointed out that there also exists an on-shell construction of gauge-invariant Lagrangian formulations for massive higher-spin supermultiplets in $\mathbb{R}^{2,1}$ and AdS_3, which were developed in [53,54]. It is obtained by combining the massive bosonic and fermionic higher-spin actions [55,56], and therefore this construction is intrinsically on-shell. The formulations given in [53–56] are based on the gauge-invariant approaches to the dynamics of massive higher-spin fields, which were advocated by Zinoviev [57] and Metsaev [58]. It is an interesting open problem to understand whether there exists an off-shell uplift of these models.

All off-shell higher-spin $\mathcal{N}=2$ supermultiplets in AdS_3, both with (2,0) and (1,1) AdS supersymmetry [1,21], are reducible gauge theories (in the terminology of the Batalin-Vilkovisky quantization [59]), similar to the massless higher-spin supermultiplets in AdS_4 [25]. The Lagrangian quantization of such theories is nontrivial. In the four-dimensional case, the quantization of the theories proposed in [25] was carried out in [60]. All off-shell higher-spin $\mathcal{N}=1$ supermultiplets in AdS_3, which we have constructed in this paper, are irreducible gauge theories that can be quantized using the Faddeev-Popov procedure [61] as in the nonsupersymmetric case, see e.g., [62,63]. This opens the possibility to develop heat kernel techniques for higher-spin theories in $\mathrm{AdS}^{3|2}$, as an extension of the four-dimensional results [16,64,65].

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APPENDIX A: NOTATION, CONVENTIONS AND $\mathcal{N}=1$ ADS IDENTITIES

We summarize our notation and conventions which follow [5]. The Minkowski metric is $\eta_{ab}=\mathrm{diag}(-1,1,1)$. The spinor indices are raised and lowered by the rule

$$\psi^\alpha = \varepsilon^{\alpha\beta}\psi_\beta, \qquad \psi_\alpha = \varepsilon_{\alpha\beta}\psi^\beta. \tag{A1}$$

Here the antisymmetric $\mathrm{SL}(2,\mathbb{R})$ invariant tensors $\varepsilon_{\alpha\beta} = -\varepsilon_{\beta\alpha}$ and $\varepsilon^{\alpha\beta} = -\varepsilon^{\beta\alpha}$ are normalized as $\varepsilon_{12}=-1, \varepsilon^{12}=1$.

We make use of real Dirac γ-matrices, $\gamma_a := ((\gamma_a)_\alpha{}^\beta)$ defined by

$$(\gamma_a)_\alpha{}^\beta := \varepsilon^{\beta\gamma}(\gamma_a)_{\alpha\gamma} = (-\mathrm{i}\sigma_2,\sigma_3,\sigma_1). \tag{A2}$$

They obey the algebra

$$\gamma_a\gamma_b = \eta_{ab}\mathbb{1} + \varepsilon_{abc}\gamma^c, \tag{A3}$$

where the Levi-Civita tensor is normalized as $\varepsilon^{012} = -\varepsilon_{012} = 1$. Some useful relations involving γ-matrices are

$$(\gamma^a)_{\alpha\beta}(\gamma_a)^{\rho\sigma} = -(\delta^\rho_\alpha\delta^\sigma_\beta + \delta^\sigma_\alpha\delta^\rho_\beta), \tag{A4a}$$

$$\varepsilon_{abc}(\gamma^b)_{\alpha\beta}(\gamma^c)_{\gamma\delta} = \varepsilon_{\gamma(\alpha}(\gamma_a)_{\beta)\delta} + \varepsilon_{\delta(\alpha}(\gamma_a)_{\beta)\gamma}, \tag{A4b}$$

$$\mathrm{tr}[\gamma_a\gamma_b\gamma_c\gamma_d] = 2\eta_{ab}\eta_{cd} - 2\eta_{ac}\eta_{db} + 2\eta_{ad}\eta_{bc}. \tag{A4c}$$

Given a three-vector A_a, it can equivalently be described as a symmetric rank-2 spinor $A_{\alpha\beta} = A_{\beta\alpha}$,

$$A_{\alpha\beta} := (\gamma^a)_{\alpha\beta}A_a, \qquad A_a = -\frac{1}{2}(\gamma_a)^{\alpha\beta}A_{\alpha\beta}. \tag{A5}$$

The relationship between the Lorentz generators with two vector indices ($M_{ab}=-M_{ba}$), one vector index (M_a) and two spinor indices ($M_{\alpha\beta}=M_{\beta\alpha}$) is as follows: $M_a = \frac{1}{2}\varepsilon_{abc}M^{bc}$ and $M_{\alpha\beta} = (\gamma^a)_{\alpha\beta}M_a$. These generators act on a vector V_c and a spinor Ψ_γ by the rules

$$M_{ab}V_c = 2\eta_{c[a}V_{b]}, \qquad M_{\alpha\beta}\Psi_\gamma = \varepsilon_{\gamma(\alpha}\Psi_{\beta)}. \tag{A6}$$

We collect some useful identities for $\mathcal{N}=1$ AdS covariant derivatives, which we denote by $\nabla_A = (\nabla_a,\nabla_\alpha)$. Making use of the (anti)commutation relation (2.12a) and (2.12b), we obtain the following identities:

$$\nabla_\alpha\nabla_\beta = \frac{1}{2}\varepsilon_{\alpha\beta}\nabla^2 + \mathrm{i}\nabla_{\alpha\beta} - 2\mathrm{i}\mathcal{S}M_{\alpha\beta}, \tag{A7a}$$

$$\nabla^\beta\nabla_\alpha\nabla_\beta = 4\mathrm{i}\mathcal{S}\nabla_\alpha, \tag{A7b}$$

$$\begin{aligned}\nabla^2\nabla_\alpha = -\nabla_\alpha\nabla^2 + 4\mathrm{i}\mathcal{S}\nabla_\alpha = 2\mathrm{i}\nabla_{\alpha\beta}\nabla^\beta + 2\mathrm{i}\mathcal{S}\nabla_\alpha \\ - 4\mathrm{i}\mathcal{S}\nabla^\beta M_{\alpha\beta},\end{aligned} \tag{A7c}$$

$$-\frac{1}{4}\nabla^2\nabla^2 = \Box - 2\mathrm{i}\mathcal{S}\nabla^2 + 2\mathcal{S}\nabla^{\alpha\beta}M_{\alpha\beta} - 2\mathcal{S}^2M^{\alpha\beta}M_{\alpha\beta}, \tag{A7d}$$

where $\nabla^2 = \nabla^\alpha\nabla_\alpha$ and $\Box = \nabla^a\nabla_a = -\frac{1}{2}\nabla^{\alpha\beta}\nabla_{\alpha\beta}$. An important corollary of (A7a) and (A7c) is

$$[\nabla_\alpha\nabla_\beta,\nabla^2] = 0 \Rightarrow [\nabla_{\alpha\beta},\nabla^2] = 0. \tag{A8}$$

The left-hand side of (A7d) can be expressed in terms of the quadratic Casimir operator of the 3D $\mathcal{N}=1$ AdS supergroup [14]:

$$\mathbb{Q} = -\frac{1}{4}\nabla^2\nabla^2 + \mathrm{i}\mathcal{S}\nabla^2, \qquad [\mathbb{Q},\nabla_A] = 0. \tag{A9}$$

We also note the following commutation relation:

$$\begin{aligned}[(\boldsymbol{\nabla}^{\underline{1}})^2(\boldsymbol{\nabla}^{\underline{1}})^2 - 4\mathrm{i}\mathcal{S}(\boldsymbol{\nabla}^{\underline{1}})^2, \boldsymbol{\nabla}^{\underline{2}}_\alpha] = 16\mathcal{S}\nabla_{\alpha\beta}\boldsymbol{\nabla}^{\underline{2}\beta} - 16\mathcal{S}^2\boldsymbol{\nabla}^{\underline{2}}_\alpha \\ - 32\mathcal{S}^2\boldsymbol{\nabla}^{\underline{2}\beta}M_{\alpha\beta} - 32\mathrm{i}\mathcal{S}^2\boldsymbol{\nabla}^{\underline{1}}_\alpha J.\end{aligned} \tag{A10}$$

Given an arbitrary superfield F and its complex conjugate $\bar{F}$, the following relation holds:

$$\overline{\nabla_\alpha F} = -(-1)^{\epsilon(F)}\nabla_\alpha\bar{F}, \tag{A11}$$

where $\epsilon(F)$ denotes the Grassmann parity of F.

APPENDIX B: COMPONENT STRUCTURE OF $\mathcal{N}=1$ HIGHER-SPIN ACTIONS

In this Appendix we will discuss the component structure of the two new off-shell $\mathcal{N}=1$ supersymmetric higher-spin theories: the transverse massless superspin-s multiplet (3.36), and the transverse massless superspin-$(s+\frac{1}{2})$ multiplet (4.28a). For simplicity we will carry out our analysis in flat Minkowski superspace. In accordance with (2.51), the component form of an $\mathcal{N}=1$ supersymmetric action is computed by the rule

$$S = \int \mathrm{d}^{3|2}z L = \frac{\mathrm{i}}{4}\int \mathrm{d}^3x D^2 L|_{\theta=0}, \qquad L = \bar{L}. \tag{B1}$$

1. Massless superspin-s action

Let us first work out the component structure of the massless integer superspin model (3.36). In the flat-superspace limit, the transverse action (3.36) takes the form

$$S^{\perp}_{(s)}[H_{\alpha(2s)},\Psi_{\beta;\alpha(2s-2)}] = \left(-\frac{1}{2}\right)^s \int \mathrm{d}^{3|2}z \bigg\{\frac{\mathrm{i}}{2}H^{\alpha(2s)}D^2H_{\alpha(2s)} - \mathrm{i}sD_\beta H^{\beta\alpha(2s-1)}D^\gamma H_{\gamma\alpha(2s-1)} - (2s-1)\mathcal{W}^{\beta\alpha(2s-2)}D^\gamma H_{\gamma\beta\alpha(2s-2)} - \frac{\mathrm{i}}{2}(2s-1)\left(\mathcal{W}^{\beta;\alpha(2s-2)}\mathcal{W}_{\beta;\alpha(2s-2)} + \frac{s-1}{s}\mathcal{W}_{\beta;}{}^{\beta\alpha(2s-3)}\mathcal{W}^{\gamma;}{}_{\gamma\alpha(2s-3)}\right)\bigg\}. \tag{B2}$$

As described in (3.38), it is possible to choose a gauge condition $\Psi_{(\alpha_1;\alpha_2\cdots\alpha_{2s-1})}=0$, such that the above action turns into

$$S^{\perp}_{(s)}[H_{\alpha(2s)},\Psi_{\beta;\alpha(2s-2)}] = \left(-\frac{1}{2}\right)^s \int \mathrm{d}^{3|2}z \bigg\{\frac{\mathrm{i}}{2}H^{\alpha(2s)}D^2H_{\alpha(2s)} - \mathrm{i}sD_\beta H^{\beta\alpha(2s-1)}D^\gamma H_{\gamma\alpha(2s-1)} - 2(s-1)\varphi^{\alpha(2s-3)}\partial^{\beta\gamma}D^\delta H_{\beta\gamma\delta\alpha(2s-3)} - \frac{2\mathrm{i}}{s}(s-1)\varphi^{\alpha(2s-3)}\Box\varphi_{\alpha(2s-3)} - \frac{\mathrm{i}(s-1)(s-2)(2s-3)}{s(2s-1)}\partial_{\delta\lambda}\varphi^{\delta\lambda\alpha(2s-5)}\partial^{\beta\gamma}\varphi_{\beta\gamma\alpha(2s-5)} + \frac{\mathrm{i}(s-1)(2s-3)}{2s(2s-1)}D_\beta\varphi^{\beta\alpha(2s-4)}D^2D^\gamma\varphi_{\gamma\alpha(2s-4)}\bigg\}. \tag{B3}$$

It is invariant under the following gauge transformations:

$$\delta H_{\alpha(2s)} = -\partial_{(\alpha_1\alpha_2}\eta_{\alpha_3\ldots\alpha_{2s})}, \tag{B4a}$$

$$\delta\varphi_{\alpha(2s-3)} = \mathrm{i}D^\beta\eta_{\beta\alpha(2s-3)}, \tag{B4b}$$

where the gauge parameter $\eta_{\alpha(2s-2)}$ is a real unconstrained superfield.

The gauge freedom (B4) can be used to impose a Wess-Zumino gauge

$$\varphi_{\alpha(2s-3)}| = 0, \qquad D_{(\alpha_1}\varphi_{\alpha_2\cdots\alpha_{2s-2})}| = 0. \tag{B5}$$

In order to preserve these gauge conditions, the residual gauge freedom has to be constrained by

$$D^\beta\eta_{\beta\alpha(2s-3)}| = 0, \qquad D^2\eta_{\alpha(2s-2)}| = 2\mathrm{i}\partial^\beta{}_{(\alpha_1}\eta_{\alpha_2\cdots\alpha_{2s-2})\beta}|. \tag{B6}$$

These imply that there remain two independent, real components of $\eta_{\alpha(2s-2)}$:

$$\xi_{\alpha(2s-2)} := \eta_{\alpha(2s-2)}|, \qquad \lambda_{\alpha(2s-1)} := \mathrm{i}D_{(\alpha_1}\eta_{\alpha_2\cdots\alpha_{2s-1})}|. \tag{B7}$$

In the gauge (B5), the independent component fields of $\varphi_{\alpha(2s-3)}$ can be chosen as

$$y_{\alpha(2s-4)} := -\frac{2s-2}{2s-1}D^\beta\varphi_{\beta\alpha_1\cdots\alpha_{2s-4}}|, \qquad y_{\alpha(2s-3)} := \frac{\mathrm{i}}{2}D^2\varphi_{\alpha(2s-3)}|. \tag{B8}$$

We define the component fields of $H_{\alpha(2s)}$ as

$$h_{\alpha(2s)} := -H_{\alpha(2s)}|, \tag{B9}$$

$$h_{\alpha(2s+1)} := \mathrm{i}\frac{s}{2s+1}D_{(\alpha_1}H_{\alpha_2\cdots\alpha_{2s+1})}|, \qquad y_{\alpha(2s-1)} := \mathrm{i}D^\beta H_{\beta\alpha_1\cdots\alpha_{2s-1}}|, \tag{B10}$$

$$F_{\alpha(2s)} := \frac{\mathrm{i}}{4}D^2H_{\alpha(2s)}|. \tag{B11}$$

Applying the reduction rule (B1) to the $\mathcal{N}=1$ action (B3), we find that it splits into bosonic and fermionic parts:

$$S^{\perp}_{(s)}[H_{\alpha(2s)},\Psi_{\beta;\alpha(2s-2)}] = S_{\mathrm{bos}} + S_{\mathrm{ferm}}. \tag{B12}$$

The bosonic action is given by

$$S_{\mathrm{bos}} = \left(-\frac{1}{2}\right)^s \int \mathrm{d}^3x \bigg\{2(1-s)F^{\alpha(2s)}F_{\alpha(2s)} + 2sF^{\alpha(2s-1)\beta}\partial^\gamma{}_\beta h_{\alpha(2s-1)\gamma} - \frac{1}{2}(s-1)h^{\alpha(2s)}\Box h_{\alpha(2s)} - \frac{(2s-1)(2s-3)}{2s(s-1)}y^{\alpha(2s-4)}\Box y_{\alpha(2s-4)} - \frac{(2s-1)(2s-3)}{4(s-1)}y^{\alpha(2s-4)}\partial^{\beta\gamma}\partial^{\delta\lambda}h_{\beta\gamma\delta\lambda\alpha(2s-4)} - \frac{(s-2)(2s-1)(2s-3)(2s-5)}{16s(s-1)^2} \times \partial_{\delta\lambda}y^{\delta\lambda\alpha(2s-6)}\partial^{\beta\gamma}y_{\beta\gamma\alpha(2s-6)}\bigg\}. \tag{B13}$$

Integrating out the auxiliary field $F_{\alpha(2s)}$ leads to

$$S_{\rm bos} = \left(-\frac{1}{2}\right)^s \frac{2s-1}{2s-2}\int \mathrm{d}^3x \bigg\{ h^{\alpha(2s)}\Box h_{\alpha(2s)} - \frac{s}{2}\partial_{\delta\lambda}h^{\delta\lambda\alpha(2s-2)}\partial^{\beta\gamma}h_{\beta\gamma\alpha(2s-2)} - \frac{2s-3}{2s}\bigg[s y^{\alpha(2s-4)}\partial^{\beta\gamma}\partial^{\delta\lambda}h_{\beta\gamma\delta\lambda\alpha(2s-4)} + 2y^{\alpha(2s-4)}\Box y_{\alpha(2s-4)} + \frac{(s-2)(2s-5)}{4(s-1)}\partial_{\delta\lambda}y^{\delta\lambda\alpha(2s-6)}\partial^{\beta\gamma}y_{\beta\gamma\alpha(2s-6)}\bigg]\bigg\}. \tag{B14}$$

This action is invariant under the gauge transformations

$$\delta_\xi h_{\alpha(2s)} = \partial_{(\alpha_1\alpha_2}\xi_{\alpha_3\cdots\alpha_{2s})}, \tag{B15}$$

$$\delta_\xi y_{\alpha(2s-4)} = \frac{2s-2}{2s-1}\partial^{\beta\gamma}\xi_{\beta\gamma\alpha_1\cdots\alpha_{2s-4}}. \tag{B16}$$

The gauge transformations for the fields $h_{\alpha(2s)}$ and $y_{\alpha(2s-4)}$ can be easily read off from the gauge transformations of the superfields $H_{\alpha(2s)}$ and $\varphi_{\alpha(2s-3)}$, respectively. Modulo an overall normalization factor, (B14) corresponds to the massless Fronsdal spin-s action $S_F^{(2s)}$ described in [14].

The fermionic sector of the component action is described by the real dynamical fields $h_{\alpha(2s+1)}$, $y_{\alpha(2s-1)}$, $y_{\alpha(2s-3)}$, defined modulo gauge transformations of the form

$$\delta_\lambda h_{\alpha(2s+1)} = \partial_{(\alpha_1\alpha_2}\lambda_{\alpha_3\cdots\alpha_{2s+1})}, \tag{B17}$$

$$\delta_\lambda y_{\alpha(2s-1)} = \frac{1}{2s+1}\partial^\beta{}_{(\alpha_1}\lambda_{\alpha_2\cdots\alpha_{2s-1})\beta}, \tag{B18}$$

$$\delta_\lambda y_{\alpha(2s-3)} = \partial^{\beta\gamma}\lambda_{\beta\gamma\alpha_1\cdots\alpha_{2s-3}}. \tag{B19}$$

The gauge-invariant action is

$$S_{\rm ferm} = \left(-\frac{1}{2}\right)^s \frac{\mathrm{i}}{2}\int \mathrm{d}^3x \bigg\{ h^{\alpha(2s)\beta}\partial_\beta{}^\gamma h_{\alpha(2s)\gamma} + 2(2s-1)y^{\alpha(2s-1)}\partial^{\beta\gamma}h_{\beta\gamma\alpha(2s-1)} + 4(2s-1)y^{\alpha(2s-2)\beta}\partial_\beta{}^\gamma y_{\alpha(2s-2)\gamma} + \frac{2}{s}(2s+1)(s-1)y^{\alpha(2s-3)}\partial^{\beta\gamma}y_{\beta\gamma\alpha(2s-3)} - \frac{(s-1)(2s-3)}{s(2s-1)}y^{\alpha(2s-4)\beta}\partial_\beta{}^\gamma y_{\alpha(2s-4)\gamma}\bigg\}. \tag{B20}$$

It may be shown that $S_{\rm ferm}$ coincides with the Fang-Fronsdal spin-$(s+\frac{1}{2})$ action, $S_{FF}^{(2s+1)}$ [14].

We have thus proved that at the component level and upon elimination of the auxiliary field, the transverse theory (B3) is equivalent to a sum of two massless models: the bosonic Fronsdal spin-s model and the fermionic Fang-Fronsdal spin-$(s+\frac{1}{2})$ model.

2. Massless superspin-$(s+\frac{1}{2})$ action

We will now elaborate on the component structure of the massless half-integer superspin model in the transverse formulation (4.28). The theory is described in terms of the real unconstrained prepotentials $H_{\alpha(2s+1)}$ and $\Upsilon_{\beta;\alpha(2s-2)}$. In Minkowski superspace, the action (4.28) simplifies into

$$S^\perp_{(s+\frac{1}{2})}[H_{(2s+1)},\Upsilon_{\beta;\alpha(2s-2)}] = \left(-\frac{1}{2}\right)^s \int \mathrm{d}^{3|2}z \bigg\{ -\frac{\mathrm{i}}{2}H^{\alpha(2s+1)}\Box H_{\alpha(2s+1)} - \frac{\mathrm{i}}{8}D_\beta H^{\beta\alpha(2s)}D^2D^\gamma H_{\gamma\alpha(2s)} + \frac{\mathrm{i}}{8}\partial_{\beta\gamma}H^{\beta\gamma\alpha(2s-1)}\partial^{\rho\delta}H_{\rho\delta\alpha(2s-1)} - \frac{\mathrm{i}}{4}(2s-1)\Omega^{\beta;\alpha(2s-2)}\partial^{\gamma\delta}H_{\beta\gamma\delta\alpha(2s-2)} - \frac{\mathrm{i}}{8}(2s-1)\big(\Omega^{\beta;\alpha(2s-2)}\Omega_{\beta;\alpha(2s-2)} - 2(s-1)\Omega_{\beta;}{}^{\beta\alpha(2s-3)}\Omega^{\gamma;}{}_{\gamma\alpha(2s-3)}\big)\bigg\}, \tag{B21}$$

with the following gauge symmetry

$$\delta H_{\alpha(2s+1)} = \mathrm{i}D_{(\alpha_1}\zeta_{\alpha_2\ldots\alpha_{2s+1})}, \tag{B22a}$$

$$\delta\Upsilon_{\beta;\alpha(2s-2)} = \frac{\mathrm{i}}{2s+1}\big(D^\gamma\zeta_{\gamma\beta\alpha(2s-2)} + (2s+1)D_\beta\eta_{\alpha(2s-2)}\big). \tag{B22b}$$

The action (B21) involves the real field strength $\Omega_{\beta;\alpha(2s-2)}$

$$\Omega_{\beta;\alpha(2s-2)} = -\mathrm{i}D^\gamma D_\beta \Upsilon_{\gamma;\alpha(2s-2)}, \qquad D^\beta \Omega_{\beta;\alpha(2s-2)} = 0. \tag{B23}$$

The gauge transformations (B22) allow us to impose a Wess-Zumino gauge on the prepotentials:

$$\begin{aligned} &H_{\alpha(2s+1)}| = 0, \qquad D^\beta H_{\beta\alpha_1\cdots\alpha_{2s}}| = 0,\\ &\Upsilon_{\beta;\alpha(2s-2)}| = 0, \qquad D^\beta \Upsilon_{\beta;\alpha(2s-2)}| = 0. \end{aligned} \tag{B24}$$

The residual gauge symmetry preserving the gauge conditions (B24) is characterized by

$$D_{(\alpha_1}\zeta_{\alpha_2\cdots\alpha_{2s+1})}| = 0, \qquad D^2\zeta_{\alpha(2s)}| = -\frac{2\mathrm{i}s}{s+1}\partial^\beta{}_{(\alpha_1}\zeta_{\alpha_2\cdots\alpha_{2s})\beta}|, \tag{B25a}$$

$$D_\beta\eta_{\alpha(2s-2)}| = D_{(\beta}\eta_{\alpha(2s-2))}| = -\frac{1}{2s+1}D^\gamma\zeta_{\gamma\beta\alpha(2s-2)}|, \tag{B25b}$$

$$D^2\eta_{\alpha(2s-2)}| = -\frac{\mathrm{i}}{2s+1}\partial^{\beta\gamma}\zeta_{\beta\gamma\alpha(2s-2)}|. \tag{B25c}$$

As a result, there are three independent, real gauge parameters at the component level, which we define as

$$\xi_{\alpha(2s)} \coloneqq \zeta_{\alpha(2s)}|, \qquad \lambda_{\alpha(2s-1)} \coloneqq -\mathrm{i}\frac{s}{2s+1}D^\beta\zeta_{\beta\alpha(2s-1)}|,$$
$$\rho_{\alpha(2s-2)} \coloneqq \eta_{\alpha(2s-2)}|. \tag{B26}$$

Let us now represent the prepotential $\Upsilon_{\beta;\alpha(2s-2)}$ in terms of its irreducible components,

$$\Upsilon_{\beta;\alpha(2s-2)} = Y_{\beta\alpha_1\ldots\alpha_{2s-2}} + \sum_{k=1}^{2s-2}\varepsilon_{\beta\alpha_k}Z_{\alpha_1\ldots\hat{\alpha}_k\ldots\alpha_{2s-2}}, \tag{B27}$$

where we have introduced the two irreducible components of $\Upsilon_{\beta;\alpha(2s-2)}$ by the rule

$$\begin{aligned} Y_{\beta\alpha_1\cdots\alpha_{2s-2}} &\coloneqq \Upsilon_{(\beta;\alpha_1\cdots\alpha_{2s-2})},\\ Z_{\alpha_1\ldots\alpha_{2s-3}} &\coloneqq \frac{1}{2s-1}\Upsilon^{\beta;}{}_{\beta\alpha_1\ldots\alpha_{2s-3}}. \end{aligned} \tag{B28}$$

The next step is to determine the remaining independent component fields of $H_{\alpha(2s+1)}$ and $\Upsilon_{\beta;\alpha(2s-2)}$ in the Wess-Zumino gauge (B24).

In the bosonic sector, we have the following set of fields:

$$h_{\alpha(2s+2)} \coloneqq -D_{(\alpha_1}H_{\alpha_2\cdots\alpha_{2s+2})}|, \tag{B29a}$$

$$y_{\alpha(2s)} \coloneqq D_{(\alpha_1}Y_{\alpha_2\cdots\alpha_{2s})}|, \tag{B29b}$$

$$z_{\alpha(2s-2)} \coloneqq -\frac{1}{s}(2s-1)D_{(\alpha_1}Z_{\alpha_2\cdots\alpha_{2s-2})}|, \tag{B29c}$$

$$z_{\alpha(2s-4)} \coloneqq -(2s-1)D^\beta Z_{\beta\alpha(2s-4)}|. \tag{B29d}$$

Reduction of the action (B21) to components leads to the following bosonic action:

$$\begin{aligned} S_{\rm bos} = \left(-\frac{1}{2}\right)^s \int \mathrm{d}^3x\Big\{ &-\frac{1}{4}h^{\alpha(2s+2)}\Box h_{\alpha(2s+2)} + \frac{3}{16}\partial_{\delta\lambda}h^{\delta\lambda\alpha(2s)}\partial^{\beta\gamma}h_{\beta\gamma\alpha(2s)}\\ &+\frac{1}{4}(2s-1)\partial_{\delta\lambda}h^{\delta\lambda\alpha(2s)}\partial^\beta{}_{(\alpha_1}y_{\alpha_2\cdots\alpha_{2s})\beta} - \frac{1}{4}(2s-1)(s-1)z^{\alpha(2s-2)}\partial^{\beta\gamma}\partial^{\delta\lambda}h_{\beta\gamma\delta\lambda\alpha(2s-2)}\\ &-\frac{1}{4}(2s-1)y^{\alpha(2s)}\Box y_{\alpha(2s)} - \frac{1}{8}(s-2)(2s-1)\partial_{\delta\lambda}y^{\delta\lambda\alpha(2s-2)}\partial^{\beta\gamma}y_{\beta\gamma\alpha(2s-2)}\\ &-(s-1)(2s-1)z^{\alpha(2s)}\Box z_{\alpha(2s)} - \frac{1}{4}(s-1)(s+2)(2s-1)(2s-3)\partial_{\delta\lambda}z^{\delta\lambda\alpha(2s-4)}\partial^{\beta\gamma}z_{\beta\gamma\alpha(2s-4)}\\ &+(s-1)(2s-1)\partial_{\beta\gamma}y^{\beta\gamma\alpha(2s-2)}\partial^\delta{}_{(\alpha_1}z_{\alpha_2\cdots\alpha_{2s-2})\delta}\\ &-\frac{s}{4}\frac{2s-3}{(s-1)(2s-1)}(4s^2-12s+11)z^{\alpha(2s-4)}\Box z_{\alpha(2s-4)}\\ &+\frac{3s}{8(s-1)(2s-1)}(s-2)(2s-3)(2s-5)\partial_{\delta\lambda}z^{\delta\lambda\alpha(2s-6)}\partial^{\beta\gamma}z_{\beta\gamma\alpha(2s-6)}\\ &+\frac{1}{4}(s+1)(2s-3)z^{\alpha(2s-4)}\partial^{\beta\gamma}\partial^{\delta\lambda}y_{\beta\gamma\delta\lambda\alpha(2s-4)}\\ &+\frac{1}{2}(s-2)(2s+1)(2s-3)\partial_{\beta\gamma}z^{\beta\gamma\alpha(2s-4)}\partial^\delta{}_{(\alpha_1}z_{\alpha_2\cdots\alpha_{2s-4})\delta}\Big\}, \end{aligned} \tag{B30}$$

which proves to be invariant under gauge transformations of the form

$$\delta_\xi h_{\alpha(2s+2)} = \partial_{(\alpha_1\alpha_2}\xi_{\alpha_3\cdots\alpha_{2s+2})}, \tag{B31a}$$

$$\delta_{\xi,\rho} y_{\alpha(2s)} = -\frac{1}{s+1}\partial^\beta{}_{(\alpha_1}\xi_{\alpha_2\cdots\alpha_{2s})\beta} - \partial_{(\alpha_1\alpha_2}\rho_{\alpha_3\cdots\alpha_{2s})}, \tag{B31b}$$

$$\delta_{\xi,\rho} z_{\alpha(2s-2)} = \frac{1}{2s(2s+1)}\partial^{\beta\gamma}\xi_{\beta\gamma\alpha(2s-2)} + \frac{1}{s}\partial^\beta{}_{(\alpha_1}\rho_{\alpha_2\cdots\alpha_{2s-2})\beta}, \tag{B31c}$$

$$\delta_\rho z_{\alpha(2s-4)} = \partial^{\beta\gamma}\rho_{\beta\gamma\alpha(2s-4)}. \tag{B31d}$$

Let us consider the fermionic sector. We find that the independent fermionic fields are

$$h_{\alpha(2s+1)} \coloneqq \frac{\mathrm{i}}{4}D^2 H_{\alpha(2s+1)}|, \tag{B32a}$$

$$y_{\alpha(2s-1)} \coloneqq \frac{\mathrm{i}}{8}D^2 Y_{\alpha(2s-1)}|, \tag{B32b}$$

$$y_{\alpha(2s-3)} \coloneqq \frac{\mathrm{i}}{2}s(2s-1)D^2 Z_{\alpha(2s-3)}|, \tag{B32c}$$

and their gauge transformation laws are given by

$$\delta_\lambda h_{\alpha(2s+1)} = \partial_{(\alpha_1\alpha_2}\lambda_{\alpha_3\cdots\alpha_{2s+2})}, \tag{B33a}$$

$$\delta_\lambda y_{\alpha(2s-1)} = \frac{1}{2s+1}\partial^\beta{}_{(\alpha_1}\lambda_{\alpha_2\cdots\alpha_{2s-1})\beta}, \tag{B33b}$$

$$\delta_\lambda y_{\alpha(2s-3)} = \partial^{\beta\gamma}\lambda_{\beta\gamma\alpha(2s-3)}. \tag{B33c}$$

The above fermionic fields correspond to the dynamical variables of the Fang-Fronsdal spin-$(s+\frac{1}{2})$ model. As follows from (B33a), (B33b) and (B33c), their gauge freedom is equivalent to that of the massless spin-$(s+\frac{1}{2})$ gauge field. Indeed, direct calculations of the component action give the standard massless gauge-invariant spin-$(s+\frac{1}{2})$ action $S^{(2s+1)}_{FF}$.

The component structure of the obtained supermultiplets is a three-dimensional counterpart of so-called (reducible) higher-spin triplet systems. In AdS_D an action for bosonic higher-spin triplets was constructed in [66] and for fermionic triplets in [67,68]. Our superfield construction provides a manifestly off-shell supersymmetric generalization of these systems. It might be of interest to extend it to AdS_4.

[1] J. Hutomo and S. M. Kuzenko, Higher spin supermultiplets in three dimensions: (2,0) AdS supersymmetry, Phys. Lett. B **787**, 175 (2018).
[2] A. Achúcarro and P. K. Townsend, A Chern-Simons action for three-dimensional anti-de Sitter supergravity theories, Phys. Lett. B **180**, 89 (1986).
[3] S. M. Kuzenko, U. Lindström, and G. Tartaglino-Mazzucchelli, Three-dimensional (p,q) AdS superspaces and matter couplings, J. High Energy Phys. 08 (2012) 024.
[4] P. S. Howe, J. M. Izquierdo, G. Papadopoulos, and P. K. Townsend, New supergravities with central charges and Killing spinors in $2+1$ dimensions, Nucl. Phys. **B467**, 183 (1996).
[5] S. M. Kuzenko, U. Lindström, and G. Tartaglino-Mazzucchelli, Off-shell supergravity-matter couplings in three dimensions, J. High Energy Phys. 03 (2011) 120.
[6] D. Butter, S. M. Kuzenko, and G. Tartaglino-Mazzucchelli, Nonlinear sigma models with AdS supersymmetry in three dimensions, J. High Energy Phys. 02 (2013) 121.
[7] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, Three-dimensional $\mathcal{N}=2$ (AdS) supergravity and associated supercurrents, J. High Energy Phys. 12 (2011) 052.
[8] D. Butter and S. M. Kuzenko, $\mathcal{N}=2$ supersymmetric sigma-models in AdS, Phys. Lett. B **703**, 620 (2011).
[9] D. Butter and S. M. Kuzenko, The structure of $\mathcal{N}=2$ supersymmetric nonlinear σ-models in AdS_4, J. High Energy Phys. 11 (2011) 080.
[10] U. Lindström and M. Roček, Scalar tensor duality and $N=1$, 2 nonlinear sigma models, Nucl. Phys. **B222**, 285 (1983).
[11] C. M. Hull, A. Karlhede, U. Lindström, and M. Roček, Nonlinear sigma models and their gauging in and out of superspace, Nucl. Phys. **B266**, 1 (1986).
[12] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, Field theory in 4D $\mathcal{N}=2$ conformally flat superspace, J. High Energy Phys. 10 (2008) 001.
[13] D. Butter, S. M. Kuzenko, U. Lindstrom, and G. Tartaglino-Mazzucchelli, Extended supersymmetric sigma models in AdS_4 from projective superspace, J. High Energy Phys. 05 (2012) 138.
[14] S. M. Kuzenko and M. Ponds, Topologically massive higher spin gauge theories, J. High Energy Phys. 10 (2018) 160.
[15] A. A. Nizami, T. Sharma, and V. Umesh, Superspace formulation and correlation functions of 3d superconformal field theories, J. High Energy Phys. 07 (2014) 022.
[16] I. L. Buchbinder and S. M. Kuzenko, *Ideas and Methods of Supersymmetry and Supergravity or a Walk Through Superspace* (IOP, Bristol, 1995), Revised Edition: 1998.

[17] S. M. Kuzenko, Supersymmetric spacetimes from curved superspace, Proc. Sci., CORFU2014 (**2015**) 140 [arXiv:1504.08114].

[18] S. M. Kuzenko, U. Lindström, M. Roček, I. Sachs, and G. Tartaglino-Mazzucchelli, three-dimensional $N=2$ supergravity theories: From superspace to components, Phys. Rev. D **89**, 085028 (2014).

[19] E. E. Donets, A. Pashnev, J. J. Rosales, and M. M. Tsulaia, $N=4$ supersymmetric multidimensional quantum mechanics, partial SUSY breaking and superconformal quantum mechanics, Phys. Rev. D **61**, 043512 (2000).

[20] E. E. Donets, A. Pashnev, V. O. Rivelles, D. P. Sorokin, and M. Tsulaia, $N=4$ superconformal mechanics and the potential structure of AdS spaces, Phys. Lett. B **484**, 337 (2000).

[21] J. Hutomo, S. M. Kuzenko, and D. Ogburn, $\mathcal{N}=2$ supersymmetric higher spin gauge theories and current multiplets in three dimensions, Phys. Rev. D **98**, 125004 (2018).

[22] S. M. Kuzenko and D. X. Ogburn, Off-shell higher spin $N=2$ supermultiplets in three dimensions, Phys. Rev. D **94**, 106010 (2016).

[23] S. M. Kuzenko, V. V. Postnikov, and A. G. Sibiryakov, Massless gauge superfields of higher half-integer superspins, Pis'ma Zh. Eksp. Teor. Fiz. **57**, 521 (1993) [JETP Lett. **57**, 534 (1993)].

[24] S. M. Kuzenko and A. G. Sibiryakov, Massless gauge superfields of higher integer superspins, Pis'ma Zh. Eksp. Teor. Fiz. **57**, 526 (1993) [JETP Lett. **57**, 539 (1993)].

[25] S. M. Kuzenko and A. G. Sibiryakov, Free massless higher-superspin superfields on the anti-de Sitter superspace, Yad. Fiz. **57**, 1326 (1994) [Phys. At. Nucl. **57**, 1257 (1994)].

[26] S. M. Kuzenko and M. Tsulaia, Off-shell massive $N=1$ supermultiplets in three dimensions, Nucl. Phys. **B914**, 160 (2017).

[27] S. J. Gates, Jr., M. T. Grisaru, M. Roček, and W. Siegel, Superspace, or one thousand and one lessons in supersymmetry, Front. Phys. **58**, 1 (1983).

[28] W. Siegel, Unextended superfields in extended supersymmetry, Nucl. Phys. **B156**, 135 (1979).

[29] S. M. Kuzenko, R. Manvelyan, and S. Theisen, Off-shell superconformal higher spin multiplets in four dimensions, J. High Energy Phys. 07 (2017) 034.

[30] I. L. Buchbinder, S. J. Gates, Jr., and K. Koutrolikos, Higher spin superfield interactions with the chiral supermultiplet: Conserved supercurrents and cubic vertices, Universe **4**, 6 (2018).

[31] J. Hutomo and S. M. Kuzenko, Non-conformal higher spin supercurrents, Phys. Lett. B **778**, 242 (2018).

[32] J. Hutomo and S. M. Kuzenko, The massless integer superspin multiplets revisited, J. High Energy Phys. 02 (2018) 137.

[33] K. Koutrolikos, P. Koči, and R. von Unge, Higher spin superfield interactions with complex linear supermultiplet: Conserved supercurrents and cubic vertices, J. High Energy Phys. 03 (2018) 119.

[34] E. I. Buchbinder, J. Hutomo, and S. M. Kuzenko, Higher spin supercurrents in anti-de Sitter space, J. High Energy Phys. 09 (2018) 027.

[35] J. F. Schonfeld, A mass term for three-dimensional gauge fields, Nucl. Phys. **B185**, 157 (1981).

[36] S. Deser, R. Jackiw, and S. Templeton, Three-Dimensional Massive Gauge Theories, Phys. Rev. Lett. **48**, 975 (1982).

[37] S. Deser, R. Jackiw, and S. Templeton, Topologically massive gauge theories, Ann. Phys. (N.Y.) **140**, 372 (1982); **185**, 406(E) (1988).

[38] S. M. Kuzenko, Higher spin super-cotton tensors and generalisations of the linear-chiral duality in three dimensions, Phys. Lett. B **763**, 308 (2016).

[39] S. M. Kuzenko and M. Ponds, Conformal geometry and (super)conformal higher-spin gauge theories, J. High Energy Phys. 05 (2019) 113.

[40] S. M. Kuzenko, J. Novak, and G. Tartaglino-Mazzucchelli, Higher derivative couplings and massive supergravity in three dimensions, J. High Energy Phys. 09 (2015) 081.

[41] L. Mezincescu and P. K. Townsend, Quantum 3D superstrings, Phys. Rev. D **84**, 106006 (2011).

[42] V. I. Ogievetsky and E. Sokatchev, Superfield equations of motion, J. Phys. A **10**, 2021 (1977).

[43] N. Berkovits and M. M. Leite, First massive state of the superstring in superspace, Phys. Lett. B **415**, 144 (1997).

[44] N. Berkovits and M. M. Leite, Superspace action for the first massive states of the superstring, Phys. Lett. B **454**, 38 (1999).

[45] R. Altendorfer and J. Bagger, Dual supersymmetry algebras from partial supersymmetry breaking, Phys. Lett. B **460**, 127 (1999).

[46] R. Altendorfer and J. Bagger, Dual anti-de Sitter superalgebras from partial supersymmetry breaking, Phys. Rev. D **61**, 104004 (2000).

[47] I. L. Buchbinder, S. J. Gates, Jr., W. D. Linch, and J. Phillips, New 4D, $N=1$ superfield theory: Model of free massive superspin-$3/2$ multiplet, Phys. Lett. B **535**, 280 (2002).

[48] I. L. Buchbinder, S. J. Gates, Jr., W. D. Linch, and J. Phillips, Dynamical superfield theory of free massive superspin-1 multiplet, Phys. Lett. B **549**, 229 (2002).

[49] T. Gregoire, M. D. Schwartz, and Y. Shadmi, Massive supergravity and deconstruction, J. High Energy Phys. 07 (2004) 029.

[50] I. L. Buchbinder, S. J. Gates, Jr., S. M. Kuzenko, and J. Phillips, Massive 4D, $N=1$ superspin 1 and $3/2$ multiplets and dualities, J. High Energy Phys. 02 (2005) 056.

[51] S. J. Gates, Jr. and S. M. Kuzenko, 4D, $N=1$ higher spin gauge superfields and quantized twistors, J. High Energy Phys. 10 (2005) 008.

[52] S. J. Gates, Jr., S. M. Kuzenko, and G. Tartaglino-Mazzucchelli, New massive supergravity multiplets, J. High Energy Phys. 02 (2007) 052.

[53] I. L. Buchbinder, T. V. Snegirev, and Y. M. Zinoviev, Lagrangian formulation of the massive higher spin supermultiplets in three dimensional space-time, J. High Energy Phys. 10 (2015) 148.

[54] I. L. Buchbinder, T. V. Snegirev, and Y. M. Zinoviev, Lagrangian description of massive higher spin supermultiplets in AdS_3 space, J. High Energy Phys. 08 (2017) 021.

[55] I. L. Buchbinder, T. V. Snegirev, and Y. M. Zinoviev, Gauge invariant Lagrangian formulation of massive higher spin fields in $(A)dS_3$ space, Phys. Lett. B **716**, 243 (2012).

[56] I. L. Buchbinder, T. V. Snegirev, and Y. M. Zinoviev, Frame-like gauge invariant Lagrangian formulation of massive

fermionic higher spin fields in AdS_3 space, Phys. Lett. B **738**, 258 (2014).

[57] Y. M. Zinoviev, On massive high spin particles in AdS, arXiv:hep-th/0108192.

[58] R. R. Metsaev, Gauge invariant formulation of massive totally symmetric fermionic fields in (A)dS space, Phys. Lett. B **643**, 205 (2006).

[59] I. A. Batalin and G. A. Vilkovisky, Quantization of gauge theories with linearly dependent generators, Phys. Rev. D **28**, 2567 (1983).

[60] I. L. Buchbinder, S. M. Kuzenko, and A. G. Sibiryakov, Quantization of higher spin superfields in the anti-de Sitter superspace, Phys. Lett. B **352**, 29 (1995).

[61] L. D. Faddeev and V. N. Popov, Feynman diagrams for the Yang-Mills field, Phys. Lett. **25B**, 29 (1967).

[62] C. Fronsdal and H. Hata, Quantization of massless fields with arbitrary spin, Nucl. Phys. **B162**, 487 (1980).

[63] M. R. Gaberdiel, R. Gopakumar, and A. Saha, Quantum W-symmetry in AdS_3, J. High Energy Phys. 02 (2011) 004.

[64] I. N. McArthur, Super b(4) coefficients, Phys. Lett. **128B**, 194 (1983); Super b(4) coefficients in supergravity, Classical Quantum Gravity **1**, 245 (1984).

[65] I. L. Buchbinder and S. M. Kuzenko, Matter superfields in external supergravity: Green functions, effective action and superconformal anomalies, Nucl. Phys. **B274**, 653 (1986).

[66] A. Sagnotti and M. Tsulaia, On higher spins and the tensionless limit of string theory, Nucl. Phys. **B682**, 83 (2004).

[67] D. P. Sorokin and M. A. Vasiliev, Reducible higher-spin multiplets in flat and AdS spaces and their geometric frame-like formulation, Nucl. Phys. **B809**, 110 (2009).

[68] A. Agugliaro, F. Azzurli, and D. Sorokin, Fermionic higher-spin triplets in AdS, Nucl. Phys. **B907**, 633 (2016).