Charged vector inflation

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We present a model of inflation in which the inflaton field is charged under a triplet of $U(1)$ gauge fields. The model enjoys an internal $O(3)$ symmetry supporting the isotropic FRW solution. With an appropriate coupling between the gauge fields and the inflaton field, the system reaches an attractor regime in which the gauge fields furnish a small constant fraction of the total energy density. We decompose the scalar perturbations into the adiabatic and entropy modes and calculate the contributions of the gauge fields into the curvature perturbations power spectrum. We also calculate the entropy power spectrum and the adiabatic-entropy cross-correlation. In addition to the metric tensor perturbations, there are tensor perturbations associated with the gauge field perturbations that are coupled to metric tensor perturbations. We show that the correction in the primordial gravitational tensor power spectrum induced from the matter tensor perturbation is a sensitive function of the gauge coupling.

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I. INTRODUCTION

Models of inflation based on a single scalar field with a flat potential are quite consistent with cosmological observations [1,2]. Among the basic predictions of models of inflation are that the primordial perturbations are nearly scale invariant, nearly adiabatic, and nearly Gaussian, in very good agreement with observations. Having said this, there is no unique realization of inflation dynamics in the context of high-energy physics or beyond the Standard Model (SM) of particle physics. For example, what is the nature of the inflaton field(s)? What mechanism keeps the inflationary potential flat enough to sustain a long enough period of inflation to solve the flatness and the horizon problems?

It is generally believed that there may exist many fields during inflation that can play some roles. If the fields are very heavy compared to the Hubble scale during inflation, then they are not expected to play important roles. However, if the fields are light or semiheavy, they can have nontrivial effects on cosmological observables such as the power spectrum and bispectrum; see, e.g., Refs. [3–5]. In addition, there is no reason that only scalar fields play important roles during inflation. Specifically, the gauge fields and vector fields are essential ingredients of the SM and any theory of high-energy physics. Therefore, it is quite natural to look for the imprints of the vector fields during inflation. One issue with the vector fields in the background is that they have preferred directions, so in general, models of inflation with background vector fields are anisotropic. The second issue with the vector fields is that, because of the conformal invariance, they are quickly diluted in an expanding background, so their effects become rapidly insignificant during inflation.

Anisotropic inflation is a model of inflation based on a $U(1)$ gauge field dynamics. To remedy the second issue mentioned above, the gauge kinetic coupling in these models is a function of the inflaton field, so the conformal invariance is broken. By choosing an appropriate form of the gauge kinetic coupling, the electric field energy density becomes nearly constant, so the gauge field survives the expansion until end of inflation [6]. In addition, the gauge field perturbations become nearly scale invariant and can take part in generating cosmological perturbations. In particular, quadrupolar statistical anisotropies are generated in these models, which can be observed in cosmic microwave background (CMB) maps. For various works on anisotropic inflation and their cosmological imprints, see Ref. [7].

The anisotropic inflation model [6] has been extended to the case in which the scalar field is charged under the $U(1)$

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gauge field in Refs. [8–10], while its isotropic realization containing a triplet of $U(1)$ gauge fields has been studied in Refs. [11,12]. In this work, we consider the isotropic extension of Ref. [6], in which the inflaton is charged under a triplet of $U(1)$ gauge fields. We show that the model has some interesting features such as containing the entropy mode in addition to the adiabatic mode and the fact that the gravitational tensor modes are sourced by the tensor modes coming from the gauge fields.

The rest of the paper is organized as follows. In Sec. II, we present our setup and study its background dynamics. In Sec. III, we study the cosmological perturbations in this setup, while the power spectra of the adiabatic and entropy perturbations and their cross-correlations are studied in Sec. IV. The tensor perturbations of the metric and the matter fields are studied in Sec. V, followed by the summaries and discussions in Sec. VI. The gauge symmetries of the setup are studied in the Appendix A, while the analysis of quadratic action is relegated to Appendix B.

II. SETUP AND BACKGROUND DYNAMICS

In this section, we introduce our setup, in which we extend the model of anisotropic inflation to the setup that can support an isotropic Friedmann–Robertson–Walker (FRW) solution. A realization of this was studied in Refs. [11,12], in which the model contains a triplet of $U(1)$ gauge fields with an additional global internal $O(3)$ symmetry. The internal $O(3)$ symmetry allows one to obtain the isotropic FRW solution [13]. In this work, we extend the setup of Refs. [11,12] to a model containing three complex scalar fields $\phi_{(a)}, a = 1, 2, 3$, charged under $U(1)_a$ gauge symmetry with gauge coupling $\mathbf{e}$. In a sense, our setup is the isotropic realization of the model of anisotropic charged inflation studied in Refs. [8–10].

A. Setup

We consider a model consisting of a triplet of $U(1)$ gauge fields that may be thought of as three independent copies of the $U(1)$ scalar electrodynamics. The desired gauge symmetry is $U(1)_a = U(1)_1 \times U(1)_2 \times U(1)_3$, and the scalar sector is defined by a triplet Φ,

$$\Phi = \begin{pmatrix} \phi_{(1)} \\ \phi_{(2)} \\ \phi_{(3)} \end{pmatrix}, \tag{2.1}$$

in which $\phi_{(a)}, a = 1, 2, 3$ are complex scalar fields that are charged under $U(1)_a$ gauge fields $\mathbf{A}^{\mu}_{(a)}$ with the covariant derivative denoted as

$$\mathbf{D}_\mu = \mathbf{1}\partial_\mu + i\mathbf{e}\mathbf{A}_\mu. \tag{2.2}$$

The gauge coupling constant $\mathbf{e}$ assigns the same charges to each scalar field.

Similarly to the original model of anisotropic inflation [6], the action of the model is given by

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}(\mathbf{D}_\mu\Phi)^\dagger(\mathbf{D}^\mu\Phi) - V(|\Phi|) - \frac{1}{4}f^2(|\Phi|)\mathrm{Tr}(\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu})\right], \tag{2.3}$$

where M_P is the reduced Planck mass, R is the Ricci scalar, $|\Phi| = \sqrt{\Phi^\dagger\Phi}$, V is the potential, f is the conformal factor, and $\mathbf{F}_{\mu\nu}$ is the field strength tensor defined in the spirit of the covariant derivative (2.2). To simplify the setup, we have assumed that V and f are only functions of the magnitude $|\Phi|$.

The details of the gauge symmetries of the model are presented in Appendix A. Gauge fields $A^{(a)}_\mu$ enjoy the associated $U(1)_a$ gauge symmetry for $a = 1$, 2, and 3. To fix the $U(1)_a$ gauge freedoms, we work in the gauge in which all scalar fields $\phi_{(a)}$ are real. In other words, we fix the $U(1)_a$ gauges by going to the unitary gauge in which the phases of the complex scalar field are set to zero. In addition, in order to obtain the isotropic FRW solution, similarly to the setup of Ref. [14], we consider a subset of the model in which $\phi_{(1)} = \phi_{(2)} = \phi_{(3)} \equiv \phi/\sqrt{3}$, where the kinetic term $(\mathbf{D}_\mu\Phi)^\dagger(\mathbf{D}^\mu\Phi)$ takes the isotropic form

$$(\mathbf{D}_\mu\Phi)^\dagger(\mathbf{D}^\mu\Phi) = \partial_\mu\phi\partial^\mu\phi + \frac{\mathbf{e}^2}{3}\phi^2 A^{(a)}_\mu A^\mu_{(a)}. \tag{2.4}$$

Putting these all together, the action (2.3) takes the following isotropic form:

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\mathbf{e}^2}{6}\phi^2 A^{(a)}_\mu A^\mu_{(a)} - V(\phi) - \frac{1}{4}f^2(\phi)F^{(a)}_{\mu\nu}F^{\mu\nu}_{(a)}\right]. \tag{2.5}$$

As expected, the action (2.5) has the same form as in models of anisotropic inflation [6], but the gauge fields here enjoy an additional internal $O(3)$ symmetry, admitting a FRW background solution. As in Ref. [6], the conformal coupling $f(\phi)$ will be chosen so as to prevent the dilution of the gauge field energy density in the inflationary background.

It is constructive to compare our model with the other inflationary models that are constructed by means of $U(1)$ gauge fields. The isotropic extension of the setup of anisotropic inflation[6] is suggested in Refs. [11,12] by means of a triplet of $U(1)$ gauge fields, while the charged extension of Ref. [6] is considered in Ref. [8]. The model considered in Refs. [11,12] has local $U(1)_a$ symmetry, while it enjoys global $O(3)$ symmetry. In this work, we have constructed the charged isotropic extension of anisotropic inflation [6]. In other words, our model is the charged generalization of Refs. [11,12] and isotropic extension of Ref. [8].

With the above discussions in mind, our setup with the action (2.5) has similarities with the model studied in Ref. [15], in which the authors extended the setup of anisotropic inflation to a model in which the inflaton field is coupled to a $SU(2)$ gauge kinetic function. In a sense, the model considered in Ref. [15] can be thought of as the charged extension of Refs. [11,12]. The authors in Ref. [15] studied the background dynamics, verifying the existence of the attractor solution and studying the shapes of anisotropies.

It is worth mentioning that we can achieve the isotropic setup with more than two gauge fields [16], so having three gauge fields is the minimal setup that we have considered in this paper. Moreover, as was mentioned above, this case can be thought of as the global limit of non-Abelian gauge field models [15].

B. Background equations

Since the action (2.5) is $O(3)$ invariant, the model admits the flat FRW cosmological background

$$ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j, \tag{2.6}$$

with the ansatz [13]

$$A_\mu^{(a)}(t) = A(t)\delta_\mu^a. \tag{2.7}$$

The model behaves like three mutually orthogonal gauge fields with $U(1)_a$ gauge symmetry, and the ansatz (2.7) assigns the same magnitudes $A(t)$ to each gauge field [17]. Note that the ansatz (2.7) is not the only solution. Indeed, one can imagine a situation in which the initial amplitudes of the gauge fields are not equal to each other, $A_\mu^{(a)}(t) \neq A_\mu^{(b)}(t)$ for $a \neq b$. In this case, the spacetime metric will be in the form of a Bianchi type I Universe. However, as shown in Ref. [18], one expects the isotropic FRW background to be the attractor solution of the system so the spacetime rapidly approaches the FRW background and the gauge field amplitudes become equal. In addition, it is shown in Ref. [16], see also Ref. [17], that with a large multiplet of $U(1)$ gauge fields and with the appropriate form of the conformal factor $f(\phi)$ the FRW solution is the attractor limit of arbitrary initial conditions with background anisotropies.

Varying the action (2.5) with respect to the gauge fields, we obtain the associated Maxwell equation

$$\partial_t(f^2 a\dot{A}) = -\frac{1}{3}\mathbf{e}^2\phi^2 aA, \tag{2.8}$$

where a dot indicates a derivative with respect to the cosmic time t.

The variation of the action (2.5) with respect to the scalar field gives the Klein-Gordon equation

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = \left(3ff_{,\phi}\dot{A}^2 - \frac{1}{3}\mathbf{e}^2\phi A^2\right)a^{-2}, \tag{2.9}$$

where $_{,\phi}$ denotes the derivative with respect to the scalar field. Note the important effects of the gauge field backreactions on the scalar field as captured by the source term on the right-hand side of the above equation.

Finally, the corresponding Einstein equations are

$$3M_P^2H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{3f^2\dot{A}^2}{2a^2} + \frac{\mathbf{e}^2\phi^2A^2}{6a^2}, \tag{2.10}$$

$$M_P^2(2\dot{H} + 3H^2) = -\left(\frac{\dot{\phi}^2}{2} - V + \frac{f^2\dot{A}^2}{2a^2} - \frac{\mathbf{e}^2A^2\phi^2}{6a^2}\right). \tag{2.11}$$

The right-hand side of Eq. (2.10) is the total energy density, while the expression in the parentheses on the right-hand side of Eq. (2.11) is the total pressure. In the absence of $\mathbf{e}$, from the above relations, we see that the pure gauge field contributions behave like radiation thanks to the conformal symmetry. Let us consider the effects of the gauge coupling $\mathbf{e}$. We see from the second term on the right-hand side of Eq. (2.9) that the interaction $\mathbf{e}^2\phi^2A^2a^{-2}$ induces a time-dependent mass for the inflaton. However, the exponential time dependence of this induced mass makes its main effect occur toward the end of inflation when the exponential growth of the gauge field has its main influence. Thus, to have a long enough period of inflation, the backreaction $\mathbf{e}^2\phi^2A^\mu A_\mu$ is negligible during much of the period of inflation, and it only controls the mechanism of the end of inflation [8,9]. In this approximation, one can easily solve the Maxwell equation (2.8) to obtain

$$\dot{A} = \frac{q_0}{a}f^{-2}, \tag{2.12}$$

where q_0 is an integration constant.

Now, as in the anisotropic inflation model [6], it is convenient to define the ratio of the energy density of the gauge fields to the energy density of the inflaton field as

$$R \equiv \frac{\rho_A}{\rho_\phi} = \frac{3q_0^2}{2V + \dot{\phi}^2}a^{-4}f^{-2}. \tag{2.13}$$

To obtain a long period of inflation with a de Sitter–like background, we expect the contribution of the gauge field to the total energy density to be small. This is because, as just mentioned above, the gauge fields' contributions are like radiation and cannot support inflation by themselves. In other words, as in conventional models of slow-roll inflation, we expect that inflation to be driven predominantly by the scalar field. As a result, we require $R \ll 1$ in order to obtain a long period of inflation.

The dynamics of the background is very similar to the setup of anisotropic inflation. During the early stage of inflation, the gauge fields do not drag enough energy from the inflaton field, so the parameter R is much smaller than the slow-roll parameters. In this limit, we can safely neglect the contributions of the gauge fields in the total energy density and pressure and solve the system as in single-field slow-roll models with

$$3M_P^2H^2 \simeq V, \qquad 3H\dot{\phi} \simeq -V_{,\phi}. \tag{2.14}$$

Therefore, in the slow-roll limit, and for a given potential $V(\phi)$, the above equations provide the solution

$$a \simeq \exp\left(-\frac{1}{M_P^2}\int_{\phi_i}^{\phi}\frac{V}{V_{,\phi}}d\phi\right). \tag{2.15}$$

Now, as inflation proceeds, the gauge fields drag more and more energy from the inflaton field via the conformal coupling $f(\phi)$. As shown in Ref. [6], the system reaches an attractor limit in which the fraction of the gauge field energy density to total energy density reaches a constant value. During the attractor stage, R becomes the order of the slow-roll parameter, and it stays nearly constant until end of inflation.

For R to reach a constant value, from Eq. (2.13), one must choose $f(\phi) \propto a(t)^{-2}$. Therefore, it is reasonable to assume that

$$f(\phi) = \exp\left(\frac{2c}{M_P^2}\int\frac{V}{V_{,\phi}}d\phi\right), \tag{2.16}$$

with a constant parameter c.

As the roles of the gauge fields become important, they backreact on the inflaton dynamics as given by the source term in Eq. (2.9). Taking into account the backreactions of the gauge fields on the inflationary trajectory fixes the relation between R and the slow-roll parameter ϵ.

The scalar field equation in the slow-roll limit is given by

$$3H\dot{\phi} = \frac{3q_0^2 f_{,\phi}}{a^4 f^3} - V_{,\phi}. \tag{2.17}$$

Using Eqs. (2.14) and (2.17), we obtain the following equation for ϕ in terms of the number of e-folds $\ln a = N$ (setting $M_P = 1$ for simplicity):

$$\phi\frac{d\phi}{dN} = -\frac{V_{,\phi}}{V} + \frac{6q_0^2c}{V_{,\phi}}e^{-4c\int(V/V_{,\phi})d\phi}e^{-4N}. \tag{2.18}$$

Now, it is suitable to rearrange Eq. (2.18) into the following form:

$$4ce^{4N}e^{4c\int(V/V_{,\phi})d\phi}\left(1 + \frac{V}{V_{,\phi}}\frac{d\phi}{dN}\right) = 24c^2{q_0}^2\left(\frac{V}{V_{,\phi}^2}\right). \tag{2.19}$$

Defining $\mathcal{G}(N) \equiv e^{4N}e^{4c\int(V/V_{,\phi})d\phi}$, the above equation takes the following form:

$$\frac{d\mathcal{G}}{dN} + 4(c-1)\mathcal{G} = 24c^2{q_0}^2\left(\frac{V}{V_{,\phi}^2}\right). \tag{2.20}$$

One can solve this differential equation in the slow-roll limit to obtain

$$\mathcal{G}(N) = \frac{6c^2q_0^2\mathcal{C}}{(c-1)}\left(\frac{V}{V_{,\phi}^2}\right)\left[1 + \frac{6c^2q_0^2\mathcal{C}}{(c-1)}\left(\frac{V}{V_{,\phi}^2}\right)e^{4N(1-c)}\right], \tag{2.21}$$

where $\mathcal{C}$ is a constant of integration. We see that for sufficiently small values of $q_0^2\mathcal{C}$ the last term in the above bracket falls off during inflation, and Eq. (2.21) implies

$$\mathcal{G}(N)^{-1} = e^{-4N}e^{-4c\int\frac{V}{V_{,\phi}}d\phi} = \frac{(c-1)}{6c^2q_0^2}\left(\frac{V_{,\phi}^2}{V}\right). \tag{2.22}$$

Consequently, ρ_A becomes nearly constant during the second phase of inflation, and after straightforward calculations, we obtain

$$R = \frac{c-1}{4c^2}\left(\frac{V_{,\phi}}{V}\right)^2. \tag{2.23}$$

Substituting Eq. (2.22) into the modified slow-roll equation (2.17), we obtain

$$3H\dot{\phi} \approx -\frac{V_{,\phi}}{c}. \tag{2.24}$$

This shows that during the second phase of inflation the effective mass squared of the inflaton field m^2 is reduced by the factor $1/c$ compared to the first stage of inflation [6].

Moreover, from Eqs. (2.10), (2.11), and (2.22), one can also obtain the slow-roll parameter as follows:

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{1}{2c}\left(\frac{V_{,\phi}}{V}\right)^2. \tag{2.25}$$

Therefore, we find

$$R = \frac{c-1}{2c}\epsilon = \frac{I}{2}\epsilon, \tag{2.26}$$

in which we have defined the parameter $I \equiv (c-1)/c$. Interestingly, the relation between R and ϵ given in Eq. (2.26) is the same as in anisotropic inflation.

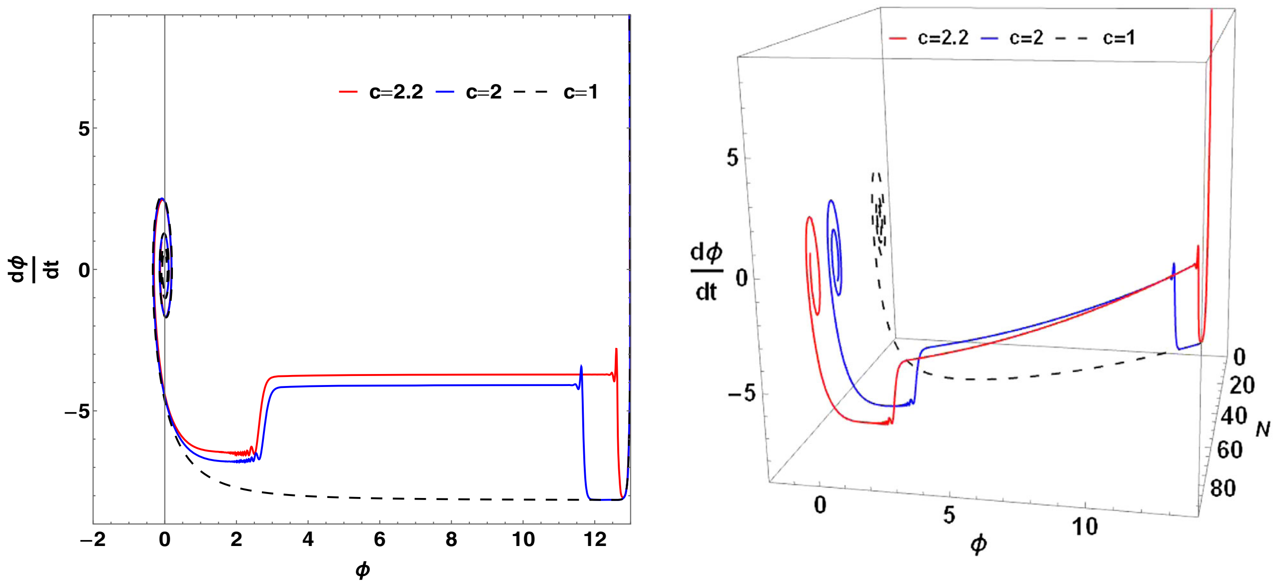

FIG. 1. Left: The phase space plot of $(\phi, \dot{\phi})$ for the potential $V = \frac{1}{2}m^2\phi^2$ with parameters $m = 10^{-6}M_P$, $\phi(0) = 12M_P$, and $\dot{\phi}(0) = 0$. We have fixed $\mathbf{e} = 0.01$ and varied the parameter c with three values $c = 1, 2$, and 2.2. The latter two values of c are too large to generate a scale-invariant power spectrum, but we have chosen them for better visualizations of the effects of gauge fields on inflation dynamics. Right: The three-dimensional plot of $(\phi, \dot{\phi})$ with respect to N for the same parameters as in the left figure.

In the left panel of Fig. 1, the phase space plot of $(\phi, \dot{\phi})$ for the potential $V = \frac{1}{2}m^2\phi^2$ for a fixed value of $\mathbf{e}$ and for three different values of c is plotted. In the right panel of Fig. 1 the behavior of $(\phi, \dot{\phi})$ as a function of the number of e-folds N is plotted. As we see from the plots, initially, the inflaton field evolves independently of the effects of the gauge field, so all three curves coincide during the first phase of inflation. However, as the gauge fields drag enough energy from the background, they kick in, and after a short transient period, the system reaches the attractor phase. The attractor phase starts sooner for the larger value of c. This is understandable, since the larger the value of c is the more energy is pumped into the gauge field from the inflaton field. We also see that the attractor phase is longer and the total number of e-folds is higher for larger values of c. This can be seen from our equations, too. Starting from $N = -\int \frac{H}{\dot{\phi}} d\phi$, and using Eq. (2.24), we obtain $N = -c\int \frac{V}{V_{,\phi}} d\phi$. Therefore, the total number of e-folds increases by increasing the value of c.

In Fig. 2, the phase space plot of $(\phi, \dot{\phi})$ (left panel) and its dependence on N (right panel) are plotted for the same potential as in Fig. 1, but this time, c is held fixed, while $\mathbf{e}$ is varied. As can be seen from the plots, $\mathbf{e}$ does not play important roles during much of the period of inflation. However, its effect becomes important during the final stage of inflation, modifying the total number of e-folds slightly. More specifically, the coupling $\mathbf{e}$ induces an effective mass $m^2 \sim \mathbf{e}^2 A^2 e^{-2N}$ for the inflaton field. When this induced mass becomes comparable to H, then the slow-roll conditions are violated, and inflation ends abruptly. During the attractor phase, $A \propto e^{(4c-1)N}$, so the induced mass scales like $\mathbf{e}^2 e^{(8c-4)N}$. Consequently, the total number of e-folds depends only logarithmically on $\mathbf{e}$. In other words, holding other parameters such as c fixed while varying $\mathbf{e}$, as in Fig. 2, the total number of e-folds changes as

$$\Delta N \sim -\frac{1}{2(2c-1)} \ln \mathbf{e}. \tag{2.27}$$

Although $\mathbf{e}$ does not play important roles during the inflation background, it has important effects on curvature perturbations power spectra and other cosmological observables.

III. COSMOLOGICAL PERTURBATIONS

In this section, we present the perturbations of our model based on action (2.5). From now on, we work with the conformal time τ defined as $d\tau = dt/a(t)$.

The metric perturbations around the background geometry (2.6) are given by

$$\delta g_{00} = 2a^2\alpha, \qquad \delta g_{0i} = a^2(\partial_i\beta + B_i),$$
$$\delta g_{ij} = a^2(2\psi\delta_{ij} + 2\partial_i\partial_j E + \partial_i F_j + \partial_j F_i + h_{ij}), \tag{3.1}$$

where α, β, ψ, and E are scalar modes; B_i and F_i are vector modes; and h_{ij} are the tensor perturbations that satisfy the following transverse and traceless conditions:

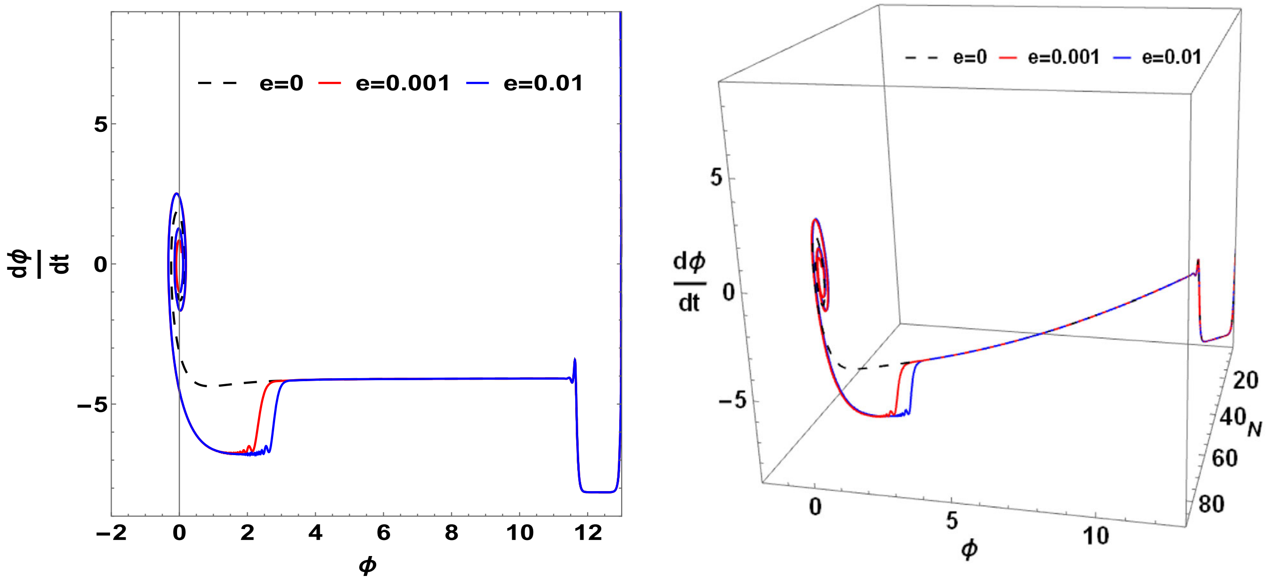

FIG. 2. Left: The phase space plot of $(\phi, \dot{\phi})$ with c held fixed at $c = 2$, while varying $\mathbf{e}$ with $\mathbf{e} = 0, 0.001$, and 0.01. Other parameters are the same as in the top figures. Right: The three-dimensional plot of $(\phi, \dot{\phi})$ with respect to N for the same parameters as in left figure.

$$\partial_i B_i = \partial_i F_i = \partial_i h_{ij} = h_{ii} = 0. \tag{3.2}$$

The gauge fields enjoy internal $O(3)$ symmetry, and the perturbations should be defined in the spirit of $O(3)$ symmetry as [13]

$$\delta A_0^{(a)} = Y_a + \partial_a Y,$$
$$\delta A_i^{(a)} = \delta Q \delta_{ia} + \partial_i(\partial_a M + M_a) + \epsilon_{iab}(\partial_b U + U_b) + t_{ia}, \tag{3.3}$$

where $(Y, \delta Q, M, U)$ are scalar modes, (Y_a, M_a, U_a) are vector modes, and (t_{ia}) label the tensor modes associated with the gauge field perturbations that are subject to the transverse and traceless conditions

$$\partial_i Y_i = \partial_i M_i = \partial_i U_i = \partial_i t_{ij} = t_{ii} = 0. \tag{3.4}$$

In addition to the above perturbations, we also have the inflaton perturbations $\delta\phi$.

The gauge freedom associated with the four-dimensional diffeomorphism invariance fixes two scalar modes and two vector modes of metric perturbations. For the scalar modes, we work in the spatially flat gauge in which

$$\psi = 0, \qquad E = 0, \tag{3.5}$$

while for the vector perturbations, we fix the gauge by setting $F_i = 0$.

Apart from the diffeomorphism invariance, the gauge fields enjoy the $U(1)_a$ gauge invariance given by Eq. (A7). But we have already fixed the $U(1)_a$ gauge in choosing the scalar fields to be real, i.e., going to the unitary gauge, yielding to the action (2.5).

In summary, after fixing the gauges associated with the diffeomorphism invariance and local $U(1)_a$ invariance, we have seven scalar degrees of freedom (d.o.f.) $(\alpha, \beta, \delta\phi, \delta Q, Y, U, M)$, eight vector d.o.f. (B_i, U_a, Y_a, M_a), and four tensor perturbations (h_{ij}, t_{ij}). In total, we have 19 physical d.o.f.

Since the model with the action (2.5) enjoys $O(3)$ symmetry, the scalar, vector, and tensor perturbations decouple at the linear order of perturbations. Moreover, since our setup is isotropic, the vector perturbations decay as usual in an expanding Universe, and we will not consider them from now on.

IV. SCALAR PERTURBATIONS

Working in the spatially flat gauge (3.5) and fixing local gauge symmetry (A7), we deal with seven scalar modes $(\alpha, \beta, Y, \delta Q, U, \delta\phi, M)$. Direct calculations shows that α and β appear with no time derivatives in the quadratic action, and therefore they can be substituted from their algebraic equations of motion. Moreover, the contributions coming from these nondynamical modes are slow-roll suppressed [9,10], and we therefore neglect them.

The quadratic action for the remaining modes $(Y, \delta Q, U, \delta\phi, M)$ is presented in Appendix B. As discussed there, the contributions of the perturbations Y and M are suppressed during much of the period of inflation and therefore can be neglected. Therefore, the quadratic action for the remaining light scalar perturbations in Fourier space is given by

$$S^{(2)} = \frac{1}{2}\int d\tau d^3k\Big\{\delta Q_c'^2 - \left(k^2 - \frac{2}{\tau^2}\right)\delta Q_c^2 + \delta\phi_c'^2 - \left[k^2 - \frac{1}{\tau^2}(2+4I)\right]\delta\phi_c^2 + U_c'^2 - \left(k^2 - \frac{2}{\tau^2}\right)U_c^2 + 8\frac{\sqrt{I}}{\tau^2}\left[2 - \frac{\mathbf{e}^2}{9H^2}\left(\frac{\tau_e}{\tau}\right)^4\right]\delta Q_c\delta\phi_c - \frac{8\sqrt{I}}{\tau}\delta Q_c'\delta\phi_c\Big\}, \tag{4.1}$$

in which a prime indicates the derivative with respect to the conformal time, τ_e is the time of the end of inflation, and we have defined the canonically normalized fields

$$\delta Q_c \equiv \sqrt{2}f\delta Q, \qquad U_c \equiv kfU, \qquad \delta\phi_c \equiv a\delta\phi. \tag{4.2}$$

We have ignored pure slow-roll corrections, i.e., terms containing the slow-roll parameters ϵ and its derivative without the factor I since they are the same as those coming from the gravitational backreactions and can be absorbed into the power spectrum in the absence of gauge fields. In addition, as we shall show later on, $I \ll 1$, so we have kept the leading terms of I in the action (4.1), which turns out to be proportional to $\sqrt{I}$.

Form the action (4.1), we see that the field U is decoupled from the other fields. In addition, it did not exist at the background level. Therefore, the field U is a pure isocurvature mode. This is unlike the mode δQ, which is the perturbations associated with the diagonal component of $A_i^{(a)}$, which also had a background component, given in Eq. (2.7). We see that both the scalar field and the diagonal component of $A_i^{(a)}$ contribute to the background energy and interact with each other. In this view, we are dealing with a multiple field model of inflation that is studied vastly in the literature. In particular, similar to the logic of Ref. [19], we expect a combination of the fields $(\delta\phi, \delta Q)$ to play the roles of the adiabatic mode and a different combination to play the role of the entropy perturbations.

A. Adiabatic and entropy decompositions

To find the adiabatic and entropy modes, we first find the comoving curvature perturbations $\mathcal{R}$ from the standard definition

$$\mathcal{R} = \psi + H\delta u, \tag{4.3}$$

where ψ measures the spatial curvature and δu is the velocity potential, which is defined as $\delta T^t_i = (\rho + p)\partial_i\delta u$. Calculating the energy-momentum tensor at the linear order of perturbations, and noting that we work in a spatially flat gauge, Eq. (3.5), the comoving curvature perturbation takes the following form:

$$\mathcal{R} = -aH\frac{\sqrt{2}fA'\delta Q_c + a\phi'\delta\phi_c + (\mathbf{e}^2/9)a^2A\phi^2Y}{2f^2A'^2 + a^2\phi'^2}. \tag{4.4}$$

We need to substitute the nondynamical perturbation Y in the above relation from Eq. (B2). As discussed in Appendix B, the contribution of Y in curvature perturbation is subleading during the inflationary stage. Therefore, to leading order, the curvature perturbation takes the following simple form:

$$\mathcal{R} = -\frac{H}{\phi'}[(1-I)\delta\phi_c - \sqrt{I}\delta Q_c]. \tag{4.5}$$

The above formula is interesting, showing that the contribution of each field into the total curvature perturbation is weighted by the fraction of the corresponding field into the total energy density [20,21]. Since $I \ll 1$, the dominant contribution into curvature perturbations is given by the inflaton field perturbations $\delta\phi$. But we expect to have subleading contributions from the diagonal component of $A_i^{(a)}$, which is given by the fraction $\sqrt{I}$ in the above formula.

Following the logic of Ref. [19], the scalar modes $\delta\phi_c$ and δQ_c can be decomposed into the adiabatic and entropy components as

$$\delta\sigma_c = \cos\theta\delta\phi_c + \sin\theta\delta Q_c, \tag{4.6}$$

$$\delta s_c = -\sin\theta\delta\phi_c + \cos\theta\delta Q_c, \tag{4.7}$$

where we have defined

$$\cos\theta \equiv \sqrt{1-I}, \qquad \sin\theta \equiv -\sqrt{I}. \tag{4.8}$$

The canonical variables $\delta\sigma_c$ and δs_σ are related to the standard adiabatic and entropy perturbations defined in Ref. [19] via

$$\delta\sigma_c = a\delta\sigma \qquad \delta s_c = a\delta s. \tag{4.9}$$

Using the decomposition defined in Eq. (4.6) into Eq. (4.5), the comoving curvature perturbations are given by

$$\mathcal{R} = -\frac{H}{\dot\phi}\cos\theta\delta\sigma. \tag{4.10}$$

In the limit $I \to 0$, we have $\cos\theta = 1$, and Eq. (4.6) gives $\delta\sigma = \delta\phi$, in which we find the well-known result $\mathcal{R} = -\frac{H}{\dot\phi}\delta\phi$ for the curvature perturbations.

Correspondingly, we define the associated normalized entropy perturbation via

$$\mathcal{S} \equiv -\frac{H}{\dot{\phi}} \cos\theta \delta s. \tag{4.11}$$

Our final aim is to find the power spectrum for the observable quantities $\mathcal{R}$ and $\mathcal{S}$. For this purpose, we rewrite the quadratic action (4.1) in terms of the adiabatic and entropy modes, yielding

$$\begin{aligned} S^{(2)} = \frac{1}{2}\int d\tau d^3k \bigg\{ & U_c'^2 - \left(k^2 - \frac{2}{\tau^2}\right) U_c^2 \\ & + \delta s_c'^2 - \left[k^2 - \frac{2}{\tau^2}\left(1 + 6I - \frac{4\mathbf{e}^2 I}{9H^2}\left(\frac{\tau_e}{\tau}\right)^4\right)\right]\delta s_c^2 \\ & + \delta\sigma_c'^2 - \left[k^2 - \frac{2}{\tau^2}\left(1 - 4I + \frac{4\mathbf{e}^2 I}{9H^2}\left(\frac{\tau_e}{\tau}\right)^4\right)\right]\delta\sigma_c^2 \\ & + \frac{8\sqrt{I}}{\tau^2}\left[2 - \frac{\mathbf{e}^2}{9H^2}\left(\frac{\tau_e}{\tau}\right)^4\right]\delta s_c \delta\sigma_c - \frac{8\sqrt{I}}{\tau}\delta s_c' \delta\sigma_c \bigg\}. \end{aligned} \tag{4.12}$$

We see that the adiabatic and entropy modes are coupled to each other with the couplings proportional to $\sqrt{I}$.

We calculate the power spectra of $\mathcal{P}_\mathcal{R}$ and $\mathcal{P}_\mathcal{S}$ and their cross-correlation $\mathcal{P}_{\mathcal{RS}}$ in the next subsections. However, before that, let us consider the perturbation U, which is a pure isocurvature mode and does not couple to other modes. Decomposing U into the creation and the annihilation operators with the Minkowski (Bunch-Davies) initial condition, we have

$$U_c(\mathbf{k}) = u(k) a_\mathbf{k} + u^*(k) a^\dagger_{-\mathbf{k}}; \qquad u(k) = \frac{i e^{-ik\tau}}{\sqrt{2k^3}\tau}(1 + ik\tau).$$

Correspondingly, the dimensionless power spectrum for $U = U_c/a$, defined as usual via $\langle U^\dagger(\tau, \mathbf{k}) U(\tau, \mathbf{k}')\rangle \equiv \frac{2\pi^2}{k^3}\mathcal{P}_U (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}')$, on superhorizon scales is given by

$$\mathcal{P}_U = \left(\frac{H}{2\pi}\right)^2. \tag{4.13}$$

The above result shows that the scalar mode U behaves like an spectator field with the amplitude $H/2\pi$.

B. Curvature perturbations power spectrum

In this subsection, we calculate the curvature perturbation power spectrum $\mathcal{P}_\mathcal{R}$. From Eq. (4.10), the power spectrum of curvature perturbation at the end of inflation τ_e is given by

$$\begin{aligned} \langle \mathcal{R}^\dagger(\tau_e, \mathbf{k}) \mathcal{R}(\tau_e, \mathbf{k}')\rangle &= \left(\frac{H}{\dot{\phi}}\right)^2 \cos^2\theta \langle \delta\sigma^\dagger \delta\sigma\rangle \\ &\equiv \frac{2\pi^2}{k^3}\mathcal{P}_\mathcal{R}(2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}'). \end{aligned} \tag{4.14}$$

The leading contribution to the curvature perturbation power spectrum comes from the adiabatic mode $\delta\sigma$. However, the adiabatic and the entropy modes are coupled to each other with the interactions given by the last two terms in the action (4.12). Therefore, we also have to calculate the corrections from the entropy mode in $\mathcal{P}_\mathcal{R}$. Since we assume $I \ll 1$, this analysis can be done perturbatively using the standard in-in formalism [22].

The two-point function for the adiabatic mode is then given by

$$\begin{aligned} \langle \delta\sigma^2(\tau_e)\rangle = & \langle 0| \left[\bar{T}\exp\left(i\int_{\tau_0}^{\tau_e} H_I(\tau'')d\tau''\right)\right] \\ & \times \delta\sigma(\tau_e)^2 \left[T\exp\left(-i\int_{\tau_0}^{\tau_e} H_I(\tau')d\tau'\right)\right]|0\rangle \\ = & \langle 0|\delta\sigma^2|0\rangle + i\langle 0|\int_{\tau_0}^{\tau_e} d\tau_1 [H_I(\tau_1), \delta\sigma^2(\tau_e)]|0\rangle \\ & - \langle 0|\int_{\tau_0}^{\tau_e} d\tau_1 \int_{\tau_0}^{\tau_1} d\tau_2 [H_I(\tau_2), [H_I(\tau_1), \delta\sigma^2(\tau_e)]]|0\rangle \\ & + \cdots, \end{aligned} \tag{4.15}$$

where $\bar{T}$ and T are the time-ordered and anti–time ordered operators and H_I is the interaction Hamiltonian. The integrals are taken from the initial time $\tau_0 \to -\infty$ when the modes are deep inside the horizon to the end of inflation $\tau_e \to 0$. The first term in the second line of Eq. (4.15) is the two-point function of the adiabatic mode in the absence of interaction determined by the free action of $\delta\sigma$ in Eq. (4.12). This gives the leading contribution to the curvature perturbations power spectrum, denoted by $\mathcal{P}_\mathcal{R}^{(0)}$, which is given by

$$\mathcal{P}_\mathcal{R}^{(0)} = \frac{H^2}{8\pi^2 \epsilon M_P^2}. \tag{4.16}$$

In obtaining the above result, we have substituted $\langle 0|\delta\sigma^2|0\rangle = H^2/2k^3$ and $(\frac{H}{\dot{\phi}})^2 \cos^2\theta = 1/2\epsilon$. To be more precise, from Eqs. (2.24) and (2.25), we find $(\frac{H}{\dot{\phi}})^2 \simeq (1 + I)/2\epsilon$. On the other hand, from Eq. (4.8), we find that $\cos^2\theta = 1 - I$ and therefore $(\frac{H}{\dot{\phi}})^2 \cos^2\theta = 1/(2\epsilon) + \mathcal{O}(I^2)$.

To calculate the corrections in the curvature perturbations power spectrum, we need to obtain the interaction Hamiltonians. In addition to the two interactions that directly couple the fields $\delta\sigma$ and δs [the last line in action (4.12) containing $\sqrt{I}$], we also have new interactions in the action from the second and third lines of Eq. (4.12) containing I. Note that we treat I as the parameter of the perturbations, so any term containing this parameter should be treated as interaction compared to the free theory. In total, we have seven interaction Hamiltonians for the scalar perturbations, $H_I^s = \sum_i^7 H_i^s$ with

$$H_1^s = -\frac{8\sqrt{I}}{\tau^2}\delta\sigma_c\delta s_c, \qquad H_2^s = \frac{4\sqrt{I}}{\tau}\delta\sigma_c\delta s_c',$$
$$H_3^s = \frac{4\mathbf{e}^2\sqrt{I}}{9H^2}\left(\frac{\tau_e^4}{\tau^6}\right)\delta\sigma_c\delta s_c, \qquad H_4^s = \frac{12I}{\tau^2}\delta\sigma_c^2,$$
$$H_5^s = -\frac{6I}{\tau^2}\delta s_c^2, \qquad H_6^s = -\frac{4\mathbf{e}^2 I}{9H^2}\left(\frac{\tau_e^4}{\tau^6}\right)\delta\sigma_c^2,$$
$$H_7^s = \frac{4\mathbf{e}^2 I}{9H^2}\left(\frac{\tau_e^4}{\tau^6}\right)\delta s_c^2. \tag{4.17}$$

Note that, because of the kinetic coupling $\delta\sigma\delta s'$, the interaction Hamiltonian is not simply $-L_I$. One has to calculate the conjugate momenta p_j corresponding to each field $\delta q_j = \{\delta\sigma, \delta s\}$ and then construct the Hamiltonian using the standard formula $H = \sum_i p_j \delta q_j' - L$. Doing this, we find that the interactions containing $\delta\sigma^2$ and δs^2 receive additional contributions compared to what one may naively construct using $H_I = -L_I$.

Let us denote the correction induced from the interactions to the adiabatic mode correlation by $\Delta\langle\delta\sigma^2\rangle$. Looking at Eq. (4.15), there are two possible ways for the interaction Hamiltonians to contribute in $\Delta\langle\delta\sigma^2\rangle$. If the contribution comes from the single Hamiltonian from the second line of Eq. (4.15), we denote it by $\Delta^{(1)}\langle\delta\sigma^2\rangle_i$; i.e., it is linear in H_i^s. On the other hand, if the contribution comes from the nested integral containing two Hamiltonians in third line of Eq. (4.15), then we denote it by $\Delta^{(2)}\langle\delta\sigma^2\rangle_{ij}$, in which the indices i and j are for $H_i^s(\tau_1)$ and $H_j^s(\tau_2)$, respectively.

The free wave function for $M_{i\mathbf{k}} = \{\delta\sigma_c(k), \delta s_c(k)\}$ with the Bunch-Davies initial condition, is given by

$$M_{i\mathbf{k}} = v(k)a_{i\mathbf{k}} + v(k)^\star a_{i-\mathbf{k}}^\dagger;$$
$$v(k) = \frac{ie^{-ik\tau}}{\sqrt{2k^3}\tau}(1+ik\tau). \tag{4.18}$$

To simplify the notation, let us pull out the factor $(2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{k}')$ and denote the corresponding correlations by Δ'. Then, the leading-order corrections in $\Delta\langle\delta\sigma^2\rangle$ are obtained as

$$\begin{aligned}\Delta'^{(1)}\langle\delta\sigma_c^2\rangle_4 &= i\int_{\tau_0}^{\tau_e} d\tau_1 [H_4(\tau_1), \delta\sigma_c^2(\tau)] \\ &= -48I\,\mathrm{Re}\left[i\int_{\tau_0}^{\tau_e} d\tau_1 \left(\frac{1}{\tau_1}\right)^2 (v(\tau_1)v^\star(\tau_e))^2\right] \\ &= \frac{8IN_e}{k^3\tau_e^2},\end{aligned} \tag{4.19}$$

$$\begin{aligned}\Delta'^{(2)}\langle\delta\sigma_c{}^2\rangle_{11} &= 512I\int_{\tau_0}^{\tau_e} d\tau_1 \int_{\tau_0}^{\tau_1} d\tau_2 \left(\frac{1}{\tau_1\tau_2}\right)^2 \\ &\quad\times \mathrm{Im}[v(\tau_1)v^\star(\tau_e)] \\ &\quad\times \mathrm{Im}[v(\tau_2)\bar{v}^\star(\tau_e)v(\tau_2)v^\star(\tau_1)] \\ &= \frac{64IN_e^2}{9k^3\tau_e^2},\end{aligned} \tag{4.20}$$

$$\begin{aligned}\Delta'^{(2)}\langle\delta\sigma_c{}^2\rangle_{12} &= -256I\int_{\tau_0}^{\tau_e} d\tau_1 \int_{\tau_0}^{\tau_1} d\tau_2 \left(\frac{1}{\tau_1^2\tau_2}\right) \\ &\quad\times \mathrm{Im}[v(\tau_1)v^\star(\tau_e)] \\ &\quad\times \mathrm{Im}[v(\tau_2)v^\star(\tau_e)v'(\tau_2)v^\star(\tau_1)] \\ &= -\frac{16IN_e^2}{9k^3\tau_e^2},\end{aligned} \tag{4.21}$$

$$\begin{aligned}\Delta'^{(2)}\langle\delta\sigma_c{}^2\rangle_{21} &= -256I\int_{\tau_0}^{\tau_e} d\tau_1 \int_{\tau_0}^{\tau_1} d\tau_2 \left(\frac{1}{\tau_1\tau_2^2}\right) \\ &\quad\times \mathrm{Im}[v(\tau_1)v^\star(\tau_e)] \\ &\quad\times \mathrm{Im}[v(\tau_2)v^\star(\tau_e)v(\tau_2)v'^\star(\tau_1)] \\ &= \frac{32IN_e^2}{9k^3\tau_e^2},\end{aligned} \tag{4.22}$$

$$\begin{aligned}\Delta'^{(2)}\langle\delta\sigma_c{}^2\rangle_{22} &= 128I\int_{\tau_0}^{\tau_e} d\tau_1 \int_{\tau_0}^{\tau_1} d\tau_2 \left(\frac{1}{\tau_1\tau_2}\right) \\ &\quad\times \mathrm{Im}[v(\tau_1)v^\star(\tau_e)] \\ &\quad\times \mathrm{Im}[v(\tau_2)v^\star(\tau_e)v'(\tau_2)v'^\star(\tau_1)] \\ &= -\frac{8IN_e^2}{9k^3\tau_e^2},\end{aligned} \tag{4.23}$$

$$\begin{aligned}\Delta'^{(2)}\langle\delta\sigma_c{}^2\rangle_{33} &= \frac{128I\mathbf{e}^4}{81H^4}\int_{\tau_0}^{\tau_e} d\tau_1 \int_{\tau_0}^{\tau_1} d\tau_2 \left(\frac{\tau_e^8}{\tau_1^6\tau_2^6}\right) \\ &\quad\times \mathrm{Im}[v(\tau_1)v^\star(\tau_e)] \\ &\quad\times \mathrm{Im}[v(\tau_2)v^\star(\tau_e)v(\tau_2)v^\star(\tau_1)] \\ &= \frac{I\mathbf{e}^4}{4851H^4k^3\tau_e^2},\end{aligned} \tag{4.24}$$

$$\begin{aligned}\Delta'^{(2)}\langle\delta\sigma_c{}^2\rangle_{31} &= -\frac{256I\mathbf{e}^2}{9H^2}\int_{\tau_0}^{\tau_e} d\tau_1 \int_{\tau_0}^{\tau_1} d\tau_2 \left(\frac{\tau_e^4}{\tau_1^6\tau_2^2}\right) \\ &\quad\times \mathrm{Im}[v(\tau_1)v^\star(\tau_e)] \\ &\quad\times \mathrm{Im}[v(\tau_2)v^\star(\tau_e)v(\tau_2)v^\star(\tau_1)] \\ &= -\frac{16I\mathbf{e}^2N_e}{189k^3H^2\tau_e^2},\end{aligned} \tag{4.25}$$

$$\Delta'^{(2)}\langle\delta\sigma_c{}^2\rangle_{32} = \frac{128I\mathbf{e}^2}{9H^2}\int_{\tau_0}^{\tau_e} d\tau_1 \int_{\tau_0}^{\tau_1} d\tau_2 \left(\frac{\tau_e^4}{\tau_1^6\tau_2}\right) \times \mathrm{Im}[v(\tau_1)v^\star(\tau_e)] \times \mathrm{Im}[v(\tau_2)v^\star(\tau_e)v'(\tau_2)v^\star(\tau_1)] = \frac{4I\mathbf{e}^2N_e}{189k^3H^2\tau_e^2}, \tag{4.26}$$

where $N_e = -\ln(-k\tau_e)$ is the number of e-folds at the end of inflation and $\Delta'^{(1)}\langle\delta\sigma_c{}^2\rangle_5 = \Delta'^{(2)}\langle\delta\sigma_c{}^2\rangle_{13} = \Delta'^{(2)}\langle\delta\sigma_c{}^2\rangle_{23} = 0$. Note that with $N_e \sim 50$–60 we have neglected the subleading corrections containing IN_e compared to IN_e^2 in the last nested integrals above.

Now, combining the above results, and neglecting the subleading IN_e contributions against the IN_e^2 contributions, the total curvature perturbation power spectrum is obtained as

$$\mathcal{P}_\mathcal{R} = \mathcal{P}_\mathcal{R}^{(0)}(1 + 16IN_e^2F(\beta)), \tag{4.27}$$

with

$$\beta \equiv \frac{\mathbf{e}^2M_P^2}{126H^2N_e}, \qquad F(\beta) \equiv 1 - \beta + \frac{9}{22}\beta^2. \tag{4.28}$$

The parameter β measures the effects of the gauge coupling $\mathbf{e}^2$. With $M_P/H \sim 10^5$, we have $\beta \gtrsim 1$ for $\mathbf{e} \gtrsim 10^{-3}$. For a large value of $\mathbf{e}$, the function $F(\beta)$ grows like β^2.

Interestingly, the correction from the gauge field dynamics in curvature perturbations in Eq. (4.27) has the same form as in Ref. [10], studied in the context of the charged anisotropic inflation model. However, in the model of Ref. [10] with a single copy of the $U(1)$ gauge field, the gauge field corrections in the power spectrum induce statistical anisotropy $\Delta\mathcal{P}_\mathcal{R}/\mathcal{P}_\mathcal{R}{}^{(0)} = g_*\cos^2(\hat{\mathbf{k}}\cdot\hat{\mathbf{n}})$ with the quadrupolar amplitude $g_* = -24IF(\beta)N_e^2$ in which $\hat{\mathbf{n}}$ is the preferred direction (direction of anisotropy) in the sky. Note that when $\mathbf{e} = \beta = 0$ then $F(\beta) = 1$ and one recovers the well-known results [9,23–25] $g_* = -24IN_e^2$. To be consistent with the observational constraints $|g_*| \lesssim 10^{-2}$ [26,27], one then requires $I \lesssim 10^{-7}$. However, in our setup with internal $O(3)$ symmetry, we have three orthogonal gauge fields with equal amplitude, so there is no statistical anisotropy. As a result, we have a less stringent constraint on the value of I.

Having calculated the corrections in the curvature perturbation power spectrum, we can also calculate the corrections in the spectral index Δn_s, given by

$$\Delta n_s = \Delta\frac{d\ln\mathcal{P}_\mathcal{R}}{d\ln k}\bigg|_* = \left(32IN_eF(\beta) + 16I\beta\left(-1 + \frac{9}{11}\beta\right)\right)\frac{dN_e}{d\ln k} = \left(32IN_eF(\beta) - 16IN_e\beta\left(-1 + \frac{9}{11}\beta\right)\right), \tag{4.29}$$

in which the subscript $*$ represents the time of horizon crossing for the mode of interest k.

To have a nearly scale-invariant power spectrum, we require Δn_s to be at the order of the slow-roll parameters. As a result, we conclude that $I \lesssim \epsilon/10N_e$. This justifies our assumption in taking $I \ll 1$. However, the above result also indicates that I is parametrically at the order $I \sim 10^{-2}\epsilon \sim 10^{-4}$, assuming that ϵ is at the order of a few percent. This is less restrictive compared to the constraint imposed on the magnitude of I in models of anisotropic inflation discussed above.

The smallness of I may raise concerns about the existence of the background attractor regime [28,29]. One may require some fine-tunings on the combination $q_0^2\mathcal{C}$ in order to neglect the last term in the brackets in (2.21). To be specific, for the chaotic inflation with $V = \frac{1}{2}m^2\phi^2$, the condition $\frac{6c^2q_0^2\mathcal{C}}{(c-1)}(\frac{V}{V_{,\phi}^2})e^{4N(1-c)} \ll 1$ requires

$$q_0^2\mathcal{C} < I\epsilon \sim \epsilon^2/N \sim \epsilon^3. \tag{4.30}$$

This indicates the level of fine-tuning required in order for the gauge field dynamics to actually reach the attractor phase.

C. $\mathcal{P}_\mathcal{S}$ and $\mathcal{P}_{\mathcal{RS}}$

In this subsection, we calculate the power spectrum of entropy mode $\mathcal{P}_\mathcal{S}$ and its cross-correlation with the curvature perturbation $\mathcal{P}_{\mathcal{RS}}$.

For the cross-correlation, we find

$$\begin{aligned}\Delta'^{(1)}\langle\delta\sigma_c\delta s_c\rangle &= i\int_{\tau_0}^{\tau_e} d\tau_1[H_1^s(\tau_1) + H_2^s(\tau_1) + H_3^s(\tau_1), \delta\sigma_c\delta s_c(\tau_e)] \\ &= 16\sqrt{I}\mathrm{Re}\left[i\int_{\tau_0}^{\tau_e} d\tau_1\left(\frac{1}{\tau_1}\right)^2 v(\tau_1)^2v_2^\star(\tau_e)^2\right] - 8\sqrt{I}\mathrm{Re}\left[i\int_{\tau_0}^{\eta_e} d\tau_1\left(\frac{1}{\tau_1}\right)v'(\tau_1)v^\star(\tau_e)v(\tau_1)v^\star(\tau_e)\right] \\ &\quad - 8\sqrt{I}\frac{\mathbf{e}^2\tau_e^4}{9H^2}\mathrm{Re}\left[i\int_{\tau_0}^{\tau_e} d\tau_1\left(\frac{1}{\tau_1}\right)^6 v(\tau_1)^2v^\star(\tau_e)^2\right] \\ &= -\frac{2\sqrt{I}N_e}{k^3\tau_e^2} + \frac{\mathbf{e}^2\sqrt{I}}{63H^2k^3\tau_e^2}.\end{aligned} \tag{4.31}$$

We see that, unlike in previous integrals, the cross-correlation is proportional to $\sqrt{I}$. The reason is that we did not have to calculate a nested integral. Correspondingly, the cross-correlation of the entropy and the curvature perturbation is given by

$$\mathcal{P}_{\mathcal{RS}} = -4\sqrt{I}\mathcal{P}_{\mathcal{R}}^{(0)} N_e (1-\beta). \tag{4.32}$$

To calculate $\mathcal{P}_{\mathcal{S}}$, we can perform similar in-in integrals as in the case of curvature perturbations in the previous subsection. However, there is a less cumbersome way to obtain $\mathcal{P}_{\mathcal{S}}$, as we describe below. Let us first look at the interaction Hamiltonians H_1^s and H_2^s, which are given by Eq. (4.17). We can perform an integration by parts and find $H_1^s + H_2^s = -\frac{4\sqrt{I}}{\tau^2}\delta\sigma_c\delta s_c - \frac{4\sqrt{I}}{\tau}\delta\sigma_c'\delta s_c$. Now, we make the identification $\delta\sigma_c \leftrightarrow \delta s_c$ with

$$H_1^s \leftrightarrow \frac{1}{2}H_1^s, \qquad H_2^s \leftrightarrow -H_2^s, \tag{4.33}$$

from which we can easily find

$$\Delta'^{(2)}\langle \delta s_c{}^2\rangle_{11} = \frac{1}{4}\times\Delta'^{(2)}\langle\delta\sigma_c^2\rangle_{11} = \frac{16IN_e^2}{9k^3\tau_e^2}, \tag{4.34}$$

$$\begin{aligned}
\Delta'^{(2)}\langle \delta s_c{}^2\rangle_{12} &= -\frac{1}{2}\times\Delta'^{(2)}\langle\delta\sigma_c^2\rangle_{12} = \frac{8IN_e^2}{9k^3\tau_e^2},\\
\Delta'^{(2)}\langle \delta s_c{}^2\rangle_{21} &= -\frac{1}{2}\times\Delta'^{(2)}\langle\delta\sigma_c^2\rangle_{21} = -\frac{16IN_e^2}{9k^3\tau_e^2},\\
\Delta'^{(2)}\langle \delta s_c{}^2\rangle_{22} &= \Delta'^{(2)}\langle\delta\sigma_c^2\rangle_{22} = -\frac{8IN_e^2}{9k^3\tau_e^2}.
\end{aligned} \tag{4.35}$$

Summing up all the above corrections, we see that they neatly cancel each other, and therefore we do not have any IN_e^2 correction to the power spectrum of the entropy mode. We have already seen that H_4^s gives corrections at the order IN_e to the curvature perturbation power spectrum, which we have neglected in comparison with the IN_e^2 corrections. Here, however, we have to consider it since there is no IN_e^2 correction. The IN_e correction to the entropy mode comes from the interaction Hamiltonian H_5^s. From Eq. (4.17), we can see that we should consider the identification

$$H_5^s \leftrightarrow -\frac{1}{2}H_4^s, \tag{4.36}$$

which implies

$$\Delta'^{(1)}\langle \delta s_c{}^2\rangle_5 = -\frac{1}{2}\times\Delta'^{(1)}\langle\delta\sigma_c^2\rangle_4 = -\frac{4IN_e}{k^3\tau_e^2}. \tag{4.37}$$

From Eq. (4.17), it is clear that the interaction Hamiltonians H_1^s and H_3^s are symmetric in $\delta\sigma_c \leftrightarrow \delta s_c$. Therefore, we simply have

$$\begin{aligned}
\Delta'^{(2)}\langle \delta s_c{}^2\rangle_{31} &= \Delta'^{(2)}\langle\delta\sigma_c^2\rangle_{31} = -\frac{16I\mathbf{e}^2N_e}{189k^3H^2\tau_e^2},\\
\Delta'^{(2)}\langle \delta s_c{}^2\rangle_{33} &= \Delta'^{(2)}\langle\delta\sigma_c^2\rangle_{33} = \frac{I\mathbf{e}^4}{4851H^4k^3\tau_e^2}.
\end{aligned} \tag{4.38}$$

The last correction to the power spectrum of the entropy mode comes from the interaction Hamiltonians H_2^s and H_3^s. Performing an integration by parts, it is easy to see that the appropriate identification will be

$$H_2^s \leftrightarrow -H_2^s - \frac{1}{2}H_1^s, \qquad H_3^s \leftrightarrow H_3^s, \tag{4.39}$$

which gives

$$\begin{aligned}
\Delta'^{(2)}\langle \delta s_c{}^2\rangle_{32} &= -\Delta'^{(2)}\langle\delta\sigma_c^2\rangle_{32} - \frac{1}{2}\times\Delta'^{(2)}\langle\delta\sigma_c^2\rangle_{31}\\
&= \frac{4I\mathbf{e}^2N_e}{189k^3H^2\tau_e^2}.
\end{aligned} \tag{4.40}$$

In the same manner, we can easily see $\Delta'^{(2)}\langle \delta s_c{}^2\rangle_{13} = \Delta'^{(2)}\langle \delta s_c{}^2\rangle_{23} = 0$.

All of these results can also be confirmed from the direct in-in calculations. Summing up all the above corrections, we find

$$\mathcal{P}_{\mathcal{S}} = \mathcal{P}_{\mathcal{R}}^{(0)}[1 - 8IN_e + 16IN_e^2(F(\beta)-1)], \tag{4.41}$$

where β and $F(\beta)$ are defined in Eq. (4.28).

V. TENSOR PERTURBATIONS

There are two different types of tensor perturbations in our model. One is the usual tensor perturbations of the metric h_{ij}. The other one is t_{ij} coming from the matter sector of the $O(3)$ gauge fields in Eq. (3.3). We therefore have four tensor modes in our model.

Using the transverse and traceless conditions, the quadratic action in Fourier space is obtained as

$$\begin{aligned}
S^{(2)} = \frac{1}{2}\int d^3k d\tau \Big\{ &\bar{h}_{ij}'^2 - \left(k^2 - \frac{2+2I\epsilon}{\tau^2}\right)\bar{h}_{ij}^2 + \bar{t}_{ij}'^2\\
&- \left(k^2 - \frac{2-5I\epsilon}{\tau^2}\right)\bar{t}_{ij}^2 + \frac{4\sqrt{I\epsilon}}{\tau^2}(\tau\bar{h}_{ij}\bar{t}_{ij}' - 2\bar{h}_{ij}\bar{t}_{ij})\\
&+ \frac{8\sqrt{I\epsilon}\mathbf{e}^2}{9\tau^2H^2\epsilon}\left(\frac{\tau_e}{\tau}\right)^4\bar{t}_{ij}\bar{h}_{ij}\Big\},
\end{aligned} \tag{5.1}$$

where we have defined the canonically normalized fields as follows:

$$\bar{h}_{ij} \equiv \frac{a}{2}h_{ij}, \qquad \bar{t}_{ij} \equiv f t_{ij}. \tag{5.2}$$

It is convenient to write the tensor modes in terms of their polarizations. To do this, we note that the traceless and transverse conditions imply $\bar{h}_{ii} = k_i\bar{h}_{ij} = \bar{t}_{ii} = k_i\bar{t}_{ij} = 0$. Consequently, we can express them in terms of the polarization tensor as $\bar{h}_{ij}=\sum_{+,\times}\bar{h}^\lambda e^\lambda_{ij}$ and $\bar{t}_{ij}=\sum_{+,\times}\bar{t}^\lambda e^\lambda_{ij}$, where we have $e^\lambda_{ii} = k^i e^\lambda_{ij} = 0$ and $e^\lambda_{ij}e^{\lambda'}_{ij} = 2\delta_{\lambda\lambda'}$.

The interaction terms in (5.1) are proportional to $\sqrt{I\epsilon}$. In the previous section, we have seen that $I \lesssim 10^{-2}\epsilon$ and therefore $\sqrt{I\epsilon} \lesssim \epsilon/10$, which is small. On the other hand, the interactions in (5.1) have the same form as the interactions in (4.12). Therefore, from our results for the scalar modes, the leading corrections in tensor correlations are at the order $I\epsilon N_e^2$.

The wave functions for the free tensor modes $N_{i\mathbf{k}} = \{\bar{h}^\lambda(k), \bar{t}^\lambda(k)\}$ are given by

$$N_{i\mathbf{k}} = n(k)a_{i\mathbf{k}} + n(k)^* a^\dagger_{i-\mathbf{k}};$$
$$n(k) = i\frac{e^{-ik\tau}}{\sqrt{2k^3}\tau}(1 + ik\tau). \tag{5.3}$$

The interaction Hamiltonians associated with the quadratic action (5.1) in the interaction picture are given by

$$H_1^t = \frac{8\sqrt{I\epsilon}}{\tau^2}\sum_{+,\times}\bar{h}^\lambda\bar{t}^\lambda, \qquad H_2^t = -\frac{4\sqrt{I\epsilon}}{\tau}\sum_{+,\times}\bar{h}^\lambda\bar{t}'^\lambda,$$
$$H_3^t = -\frac{8\sqrt{I\epsilon}\mathbf{e}^2}{9\tau^2H^2\epsilon}\left(\frac{\tau_e}{\tau}\right)^4\sum_{+,\times}\bar{h}^\lambda\bar{t}^\lambda,$$
$$H_4^t = \frac{2I\epsilon}{\tau^2}\sum_{+,\times}\bar{h}^\lambda\bar{h}^\lambda, \qquad H_5^t = \frac{5I\epsilon}{\tau^2}\sum_{+,\times}\bar{t}^\lambda\bar{t}^\lambda. \tag{5.4}$$

Similar to the analysis of entropy power spectrum in Sec. IV C, we do not need to explicitly perform the cumbersome in-in calculations since we can simply model the above interaction Hamiltonians to those we had in the case of scalar perturbations given in Eq. (4.17) via the following identifications:

$$H_1^t \leftrightarrow -\sqrt{\epsilon}H_1^s, \qquad H_2^t \leftrightarrow -\sqrt{\epsilon}H_2^s,$$
$$H_3^t \leftrightarrow -\frac{2}{\sqrt{\epsilon}}H_3^s, \qquad H_5^t \leftrightarrow -\frac{5}{6}\epsilon H_5^s. \tag{5.5}$$

Using the above identifications and the results obtained from Eqs. (4.20)–(4.26), we can easily obtain the nonzero corrections to the power spectrum of the tensor modes as

$$\Delta'^{(2)}\langle(\bar{h}^\lambda)^2\rangle_{11} = \epsilon\Delta'^{(2)}\langle\delta\sigma_c^2\rangle_{11} = \frac{64I\epsilon N_e^2}{9\tau_e^2k^3},$$
$$\Delta'^{(2)}\langle(\bar{h}^\lambda)^2\rangle_{12} = \epsilon\Delta'^{(2)}\langle\delta\sigma_c^2\rangle_{12} = -\frac{16I\epsilon N_e^2}{9\tau_e^2k^3},$$
$$\Delta'^{(2)}\langle(\bar{h}^\lambda)^2\rangle_{21} = \epsilon\Delta'^{(2)}\langle\delta\sigma_c^2\rangle_{21} = \frac{32I\epsilon N_e^2}{9\tau_e^2k^3},$$
$$\Delta'^{(2)}\langle(\bar{h}^\lambda)^2\rangle_{22} = \epsilon\Delta'^{(2)}\langle\delta\sigma_c^2\rangle_{22} = -\frac{8I\epsilon N_e^2}{9\tau_e^2k^3},$$
$$\Delta'^{(2)}\langle(\bar{h}^\lambda)^2\rangle_{33} = \frac{4}{\epsilon}\Delta'^{(2)}\langle\delta\sigma_c^2\rangle_{33} = \frac{4\mathbf{e}^4(I/\epsilon)}{4851H^4k^3\tau_e^2},$$
$$\Delta'^{(2)}\langle(\bar{h}^\lambda)^2\rangle_{31} = 2\Delta'^{(2)}\langle\delta\sigma_c^2\rangle_{31} = -\frac{32\mathbf{e}^2IN_e}{189H^2k^3\tau_e^2},$$
$$\Delta'^{(2)}\langle(\bar{h}^\lambda)^2\rangle_{32} = 2\Delta'^{(2)}\langle\delta\sigma_c^2\rangle_{32} = \frac{8\mathbf{e}^2IN_e}{189H^2k^3\tau_e^2},$$

with $\Delta'^{(2)}\langle(\bar{h}^\lambda)^2\rangle_{13} = \Delta'^{(2)}\langle(\bar{h}^\lambda)^2\rangle_{23} = 0$.

Summing up all the above corrections, we find

$$\Delta'^{(2)}\langle\bar{h}^{\lambda\dagger}\bar{h}^{\lambda'}\rangle = \frac{8I}{k^3\tau_e^2}\left(\epsilon N_e^2 - \frac{\mathbf{e}^2N_e}{63H^2} + \frac{(\mathbf{e}^4/\epsilon)}{9702H^4}\right)\delta_{\lambda\lambda'}. \tag{5.6}$$

Note the important effect that the charge coupling interaction induces $1/\epsilon$ enhancement to the tensor power spectrum, which is the specific feature of this model. This is similar to the results obtained in the model of charged anisotropic inflation [10] in which the statistical anisotropy induced in the tensor power spectrum is more pronounced compared to statistical anisotropy induced in the scalar power spectrum.

To calculate the power spectrum of the gauge field tensor mode, we note that it appears exactly the same as in the entropy mode. Therefore, upon making the appropriate identifications of the interaction Hamiltonians, we find the results

$$\Delta'^{(2)}\langle(\bar{t}^\lambda)^2\rangle_{11} = \frac{64I\epsilon N_e^2}{9k^3\tau_e^2}, \quad \Delta'^{(2)}\langle(\bar{t}^\lambda)^2\rangle_{12} = -\frac{16I\epsilon N_e^2}{9k^3\tau_e^2},$$
$$\Delta'^{(2)}\langle(\bar{t}^\lambda)^2\rangle_{21} = -\frac{64I\epsilon N_e^2}{9k^3\tau_e^2}, \quad \Delta'^{(2)}\langle(\bar{t}^\lambda)^2\rangle_{22} = \frac{16I\epsilon N_e^2}{9k^3\tau_e^2},$$
$$\Delta'^{(2)}\langle(\bar{t}^\lambda)^2\rangle_{33} = \frac{4\mathbf{e}^4(I/\epsilon)}{4851H^4k^3\tau_e^2}, \quad \Delta'^{(2)}\langle(\bar{t}^\lambda)^2\rangle_{31} = -\frac{32IN_e\mathbf{e}^2}{189k^3H^2\tau_e^2},$$
$$\Delta'^{(2)}\langle(\bar{t}^\lambda)^2\rangle_{32} = \frac{8IN_e\mathbf{e}^2}{189k^3H^2\tau_e^2}, \quad \Delta'^{(1)}\langle(\bar{t}^\lambda)^2\rangle_5 = \frac{10I\epsilon N_e}{3k^3\tau_e^2},$$

with $\Delta'^{(2)}\langle(\bar{t}^\lambda)^2\rangle_{13} = \Delta'^{(2)}\langle(\bar{t}^\lambda)^2\rangle_{23} = 0$.

Summing the above corrections, we see that they cancel one another and, similar to the case of $\mathcal{P}_\mathcal{S}$, there is no $I\epsilon N_e^2$ correction to the two-point function of $\bar{t}^\lambda$, and we have to keep the $I\epsilon N_e$ corrections.

What remains is the cross-correlation between $\bar{h}^\lambda$ and $\bar{t}^\lambda$. Keeping the above identifications in mind, looking at Eq. (4.31), we see that the first term in the last line comes from the interaction Hamiltonians H_1^s and H_2^s in Eq. (4.17), while the second term comes from H_3^s in Eq. (4.17). Therefore, from the identifications (5.5), we easily find

$$\Delta'^{(1)}\langle\bar{h}^\lambda\bar{t}^\lambda\rangle = \frac{2\sqrt{I\epsilon}}{k^3\tau_e^2}\left[N_e - \frac{(\mathbf{e}^2/\epsilon)}{63H^2}\right], \tag{5.7}$$

which can also be justified from the direct in-in calculation.

Having obtained the two-point function of $\bar{h}^\lambda$ and $\bar{t}^\lambda$ and their cross-correlation, we can obtain the power spectra. The power spectra of the gravitational tensor modes as usual are defined via

$$\sum_{+,\times}\langle h^{\lambda\dagger}(\tau,\mathbf{k})h^{\lambda'}(\tau,\mathbf{k}')\rangle = 2\langle h^{\lambda\dagger}h^\lambda\rangle \equiv \frac{2\pi^2}{k^3}\mathcal{P}_h(2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{k}'). \tag{5.8}$$

From Eq. (5.2), we have $\langle h^{\lambda\dagger}h^\lambda\rangle = \frac{4\langle\bar{h}^{\lambda\dagger}\bar{h}^\lambda\rangle}{a^2}$, and after substituting from Eq. (5.6), we obtain the expression for the power spectrum of the gravitational tensor modes,

$$\mathcal{P}_h = \mathcal{P}_h^{(0)}(1 + 16I\epsilon N_e^2 F(\hat{\beta})), \tag{5.9}$$

in which

$$\mathcal{P}_h^{(0)} \equiv \frac{2H^2}{\pi^2} \tag{5.10}$$

is the standard tensor power spectrum for gravitons. The function $F(\hat{\beta})$ is defined as in Eq. (4.28) with the new dimensionless parameter $\hat{\beta}$ given in terms of β as

$$\hat{\beta} \equiv \frac{2\beta}{\epsilon}. \tag{5.11}$$

Interestingly, the corrections induced from the gauge field dynamics in the gravitational tensor power spectrum in Eq. (5.9) have the same form as statistical anisotropy induced in the tensor power spectrum in the model of charged anisotropic inflation [10]. As discussed before, with $\mathbf{e} \gtrsim 10^{-3}$, we have $\beta \gtrsim 1$, and therefore one can easily have $\hat{\beta} \gtrsim 100$. For our perturbative approach to be valid, we require that $16I\epsilon N_e^2 F(\hat{\beta}) \ll 1$. Using the form of the function $F(\hat{\beta})$ and the definition of $\hat{\beta}$, this is translated into

$$\mathbf{e} \lesssim \frac{10H}{M_P}\left(\frac{\epsilon}{16I}\right)^{1/4} \sim 10^{-3}, \tag{5.12}$$

in which the approximations $I \lesssim 10^{-4}, \epsilon \sim 10^{-2}$, and $H/M_P \sim 10^{-5}$ have been used to obtain the final result. In conclusion, for $\mathbf{e} > 10^{-3}$ or so, the corrections induced from the gauge field into the gravitational tensor power spectrum become large, and our perturbative approximations break down. This conclusion is in line with the result obtained in Ref. [10].

Similarly, for $\mathcal{P}_t$ and $\mathcal{P}_{ht}$, we find

$$\mathcal{P}_t = \mathcal{P}_h^{(0)}\left[1 + \frac{20}{3}I\epsilon N_e + 16I\epsilon N_e^2(1 - F(\hat{\beta}))\right], \tag{5.13}$$

$$\mathcal{P}_{ht} = 4\sqrt{I\epsilon}\mathcal{P}_h^{(0)}N_e(1-\hat{\beta}). \tag{5.14}$$

We see interesting similarities between $\mathcal{P}_t$ and $\mathcal{P}_S$ in Eq. (4.41) and between $\mathcal{P}_{ht}$ and $\mathcal{P}_{S\mathcal{R}}$ in Eq. (4.32).

Having calculated the curvature perturbation and the gravitational tensor power spectra in Eqs. (4.27) and (5.9), the ratio of the tensor to scalar power spectra, denoted by the parameter r, is given by

$$r \simeq 16\epsilon(1 - 16IN_e^2 F(\beta) + 16I\epsilon N_e^2 F(\hat{\beta})). \tag{5.15}$$

For large enough $\hat{\beta}$, the last term above dominates over the second term, and we will have a positive contribution for r, modifying the standard result $r = 16\epsilon$ in single-field slow-roll models of inflation. For example, if we take $\mathbf{e}$ such that $\hat{\beta} \sim 10$, then the last term above is at the order of unity in the chaotic model. A large value of r is disfavored in light of the recent constraint $r \lesssim 0.07$ [30].

VI. CONCLUSIONS

In this work, we considered a model of inflation containing three complex scalar fields charged under $U(1)_a$ gauge symmetry with gauge coupling $\mathbf{e}$. The corresponding gauge fields $A_\mu^{(a)}$ enjoy an internal $O(3)$ symmetry associated with the rotation in field space. In a sense, this model is a hybrid of models of anisotropic inflation and models based on non-Abelian gauge fields [31–37]. Similar to anisotropic inflation models, with appropriate coupling of the gauge fields to the inflaton field, the system reaches an attractor phase in which the energy density of the gauge fields reaches a constant fraction of the total energy density and the gauge field perturbations become scale invariant.

We have decomposed the scalar perturbations into the adiabatic and entropy modes. The corrections from the gauge fields to the curvature perturbations are given by Eq. (4.27), in which the effects of gauge coupling are captured by the function $F(\beta)$. As expected, it has the same structure as in models of anisotropic inflation, i.e., being proportional to IN_e^2. However, because of the background isotropy, no quadrupolar statistical anisotropy is generated. We have also calculated the corrections in the spectral index. Requiring a nearly scale-invariant curvature perturbation power spectrum requires $I \lesssim \epsilon/10N_e \sim 10^{-4}$.

This should be compared to models of anisotropic inflation in which the amplitude of quadrupolar anisotropy g_* is given by $g_* = 24IN_e^2$ and demanding $|g_*| \lesssim 10^{-2}$ from CMB observations requires $I \lesssim 10^{-7}$.

We have calculated the tensor power spectra of the model. In addition to tensor perturbations coming from the metric sector, we also have new tensor perturbations from the gauge fields sector. The interactions between the matter and metric tensor perturbations induce corrections into the primordial gravitational wave spectra given by Eq. (5.9). We have shown that the effects of gauge coupling $\mathbf{e}$ are more pronounced in the tensor power spectrum, controlled by the function $F(\hat{\beta})$. For example, in the simple model of chaotic inflation with $H/M_P \sim 10^{-5}$, we require $\mathbf{e} \lesssim 10^{-3}$ in order for the corrections in the tensor power spectrum to be perturbatively under control. This originates from the interaction $\mathbf{e}^2 g^{\mu\nu} A_\mu^{(a)} A_\nu^{(a)} \phi^2$ as in the Higgs mechanism. In the large field model with $\phi > M_P$, large interactions between the tensor perturbations and gauge field perturbations are generated, which induces large corrections in the tensor power spectrum. We also calculated the power spectrum of the matter tensor perturbation and the cross-correlation between the matter and metric tensor perturbations, given, respectively, by Eqs. (5.13) and (5.14).

One shortcoming of our analysis is that in order to simplify the setup we have restricted ourselves to the subset of the model in which $\phi_{(1)} = \phi_{(2)} = \phi_{(3)} \equiv \phi/\sqrt{3}$. This requires some levels of fine-tuning. However, similar to the analysis of Ref. [16], one expects that the isotropic FRW background is an attractor solution at least in some corners of model parameters, so we may assume $\phi_{(1)} = \phi_{(2)} = \phi_{(3)} = \phi(t)/\sqrt{3}$ at the background level. However, to simplify the analysis further, we impose a stronger condition and assume that these scalar fields behave similarly at the level of perturbations, i.e., $\delta\phi_{(1)} = \delta\phi_{(2)} = \delta\phi_{(3)} = \delta\phi(t, \mathbf{x})/\sqrt{3}$. If we do not take this simplification into account, we will find three entropy modes, whereas in our simplified setup studied here, the three entropy modes are treated as identical. While we expect that the structure of the main results obtained here will remain unchanged, it is an important question to study the general case in which all three entropy modes are turned on.

There are a number of directions in which the current study can be extended. One natural question is the non-Gaussianity of the model. In particular, in models of anisotropic inflation, large anisotropic non-Gaussianities are generated. Correspondingly, we expect observable local-type non-Gaussianity to be generated in our model. In addition, there will be cross-correlation between tensor-scalar-scalar correlations that may have observable implications such as for the fossil effects [38–44]. Another open question in our model is the reheating mechanism, which is not specified. One simple mechanism, as in the standard mechanism of reheating, is that at the end of inflation the gauge fields simply transfer all their energies to conventional radiation, i.e., photons and other d.o.f. in the Standard Model. Another option is that the gauge fields do not decay. In this case, their energy density has the form of radiation, which will be quickly diluted in subsequent expansion of the Universe. Another open question in our setup is the roles of the entropy perturbations. This question is also linked to the previous question about the mechanism of reheating. Observationally, there are stringent constraints on entropy perturbations. Therefore, the model should not generate too many entropy perturbations. To study this question, we have to specify how the reheating mechanism works in this model and whether or not the gauge fields decay to photons, baryons, etc. Finally, in this work, we did not elaborate on the observational implications of the model. It is an interesting question to study the predictions of the model for the CMB temperature perturbations and polarizations. The contributions of the entropy modes and the corrections in the primordial tensor power spectrum can have interesting observational implications in light of the Planck CMB data.

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APPENDIX A: GAUGE SYMMETRIES OF THE MODEL

Here, we study the gauge symmetries of the model in some details.

We have three independent gauge fields $A_\mu^{(a)}$ with gauge symmetry $U(1)_a$, and therefore we should demand that the three generators τ_a of the algebra $u(1)_a$ be independent. In the matrix notation, we choose the following representation:

$$\tau_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{A1}$$

The above matrices are clearly independent and further satisfy

$$\tau_a \tau_b = \tau_a \delta_{ab}. \tag{A2}$$

Moreover, the generators in Eq. (A1) satisfy the Abelian algebra

$$[\tau_a, \tau_b] = 0. \tag{A3}$$

The field strength tensor associated with three copies of gauge fields is given by

$$i\mathbf{e}\mathbf{F}_{\mu\nu} = [\mathbf{D}_\mu, \mathbf{D}_\nu]. \tag{A4}$$

Substituting Eq. (2.2) into Eq. (A4) and then using Eq. (A3), we find

$$F^{(a)}_{\mu\nu} = \partial_\mu A^{(a)}_\nu - \partial_\nu A^{(a)}_\mu, \tag{A5}$$

where as usual $\mathbf{F}_{\mu\nu} = F^{(a)}_{\mu\nu}\tau_a$ and $\mathbf{A}_\mu = A^{(a)}_\mu \tau_a$.

Because of the Abelian structure (A3) of the algebra $u(1)_a$, the gauge coupling $\mathbf{e}$ did not appear in the above curvature tensor, which confirms that we deal with three independent copies of $U(1)$ gauge fields.

The model (2.3) is invariant under the $U(1)_a$ gauge symmetry

$$\Phi \to \exp(i\mathbf{\Lambda})\Phi, \qquad \mathbf{A}_\mu \to \mathbf{A}_\mu - \frac{1}{\mathbf{e}}\partial_\mu \mathbf{\Lambda}, \tag{A6}$$

where $\mathbf{\Lambda}$ is a general matrix in the field space. More specifically, the matrix Λ can be expressed in terms of the basis as $\mathbf{\Lambda} = \lambda^{(a)}\tau_a$, which, after substituting from Eq. (A1), takes the form $\mathbf{\Lambda} = \mathrm{diag}(\lambda^1, \lambda^2, \lambda^3)$. The gauge transformations (A6) then imply

$$\phi_{(a)} \to \exp(i\lambda^{(a)})\phi_{(a)}, \qquad A^{(a)}_\mu \to A^{(a)}_\mu - \frac{1}{\mathbf{e}}\partial_\mu \lambda^{(a)}. \tag{A7}$$

As expected, each copy of the gauge fields $A^{(a)}_\mu$ enjoys $U(1)$ gauge symmetry. To fix the $U(1)_a$ gauge freedoms, we work in the unitary gauge in which the phases of the complex scalar field are set to zero and all scalar fields $\phi_{(a)}$ are real.

We are interested in isotropic FRW solution, so let us check if this solution can be supported in our setup. The Maxwell kinetic term in the action (2.3) takes the component form $F^{(a)}_{\mu\nu}F^{\mu\nu}_{(a)}$, where we have used the fact that $\mathrm{Tr}(\tau_a\tau_b) = \delta_{ab}$, as can easily be deduced from Eq. (A1). We see that the Maxwell kinetic term enjoys an internal $O(3)$ symmetry; i.e., it is invariant under an $O(3)$ rotation in field space $A^{(a)}_\mu \to R^{(a)}_{(b)} A^{(b)}_\mu$, where $R^{(a)}_{(b)}$ are the components of the $O(3)$ rotation matrices. Therefore, the Maxwell term can support an isotropic FRW background solution. On the other hand, the kinetic term of the scalar sector in the unitary gauge where all $\phi_{(a)}$ are real is given by

$$\begin{aligned}(\mathbf{D}_\mu\Phi)^\dagger(\mathbf{D}^\mu\Phi) &= \partial_\mu\Phi^\dagger\partial^\mu\Phi + \mathbf{e}^2\Phi^\dagger\mathbf{A}^\dagger_\mu\mathbf{A}_\mu\Phi \\ &\quad + i\mathbf{e}(\partial^\mu\Phi^\dagger\mathbf{A}_\mu\Phi - \Phi^\dagger\mathbf{A}^\dagger_\mu\partial^\mu\Phi) \\ &= \partial_\mu\phi_{(a)}\partial^\mu\phi_{(a)} + \mathbf{e}^2\phi^2_{(a)}A^{(a)}_\mu A^\mu_{(a)},\end{aligned} \tag{A8}$$

where in the second line we have substituted from Eq. (2.1) and the summation rule on the repeated index a is understood.

The term $\phi^2_{(a)}A^{(a)}_\mu A^\mu_{(a)}$ in Eq. (A8) is not invariant under internal $O(3)$ rotation, so in general, an isotropic FRW background may not be supported by this model. As mentioned in the main text, in order to obtain an isotropic solution, we consider a subset of the model in which $\phi_{(1)} = \phi_{(2)} = \phi_{(3)} \equiv \phi/\sqrt{3}$, upon which the kinetic term (A8) takes the isotropic form [14]

$$(\mathbf{D}_\mu\Phi)^\dagger(\mathbf{D}^\mu\Phi) = \partial_\mu\phi\partial^\mu\phi + \frac{\mathbf{e}^2}{3}\phi^2 A^{(a)}_\mu A^\mu_{(a)}. \tag{A9}$$

Plugging this in the starting action (2.3) yields the reduced action Eq. (2.5).

APPENDIX B: QUADRATIC ACTION FOR SCALAR PERTURBATIONS

Here, we present the quadratic action of the scalar perturbations. As discussed in the main text, we neglect the gravitational backreactions from the nondynamical fields (α, β).

Going to the Fourier space $\delta X(\tau, x) = \int \frac{d^3k}{(2\pi)^3}\delta X_k(\tau)e^{i\mathbf{k}.\mathbf{x}}$ and plugging the perturbations defined in Eqs. (3.1) and (3.3) into the action (2.5) and performing some integration by parts, it is cumbersome but straightforward to show that the quadratic action for the scalar modes is given by

$$\begin{aligned}S^{(2)} = \int d\tau d^3k \Bigg[&\frac{1}{2}a^2\delta\phi'^2 - \left(\frac{1}{2}a^4V'' + \frac{1}{2}a^2A^2\mathbf{e}^2 + \frac{a^2k^2}{2} - \frac{3}{2}fA'^2f'' - \frac{3}{2}A'^2f'^2\right)\delta\phi^2 + \frac{3}{2}f^2\delta Q'^2 - \left(\frac{1}{2}a^2\mathbf{e}^2\phi^2 + f^2k^2\right)\delta Q^2 \\ &+ \frac{1}{2}k^4f^2M'^2 - \frac{1}{6}\mathbf{e}^2k^4a^2\phi^2M^2 + f^2k^2U'^2 - \left(\frac{1}{3}a^2\mathbf{e}^2k^2\phi^2 + f^2k^4\right)U^2 + Y^2\left(\frac{1}{6}a^2\mathbf{e}^2k^2\phi^2 + \frac{f^2k^4}{2}\right) \\ &+ Y(f^2k^2\delta Q' - k^4f^2M' + 2fk^2A'f'\delta\phi) + 6fA'f'\delta Q'\delta\phi - 2a^2A\mathbf{e}^2\phi\delta Q\delta\phi \\ &+ \frac{1}{3}\mathbf{e}^2a^2k^2(2A\phi\delta\phi M + \phi^2\delta QM) - 2k^2f'fA'\delta\phi M' - k^2f^2\delta Q'M'\Bigg],\end{aligned} \tag{B1}$$

where we have represented the amplitude of the Fourier modes $\delta X_k(\tau)$ with $\delta X(\tau)$ and a prime indicates the derivative with respect to the conformal time τ.

From the above action, we see that the mode Y is nondynamical and can be solved from its equation of motion as

$$Y = -\frac{3f(2\delta\phi A' f' + f(\delta Q' - k^2 M'))}{a^2 \mathbf{e}^2 \phi^2 + 3f^2 k^2}. \tag{B2}$$

We can substitute the above solution into the action (B1). Before doing this, we note that in the denominator of (B2) we can neglect $\mathbf{e}^2 a^2 \phi^2$ in comparison with $3f^2k^2$. To see this, let us find time τ_c when these two terms become comparable

$$-\tau_c = \left(-\frac{\mathbf{e}\phi}{Hk}\right)^{\frac{1}{3c}} (-\tau_e)^{\frac{2}{3}}. \tag{B3}$$

The ratio of the second term compared to the first term scales as $\frac{\mathbf{e}^2 a^2}{f^2} \sim \frac{e^2 H^2 \tau_e^{4c}}{\tau^6}$. Hence, during the early stage of inflation in which $|\tau| \gg |\tau_e|$, the second term is negligible compared to the first term. Then, the effect of gauge coupling $\mathbf{e}$ is subdominant at this stage, and the leading interactions comes from $f(\phi)^2 F^2$. However, as inflation proceeds, the effect of the second term becomes important, and the interaction $\mathbf{e}^2\phi^2 A^2$ dominates only near the time of the end of inflation. Therefore, neglecting $a^2\mathbf{e}^2\phi^2$ in comparison with $3f^2k^2$ in Eq. (B2) and then substituting the result into the action (B1), we find

$$\begin{aligned} S^{(2)} = \int d\tau d^3k \Bigg[&\frac{1}{2} a^2 \delta\phi'^2 - \left(\frac{1}{2} a^4 V'' + \frac{1}{2} a^2 A^2 \mathbf{e}^2 + \frac{1}{2} f'^2 A'^2 - \frac{2}{3} \frac{\mathbf{e}^2 a^2 \phi^2 f'^2 A'^2}{k^2 f^2} + \frac{a^2 k^2}{2} - \frac{3}{2} f A'^2 f''\right) \delta\phi^2 \\ &+ f^2 \left(1 + \frac{1}{6k^2 f^2} \mathbf{e}^2 a^2 \phi^2\right) \delta Q'^2 - f^2 k^2 \left(1 + \frac{1}{2f^2 k^2} a^2 \mathbf{e}^2 \phi^2\right) \delta Q^2 \\ &+ \frac{1}{6} \mathbf{e}^2 k^2 a^2 \phi^2 (M'^2 - k^2 M^2) + f^2 k^2 \mathcal{U}'^2 - f^2 k^4 \left(1 + \frac{1}{3f^2 k^2} a^2 \mathbf{e}^2 \phi^2\right) \mathcal{U}^2 \\ &+ 4 f A' f' \left(1 + \frac{1}{6k^2 f^2} \mathbf{e}^2 a^2 \phi^2\right) \delta Q' \delta\phi - 2a^2 A \mathbf{e}^2 \phi \delta Q \delta\phi \\ &+ \frac{\mathbf{e}^2}{3} a^2 \phi k^2 (2A\delta\phi M + \phi \delta Q M) - \frac{2}{3f} \mathbf{e}^2 a^2 \phi^2 f' \mathcal{A}' \delta\phi M' - \frac{1}{3} \mathbf{e}^2 a^2 \phi^2 \delta Q' M' \Bigg]. \end{aligned} \tag{B4}$$

We now consider the field redefinition $\bar{M} = k^2 M - \delta Q$ in terms of which the above action takes the following form:

$$\begin{aligned} S^{(2)} = \int d\tau d^3k \Bigg[&\frac{1}{2} a^2 \delta\phi'^2 - \left(\frac{1}{2} a^4 V'' + \frac{1}{2} a^2 A^2 \mathbf{e}^2 + \frac{1}{2} f'^2 A'^2 - \frac{2}{3} \frac{\mathbf{e}^2 a^2 \phi^2 f'^2 A'^2}{k^2 f^2} + \frac{a^2 k^2}{2} - \frac{3}{2} f A'^2 f''\right) \delta\phi^2 + f^2 \delta Q'^2 \\ &- f^2 k^2 \left(1 + \frac{1}{3f^2 k^2} a^2 \mathbf{e}^2 \phi^2\right) \delta Q^2 + \frac{1}{6k^2} \mathbf{e}^2 a^2 \phi^2 (\bar{M}'^2 - k^2 \bar{M}^2) + f^2 k^2 U'^2 - f^2 k^4 \left(1 + \frac{1}{3f^2 k^2} a^2 \mathbf{e}^2 \phi^2\right) U^2 \\ &+ 4 f A' f' \delta Q' \delta\phi - \frac{4}{3} a^2 A \mathbf{e}^2 \phi \delta Q \delta\phi + \frac{2\mathbf{e}^2}{3} a^2 \phi A \delta\phi \bar{M} - \frac{2\mathbf{e}^2}{3k^2 f} a^2 \phi^2 f' A' \delta\phi \bar{M}' \Bigg]. \end{aligned} \tag{B5}$$

The advantages of working with $\bar{M}$ are that not only does the quadratic action take a more simple form but also that this mode is heavy during most of the inflationary era and we can therefore neglect it. To see this, we compare the two scalar modes δQ and $\bar{M}$ in the above action as

$$\frac{L_{\delta Q^2}}{L_{\bar{M}^2}} \sim \frac{k^2 f^2}{\mathbf{e}^2 a^2 \phi^2} \gg 1, \tag{B6}$$

which clearly shows that the contribution from the mode $\bar{M}$ is negligible during much of the period of inflation.

Now, neglecting the subleading slow-roll corrections containing ϵ and its derivative and working to linear order in I, we obtain the action (4.1). In principle, we could calculate the quadratic action nonperturbatively in terms of the parameter I (i.e., to all orders in powers of I). However, as demonstrated in Sec. IV B, requiring nearly scale-invariant corrections from the gauge field into the curvature perturbation power spectrum requires $I \ll 1$, justifying our approximation in keeping only terms linear in I in the quadratic action (4.1).

In obtaining the action (4.1), we have used the following formula:

$$V \simeq 3H^2\left(1 - \frac{\epsilon}{6}(I+2)\right), \tag{B7}$$

$$A' = \sqrt{I\epsilon}(-\tau)^{-1}\frac{a}{f}, \qquad \mathbf{e}\phi A = \mathbf{e}\frac{\sqrt{2I}}{3}\frac{a}{f}, \tag{B8}$$

$$f = (\tau/\tau_e)^2, \tag{B9}$$

$$\phi = \sqrt{2/\epsilon}. \tag{B10}$$

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