Correspondences of matter fluctuations in semiclassical and classical gravity for cosmological spacetime

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A correspondence between fluctuations of minimally coupled scalar fields and that of an effective perfect fluid is shown to exist. A similar correspondence between the stress tensors themselves is known and widely used in literature. Using recent results obtained in semiclassical stochastic gravity for the fluctuations of the quantum stress tensor, we obtain this correspondence, which is argued to be of fundamental importance to statistical analysis of systems in curved spacetime. We show that the scalar field fluctuations are related to covariances of energy density and pressures of the effective perfect fluid. Such a correspondence between the semiclassical and classical fluctuations therefore, is expected to give insight to the mesoscopic phenomena for gravitating systems and would further enhance the perturbative analysis for cosmological spacetimes and astrophysical objects in the decoherence limit. A kinetic theory in curved spacetime may find useful insights from such correspondences in future work.

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In this paper, inspired by the correspondence between stress tensor for scalar fields and that for general fluid as in a hydrodynamic limit [1,2], we attempt to establish a similar correspondence between the fluctuations of the two in gravitating bodies. This is an enhanced result that we obtain by using an exact form of the noise kernel [3,4] defining fluctuations of a quantum field, and relate it to those of perfect fluid stress tensor in the classicalized limit. This is the first attempt to show any such relation between quantum and classical fluctuations of the two stress tensors. The semiclassical Einstein-Langevin equation, which sets the stage for semiclassical stochastic gravity [4,5], is aimed at studying the induced perturbations of the metric which are in the low energy limit equivalent to quantum perturbations of the metric as would be obtained in a viable theory of quantum gravity. This is of significance to very early universe, where the details of the quantum structure of spacetime would be of interest. The correspondence between the noise kernel due to quantum fields and that of a fluid model in classicalized limit as presented in this paper is aimed for later stages in the evolution of the universe and inflationary period, where the classicalized effects would be of significance for a similar Einstein Langevin equation. This would lead to statistical analysis of extended structure of spacetime in the classicalized limit. Thus in this theme, hydrodynamic approximation of the matter fields would make the solution of the Einstein Langevin equation much simpler mathematically, which for the quantum matter fields is more difficult. The noise kernel in the hydrodynamic limit then can be used as the central quantity in a likewise classicalized Einstein Langevin equation and thus lays foundations for a whole theme to work upon in cosmology.

Of interest toward possible applications of such a correspondence would thus be the cosmological and astrophysical spacetimes and their equilibrium and non-equilibrium statistical physics in different configurations.

It is known that a stress tensor for a scalar field can be approximated by a general fluid [1], thus

$$T_{ab}(x) = \phi_{;a}\phi_{;b} - \frac{1}{2}g_{ab}\phi^{;c}\phi_{;c} - \frac{1}{2}g_{ab}m^2\phi^2 + \xi(g_{ab}\Box - \nabla_a\nabla_b + G_{ab})\phi^2 \tag{1}$$

where the Klein Gordon field ϕ satisfies

$$(\Box + m^2 + \xi R)\phi = 0 \tag{2}$$

has a correspondence with

$$T_{ab}(x) = u_a u_b(\epsilon + p) - g_{ab}p + q_a u_b + u_a q_b - \pi_{ab} \tag{3}$$

where ϵ and p are the energy density and pressure of the fluid, u_a the four-velocity, and q_a and π_{ab} the heat flux and anisotropic stress. We in this article restrict to the case $\xi = 0$ in Eq. (1) which corresponds to taking $q_a = 0$ and $\pi_{ab} = 0$ in (3) so that it reduces to a perfect fluid stress tensor.

A quantum scalar field similarly can be described by such a hydrodynamic analogy [6]. Thus for systems whose fundamental description involves quantum fields, there is a

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local thermal equilibrium limit where the system may be described as a fluid. Also when decoherence of the quantum states is effective in the quantum to classical transition of the system such as in stochastic inflation this hydrodynamic approximation can be seen to be applicable. Though these correspondences are open to detailed investigations [7], they form the base of many well established studies.

Here we go a step further and obtain a correspondence for the fluctuations of the two stress tensors by using the exact form of the semiclassical noise kernel as is given in [2,3] for quantum fields.

The quintessence field for the dark energy is another example where such a correspondence can be useful for perturbative studies and a stochastic analysis. This can also be applicable for studying backreaction on the metric perturbations during the later stages in inflationary era, after decoherence effects, due to squeezing of states [6,8] become important.

We begin with the noise kernel expression as used in semiclassical stochastic gravity which is defined as

$$8N_{abcd}(x,x') = \langle\{\hat{T}_{ab}(x), \hat{T}_{c'd'}(x')\}\rangle - 2\langle\hat{T}_{ab}(x)\rangle\langle\hat{T}_{c'd'}(x')\rangle \tag{4}$$

where $\hat{T}_{ab}$ denotes the quantum stress tensor, obtained by raising ϕ in (1) to an operator.

The expectation of such a quantum stress tensor $\langle\hat{T}_{ab}(x)\rangle$ after regularization is used as the matter content in the semiclassical Einstein's equations. A step further, the fluctuations of the same via a noise kernel (4) form the central quantity of importance in the theory of semiclassical stochastic gravity as mentioned earlier. An elaborate procedure using point splitting formalism to deal with ill-defined operators $\hat{\phi}^2$ gives explicit expression for the noise kernel as is given in [2,3]. For mathematical simplicity of the correspondence we consider here the minimally coupled scalar field, thus

$$\begin{aligned}8\tilde{N}_{abcd}(x,x') = {} & (G_{;c'b}G_{;d'a} + G_{;c'a}G_{;d'b}) + 2g_{ab}G^{p}_{;c'}G_{;d'p} \\ & - g_{c'd'}(G_{;p'b}G^{p'}_{;a} + m^2 G_{;a}G_{;b}) \\ & + g_{ab}g_{c'd'}\frac{1}{2}[G_{;p'q}G^{;p'q} \\ & + m^2\{G_{;p'}G^{;p'} + G_{;p}G^{;p} + m^2G^2\}]\end{aligned} \tag{5}$$

where $G \equiv G(x,x') = \langle\hat{\phi}(x)\hat{\phi}(x')\rangle$ are the Wightman functions for the quantum field.

This noise kernel in the decoherence limit can be written simply with the quantum stress tensor in (4) replaced by a classical stress tensor,

$$\begin{aligned}8N_{abcd}(x,x') &= 2(\langle T_{ab}(x)T_{c'd'}(x')\rangle - \langle T_{ab}(x)\rangle\langle T_{c'd'}(x')\rangle) \\ &= 2\mathrm{Cov}[T_{ab}(x)T_{cd}(x')]\end{aligned} \tag{6}$$

Here the classicalized field ϕ acts as a random variable and has a distribution. Thus the averages above in the classical limit are statistical averages. The stress tensor then reads

$$T_{ab} = \phi_{;a}\phi_{;b} - g_{ab}\mathcal{L}_\phi \tag{7}$$

where

$$\mathcal{L}_\phi = \phi^{;c}\phi_{;c} - V(\phi)$$

in what follows we consider a specific form of the potential $V(\phi) = -\frac{1}{2}m^2\phi^2$. However equivalent treatment to other cases can be given easily. The corresponding effective perfect fluid stress tensor is then given by

$$T_{ab} = u_a u_b(p+\epsilon) - g_{ab}p \tag{8}$$

such that

$$\begin{aligned}\epsilon &= \frac{1}{2}\phi_{;c}\phi^{;c} + V(\phi) \\ p &= \frac{1}{2}\phi_{;c}\phi^{;c} - V(\phi) \\ u_a &= [\partial_c(\phi^2)\partial^c(\phi^2)]^{-1/2}\partial_a\phi^2\end{aligned} \tag{9}$$

It is important to realise that, here the randomness to the stress tensor is not imparted by any classical particles performing random motion in the fluid, but the fluid model is fundamentally different than what is proposed in [9]. For the case discussed in this paper, it is the scalar field distribution, which accounts for the stochastic behavior of the stress tensor. Further, the effective noise kernel, in terms of the perfect fluid variables can be obtained by using (8) in (6) as follows

$$\begin{aligned}4N_{abcd}(x,x') = \mathrm{Cov}[T_{ab}(x)T_{cd}(x')] = {} & u_a u_b u_c u_d\{\mathrm{Cov}[\epsilon(x)\epsilon(x')] + \mathrm{Cov}[\epsilon(x)p(x')] + \mathrm{Cov}[p(x)\epsilon(x')] + \mathrm{Cov}[p(x)p(x')]\} \\ & - u_a u_b g_{cd}\{\mathrm{Cov}[\epsilon(x)p(x')] + \mathrm{Cov}[p(x)p(x')]\} - g_{ab}u_c u_d\{\mathrm{Cov}[p(x)\epsilon(x')] \\ & + \mathrm{Cov}[p(x)p(x')]\} + g_{ab}g_{cd}\mathrm{Cov}[p(x)p(x')].\end{aligned} \tag{10}$$

Comparing Eqs. (10) and (5) is straightforward since one can trace the term by term correspondences as below,

$$
\begin{aligned}
&G_{;c'b}G_{;d'a} + G_{;c'a}G_{;d'b} \to 2u_b u_b u_c u_d\{\mathrm{Cov}[\epsilon(x)\epsilon(x')] + \mathrm{Cov}[\epsilon(x)p(x')] + \mathrm{Cov}[p(x)\epsilon(x')] + \mathrm{Cov}[p(x)p(x')]\}\\
&G^{p}_{;c'}G_{;d'p} \to -u_c u_d\{\mathrm{Cov}[p(x)\epsilon(x')] + \mathrm{Cov}[p(x)p(x')]\}\\
&G_{;p'b}G^{p'}_{;a} + m^2 G_{;a}G_{;b} \to 2u_a u_b\{\mathrm{Cov}[\epsilon(x)p(x')] + \mathrm{Cov}[p(x)p(x')]\}\\
&\frac{1}{2}[G_{;p'q}G^{;p'q} + m^2\{G_{;p'}G^{;p'} + G_{;p}G^{;p} + m^2 G^2\}] \to 2\mathrm{Cov}[p(x)p(x')].
\end{aligned}
\tag{11}
$$

We see, that the two point covarainces of pressure and density capture the microscopic nature of the fluid in a statistical description which are in turn related to quantum fluctuations inherently. This is the central feature of the correspondence that we obtain here.

In addition to being used in the perturbative theory as the noise source, these fluctuations characterize the microscopic effects in the matter fields coupled to a spacetime of interest. The significance of these fluctuations also lies in realising their importance for compact astrophysical objects which are coupled to say thermal fields as discussed in [10,11] and are of interest to collapsing clouds, towards critical phases and end states of collapse.

Our results indicate that, fluctuations of quantum fields induce mesoscopic classicalized effects in the fluid description of the same model, given by covariances of pressures and energy density in the background spacetime.

Since the noise kernel $\tilde{N}_{abcd}(x,x')$ due to quantum fluctuations, is also known to be a classical stochastic source in nature, one can raise a question about the correspondence worked out here. The importance of what we have shown here, lies in realizing that the quantum sourced noise can be captured partially via the fluctuations of classical parameters of an effective perfect fluid model of matter.

In the future it would be interesting to obtain such a correspondence for nonminimally coupled fields and imperfect fluids, thus taking into account heat flux and dissipation. The usefulness of such a correspondence, that of noise in the system lies on one hand, in aiding to study the microscopic structure and its connections with kinetic theory in curved spacetime [6] which is a developing area of research, while on the other hand can be used as a source for perturbative analysis of gravitating systems. The perturbative analysis using the semiclassical Einstein Langevin equation is an elaborate approach, however solutions of the semiclassical Einstein equations pose a challenge due to the presence of quantum stress tensor and semiclassical noise kernel. Thus a simplified version of matter fields in terms of hydrodynamic limit, may be useful in finding solutions of the same more easily, though these would put restriction regarding the classicalizing of the system. The details of this restriction would follow in future developments. Characterizing noise and dissipation in the system in the decoherence limit is a first step towards this.

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