

## Screening and degenerate kinetic self-acceleration from the nonlinear freedom of reconstructed Horndeski theories

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We have previously presented a reconstruction of Horndeski scalar-tensor theories from linear cosmological observables. It includes free nonlinear terms which can be added onto the reconstructed covariant theory without affecting the background and linear dynamics. After discussing the uniqueness of these correction terms, we apply this nonlinear freedom to a range of different applications. First we demonstrate how the correction terms can be configured to endow the reconstructed models with screening mechanisms such as the chameleon, k-mouflage, and Vainshtein effects. A further implication is the existence of classes of Horndeski models that are degenerate with standard cosmology to an arbitrary level in the cosmological perturbations. Particularly interesting examples are kinetically self-accelerating models that mimic the dynamics of the cosmological constant to an arbitrary degree in perturbations. Finally, we develop the reconstruction method further to the level of higher-order effective field theory, which under the restriction to a luminal propagation speed of gravitational waves introduces two new free functions per order. These functions determine the corresponding correction terms in the linearly reconstructed action at the same order. Our results enable the connection of linear cosmological constraints on generalized modifications of gravity and dark energy with the nonlinear regime and astrophysical probes for a more global interpretation of the wealth of forthcoming cosmological survey data.

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### I. INTRODUCTION

The observation of the late-time accelerated expansion of the Universe [1,2] has led to a large number of theoretical models that attempt to explain it. To date, the  $\Lambda$  cold dark matter ( $\Lambda$ CDM) model, consisting of a cosmological constant  $\Lambda$  and dark matter treated as a cold pressureless fluid, remains the most successful among them [3]. Despite its simplicity there remain plenty of open questions. One of the most pressing issues is the large contribution to  $\Lambda$  that should arise from the quantum corrections to the various matter fields in the Universe [4,5]. Taken with the fact that the fundamental nature of dark matter also remains a mystery, these problems have encouraged a great deal of model-building beyond  $\Lambda$ CDM. Many of these models involve an additional scalar degree of freedom (d.o.f.) that may drive the acceleration even in the absence of a cosmological constant [6–8]. The scalar field can be thought of as an additional exotic contribution to the matter sector or the low-energy effective description of a modification to general relativity (GR) which acts on cosmological scales. Scalar fields typically arise through a symmetry breaking mechanism from a UV-complete theory. The Higgs field is an example of this and its

presence in the Standard Model of particle physics provides motivation to study the effects of scalar fields on gravitational dynamics.

Incorporating a scalar field into GR is not a trivial task. Higher derivatives can easily enter the field equations of motion leading to extra propagating d.o.f. and an unbounded Hamiltonian. This is a consequence of the Ostrogradsky theorem [9]. In 1974 Horndeski identified the unique scalar-tensor theory in four dimensions that leads to at most second-order equations of motion and thus avoids the Ostrogradsky ghost [10]. The theory was later rediscovered by generalizing Galileons to curved spacetime [11,12]. Note that it is possible to have stable higher-order theories [13,14], which shall, however, not be considered in this work.

A useful approach to a unified treatment of various dark energy and modified gravity models is provided by the effective field theory (EFT) of dark energy. The formalism was originally developed in the context of inflation [15,16] before its application to late-time cosmology [17–27]. It features a systematic order-by-order expansion in the cosmological perturbations and proves to be a useful tool for the unified description of the cosmological effects of Horndeski theory.

The trade-off for generality is that this EFT formalism is restricted by definition to certain length scales, usually just the cosmological background and linear perturbations. Recently there has been some work in extending the expansion to higher-order perturbations [28,29]. An alternative approach is to start from the full covariant action. The loss of generality in this approach is now traded for the applicability on a much broader range of length scales, allowing nonlinear effects such as screening to be studied. We have recently presented [30,31] a reconstruction from the EFT of dark energy on the level of the background and linear perturbations to the class of Horndeski theories that give rise to the particular set of given EFT functions. With this covariant action it becomes feasible to generally connect the nonlinear regime to that of the background and linear scales. This link shall be the focus of this paper.

More precisely, within the reconstructed theory of Refs. [30,31] there are correction terms that account for the nonlinear freedom that exists between Horndeski theories that are degenerate at the level of the background and linear perturbations. Specification of these correction terms allows one to move between linearly degenerate theories. We first discuss the uniqueness of the correction terms in the reconstructed theory. Applying the recent constraint on the equality between the speeds of light and of gravitational waves [32] we show that the number of free functions that are present at higher order in the EFT of dark energy is significantly reduced to two per order in perturbation theory. This then implies that the nonlinear freedom is uniquely specified by the nonlinear correction terms. It is worth noting that out of the four new EFT functions found in Ref. [33] at second order in the cosmological perturbations of Horndeski theory, the two functions dominating in the subhorizon regime vanish for a luminal speed of gravity, and the impact of our nonlinear correction terms on the weakly nonlinear regime of structure formation remains to be examined in detail. Note that Ref. [34] showed that a subclass of Horndeski models that was previously considered to be ruled out by the luminal speed of gravity constraint can survive at the background level. However, this loophole breaks down at the level of the perturbations reducing the remaining freedom in Horndeski to that of Refs. [35,36] which we restrict to in this paper.

As an initial demonstration of the implications of the correction terms, we show how this nonlinear freedom can be used to endow a reconstructed theory with a screening mechanism. Due to the tight Solar-System constraints on deviations from GR [37] it is necessary for a large-scale modification of GR to employ a screening mechanism that suppresses the effects of a fifth force on small scales. These screening mechanisms fall into one of three categories [8]: those that screen through deep gravitational potentials such as the chameleon [38] or symmetron mechanisms [39], those that screen through first derivatives of the potentials such as k-mouflage models [40], or those that screen

through second derivatives as for the Vainshtein mechanism [41].

A simple scaling method was developed in Refs. [36,42] to determine whether a given theory possesses an Einstein gravity limit. We present an application of this scaling method to the reconstructed theory and demonstrate with three examples that there is enough freedom in the nonlinear regime of a reconstructed theory to obtain, in principle, any of these three screening mechanisms.

A further interesting consequence that arises when considering theories built from the correction terms is that it is simple to construct theories that are indistinguishable from  $\Lambda$ CDM to arbitrary level in cosmological perturbations. Only observations in the nonlinear regime can be used to distinguish them from  $\Lambda$ CDM. Such degenerate theories may be built from kinetic terms alone without including a cosmological constant, hence providing a kinetic self-acceleration effect.

Finally, we present a reconstruction from the nonlinear EFT back to the space of manifestly covariant theories. This follows a similar structure to the background and linear reconstruction and in principle provides a method for obtaining a Horndeski theory reconstructed from a range of different length scales from the background to the nonlinear regime.

The paper is organized as follows. In Sec. II we briefly review Horndeski scalar-tensor gravity, the EFT formalism, the reconstruction method from linear EFT to Horndeski gravity, and the nonlinear freedom available for the reconstructed theories. The uniqueness of the nonlinear correction terms in the reconstructed action is examined in Sec. III. Section IV briefly reviews the scaling method and discusses how the nonlinear freedom in the reconstructed scalar-tensor theories can be used to implement screening effects due to large gravitational potentials and large first or second derivatives of the potential. In Sec. V we discuss how the nonlinear freedom can be used to construct models that accelerate the cosmic expansion without a cosmological constant with a suitable choice of kinetic terms, yet are degenerate with standard cosmology at the background level or even to the arbitrary level of perturbations. The derivation of a third-order reconstruction is presented in Sec. VI along with a discussion of the extension to  $n$ th order. Finally, we provide conclusions on the results in Sec. VII.

## II. RECONSTRUCTED SCALAR-TENSOR THEORIES

For the benefit of the unfamiliar reader we shall briefly review Horndeski gravity in Sec. II A before discussing the reconstruction from the EFT of dark energy and modified gravity to manifestly covariant theories in Sec. II B. Sec. II C then examines the nonlinear freedom in this reconstruction. The free nonlinear correction terms available will then be applied to screening in Sec. IV, to the

formulation of degenerate kinetic self-acceleration effects in Sec. V and finally to the connection to higher-order EFT in Sec. VI.

### A. Horndeski gravity

The most general scalar-tensor theory in four dimensions that yields at most second-order equations of motion is given by the Horndeski action [10–12]

$$S = \sum_{i=2}^5 \int d^4x \sqrt{-g} \mathcal{L}_i, \quad (1)$$

where the Lagrangian densities  $\mathcal{L}_i$  are defined as

$$\mathcal{L}_2 \equiv G_2(\phi, X), \quad (2)$$

$$\mathcal{L}_3 \equiv G_3(\phi, X) \square \phi, \quad (3)$$

$$\begin{aligned} \mathcal{L}_4 \equiv & G_4(\phi, X) R \\ & - 2G_{4X}(\phi, X) [(\square \phi)^2 - (\nabla^\mu \nabla^\nu \phi)(\nabla_\mu \nabla_\nu \phi)], \end{aligned} \quad (4)$$

$$\begin{aligned} \mathcal{L}_5 \equiv & G_5(\phi, X) G_{\mu\nu} \nabla^\mu \nabla^\nu \phi \\ & + \frac{1}{3} G_{5X}(\phi, X) [(\square \phi)^3 - 3(\square \phi)(\nabla_\mu \nabla_\nu \phi)(\nabla^\mu \nabla^\nu \phi) \\ & + 2(\nabla_\mu \nabla_\nu \phi)(\nabla^\sigma \nabla^\nu \phi)(\nabla_\sigma \nabla^\mu \phi)], \end{aligned} \quad (5)$$

with  $X \equiv g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$ . A restriction to the class of Horndeski theories with luminal speed of gravity simplifies the action (1) considerably to [36]

$$\mathcal{L}_2 \equiv G_2(\phi, X), \quad (6)$$

$$\mathcal{L}_3 \equiv G_3(\phi, X) \square \phi, \quad (7)$$

$$\mathcal{L}_4 \equiv G_4(\phi) R, \quad (8)$$

where  $\mathcal{L}_5$  can be set to zero. By varying this reduced Horndeski action in Eqs. (6) to (8) with respect to the metric and the scalar field, one obtains the metric and scalar field equations [12,36]. They will be needed solely in Sec. IV, and the explicit expressions are given in the Appendix.

### B. Reconstruction from linear effective field theory

The effects of Horndeski theory on the cosmological background evolution and the linear perturbations can be described in a convenient manner by adopting the EFT of dark energy [20–25]. The relevant action is constructed with the usual spirit of EFT by writing down every operator, in this case the cosmological perturbations, which is consistent with the symmetries imposed on the theory. Time diffeomorphism symmetry is broken in the EFT of dark energy and so every operator which remains invariant under spatial diffeomorphisms is employed. The scalar

field is then the pseudo Nambu–Goldstone boson of broken time translational symmetry.

At the level of the background and linear perturbations the EFT action [20,21] in the notation of Ref. [43] is given by

$$S = S^{(0,1)} + S^{(2)} + S_M[g_{\mu\nu}, \Psi_m], \quad (9)$$

$$S^{(0,1)} = \frac{M_*^2}{2} \int d^4x \sqrt{-g} [\Omega(t) R - 2\Lambda(t) - \Gamma(t) \delta g^{00}], \quad (10)$$

$$\begin{aligned} S^{(2)} = & \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_*^4(t) (\delta g^{00})^2 - \frac{1}{2} \bar{M}_1^3(t) \delta K \delta g^{00} \right. \\ & \left. - \bar{M}_2^2(t) \left( \delta K^2 - \delta K^{\mu\nu} \delta K_{\mu\nu} - \frac{1}{2} \delta R^{(3)} \delta g^{00} \right) \right]. \end{aligned} \quad (11)$$

The set  $\{\Omega(t), \Lambda(t), \Gamma(t), M_*^4(t), \bar{M}_1^3(t), \bar{M}_2^2(t)\}$  of the free time-dependent coefficients can be derived for a particular choice of the Horndeski functions  $G_i$  [20,22]. There are alternative bases for the EFT coefficients. A frequently adopted set was introduced by Ref. [26], which is related to the coefficients in Eqs. (10) and (11) by a linear transformation

$$\alpha_M \equiv \frac{M_*^2 \Omega' + 2(\bar{M}_2^2)'}{M_*^2 \Omega + 2\bar{M}_2^2}, \quad (12)$$

$$\alpha_B \equiv \frac{M_*^2 H \Omega' + \bar{M}_1^3}{2H(M_*^2 \Omega + 2\bar{M}_2^2)}, \quad (13)$$

$$\alpha_K \equiv \frac{M_*^2 \Gamma + 4M_*^4}{H^2(M_*^2 \Omega + 2\bar{M}_2^2)}, \quad (14)$$

$$\alpha_T \equiv -\frac{2\bar{M}_2^2}{M_*^2 \Omega + 2\bar{M}_2^2}, \quad (15)$$

where primes denote derivatives with respect to  $\ln a$ . Furthermore  $\alpha_M$  denotes the Planck mass evolution rate,  $\alpha_B$  is related to the coupling between the metric and the scalar field,  $\alpha_K$  arises as a coefficient of the kinetic term for the scalar field, and  $\alpha_T$  is the deviation of the speed of gravitational waves from that of light, now determined to be vanishing at late times [32] (also see Refs. [44,45] for forecasted implications). A further set of EFT functions was recently introduced in Ref. [46] with  $\alpha_T = 0$  to avoid stability issues associated with the previous EFT bases.

Different models such as the cubic Galileon [47,48], quintessence [49], and  $k$ -essence [50,51] in general give different functional forms for this set [20]. In particular, Horndeski theories with luminal speed of gravity imply  $\bar{M}_2^2(t) = 0$ . The EFT functions can then directly be related to effective descriptions of a modified Poisson equation and gravitational slip [25,43,52–57] that are probed by cosmological observations [58].

In order to link observational constraints on these effective modifications to theoretical constraints on fundamental theories it is useful to formulate a mapping from EFT back to the space of physical covariant theories. Such a reconstruction was developed in Refs. [30,31]. It determines the class of Horndeski theories reconstructed from a given set of EFT coefficients that are degenerate at the level of the cosmological background and the linear perturbations. Specifically, the reconstruction is given by

$$G_2(\phi, X) = -M_*^2 U(\phi) - \frac{1}{2} M_*^2 Z(\phi) X + a_2(\phi) X^2 + \Delta G_2, \quad (16)$$

$$G_3(\phi, X) = b_0(\phi) + b_1(\phi) X + \Delta G_3, \quad (17)$$

$$G_4(\phi, X) = \frac{1}{2} M_*^2 F(\phi) + c_1(\phi) X + \Delta G_4, \quad (18)$$

$$G_5(\phi, X) = \Delta G_5, \quad (19)$$

where each term in the reconstruction such as  $U(\phi)$  and  $Z(\phi)$  is dependent on a particular combination of EFT functions. For completeness, the full list of expressions is provided in Table I. The  $\Delta G_i$  functions denote nonlinear correction terms that characterize the degenerate class of Horndeski theories. In particular, the correction terms can be used to move between different theories that only differ at the nonlinear level.

### C. Nonlinear freedom

Under the assumption of the luminal speed of gravity [32] we shall show in Sec. III that the unique nonlinear correction terms in the reconstructed theory are specified by

$$\Delta G_{2,3} = \sum_{n>2} \xi_n^{(2,3)}(\phi) \left(1 + \frac{X}{M_*^4}\right)^n, \quad (20)$$

where  $\Delta G_{4,5} = 0$  and  $\xi_n^{(i)}(\phi)$  are free functions of the scalar field, reflecting the large d.o.f. that exists on nonlinear scales without affecting linear scales. These terms arise from noting that in the unitary gauge with the foliation

$\phi = tM_*^2$  the kinetic term of the scalar field becomes  $X = (-1 + \delta g^{00})M_*^4$ . Equation (20) is therefore an expansion in  $(\delta g^{00})^n$ .

The freedom in the correction term (20) may be exploited to endow the reconstructed theories with some desired nonlinear features without affecting linear theory. In particular,  $\xi_n^{(i)}(\phi)$  can be designed to implement a screening mechanism (Sec. IV) or even to hide a kinetic self-acceleration effect of the cosmic background expansion to an arbitrary level of nonlinear perturbations (Sec. V).

### III. UNIQUENESS OF THE $\Delta G_i$ CORRECTIONS

Due to the importance of the  $\Delta G_i$  nonlinear correction terms for the applications of interest in Secs. IV, V, and VI, we shall first investigate to what extent these terms are the unique corrections to the reconstructed Horndeski action in Eqs. (16) to (19).

Recall that the correction terms in (20) were inferred from the requirement that in covariant language  $\delta g^{00} = 1 + X/M_*^4$ . Successive powers of  $1 + X/M_*^4$  therefore yield corrections that do not affect lower-order perturbations, in particular, the background or linear theory. However, there are of course other operators which can be added to the EFT which will not affect the background and linear dynamics such as  $\delta K^3$  and  $(\delta R^{(3)})^3$ . In principle a term such as  $\delta K^3$  could be added to the EFT action, which would affect the dynamics of the second-order perturbations. Note, however, that for the same reason that  $\delta K^2$  only appears in combination with  $\delta K_{\mu\nu} \delta K^{\mu\nu}$  after  $\mathcal{L}_4$  is written in the unitary gauge and expanded in the perturbations, it is not possible to simply add  $\delta K^3$  as there are no terms in the Horndeski action that give rise to this term alone. More specifically, on the cosmological background  $K_{\mu\nu} = H h_{\mu\nu}$ , the perturbation  $\delta K = K - 3H$  must appear in the combination

$$K^3 - 3K K_{\mu\nu} K^{\mu\nu} + 2K_{\mu\nu} K^{\mu\sigma} K^\nu{}_\sigma, \quad (21)$$

which gives rise to a number of nonlinear operators in the EFT action involving  $\delta K_{\mu\nu}$  [28,29]. The only term in the Horndeski action that gives rise to such a combination

TABLE I. The Horndeski functions  $G_i(\phi, X)$  reconstructed from the effective field theory of dark energy at the level of the cosmological background evolution and linear perturbations. The primes indicate a derivative with respect to  $\phi$ . See Ref. [30] for the derivation.

$U(\phi) = \Lambda + \frac{\Gamma}{2} - \frac{M_+^2}{2M_*^2} - \frac{9H\bar{M}_1^3}{8M_*^2} - \frac{(\bar{M}_1^3)'}{8} + \frac{M_+^2(\bar{M}_2^2)'}{4} + \frac{7(\bar{M}_2^2)'H}{4} + \bar{M}_2^2 H' + \frac{9H^2\bar{M}_2^2}{2M_*^2}$	
$Z(\phi) = \frac{\Gamma}{M_*^4} - \frac{2M_+^2}{M_*^6} - \frac{3H\bar{M}_1^3}{2M_*^6} + \frac{(\bar{M}_1^3)'}{2M_*^4} - \frac{(\bar{M}_2^2)'}{M_*^2} - \frac{H(\bar{M}_2^2)'}{M_*^4} - \frac{4H'\bar{M}_2^2}{M_*^4}$	
$a_2(\phi) = \frac{M_+^2}{2M_*^6} + \frac{(\bar{M}_1^3)'}{8M_*^6} - \frac{3H\bar{M}_1^3}{8M_*^6} - \frac{(\bar{M}_2^2)'}{4M_*^4} + \frac{H(\bar{M}_2^2)'}{4M_*^6} + \frac{H'\bar{M}_2^2}{M_*^6} - \frac{3H^2\bar{M}_2^2}{2M_*^6}$	
$b_0(\phi) = 0$	
$F(\phi) = \Omega + \frac{\bar{M}_2^2}{M_*^2}$	
	$b_1(\phi) = \frac{2H\bar{M}_2^2}{M_*^6} - \frac{(\bar{M}_2^2)'}{M_*^4} + \frac{\bar{M}_1^3}{2M_*^6}$
	$c_1(\phi) = \frac{\bar{M}_2^2}{2M_*^4}$

is in  $\mathcal{L}_5$ . Following the spirit of EFT one may add these nonlinear operators because they are consistent with the symmetries that we have imposed, but the theory which is underlying such a combination generally violates the luminal speed of gravity constraint [36] such that we will omit these terms. By the use of the Gauss-Codazzi relation

$$R^{(3)} = R - K_{\mu\nu}K^{\mu\nu} + K^2 - 2\nabla_\nu(n^\nu\nabla_\mu n^\mu - n^\mu\nabla_\mu n^\nu), \quad (22)$$

relating the three-dimensional Ricci scalar  $R^{(3)}$  to the four-dimensional Ricci scalar  $R$  and  $K_{\mu\nu}$ , one can furthermore see that adding on higher powers of  $R^{(3)}$  to the EFT in a similar manner will inevitably introduce higher powers of  $\delta K$ , and the previous argument applies. The same logic also requires  $\Delta G_4$  and  $\Delta G_5$  to vanish and the nonlinear freedom is now completely specified by Eq. (20).

An alternative perspective on this argument is to consider a covariant form of the extrinsic curvature tensor or for simplicity its trace

$$K = -\nabla_\mu \left( \frac{\partial_\mu \phi}{\sqrt{-X}} \right). \quad (23)$$

By expressing the denominator in terms of the metric perturbations, Taylor expanding and performing the replacement of  $\delta g^{00}$  with  $1 + X/M_*^2$ , one obtains in schematic form

$$K = \square\phi + F(X, \nabla_\mu\phi, \nabla_\mu X), \quad (24)$$

where  $F(X, \nabla_\mu\phi, \nabla_\mu X)$  is some complicated function of the scalar field and the derivatives of the scalar field obtained after the expansion, the precise form of which is not relevant to the discussion. Taking higher powers of  $\delta K$  and making use of Eq. (24) will lead to terms such as  $(\square\phi)^n$ . Such expressions belong either to Horndeski models with nonluminal speed of gravitational waves or beyond-Horndeski theories. Reversing the logic, it is necessary to start from such a model in order to obtain a nonlinear correction involving a higher power of  $\delta K$ . Therefore, any correction terms to the EFT of dark energy that make use of the operators  $(\delta K)^n$  with  $n \geq 2$  and  $R^{(3)}$  will reconstruct a theory that has a nonvanishing  $G_{4X}$  or  $G_5$  term or a beyond-Horndeski model.

For Horndeski models with a luminal speed of gravity, the only nonlinear operators that appear at  $n$ th order are therefore

$$(\delta g^{00})^n, \quad (\delta g^{00})^{n-1} \delta K, \quad (25)$$

which add two new independent EFT functions per order in the perturbations. More explicitly, the  $n$ th order contribution to the EFT action with  $n \geq 3$  is given by

$$\delta \mathcal{S}^{(n)} = \int d^4x \sqrt{-g} \sum_{i=3}^n [\bar{M}_i^4(t) (\delta g^{00})^i + \bar{M}_i^3(t) (\delta g^{00})^{i-1} \delta K], \quad (26)$$

where each  $\bar{M}_i^3(t)$  and  $\bar{M}_i^4(t)$  are the two free functions that contribute at  $i$ th order in the action. This is a logical extension to  $n$ th order of the first two operators which appear in  $\mathcal{S}^{(2)}$  in Eq. (11), namely  $(\delta g^{00})^2$  and  $\delta g^{00} \delta K$ .

#### IV. NONLINEAR FREEDOM FOR SCREENING

As a first application of the free nonlinear correction term in Eq. (20) in the reconstructed scalar-tensor action we shall consider the realization of screening mechanisms that are required to recover GR in the well-tested Solar-System regime [37]. For this purpose, we shall employ the scaling method of Refs. [36,42] (also see applications in Refs. [59–61]) that allows an efficient identification of the existence of Einstein gravity regimes for a particular choice of Horndeski functions. We briefly review the method (Sec. IV A) and then apply it for a characterization of the nonlinear correction terms  $\Delta G_i$  that realize screening by large gravitational potentials  $\Phi_N > \Lambda$  for some threshold  $\Lambda$  (Sec. IV B), large first derivatives  $\nabla\Phi_N > \Lambda$  (Sec. IV C), or large second derivatives  $\nabla^2\Phi_N > \Lambda$  (Sec. IV D) [8].

##### A. Scaling method

The scaling method was developed in Refs. [36,42] to efficiently determine whether a given Horndeski theory possesses an Einstein gravity limit. It proceeds as follows. At the level of the field equations the scalar field  $\phi$  is expanded in terms of a field perturbation  $\psi$  as

$$\phi = \phi_0(1 + \alpha^q \psi), \quad (27)$$

where  $\phi_0$  denotes the background value and  $\alpha$  is the theoretical parameter relevant to the expansion. For example, it could be the speed of light or the coupling of a Galileon interaction term. After performing this expansion, the scalar field equation of the Horndeski model [see Eq. (A2)] takes the generic form

$$\alpha^{s+mq} F_1(\psi, \tilde{X}) + \alpha^{t+nq} F_2(\psi, \tilde{X}) = \frac{T}{M_*^2}, \quad (28)$$

where  $s, m, t, n \in \mathbb{N}$  and  $\tilde{X} = \partial_\mu \psi \partial^\mu \psi$ . Now consider the limit of  $\alpha \rightarrow \infty$  or  $\alpha \rightarrow 0$ . As the right-hand side of Eq. (28) is independent of  $\alpha$ , the leading-order term on the left-hand side must also be independent of  $\alpha$  to balance the equation. This restricts the possible values of the exponent  $q$ . Therefore there must be at least one term which scales as  $\alpha^0$  with every other term involving nonzero powers of  $\alpha$  vanishing in the  $\alpha \rightarrow 0$  or  $\alpha \rightarrow \infty$  limit. For example, choosing  $q = -s/m$  and taking the  $\alpha \rightarrow \infty$  limit requires

$-s/m < -t/q$ , so that  $t + n(-s/m) < 0$  and the dominating terms in the field equation become

$$F_1(\psi, \tilde{X}) = \frac{T}{M_*^2}. \quad (29)$$

If in a given  $\alpha$  limit the metric field equations reduce to the Einstein equations after performing the expansion (27), then the corresponding scalar field equation applies to the screened limit where the fifth force is suppressed. To ensure consistency the value of  $q$  chosen to obtain a screened limit must be the same in both the scalar and the metric field equations. Note that there may also be terms that involve powers of  $\alpha$  that do not depend on  $q$ . Depending on whether they are raised to a positive or a negative power they will diverge or vanish in either limit of  $\alpha$ . If they vanish, then this is not an issue, but if they diverge, extra care must be taken. For example, it may be important to use the freedom in the  $\Delta G_i$  terms to remove any divergences which arise in either limit.

In the following we present the recovery of three distinct screening mechanisms by suitable choices of  $\Delta G_i$ . Drawing on the distinction discussed in Ref. [8] this will encompass the known screening mechanisms: (i) by large gravitational potentials  $\Phi_N > \Lambda$  for some threshold  $\Lambda$  (Sec. IV B), (ii) by large first derivatives  $\nabla\Phi_N > \Lambda$  (Sec. IV C), and (iii) by large second derivatives  $\nabla^2\Phi_N > \Lambda$  (Sec. IV D). We shall find that there is more than sufficient freedom in the nonlinear sector to, in principle, endow the reconstructed theory with a particular screening mechanism regardless of the constraints of the background and the linear perturbations. Importantly, however, while this generally implies the existence of Einstein gravity limits in the deeply nonlinear regime, this does not guarantee that a given observed region is nonlinear enough for the screening mechanism to be activated. The numerical value of the screening scale needs to be computed separately and ultimately decides whether a theory is compatible with stringent Solar-System tests. It is not surprising that screening mechanisms can be added to linearly reconstructed models as they are inherently nonlinear effects. It is, however, important to verify this explicitly.

## B. Large field value screening

As a first example we consider the implementation of a screening effect by large field values  $\Phi_N > \Lambda$ . More specifically, we will focus on the chameleon mechanism [38,62]. We shall first cast the reconstructed theory into the Brans-Dicke representation with  $F(\phi) = \phi/M_*$  (see Sec. III in Ref. [31]). With this choice we have that  $\Gamma = \phi/M_*$  and  $\Xi = 1$  in Eqs. (A1) and (A2). By making use of the freedom in  $\Delta G_i$  it is possible to add a term to  $G_2$  that sets the  $q$ -value to be arbitrarily positive or negative. To see this let us begin with the full reconstructed Horndeski action in Eqs. (16)–(18) with a  $\Delta G_2$  term that takes the form

$$\Delta G_2 = \xi(\phi) \left(1 + \frac{X}{M_*^4}\right)^n, \quad (30)$$

with  $n \geq 3$  and  $\xi(\phi)$  given by

$$\xi(\phi) = M_*^2 U(\phi) - \frac{\lambda^{-N}}{2} (\phi - \phi_{\min})^k, \quad (31)$$

where  $\lambda$  is a coupling parameter,  $N$  and  $k$  are both positive integers,  $U(\phi)$  is the reconstructed potential in Eq. (16), and  $\phi_{\min}$  denotes the minimum value of the second contribution to the potential in Eq. (31). No other  $\Delta G_i$  terms are necessary as they all contain derivative terms which vanish in the screened limit. We shall take the scaling parameter  $\alpha$  to be the coupling  $\lambda$ .

This choice cancels the potential obtained from the linear reconstruction and replaces it with a power-law potential that takes a similar form to the chameleon screening example in Refs. [36,63] but with  $\alpha \rightarrow \alpha^{-N}$ . It is with a suitable choice of  $N$  that no derivative terms contribute in the screening limit. In this limit we then obtain the Einstein equation

$$\frac{\phi}{M_*} R_{\mu\nu} = -\mathcal{T}_{\mu\nu}^{(2)} + \left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)/M_*^2 + H_m[\nabla_\mu\phi], \quad (32)$$

where  $\mathcal{T}_{\mu\nu}^{(2)}$  is defined in Eq. (A8) and  $H_m[\nabla_\mu\phi]$  represents all the terms that involve derivatives of  $\phi$  in the metric field equation, the precise form of which is not relevant as we shall find that they disappear in the  $\alpha \rightarrow 0$  limit of interest. Taking the trace of Eq. (32) leads to  $\phi R/M_* = -\mathcal{T}^{(2)}$  which, noting that  $\mathcal{T}^{(2)} = 2G_2/M_*^2$ , gives a relation between  $R$  and  $G_2$ . The scalar field equation is given by

$$-\frac{2\phi}{M_*^2}(G_{2\phi} + G_{4\phi}R) + \mathcal{T}^{(2)} + H_s[\nabla_\mu\phi] = -T/M_*^2, \quad (33)$$

where  $H_s[\nabla_\mu\phi]$  represents all the terms in the scalar field equation involving derivatives of  $\phi$  which will disappear in the  $\alpha \rightarrow 0$  limit. With the choice of  $\Delta G_2$  in Eq. (30) there is no contribution from the reconstructed potential  $U(\phi)$  to the scalar field equation. After eliminating  $R$  and  $\mathcal{T}^{(2)}$  in favor of  $G_2$  the scalar field equation becomes

$$\alpha^{-N}(\phi - \phi_{\min})^{k-1}[\phi k - 2(\phi - \phi_{\min})] + H_s[\nabla_\mu\phi] = -T. \quad (34)$$

Applying the scaling method with the scalar field now expanded in terms of  $\psi$  as in Eq. (27), we examine the set of  $q$  values which leave nonvanishing terms on the left-hand side of Eq. (34) in the  $\alpha \rightarrow 0$  limit. As  $\alpha \rightarrow 0$  it is necessary to take the largest  $q$  value from this set after the scaling in Eq. (34). Disregarding the derivative terms in  $H_s[\nabla_\mu\phi]$ , we find that  $q$  takes one of two possible values

$$q \in \left\{ \frac{N}{k-1}, \frac{N}{k} \right\}. \quad (35)$$

We must take  $q = N/(k-1)$  as it is the largest in the set of  $q$  values from  $G_2$ . The integer  $N$  can then be chosen in Eq. (31) to be arbitrarily large. In the limit of  $\alpha \rightarrow 0$  this will send all terms involving spacetime derivatives of  $\phi$  to zero, justifying the original choice of  $\xi(\phi)$ . This is important as in principle the value of  $n$  in Eq. (30) is only bounded from below by the requirement that it is a nonlinear correction. All the terms involving derivatives of the scalar field scale as  $X^m = \phi_0^{2m} \alpha^{2mN/(k-1)} \tilde{X} \rightarrow 0$  as  $\alpha \rightarrow 0$  with  $m = \{1, \dots, n\}$ .

Now we expand the scalar field around the minimum of the potential such that  $\phi_{\min} \approx \phi_0$ . This then implies that  $\phi - \phi_0 = \phi_0 \alpha^q \psi$ . The remaining terms in the scalar field equation for  $\alpha \rightarrow 0$  relate the local value of the scalar field to the matter density as

$$\psi = \left( \frac{-T}{\phi_0^k k} \right)^{\frac{1}{k-1}}, \quad (36)$$

which recovers the chameleon screening effect for  $k < 1$ . The metric field equation in the same limit reduces to

$$\phi_0 R_{\mu\nu} = \left( T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right) / M_*, \quad (37)$$

recovering the standard Einstein equation with a rescaled Planck mass set by the background field value  $\phi_0$ . Therefore we have implemented a chameleon mechanism in a scalar-tensor action that is reconstructed from an arbitrary cosmological background evolution and linear perturbations by adding a suitable choice of  $\Delta G_2$ . Whether the screening effect operates in the Solar System to comply with stringent local tests of gravity needs to be checked numerically for a given reconstructed model.

### C. First-derivative screening

Next we examine the implementation of a screening effect that operates through large first derivatives  $\nabla \Phi_N > \Lambda$ . More specifically, we focus on the k-mouflage screening effect [40,64]. We may simply choose here the scaling parameter  $\alpha$  to be the kineticity function  $\alpha_K$  and take the  $\alpha \rightarrow \infty$  limit. EFT functions such as  $\alpha_K$  are typically parametrized as  $\alpha_{K0} f(a)$  where  $f(a)$  is some function of the scale factor with  $f(a=1) \equiv 1$ . Often this is simply a power of the scale factor or the evolution of the dark energy density normalized to the present value  $\Omega_{\text{DE}}(a)/\Omega_{\text{DE}0}$ . This ensures that the effects of the modifications only become relevant at late times. We shall take here the scaling parameter to correspond to the value of  $\alpha_K$  today,  $\alpha = \alpha_{K0}$ . It is also possible to take  $\alpha_{B0}$  or  $\alpha_{M0}$  as the scaling parameter but as the reconstruction depends differently on these EFT parameters this will lead to different

behavior in the screened limit (see Sec. IV D). Taking  $\alpha$  to be  $\alpha_{K0}$ , we see that as the reconstructed action is linear in the EFT functions we have from Table I that each term scales as  $U(\phi) \sim \alpha$ ,  $Z(\phi) \sim \alpha$ ,  $a_2(\phi) \sim \alpha$ , and  $b_1(\phi) \sim \alpha^0$ , which follows from the fact that  $\bar{M}_1^3$  is independent of  $\alpha_K$  (see Table II in Ref. [30] for the full set of relations between the EFT coefficients of the different bases). With this choice we have that the terms in  $G_2$  will scale as  $\alpha^{1+nq}$  for some integer  $n$  but those in  $G_3$  will scale as  $\alpha^{nq}$ .

In order to obtain an Einstein field equation it is necessary to remove the potential to avoid divergences in the  $\alpha \rightarrow \infty$  limit. This also makes physical sense as the screening mechanism in this case operates via the kinetic terms. We shall also remove all of the dependence on the canonical kinetic term linear in  $X$  to ensure that the screening operates through higher powers of  $X$ . To this end, we choose  $\Delta G_2 = \Delta G_2^{(1)} + \Delta G_2^{(2)}$ , where

$$\Delta G_2^{(1)} = \frac{1}{2} M_*^6 Z(\phi) \left( 1 + \frac{X}{M_*^4} \right)^4 - \frac{1}{2} M_*^6 Z(\phi) \left( 1 + \frac{X}{M_*^4} \right)^3, \quad (38)$$

$$\Delta G_2^{(2)} = 2M_*^2 U(\phi) \left( 1 + \frac{X}{M_*^4} \right)^3 - M_*^2 U(\phi) \left( 1 + \frac{X}{M_*^4} \right)^6. \quad (39)$$

These nonlinear corrections ensure that every term in  $G_2$  is now at least proportional to  $X^2$  or greater. With this choice the relevant term in the scalar field equation is

$$\nabla^\mu J_\mu^{(2)} = -G_{2XX} \nabla^\mu X \nabla_\mu \phi - X G_{2X\phi}, \quad (40)$$

where  $J_\mu^{(2)}$  is defined in Eq. (A6). The first term on the right-hand side in Eq. (40) scales as  $\alpha^{1+3q}$ , which sets the minimum  $q$  value to be  $q = -1/3$ . As every term in  $G_3$  scales as  $\alpha^{nq}$  with  $n > 0$ , this will send every term involving  $G_3$  to zero in the  $\alpha \rightarrow \infty$  limit. This particular  $q$ -value will also ensure that  $T_{\mu\nu}^{(i)} \rightarrow 0$  as  $\alpha \rightarrow \infty$  so that the metric field equation reduces to the standard Einstein field equation. The resulting scalar field equation corresponds to a k-mouflage model

$$\xi(\phi) \partial^\mu X \partial_\mu \phi = -\frac{T}{M_*^2}, \quad (41)$$

with

$$\xi(\phi) = a_2(\phi) + \frac{9Z(\phi)}{2M_*^2} - \frac{9U(\phi)}{M_*^8}. \quad (42)$$

### D. Second-derivative screening

Finally, we consider the implementation of screening through large second derivatives  $\nabla^2 \Phi_N > \Lambda$ , more explicitly the realization of the Vainshtein mechanism in the

$\alpha \rightarrow \infty$  limit where the scaling parameter  $\alpha$  is taken to be  $\alpha_B$  only. The procedure is similar to Sec. IV C. In this case  $U(\phi) \sim \alpha$ ,  $Z(\phi) \sim \alpha$ ,  $a_2(\phi) \sim \alpha$  as before, but in contrast to Sec. IV C,  $b_1(\phi) \sim \alpha$ , which follows from the fact that  $\bar{M}_1^3 \propto \alpha_B$ . We begin by adding on the nonlinear counterterms in Eqs. (38) and (39) to ensure the  $X$  dependence of  $G_2$  is at least  $X^2$ .

It turns out that the important term in the scalar field equation which gives rise to a nontrivial equation of motion and Vainshtein screening is  $\nabla^\mu J_\mu^{(3)}$  where  $J_\mu^{(3)}$  is given in Eq. (A7). Plugging in the expression in Eq. (17), we have that

$$\nabla^\mu J_\mu^{(3)} = b_1(\phi)S^{(4,2)} + H_s[\nabla_\mu \phi], \quad (43)$$

where again  $H_s[\nabla_\mu \phi]$  represents all of the terms involving derivatives of  $\phi$  that will vanish in the  $\alpha \rightarrow \infty$  limit. Furthermore  $S^{(4,2)}$  is a term that involves four derivative operators and two powers of the scalar field, which is given explicitly by

$$S^{(4,2)} = (\square\phi)^2 + \partial_\mu \phi \partial^\mu \square\phi + \square X. \quad (44)$$

These terms each scale as  $\alpha^{1+2q}$  requiring a  $q$ -value of  $-1/2$  to ensure independence of  $\alpha$  on the left-hand side. As we have also ensured that  $G_2$  starts at least at  $X^2$ , scaling as  $\alpha^{4q}$  with  $q = -1/2$ , these higher-derivative terms will disappear in the  $\alpha \rightarrow \infty$  limit. The scalar field equation in this limit then becomes

$$\frac{\phi_0^3}{M_*} b_1(\phi_0)[(\square\psi)^2 + \partial_\mu \psi \partial^\mu \square\psi + \square\tilde{X}] = -\frac{T}{M_*^2}, \quad (45)$$

where  $\tilde{X} \equiv \partial_\mu \psi \partial^\mu \psi$ . This is a typical scalar field equation involving higher derivatives of  $\psi$  expected for Vainshtein screening. It is necessary to ensure that the standard Einstein equation is obtained in the same limit in the metric field equations so that we can be sure this is the screened limit.

Having already set  $q = -1/2$  from the scalar field equation and ensured that  $G_2$  starts at  $X^2$  with  $\Delta G_2^{(1)}$  and  $\Delta G_2^{(2)}$ , every term  $\mathcal{T}_{\mu\nu}^{(i)}$  in the metric field equation (A1) vanishes in the  $\alpha \rightarrow \infty$  limit. For example, the first term in  $\mathcal{T}_{\mu\nu}^{(4)}$  scales as

$$G_{4\phi}S^{(2,1)} \sim \alpha^{2q} \sim \alpha^{-1} \rightarrow 0, \quad (46)$$

and the first one in  $\mathcal{T}_{\mu\nu}^{(3)}$  scales as

$$\frac{2}{M_*^2} G_{3X}S^{(4,3)} \sim \alpha^{1+3q} \sim \alpha^{-\frac{1}{2}} \rightarrow 0. \quad (47)$$

With the choice of the Brans-Dicke representation of  $F(\phi) = \phi/M_*$  we have that  $\Gamma = \phi_0/M_*$  and  $\Xi = 1$ , and the metric field equation reduces to Eq. (37).

To summarize, by choosing  $\alpha_{B0}$  as the scaling parameter and removing the constant and linear terms in  $X$  from  $G_2$  one can obtain the standard Einstein field equation with a rescaled Planck mass and a scalar field equation involving second derivatives in  $\psi$  as expected in the case of Vainshtein screening.

## V. NONLINEAR FREEDOM FOR DEGENERATE KINETIC SELF-ACCELERATION

As a further application of the nonlinear freedom in reconstructed scalar-tensor theories, we demonstrate how the correction term in Eq. (20) can be configured to construct scalar-tensor theories that are degenerate with standard cosmology to an arbitrary level of cosmological perturbations (Sec. VA). As a particular interesting example we show how this allows for models that accelerate the Universe without a cosmological constant yet remain dynamically degenerate with  $\Lambda$ CDM through a suitable configuration of the kinetic terms (Sec. VB).

### A. Perturbative degeneracy with $\Lambda$ CDM

An important implication of Eq. (20) is that it is possible to use the  $\Delta G_i$  terms to write down a Horndeski theory that possesses a highly nontrivial form for the nonlinear perturbations yet reduces to  $\Lambda$ CDM on the background, where the correction terms vanish. This degeneracy may even be extended to an arbitrary level of perturbations. The existence of such classes of theories is a natural consequence of the reconstruction being an expansion in  $(1 + X/M_*^4)^n$  with  $n \in \mathbb{N}$ . One can therefore construct theories whose physical effects only become relevant at a particular level of higher-order perturbations characterized by the power  $n$ .

To see how this works in practice let us choose, for example,

$$G_2 = -M_*^2 \Lambda + \xi_n^{(2)}(\phi) \left(1 + \frac{X}{M_*^4}\right)^n, \quad (48)$$

with  $G_3 = 0$ ,  $G_4 = M_*^2/2$ , and  $n \geq 3$ . After performing an ADM decomposition with  $\phi = tM_*^2$  the second term in Eq. (48) becomes  $\xi_n^{(2)}(t)(\delta g^{00})^n$ . On the background and linear scales therefore there will be no effects arising from the noncanonical kinetic terms and it will appear to be exactly  $\Lambda$ CDM. Note that this argument does not rely on the specific foliation adopted as we shall verify shortly for a specific example, but for now simply note that any nonzero perturbations that arise from another choice of foliation must be pure gauge. At the nonlinear level Eq. (48) departs from  $\Lambda$ CDM, and we have discussed the mapping of the  $\xi_n^{(2)}(t)$  functions onto nonlinear EFT functions in Sec. VI. It is also possible to write a theory with  $G_2 = \Lambda$  and

$$G_3 = \xi_n^{(3)}(\phi) \left(1 + \frac{X}{M_*^4}\right)^n. \quad (49)$$

In an equivalent manner this corresponds to a Galileon theory that can only be distinguished from  $\Lambda$ CDM on nonlinear scales. Combinations of  $\Delta G_2$  and  $\Delta G_3$  can also be used to construct more nontrivial theories.

For clarity we shall provide an explicit example of this degeneracy and compute the background equations of motion and check that the expansion is indeed matching that of  $\Lambda$ CDM. A more detailed analysis, including the investigation of possible instabilities and perturbative effects, will be the subject of further analysis. For simplicity, we shall focus here only on the degeneracy at the level of the background and not for the linear perturbations. Hence, we take  $n = 2$  in Eq. (48) so that

$$\begin{aligned} G_2 &= -M_*^2 \Lambda + \xi(\phi) \left(1 + \frac{X}{M_*^4}\right)^2 \\ &= -M_*^2 \Lambda + \xi(\phi) + 2\xi(\phi)X/M_*^4 + \xi(\phi)X^2/M_*^8, \end{aligned} \quad (50)$$

where  $\xi(\phi)$  is a free function of  $\phi$ . Not making any assumptions about the spacelike foliation we now put this equation into the unitary gauge by setting the scalar field to be just a function of time. With  $X = (-1 + \delta g^{00})\dot{\phi}^2$  we have at linear order

$$\begin{aligned} G_2 &= -M_*^2 \Lambda + \xi(t) + \frac{2\xi(t)X_0}{M_*^4} + \frac{\xi(t)X_0^2}{M_*^8} \\ &\quad - \left[ \frac{2\xi(t)X_0}{M_*^4} + \frac{2\xi(t)X_0^2}{M_*^8} \right] \delta g^{00}, \end{aligned} \quad (51)$$

where  $X_0$  is the value that  $X$  takes on the background, i.e.,  $X_0 = -\dot{\phi}^2$ . This gives an explicit expression for the EFT functions  $\Lambda(t)$  and  $\Gamma(t)$  in the unitary gauge expansion of  $G_2$  in Eq. (51), where the first line corresponds to  $-M_*^2 \Lambda(t)$  and the second line to  $-M_*^2 \Gamma(t)/2$ . The Friedmann equations in the EFT formulation are given by [20,25,43]

$$\Gamma(a) + \Lambda(a) = 3H^2 - \frac{\rho_m}{M_*^2}, \quad (52)$$

$$\Lambda(a) = 2HH' + 3H^2, \quad (53)$$

where we have set the nonminimal coupling parameter  $\Omega = 1$  and we parametrize the time  $t$  in terms of the scale factor  $a$ . With the expressions for  $\Gamma(a)$  and  $\Lambda(a)$  obtained from Eq. (51) one can take linear combinations of the Friedmann equations (52) and (53) to eliminate the dependence on the background expansion  $H$  and obtain a field equation for the background value of the scalar field. This is determined from the resulting expression

$$\Gamma(a) + \frac{1}{3}[\Gamma(a) + \Lambda(a)]' = 0 \quad (54)$$

to be

$$\left[ \frac{4\xi(a)X_0}{M_*^{10}} + \frac{\xi'(a)}{M_*^{10}}(X_0 - M_*^4/3) \right] (X_0 + M_*^4) + \quad (55)$$

$$X_0' \left[ \frac{2\xi(a)}{3M_*^6} + \frac{2\xi'(a)X_0}{M_*^{10}} \right] = 0, \quad (56)$$

which is the nontrivial Klein-Gordon scalar field equation. It has a trivial solution  $X_0 = -M_*^4$ . More complicated solutions to the background scalar field equation will be explored in the future. From  $X_0 = -M_*^4$ , one immediately recognizes in Eq. (50) that  $G_2(X_0) = -\Lambda$ , and hence the recovery of the  $\Lambda$ CDM background expansion. Alternatively, once the solution to the background evolution of the scalar field has been obtained it is possible to derive the equation-of-state parameter for the resulting  $k$ -essence model given by [65]

$$w(a) = \frac{-M_*^2 \Lambda + \xi(\phi)(1 + X/M_*^4)^2}{M_*^2 \Lambda - \xi(\phi)(1 + X/M_*^4)(1 - 3X/M_*^4)}. \quad (57)$$

After inserting the background solution  $X = X_0 = -M_*^4$  one obtains  $w = -1$ , confirming that the background expansion is indeed matching that of  $\Lambda$ CDM.

## B. Degenerate kinetic self-acceleration

To highlight the implications of the perturbative degeneracy, we will now study a particularly interesting example of Eq. (48). Let us consider a class of models specified by  $\xi(\phi) = M_*^2 \Lambda_\phi$  in Eq. (50). The subscript  $\phi$  indicates that  $\Lambda_\phi$  is a coupling parameter in the higher-order kinetic terms of the scalar field  $\phi$ . Equation (50) then becomes

$$G_2 = -M_*^2 \Lambda_{\text{GR}} + M_*^2 \Lambda_\phi \left(1 + \frac{X}{M_*^4}\right)^2, \quad (58)$$

where we defined  $\Lambda \equiv \Lambda_{\text{GR}}$ . We also set  $G_4 = 1$  and  $G_3 = 0$  and stress that any contributions to  $\Lambda_{\text{GR}}$  from quantum corrections of matter fields in this discussion are neglected. If we now set  $\Lambda_\phi = \Lambda_{\text{GR}}$ , this model exhibits the particular feature of having no explicit cosmological constant. The model is now simply

$$G_2 = 2\Lambda_\phi X/M_*^2 + \Lambda_\phi X^2/M_*^6. \quad (59)$$

However, the observed cosmological constant  $\Lambda_{\text{obs}}$  in the cosmological background of this model remains  $\Lambda_{\text{obs}} = \Lambda_\phi = \Lambda_{\text{GR}}$ . An alternative approach is to start with the model

$$G_2 = 2\Lambda_\phi X/M_*^2 + \Lambda_\phi X^2/M_*^6 - 2M_*^2\Lambda_{\text{GR}}, \quad (60)$$

and then set  $\Lambda_{\text{GR}} = 0$ . In summary, in one interpretation the coupling  $\Lambda_\phi$  is tuned to match a nonvanishing  $\Lambda_{\text{GR}}$  that corresponds to the observed  $\Lambda_{\text{obs}}$  or  $\Lambda_{\text{GR}} = 0$  and  $\Lambda_\phi = \Lambda_{\text{obs}}$ .

With either interpretation these models generate a *kinetic self-acceleration* effect that is degenerate with the cosmological constant to the  $(n-1)$ th order of cosmological perturbations. While this may certainly be viewed as an engineered self-acceleration effect, it also raises more general questions about the genuineness of a kinetic self-acceleration that resembles a cosmological constant for observational compatibility. We note that a similar expansion to Eq. (58) can be performed for  $G_3$  with similar implications. For instance, one may consider a kinetic gravity braiding model with nontrivial  $G_2$  and  $G_3$ . By combining power series of  $(1 + X/M_*^4)^n$  in  $G_2$  and  $G_3$  that only contribute at the  $(n-1)$ th order in cosmological perturbations, one can choose the coefficients of  $G_2$  and  $G_3$  in an expansion in  $X$  to cancel off to just leave a term  $X^n$  in  $G_2$  and  $G_3$  for arbitrarily large  $n$ . Greater values of  $n$  then correspond to models which are more difficult to distinguish from  $\Lambda$ CDM and for which nonlinear data must be used for their discrimination. This may shed some light on the results of Ref. [66], where better agreement with  $\Lambda$ CDM at the linear level was likewise found for kinetic gravity braiding models with  $G_3 \propto X^n$  for large  $n$  but adopting a canonical  $G_2$  instead, which is not feasible with using  $\Delta G_i$  corrections only.

We shall leave a more detailed examination of the genuineness of kinetic self-acceleration that closely matches  $\Lambda$ CDM phenomenology to subsequent work. It is worth noting, however, that a further interesting consequence of  $\Lambda_{\text{obs}}$  being interpreted as a coupling rather than a bare constant is that it may be possible to render the acceleration effect in Eq. (58) technically natural as it can now enter as a coefficient to an irrelevant operator rather than as a nonrenormalizable constant [67,68]. The details shall also be studied further in forthcoming work. At a more practical level, we emphasize that these models have the interesting property that discriminatory effects of this type of cosmic acceleration are left exclusively to the nonlinear observational regime.

## VI. HIGHER-ORDER RECONSTRUCTION

With the higher-order EFT expansion in Eq. (26) and the freedom in the nonlinear sector having been significantly reduced by the restriction to a luminal speed of gravity, it becomes straightforward to perform a  $n$ th order reconstruction of the corresponding class of Horndeski theories by fixing the  $\Delta G_i$  functions order-by-order in terms of the nonlinear EFT functions  $\bar{M}_i^{3,4}$ . We shall now see how this extra information modifies the reconstruction

from the background and linear scales by adding in the new free functions and slightly changing the dependence on the linear EFT functions. We shall elaborate on this explicitly for the case of  $i = 3$  before outlining the general  $n$ th order case.

Let us begin by noting that in the unitary gauge a term that takes the form  $\xi(\phi)X^m\Box\phi$  becomes

$$\begin{aligned} \xi(\phi)X^m\Box\phi &= \mp \frac{2m}{2m+1} \xi(\phi)(-X)^{m+\frac{1}{2}}K \\ &\pm \frac{1}{2m+1} \xi'(\phi)(-X)^{m+1}, \end{aligned} \quad (61)$$

where the sign differences on the top and bottom indicate even or odd  $m$ , respectively, and the prime denotes a derivative with respect to  $\phi$ . After expanding Eq. (61) in the unitary gauge there will be several terms that contribute and that can be mapped onto the operators in Eq. (26).

We shall proceed along the same lines as Ref. [30] to obtain a corresponding covariant action. To begin, by using the replacement  $\delta g^{00} = 1 + X/M_*^2$  the  $(\delta g^{00})^3$  operator becomes

$$\bar{M}_3^4(t)(\delta g^{00})^3 = \bar{M}_3^4(\phi) \left( 1 + \frac{3X}{M_*^4} + \frac{3X^2}{M_*^8} + \frac{X^3}{M_*^{12}} \right). \quad (62)$$

This contributes to  $U(\phi)$ ,  $Z(\phi)$ ,  $a_2(\phi)$  along with a new, now necessarily nonvanishing contribution to the coefficient of  $X^3$  that we call  $a_3(\phi)$ . Let us now derive the covariant action which gives rise to the following expansion in the unitary gauge:

$$\bar{M}_1^3(t)\delta g^{00}\delta K + \bar{M}_3^3(t)(\delta g^{00})^2\delta K. \quad (63)$$

We shall take the case of  $m = 1, 2$  in Eq. (61) for simplicity and begin with the combination

$$G_3 = b_1(\phi)X\Box\phi + b_2(\phi)X^2\Box\phi + \Delta G_3^{(4)}, \quad (64)$$

where  $\Delta G_i^{(4)}$  indicates that the nonlinear corrections now start at fourth order. We transform Eq. (64) into the unitary gauge and then solve for  $b_1(\phi)$  and  $b_2(\phi)$  in terms of the EFT functions. It is necessary to have two independent functions in the covariant expansion as there are two independent EFT functions. At third order in the perturbations we obtain

$$G_3 \supset -b_1(\phi)M_*^6\delta g^{00}\delta K + \frac{1}{4}b_1(\phi)M_*^6(\delta g^{00})^2\delta K \quad (65)$$

$$+2b_2(\phi)M_*^{10}\delta g^{00}\delta K - \frac{3}{2}b_2(\phi)M_*^{10}(\delta g^{00})^2\delta K, \quad (66)$$

where for the sake of clarity we have not shown the terms which are independent of  $\delta K$ . We then require that

$$-b_1(\phi)M_*^6 + 2b_2(\phi)M_*^{10} = \bar{M}_1^3(\phi), \quad (67)$$

$$b_1(\phi)M_*^6 - 6b_2(\phi)M_*^{10} = 4\bar{M}_3^3(\phi). \quad (68)$$

This system of equations can be straightforwardly solved to obtain  $b_1(\phi)$  and  $b_2(\phi)$ . The results are shown in Table II along with the contributions to  $G_2$ .

Importantly, this method can straightforwardly be extended to higher orders, where at each order it is necessary to invert an  $n \times n$  matrix to obtain the corresponding EFT coefficients in terms of covariant functions in  $G_3$ . It is then possible to derive a reconstruction from the  $\bar{M}_i^4, \bar{M}_i^3$  terms which proceeds in exactly the same manner as discussed for  $n = 3$ . It is also important to stress that a different combination of the terms in Eq. (61) with different choices of  $m$  could have been chosen to develop the reconstruction. From the structure of Eq. (61) there will always be terms involving  $(\delta g^{00})^n \delta K$  to arbitrary order for any  $m$  which can be used as the basis for deriving the reconstructed theory. There is therefore a degeneracy in the space of models which go as  $X^m \square \phi$  on the behavior of the background and perturbations.

The reconstructed Horndeski theory that covers the background, linear-order, and second-order cosmological perturbations is given by

$$G_2(\phi, X) = -M_*^2 U(\phi) - \frac{1}{2} M_*^2 Z(\phi) X + a_2(\phi) X^2 + a_3(\phi) X^3 + \Delta G_2, \quad (69)$$

$$G_3(\phi, X) = b_0(\phi) + b_1(\phi) X + b_2(\phi) X^2 + \Delta G_3, \quad (70)$$

$$G_4(\phi, X) = \frac{1}{2} M_*^2 F(\phi). \quad (71)$$

The precise form of each term written in terms of the EFT functions is presented in Table II. Note that now that we have extended the reconstruction to nonlinear order it is necessary to include higher powers of  $X$  in the reconstruction, in both  $G_2$  and  $G_3$ . In the same manner, if we were to extend the reconstruction to  $(n - 1)$ th order in cosmological perturbations it would introduce terms of the form  $X^n$  in  $G_2$  and  $G_3$ .

Finally, it is also of interest to examine what effect these higher-order perturbations have on the physical EFT basis recently introduced in Refs. [31,46]. It consists of parametrizing the EFT formalism in terms of inherently stable basis functions: The effective Planck mass squared  $M^2$ , the sound-speed squared  $c_s^2$ , the kinetic energy of the scalar field  $\alpha$ , and the background expansion  $H(t)$ , along with  $\alpha_{B0}$ . Any constraints placed on these parameters are guaranteed to satisfy the conditions for avoiding ghost and gradient instabilities, which otherwise must be checked independently for other bases. For higher-order perturbations, note that by shifting the time coordinate infinitesimally such that  $t \rightarrow t + \pi$  the important operators for our purpose in the EFT action change in accordance with the following Stückelberg transformations [20,21]:

$$g^{00} \rightarrow g^{00} + 2g^{0\mu} \partial_\mu \pi + g^{\mu\nu} \partial_\mu \partial_\nu \pi, \quad (72)$$

$$\delta K \rightarrow \delta K - 3\dot{H}\pi - a^{-2} \square \pi, \quad (73)$$

where  $\pi$  is interpreted as the extra scalar d.o.f. which was hidden when the action was written in the unitary gauge. Higher-order operators will introduce new terms in the perturbative expansion, for example in the coefficients of  $\dot{\pi}^2$ , which may alter the stability conditions of the theory. As the physical basis for the EFT functions is defined through the coefficients of such terms, this implies that these higher-order operators act to correct the lower-order EFT functions. For example, the sound speed will now depend on these higher-order EFT functions, and so the linear stability may be affected by what occurs at the nonlinear level. Physically this makes sense. If one has a second-order perturbation which is unstable, it will produce a runaway effect such that it will grow to affect the linear and background scales. In other words, the perturbations of the perturbations must be kept under control if the theory is to be completely stable. The stability of the full theory can of course be computed at the level of the covariant action. EFT naturally splits up the dynamics of the different length scales, and in order to obtain a theory that is stable, this stability must be kept at all orders in the EFT expansion. Note, however, that Ref. [29] checked the stability conditions of a general beyond Horndeski action at fourth order, finding them to be automatically satisfied once the

TABLE II. Contributions to the reconstructed Horndeski action arising from the nonlinear corrections in the EFT action at third order. The reconstruction can easily be expanded to arbitrary higher order.

$U(\phi) = \Lambda + \frac{\Gamma}{2} - \frac{M_*^4}{2M_*^2} - \frac{3H\bar{M}_1^3}{2M_*^2} + \frac{3H\bar{M}_3^3}{M_*^2} - \frac{3(\bar{M}_1^3)'}{20} + \frac{(\bar{M}_3^3)'}{5} - \frac{\bar{M}_3^4}{M_*^2}$	
$Z(\phi) = \frac{\Gamma}{M_*^4} - \frac{2M_*^4}{M_*^6} - \frac{3H\bar{M}_1^3}{M_*^6} + \frac{12H\bar{M}_3^3}{M_*^6} + \frac{3(\bar{M}_1^3)'}{5M_*^4} - \frac{4(\bar{M}_3^3)'}{5M_*^4} - \frac{6\bar{M}_3^4}{M_*^6}$	
$a_2(\phi) = \frac{M_*^4}{2M_*^8} - \frac{3H\bar{M}_1^3}{M_*^8} + \frac{(\bar{M}_1^3)'}{5M_*^6} - \frac{3(\bar{M}_3^3)'}{5M_*^6} + \frac{3\bar{M}_3^4}{M_*^8}$	
$a_3(\phi) = \frac{(\bar{M}_1^3)'}{40M_*^{10}} - \frac{(\bar{M}_3^3)'}{5M_*^{10}} + \frac{\bar{M}_3^4}{M_*^{12}}$	
$F(\phi) = \Omega$	
	$b_1(\phi) = \frac{3\bar{M}_1^3}{4M_*^6} - \frac{2\bar{M}_3^3}{M_*^6}$
	$b_2(\phi) = \frac{\bar{M}_1^3}{8M_*^{10}} - \frac{\bar{M}_3^3}{M_*^{10}}$

linear stability conditions were met. It is an open question whether this result can be generalized to our expansion in Eq. (26) at  $n$ th order. We leave a discussion of these issues for future work.

## VII. CONCLUSIONS

Constraining models beyond  $\Lambda$ CDM are worthwhile and promising endeavors of modern cosmology. We are about to see an enormous influx of observational data from surveys such as Euclid [69,70] and LSST [71], which will provide percent-level constraints on the cosmological parameters. The outcome of these surveys will be twofold. On the one hand, the Universe turns out to be consistent with  $\Lambda$ CDM, which will motivate a more directed effort in tackling the cosmological constant problem (see, e.g., Refs. [72–85]). On the other hand, if recent observational tensions [3,86,87] persist, then that will be strong evidence that the theory describing the Universe on cosmological scales requires revision and potentially will go beyond a cosmological constant. Constraints on deviations from GR are obtained on a broad range of different length scales, and a potential new theory acting on large cosmological scales must also be consistent with observations at nonlinear scales.

In this paper we have discussed how in generalized scalar-tensor theories observations made at the level of the background and the linear perturbations may be connected with the nonlinear regime and vice versa. This is made possible through the reconstruction of covariant Horndeski theory from the EFT of dark energy [30,31]. The reconstructed theories are degenerate to linear order in cosmological perturbations and differ only by nonlinear correction terms  $\Delta G_i$ . We first explored the uniqueness of these correction terms. At  $n$ th order in perturbation theory the number of EFT operators that one can write down which are consistent with the symmetry of broken time diffeomorphisms becomes unmanageable. However, we have argued that by restricting to Horndeski theories that respect the GW170817 constraint of luminal speed of gravity [32,36], the number of free functions that enter the EFT expansion at each order is limited to two. The two correction terms at  $n$ th order can then be related to the free functions  $\xi_n^{(2,3)}(\phi)$  specifying  $\Delta G_2$  and  $\Delta G_3$ .

As a first application of the nonlinear correction terms, we have considered the implementation of screening mechanisms. With the reconstructed covariant theory it is possible to apply techniques that have been developed [36,42] to identify the existence of Einstein gravity limits within a given Horndeski theory. With the use of these methods we have demonstrated that there is enough freedom on nonlinear scales to employ a particular type of a screening mechanism by a suitable configuration of the correction terms. More specifically, we have provided the examples of realizing chameleon, k-mouflage, and Vanshtein mechanisms.

A further consequence of the reconstruction method concerns the identification of a class of models that is degenerate with  $\Lambda$ CDM at the level of the background and linear perturbations but departs from it at an arbitrary order of nonlinear perturbations. A subclass of these models further exhibits kinetic self-acceleration, where the background expansion is accelerating exactly as  $\Lambda$ CDM but there is no explicit cosmological constant written in the theory. The acceleration is instead driven by the kinetic terms. An immediate consequence of the existence of such models is that even if the background expansion and linear matter power spectrum is measured to agree with  $\Lambda$ CDM from the next generation of surveys, the degenerate alternatives may not generally be excluded. Moreover, a theoretically appealing aspect of these models is that, with the cosmological constant now acting as a coefficient of kinetic terms rather than a bare constant, it may be possible to render it technically natural. These implications warrant a more detailed study of these models. Finally, the same techniques that were employed in the development of the reconstruction of the Horndeski action to linear order in cosmological perturbations were utilized here to derive a reconstructed theory that includes the nonlinear EFT functions. For given constraints on these functions this enables a reconstruction of the Horndeski theory across a broad range of length scales, which may be supplemented with a restriction of the allowed forms of  $\Delta G_i$  to those that employ a screening mechanism.

There remain many further applications to be examined for the nonlinear sector of the reconstruction method. For example, obtaining the stability conditions is an important step in understanding the viability of the sampled models in parameter estimation analyses, and it is as yet unclear what effect the nonlinear correction terms have on the stability of the theory. There may also be a more physical basis for the correction terms such as that presented in Ref. [46] for linear perturbations, which automatically satisfies the stability constraints at the nonlinear level. We leave such considerations to upcoming studies.

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## APPENDIX: HORNDESKI FIELD EQUATIONS WITH $\alpha_T = 0$

For completeness, we shall present here the metric and scalar field equations that are obtained from varying  $g_{\mu\nu}$

and  $\phi$  in Eqs. (6) to (8). Although the structure of these equations is complicated, the relevance for the application in Sec. IV is simply the number of spacetime derivatives and powers of the scalar field that enter into each of the field equations. The metric field equation is given by [36]

$$\Gamma R_{\mu\nu} = -\sum_{i=2}^4 \mathcal{T}_{\mu\nu}^{(i)} + \left( T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right) / M_*^2, \quad (\text{A1})$$

and the scalar field equation is given by

$$\Gamma \sum_{i=2,3,4} (\nabla^\mu J_\mu^{(i)} - P_\phi^{(i)}) + \Xi \sum_{i=2}^4 \mathcal{T}^{(i)} = -\frac{T}{M_*^2} \Xi, \quad (\text{A2})$$

where  $\Gamma \equiv 2G_4/M_*^2$  and  $\Xi \equiv 2G_{4\phi}/M_*^2$ , and

$$P_\phi^{(2)} = \frac{2}{M_*^2} G_{2\phi}, \quad (\text{A3})$$

$$P_\phi^{(3)} = \frac{2}{M_*^2} \nabla_\mu G_{3\phi} \nabla^\mu \phi, \quad (\text{A4})$$

$$P_\phi^{(4)} = \frac{2}{M_*^2} G_{4\phi} R, \quad (\text{A5})$$

$$J_\mu^{(2)} = -G_{2X} \nabla_\mu \phi, \quad (\text{A6})$$

$$J_\mu^{(3)} = -G_{3X} \square \phi \nabla_\mu \phi + G_{3X} \nabla_\mu X + 2G_{3\phi} \nabla_\mu \phi, \quad (\text{A7})$$

$$\mathcal{T}_{\mu\nu}^{(2)} = -\frac{1}{M_*^2} G_{2X} \nabla_\mu \phi \nabla_\nu \phi + \frac{1}{2M_*^2} g_{\mu\nu} (X G_{2X} + 2G_2), \quad (\text{A8})$$

$$\mathcal{T}_{\mu\nu}^{(3)} = \frac{2}{M_*^2} G_{3X} \mathcal{S}^{(4,3)} + G_{3\phi} \nabla_\mu \phi \nabla_\nu \phi, \quad (\text{A9})$$

$$\mathcal{T}_{\mu\nu}^{(4)} = G_{4\phi} \mathcal{S}^{(2,1)} + G_{4\phi\phi} \mathcal{S}^{(2,2)}. \quad (\text{A10})$$

Note that  $J_\mu^{(4)} = 0$ . The  $\mathcal{S}^{(i,j)}$  notation indicates a term that contains  $i$  spacetime derivatives and  $j$  powers of the scalar field. As discussed in Sec. IV, knowledge of these quantities is sufficient to determine whether a given term will become dominant or subdominant in a screened or unscreened limit, not its precise functional form. We refer the reader to the Appendix of Ref. [36] for the explicit expressions but note the different definitions of the  $G_i$  functions and  $X$ .

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